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FINANCIAL INNOVATION IN MULTI-PERIOD ECONOMIES*

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ABSTRACT

I present an attempt to construct multi-period, finite horizon extensions to the well -known two- period financial innovations literature. I first extend the definition of competitive equilibrium with innovations. It is shown that, with a dominating household type, it is impossible to observe a set of complete Arrow securities with a positive amount issued for each of them in equilibrium, either at the last date or at any date.

RESUMEN

En este trabajo se presenta una primer extensión del modelo standard de equilibrio general con innovación financiera endógena a economías de más de dos períodos (con horizonte finito). En primer lugar se extiende la definición de equilibrio competitivo con innovaciones financieras, incluyendo la posibilidad de introducción de nuevos productos financieros en períodos posteriores al inicial. Se demuestra que, si existe un tipo dominante de inversor (con la mayor tasa marginal de sustitución de todos los consumidores), es imposible observar en equilibrio un conjunto completo de activos de Arrow ofrecidos por los innovadores, tanto en el último período como en cualquier otro intermedio.

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1. Introduction

The question of how financial structure arises endogenously is still a puzzling feature in economic theory. The traditional incomplete markets literature assumes an exogenous financial structure[1]. Nevertheless, the empirical evidence about the continuous financial innovation in the past two decades implies that the traditional approach cannot tell much about why it was observed an impressing amount of new financial products in the last twenty years. This evidence is summarized by Duffie and Rahi (1995). Hence, a new theory that makes the asset structure in some way endogenous is needed to explore the reasons for this evidence.

Duffie and Jackson (1989) constitutes the first paper known in this area. It presents a problem of innovation in futures. They assume that the objective of the exchange is to maximize volume, so that the amount of commissions paid to each of the exchange members is maximized. They evaluate different possible Nash equilibria for the innovation problem. The main result is that the optimal monopolistic contract in a two period economy is Pareto optimal. They also construct an example of a Nash equilibrium in which the contract design does not lead to a Pareto efficient allocation. They present another example for a multi-period economy in which the monopolistic case can be Pareto suboptimal as well. In any of these cases, though, the main assumption is volume maximization as the innovator's objective, with the implicit assumption of having commissions that are positively related with the amount of trade. But this last feature was not explicitly modeled.

Parallel to this last framework, the early works by Allen and Gale (1988, 1991, 1994) focus on models in which the innovators explicitly maximize some profit function and also consumers maximize their utilities subject to standard budget constraints in equilibrium. They add short sales restrictions as incentives to issue new assets. They model production economies in which the firms issue securities to finance production plans. In Allen and Gale (1988), where no short sales are allowed, they prove the existence and constrained efficiency of the competitive equilibrium with innovation, in an economy with only one type of firm. In that case, under suitable conditions, they show that any set of innovated assets is equivalent to a pair of "extreme securities". In Allen and Gale (1991), they extend their result to an environment with more than one type of firm, departing from the perfect competitive analysis to get into more strategical considerations. This actually allowed for short sales in equilibrium.

Several new approaches were then developed in pure-exchange and production economies. Some of the papers explore the need for collateral in the process of innovation. Important contributions were Pesendorfer (1995) and Chen (1995). Pesendorfer provides a definition of a *competitive innovation equilibrium* and proves its existence. Under this equilibrium definition, households and

[1] For a review of the standard GEI literature, see Magill and Quinzii (1996) for an introductory level, and Magill and Shaffer (1991), for a more advanced treatment.

innovators optimize their respective objective functions while markets for goods, standard securities and innovated securities clear. He also shows the constrained efficiency of the equilibrium, as well as the possibilities of having redundant newly issued assets that do not span the state space. One of the most important characteristics of his model is the non-linearity of equilibrium asset prices (for the newly introduced financial products). This is a consequence of (fixed) marketing costs of those new assets.

On the other hand, Chen (1995) characterizes the pricing of innovated securities, showing that they are sub-linear in the payoff structure. He also shows through examples why there may be such incentives to introduce new securities and which role the “trading frictions’’ play in financial innovation (generalizing the no-short sales case done by Allen and Gale, 1988). He detects cases in which there can be profitable innovation, introducing a very small amount of a new financial product into the market.

More recently, an asymmetric –information – based security design literature has been developed. The paper by Rahi (1996) studies the problem of an informed risk-averse entrepreneur that decides the type of asset to issue. The main result is that the insider does not float a security that gives the entrepreneur any informational advantage. In other words, the asset is such that a fully revealing rational expectations equilibrium arises. This result is mainly driven by the presence of adverse selection in the secondary market. Another important contribution along this line is the work by DeMarzo and Duffie (1999). In this paper the issuer is also an insider. However this agent needs to back the security with specified assets. On the other hand, inside information may cause illiquidity in the sense of a downward sloping demand curve for the innovation. Thus the design of the security implies a trade-off between cost of retention of cash non included in the asset and the liquidity cost of including those cash flows.

On the other hand, there exists a branch of the more traditional GEI literature where the focus is the effect of an *exogenous* introduction of a new security on equilibrium prices and allocations. This has mainly been motivated by the example in Hart (1975) who shows a case in which introducing an asset in an incomplete markets economy actually implies a Pareto worsening move. This literature then is mainly interested in showing how *special* was that Hart’s example. Some representative work on this has been done by Elul (1995, 1999), Cass and Citanna (1998) and Calvet*et al.* (2000). However, as stated above, all these papers take the financial structure as exogenous and only considers exogenous introductions of assets.

All the approaches mentioned in the previous paragraphs were studied in two-period economies. The obvious problem in such frameworks is that there is no way of differentiating “short - term’’ from “long-term’’ securities. Real-world capital markets, on the other hand are characterized by the presence of very different maturity assets. Moreover, Duffie and Rahi (1995) present some evidence of financial innovation in real world markers. Those new assets have had very different maturities. It is clear that a model with a two period economy is insufficient to study the economic rationale for these different lengths and maturity dates of financial innovations.

In this paper, I attempt to present a first extension of the two-period analysis to multi-period economies. In a finite horizon economy which last at least for three periods, there is a potential for a richer variety of innovations. My first goal is to introduce an equilibrium definition, named *multi-period financial innovation* equilibrium. Here consumers choose optimally their portfolio of a riskless bond and new financial products at each node of the economy, subject to short sale constraints (as in Allen and Gale, 1988). In this equilibrium the so-called financial innovators (whose profits are derived from issuing new financial products) also maximize their discounted expected benefits. The innovators may intervene in the riskless bond market, purchasing part of the bonds that consumers have (as part of their endowment) and "spliting them up'' into new securities, similar to Pesendorfer (1995). The innovators have limited liability. Then at every period and every history profits must be non-negative. Innovators can purchase the riskless bond and pay dividends on old issues by either issuing more new assets or getting the payoffs from the last-period riskless bond holdings. The innovator cannot borrow.

This extension is not trivial at all, and so only some very preliminary results are obtained within this framework. I first check that with risk-neutral consumers, there is no room for innovation. The intuition is obviously that the main objective of issuing new securities in this economy is to improve risk-sharing of consumers. Then, we could expect that risk neutral consumers have just no need of risk sharing at all. Hence an introduction of a new asset (unless it gives a higher expected return) does not improve welfare for any investor.

