CUADERNOS DE ECONOMÍA, VOL. 45 (MAYO), PP. 3-28, 2008

# OPTIMAL BIDDING IN THE MEXICAN TREASURY SECURITIES PRIMARY AUCTIONS: RESULTS OF A STRUCTURAL ECONOMETRIC APPROACH*

SARA CASTELLANOS
*Banco de México*
MARCO OVIEDO
*Secretaría de Hacienda y Crédito Público, México*

*This analysis of the Mexican Treasury securities primary auctions suggests that the uniform format yields higher revenues than the discriminatory format. It applies the structural econometric model proposed by Février, Préget, and Visser (2004). This model's main advantage is that it allows us to (i) estimate the parameters that characterize the distribution function of the securities' marginal value and the conditional distribution of the signals given the securities' value; (ii) derive optimal bids and equilibrium prices of alternative auction mechanisms; and (iii) compare revenues. The uniform format's revenue superiority seems to be due to market uncertainty, defined as an environment with noisier value signals.*

*JEL:* C52, C72, D44
*Keywords:* Treasury Securities, Share Auction, Mexico.

## 1. INTRODUCTION

In this paper, we apply the structural econometric model of the share auction proposed by Février, Préget, and Visser (2004) –here after FPV– to analyze the Mexican Treasury securities primary auctions. Our motivation is twofold. On

* We deeply thank Philippe Février and Michael Visser for sharing with us their experience in programming and estimating the structural econometrics model of the share auction, Gerardo Hernández, for his assistance in preparing the data base, and Gerardo Gómez, for his invaluable help throughout different stages of this project. Useful comments were provided by the participants of the economics workshops at Banco de México, ITAM, the 2003 Latin American Meeting of the Econometric Society and the VIII Meeting of the Network of America Central Bank Researchers, specially, by Elías Albagli, Daniel Garcés, Phillip Haile, David Levine and Carlos Pérez Verdía. Lastly, we thank the valuable suggestions of an anonymous referee, that enriched this article and helped us giving its final form. All mistakes are the authors' responsibility. Opinions expressed in this document do not represent those of Banco de México.
Email: sgcastel@banxico.org.mx; marco_oviedo@hacienda.gob.mx

the one hand, we ought to mention that the intention to maximize the treasury's sales revenue is an important objective; however, due to the huge sums of money involved, the sales agencies are very sensitive to the need to avoid unnecessary responses to format changes that could drive investors out of the markets[1]. As a result, there have been hardly any "natural experiments" involving auction format switches up to now. The treasury securities markets' survey conducted by Bartolini and Cottarelli (1994) reported that within a sample of 77 countries only 7 of them –Belgium, Tanzania, France, Gambia, Italy, Mexico and the United States– had switched format, namely from the uniform to the discriminatory format. Furthermore, not many other format changes have occurred since then[2]. We think that this consideration favors the use of structural econometric models in order to compare auctions' revenue-generating properties, because they do not require to obtain results under different auction settings.

Our second motivation is that of comparing the findings of the structural model with those of the reduced form equations based on the results of "natural experiments" with a view to assess their consistency. The Mexican case readily lends itself for this purpose because there are two previous empirical studies using that technique. One of them is Umlauf (1993) –perhaps one of the best known auction studies– that analyzes the auctions of *Certificados de la Tesorería de la Federación* (CETES) carried out during the period 1986-1991. This study's best-known conclusion is that after Mexico instituted discriminatory auctions instead of uniform pricing auctions in 1989, bidders' profits decreased and seller's revenues increased accordingly. The other study is Laviada *et al.* (1997) who have reached the same conclusion in their analysis of the period 1995-1997, which covers the change back to the discriminatory auction format that took place in November of 1995. This is the auction format that has been used to sell the CETES since then (of course the problems of interpreting parameters obtained from reduced form equations, best summarized as the Lucas' critique, ought to be regarded as a severe warning against drawing conclusions on what policymakers should have done in the light of these two studies' results)[3].

[1] For instance, in September 1991, in the wake of Solomon Brothers' admissions of deliberate and repeated violations of Treasury auction rules, the Treasury Department, the Federal Reserve and the Securities Exchange Commission undertook a joint review of the government securities market. Among a broad range of issues, the report addressed the need to (i) strengthen enforcement of Treasury's auction rules, (ii) automate the auctions, (iii) introduce potential changes in Treasury's auction technique and debt management policies, and (iv) define the role of primary dealers. In such Joint Report, the three agencies considered that any degradation in the smooth functioning of the government securities market would result in higher costs to the taxpayer; at that time, they estimated that an increase in financing costs of only one basis point would cost taxpayers over $ 300 million each year.

[2] Perhaps the best-documented format switch occurred after 1994, this time from the discriminatory to the uniform one, occurred again in the United States in 1999, after carrying out an explicit series of experiments on auction formats. For details, see Malvey *et al.* (1995) and Malvey and Archibald (1998).

[3] Nonetheless, the Mexican Treasury has been using the uniform format to issue securities with maturities longer than a year and a fixed rate at least since 2001.

Assessing consistency between the two methods may be useful because constructing structural theoretical models or estimating their empirical counterpart are not easy tasks. In what respects treasury securities auctions, the time lag existing between Robert Wilson's (1979) proposal of the share auction model and any empirical counterparts that we can estimate is a very good illustration of this. Therefore, reduced forms will remain in use as a first approximation to understand complex economic settings and integrate theory and econometrics in future structural models[4].

For our analysis, we estimate the FVP's structural econometric model, which is an empirical counterpart to Wilson's share auction model and which many authors consider a good theoretical approximation of the treasury securities auction's context[5]. We use a data set construed based on the public results of 180 CETES primary auctions carried out between January 2001 and April 2002, and published on Banco de México's website. The data includes: (i) securities' characteristics, (ii) summary statistics of auctions, and (iii) anonymous distribution of prices and quantities of asked and allocated bids. As several other central banks publish auction results in the same fashion (given that they face similar restrictions in what respects revealing bidders' identity and storing data), this approach may also be applicable in other cases.

Before briefly summarizing our results, let us point out one distinctive feature of the FPV model. Its statistical inference method relies on the Euler condition implied by the optimization problem of a bidder in a discriminatory price auction; so although we may assume that an equilibrium strategy exists and all bidders use it, it is not necessary to know the equilibrium's explicit form. However, the method requires a parametric framework to evaluate and compare the auctions' performance, with the advantage that this always enables us to rank auctions in terms of their revenue. In contrast, structural models that are distribution free and solve for the equilibrium bidding strategies usually require bidder-specific data (Armantier and Sbai, 2003; Hortacsu, 2002 and 2002a; Kang and Puller, 2007), which in turn is more difficult to obtain[6].

Our estimation results of the FPV model suggest, once again, that in Mexico the uniform price auction produces more revenues than the discriminatory price auction. Revenues from the CETES discriminatory auctions carried out during the period from January 2001 to April 2002 totaled 79,767.05 billion pesos. Contrasting with this, revenues from the corresponding hypothetical uniform auctions are 80,918.47 billion pesos. This difference in revenue is statistically significant. When we disaggregate the data by maturity, we also find that the discriminatory price auction yields higher revenues than the uniform price auction in the case of short term 28-day CETES,

[4] See, for example, Orellana *et al*. (2007).

[5] Back and Zender (1993) and Bikhchandani and Huang (1993) provide more detailed explanations on the similarities between treasury securities auctions and Wilson's share auction model.

[6] In fact, a handbook for developing government bond markets of the World Bank and the IMF contains the following recommendation: "As much aggregate information as possible should be disclosed after the auction. No information should be disclosed that might identify individual bidders." (Page 162).

while for the longer term –91, 182 and 364-day CETES– the uniform price format yields the highest revenue. The revenue ranking runs counter to FPV's findings for French Treasury securities auctions (even though the two countries' auctions have several features in common), but coincides with the previous results for the 28-day CETES auctions obtained through the reduced equation technique.

To investigate whether the driving force behind the above-mentioned results relates to the type of bid shading that occurs in response to market uncertainty, we resort to four exercises that use the results of both the structural and reduced form estimations to greatest possible advantage. First, we compare the conditional variance of the value obtained in our exercise, which we can interpret as a higher degree of uncertainty in the good's value, as compared to the one obtained by FPV, and find that ours is considerably larger. Second, we look at the relationship between the gains of employing the uniform format and the volatility of the securities resale price for the 28-day CETES in our estimations, as well as in the results of Umlauf (1993) and Laviada *et al.* (1997) that cover different periods. We find that this relationship –that can also be ascribed to market uncertainty– is positive. Third, the cross maturity comparison of our estimations shows the same pattern. Fourth, a simulation re-estimating FPV's model using a value signal constructed with a higher variance (in effect, noisier) than the original data, shows that: (i) parameters obtained are consistent with the signals being less informative; and (ii) revenues obtained from the hypothetical uniform auctions exceed those from the observed discriminatory auctions by an even larger proportion than before. Thus, the four exercises point to market uncertainty having a role in the results.

