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Modeling Nonnegativity via Truncated Logistic and Normal Distributions: An Application to Ranch Land Price Analysis

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This study presents an empirical method of modeling the nonnegativity of dependent variables using truncated logistic and normal disturbance distributions. The method is applied in estimating a ranch land hedonic price function. Results show that the degree of truncation is significant.

Key words: logistic and normal distributions, model misspecification, nonnegativity, ranch land prices, truncation.

Introduction

The dependent variables being modeled in empirical economic analyses are oftentimes nonnegative random variables in nature. Examples in the agricultural economics literature abound, and include a myriad of past studies modeling quantities demanded or supplied, price dependent inverse demand or supply functions, and hedonic price functions. In these cases, any distribution assumed for the disturbance term in a regression-type model that allows negative dependent variable values to occur with nonzero probability is a model misspecification, a priori. A popular assumption for the disturbance distribution in applied work is some member of the normal family of distributions, but this family of distributions is a priori incorrect if it is utilized in modeling cases where the dependent variable of a regression model with additive disturbances is nonnegatively valued. In fact, any family of disturbance distributions having the real line for its support will be similarly a priori inappropriate unless the distributions are subsequently truncated from below in order that the nonnegativity of the dependent variable be properly represented.

Presumably an implicit assumption made in past studies is that, although normality (or any other distribution having the real line for its support) is literally incorrect as the disturbance distribution, the offending lower tail of the distribution associated with negative dependent variable values, and upper-bounded by the truncation point, has near-zero probability regardless of the value of the explanatory variables in the data set being analyzed, and, hence, regardless of the expected value of the dependent variable. This implicit or explicit assumption of stochastic irrelevance of negatively valued dependent variable values presumes knowledge of the location of the regression function surface relative to the spread of the disturbance distribution over all points in the data set, which is information that, in actuality, is generally unavailable to the researcher a priori. Furthermore, even if the negative dependent variable values were to occur with "small probability" under a given disturbance distribution, such as the normal distribution, there is ambiguity relating to how small such probabilities need to be in order for the truncation bias introduced into the estimates of model parameters to be inconsequential. Ultimately,

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the primary issue of concern to the researcher is whether the form of the actual disturbance distribution that is enforcing nonnegativity also has significant implications for the functional specification of the conditional expectation function being estimated in a regression model, as could be the case, for example, for a normal disturbance distribution.

Given recent advances in nonlinear estimation techniques, and nonnested and conditional moment hypothesis testing procedures, systematic investigations of the empirical relevance and implications of the nonnegativity/truncation assumption are now well within the range of feasibility. The purpose of the current study is to present the results of one such explicit investigation. Specifically, within the context of a rangeland hedonic price analysis, we provide statistical evidence against the assumption of an untruncated normal or logistic distribution for the disturbance term, and we document the practical significance of truncation bias with respect to predicting hedonic land values. Our results coincide with the results of Xu, Mittelhammer, and Barkley (1991, 1993), who demonstrated the significance of the truncation effect in another empirical investigation. The results of the current study, together with the previous work of Xu, Mittelhammer, and Barkley (1991, 1993), suggest that the truncation effect may not be as innocuous as many researchers would like to believe, and lead to important questions regarding the appropriate functional specification of regression functions.

The remainder of the article is organized as follows. First, the pertinent statistical theory underlying the specification of truncated regression models based on a normal or logistic disturbance distribution is presented. Next, the results of an application of the theory to a hedonic model of ranch land sales are discussed. Finally, implications of the results and suggested directions for future research are presented.

Truncation Models Based on the Logistic and Normal Distributions

Let a statistical model for the random variable Y be represented in standard regression form as

$$Y = g(X; \beta) + \mu = g(\cdot) + \mu, \tag{1}$$

where $g(\cdot)$ is a differentiable function of the explanatory variables (X) and the vector of unknown parameters to be estimated (β). Assume initially that the error term μ has either a logistic or normal distribution, each with mean zero and scale parameter τ, as

$$f(\mu) = \frac{\exp(-\mu/\tau)}{\tau(1 + \exp(-\mu/\tau))^2}, \quad \text{or} \tag{2}$$
$$f(\mu) = \frac{1}{\sqrt{2\pi\tau^2}}\exp(-.5(\mu/\tau)^2),$$

respectively. For empirical modeling purposes, we examine the logistic distribution as a potential alternative to the normal distribution because it is analytically more tractable than the normal, while providing similar probabilistic properties. The major difference between the two distributions is that the logistic distribution has slightly thicker tails (Amemiya, p. 269; Johnson and Kotz, pp. 5–6).

