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FIRM AND CORPORATE BOND VALUATION: A SIMULATION DYNAMIC PROGRAMMING APPROACH*

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This paper analyzes corporate bond valuation of a straight bond, and the convertibility feature, when interest rates are stochastic and the firm value is determined by the interaction of a series of stochastic variables. The sensitivity of the corporate debt value to some key parameters is also explored. The methodology applied here is based on a hybrid of simulation and dynamic programming proposed by Raymar and Zwecher in 1997 to value financial American-type options. This methodology proves to be extremely efficient to value American-type options when the sources of uncertainty are numerous.

JEL: C15, G13, C32

Keywords: Valuation, Options, Bond, Equity

1. INTRODUCTION

This paper is concerned with the valuation of different types of corporate bonds, in an scenario characterized by many stochastic variables interacting to determine the value of the assets of a company, and where interest rates are also assumed to follow a stochastic process. The valuation methodology proposed here is a hybrid of simulation and dynamic programming and corresponds to an extension of the method proposed by Raymar and Zwecher in 1997 to value financial American-type options. The main advantage of the proposed methodology is that it proves to be extremely efficient to value American-type options when the sources of uncertainty are numerous.

The paper is organized as follows: In Section 2 a review of the related literature in options valuation methodologies and corporate bond valuation methods are presented. Section 3 contains a description of the methodology applied here, with a description of the process followed by the different stochastic varia-

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bles considered in the model, and the simulation-dynamic programming algorithm is then described. In Section 4 the methodology is employed to value corporate bonds with different characteristics. The estimation of key parameters and some sensitivity analysis is also considered. Section V concludes the paper.

2. RELATED LITERATURE REVIEW

In this section I present a review of the related literature in two major subjects: American options valuation methodologies, and corporate bond valuation methods.

2.1. Related Literature in American Options Valuation

Methods to value options developed since Black and Scholes (1973) include lattice methods such as the binomial and trinomial methods; techniques based on solving partial differential equations, integral equations, or variational inequalities; and Monte Carlo or other related methods where the valuation is made by simulating the risk neutral process followed by the underlying security and other state variables affecting the payoff of the option.[1] Up until recently, most of the applications developed to value real options had focused on the use of lattice and partial differential equation methods. Before the 1990s, simulation methods had only been used to value European options, i.e. those where the holder does not need to, or is not able to make any decisions until maturity.

The lattice and partial differential methods share two common drawbacks: their computing requirements in terms of memory and computation time grow exponentially with the dimension of the problem, and they can only be used to analytically solve problems where the sources of uncertainty are at most two or three. Simulation methods on the other hand can easily handle higher dimensional problems, but their forward approach was until recently believed to be incompatible with the backwards recursion approach that is needed to solve for the optimal exercise policies that pricing American options requires.

In the 1990s a series of procedures were developed that allow using simulation to value American-type options. Tilley (1993), Barraquand and Martineau (1995), Boyle, Broadie, and Glasserman (1997), Raymar and Zwecher (1997), Broadie and Glasserman (1997a), and Broadie and Glasserman (1997b) present several simulation and dynamic programming methodologies that work well to value American options when the source of uncertainty is only one.

Broadie and Glasserman (1997a) propose a pricing method based on generating random trees of the underlying assets and the state variables, and the posterior application of dynamic programming to value the options. This “simulated

[1]Examples of those methodologies are found in Cox, Ross, and Rubinstein (1979), Cox and Rubinstein (1985), Black and Scholes (1973), Boyle (1988), Hull (1997), and Neftcy (1996).

tree" method works well in the multidimensional case, but because each branch emanating from a single node will generate an independent sub-tree, the algorithm proposed by the authors leads to computation time requirements that grow exponentially with the number of exercise opportunities. Raymar and Zwecher (1997) extend the methodology developed by Barraquand and Martineau (1995) considering partitions based on two factors instead of one. They show that this extension substantially reduces the problem originally faced by Barraquand and Martineau (1995) because the two factors together become an almost sufficient statistic for the exercise decisions.

Finally, Longstaff and Schwartz (2001) made a very interesting contribution to the development of the simulation-based methodologies to value American type options by proposing a methodology that combines simulation and ordinary least square (OLS) techniques. This methodology has the additional advantage of providing a complete specification of the optimal exercise strategy of the option.

Two papers that implement versions of the simulation and dynamic programming methodology to value firms are Cortazar and Schwartz (1998) and Castillo (2000). Cortazar and Schwartz (1998) applies the methodology developed by Barraquand and Martineau to price an undeveloped oil field, considering the option to postpone the initial investment as the only flexibility available to the investor.

Castillo (2000) adapts the methodology proposed by Raymar and Zwecher (1997) to the problem of pricing a mine that offers the options to close, reopen and abandon not at every point in time, but with some discrete frequency. The methodology is shown to work well when the sources of uncertainty in the value of the mine are many.

2.2. Related Literature in Bond Valuation

Many of the models that have been developed to value bonds assume that interest rates are deterministic, and consider that the only stochastic variable to be considered is the value of the assets of the firm issuing those bonds. Merton (1974) values both a risky zero-coupon bond and a callable coupon bond; Brennan and Schwartz (1977a) propose how to value callable and convertible debt; Black and Cox (1976) and Geske (1977) value coupon paying bonds when sales of certain assets are restricted, solving for the optimal default policy from the equity holders point of view.