The paper proceeds as follows: Section 2 describes the CETES auction framework and the dataset. Section 3, for the sake of completeness, presents Wilson's (1979) share auction model and the FPV (2004) estimation technique. Section 4 addresses the estimation results and the revenue comparisons between the discriminatory and the uniform format. Section 5 performs exercises to explore the possible impact of market uncertainty in the results. Finally, section 6 summarizes some conclusions and possible extensions.

## 2. A Description of CETES' Auctions and Data

The sales mechanism of the Mexican Treasury securities has been modified several times since the CETES were first issued in 1978. For purposes of our analysis, we focus our attention on the institutional framework that prevailed in our data period, which is between January 2001 and April 2002:

- Only brokerage houses, banks and investment funds based in Mexico can bid and acquire treasury securities[7].

[7] Other agents specifically authorized by Banco de México, the Central Bank, can also bid and buy treasury securities.

- Banco de México publishes on its website the primary auction announcement, after 12:00 a.m. of the last market day of the week immediately before the auction takes place[8, 9]. It provides information as to the securities and auction's characteristics: securities' date of issue, announcement number, and issue's identification number, and auction format and maximum amount tendered.
- Primary auctions can be of either the uniform price or the discriminatory price format (the latter was the one in place during the period under study).
- Bidding for CETES is only through competitive bids, indicating the amount and discount rate at which the bidder is willing to buy the securities tendered[10]. Each bidder may submit one or more bids in the same auction. Bidders are to submit their bids no later than 13.30 p.m. on the second market day immediately before the securities' issue date.
- The sum of any bidder's quantity bids for any auction must not exceed 60% of the maximum amount tendered.
- All bids are obligatory and irreversible for the bidder. If a bidder does not pay for the securities allocated to him in full, Banco de México can cancel the sale for the unpaid securities amount. It can also ban the bidder from participating in subsequent auctions.
- The weighted allocation rate is determined based on the allocated bids.
- At any auction, the Treasury can determine the maximum discount rate at which it is willing to place the auction securities. Higher discount rates are not met in those cases (though, this right was not exercised in any of the auctions of the period under study)[11].
- Banco de México notifies the auctions' results to each bidder no later than 10:30 a.m. of the market day immediately after the auctions take place through the bank's service counter to meet account holders needs[12]. In addition, it announces the auctions' general results no later than 18:30 p.m. of the day of the auction through its website.
- The Security Safe Custody Institute delivers the securities allocated through each bidder's account[13] on the issue date. Brokerage houses and banks must pay for the securities through the institute's system. Other institutions must pay for the securities through a brokerage house or a bank.

[8] Mexico's central bank website address is http://www.banxico.org.mx.
[9] These announcements, in turn, follow the quarterly issuance calendar of the Ministry of Finance.
[10] Discount rates must be expressed in percentage points, up to two decimal points, in yearly terms and based on years of 360 days.
[11] After September 2002 the rule is that the Treasury only can declare the whole auction deserted if discount rates are too high, but this new rule has not been used either.
[12] Sistema de Atención a Cuentahabientes del Banco de México (SIAC-Banxico), in Spanish.
[13] Instituto para el Depósito de Valores (S. D. INDEVAL). INDEVAL is the only firm in México authorized to operate as a depository of securities. The services it must provide include custody, administration and transfer of securities, as well as operation compensation and liquidation.

Although the main CETES auction features agree in very broad terms with Wilson's (1979) share auction model and the statistical inference method of FPV, so we can assume that bidders are risk neutral, and the good on sale is divisible and has the same value for all bidders –in spite that it is unknown at the start of the auction–, there is one institutional feature of the CETES auctions that does not match those models. As in many security markets, after the primary auction there is a buy option for market makers that permits them to acquire more securities by placing non-competitive bids[14]. Actually, this mechanism is also in place for the French Treasury auctions analyzed by FPV. As a point in favor of this exercise, we can state that there are two aspects of the CETES auctions making them resemble Wilson's model much more than the French Treasury auctions. These aspects are: i) CETES bidders at the primary auction are only allowed to submit competitive bids; and ii) the securities allocated through the market makers' buy option represent a much smaller proportion of the total number of securities' that are placed by the Mexican Treasury as compared to those placed by the French Treasury.

CETES are zero coupon bonds issued and liquidated by the Federal Government at the maturity date. Even today, they are among the most important public debt instruments of the Federal Government, with a high preponderance in the Mexican money and stock markets. The most common maturity dates have been 28, 91, 182 and 364 days. Our database derives from the general results of 180 CETES primary auctions that Banco de México publishes weekly at its website. The data includes (i) securities' characteristics, (ii) auction's summary statistics, and (iii) anonymous distribution of prices and quantities of both asked and allocated bids. During the period under study, 28 and 91-day CETES were auctioned weekly, 182-day CETES were auctioned every 2 weeks, and 364-day CETES every 4 weeks[15]. In turn, the source of the CETES secondary market prices is the price vector that Banco de México publishes on its website.

[14] We sketched a two-stage model that takes into account this buy option and estimated the FPV model using only a sample that would be consistent with Wilson's share auction model; that is, the sample of primary auctions after which the market makers' buy option was empty. This exercise suggests that the characterization of the CETES as a share auction was adequate because the securities allocations through the market makers' buy option have been a small proportion of the total amount issued by the Mexican Treasury. However, the potential asymmetry among bidders that the market makers' buy option may introduce did not affect the estimated parameters substantially across the samples. A more detailed description of Mexico's market makers mechanism, the model sketch and the estimation results are available in Spanish in the working paper version of this paper, Castellanos and Oviedo, 2004.

[15] CETES issues with maturity of 27, 90, 168, 182, 335 or 363 days, that result from computing the securities' maturity according to the number of market days and from the practice of "reopening" the 182 and 235 days issues to improve their liquidity, for presentation purposes are grouped with the closest of the 4 basic issues.

TABLE 1
OVERALL INFORMATION ABOUT THE AUCTIONS

| | Number | Percentage (%) |
|---|---|---|
| CETES | | |
| Number of Auctions | 180 | |
| 28-day | 65 | 36.11 |
| 91-day | 65 | 36.11 |
| 182-day | 33 | 18.33 |
| 364-day | 17 | 9.45 |
| Number of Bidders | 3,581 | |
| Number of Bids | 13,393 | |
| Allocated totally or partially | 4,506 | 33.64 |
| Not allocated | 8,887 | 66.36 |
| Total amount issued by the Treasury (in thousands of pesos) | 879,249,141 | |
| Non-competitive bids in the buy option for market makers | 55,860,991 | 6.35 |
| Competitive bids in the primary auction | 823,388,150 | 93.65 |

Source: Author's own calculations based on public results of 180 CETES primary auctions carried out between January 2001 and April 2002.

Table 1 shows the basic statistics of our dataset, involving 3,581 "different" auction bidders that submitted 13,392 competitive bids totaling approximately 2,675,255 million pesos. Of these bids, 33.64% were allocated either totally or partially and 66.36% were rejected. The total amount of CETES issued by the Treasury is approximately 879,249 million pesos. Therefore, 93.65% of this amount was placed through competitive bids in the primary auction and only 6.35% was placed through market makers' non-competitive bids in the buy option. FPV report that these last two figures for the French Treasury securities are 91% and 8%, respectively (with the 1% residual placed through non-competitive bids received in the primary auctions). Hence, their argument that this amount of non-competitive bidding is too small to have an effect on the assumptions that support their estimation method can also hold in the case of the CETES auctions.

Table 2 shows summary statistics per auction of the variables suggested by FPV for the empirical estimation. Statistics calculated for the whole sample are comparable to the French securities auction data as reported by the authors of the FPV model. The most obvious difference between the two samples relates to the securities' average maturity, which in Mexico is shorter than one year and in France is longer than 10 years. In general, the longer that the securities' maturity dates are, the higher the nominal yield is and the lower the secondary market price is. Therefore, for similar maturity dates, securities' secondary market prices seem to be higher in Mexico than in France. On the other hand, the variables for number of bidders, number of bids and cover (defined as the ratio of total amount of quantity bids to total amount issued by the Treasury), which measure the degree of auction competition, do not vary much across CETES with different maturity.