In applications involving nonnegatively valued dependent variables, it is commonly assumed, either explicitly or implicitly, that $P(Y < 0) = P(\mu < -g(\cdot))$ is negligible and $E\mu = [0]$, so that under standard regularity conditions (e.g., Amemiya, pp. 127–35), nonlinear least squares estimation procedures applied to (1) result in consistent and asymptotically normal estimates of the parameter vector β. Under the assumption of a disturbance distribution having the real line for its support, such as either the normal or logistic distribution, this is tantamount to assuming that truncation of the distribution from below is negligible or irrelevant. In order to define a statistical model context in which a straightforward assessment can be made of the validity of $P(\mu < -g(\cdot)) \approx 0$ and $E\mu = [0]$, and thus the significance of the truncation effect, assume that the dependent variable Y in (1) is truncated at zero, i.e., $Y \geq 0$. It can be shown (appendix A), under the distributional assumptions in (2), that the expected value of the truncated distribution for Y is given, respectively, by

$$E(Y \mid Y \geq 0) = \tau(1 + \exp(-g(X; \beta)/\tau))\ln(1 + \exp(g(X; \beta/\tau))), \quad \text{or} \tag{3}$$
$$E(Y \mid Y \geq 0) = g(X; \beta) + \tau\frac{\phi(-g(X; \beta)/\tau)}{\Phi(g(X; \beta)/\tau)},$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are, respectively, the probability density and the cumulative density function of the standard normal distribution. Thus, a specification of the truncated version of model (1) in a form suitable for nonlinear least squares estimation of the parameter vector β and scale parameter τ is

$$Y = E(Y \mid Y \geq 0) + \epsilon = H(X; \beta, \tau) + \epsilon, \tag{4}$$

where $E(\epsilon) = 0$, and $H(X; \beta, \tau)$ represents the conditional expectation of Y either for a logistic or normal distribution for μ.

It can be shown that each of the functional specifications for $H(X; \beta, \tau)$ in (3) subsumes the standard untruncated specification for the expected value of Y, i.e., $EY = g(X; \beta)$, as a special limiting case. In particular, in either case (appendix B),

$$\lim_{\tau \to 0} H(X; \beta, \tau) = g(X; \beta). \tag{5}$$

This nesting of the standard untruncated functional specification for the expected value of Y within specification (4) allows an asymptotic one-sided t-test to be used to test the significance of the null hypothesis (H_0: $\tau = 0$) versus the alternative hypothesis (H_a: $\tau > 0$), thereby testing the statistical significance of the truncation effect. A significant t-statistic provides statistical evidence against an untruncated error distribution specification for the model. The test statistic would be calculated in the usual way as $\hat{\tau}/(\widehat{\text{Var}}(\tau))^{.5}$, where $\hat{\tau}$ is the estimate of τ, and $\widehat{\text{Var}}(\tau)$ is a consistent estimate of the asymptotic variance of $\hat{\tau}$ provided by a nonlinear least squares estimation of (4).[1]

In order to characterize the effect of truncation on the marginal effects of the explanatory variables in (1), first note that differentiation of $H(X; \beta, \tau)$ with respect to the ith explanatory variable, X_i, yields a functional relationship between $\partial H(X; \beta, \tau)/\partial X_i$ and $\partial g(X; \beta)/\partial X_i$. In particular, defining

$$D(g(X; \beta)/\tau) = 1 - (\exp(-g(\cdot)/\tau)\ln(1 + \exp(g(\cdot)/\tau)), \quad \text{or}$$

$$D(g(X; \beta)/\tau) = 1 - (g(\cdot)/\tau)\frac{\phi(g(\cdot)/\tau)}{\Phi(g(\cdot)/\tau)} - \left(\frac{\phi(g(\cdot)/\tau)}{\Phi(g(\cdot)/\tau)}\right)^2$$

for the logistic and normal distribution, respectively, it follows that

$$\frac{\partial H(X; \beta, \tau)}{\partial X_i} = D\left(\frac{g(X; \beta)}{\tau}\right)\frac{\partial g(X; \beta)}{\partial X_i}. \tag{6}$$