Fischer, Heinkel, and Zechner (1989a,b), Leland (1994), Leland and Toft (1996), Leland (1998), and Goldstein, Ju, and Leland (2000) find the optimal default policy and the optimal call policy of debt while solving for the optimal capital structure of a firm. Papers by Anderson and Sundaresan (1996), Huang (1997), Acharya, Huang, Subrahmanyam, and Sundaram (1999), and Fan and Sundaresan (2000) introduce costs to the liquidation decision and consider bankruptcy as a bargaining game.

A second group of models are the ones allowing for stochastic interest rates. Brennan and Schwartz (1977b) and Courtadon (1982) look at callable debt that is assumed to be free of default risk. Amin and Jarrow (1992), Jorgensen (1997), Ho, Stapleton, and Subrahmanyam (1997), and Peterson, Stapleton and Subrahmanyam (1998) also look at valuation of non-defaultable bonds issued by firms, when interest rates are recognized as stochastic.

Finally, there is a series of papers where interest rates are assumed to be stochastic, and bankruptcy is allowed. Brennan and Schwartz (1980), Nielsen, Saa-Requejo, and Santa-Clara (1993), Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995), Briys and de Varenne (1997), and Collin - Dufresne and Goldstein (2001) include exogenous bankruptcy rules in the form of critical asset values or critical payout levels.

Ramaswamy and Sundaresan (1986), Jarrow, Lando, and Turnbull (1997), Madan and Unal (1993), Jarrow and Turnbull (1995), Duffie and Huang (1996), Duffie and Singleton (1999) and Das and Sundaram (2000) also assume stochastic interest rates, and model bankruptcy through a stochastic credit spread or hazard rate. Acharya and Carpenter (2002) consider the valuation of corporate bonds under stochastic interest rates and assets value, assuming endogenously determined bankruptcy, through a two-factors binomial model.

3. DESCRIPTION OF THE METHODOLOGY

In this section the stochastic processes followed by the value of the assets of the firm and by the interest rate are presented. The simulation and dynamic programming algorithm applied to value corporate debt with different features is also described.

3.1. The Stochastic Processes

The value of the equity and debt of a firm will be modeled here as a function of both a series of stochastic variables determining the value of the company's assets, and a stochastic interest rate. The value of the assets of a firm and its evolution over time, V_t is assumed to be determined by the interaction of a series of stochastic variables. This could be modeled by considering stochastic variables affecting the positive and negative cash flows the assets will generate in the future (expected inflows and expected outflows) and discounting those cash flows to obtain their present value at time t, or as will be done here, by assuming that the total value of the assets of the firm results from adding the value of N different assets V_t^n, with:

(1) $$V_t = \sum_{n=1}^{N} V_t^n$$

And assuming that the value of each of these N assets follows a stochastic process as the one described in the following equation:

(2) $$\frac{dV_t^n}{V_t} = \left(m_t^n - d_t^n\right)dt + s_t^n dz_t^n$$

where m^n corresponds to the total expected return of the n asset, d^n is the expected dividend the n asset would pay, s^n is the standard deviation of the n asset value, and dz^n is the increment to a standard Gauss-Wiener process.[2]

The interest rate is assumed to follow a mean reverting process such as the one described by Cox, Ingersoll, and Ross (1985), and shown here:

(3) $$dr_t = k\left(a - r\right)dt + s_{N+1}\sqrt{r_t}\,dz_t^{N+1}$$

where a represents the long run value towards the interest rate is supposed to revert and k represents the velocity of adjustment to that long run value, s^{N+1} is the standard deviation of the interest rate value and dz^{N+1} is the increment to a standard Gauss-Wiener process. The possibility of correlation between each pair of the *N+1 dz* increments is also considered. The processes to be used in the simulations are not exactly the previous ones, but the risk-adjusted and discrete versions of the continuous time processes presented here. The processes are risk-adjusted so we can discount cash flows at the risk free rate.[3]

3.2. The Simulation and Dynamic Programming Methodology

The methodology can be applied following two steps. The first step consists of doing a preliminary set of Z_1 simulations based on the risk neutral discrete version of the stochastic processes followed by the value of each of the N assets V_t^n composing the total assets of a firm, and using those simulations to obtain Z_1 simulated values for the total value of the assets of the firm V_t. At the same time a set of Z_1 preliminary simulations of the interest rate r_t is generated.

The simulations are performed simultaneously, and they generate a set of T matrixes. Each of those T matrixes has a dimension of $(Z_1 * 2)$, and contains a simulated pair of values for V_t and r_t in each row. Each matrix contains the simulated values corresponding to one point in time t. Time has been partitioned in T periods and each point in time will be named $t = 1,\ldots,T$. The following diagram shows the matrixes, and their content.

[2]Actually V_t will be computed as the sum of the values of the N assets composing the firm, but after discounting the coupons the firm is supposed to pay debtholders at time t.