The remaining results show sufficient conditions on consumption allocations such that they cannot be supported by a complete set of Arrow securities. It is well known that this type of asset structure is the simplest example of contingent contracts specified in many different applications in macroeconomics and finance. In particular, if one type of consumer has the highest intertemporal marginal rate of substitution for each state, then the equilibrium with endogenous financial structure cannot be supported by a complete set of Arrow securities. If those marginal rates of substitutions are high enough, then there can be no Arrow security issued in such an equilibrium. These results constitute at least a *guide* that provides a better understanding of how the equilibrium consumption allocation cannot look like if Arrow securities are available, a fact that was ignored in the traditional literature with exogenous set of Arrow securities. However, this proposition is also certainly special. It still leaves open the question of what the conditions must be in order to obtain allocations that can be supported by some arbitrary asset structure that at the same time is optimal for the innovator.

It is important to note that there has been some recent contributions that refer to financial innovation in multi period economies. The work by Bettzuge and Hens (2001) consists in a model which is a sequence of static CAPM economies (a sequence of successive generations who live only two periods). The main focus of the authors is the survival of newly introduced assets in the long run. They analyze this by assuming some transition function mapping participation rates from period t to participation rates in t+1. This provides an equilibrium evolutionary process governing the amount of trading in each new asset. The model however does not model explicitly how the assets are introduced. A paper by Willen (1999) provides a CARA-Normal framework with finite horizon to evaluate the welfare effects of financial innovation. Apart from the CARA-Normal assumption, this paper still assumes an exogenous introduction of new assets.

In section 2 the economy is introduced, while in section 3 the equilibrium concept is defined. In section 4 the results for risk neutral consumers are presented. In section 5 the characterizations of consumption allocations that cannot be supported by Arrow securities are provided. Finally, in section 6, conclusions and possible directions for future research are proposed.

2. The Economy

Assume a pure - exchange economy that lasts for T+1 discrete time periods, where $2 \leq t < \infty$. At every period $t \geq 1$, there exists a random variable whose realization is denoted by s_t, that take values on a finite set S. There is a unique non-storable consumption good at each period, whose spot price is normalized to one. In the first place I introduce the features about the financial structure and in particular the characteristics of innovators.

2.1. Standard securities

Following Pesendorfer (1995), I assume that there is a riskless bond. Note that I simplified the original structure, since he assumed a more general set of standard securities. In the present structure, the bond lasts for the T period. At each particular node s^t the standard security pays off exactly one unit of the consumption good. I denote $q(s^t)$ as the price of the standard asset at period t and history s^t.

2.2. Financial Innovators

Assume a continuum of identical financial innovators, so that we do not have to consider an inter-dealer financial market (the one in which only innovators will trade). The representative innovator can issue at every node s^t a number $K(s^t)$ of assets. In equilibrium $K(s^t)$ must be such that no innovator at any of the nodes has an incentive to introduce an additional security. Each product issued at s^t is denoted as $k(s^t)$. With an abuse of notation, I also denote $K(s^t)$ as the set of innovations produced at node s^t. Each product $k(s^t)$ gives payoffs in consumption goods at every possible subsequent node, denoted as $d_{k(s^t)}(s^t, s^j)$, where

$j \leq t+1$. I assume that $d_{k(s^t)}(s^t, s^j) \geq 0$ for all t and j. Note that from here we can define an asset by its length from the dividend structure. I say that an asset issued at node s^t lasts for τ periods, where $t + \tau \leq T$, if for any $r \geq \tau + t + 1$, we have that $d_{k(s^t)}(s^t, s^{r-t}) = 0$. This means that the only possible periods in which this asset gives positive payoffs are t+1, t+2, ... , τ. In the definition of equilibrium I will be using, this is enough to characterize the length of an asset. Since there is a problem of indeterminate scaling, I assume, for every s^t, s^j, for every t and j:

$$(1) \qquad \sum_{s \in S} d_{k(s^t)}(s^t, s^j, s) \leq B$$

where B is some large positive integer. Note that this includes the case of no innovation:

$$(2) \qquad d_{k(s^t)}\left(s^t, s^j, s\right) = 0, \quad \forall_s \in S$$

An asset structure (or innovation structure) is defined as a collection of dividend matrix payoffs in the following way. I denote D^0 as the collection of matrix payoffs of the innovations issued at period 0, that is:

$$(3) \qquad D^0 = \left\{ D_1^0 ; \left\{ D_{s^1}^0 \right\}_{s^1 \in S}, \ldots, \left\{ D_{s^{T-1}}^0 \right\}_{s^{T-1} \in S^{T-1}} \right\}$$

Here $D_{s^t}^0$ denotes the dividend payoffs in period t+1, if the particular history s^t, up to period t, is realized, paid by the innovations issued at 0. The elements $D_{s^t}^0$ are K(0) x S matrices whose entries satisfy the normalization restrictions above. The elements of each row either add up to one or are S-dimensional 0 vectors. In a similar way:

$$(4) \qquad D\, s^t = \left\{ D_1^{s^t} ; \left\{ D_{(s^t, s)}^{s^t} \right\}_{s \in S} ; \ldots ; \left\{ D_{(s^t, s^{T-t-1})}^{s^t} \right\}_{s^{T-t-1} \in S^{T-t-1}} \right\}$$

denotes the collection of $K(s^t)$ x S matrices with the payoffs corresponding to the innovations issued at date t. Then a financial structure is given by

$$(5) \qquad D = \left\{ D^0, \left\{ D^{s^1} \right\}_{s^1 \in S}, \ldots, \left\{ D^{s^2} \right\}_{s^2 \in S^2}, \ldots, \left\{ D^{s^{T-1}} \right\}_{s^{T-1} \in S^{T-1}} \right\}$$

Note that since each matrix $D^{s^t}_{(s^t,s^j)}$ has rows that lie on a compact subset of $\Re_+$, then the matrix $D^{s^t}_{(s^t,s^j)}$ is compact on $\Re_+^{K(s^t)xS}$, and so D^{s^t} is a compact subset of $\Re_+^{K(s^t)xSx(1+S+S^2+..+S^{T-1})}$.

Let us denote $\theta_{k(s^t)}$ as the total number of shares of security $k(s^t)$ issued at s^t. I assume that this amount is a non-negative real number. I also assume that issuing $\theta_{k(s^t)}$ shares at s^t implies a cost equal to $C\left[D^{s^t}\right]\theta_{k(s^t)}$. Here C is a continuous function mapping $\Re_+^{K(s^t)x(S+S^2+..+S^{T-t-1})}$ to $\Re_{++}$. This cost function represents the "issuing technology'' possessed by the innovator. It states how much resources should be devoted to place a new financial product in the market. These resources include items such as research expenditures, marketing expenditures and similar types. This is a special case of the cost function in Pesendorfer (1995), since he assumed that the total issuing costs were decomposed in two parts, one independent of the amount of payoffs (associated with marketing costs) and the other component exactly as in our case. Here I am not dealing with fixed marketing costs, although its inclusion is a feature that could be explored in the future.