The $D(\cdot)$ function represents a proportionality factor relating marginal effects of explanatory variables under the truncated and untruncated regression models. Regarding the range of the function D, note that for $(g(X; \beta)/\tau) \in [0, \infty)$, $D(g(X; \beta)/\tau)$ is a monotonically increasing function such that $D(g(X; \beta)/\tau) \in [.3069, 1]$ for the logistic distribution and $D(g(X; \beta)/\tau) \in [.3634, 1]$ for the normal distribution. It follows that

$$\frac{\partial H(X; \beta, \tau)}{\partial X_i} \leq \frac{\partial g(X; \beta)}{\partial X_i},$$

the closer $(g(X; \beta)/\tau)$ is to zero, the larger $\partial g(X; \beta)/\partial X_i$ is relative to $\partial H(X; \beta, \tau)/\partial X_i$, and as $(g(X; \beta)/\tau) \to \infty$, $|\partial g(X; \beta)/\partial X_i| \to |\partial H(X; \beta, \tau)/\partial X_i|$. This is in accordance with the fact that both $P(Y \geq 0) = (1 + e^{-g(X;\beta)/\tau})^{-1}$ for the logistic distribution and $P(Y \geq 0) =$

Table 1. Definitions of Variables as Used in the Statistical Analysis

Variable	Definition
PRICEAUY	Ranch sale price on a $/AUY basis.
BLMAUY	AUY carrying capacity from BLM land.
FSAUY	AUY carrying capacity from USFS land.
STATAUY	AUY carrying capacity from state trust land.
PERBLM	Percentage of total ranch carrying capacity from BLM land, i.e., (*BLMAUY*/*TOTAUY*)•100.
PERFS	Percentage of total ranch carrying capacity from USFS land, i.e., (*FSAUY*/*TOTAUY*)•100.
PERST	Percentage of total ranch carrying capacity from state trust land, i.e., (*STATAUY*/*TOTAUY*)•100.
SIZE	Size of the ranch purchased in sections, including both deeded and leased lands.
ACCULTAU	Acres of cultivated land included with the ranch sale on $/AUY basis.
HBVALAUY	Appraised value of houses and buildings included with the ranch sale on $/AUY basis.
PROD	Average rangeland productivity, computed as the total number of AUY included in the sale, divided by the total number of sections sold.
TIME	Time trend variable defined as the number of years following January 1979 that the range sold, i.e., January 1982 = 3; July 1988 = 9.5.
COSTAUY	Cost advantage per AUY of carrying capacity on the ranch.

$\Phi(g(X;\beta)/\tau)$ for the normal distribution are monotonically increasing in $(g(X;\beta)/\tau)$, so that as $P(Y \geq 0) \rightarrow 1$, and thus as $(g(X;\beta)\tau) \rightarrow \infty$, the truncation effect eventually vanishes.

The preceding discussion indicates that for the truncation effect on the marginal impact of the explanatory variables to be negligible, it must be the case that the value of $g(X;\beta)$ relative to the scale parameter τ is sufficiently large so that $D(g(X;\beta)/\tau) \approx 1$ for all values of the explanatory variables X in the data being analyzed. The assumption would appear to be quite stringent, especially since at the outset of the analysis, it is typically the case that neither $g(X;\beta)$ nor τ are known to the researcher. A prudent research strategy would be to test the significance of the truncation effect, as we illustrate in the next section.

An Application to a Ranch Land Market Hedonic Price Function

Empirical Model

The data set used in this study is identical to the ranch land sales data for New Mexico reported in Torell and Doll. Detailed information on the data can be obtained from that source. Data characteristics are summarized briefly as follows. Sales judged to be substantially influenced by nonagricultural factors were deleted. Sales with capacity below 25 AUY[2] also were deleted. The value of livestock and machinery was excluded from the sale price. The final data set included 452 ranch land sales from January 1979 to December 1988 in New Mexico. Variable definitions are provided in table 1.

