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AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY
A Comparison of ARCH and Random Coefficient Models

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In this paper it is shown that the popular Autoregressive Conditional Heteroscedasticity (ARCH) models are closely related to more traditional random coefficient models. It is demonstrated that for every ARCH model a simple random coefficient model can be formulated which implies exactly the same conditional variance pattern for the variable of interest.

1. Introduction

In an influential paper, Engle (1982) presented a new class of stochastic processes called Autoregressive Conditional Heteroscedasticity (ARCH) processes. ARCH processes are mean zero, serially uncorrelated processes with non-constant variances. For these processes the recent past carries information about the one-period forecast variance.

The general ARCH model of the order p has the following conditional probability density function:

$$y_t \mid I_{t-1} \sim \mathrm{N}(0, h_t), \tag{1}$$

with

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2. \tag{2}$$

Here y_t is our variable of interest, I_{t-1} is the information set that is available at time $t-1$ and the α_is are fixed parameters. The expression (2) for h_t captures the nature of the conditional heteroscedasticity that is allowed for in the family of ARCH models. In section 2 of this paper, the random coefficient model is introduced. In section 3, it will be shown that the two classes of models are closely related and that for every ARCH model a simple random coefficient model can be formulated which exhibits exactly the same conditional variance pattern. Section 4 contains some concluding remarks.

2. Random coefficient models

Hildreth and Houck (1968) considered the problem of estimating parameters in random coefficient models of the following general form:

$$y_t = x_t'\beta_t, \tag{3}$$

$$\beta_t = \bar{\beta} + u_t, \qquad u_t \sim \text{NID}(0,Q). \tag{4}$$

Here x_t is a vector of explanatory variables, β_t is a vector of random coefficients with mean $\bar{\beta}$ and variance-covariance matrix Q. If the first element of x_t is defined to be unity, it is unnecessary to add a disturbance term to (3) since such a term is implicitly contained in (4), attached to the coefficient of the constant term. Random coefficient models can be used in a variety of contexts whenever parameters can be assumed to fluctuate randomly about a fixed, but unknown, mean.

3. Comparing ARCH and random coefficient models

In the ARCH model presented in eqs. (1) and (2) y_t has the following conditional first and second moments:

$$\text{E}[y_t \mid I_{t-1}] = 0, \tag{5}$$

$$\text{var}[y_t \mid I_{t-1}] = \text{E}\left[y_t^2 \mid I_{t-1}\right] = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2. \tag{6}$$

Now consider a specialized version of the random coefficient model. The following restrictions are imposed on eqs. (3) and (4):

$$x_t' = \left[1, y_{t-1}, y_{t-2}, \ldots, y_{t-p}\right], \tag{7}$$

$$\bar{\beta} = 0, \tag{8}$$

and Q is a diagonal matrix of order $p+1$ with the following diagonal elements:

$$Q_{11} = \sigma^2, \tag{9}$$

$$Q_{ii} = \sigma_{i-1}^2, \qquad i = 2, 3, \ldots, p+1. \tag{10}$$

In this specialized version of the random coefficient model y_t has the following conditional first and second moments:

$$\text{E}[y_t \mid I_{t-1}] = 0, \tag{11}$$

$$\text{var}[y_t \mid I_{t-1}] = \text{E}\left[y_t^2 \mid I_{t-1}\right] = \sigma^2 + \sum_{i=1}^{p} \sigma_i^2 y_{t-i}^2. \tag{12}$$

Note that the conditional means in (5) and (11) are both zero and that the conditional variances in

(6) and (12) have the same linear structure. If we set $\alpha_0 = \sigma^2$ and $\alpha_i = \sigma_i^2$ for $i = 1, 2, \ldots, p$, the ARCH model has exactly the same conditional variance as the random coefficient model.

4. Concluding remarks

In this paper we have shown that the popular ARCH models are closely related to traditional random coefficient models. For every ARCH model a random coefficient model can be found which implies exactly the same pattern for the conditional variance of the variable of interest.

Engle (1982) shows that the pth order ARCH model satisfies certain regularity conditions if $\alpha_0 > 0$ and $\alpha_1, \alpha_2, \ldots, \alpha_p \geq 0$. These inequalities are automatically satisfied in the context of the random coefficient model.

References

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