TABLE 2
CETES SUMMARY STATISTICS PER AUCTION
(January 2001-April 2002)

| Statistic | Number of bidders | Number of bids | Amount issued by the Treasury (Thousands of pesos) | Secondary market price | Nominal yield* | Maturity of the security | Cover |
|---|---|---|---|---|---|---|---|
| All CETES | | | | | | | |
| Mean | 19.46 | 73.92 | 4,538,043 | 96.84 | 10.30 | 109.18 | 3.24 |
| Standard Deviation | 6.13 | 19.59 | 674,815 | 3.14 | 3.48 | 94.22 | 0.90 |
| Max | 91 | 145 | 5,200,000 | 100 | 18.38 | 364 | 7.31 |
| Min | 12 | 35 | 3,300,000 | 84.81 | 5.26 | 27 | 1.68 |
| Observations | 180 | 180 | 180 | 180 | 180 | 180 | 180 |
| 28-day CETES | | | | | | | |
| Mean | 18.90 | 72.12 | 4,500,000 | 99.30 | 9.13 | 28.00 | 3.02 |
| Standard Deviation | 2.70 | 16.54 | 0 | 0.25 | 3.07 | 0.28 | 0.81 |
| Max | 27.00 | 107.00 | 4,500,000 | 100.00 | 16.61 | 29.00 | 5.66 |
| Min | 15.00 | 42.00 | 4,500,000 | 98.74 | 5.65 | 27.00 | 1.70 |
| Observations | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 91-day CETES | | | | | | | |
| Mean | 19.63 | 78.71 | 5,200,000 | 97.62 | 9.74 | 91.00 | 3.60 |
| Standard Deviation | 3.41 | 18.68 | 0 | 0.78 | 2.93 | 0.28 | 0.87 |
| Max | 29.00 | 128.00 | 5,200,000 | 100.00 | 17.01 | 92.00 | 6.37 |
| Min | 13.00 | 35.00 | 5,200,000 | 95.80 | 5.92 | 90.00 | 2.32 |
| Observations | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 182-day CETES | | | | | | | |
| Mean | 18.61 | 68.86 | 3,300,000 | 94.91 | 10.59 | 178.96 | 3.38 |
| Standard Deviation | 3.11 | 20.39 | 0 | 1.41 | 2.75 | 5.83 | 1.17 |
| Max | 25.00 | 112.00 | 3,300,000 | 97.51 | 16.53 | 182.00 | 7.31 |
| Min | 13.00 | 40.00 | 3,300,000 | 92.11 | 6.49 | 168.00 | 1.78 |
| Observations | 33 | 33 | 33 | 33 | 33 | 33 | 33 |
| 364-day CETES | | | | | | | |
| Mean | 19.00 | 78.00 | 4,646,154 | 89.16 | 11.20 | 348.77 | 3.02 |
| Standard Deviation | 3.34 | 27.70 | 161,325 | 2.12 | 2.18 | 14.52 | 0.97 |
| Max | 25.00 | 145.00 | 5,000,000 | 92.23 | 15.36 | 364.00 | 4.45 |
| Min | 14.00 | 43.00 | 4,500,000 | 85.61 | 8.34 | 335.00 | 1.68 |
| Observations | 17 | 17 | 17 | 17 | 17 | 17 | 17 |

Note: * Weighted allocation rate of the primary auction.
Source: Author's own calculations based on the public results of 180 CETES primary auctions carried out between January 2001 and April 2002.

Table 3 shows the summary statistics per bidder and per bid. In each auction, each bidder submitted four competitive bids on average; that is, four combinations of amount and discount rate at which they are willing to buy the securities. According to FPV, bidders in France and Portugal submit three bids on average, while in Turkey they submit seven bids on average. If a bidder distributes his individual demand into a larger number of bids as an optimal strategy to cope with market value uncertainty, these numbers suggest that the bidders participating in the Mexican auctions perceive a more uncertain environment than those participating in the French or Portuguese auctions, but less uncertain than those participating in the Turkish auctions[16].

TABLE 3
SUMMARY STATISTICS PER BIDDER OR PER BID IN THE CETES AUCTIONS

| Variable | Mean | Standard Deviation | Maximum | Minimum | Observations |
|---|---|---|---|---|---|
| Number of bids per bidder | 3.85 | 0.75 | 7.69 | 0.64 | 3,581 |
| Demanded quantity per bidder (Thousands of pesos) | 770,628 | 215,108 | 1,461,471 | 205,625 | 3,581 |
| Demanded quantity per bid (Thousands of pesos) | 204,300 | 61,088 | 418,286 | 116,342 | 13,393 |
| Allocated bids per winning bidder | 2.04 | 0.82 | 4.09 | 0.12 | 4,506 |
| Allocated quantity per winning bidder (Thousands of pesos) | 576,622 | 531,294 | 2,600,000 | 183,333 | 4,506 |
| Allocated quantity per winning bid (Thousands of pesos) | 432,918 | 571,619 | 2,600,000 | 64,706 | 4,506 |
| Price bid | 96.68 | 3.19 | 99.57 | 84.55 | 13,393 |
| Highest price bid – Lowest price bid | 0.38 | 0.42 | 2.58 | 0.04 | 13,393 |

Source: Author's own calculations based on the public results of 180 CETES primary auctions carried out between January 2001 and April 2002.

In turn, the average quantity bid per bidder is 770.63 million pesos, and the average winning number of bids per winning bidder is 576.62, so each winning bidder receives on average 74.82% of his quantity of bids. However, the rest of the data does not support this winning expectation. The mean and standard deviation of the demanded quantity per bid are 204.29 and 61.09, respectively, while those

[16] See Gordy (1996) for a more detailed presentation of this idea.

of the allocated quantity per winning bid are 432.92 and 571.62, respectively. Since these distributions of variables are truncated at zero, the latter statistics seem more consistent with a pattern of asymmetric information among the bidders[17]. Specifically, it would seem that bidders submitting large bids have more information about the good's value than bidders that submit small bids. Therefore, large bids win more often than small bids[18]. Although such asymmetry is not consistent with Wilson's model and we find it somewhat awkward, since we have no way of telling large from small bidders in our dataset we must assume symmetry for the rest of the analysis.

## 3. THEORETICAL MODEL AND ECONOMETRIC TECHNIQUE

For the sake of completeness, this section briefly outlines Wilson's share model as presented in FPV along with the latter's econometric technique. However, readers are encouraged to refer to FPV's article for a more detailed discussion about the model's assumptions and properties[19]. Let us consider the auction of a perfectly divisible good among $n \geq 2$ risk neutral bidders. The good's value is the same for all bidders but unknown when the auction starts. We assume that the good's value follows a distribution function $F_v(v) = \Pr(V < v)$. Before the auction, each bidder $i = 1, \ldots, n$ receives a private signal about the good's value. This signal is a realization of the random variable $S_i$. We assume signals $S_1, \ldots, S_n$ to be independently and identically distributed given $V$.

The distribution of $S_i$ given $V$ is the same for all bidders and denoted as $F_{S|V}(s \mid v) = \Pr(S_i \leq s \mid V = v)$. The signal received by each bidder is observed only by him and not by either the seller or the rest of the bidders. The number of bidders, $n$, and the distributions $F_v(.)$ and $F_{S|V}(.)$ are common knowledge.

Each bidder must submit his bid, consisting of the fraction of the good that he requests at each price, to the seller. The price and quantity combinations constitute his individual demand. Adding up all individual demands, the seller can determine the market equilibrium price; that is, the price at which aggregate demand adds up to one.

Let us define $x_i(.,.)$ as bidder *i's* strategy in the primary auction. This strategy is a function of the good's price $p$ and of the signal $s_i$, so that when bidder $i$ gets the signal $S_i = s_i$, his bid specifies that he will demand a share $x_i(p, s_i)$. In a symmetric optimal strategies equilibrium $x_i(p, s_i) = x_i(.,.)$ for all $i$.

17 Notice that asymmetry across bidders may also be the result of different costs of obtaining or placing customers offers.

18 For the 28-day CETES auctions of the period 1986-1991, see Umlauf (1993), whose data permits distinguishing bidders' sizes and also provides evidence suggesting that there is asymmetric information between large and small bidders. However, it should be noticed that, due to the consolidation of the banking industry in Mexico, there are fewer but larger banks in the period analyzed.

19 More details about the asymptotic properties of FPV's semi parametric two-stage estimator are also available in Newey and McFadden (1994).