Using the specification of Torell and Doll, a hedonic model of per AUY sale price (*PRICEAUY*) of ranches was specified in the context of equation (1) as follows:

$$\begin{aligned} g(X;\beta) = {} & \beta_0 + \beta_1 PERBLM + \beta_2 PERFS + \beta_3 PERST + \beta_4 SIZE + \beta_5 SIZE^2 \\ & + \beta_6 ACCULTAU + \beta_7 HBVALAUY + \beta_8 PROD + \beta_9 TIME \\ & + \beta_{10} TIME^2 + \beta_{11} TIME^3 + \beta_{12} COSTAUY + \beta_{13}(TIME)(PERBLM) \\ & + \beta_{14}(TIME)(PERFS) + \beta_{15}(TIME)(PERST). \end{aligned} \tag{7}$$

The corresponding functional form, incorporating the truncation effect via equation (3), and expressed in a form suitable for nonlinear least squares estimation, is

$$(8)\qquad PRICEAUY = \tau(1 + \exp(-g(X; \beta)/\tau))\ln(1 + \exp(g(X; \beta/\tau)) + \epsilon, \qquad \text{or}$$

$$PRICEAUY = g(X; \beta) + \tau\frac{\phi(-g(X; \beta)/\tau)}{\Phi(g(X; \beta)/\tau)} + \epsilon$$

for the logistic or normal distribution, respectively, where $\beta = \{\beta_0, \beta_1, \ldots, \beta_{15}\}$ and τ denotes parameters to be estimated. All variables on the right-hand side of (7) are local and specific ranch characteristics except *COSTAUY* and *TIME*. These latter two variables capture the effects on ranch land prices of economic variables as they changed or were anticipated to change through time (Torell and Doll).

The models represented by (7) and (8) were estimated using the MODEL procedure in the SAS/ETS package (SAS Institute, Inc.). Nonlinear least squares estimation results are presented in table 2. The results reported under the heading "Model I" use all of the variables specified in (7), while the results reported under the heading "Model II" exclude two slope shifters (*TIME·PERFS* and *TIME·PERST*) that were not statistically significant in model I.

The null hypothesis that the truncation effect is insignificant was rejected at the .01 level of type I error in models I and II, using either the logistic or normal distribution. Thus, the effect of truncation is judged to be significant using either distributional assumption, and it is concluded that the specification of the expectation of ranch land sales given by (7), which does not account for truncation, is inappropriate.

A nonnested *P*-test (Davidson and MacKinnon; MacKinnon, White, and Davidson) was used to test which, if any, of the truncated models were appropriate for representing the prices. Let two alternative model specifications be given by

$$(9)\qquad H_1\colon Y = L(X, \beta) + \epsilon_1, \qquad \text{and}$$

$$H_2\colon Y = N(Z, \gamma) + \epsilon_2.$$

The *P*-test of the appropriateness of specification H_1 can be accomplished in three steps. First, the parameters of both models H_1 and H_2 are estimated using appropriate econometric techniques. Second, a general compound model is estimated in the form

$$(10)\qquad Y - L(X, \hat{\beta}) = \alpha[N(Z, \hat{\gamma}) - L(X, \hat{\beta})] + \hat{F}b + \epsilon_3,$$

where $\hat{F}$ represents the gradient vector of $L(X, \beta)$ evaluated at $\hat{\beta}$, and α and b (a vector) are parameters to be estimated. Finally, a *t*-test of the null hypothesis, H_1: $\alpha = 0$, is conducted using the results from estimating model (10). If H_1 ($\alpha = 0$) is accepted, then specification H_1 is deemed compatible with the data at the chosen level of type I error. In order to test the appropriateness of H_2, the roles of $L(X, \beta)$ and $N(Z, \gamma)$ are reversed in the *P*-test procedure. It is possible for either, neither, or both $L(X, \beta)$ and $N(Z, \gamma)$ to be compatible with the data.

The outcomes of the *P*-tests were such that the truncated specification based on the logistic distribution is rejected as being compatible with the data at any conventional level of type I error (table 3). The truncated model based on the normal distribution is judged to be data compatible at any level of type I error $< .13$.