[3]The risk adjusted process for the value of individual assets is: $dV_t^n / V_t = \left(r_t - d_t^n\right)dt + s_t^n dz_t^n$, where the total expected return of the individual asset has been replaced by the risk free interest rate. The risk adjusted process for the interest rate is: $dr_t = k\left(\hat{a} - r\right)dt + s_{N+1}\sqrt{r_t}\,dz_t^{N+1}$, where $\hat{a}$ represents the risk adjusted long run interest rate.

$$\begin{bmatrix} V_{1,1} & ; & r_{1,1} \\ \vdots & & \vdots \\ V_{1,Z_1} & ; & r_{1,Z_1} \end{bmatrix} \quad \begin{bmatrix} V_{2,1} & ; & r_{2,1} \\ \vdots & & \vdots \\ V_{2,Z_1} & ; & r_{2,Z_1} \end{bmatrix} \quad \cdots\cdots\cdots \quad \begin{bmatrix} V_{T,1} & ; & r_{T,1} \\ \vdots & & \vdots \\ V_{T,Z_1} & ; & r_{T,Z_1} \end{bmatrix}$$

$$t=1 \qquad\qquad t=2 \qquad\qquad \cdots\cdots\cdots \qquad\qquad t=T$$

At each point in time t, the Z_1 simulated pairs of values for values for V_t and r_t are sorted by their V_t values in increasing order and used to partition the "total value of the firm at t" space into K_1 bins, that are indexed $k_1 = 1, \ldots K_1$, in a way that each bin contains the same number of pairs of simulations, and finally the average value of the last simulated V_t included in one bin and the first simulated V_t included in the next bin are recorded. This means that if Z_1 pairs of simulations are made at this step, each bin will contain Z_1/K_1 pairs of simulations, and (K_1+1) border values will be saved at each point in time t.[4]

At each point in time t, and within each of the original K_1 bins, the Z_1/K_1 pairs of values simulated are now sorted by their r_t values in decreasing order and used to partition the "interest rate at t" space into K_2 bins, that are indexed $k_2=1,\ldots K_2$, in a way that each bin contains the same number of pairs of simulations. Finally the average value of the last simulated r_t included in one bin and the first simulated r_t included in the next bin are recorded. This means that out of the Z_1 pairs of simulations performed originally each of the (K_1*K_2) bins will end up with $Z_1/(K_1*K_2)$ pairs of simulations, and that $(K_1+1)*(K_2+1)$ average values representing the borders of the bins will be saved at each point in time t.

Once the (K_1*K_2) partitions are constructed, a new set of Z_2 simulations is made with the objective of computing the risk adjusted transition probabilities and the average asset's and interest rate's values per partition. First, for each (k_1,k_2,t) bin, the number of times that a simulated path falls into that bin is recorded as $a(k_1,k_2,t)$, and the sum of the X values of V of those simulations is computed as follows:

$$c(k_1,k_2,t) = \sum_{x=1}^{X} V_x \tag{4}$$

Where the X values added are the total asset values of the paths that fall into that bin. Also, for each pair of bins (k_1,k_2) at time t and (l_1,l_2) at time $t+1$, the number of paths that fall into both bins is recorded as $b(k_1,k_2,l_1,l_2,t)$. Notice how the risk adjusted transition probabilities from bin (k_1,k_2) at time t and bin (l_1,l_2) at time $t+1$ are then obtained as

$$p(k_1,k_2,l_1,l_2,t) = b(k_1,k_2,l_1,l_2,t)\,/\,a(k_1,k_2,t) \tag{5}$$

[4]Those $K+1$ border values correspond to the K-1 average values, the maximum value of the first bin, and the minimum value of the last bin.

The only difference of computing the debt value at t=0 and computing the debt value at any other period *t=1* to *t=T-1* is that the formula that must be used to compute the risk neutral probabilities at t=0 should be:

$$p(k_1 = 1, k_2 = 1, l_1, l_2, t = 0) = a(l_1, l_2, t = 1) / Z_2 \tag{6}$$

This happens because at *t=0* all of the Z_2 simulated paths of *V* and *r* start from the same and only bin considered at that period.

Finally, the two-dimensional dynamic programming problem can be solved. The way the problem is solved will be determined by the presence or absence of special features such as convertibility or callability in the debt contracts. We would start from *T* which will be defined as the period when debt matures. At that point the firm has to verify if debt can be paid or if the company will default. Assuming the company only has straight debt;[5] that the firm has promised to pay an amount *D* at maturity and coupons of size d with some periodicity, the firm will verify if it is able to pay the debt and pay it if it can or give away the firm´s assets to the bondholders. In the case the company goes bankrupt, the bondholder will receive an a proportion of the firm´s assets. The a proportion will be represented by a number between 0 and 1 and is the way that direct bankruptcy costs are considered in the model.[6] The value of debt at T in each bin will be computed as:

$$\begin{array}{ll} B(k_1, k_2, T) = d + D & \text{if } V(k_1, k_2, T) \geq d + D \quad \text{and} \\ B(k_1, k_2, T) = a \; V(k_1, k_2, T) & \text{if } V(k_1, k_2, T) < d + D \end{array} \tag{7}$$

Where $V(k_1, k_2, T)$ corresponds to the value of the company in the corresponding (k_1, k_2, T) bin, and it is computed as:

$$V(k_1, k_2, T) = c(k_1, k_2, T) / a(k_1, k_2, T) \tag{8}$$

This means that the debt value $B(k_1, k_2, T)$ at each bin (k_1, k_2, T) will be either the sum of the last coupon plus the principal, when the company has assets to pay it, or it will become the value of the assets of the firm net of direct bankruptcy costs, when the firm has not enough assets to avoid bankruptcy.