Along with this notation, the equation that defines the market equilibrium of the primary auction under the uniform price format as a function of the equilibrium price $p^0$ is written as:

$$(1)\qquad \sum_{j\neq i} x\left(p^0, s_j\right) + y\left(p^0, s_i\right) = 1$$

This equation depends on bidder *i's* signal and on the signals received by each one of the other bidders, which are unknown to bidder *i*. As a result, the equilibrium price $p^0$ is also unknown to bidder *i*. However, since bidder *i* know the probability distribution function from which he extracts signals and the function $x_i$ $(p, s_i)$, he can determine the conditional distribution of the random variable $P^0$:

$$(2)\qquad H(p; v, y) = \Pr\left\{P^0 \leq p \middle| V = v, y(p, s_i) = y, S_i = s_i\right\}$$
$$= \Pr\left\{\sum_{j\neq i} x(p, S_j) \leq 1 - y \middle| V = v, y(p, si) = y, S_i = s_i\right\}$$
$$= \Pr\left\{\sum_{j\neq i} x(p, S_j) \leq 1 - y \middle| V = v\right\}$$

If a uniform price auction format is employed, bidder *i*'s expected benefit when he resorts to strategy $y$ (.,.) and the good's value and equilibrium price are, respectively, $v$ and $p^0$ is:

$$(3)\qquad E\left\{\int_0^{\infty} (V - p) y(p, s_i) dH(p; V, y(p, s_i)) \middle| S_i = s_i\right\}$$

where the expected value is with respect to $V$ given $S_i = s_i$. The strategy $x$ (.,.) indeed is optimal if the maximum of equation (3) is attained at $y$ (.,.) = $x$ (.,.). A solution to this optimization can be characterized by resorting to calculus of variations. The necessary condition for a maximum is that for all $p \in [0, \infty]$:

$$(4)\qquad E\left\{(V - p)\frac{\partial H(p; V, y)}{\partial p} + x(p, s_i)\frac{\partial H(p; V, y)}{\partial y} \middle| S_i = s_i\right\} = 0$$

where partial derivatives of $H$ with respect to $p$ and $y$ are evaluated at $y = x(p, s_i)$. On the other hand, if a discriminatory price auction format is employed, bidder *i's* expected benefit becomes:

$$(5)\qquad E\left\{\int_0^{\infty}\left[(V - p) y(p, s_i) - \int_p^{p_{\max}} y(u, s_i) du\right] dH\left(p; V, y(p, s_i)\right) \middle| S_i = s_i\right\}$$

The Euler equation derived to maximize this expression is:

(6) $$E\left\{(V-p)\frac{\partial H(p;V,y)}{\partial p}-H(p;V,y)\middle|S_i=s_i\right\}=0$$

and it has a corresponding empirical counterpart, as derived by FPV, written as:

(7) $$E\left\{(n-1)\cdot\left(E(V\middle|S_1=s_1,\ldots,S_n=s_n)-p\right)\cdot 1\left\{P^0\le p\right\}\right\} \\ -E\left\{(p-P^0)\cdot 1\left\{P^0\le p\right\}\right\}=0$$

where the first expected value is with respect to the signals $S_1,\ldots, S_n$ (the random variable $P^0$ only depends on these signals), the second one is with respect to $V$ given $S_1,\ldots, S_n$, the third one is with respect to $P^0$, and $1\ \{\cdot\}$ is the indicator function. This condition is satisfied for all $p \in [0, \infty]$. Then the objective is to find an estimator of $\theta^0$, the true value of $\theta$, defined as the minimum of the empirical counterpart of the Euler condition derived from the bidders utility maximization problem. Using this condition to estimate the structural parameters of the model makes it essential to find a way to compute the conditional expectation $E\left(E(V\middle|S_1=s_1,\ldots,Sn=sn\right)$, which is an unbiased estimate of V. The problem is that we do not observe the signals of the bidders $s_1,\ldots, s_n$. But we do observe their bids, so to overcome this problem FPV suggest assuming that bidders' strategies are strictly decreasing in their signal, $s_i$. With this assumption, the authors can use observed bid functions instead of the unobserved signals to form the above conditional expectation. In particular, if bid strategies, $x\ (p, s_i)$, are strictly decreasing in the signals, $s_i$, then the quantiles of $x\ (p, s_i)$ can be equated to quantiles of $s_i$ to "invert" an observed bid $x\ (p, s_i)$ to find its corresponding signal, $s_i$. This could well be deemed a very strong assumption that ought to be tested against data. Unfortunately, our data is not rich enough to permit it, so we adopt this working assumption and leave its testing for future research.

The estimation is carried through a two stage semi-parametric method that exploits the results of a set of $L$ auctions that exhibit observed heterogeneity across them in terms of a number of participants $N_l$ and of a vector of auction characteristics $Z_l$[20]. For auction $l$ with characteristics $z_l$ and $n_l$ bidders, the Euler equation can be rewritten in terms of auction-specific variables as:

(8) $$0=E\left\{(n_l-1)\cdot(E(Vl\middle|S_{1l}=s_{1l},\ldots S_{nl}=s_{nl})-p)\cdot 1\left\{P_l^0\le p\right\}\middle|N_l=n_l,Z_l=z_l\right\} \\ -E\left\{(p-P_l^0)\cdot 1\left\{P_l^0\le p\right\}\middle|N_l=n_l,Z_l=z\right\}$$

[20] To this end, the random variables $(N_l, Z_l)$, $l = 1, \ldots, L$, are assumed to be independently and identically distributed. The good's value in the 1-th auction, $V_l$ and the signal received by bidder $i$ in the auction $l$, $S_{il}$ (dependent on $V_l$), are assumed to be dependent of $Z_l$ and independent of $N_l$. The value realizations of $V_1,..,V_L$, conditional on $Z_l$, are independently and identically distributed. $S_{1l},..S_{nl}$ are independent conditional on $Z_l$ and $V_l$, and the signals $S_{il}$ and $S_{il'}$ are also independent conditional on $Z_l$ and $Z_l'$ for all $l \neq l'$. In addition, the respective conditional distribution functions of $V_l$, and $S_{il}$ are denoted $F_{V|Z}(\cdot \mid z;\ \theta_1)$ and $F_{S|V,Z}(\cdot \mid v, z;\ \theta_2)$, where $\theta_1$ and $\theta_2$ are parameter vectors. From these two distributions, the one for $S_{il}$ given $Z_l = z$, $F_{S|Z}(\cdot \mid z;\ \theta)$, where $\theta = (\theta_1', \theta_2')'$, can be determined.

where the random variable $P_l^0$ represents the equilibrium price at auction $l$ and the first expected value is taken with respect to $S_{il},\ldots, S_{nl,l}$ given $N_l = n_l$, and $Z_l=z_l$. This condition must hold for all $p \in [0, \infty]$ and all $l = 1,\ldots, L$.

In Stage 1 the distribution of optimal bids, $G(x \mid n, z; p)$, is estimated non parametrically from the observed bids using Kernel estimation methods[21]. Let $K(.,.)$ be a Kernel and $h_N$ and $h_Z$ be the bandwidth parameters –*i.e.*, $h_Z$ being the vector of bandwidth parameters for each characteristic $z$-. Then a non-parametric estimator of the distribution of optimal bidding strategies $G(\cdot \mid \cdot, \cdot, ; p)$ is:

$$(9) \qquad \hat{G}(x|n,z;p) = \frac{\sum_{l=1}^{L} \frac{1}{n_l}\sum_{i=1}^{nl} 1\{x_{ilp} \le x\} K\left(\frac{n-n_l}{h_N}, \frac{z-z_l}{h_Z}\right)}{\sum_{l=1}^{L} K\left(\frac{n-n_l}{h_N}, \frac{z-z_l}{h_L}\right)}$$

Once this distribution function is obtained, for any $\theta$, the unobserved signals appearing in the above equation are replaced with the estimated inverse demand functions (since $\tilde{x}^{-1}\left(x,p,S_{il},n,z;\theta\right) = F^{-1}{}_{S|Z}\left(1-\hat{G}\left(x|n,z;p\right)|z;\theta\right)$ and also $s_{il} = x^{-1}(x,p,S_{il},n,z;\theta^0))$[22]. Then, the following empirical counterpart for the right hand side of equation (8) is considered:

$$(10) \qquad m(x_{11p},\ldots,x_{nlLp},n_1,\ldots,n_L,p_1{}^0,\ldots,p_L{}^0,z_1,\ldots,z_L,p;\theta) =$$
$$\sum_{l=1}^{L}\Big[\Big(E(V_l|S_{1l} = \tilde{x}^{-1}(x_{1lp},p,n_l,z_l;\theta),\ldots,S_{nll} =$$
$$\tilde{x}^{-1}(x_{nllp},p,n_l,z_l;\theta),N_l = n_l, Z_l = z_l) - p\Big)$$
$$\times(n_l - 1)1\left\{p_l{}^0 \le p\right\} - (p - p_l{}^0)1\left\{p_l{}^0 \le p\right\}\Big]$$

Stage 2 consists of minimizing with respect to $\theta$ the squared sum of a fixed number T of empirical moments (FVP choose T equal to the number of auctions in the sample)[23]. In effect:

$$(11) \qquad \hat{\theta} = Arg\min_{\theta}\sum_{t=1}^{T} m^2(x_{11pt},\ldots,x_{n_l Lp_t},n_1,\ldots,n_L,p_1{}^0,\ldots,p_L{}^0,z_1,\ldots,z_L,p_t;\theta)$$

At the first estimation stage, the optimal bidding strategies' distribution function is estimated using the *Epanechnikov* Kernel. This kind of estimation requires a vector of observations, denoted as $z = (z^1, z^2, z^3)$, to evaluate the Kernel for each of the variables $z$. Hence, the Kernel estimator is defined as follows:

[21] Pagan and Ullah (1999).

[22] This follows from the definition of $G\left(x|n,z;p\right) = \Pr\left(x\left(p,S_{il},N_l,Z_l;\theta^0\right) \le x|N_l = n, Z_l = z\right)$, assuming that $S_{il}$, and $N_l$ are conditionally independent, when the optimal strategy is a decreasing function of the signal.