I define the price of an asset issued at period t in the market at period t+j as $r_{k(s^t)}(s^t,s^j,D^{s^t})$. This defines the number of goods that an agent must give up at node s^{t+j} to buy one unit of the $k(s^t)$ asset. The way this is given is described below, but it is essentially the marginal willingness to pay of the "most interested'' consumer type. This means that the price for a potential new financial products is given by the household who values it more, that is, the one with the highest intertemporal marginal rate of substitution adjusted by the payoffs d. This follows the classical "rational conjecture'' adopted by most of the literature.

On the other hand, I assume that the innovator backs the new issues using the standard riskless security. Denote $\psi(s^t)$ as the portfolio of the riskless bond traded by the innovator at node s^t. This is the only extra-source of income perceived by the innovator in order to finance the new financial products.

2.3. Households

I assume a continuum of households. There are I types, each measure 1. I also assume that $I < \infty$. At every node s^t, the agent receives an endowment in consumption good equal to $w^i(s^t) > 0$. Let us assume for simplicity that for every t, every st, and for all i, we have $\sup_i \left|\omega^i(s^t)\right| \leq M$, for some large but finite positive

M. I also assume that each household has a standard Von-Neumann-Morgestern, time-state separable utility function:

(6) $$U^i = \sum_{t=0}^{T} \beta_i^t \sum_{s^t \in S^T} u_i\left(c^i\left(s^t\right)\right)\mu_t\left(s^t\right)$$

where $\mu_t(s^t)$ denotes the probability of occurrence of history s^t, and $c^i(s^t)$ denotes the consumption of type i household at node s^t. I will suppose for simplicity that $\mu_t(s^t)$ is common across types for any event s^t, that is, rational expectations hold all the time. The per-period utility given by $u^i(c^i(s^t))$, strictly is C^2, strictly increasing, strictly concave and satisfies the usual Inada conditions for every i. Hence I assure interior solutions for at least one agent.

At every node s^t, the type i consumer can trade in the spot market for the consumption good, and also in all open asset markets. In this last case the i-th household can trade the riskless standard security and also the set of existing innovations up to and including the node s^t. I denote $z^i(s^t)$ as the portfolio of the standard security and denote $y^i_{k(s^t)}(s^{t+j})$ as the portfolio of the innovation $k(s^t)$ issued at s^t held by i at node s^{t+j}. An important assumption done here is that:

(7) $$z^i(s^t) \geq -Z$$

(8) $$y^i_{k(s^t)}\left(s^{t+j}\right) \geq 0$$

with $Z > 0$ and for all i, s^t, $k(s^t)$, and s^{t+j}. This is essentially the same framework as in Chen (1995) . Notice that this includes the absence of short sales, as in Allen and Gale (1988). The presence of short sales constraints in competitive markets would be especially important for existence of equilibrium. Since the objective of the paper does not include showing the existence of an equilibrium I defer this task to future research.

The household faces the following sequence of budget constraints.

(9) $$c^i(s^0)+q(s^0)z^i(s^0)+r_{k(s^0)}(s^0)\cdot y^i_{K(s^0)}(s^0) \leq \omega_0^i$$

(10) $$c^i(s^t)+q(s^t)z^i(s^t)+\sum_{j=0}^{t} r_{K(s^j)}(s^t)\cdot y^i_{K(s^j)}(s^t)$$

$$\leq \omega^i(s^t)+z^i(S^{t-1})\left[q(s^t)+1\right]+\sum_{j=0}^{t-1}\left[r_{K(s^j)}(s^t)+d_{K(s^j)}(s^t)\right]\cdot y_{K(s^j)}(s^{t-1}),$$

$$t \leq T-1$$

and for T:

$$(11)\qquad c^i(s^T) \le \omega_i(s^T) + z^i(s^{T-1}) + \sum_{j=0}^{T-1} d_{K(s^j)}(s^T)\cdot y_{K(s^j)}(s^{T-1})$$

where the inner product for the innovations denotes the sum over all innovations of the values of the portfolios and payoffs.

2.4. Prices for innovations

The innovators must "guess'' the price for their products whenever they put them in the market. That is, the price $r_{k(s^t)}(s^t, D^{s^t})$ must be inferred by the innovators, since they are not marketed assets at the time of the issue. There is no past information based on the market for a new issue. As in the literature, I use the assumption of *rational conjecture* for the price $r_{k(s^t)}(s^t)$, which is usual in the literature. The innovators guess exactly what the price is in equilibrium. In our case, the rational conjecture takes the following form.

(12)

$$r_{k(s^t)}(s^t, D^{st}) =$$

$$\max_{i\in I}\left\{\beta_i \sum_{s\in S}\frac{u_i'(c^i(s^t,s))}{u_i'(c^i(s^t))}\left[d_{k(s^t)}(s^t,s)+r_{K(s^t)}\left(s^t,s,D^{s^t}_{s^{t+1}},\dots,D^{s^t}_{s^{T-1}}\right)\right]\frac{\mu_{t+1}(s^t,s)}{\mu_t(s^t)}\right\}$$

This holds for all s^t, and all t. The dependence of r with respect to D^{st} comes also through how $r_{k(s^t)}(s^{t+1}, D^{s^{t+1}})$ depends on $D^{s^{t+1}}$[2]. This rational conjecture equilibrium holds for all the periods, so that in the last period:

(13)

$$r_{k(s^t)}(s^{T-1}, D^{s^t}_{s^{T-1}}) = \max_i\left\{\beta_i \sum_{s\in S}\frac{u_i'(c^i(s^{T-1},s))}{u_i'(c^i(s^{T-1}))}\left[d_{K(s^t)}(s^{T-1},s)+r_{K(s^t)}\right]\frac{\mu_T(s^{T-1},s)}{\mu_{T-1}(s^{T-1})}\right\}$$

[2] One could argue that the innovators could also change the price by changing the quantity of the new asset. Nevertheless, this contradicts assumption of perfect competition, which assumes that no individual agent can change the price with its own quantity decision. The reason of why the individual dividends do affect the price is that, changing the payoffs, an individual innovator could attract a large number of investors (as long as the change in payoffs imply a rise in utilities for investors), increasing then the profits for the new assets producers.

3. EQUILIBRIUM

The equilibrium concept presented here is just an extension of the Allen and Gale (1988) and Pesendorfer (1995) definitions, in which all the consumers optimize, the "representative" innovator optimize and markets clear.