Testing the Adequacy of the Truncated Normal Specification

Clearly, a large number of alternative functional specifications of the hedonic price function could have been analyzed in this study. Furthermore, given the complexity of land markets, the correct number and types of variables to choose in specifying a hedonic model are never obvious. Finally, the analyst always is working with data that are limited in terms of the number of variables and observations available, and in quality. An important question is whether the chosen empirical model represents adequately the conditional

Table 2. Estimates of Truncated and Untruncated Models

Parameter	Variable	Model I: Ignoring Truncation	Model I: Truncation via Logistic Distr.	Model I: Truncation via Normal Distr.	Model II: Ignoring Truncation	Model II: Truncation via Logistic Distr.	Model II: Truncation via Normal Distr.
*B*0	*INTERCEPT*	3,333.49*	3,400.67*	3,414.59*	3,348.54*	3,426.66*	3,438.85*
		(107.74)	(137.02)	(134.85)	(104.35)	(134.25)	(132.63)
*B*1	*PERBLM*	−36.82*	−42.48*	−42.10*	−38.85*	−45.91*	−45.53*
		(5.47)	(7.04)	(7.00)	(4.71)	(6.50)	(6.45)
*B*2	*PERFS*	−28.60*	−34.10*	−33.941*	−30.64*	−37.78*	−37.63*
		(5.62)	(7.31)	(7.28)	(4.78)	(6.68)	(6.65)
*B*3	*PERST*	−26.28*	−29.88*	−29.79*	−30.30*	−37.37*	−37.22*
		(7.50)	(9.28)	(9.25)	(5.13)	(7.12)	(7.08)
*B*4	*SIZE*	−3.72*	−4.82*	−4.75*	−3.67*	−4.72*	−4.66*
		(1.00)	(1.24)	(1.21)	(.99)	(1.23)	(1.20)
*B*5	*SIZE*^{3}	.005*	.007*	.007*	.005*	.006*	.006*
		(.002)	(.002)	(.002)	(.002)	(.002)	(.002)
*B*6	*ACCULTAU*	85.12*	90.86*	89.91*	85.71*	91.58*	90.67*
		(28.08)	(28.33)	(28.11)	(28.03)	(28.29)	(28.08)
*B*7	*HBVALAUY*	1.18*	1.22*	1.21*	1.18*	1.22*	1.21*
		(.13)	(.13)	(.13)	(.13)	(.13)	(.130)
*B*8	*PROD*	−11.43*	−20.51*	−20.66*	−11.37*	−20.07*	−20.24*
		(1.47)	(4.46)	(4.47)	(1.47)	(4.40)	(4.40)
*B*9	*TIME*	739.66*	851.73*	845.83*	738.31*	845.35*	840.48*
		(81.54)	(99.78)	(97.51)	(81.19)	(98.98)	(96.64)
*B*10	*TIME*^{2}	−166.61*	−187.99*	−186.66*	−167.99*	−189.67*	−188.46*
		(19.69)	(23.59)	(23.11)	(19.60)	(23.47)	(22.98)
*B*11	*TIME*^{3}	9.09*	10.20*	10.13*	9.23*	10.39*	10.33*
		(1.33)	(1.56)	(1.53)	(1.32)	(1.54)	(1.52)
*B*12	*COSTAUY*	23.11*	32.36***	32.69**	29.81**	44.18**	40.45*
		(16.60)	(20.29)	(20.24)	(14.00)	(17.86)	(17.84)
*B*13	*TIME•PERBLM*	.96*	.72**	.68**	.93*	.66***	.61***
		(.28)	(.41)	(.41)	(.27)	(.40)	(.41)
*B*14	*TIME•PERFS*	.26	.28	.27	—	—	—
		(.49)	(.70)	(.71)		—	
*B*15	*TIME•PERST*	−.33	−.73	−.71	—		—
		(.47)	(.60)	(.60)			
τ		—	580.77*	978.74*	—	571.55*	965.05*
			(115.11)	(183.44)		(115.49)	(181.33)
Root MSE		530	524	523	529	524	523
Adj. R^2		.7880	.7931	.7936	.7887	.7932	.7938

Notes: The number of observations is 452, variables are as defined in the text, the mean of the dependent variable is 2,636, and values in parentheses are standard errors. Single, double, and triple asterisks (*) indicate significance at $\alpha = .01$ or higher, significance between $\alpha = .05$ and $\alpha = .01$, and significance between $\alpha = .15$ and $\alpha = .05$, respectively.