[23] Though, FPV note that since equation (9) is satisfied for an infinite number of prices, there exists an infinite number of moments and, for each of these theoretical moments, there exists an empirical counterpart with the form of equation (10).

(12) $$K\left(\frac{n-n_l}{h_N},\frac{z-z_l}{h_Z}\right)=K\left(\frac{n-n_l}{h_N}\right)K\left(\frac{z^1-z_{1l}}{h_{1Z}}\right)K\left(\frac{z^2-z_{2l}}{h_{2Z}}\right)K\left(\frac{z^3-z_{3l}}{h_{3Z}}\right)$$

where $K(u) = 0.75\ (1 - u^2)1\{|u|\leq 1\}$ and $h_N$, $h_{1Z}$, $h_{2Z}$, and $h_{3Z}$ are bandwidth parameters[24].

Next, it is necessary to choose the parametric specifications for the signal and valuation distribution functions. The distribution function of $V_l$ given $Z_l = z_1$ is:

(13) $$F_{V|Z}\left(v\left|z_l;\theta_1\right.\right)=\int_0^v \gamma u^{\gamma-1}\frac{\beta_l^{\alpha_l}}{\Gamma(\alpha_l)}u^{\gamma(\alpha_l-1)}\exp\left[-\beta_l u^{\gamma}\right]du$$

where $\alpha_l = (1, z_l)\cdot\alpha$ and $\beta_l = (1, z_l)\cdot\beta\cdot\Gamma(\cdot)$ is the gamma function, $\alpha$ and $\beta$ are parameter vectors of 4×1 dimension, and $\gamma$ is a scalar. If $\gamma = 1$ the distribution described in equation (13) is a gamma distribution with parameters $\alpha_l$ and $\beta_l$, while if $\gamma \neq 1$ then $V_l^{\gamma}$ follows a gamma distribution with parameters $\alpha_l$ and $\beta_l$. Note also that $\theta_l = (\alpha', \beta', \gamma)$.

The probability distribution of $S_{il}$ given $V_l = v$ and $Z_l = z$, is specified with the exponential distribution:

(14) $$F_{S|V,Z}\left(s\left|v_l,z_l;\theta_2\right.\right)=1-\exp\left[-sv_l^{\gamma}\right]$$

where $\gamma$ is the same parameter that appears in the conditional distribution of $V_l$. In this case, the conditional expected value and the conditional variance of $S_{il}$ are independent of $z_l$. So the complete vector of parameters is: $\theta = (\alpha', \beta', \gamma)$; that is $\theta$, which has 9×1 dimension.

Under these two specifications proposed by FPV, the conditional expectation of $V_l$ that appears in the empirical moment *m(.)* is:

$$E\left(V_l\left|S_{1l}=\tilde{x}^{-1}(x_{1lp},p,n_l,z_l;\theta),\ldots,S_{nll}=\tilde{x}^{-1}(x_{nllp},p,n_l,z_l;\theta),N_l=n_l,Z_l=z_e\right.\right)=$$

(15) $$=\frac{\Gamma\left(n_l+\alpha_l+\frac{1}{\gamma}\right)}{\Gamma(n_l+\alpha_l)}\frac{1}{\left(\beta_l+\sum_{i=1}^{n_l}\beta_l\left[\frac{1}{\hat{G}\frac{1}{\alpha_l}\left(x_{ilp}\left|n_l,z_l;p\right.\right)}-1\right]\right)^{\frac{1}{\gamma}}}$$

[24] To calculate this expression, as FPV we resort to the rule of thumb to define the bandwidth parameters as follows: $h_i = 2.214s / L^{1/7}$, for $i = \{N, Z\}$, where s is the standard deviation of variable $i$ and $L$ is the number of observations. According to $h_i$, bandwidth parameters differ across the variables if they show different variability in the data.

Once this expression is appropriately substituted into equation (11), we can proceed with the second stage estimation through generalized methods of moments of $\theta^0$, the true value of $\theta$. FPV select the value of T equal to the number of auctions or stop-out prices. The corresponding standard errors are computed with the asymptotic variance-covariance matrix derived in the Appendix C of FPV.

Given these estimated values of $\theta$ and using equation (11), $E(V_l \mid Z_l = z_l)$ can be computed as follows:

$$(16)\qquad E\left(V_l \middle| Z_l = z_l\right) = \int_0^\infty v f\left(v \middle| z\right) dv = \frac{\Gamma\left(\alpha_l + \frac{1}{\gamma}\right)}{\Gamma(\alpha_l)} \bullet \beta_l^{-\frac{1}{\gamma}}$$

FPV's specifications have the property of allowing us to obtain closed form solutions for optimal strategies and equilibrium prices in the uniform price auction. In fact, Wilson's share auction with the uniform format has multiple equilibria and – depending on which equilibrium is played – the price in the auction may be anything between the seller's reservation price and the "true value" (Back and Zender, 1993). Despite this, the optimal strategy derived by FVP actually is the unique equilibrium strategy in the class of demand functions that are linear in the individual signal's value, under the above distributional assumptions[25]:

$$(17)\qquad x\left(p, s_{il}, n_l, z_l; \theta\right) = \left[1 - \left\{\frac{\beta_l}{n_l} + s_{il}\right\}\left\{\frac{\Gamma(n_l + \alpha_l)}{\Gamma\left(n_l + \alpha_l + \frac{1}{\gamma}\right)} \frac{1+\gamma}{\gamma} p\right\}^{\gamma}\right] \bullet \frac{1}{n_l - 1}$$

That is, the closed-form expressions of the optimal strategy in a uniform auction that result from the parametric specifications (13) and (14). Then the equilibrium stop-out price at the *l–th* uniform price auction, as a function of the estimated signals and parameters is:

$$(18)\qquad \begin{aligned} p_l^0 &= \frac{1}{1+\frac{1}{\gamma}} E\left(V_l \middle| S_{1l} = s_{il}, \ldots, S_{n_l l} = s_{nl}, Z_l = z_l\right) = \\ &= \frac{1}{1+\frac{1}{\gamma}} \frac{\Gamma(n_l + \alpha_l)}{\Gamma\left(n_l + \alpha_l + \frac{1}{\gamma}\right)} \frac{1}{\left(\beta_l + \sum_{i=1}^{n_l} s_{il}\right)^{\frac{1}{\gamma}}} \end{aligned}$$

[25] Such class of linear demand functions has the form $x\left(p, S_{il}, n_l, z_l; \theta\right) = a(p, n_l, z_l; \theta) + b(p, n_l, z_l; \theta) s_l$, with the only restriction that functions $a(.,.,.;.)$ and $b(.,.,.;.)$ are such that $x(.,.,.,.;.)$ is decreasing in $p$ and $s_{il}$.

Therefore, we can obtain the stop-out price replacing $\theta$ and $s_{il}$ in the above expression by their estimates. Then we can compute the hypothetical revenue from uniform price auction $l$ as the product of the equilibrium price times the amount of bonds auctioned, and the total hypothetical income under the uniform auction derives from the summation over all the sample auctions.

## 4. RESULTS

### 4.1 Parameters

For our estimations, the dimension of $z_1$ is equal to 3 and includes the secondary market price (in pesos), the maturity (in days) and the nominal yield (in percentage) as shown in Table 2[26]. Our calculated values are $h_N = 20.6216$, $h_{1_z} = 3.3344$, $h_{2_z} = 3.6252$, and $h_{3_z} = 99.8950$. Notice that these values agree with the data of Table 5, because the number of bidders and nominal yield exhibit a higher variance than the secondary market price and maturity.

The number of moments we chose to estimate equation 10 is T = 180.[27] Table 4 shows our estimates. All parameters are significant and different from zero at 5% confidence level.

TABLE 4
SECOND STAGE ESTIMATE OF $\theta$

| Estimate of alpha: | Coefficient | Standard Error |
|---|---|---|
| Constant | –15.2771 | 0.97630 |
| Secondary market price | 148.4281 | 0.92758 |
| Nominal yield | –12.5593 | 0.11267 |
| Maturity (Divided by 364) | –4.7492 | 0.43610 |
| Estimate of beta: | | |
| Constant | –29.8005 | 0.63808 |
| Secondary market price | 151.1429 | 0.60659 |
| Nominal yield | 12.9369 | 0.07355 |
| Maturity (Divided by 364) | 0.3889 | 0.28267 |
| Gamma | 118.7335 | 0.66655 |

We evaluate the derivatives of equation 16 with respect to each of the variables at the sample mean of the characteristics. The values obtained for the derivatives with respect to secondary market price, nominal yield, and maturity are –0.0804,

[26] The nominal yield is the weighted average rate of allocation.
[27] In FPV, T = 45. Since this number may be deemed somewhat small for a GMM estimation, we decided to take into account the information of all auctions in our dataset.