In equilibrium, each innovator will be solving the following problem:

$$(14) \quad \max_{\theta,D,\psi} \left\{ \sum_{t=0}^{T} \delta^t \sum_{s^t} \left[\pi(s^t) \mu_t(s^t) \right] \right\}$$

subject to:

$$(15) \quad \pi(s^t) \geq 0$$

for any t, any s^t, where:

$$(16) \quad \pi(s^t) = \left[r_{K(s^t)}(s^t, D^{st}) - C(D^{st}) \right] \cdot \theta_{K(s^t)} + \psi(s^{t-1})(1+q(s^t)) - \sum_{j=0}^{t-1} \theta_{K(s^j)} \cdot d_{K(s^j)}(s^j, s^{t-j}) - q(s^t)\psi(s^t)$$

for all $s^t, 0 \leq t \leq T-1$ and:

$$(17) \quad \pi(s^T) = \psi(s^{T-1}) - \sum_{j=0}^{T-1} \theta_{K(s^j)} \cdot d_{K(s^j)}(s^j, s^{T-j})$$

for all s^T. Here $\pi(s^t)$ denotes the period t, node s^t payoff of the innovator. She chooses a collection of matrices D satisfying the restrictions above:

$$(18) \quad \sum_{s \in S} d_{K(s^t)}(s^t, s^j, s) \leq B$$

for some large B >I, and for any s^t, s^j, t and j. Also, let us restrict the choice of θ and ψ on compact sets Θ and Ψ such that in fact, in equilibrium, the compactness will not be an issue.

The restriction (15) for the one-period innovator 's profits is called *delivery constraint*. This implies that the innovator must pay the purchases of riskless bond portfolios and the dividends to the owners of the innovators delivering the corresponding amount of the consumption good. This at the same time comes from the payoffs of the riskless bonds purchased one period before and the revenue for the new innovations at any node s^t. In a sense, this constitutes a short sale constraint to the innovators.

The households will be maximizing their utility functions (6) subject to the budget constraints (9), (10) and (11). This is the household problem.

The market clearing conditions are given by:

(19) $$\sum_{i=1}^{I} z^i(s^t) + \psi(s^t) = 0, \quad \forall s^t$$

(20) $$\sum_{i=1}^{I} y^i_{K(s^t)}(s^j) = \theta_{K(s^t)}, \quad \forall s^t, \forall s^j, j \geq t$$

in the asset markets and

(21) $$\sum_{i=1}^{I} c^i(s^t) = \sum_{i=1}^{I} \omega^i(s^t)$$

in the goods market.

This allows to define a competitive equilibrium with financial innovation.

Definition 1

A competitive equilibrium with financial innovation is a set of consumption allocations $\{c^i(s^t)\}_{i,t}$, a set of portfolios held by consumers, $\left\{z^i(s^t), y^i_{K(s^t)}(s^j)\right\}$, a collection of portfolios of standard securities held by the innovators $\{\psi(s^t)\}$, an amount of innovated assets, $\left\{\theta_{K(s^t)}\right\}_{s^t}$, a collection of payoff matrices D, and a price system $q(s^t)$, $r_{K(s^t)}(s^j)$, $j \geq t$, such that

(1) The allocation, $\{c^i(s^t)\}_{i,t}$ $\left\{z^i(s^t), y^i_{K(s^t)}(s^j)\right\}$ solves the household's problem as stated before, taken as given the price system and the dividend payoff matrices for each asset.

(2) The collection $\{\psi(s^t)\}$, $\left\{\theta_{K(s^t)}\right\}_{s^t}$ and D maximizes the representative innovator objective function, also taking as given the price system.

(3) Market clearing conditions hold.

Remark 1

Though this concept of equilibrium is close in spirit to the one given in Pesendorfer (1995), I do include no marketing plan. The reason is that, due to the assumption that each type of consumer has Lebesgue measure one, then each type will be choosing a portfolio y^i of innovated securities, and so the marketing plan is trivial: in Pesendorfer's case, the marketing plan was induced by the Lebesgue measure on the space of consumers. But in our case that is just one for each i. Then we do not need to introduce any other element in the definition. Second, we use the pricing rule by Allen and Gale (1988). This is because we are assuming the absence of fixed cost of marketing as it was assumed in Pesendorfer. This means that there are no complementarities between assets. This makes simple the pricing for innovated securities, and it can be shown that sub-linearity (see Chen, 1995) is also true even for the multiperiod case.

4. The Case of Risk Neutral Households

When the households have linear utility functions, then the pricing of innovations becomes straightforward. Indeed, for an innovation issued at node s^t the following is true.

$$(22) \qquad r_{K(s^t)}(s^t, D^{st}) = \sum_{j=0}^{T-t} \bar{\beta}^j \sum_{s^{t+j}} d_{K(s^t)}(s^{t+j})\mu_t(s^{t+1})$$

where

$$(23) \qquad \bar{\beta} \equiv \max_{i \in I} \{\beta_i\}$$

and $\mu_t(s^{t+1})$ denotes the conditional probability for the event s^{t+1} conditioned on information available up to time t.

From this it is clear that the price inferred by the innovator is linear in payoffs. Then the total per-period return for the innovator is as follows.

$$(24) \qquad \pi(s^t) = \left[\sum_{j=0}^{T-t} \bar{\beta}^l \sum_{s^{t+1}} d_{K(s^t)}(s^{t+1})\mu_t(s^{t+1}) - C(D^{s^t})\right] \bullet \theta_{K(s^t)}$$
$$+ \psi(s^{t-1})(1+q(s^t))$$
$$- \sum_{j=0}^{t-1} \theta_{K(s^j)} \bullet d_{K(s^j)}(s^j, s^{t-j}) - q(s^t)\psi(s^t)$$

This holds for $0 \le t \le$ T- 1, where $\pi(s^T)$ is defined as before.

In this case, the following first - order conditions characterize the optimum amount of supply for innovations:

$$(25)\quad \sum_{l=0}^{T-t} \bar{\beta}^{j} \sum_{s^{t+1}} d_{K(s^t)}(s^{t+1})\mu_t(s^{t+1}) - C_{K(s^t)}(D^{s^t})$$

$$-\sum_{l=0}^{T-t} \delta^{1} \sum_{s^{t+1}} d_{K(s^t)}(s^{t+1})\mu_t(s^{t+1})$$

$$\leq 0$$

$$= 0 \text{ if } \theta_{k(s^t)} > 0$$

for all k (s^t). This can be reduced to:

$$(26)\quad \sum_{l=0}^{T-t} \left(\bar{\beta}^{1} - \delta^{1}\right) \sum_{s^{t+1}} d_{K(s^t)}(s^{t+1})\mu_t(s^{t+1})$$

$$\leq C_{K(s^t)}(D^{s^t})$$

$$= C_{K(s^t)}(D^{s^t}), \text{ if } \theta_{K(s^t)} > 0$$

where $C_{K(s^t)}(D^{s^t})$ is the k(s^t) component of the vector $C(D^{s^t})$. From here it is easy to show the following proposition. (Proofs of this and all propositions are in the appendix)

Proposition 2

Suppose $C_{K(s^t)}(D^{s^t})$ *> 0, for all* s^t*, all k(*s^t*), and also* $\bar{\beta} \leq \delta$. *Then no innovation is possible under these assumptions.*

The proof is a direct application of the first order conditions stated before. The intuition is also very simple. Since agents are risk neutral, they do not have risk sharing purposes. The only possible motive of asset accumulation is future consumption. Hence, if the innovator is patient enough so that she has a higher discount factor (or equivalently, a lower discount rate) then she is not interested in selling any new asset in the market. Households are more impatient and they have less desire to save. The more impatient the risk neutral consumers are, the less savings they want to do, and the less room is to have financial innovations.