Table 3. *P*-Tests of Model Specification

Hypothesis	t-Value	Probability $\|t\| > t$-Value
Logistic vs. Non-Logistic	3.01	.004
Normal vs. Non-Normal	1.52	.129

expectation of land prices relative to the vector of explanatory variables actually used in the model.

The Bierens conditional moments test provides a mechanism for addressing this question. In the current context, the null hypothesis—H_0: $E(Y \mid X, Y \geq 0) = H(X; \beta, \tau)$ for some β and τ—is tested using the Bierens procedure. The Bierens test is consistent against all departures from the null hypothesis. That is, it is a test designed to reject H_0 with probability $\rightarrow 1$ as sample size $\rightarrow \infty$ for *any* hypothesis alternative to H_0. Here, the test is based on the statistic

$$\text{(11)} \qquad \hat{M}(t) = (1/n) \sum_{j=1}^{n} (Y_j - H(X; \hat{\beta}, \hat{\tau}))\exp(t'\xi(X_j)),$$

where $\xi(X_j) = [\tan^{-1}(X_{j1}), \ldots, \tan^{-1}(\xi_{jm})]'$; X_{jk} is the jth observation on the kth explanatory variable; and $\tan^{-1}(Z)$ represents the arctangent of Z (Bierens, pp. 1445–46). Bierens showed that $\sqrt{n}\hat{M}(t) \rightarrow N[0, s^2(t)]$ as $n \rightarrow \infty$, and defined a consistent estimator of $s^2(t)$ to form the statistic $\hat{W}(t) = n[\hat{M}(t)]^2/\hat{s}^2(t)$, which converges asymptotically to a chi-square random variable with one degree of freedom under H_0 and approaches infinity with probability one as $n \rightarrow \infty$ when H_0 is false. The choice of t and the calculation of $\hat{s}^2(t)$ are discussed in appendix C.

An outcome of Bierens' conditional moments test was calculated to be $W(t) = 2.51$, which has a probability value of .114. Thus, the null hypothesis that the conditional expectation of land prices is represented adequately by the estimated truncated normal hedonic model is accepted at conventional levels of type I error. This is not to say that all relevant factors affecting land values have been taken into account. Rather, the test results suggest that given the regressors (X) actually used in the empirical model, the estimated truncated normal model provides an adequate representation of $E(Y \mid X, Y \geq 0)$.

For additional perspective on the adequacy of the truncated normal model, an untruncated model with heteroskedastic disturbances was examined as an alternative for representing the nonnegativity of the dependent variable. The idea was to allow the variance of the disturbance term to decrease sufficiently for small values of the regression function, and thereby utilize the decreased spread of the disturbance distribution for modeling nonnegativity of Y. Harvey's heteroskedastic regression model,

$$\text{(12)} \qquad Y_t = g(X_t; \beta) + \mu_t$$
$$\sigma_t^2 = \text{var}(\mu_t) = \exp\left(\alpha_0 + \sum_{j=1}^{k} X_{jt}\alpha_j\right),$$

was estimated via the SHAZAM econometrics computer program (White et al.), where $g(X_t; \beta)$ was specified as indicated in (7) with the last two variables removed (i.e., the same variables as were in model II of table 2). Then α's represent parameters to be estimated in the variance function, and X_{jt} refers to the jth explanatory variable in the $(k \times 1)$ vector, X_t. All but two of the estimated coefficients of the variance equation were statistically significant at the .05 level, and the overall heteroskedastic model appeared to fit the data reasonably well ($R^2 = .77$).[3]

A nonnested J-test was used to test the hypothesis that the truncated normal model was compatible with the data using the heteroskedastic model as an alternative model for land price determination. Specifically, the compound model,

$$Y = (1 - \alpha)H(X; \beta, \tau) + \alpha g(X; \hat{\beta}) + \epsilon, \tag{13}$$

was estimated via nonlinear least squares, where $g(X; \hat{\beta})$ represents the predicted values of Y generated from the fitted heteroskedastic model, and then the hypothesis $\alpha = 0$ was tested using an asymptotic t-test.[4] The estimated value of α was .073 with a standard error of .297, resulting in a t-value of .247, which is insignificant at any reasonable level of type I error. Thus, contrasted with the heteroskedastic model, and consistent with the Bierens test, the truncated normal model still is judged to be compatible with the data.