–0.1769, and -0.1265, respectively[28]. Although the first sign is not very intuitive, the last two are because we usually expect securities' value to increase as the secondary market price is higher and the nominal yield and maturity are lower.

### 4.2 Conditional mean and variance.

The average estimated expected value given the signals, $E\left(V_l \middle| S_{1l} = s_{il}, \ldots, S_{nll} = s_{nl}, Z_l = z_l\right)$, is equal to 0.9910 and the average value, $E\left(V_l \middle| Z_l = z_l\right)$, is equal to 1.0004. In turn, the average spread between $E\left(V_l \middle| S_{1l} = s_{il}, \ldots, S_{nll} = s_{nl}, Z_l = z_l\right)$ and the stop-out price is 0.0237, while the spread between $E\left(V_l \middle| Z_l = z_l\right)$ and the stop-out price is 0.0332[29]. Notice that it seems more natural that the spread increases with the good's value because, if the good is valuable, competition should be stronger and the resulting stop-out price should be lower. However, the data indicate that this is the case only for the average expected value.

### 4.3 Comparing revenues

The total hypothetical revenue obtained is 80,918.48 billion pesos, while the revenue observed in the discriminatory auction is 79,767.05 billion pesos. Hence, if the Federal Government had used the uniform price mechanism to auction its securities instead of the discriminatory price mechanism, it would have raised an additional 1,151.42 billion pesos, that is, the revenue would have been 1.44% higher.

In order to test the significance of these estimates, we calculate the bootstrapped confidence intervals of the difference in revenue per auction (Table 5)[30], and find a significant difference between the discriminatory and the uniform auction. The bootstrapped mean of the difference is approximately 6 million pesos, with an upper bound of 8.44 million pesos and a lower bound of 4.50 million pesos.

[28] These values are lower in magnitude than those calculated by FPV for the French securities auctions. The difference in magnitude of these results seems to be related to the magnitude of gamma and of the constants. For instance, both of the two gammas calculated in this exercise are higher than the one estimated in FPV.

[29] Estimations of $E\left(V_l \middle| S_{1l} = s_{il}, \ldots, S_{nll} = s_{nl}, Z_l = z_l\right)$, the secondary market prices, the stop-out prices, and $E\left(V_l \middle| Z_l = z_l\right)$ for all auctions, computed from the estimators obtained for each sample, are omitted for the sake of briefness but are available from the authors upon request.

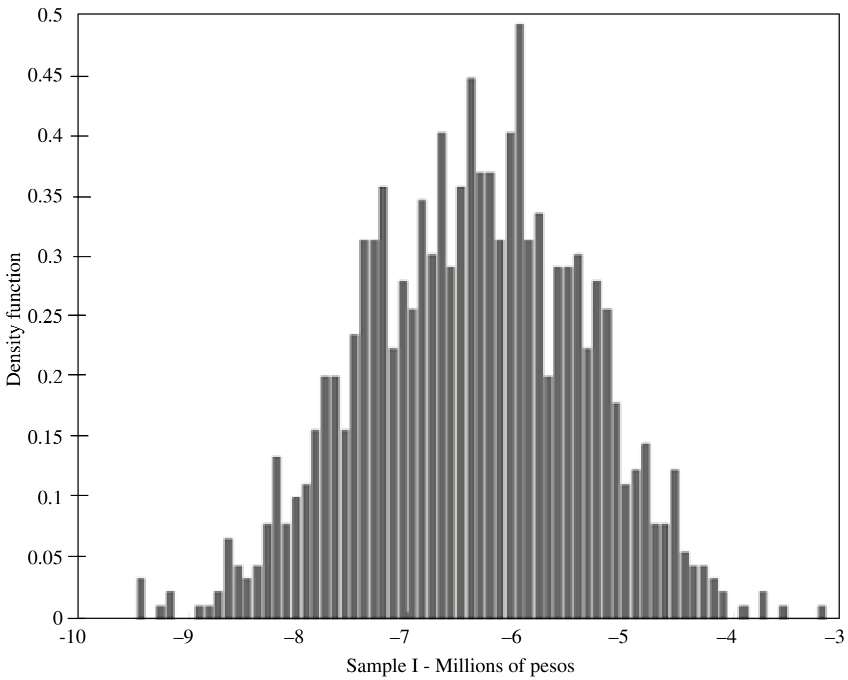
[30] The bootstrap procedure was carried out by making random sub-samples of the different auctions several times and calculating the difference in revenue each time given the estimators. With this, we were able to construct the distribution of the difference in revenue and its interval at 95% of this distribution. A difference in revenue equal to zero was the null hypothesis, which was rejected.

TABLE 5
MEAN OF DIFFERENCE IN REVENUE – DISCRIMINATORY MINUS ESTIMATED UNIFORM
(In millions of pesos)

| Mean | Mean Bootstrap | Confidence interval (95%) | |
|---|---|---|---|
| | | Lower bound | Upper bound |
| –6.3968 | –7.4383 | –8.4476 | –4.5051 |

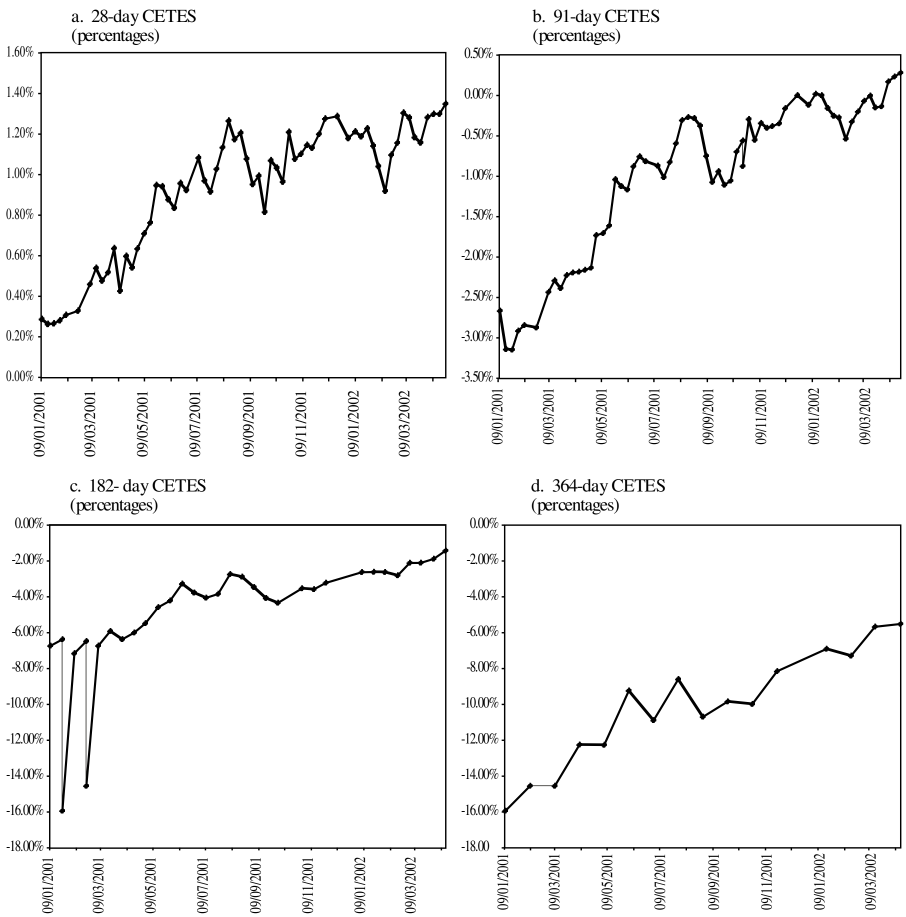
In addition, we calculated the bootstrapped interval several times and found that their figures do not change across calculations. The aggregated difference seems small, but it is considerably negative in each auction. The estimated density function for the difference in revenue is shown in Figure 1.

FIGURE 1
BOOTSTRAP DENSITY FUNCTION OF THE DIFFERENCE IN REVENUE BETWEEN THE DISCRIMINATORY AND THE UNIFORM AUCTION FORMATS
(Millions of pesos)



There are two other interesting features of these results. First, the uniform auction's revenue superiority differs across CETES with different maturity. Second, for all maturities this revenue superiority diminishes throughout the analysis period. Actually, in the case of the 28-day CETES, the discriminatory scheme obtains higher revenues than the uniform one. Benefits derived from the discriminatory auction increase through time, from 0.3% to 1.35%. For the rest of the CETES, the uniform price auction is higher in revenue. In the 91-day CETES auctions, the benefits involved in implementing a uniform price auction go from 2.66% to a loss of 0.28%. In the 182-day CETES auctions, this benefit falls from 7% to 1.4%. Finally, benefits from selling the 364-day CETES, when they are at their highest, decrease from 10% to 5% (See Figure 2).