This is also true even when the less impatient type of consumer has the same degree of impatience than the innovator (that is, $\overline{\beta} = \delta$). This is because there is a cost associated with the issuing of new assets. At first sight there would be a possibility of having a market opened for a new set of securities. The marginal value of those assets between the supply and the demand side are equal. The presence of issuing costs inhibits the former of introducing those assets.

On the other hand, we can show that we can obtain an equilibrium from "accommodating'' the cost function and the factor δ.

Proposition 3

Suppose that $\delta < \overline{\beta}$ *, and* $C_{K(s^t)}(D^{s^t}) > 0$*. Then there is no financial innovation equilibrium in this economy for any finite T, with positive amount of new financial securities.*

The proof of this is available upon request. It has a straightforward, almost obvious intuitive appealing. Since risk neutral consumers have no interest in sharing risk, there is no incentive to buy financial assets except to translate consumption through time. Hence the only relevant parameter is the (subjective) discount rate. In a sense, the innovator cannot be differentiated from the rest of the consumers, so that the former becomes "one more type" of household. Therefore the only agent interested in transferring consumption through time is that one with higher degree of patience.

5. An Impossibility Result with Risk Averse Households and i.i.d. Shocks

The literature on financial innovation and incomplete markets has often studied how the financial innovators could improve the *risk sharing* of consumers. This issue is however completely hidden with risk neutral consumers, as mentioned earlier. Hence it is obvious to generalize the analysis of the last section to the risk - aversion case. Nevertheless, a general framework is somehow cumbersome and requires a very detailed analysis. This section then provides some characterization-of-equilibrium results.

Let the economy has the property of having equally likely states at each period. That is:

$$\frac{\mu_{t+1}(s^t, s)}{\mu_t(s^t)} = \pi(s) \tag{27}$$

for all t, for all s^t, for all s in S with $0 < \pi(s) < 1$. Here $\mu_t(s^t, s)$ denotes the probability, conditional to the realization of s^t and the information available at that node, of occurrence of the event s at t+1.

I start considering different asset structures as candidates for equilibrium innovations. I consider the possibility of having an equilibrium with the following asset structure. The innovator issues at each node s^t a complete set of Arrow securities that pay 1 unit of the consumption good if the realization at next period is s and 0 otherwise. That is, the payoff for the innovation $k(s^t) = [s^t, s]$ is defined as:

$$(28) \qquad d_{[s^t,s]} = \begin{cases} 1 \text{ if s occurs} \\ 0 \text{ otherwise} \end{cases}$$

We will show that there is no possible financial innovation equilibrium with a strictly positive amount of *all* of the Arrow securities issues for some special condition at the one-before-last node s^{T-1}:

Proposition 4

Suppose an allocation such that for some s^{T-1}:

$$(29) \qquad \max_{i \in I} \left\{ \beta_i \pi(s) \sum_{s \in S} \frac{u_i'\left[c^i(s^{T-1},s)\right]}{u_i'\left[(c^i(s^{T-1})\right]} \right\} = \beta_{i^*} \pi(s) \frac{u_i'\left[c^{i^*}(s^{T-1},s)\right]}{u_{i^*}'\left[(c^{i^*}(s^{T-1})\right]}$$

for all s

That is, for s^{T-1}, there is one type of consumers with the maximum intertemporal marginal rate of substitution for any of the possible states in the last period. If $C(D^{s^{T-1}}) >> 0$, for all s, then the allocation $c^i(s^t)$ cannot be supported by a strictly positive amount of Arrow Securities for the last period of the economy.

The intuition of this result is as follows. The presence of a consumer whose equilibrium intertemporal MRS is the greatest for all states in period T implies that a unique consumer would be the buyer of all Arrow securities. But this would imply this agent only needs to smooth consumption between T-1 and T. However, a cheaper way of attaining this is by buying the riskless bond.

This last result can be slightly generalized in the following way.

Proposition 5

Assume S ³ 2. Suppose also that for all s^t, where % t £ T-1, there is an allocation and some $i^*(s^t)$ such that the following condition is satisfied:

(A) $$\max_{i \in I} \left\{ \beta_i \, \pi(s) \frac{u_i'\left[c^i(s^t,s)\right]}{u_i'\left[(c^i(s^t)\right]} \right\} = \beta_{i^*(s^t)} \, \pi(s) \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[(c^{i^*(s^t)}(s^t)\right]}$$

for all s

Moreover, suppose that at least for a some pair of states s and s' it is true that

(B) $$\frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[(c^{i^*(s^t)}(s^t)\right]} \neq \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s')\right]}{u'_{i^*(s^t)}\left[(c^{i^*(s^t)}(s^t)\right]}$$

and that

(C) $$\beta_{i^*(s^t)} \sum_{s \in S} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[(c^{i^*(s^t)}(s^t)\right]} \pi(s) < 1$$

for every t. Then, the consumption allocation cannot be supported by a complete set of Arrow securities supplied in positive amounts by the innovators for all s^t.

The interpretation or these propositions is clear. If a complete set of Arrow securities were available and if consumer i^* has the highest intertemporal marginal rate of substitution, this consumer must be the one that purchases all the securities. But since this is true for every state, then consumer i^* is finally purchasing a bond, which is costly. However there exists a riskless bond which is already available at no extra cost[3].

These two results lead to the following natural corollary:

Corollary 6

If all the households are identical, if conditions (B) and (C) of proposition 5 are satisfied, and if costs of issuing assets are positive, then there is no financial innovation competitive equilibrium with a positive amount of complete set of 1-period Arrow securities traded at any of the nodes s^t.

Proof

This follows from propositions 4 and 5, since the condition for the latter is clearly satisfied in a representative household economy.

3 I thank one of the referees for providing this nice interpretation.

In fact, this result can be easily generalized to state that under the same conditions we have that in fact any consumption allocation characterized by inequality (29) implies that it cannot be an equilibrium allocation with an incomplete set of Arrow securities issued by the innovators.

Proposition 7

Assume condition (29) is true. Then it is impossible that such consumption allocation may be implemented through an incomplete set of Arrow securities endogenously issued by the financial innovators at T-1. If condition (A) from proposition 5 also holds for all s, and if

$$(D) \qquad \beta_{i^*(s^t)}\,\pi(s)\frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[(c^{i^*(s^t)}(s^t)\right]} \geq \delta$$

for all s, then it is impossible that this allocation can be supported by an incomplete set of Arrow securities issued at any t by the financial intermediaries.

The proof is presented in the Appendix. This proposition then states that the conditions before also block the possibility of having any set of Arrow securities as an equilibrium financial structure. Note that condition D states that the maximum MRS for every state is at least equal to the innovator's discount factor. This result may seem surprising since intuitions says that a sufficiently high intertemporal MRS would be enough to provide an incentive to introduce securities in order to improve risk sharing. However, again, given that the highest MRS belongs to the same person (for every future state), if the financial innovator purchases riskless bonds to back Arrow securities, the bond price would be δ. But then it would be cheaper for the consumer with highest MRS to purchase the bond rather than Arrow securities.