The roles of $H(\cdot)$ and $g(\cdot)$ were reversed in (13), and the new compound model was estimated via nonlinear generalized least squares using the heteroskedastic structure indicated in (12) to represent the disturbance term variances. The estimated value of α was .509 with a standard error of $.247 \times 10^{-3}$, resulting in a t-value of 2,056.3, which is clearly significant. Thus, contrasted against the truncated normal model, the heteroskedastic model is soundly rejected as an appropriate specification for the hedonic price function.

Results Summary

As in Torell and Doll, the total ranch value (*PRICETOT*) can be calculated as *PRICETOT* = *PRICEAUY* · *TOTAUY*, where *TOTAUY* refers to the total number of AUYs available from the purchased ranch land, consisting of AUYs generated from deeded land, Bureau of Land Management (BLM) land, U.S. Forest Service (USFS) land, and New Mexico state trust land. Differentiating *PRICETOT* with respect to AUYs obtained from each land type gives the equation of marginal value for each land type. For example, using model II, recalling the variable definitions given in table 1, and using result (6), the equation for the marginal value of deeded land is given by

$$\frac{\partial PRICETOT}{\partial DEEDAUY} = D\left(\frac{g(X;\beta)}{\tau}\right)\cdot(\beta_0 + \beta_4 SIZE + \beta_5 SIZE^2 + \beta_8 PROD + \beta_9 TIME + \beta_{10} TIME^2 + \beta_{11} TIME^3),$$

where *DEEDAUY* refers to the number of AUYs obtained from deeded land. The marginal value of deeded land is seen to depend on ranch size, productivity, unspecified factors captured in the trend variables of the model, and the truncation effect.

Regarding the practical significance of the truncation effect in this application, note that the values of $D(g(X; \beta)/\tau)$ were .9769 and .9676 evaluated at the mean level of predicted sale prices, and .8899 and .7233 evaluated at 50% of the mean level of predicted price from model II based on truncation using the logistic and normal distributions, respectively. The effect becomes substantially more pronounced the lower the predicted sale price. Figure 1 is an explicit illustration of the magnitude of the truncation effect, providing the estimated trends of marginal value of deeded land in dollars per AUY with or without truncation, calculated at the mean levels of variables for each year. Note that the value of $D(\cdot)$ is also calculated at the mean levels of variables for each year. As can be seen, the estimated marginal values of deeded land are mostly larger when generated from the model ignoring truncation than from the truncated models, especially at low levels of predicted sale prices of ranch land, as observed after 1986 when New Mexico ranch land values reached their lowest value in nearly 15 years.

The marginal implicit values of AUY land are provided in table 4. These values are estimates of the amount by which the price of an average (BLM) AUY is discounted (relative to deeded land) as the proportion of BLM leased land increases by 1%. As can be seen from these estimates, the marginal implicit values of a BLM grazing permit estimated from the model ignoring truncation were smaller in magnitude (less negative)