FIGURE 2
DIFFERENCE IN REVENUE BETWEEN THE (OBSERVED) DISCRIMINATORY AUCTION AND THE (HYPOTHETICAL) UNIFORM AUCTION



The date after which the revenues from discriminatory auctions begin to rise in a noticeable manner is May 2001; this date corresponds to the implementation of new rules for non-competitive bidding in the market makers' buy option. Since we cannot explain this phenomenon within the frame of the analytical setting of this paper, let us only state a conjecture regarding the effects of this change. These new rules determine the maximum quantity allocation as a function of these agents' competitive bids submitted in the primary auction, featuring a small interval around the lowest price (highest rate) that receives an allocation at the primary auction, instead of the total amount allocated to it. Setting rules that promote stronger competition among market makers may have contributed to a more aggressive bidding in the primary auction and, therefore, the differential in revenue between the two auction formats may have fallen. Alternatively, the market making mechanism may have been becoming more effective in disseminating information across the secondary market.

## 5. Exploring the Role of Market Uncertainty

### 5.1 A comparison with the previous results for CETES and French Treasury securities auctions

In this model, one possible reason why the uniform price auction may produce higher sales revenue than the discriminatory one is that the conditional variance of the value obtained in this exercise is considerably higher than the one that FPV obtain. We may interpret this as a higher degree of uncertainty in the good's value, which would be a reason for more cautious bidding in the discriminatory auctions in Mexico than those in France. In this sense, the values of $\alpha_l$ and $\beta_l$ evaluated at the sample mean of $z$ can be seen in Table 6. It is important to remember that in this case $V_l^\gamma$ follows a gamma distribution with parameters $\alpha_l$ and $\beta_l$.

TABLE 6
$\alpha_l$, $\beta_l$, CONDITIONAL MEAN, VARIANCE AND VARIATION COEFFICIENT OF $V^\gamma$

| | $\alpha_l$ | $\beta_l$ | Mean ($\alpha_l/\beta_l$) | Variance ($\alpha_l/\beta_l^2$) | Variation coefficient |
|---|---|---|---|---|---|
| CETES auctions of January 2001-April 2002 | 125.68 | 117.97 | 1.0653 | 0.0090 | 0.0891 |
| Février,Préget, and Visser (2004) | 3045.04 | 848.72 | 3.5878 | 0.0042 | 0.0181 |

Note: Evaluated at the characteristics' sample mean.

According to the table's data, the distribution of $V_l^\gamma$ in our sample exhibits a higher variance than the one obtained in FPV. We can appreciate this higher dispersion in a better manner by looking at the coefficients of variation.

Therefore, we can state that the Mexican market shows more value uncertainty than the French market.

Let us now compare our findings with the previous ones of Umlauf (1993) and Laviada *et al.* (1997) for the 28-day CETES auctions. We construct the variance of the daily funding rate with government securities over the five-day period leading to and including the day on which the auction is conducted –that is, the variable used to proxy resale risk and information dispersion in those studies– for the periods examined in each of the three studies. For the first two periods, we calculate the revenue of the discriminatory format as the product of the amount issued times the average allocation price. Next, we estimate the revenue of the hypothetical uniform auction as the amount issued times the sum of the average allocation price plus the positive mark-up per bid in the uniform auction with respect to the discriminatory format reported by those authors. Finally, we subtract the former from the latter to obtain the difference in revenue. Table 7 shows a positive relationship between the gains of using the uniform format and market uncertainty; that is, the gain is positive in the auctions examined by Umlauf and Laviada *et al.*, and negative in those that we examined. In turn, we may connect this to higher market volatility in those samples than in ours.

TABLE 7
AUCTION REVENUE AND MARKET VOLATILITY COMPARISON WITH PREVIOUS REDUCED FORM ESTIMATIONS FOR 28-DAY CETES

| Analysis Date | Auction format dummy variable | Discriminatory Auctions' observed revenue (millions of pesos) | Uniform Auctions' Hypothetical Revenue (millions of pesos) | Revenue Difference (%) | Variance of the funding rate |
|---|---|---|---|---|---|
| Aug 1986- May 1991 (Umlauf, 1993) | 2.44$^{bp}$ | 74 | 74 | 0.000% | 0.160 |
| Jun 1995- Mar 1997 (Laviada *et al.*, 1997) | 18.96$^{bp}$ | 8,558 | 8,558 | 0.002% | 3.498 |
| Jan 2001-Apr 2002 (present) | – | 29,657,590 | 29,398,573 | –0.873% | 0.096 |

Next, we look for this same positive relationship in our cross maturity results. As different resale price volatility across CETES maturities are needed for this exercise, we construct the variance of resale price with the *Enlaces Prebon* CETES secondary market price index (IEP index)[31]. In addition, we only look

[31] Enlaces Prebon is one of the main inter-dealer brokerage firms operating in the Mexican Stock Market. The IEP index for CETES corresponds to the mean market interest rate at 12:15 a.m., determined through a survey to 12 participating institutions. The three highest and three lowest reported

at 17 auctions of each maturity date, because the 364-day CETES are auctioned monthly. Table 8 once again shows the positive relationship between the gains derived from using the uniform auction format and market volatility in CETES with maturities of 28, 91 and 182 days. We think that this is due to a low transaction volume problem affecting issues with longer maturities to a greater extent than the resale market for the latter securities that is, in fact, less uncertain than those of the shorter term maturities[32].

TABLE 8
AUCTION REVENUE AND MARKET VOLATILITY
COMPARISON ACROSS MATURITIES

| CETES Maturity | Discriminatory Auctions' observed revenue (millions of pesos) | Uniform Auctions' hypothetical revenue (millions of pesos) | Revenue Difference (%) | Variance of the IEP index |
|---|---|---|---|---|
| 28 days | 7,572.253 | 7,518.962 | –0.70% | 0.052 |
| 91 days | 8,564.527 | 8,690.297 | 1.47% | 0.064 |
| 182 days | 5,321.119 | 5,511.703 | 3.58% | 0.067 |
| 364 days | 7,141.314 | 7,807.403 | 9.33% | 0.046 |

## 5.2 Simulation exercise with noisier value signals

In this section, we test whether market uncertainty affects bidding within the structural model framework. First, we generate a more volatile series of the secondary market price, and use it to estimate stage 1 signals' distribution and to generate new model parameters.

The new series of secondary market prices is modeled as the observed secondary market price with an AR(1) process -conditional on the CETES maturity- plus *i.i.d.* shocks. This model yields a variance of shocks of 2.55 and an autoregressive parameter $\rho = 0.091$. Next, we use the AR(1) approximation method proposed by Tauchen (1986) to simulate 180 new data of the secondary market price. We assume that the new series has the same variance as that for the period between June 1995 and March 1997 analyzed by Laviada *et al.* (1997); that is, a variance of the daily funding rate equal to 3.49 according to Table 14.1. Though this variance is 55% higher than the one observed in our data set, we can still regard it as a conservative simulation in view of the Mexican market experience.

rates are eliminated, so the average rate is constructed from the remaining six reports. The index is constructed for CETES with 28, 91, 182, and 364 days maturity since June of 1996.

32 IEP indexes are perception indexes, not executable indexes (there is no intention to buy or sell securities at the quoted rates). While this is probably the only public source of CETES secondary prices that covers our analysis period, this aspect may be a disadvantage for our purpose.

Table 9 shows the resulting parameters. We can observe a higher estimated value of the parameter $\gamma$, which can be interpreted as consistent with a setting in which the bidders face less informative signals. In fact, as the value of $\gamma$ increases $V^{\gamma}$ decreases (recall that $0 < V < 1$), the distribution of signals, $F_{S|V,Z}\left(s\,|\,v_l, z_l; \theta_2\right) = 1 - \exp\left[-s v_l^{\gamma}\right]$ collapses. On the other hand, as this happens we would expect higher revenues from the uniform price. This is precisely what we find: the new total hypothetical revenue obtained from the uniform auction now is 81,506.33 billion pesos, which not only is 2.1% higher than the revenue observed from the discriminatory auctions, but it also exceeds by 0.7% the uniform auction revenue that was obtained before.

TABLE 9
SECOND STAGE ESTIMATE OF $\theta$ USING A SIMULATED SECONDARY MARKET PRICE SERIES DISTRIBUTED WITH MEAN 3.70 AND VARIANCE 3.49

| Estimate of alpha: | Coefficient | Standard Error |
|---|---|---|
| Constant | 327.1935 | 0.00000015 |
| Secondary market price | 162.8496 | 0.00001419 |
| Nominal yield | 20.2399 | 0.00000151 |
| Maturity | 447.9777 | 0.00000002 |
| Estimate of beta: | | |
| Constant | 5.4844 | 0.00003000 |
| Secondary market price | 34.3807 | 0.00290531 |
| Nominal yield | 82.8601 | 0.00030871 |
| Maturity | 2031.6185 | 0.00000309 |
| Gamma | 745.6563 | 14.10575237 |

## 6. CONCLUSIONS

The share auction framework supporting the structural estimation method of FPV seems to provide an adequate characterization of Mexico's CETES auctions during the period under analysis. Although the coefficients are significant and have a plausible size, the estimated value of the securities does not seem to be too sensitive to changes in the auction characteristics considered. Moreover, the sign of some of the coefficients are not very intuitive and differ across the samples analyzed. While some small sample bias may explain these findings, a selection criterion may be needed to enable us to choose from among a set of several possible exogenous variables those that can best describe the auction heterogeneity. Such development may contribute to raise the power of the estimation procedure and extend its applicability to other securities for which there is less data available than in the case of the zero coupon bonds, particularly in what respects market prices. On the other hand, FPV's estimation approach relies on specific distribution functions that yield tractable analytical solutions. However, despite this we think that experimenting with other parametric specifications of the value and the individual signals may also be desirable in the future, both to assess the method's robustness and, more generally, to study other game theoretic models for which no explicit strategies can be found (due to either complexity or lack of data).