Now we can go to *T-1* and verify at each bin if the company would pay the corresponding coupon[7] maturing at that period, and keep the company (and the

[5]Straight debt will be defined here as debt without convertibility, callability, or other options.

[6]For example, an a of 0.9 would mean that the direct bankruptcy costs facing the company represent *(1-a)*, which in this case is 10% of the total value of the assets. The bondholder would only receive 90% of the value of the assets in case the company goes bankrupt. Indirect bankruptcy costs that arise before the company goes bankrupt, and only because the probability of default in the future becomes high enough, are not considered here. Given the importance that indirect bankruptcy costs can have, we should attempt to include them in an improved version of this model.

[7]Not necessarily at *T-1* or at any other t period the company would have to pay a coupon. Coupons are paid with some frequency, usually every six months, and the length of a period could be much less than that.

compromise to pay a coupon and the principal at *T*) alive.[8] The value of debt at *T-1* or at any other *t* period can be computed for each bin (k_1, k_2, t) as:

$$B(k_1,k_2,t) = d + E_{t,k1,k2}\left[B(t+1)\right] \quad \text{if} \quad V(k_1,k_2,t) \geq d \tag{9}$$
$$B(k_1,k_2,t) = a \;\; V(k_1,k_2,t) \quad \text{if} \quad V(k_1,k_2,t) < d$$

The interpretation is that the firm will go bankrupt and give away the company's assets if the value of those assets, computed in a particular bin (k_1,k_2,t) as $V(k_1,k_2,t)$ is not big enough to pay the coupon maturing at t, but the company will not go bankrupt if it has enough assets to pay that coupon. The expected value at t of debt at $t+1$, defined here as $E_{t,k1,k2}\left[B(t+1)\right]$ is computed using the risk-neutral transition probabilities defined in equation (5), as follows:

$$E_{t,k1,k2}\left[B(t+1)\right] = \sum_{l_1=1}^{K_1}\sum_{l_2=1}^{K_2} p(k_1,k_2,l_1,l_2,t) * B(l_1,l_2,t+1) * e^{-\bar{r}(k_1,k_2,t)\,Dt} \tag{10}$$

Where $\bar{r}(k_1,k_2,t)$ corresponds to the average interest rate computed for that bin at time *t*, and represents the discount rate that should be used to compute the present value of the bond at that point in time.

4. APPLICATION OF THE METHODOLOGY TO VALUING DIFFERENT TYPES OF DEBTS

In this section we apply the methodology previously described to obtain the value of the debt of a company, assuming first that the bond has no special features and considering then a convertible bond. We also explore how sensitive the results are to the values of the main parameters of the model.

4.1. Straight Debt

The first case analyzed here corresponds to a straight bond issued by a company that has total assets with a value of \$150 today, composed by $N = 4$ different assets, each of them with an initial value of \$37.5, and a volatility of assets represented by a standard deviation of 0.15.[9] The correlation coefficient between

[8]Actually a key determinant of debt value will be the default rule applied by the firm and the debt holders. Debt holders usually will not be able to force bankruptcy in period $t < T$ (before debt maturity) unless the company proves to be unable of paying even the coupons maturing in that period.

[9]The number o assets to be used was arbitrary. We just wanted to have an example where the number of sources of uncertainty would make impossible to solve the problem using the other methodologies available.

each pair of assets is assumed to be 0.5. The bond issued by the company is assumed to promise a payment of $100 10 years from now, and coupons of $9.42 from years 1 to 10. The initial value of the interest rate is assumed to be 0.09. The standard deviation of the interest rate is 0.10 and its correlation coefficient with each of the assets is assumed to be 0 initially. Table 1 outlines the values of the parameters describing the processes followed by the assets and the interest rate. It also presents the results of the bond valuation in the initial case.[10] The value of the bond under these conditions results to be $98.69, and the likelihood of default is 15.42%.

Figures 1 to 3 present the sensitivity of the value of the bond to changes in some key parameters. Figure 1 shows the sensitivity of the value of the bond to changes in the degree of correlation among the individual assets composing the total assets of the company. As we would expect, Figure 1 shows that the higher the positive correlation between pairs of individual assets the lower the value of the bond. A higher positive correlation makes higher the probability of all the individual assets presenting a low value at the same time, which would result in problems to pay to the bond holders.

In other words, the explanation comes from the relationship between correlation of individual assets and volatility of total assets. The higher the correlation between individual assets, the higher should be the volatility of total assets. This in turn should result in a lower value to the bond.

In a related result, Figure 2 shows the relationship between bond value and volatility of total assets. The higher the volatility of total assets, the lower the value of the bond of the company. This is an expected result since more volatility in the upper side has no positive effect to the bond holder, but more volatility in the lower side would result in smaller payments for the bond holder.

Figure 3 presents the sensitivity of the value of the bond to the degree of correlation between assets and the interest rate. The higher the degree of positive correlation between these pairs of variables, the lower the value of the bond. This is an expected result because if correlation is positive and high, then those events when cash flow payments for the bond holder are high will be discounted at high interest rates.