The results presented above may suggest that conditions A through C may be enough to eliminate an equilibrium with *any* type of financial innovation, not only Arrow securities. However, condition A only states that for every node s^t there is some household with the highest MRS between t and t+1 for all s in S. But the name of the household $i^*(s^t)$ may change through different nodes of the tree. Hence generalizing the Arrow securities impossibility results to the general case may require stronger conditions such as the fact that the household with the highest intertemporal MRS is always the same across all nodes. However this is not an obvious question, leaving this investigation for future research.

This set of results can be summarized in the following way. They provide a set of sufficient conditions that show when it is not possible to observe innovation equilibria with either any set (complete or incomplete) of Arrow securities (in addition to the riskless asset) at any node. The main intuition is that for Arrow securities to be issued in equilibrium the risk sharing needs by consumers must be non trivial.

Clearly, pure intertemporal consumption smoothing by itself cannot imply a positive amount of Arrow securities issued. These results are important in the sense of giving conditions under which we should not expect (a complete set of) short term securities to appear in the market.

6. Conclusions and Future Research

In this paper I propose an equilibrium concept for a multi - period economy with endogenous financial innovation, called financial innovation equilibrium. The special feature in this equilibrium is that financial innovators choose which markets for new financial products to open (including date and state), the amount supplied for each of them, and their payoffs (and in this way, indirectly, the duration of the innovations). The prices for these new products are assumed to be rationally conjectured directly from the preferences of the households in this economy. This is also consistent with the standard GEI models with financial innovation. I checked formally the statement that, with risk-neutral households, no financial innovation occurs. This is due to absence of incentives for risk sharing. I also provide some preliminary characterization with risk averse agents. In particular, sufficient conditions are provided so that we cannot support allocations with a complete set of short lived Arrow securities, either in the last period of the economy, or in any period.

This is only a preface of a long term research program. The next natural step is to show different cases which could show the different possible asset structures that can arise in equilibrium within this framework. Especially important is to provide sufficient conditions to get different equilibria with very different asset durations, as observed in the data. The conjecture is that these conditions will be strongly cost-of-the-innovators dependent. I leave this for future research.

Another task is the extension of these economies to infinite periods. It is known that with finite periods, rational expectations and no informational asymmetries, the pricing of any asset in a competitive framework is totally determined by the fundamentals, even with incomplete markets. This can clearly disappear in the presence of infinite horizon. Thus, we could have that the pricing of new financial securities are not determined uniquely by the payoffs they will offer in the future periods, but instead some "bubble" effect can arise, and then the "rational conjecture" is not as simple as in the finite horizon case. The presence of bubbles under incomplete markets has been explored by Santos and Woodford (1997) among others. This work suggests that some innovators could be tempted to place new financial products whose payoffs are relatively much smaller than the price the households are willing to pay, due to a bubble. This issue requires also future work and its exploration is also part of the research program.

This paper assumes that the driving force for new assets is the need for improving risk sharing. This is not clearly the unique motivation for more securities to show up in the market. Clearly there are other phenomena that justify the subsequent introduction of new assets. For example, tax arbitrage may also be another reason for new assets. However a model that only focuses on tax arbitrage without other motivations (e.g., without risk sharing motives), provided that the tax system is assumed exogenous, does not seem too interesting. More important is the study of the interaction between taxes, risk sharing and financial innovation. This is also very relevant for the analysis of optimal taxation, which is also left for future research.

There are other potential sources for financial innovation that seem more involved. First, market illiquidity may call for financial innovation, specially in the form of securitization. However, market liquidity is usually an endogenous variable strongly affected by informational issues, as stressed, for instance, by Grossman and Miller (1988) (see also chapter 8 in O'Hara, 1995). Modelling this in a true dynamic model is beyond the scope of this paper, but it must also be emphasized that this task is not trivial at all. Asymmetric information affecting liquidity may be resolved through time independently of financial innovation, and this process is itself quite complicated. Similar points can be made if one consider the case of agency costs. Financial innovation may help in reducing these costs through time, but they could also be reduced through reputation and other effects alike. Modelling this requires to work with multiperiod versions of agency costs models of corporate finance, which is clearly beyond the scope of the current paper.

7. Proofs

Proof of proposition 2

First of all, for all the households types such that $\beta_i < \overline{\beta}$, they will be just consuming their endowments, since they value any of the assets strictly less than how the "less impatient type'' value them. So for those i, we should have $c^i (s^t) = \omega^i (s^t)$. Then, by non-arbitrage, the price of the riskless security is equal to:

(30) $$q(s^t) = \sum_{t=0}^{T-t} \overline{\beta}^1$$

for any t. This is because otherwise, the demand for riskless bond done by the type with highest β is unbounded. We also have that:

(31) $$\psi(s^t) = 0$$

for every s^t, since $\delta < \overline{\beta}$. In particular, this implies that

(32) $$\psi(s^{T-1}) = 0$$

But then, since at time T we cannot have any innovation (for any possible innovation done at date T, we have that $r_{K(s^T)=0}$, then the condition:

$$\text{(33)} \qquad \psi(s^{T-1}) \geq \sum_{j=0}^{T-1} \theta_{K(s^j)} \bullet d_{K(s^j)}(s^j, s^{T-j})$$

together with $d_{K(s^j)}(s^j, s^{T-j}) \geq 0$ for every (s^j, s^{T-j}), every j, implies that for every j, $d_{K(s^j)}(s^j, s^{T-j}) = 0$. This in particular gives that for any innovation introduced at time T-1 we have:

$$\text{(34)} \qquad r_{k(s^{T-1})}(s^{T-1}) = \overline{\beta} \sum_{s \in S} d_{k(s^{T-1})}(s^{T-1}, s)$$

Since $C(D^{s^{T-1}}) > 0$, the first order condition implies that there is no positive (profitable) innovation at time T-1. We proceed using induction. Suppose the innovator does not issue any new financial product from date t to T-1. Hence we must prove that there will be no innovation at period t-1. By the inductive argument, it is clear that the absence of innovation from t to T-1 is implied by:

$$\text{(35)} \qquad d_{k(s^h)}(s^h, s^{h+j}) = 0$$

for h = t, t+1, ..., T-1 and j = 1, 2,...,T-h. Then we have that:

$$\text{(36)} \qquad r_{k(s^h)}(s^h) = 0$$

h = t, ..., T-1. At date t-1, we have that, if there is some innovation, we must have for any innovation introduced at t-1, the following must be true,

$$d_{k(s^{t-1})}(s^{t-1}, s^{t-1+j}) = 0$$

for j = 2,...,T-1. This is because $r_{k(s^h)}(s^h) = 0$ for h = 2,...,T-1, and also $\psi(s^t) = 0$ for every t, then we have that, from the delivery constraint:

(37) $$\left[r_{K(s^h)}(s^h, D^{s^h}) - C(D^{s^h})\right] \bullet \theta_{K(s^h)} + \psi(s^{h-1})$$

$$\geq \sum_{j=0}^{h-1} \theta_{K(s^j)} \bullet d_{K(s^j)}(s^j, s^{h-j})$$

$$h = t, \ldots, T-1$$

to be true, together with non-negativity of returns. In fact this also implies that

(38) $$d_{k(s^{t-1})}(s^{t-1}, s^{t-1+l}) = 0$$

for l = 1,2,...,T- (t - 1). But on the other hand

(39) $$r_{k(s^{t-1})}(s^{t-1}) = \sum_{t=0}^{T-(t-1)} \overline{\beta}^l \sum_{s^l} d_{k(s^{t-1})}(s^{t-1}, s^{t-1+l})$$

Hence $r_{k(s^{t-1})}(s^{t-1}) = 0$. But since $C(D^{s^{T-1}}) > 0$, the first order conditions show that the optimal amount of new financial products is equal to 0 for s^{t-1}. By the inductive hypothesis, it is true for any s^t, any t, concluding the proof.