Our results indicate that the uniform price auction produces higher CETES sales revenue than the discriminatory price auction during the period studied. The difference in revenue is 1.68% and is statistically significant. However, some back-of the envelope calculations could well be enough to give us some grasp of this finding's economic significance. The average issue size of the sample constituted by 180 auctions is 4,500 million pesos. This is equal to 2.5% of the overall outstanding CETES debt issued by the Mexican Treasury during the period under analysis (that is, each week the CETES debt is adjusted by 2.5%). Outstanding CETES debt represents 3% of Mexico's GDP. Therefore, the difference in revenue is grossly 0.22% of GDP (0.03*0.025*0.0168*180). Coincidentally, according to the Mexico's Ministry of Finance, the primary fiscal deficit target for the end of the year stated in the 2005 Economic Program is precisely 0.22% of GDP![33] This evidence confirms the previous estimations with reduced form estimations for 28-day CETES auctions and provides a robustness check on FPV's structural estimation method (which obtained the opposite revenue ranking for French Treasury securities). In addition, we find new evidence that suggests that CETES' market volatility has diminished across the analyzed episodes, which would diminish some of the benefits of using the uniform auction format instead of the discriminatory one[34, 35]. As a result, the difference in revenue between auction formats that we find in this study is lower than in Umlauf's or Laviada's.

We also detect that the difference in revenue between the two auction formats varies across CETES with different maturities. The discriminatory format produces higher revenue than the uniform format in the 28-day securities auctions, while the uniform format produces higher revenue than the discriminatory format in the 91, 182, and 364 -day securities auctions. This positive relationship between the gains of the uniform format and the securities' maturity coincides

[33] Source: 2005 Economic Program, Ministry of Finance, Mexico, January 3, 2005.

[34] Besides market uncertainty, another common argument for the existence of a difference in revenue across the discriminatory and the uniform auctions of treasury securities refers to the possibility of collusion. Notice that in the case of Mexico, the existence of resale price uncertainty is very agreeable with this study's results. Although we do not look directly into the issue of collusion in repeated auctions, the usual argument is that the uniform format is more susceptible to this problem, which would predict the opposite differences in revenue that we would get given the frequency, size, and bidder participation conditions of the CETES auctions (see Table 3). Quantity uncertainty, another common argument for auction differences in revenue, is not an applicable argument in this case either, due to the fact the Mexican Treasury has been publishing, for some years already, a quarterly debt issuance calendar in advance.

[35] In fact, according to a recent BIS report decreasing bond market volatility is a trend that is observed not only in Mexico, but also among several other emerging economies (for more details, see "Financial stability and local currency bond markets", Committee on the Global Financial System Papers No 28, BIS, 2007, or Eichengreen, Borensztein and Panizza, 2008).

with the practice, by the Mexican Treasury as well as by other countries' debt issuing agencies, of selling securities with short maturity with the discriminatory format and long-term securities with the uniform auction format[36].

Finally, the results obtained in the simulation exercise shed some light on the relevance of market uncertainty when a country chooses an auction technique to sell bonds. In this particular example, a bond sale in a market with higher level of uncertainty, depicted by private signals featuring higher variance, will yield more revenue under a uniform format. However, within the frame of a particular technique, it may be interesting to consider uncertainty with respect to other characteristics, such as, for instance, the issue size or maturity. Some recent theoretical models have not only proposed that quantity uncertainty can affect revenue ranking between uniform and discriminatory format, but also that some debt issuing agencies deliberately introduce quantity uncertainty in their auctions; this is because as bond markets develop, issuing agencies are moving towards publishing their calendars in advance. Furthermore, some securities are being issued with greater frequencies than others. While these aspects are perceived as desirable, the methodology used in this paper, that is, estimations with FPV's structural model complemented by the kind of simulation exercise that we performed in section 5.2, may produce some evidence as to the extent to which such aspects may matter. These empirical results set a useful baseline for further research in other developing markets.

## REFERENCES


Armantier, O. and E. Sbai (2003), "Estimation and Comparison of Treasury Auction Formats when Bidders are Asymmetric", mimeograph, State University of New York at Stony Brook and GREMAQ, Université de Toulouse, May 2003.

Back K. and J.F. Zender (1993), "Auctions of divisible goods: on the rationale for the treasury experiment". *Review of Financial Studies* 6 (4): 733-764.

Bartolini, L. and C. Cottarelli (1994), "Treasury Bill Auctions: Issues and Uses", IMF Working Paper 94/135.

Bikhchandani, S. and C. Huang (1993), "The economics of Treasury securities markets", *Journal of Economic Perspectives*, 7, 117-134.

Castellanos, S. and M. Oviedo (2004), "Análisis de las posturas óptimas en las subastas primarias de títulos gubernamentales: resultados de un enfoque econométrico estructural", Documento de Investigación. Nº 2004-7.

Department of the Treasury, Securities and Exchange Commission, and Board of Governors of the Federal Reserve System, *Joint Report on the Government Securities Market*, United States of America, January 1992.

Eichengreen, B., E. Borensztein and U. Panizza (eds.) (2008), "Bond Markets in Latin America: On the Verge of a Big Bang?", MIT Press, forthcoming.

Février, P., R. Préget and M. Visser (2004), "Econometrics of Share Auctions", mimeograph, University of Chicago and INRA.


[36] Sareen (2004) provides a brief cross-country survey of debt issuing practices.

Gordy, M. B. (1996), "Hedging Winner's Curse with Multiple Bids: Evidence from the Portuguese Treasury Bill Auction", Bank of Portugal, Research and Statistics Department WP 21-96, Dec. 1996.

Hortacsu, A. (2002), "Mechanism Choice and Strategic Bidding in Divisible Good Auctions: An Empirical Analysis of the Turkish Treasury Auction Market", mimeo, University of Chicago.

Hortacsu, A. (2002a), "Bidding Behavior in Divisible Good Auctions: Theory and Evidence from the Turkish Treasury Auction Market", mimeo, University of Chicago.

Laviada, B. and F. Laviada (1997), "Análisis del Mecanismo de Subastas de Valores Gubernamentales en México", Tesis de Licenciatura en Economía, Universidad Iberoamericana.

Kang, B. S. and S. L. Puller (2007), "The Effect of Auction Format on Efficiency and Revenue in Divisible Goods Auctions: A Test Using Korean Treasury Auctions", mimeograph, Texas A&M University.

Mazón, C. and S. Núñez (1999), "On the optimality of treasury bond auctions: the Spanish case" Bank of Spain, Research Document Nº 9905.

Malvey, P., C. Archibald and S. Flynn (1995), "Uniform-Price Auctions: Evaluation of the Treasury Experience". Report of the U.S. Department of the Treasury.

Malvey, P. and C. Archibald (1998), "Uniform-Price Auctions: Update of the Treasury Experience". Report of the U.S. Department of the Treasury.

Newey, W. K. and D. L. McFadden (1994), "Large sample estimation and hypothesis testing", in R. F. Engle and D. L. McFadden, eds., *Handbook of Econometrics*, Vol. 4, 2113-2245.

Orellana, W., B. Fernández and V. Fernández (2007), "Subasta electrónica interactiva y subasta a sobre cerrado: Un análisis comparativo de los resultados en Bolivia", Documento de Investigación, Banco Central de Bolivia.

Pagan, A. and A. Ullah (1999), *Nonparametric Econometrics*. Cambridge University Press.

Sareen, S. (2004), "Cross-Country Comparison of Models for Issuance of Government Securities", Mimeo. Department of Economics Duke University.

Tauchen, G. (1986), "Finite State Markov Chain Approximations". *Economic Letters*, 20, 17-181.

The World Bank and The International Monetary Fund (Editors), "Developing Government Bond Markets A Handbook", 2001.

Umlauf, S. R. (1993), "An empirical study of the Mexican Treasury bill auction", *Journal of Financial Economics*, 33, 313-340.

Wilson, R. (1979), "Auctions of Shares", *The Quarterly Journal of Economics*, Vol. 93, Issue 4, 1979, 675-689.