Figure 4 shows sensitivity of the bond value to the interest rate volatility. The value of the bond increases slightly as the volatility of the interest rate increases, which is an expected result given the convex relationship between bond value and the interest rate.

[10]The valuation method utilized here, as explained in Raymar and Zwecher (1997), generates an upward bias that disappears when enough simulations are considered. The results reported here were obtained using Z_1 = 50.000 simulations. Those 50.000 simulations were assigned to K_1* K_2 = 500 bins. The "total value of the firm at *t*" space was partitioned first into K_1 = 50 bins. Then the "interest rate at space *t*" space was partitioned into K_2 = 10 bins. In the second step we used Z_2 = 500.000 simulations., to avoid the upward bias. Raymar and Zwecher (1997) proved that the methodology would work properly if 10.000 simulations or more were used.

TABLE 1
MAIN ASSUMPTIONS FOR CORPORATE BOND VALUATION MODEL

Assumptions:	
Number of assets	4.0
Initial value of each asset (in $)	37.5
Initial value total assets (in $)	150.0
Parameters process followed by assets:	
Expected dividend	0.0
Standard deviation of assets value	0.15
Parameters process followed by interest rate:	
Initial value interest rate	0.09
Long run interest rate	0.09
Velocity of adjustment	0.5
Standard deviation of interest rate	0.1
Corporate bond characteristics:	
Coupon rate	0.0942
Principal (in $)	100
Maturity (in years)	10
Correlation (asset i, asset j)	0.5
Correlation (asset i, interest rate)	0.0
Results:	
Corporate bond value	98.69
Likelihood of default	0.1542