Proof of proposition 4

Suppose by means of contradiction that we have some allocation $\left\{\left\{c^i(s^t)\right\}_{t=0}^T\right\}_{i=1}^I$ and some price system $\left(\left\{q(s^t), r_{[s^t,s]}\right\}_{t=0}^{T-1}\right)_{s\in S}$ such that for every s^{T-1}, for every s, $\theta_{[s^{T-1,s}]} > 0$. Since at period T, we have that $r_{[s^{T,s}]} = 0$, and since $C(D^{s^T}) >> 0$, then we must have from the delivery constraint that $\psi(s^{T-1}) > 0$. (Otherwise either the payoffs or the amount of past issues should be zero, leaving us with the same argument as in the risk-neutral case). This means that the price that the innovator pays is at least equal to the maximum price that the households are willing to pay for the riskless bond. Formally:

(40) $$q(s^{T-1}) \geq \max_{i\in I}\left\{\beta_i \pi(s) \sum_{s\in S}\left[\frac{u'(c(s^{T-1}, s))}{u'(c(s^{T-1}))}\right]\right\}$$

Also note that, since:

(41) $$\max_{i \in I} \left\{ \beta_i \pi(s) \frac{u_i'\left[c^i(s^{T-1},s)\right]}{u_i'\left[c^i(s^{T-1})\right]} \right\}$$

$$= \beta_{i^*} \pi(s) \frac{u_{i^*}'\left[c^{i^*}(s^{T-1},s)\right]}{u_{i^*}'\left[c^{i^*}(s^{T-1})\right]}$$

for all s

then:

(42) $$q(s^{T-1}) \geq \sum_{s \in S} \beta_{i^*} \pi(s) \frac{u_{i^*}'\left[c^{i^*}(s^{T-1},s)\right]}{u_{i^*}'\left[c^{i^*}(s^{T-1})\right]}$$

From the non-arbitrage condition for the innovator we must have:

(43) $$q(s^{T-1}) = \delta$$

Then we have

(44) $$\delta \geq \sum_{s \in S} \beta_{i^*} \pi(s) \frac{u_{i^*}'\left[c^{i^*}(s^{T-1},s)\right]}{u_{i^*}'\left[c^{i^*}(s^{T-1})\right]}$$

On the other hand, we know that the price for each of the Arrow securities issued at date T-1, is

(45) $$r\left[s^{T-1},s\right] = \max_{i \in I} \left\{ \beta_i \pi(s) \sum_{s \in S} \left[\frac{u'(c(s^{T-1},s))}{u'(c(s^{T-1}))} \right] \right\}$$

$$= \beta_{i^*} \pi(s) \left[\frac{u_{i^*}'(c^{i^*}(s^{T-1},s))}{u_{i^*}'(c^{i^*}(s^{T-1}))} \right]$$

for every s, every s^{T-1}. From profit maximization (for the innovators) we have that

(46) $$r_{[s^{T-1},s]} = C_s\,(D^{s^{T-1}}) + \delta$$

since $\theta_{[s^{T-1},s]} > 0$. Since $C_s(D^{s^{T-1}}) > 0$, we must have $r_{[s^{T-1},s]} > \delta$. We also have that for i^*, there is some $\bar{s}$ in S such that:

(47) $$\sum_{s\in S} \beta_{i^*}\,\pi(s) \frac{u'_{i^*}\left[c^{i^*}(s^{T-1},s)\right]}{u'_{i^*}\left[c^{i^*}(s^{T-1})\right]} \geq \beta_{i^*}\,\pi(\bar{s}) \frac{u'_{i^*}\left[c^{i^*}(s^{T-1},\bar{s})\right]}{u'_{i^*}\left[c^{i^*}(s^{T-1})\right]}$$

The reason is simple. If not, then for all s, we should have

(48) $$\sum_{s\in S} \beta_{i^*}\,\pi(s) \frac{u'_{i^*}\left[c^{i^*}(s^{T-1},s)\right]}{u'_{i^*}\left[c^{i^*}(s^{T-1})\right]} < \beta_{i^*}\,\pi(\bar{s}) \frac{u'_{i^*}\left[c^{i^*}(s^{T-1},\bar{s})\right]}{u'_{i^*}\left[c^{i^*}(s^{T-1})\right]}$$

Adding over s we have

$$\sum_{s\in S}\sum_{s\in S} \beta_{i^*}\,\pi(s) \frac{u'_{i^*}\left[c^{i^*}(s^{T-1},s)\right]}{u'_{i^*}\left[c^{i^*}(s^{T-1})\right]} = \beta_{i^*} S \sum_{s\in S} \frac{u'_{i^*}\left[c^{i^*}(s^{T-1},s)\right]}{u'_{i^*}\left[c^{i^*}(s^{T-1})\right]}\pi(s)$$

$$< S\beta_{i^*}\,\pi(\bar{s}) \frac{u'_{i^*}\left[c^{i^*}(s^{T-1},s)\right]}{u'_{i^*}\left[c^{i^*}(s^{T-1})\right]}$$

and since S > 1, and $\pi(\bar{s}) < 1$, this is a clear contradiction. Thus

$$\delta = q(s^{T-1}) \geq \left\{\beta_{i^*} \sum_{s\in S} \pi(s) \left[\frac{u'_{i^*}(c^{i^*}(s^{T-1},s))}{u'_{i^*}(c^{i^*}(s^{T-1}))}\right]\right\}$$

$$> \beta_{i^*}\,\pi(\bar{s}) \frac{u'_{i^*}(c^{i^*}(s^{T-1},\bar{s}))}{u'(c^{i^*}(s^{T-1}))} = r_{[s^{T-1},\bar{s}]}(s^{T-1})$$

Hence

$$(49) \qquad \delta > r\left[s^{T-1}, \bar{s}\right]$$

So we have a contradiction since we assumed that for $\theta\left[s^{T-1}, s\right] > 0$ for *every s* to be true we must have the opposite inequality. Hence we cannot have a complete set of Arrow Securities issued at any of the s^{T-1} nodes.