FIGURE 1
BOND VALUE AS A FUNCTION OF CORRELATION BETWEEN PAIRS OF ASSETS

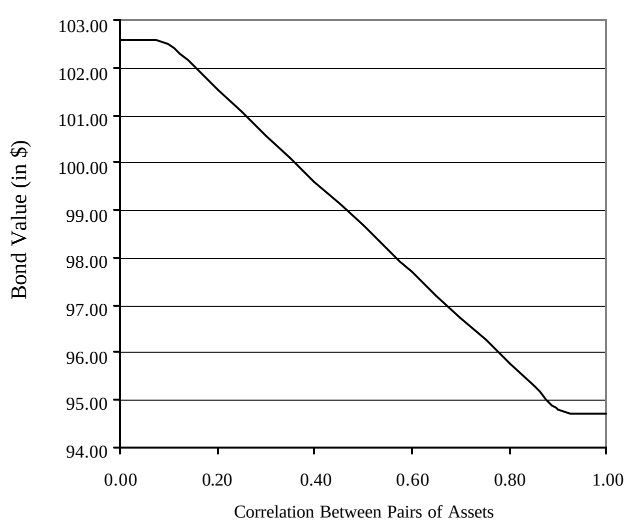


FIGURE 2
BOND VALUE AS A FUNCTION OF VOLATILITY (SD) OF TOTAL ASSETS

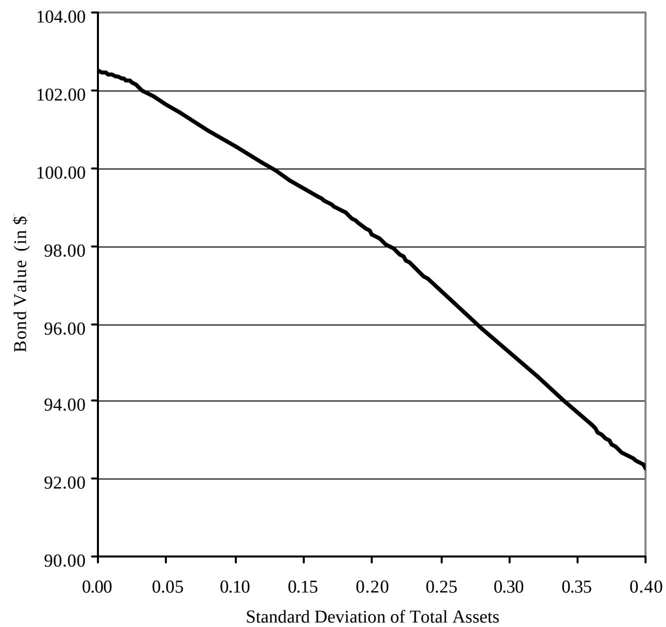


FIGURE 3
BOND VALUE AS A FUNCTION OF CORRELATION BETWEEN INTEREST RATE AND ASSETS

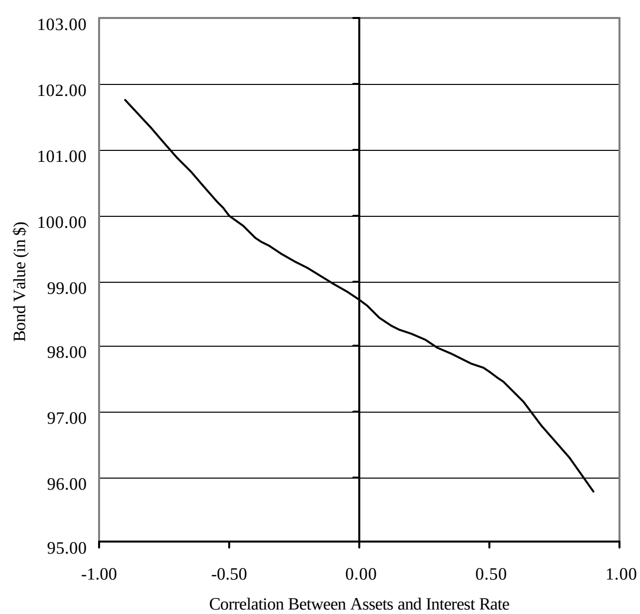
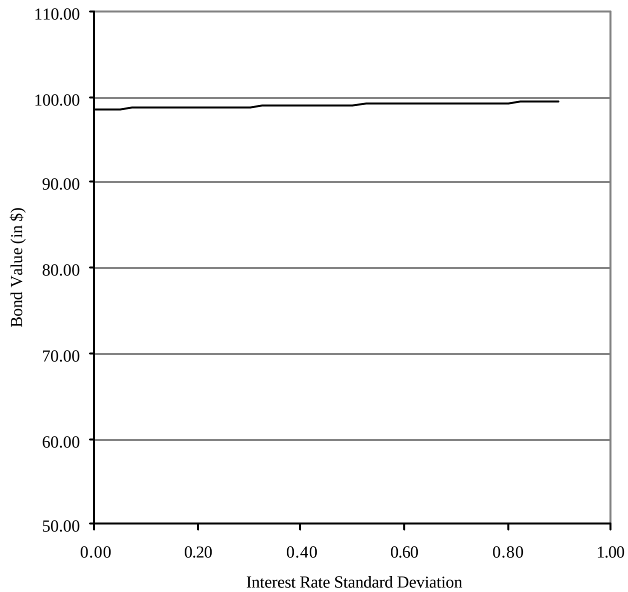


FIGURE 4
BOND VALUE AS A FUNCTION OF INTEREST RATE VOLATILITY



4.2. Convertible Debt

In this subsection I value a bond with the same characteristics as the one valued in the previous section, but with an additional convertibility feature which allows the bond holder at any time to become a shareholder of the firm, if he wants to. The valuation methodology remains mainly unchanged, but equations (6) and (8) are replaced by equations (11) and (12) to incorporate the convertibility feature.[11]

Equation (11) shows how the value of the bond will be computed at debt maturity, defined here as period T. It shows that if the value of the last coupon and the principal is smaller than the value of the β fraction of the company the debt holder would receive by converting his debt in equity, then the conversion would happen.

$$
(11)\quad \begin{array}{ll} B(k_1,k_2,T) = Max\{d + D; b\,V(k_1,k_2,T)\} & if\ \ V(k_1,k_2,T) \geq d + D\ \ and \\ B(k_1,k_2,T) = a\,V(k_1,k_2,T) & if\ \ V(k_1,k_2,T) < d + D \end{array}
$$

[11]When convertibility is considered, the issue of how often the company would be able to convert the debt into equity should affect the value of the bond. We discretized time assuming that a period corresponds to 6 months, to match it with the frequency coupons are paid. Then we tried with three months periods and a month period, and those changes did not have an impact in the results.

Equation (12) shows how the conversion could actually occur before the expiration of the debt, if at any point in time t the debt holder decides that it is better to become an equity holder of the firm.

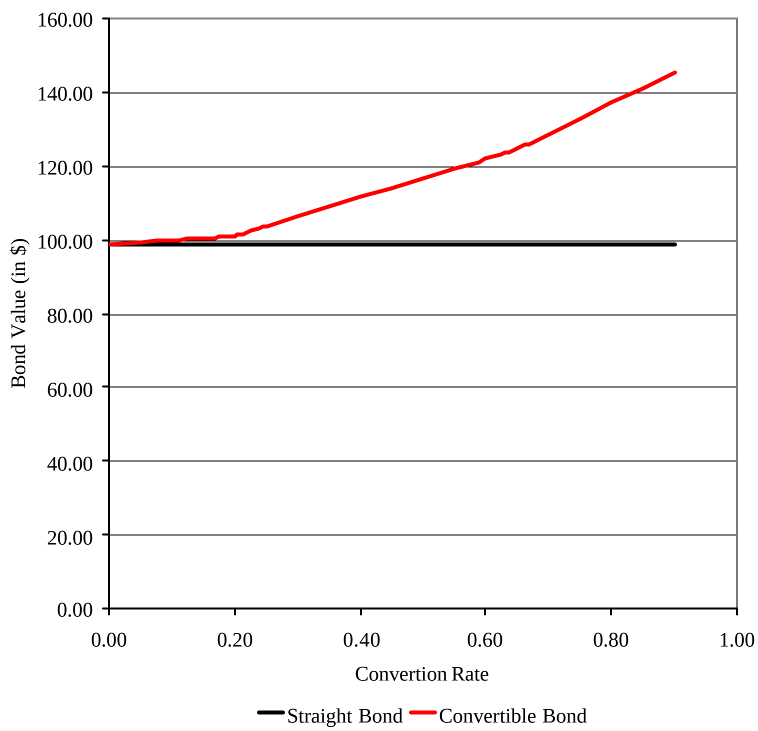
(12)

$$B(k_1,k_2,t)=Max\left\{d+E_{t,k1,k2}\left[B(t+1)\right];d+b\,V(k_1,k_2,t)\right\}\;\; if\,V(k_1,k_2,t)\geq d$$

$$B(k_1,k_2,t)=\;a\,V(k_1,k_2,t)\qquad\qquad if\,V(k_1,k_2,t)<d$$

Figure 5 shows how the value of the convertible bond compares with the value of the non-convertible bond described in the previous section, and how the value of the convertible bond increases with the fraction of the equity the debt holder would receive upon conversion. There we can appreciate how the convertibility feature could potentially add a significant amount of value to a bond.