Proof of proposition 5

In this case, we can have two possibilities. The first corresponds to the case in which the innovator buys at s^t some positive amount of the riskless bond. However, since the condition implies that

$$(50) \qquad \max_{i \in I}\left\{\beta_i \sum_{s \in S} \pi(s)\left[\frac{u_i'\left[(c^i(s^t, s)\right]}{u_i'\left[(c^i(s^t)\right]}\right]\right\}$$

$$= \beta_{i^*}(s^t) \sum_{s \in S} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t, s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]} \pi(\bar{s})$$

Then, we must have that

$$(51) \qquad q(s^t) = \delta + \delta \sum_{s \in S} q(s^t, s)\, \pi(s)$$

$$\geq \beta_{i^*(s^t)} \sum_{s \in S} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t, s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]} (1 + q(s^t, s))\, \pi(s)$$

This implies either

$$(52) \qquad \delta \geq \beta_{i^*(s^t)} \sum_{s \in S} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t, s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]} \pi(s)$$

or

$$(53)\qquad \delta \sum_{s\in S} q(s^t,s)\,\pi(s) \geq \beta_{i^*(s^t)} \sum_{s\in S} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]}\, q(s^t,s))\,\pi(s)$$

In the first case, we have for some $\bar{s}$ that:

$$(54)\qquad \delta \geq \beta_{i^*(s^t)} \sum_{s\in S} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]}\,\pi(s)$$

$$\geq \beta_{i^*(s^t)} \sum_{s\in S} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,\bar{s})\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]}\,\pi(\bar{s})$$

$$= r_{\left[\bar{s},s^t\right]}(s^t)$$

for the same reason as in the proof of Proposition 4. Since $C_{\bar{s}}(D^{s^t})>0$, this cannot happen whenever $\theta_{\left[s^t,\bar{s}\right]}>0$. Then we have that the first inequality cannot be possible.

Suppose that the second inequality holds. But since $C_s(D^{s^t})>0$ for all s in S, if we want $\theta_{\left[s^t,s\right]}>0$ for all s we must have that $r_{\left[s,s^t\right]}(s^t)>\delta$ for all s. But this last inequality implies

$$(55)\qquad \beta_{i^*(s^t)} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]}\,\pi(s)>\delta$$

for all s and then

$$\text{(56)} \qquad \beta_{i^*(s^t)} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]} q(s^t,s)\,\pi(s) > \delta q(s^t,s)$$

$$\forall s \in S$$

Adding over $s \in S$ we have that

$$\text{(57)} \qquad \beta_{i^*(s^t)} \sum_{s\in S} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]} \pi(s)$$

$$> \delta \sum_{s\in S} q(s^t,s) > \delta \sum_{s\in S} q(s^t,s)\,\pi(s)$$

contradicting the inequality we originally started with. Then we cannot have that the allocation satisfying the condition (A) be supported by a complete set of Arrow securities supplied in positive amounts for the case in which the innovators buy some positive amount of the riskless bond.

If the innovators have 0 of riskless bond at s^t, then we must have that for every state then innovators must finance the payments of the Arrow securities issued at s^t with the receipts from the Arrow securities issued at s^{t+1}. Clearly again we may have $\psi(s^{t+1}) > 0$ (in whose case the same argument as before applies) or $\psi(s^{t+1}) = 0$, in whose case the s^{t+1} issues will be financed by those in period t+2, and so on. Therefore, if $\psi(s^\tau)$ for τ greater than t and smaller than T-1 are zero, then it must be true that $\psi(s^{T+1}) > 0$ to satisfy the delivery constraint, but then condition (A) implies a contradiction for t = T-1. If for some τ less than T-1 it is true that $\psi(s^\tau) > 0$ then the argument in the paragraph above applies.

Proof. of proposition 7.

In case that condition (29) holds we can argue by contradiction. Suppose there exists in equilibrium an incomplete set of Arrow securities issued at date T-1. Call S_T the subset of states in S such that for every s in S_T the amount $\theta\left[s^{T-1}, s\right]$ is strictly positive. Therefore for every s in S_T it must be true that

$r\left[s^{T-1},s\right] = C(s^{T-1},s) + \delta$. By the same arguments as in proposition 4 it must also happen that $\psi(s^{T-1}) > 0$ and so $q(s^{T-1}) = \delta$. On the other hand, by those arguments it was also true that

$$q(s^{T-1}) = \delta \geq \max_i \left\{ \sum_{s\in S} \beta_i \frac{u'(c^i(s^{T-1},s))}{u'(c^i(s^{T-1}))}\pi(s) \right\} = \sum_{s\in S} \beta_i \frac{u'(c^{i^*}(s^{T-1},s))}{u'(c^{i^*}(s^{T-1}))}\pi(s)$$

by condition (29). The rational conjecture also implies that $r\left[s^{T-1},s\right] = \beta_{i^*} \frac{u'(c^{i^*}(s^{T-1},s))}{u'(c^{i^*}(s^{T-1}))}\pi(s)$ for every s in S_T. If $C(s^{T-1},s) > 0$ then

$$\beta_{i^*} \frac{u'(c^{i^*}(s^{T-1},s))}{u'(c^{i^*}(s^{T-1}))}\pi(s) > q(s^{T-1}) \geq \sum_{s\in S} \beta_{i^*} \frac{u'(c^{i^*}(s^{T-1},s))}{u'(c^{i^*}(s^{T-1}))}\pi(s).$$ Therefore

(58) $$u'(c^{i^*}(s^{T-1},s))\,\pi(s) > \sum_{s\in S} u'(c^{i^*}(s^{T-1},s))\,\pi(s)$$

but since u'(c) > 0 and $\pi(s)$ for all s, this cannot happen. Therefore $r\left[s^{T-1},s\right] = C(s^{T-1},s) + q(s^{T-1})$ for s in S_T cannot happen. But then no Arrow security can support this consumption allocation as an equilibrium in T.

This can also be generalized to any other period t provided that conditions (A) and (D) hold. From A we know that

(59) $$\max_{i\in I} \left\{ \beta_i \frac{u_i'\left[(c^i(s^t,s)\right]}{u_i'\left[c^i(s^t)\right]}\pi(s) \right\} = \beta_{i^*(s^t)} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]}\pi(s)$$

for all s in S. From (D) we also know that

(60) $$\beta_{i^*(s^t)} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]}\pi(s) \geq \delta$$

and so

(61) $$\beta_{i^*(s^t)} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]}\pi(s) \geq \delta\pi(s)$$

$$\text{(62)} \quad \beta_{i^*(s^t)} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]} \pi(s)\, q(s^t,s) > \delta q(s^t,s)\, \pi(s)$$

Therefore

$$\text{(63)} \quad \beta_{i^*(s^t)} \sum_{s\in S} \frac{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t,s)\right]}{u'_{i^*(s^t)}\left[c^{i^*(s^t)}(s^t)\right]} (1+q(s^t,s))\pi(s) > \delta + \delta \sum_{s\in S} q(s^t,s)\, \pi(s)$$

Since this is true for all t, all s^t then $\psi(s^t) = 0$ for all t and s^t. However if this is the case the delivery constraint cannot be satisfied since the innovator can never finance the payments over the Arrow securities. This finishes the proof.

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