FIGURE 5
VALUE OF CONVERTIBLE BOND AS A FUNCTION OF CONVERTION RATE



5. CONCLUSIONS

In this paper I adapt a methodology recently developed to price American-type options, which combines simulation techniques with dynamic programming. I apply this methodology to value straight corporate debt, and also corporate debt with special features such as convertibility.

One of the interesting features of the model is that it allows assuming that the value of the assets of the company is being determined by the interaction of a series of stochastic variables and it also allows assuming that the value of those corporate debts can be modeled as a function of the company's assets values and an stochastic interest rate. At the same time it allows valuing the debt of a company as an American option where the default and convertibility features allow the debt holder exercising his options before maturity. The methodology also allows exploring how sensitive are corporate debt and equity of a company to changes in some key parameters such as volatility of assets, correlation between pairs of assets, volatility of the interest rate, and correlation between the interest rate and the value of assets.

Testing the validity of the proposed valuation model would require knowing the true value of a certain security, but since there are no other models available to compute true security values, we can not follow this line of action.[12] The other possibility would be to test the model by comparing the results we can achieve to the market value of a security or of a group of securities. An extension of this paper will explore those two paths.

REFERENCES

Acharya, V. and J. Carpenter (2002), "Corporate Bond Valuation and Hedging with Stochastic Interest Rates and Endogenous Bankruptcy", *Review of Financial Studies* 15: 1355 -1383.

Acharya, V.; J.Z. Huang; M.G. Subrahmanyam and R.K. Sundaram (1999), "Costly Financing, Optimal Payout Policies, and the Valuation of Corporate Debt, Working Paper, New York University.

Amin, K., and R.A. Jarrow (1992), "Pricing options on Risky Assets in a Stochastic Interest Rate Economy", *Mathematical Finance* 2: 217-237.

Anderson, R.W. and S. Sundaresan (1996), "Design and Valuation of Debt Contracts", *Review of Financial Studies* 9: 37-68.

Barraquand, J. and D. Martineau (1995), "Numerical Valuation of High Dimensional Multivariate American Securities", *Journal of Financial and Quantitative Analysis* 30: 383-405.

Black, F. and J. C. Cox (1976), "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions", *Journal of Finance* 31: 351-367.

[12]The other model available to value American-type options when sources of uncertainty are numerous would be the one proposed by Longstaff and Schwartz (2001), but this alternative model belongs to the same family of models that combines simulation and dynamic programming as the one presented here.

Black, F. and M. Scholes (1973), “The Pricing of Options and Corporate Liabilities”, *Journal of Political Economy* 28: 637-654.

Boyle, P. (1988), “A Lattice Framework for Option Pricing with Two State Variables”, *Journal of Financial and Quantitative Analysis* 23: 1-12.

Boyle, P.; M. Broadie and P. Glasserman (1997), “Monte Carlo Methods for Security Pricing”, *Journal of Economic Dynamics and Control* 21, 8 and 9: 1267-1322.

Brennan, M.J. and E.S. Schwartz (1977a), “Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion”, *Journal of Finance* 32: 1699-1715.

Brennan, M.J. and E.S. Schwartz (1977b), “Savings Bonds, Retractable Bonds and Callable Bonds, *Journal of Financial Economics* 5: 67-88.

Brennan, M.J. and E.S. Schwartz (1980), “Analyzing Convertible Bonds”, *Journal of Financial and Quantitative Analysis* 15: 907-929.

Brennan, M.J. and E.S. Schwartz (1985), “Evaluating Natural Resource Investments”, *Journal of Business* 58: 133-158.

Broadie, M. and P. Glasserman (1997a), “Pricing American-Style Securities Using Simulation”, *Journal of Economic Dynamics and Control* 21, 8 and 9: 1323-1352.

Broadie, M. and P. Glasserman (1997b), “Monte Carlo Methods for Security Pricing ”. *Journal of Economic Dinamics and Control* 21, 8 and 9: 1267-1321.

Brys, E., and F. de Varenne (1997), “Valuing Risky Fixed Rate Debt: An Extension”, *Journal of Financial and Quantitative Analysis* 32: 239-248.

Castillo-Ramíez, A. (2000), “An Application of Natural Resource Evaluation using a Simulation - Dynamic Programming Approach”, *Journal of Computational Finance*, 3, (2): 91-107.

Collin - Dufresne, P. and R.S. Goldstein (2001), “Do Credit Spreads Reflect Stationary Leverage Ratios?”, *Journal of Finance*, 56, 5: 1929-1957.

Cortazar, G. and S.E. Schwartz (1998), “Monte Carlo Evaluation of an Undeveloped Oil Field”, Working Paper 1-98, Anderson School, UCLA.

Courtadon, G. (1982), “The Pricing of Options on Default-Free Bonds”, *Journal of Financial and Quantitative Analysis* 17: 75-100.

Cox, J.C. and M. Rubinstein (1985), *Options Markets*, First Edition, Prentice Hall. Englewood Clifss, New Jersey 07632, USA.

Cox, J.C.; J.E. Ingersoll and S.A. Ross (1985), “A Theory of the Term Structure of Interest Rates”, *Econometrica* 53: 385-407.

Cox, J.C.; S.A. Ross and M. Rubinstein (1979), *Journal of Financial Economics* 7: 229-263.

Das, S.R. and R.K. Sundaram (2000), “A Discrete-Time Approach to Arbitrage-Free Pricing of Credit Derivatives, *Journal of Management Science* 46, 1: 46-63.

Duffie, D. and M. Huang (1996), “Swap Rates and Credit Quality”, *Journal of Finance* 51: 921-949.

Duffie, D. and K. Singleton (1999), “Modeling Term Structures of Defaultable Bonds, *Review of Financial Studies* 12: 687-720.

Fan, H. and S. Sundaresan (2000), “Debt Valuation, Strategic Debt Service, and Optimal Dividend Policy”, *Review of Financial Studies* 13: 1057-1099.

Fischer, E.O.; R. Heinkel and J. Zechner (1989a), “Dynamic Capital Structure Choice: Theory and Tests”, *Journal of Finance* 44: 19-40.

Fischer, E.O.; R. Heinkel and J. Zechner (1989b), “Dynamic Recapitalization Policies and the Role of Call Premia and Issue Discounts”, *Journal of Financial and Quantitative Analysis* 24: 427-446.

Geske, R. (1977), "The Valuation of Corporate Liabilities as Compound Options", *Journal of Financial and Quantitative Economics* 12: 541-552.

Goldstein, R.S.; N. Ju and H. Leland (2000), "An EBIT-based Model of Dynamic Capital Structure", *Journal of Business*, 74, 4: 483-512.

Ho, T.S.; R.C. Stapleton and M.G. Subrahmanyam (1997), "The Valuation of American Options on Bonds", *Journal of Banking and Finance* 21: 1487-1513.

Huang, J.Z. (1997), "The Option to default and Optimal Debt Service", Working Paper, New York University.

Hull, J. C. (1997), *Options, Futures, and Other Derivatives*, Third Edition, Prentice Hall, Upper Sadle River, NJ07 458, USA.

Jarrow, R.A.; D. Lando and S. Turnbull (1997), "A Markov Model for the Term Structure of Credit Spreads, *Review of Financial Studies*, 10: 481-523.

Jarrow, R.A. and S.Turnbull (1995), "Pricing Derivatives on Financial Securities Subject to Default Risk", *Journal of Finance* 50: 53-86.

Jorgensen, P.L. (1997), "American Bond Option Pricing in One-Factor Dynamic Term Structure Models, *Review of Derivatives Research* 1: 245-267.

Kim, I.J.; K. Ramaswamy and S. Sundaresan (1993), "Does Default Risk in Coupons Affect the Valuation of Corporate Bonds?: A Contingent Claims Model", *Financial Management*, special issue on Financial Distress, 40: 117-131.

Leland, H. (1994), "Risky Debt, Bond Covenants, and Optimal Capital Structure", *Journal of Finance* 49: 1213-1252.

Leland, H. (1998), "Agency Costs, Risk Management, and Capital Structure", *Journal of Finance* 53: 1213-1243.

Leland, H. and H. Toft (1996), "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads", *Journal of Finance* 51: 987-1019.

Longstaff, F.A. and E.S. Schwartz (1995), "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt", *Journal of Finance* 50: 789-819.

Longstaff, F.A. and E.S. Schwartz (2001), "Valuing American Options by Simulation: A Simple Least Squares Approach", *Review of Financial Studies*, 14, 1: 113-147.

Madan, D. and H. Unal (1993), "Pricing the Risk of Default", Working Paper 93-5: 1-21, University of Maryland.

Merton, R.C. (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates", *Journal of Finance* 29: 449-470.

Neftcy, S. (1996), *An Introduction to the Mathematics of Financial Derivatives*, Academic Press, San Diego, California, USA.

Nielsen, L.T.; J. Saa-Requejo and P. Santa-Clara (1993), "Default Risk and Interest Rate Risk: The Term Structure of Default Spreads", Working Paper 05-93 INSEAD.

Peterson, S.; R.C. Stapleton and M.C. Subrahmanyam (1998), "A Two-Factor Log-normal Model of the Term Structure and the Valuation of American-Style Options on Bonds, Working Paper, New York University.

Ramaswamy, K. and S. Sundaresan (1986), "The Valuation of Floating-Rate Instruments: Theory and Evidence", *Journal of Financial Economics* 17: 251-272.

Raymar, S. and M.J. Zwecher (1997), "Monte Carlo Estimation of American Call Options on the Maximum of Several Stocks", *Journal of Derivatives* 5, 1: 7-23.

Tilley, J.A. (1993), "Valuing American Options in a Path Simulation Model", *Transactions of the Society of Actuaries*, 45: 83-104.