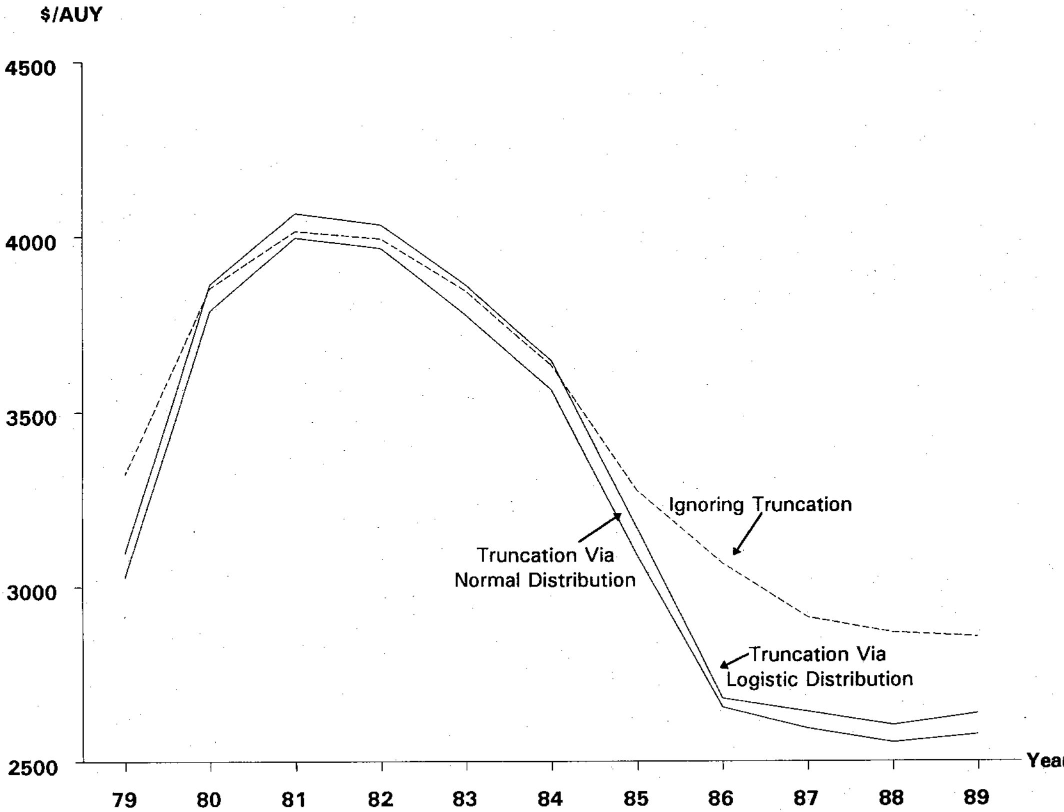

Figure 1. Estimated trend of marginal value of deeded land, 1979-89

than the estimates from the models incorporating truncation. The estimated values from the two models incorporating truncation are very close in each of the 10 years considered.

For the public land policy questions addressed by Torell and Doll, the important model parameters were the price discount for public lands relative to deeded land (β_1 through β_3, and β_{13} through β_{15}), and the rate at which past underpricing of public land forage has been capitalized into the value of public land ranches (β_{12}). Our results suggest that the

Table 4. Comparison of Estimated Marginal Implicit Values of the BLM Land with/without Truncation, 1979-89

	Marginal Implicit Value		
Year	Ignoring Truncation	Truncation via Logistic Distr.	Truncation via Normal Distr.
	($/AUY)		
1979	−38.54	−42.20	−42.66
1980	−37.52	−43.60	−44.08
1981	−36.63	−43.23	−43.65
1982	−35.71	−42.63	−43.07
1983	−34.72	−41.28	−41.96
1984	−33.84	−40.75	−41.45
1985	−32.97	−39.79	−40.56
1986	−32.00	−35.99	−36.44
1988	−30.21	−35.49	−35.98
1989	−29.61	−35.44	−36.05

lack of explicit representation of the truncation effect in Torell and Doll's 1991 model may have led to some distortion in the parameter estimates of key interest to the policy questions addressed by these authors.

Concluding Comments

Given the prevalence of nonnegatively constrained dependent variables in the econometric models of various aspects of the agricultural economy, and given the increasingly available computational ability to perform nonlinear parameter estimation, the time may be right for a more systematic evaluation of the need for explicitly incorporating the effects of the nonnegativity constraint into the specification of model structures. One method of explicitly modeling the nonnegativity constraint is provided by the truncated logistic and normal distribution specifications presented in this article. The method is straightforward to implement using a nonlinear least squares algorithm, and allows a direct test of the significance of the truncation effect. Of course, modeling truncation via distributions other than the logistic or normal can be pursued following an approach analogous to the one used in this study. It also may be possible to represent nonnegativity of the dependent variable without the use of truncated distributions by choosing a disturbance distribution that does not have the real line for its support but, rather, has support that exhibits a finite lower bound equal to or exceeding the critical value of $-g(X; \beta)$. However, in the absence of a priori knowledge relating to the value of β or the form of $g(\cdot)$, this latter approach may not be straightforward. Furthermore, one might attempt to model nonnegativity by utilizing a set of disturbance distributions whose variances are some function of the location of the regression function (i.e., functionally heteroskedastic disturbances) in an attempt to lessen the influence of the lower tail of the disturbance distribution. One might also combine the aforementioned approaches. In any case, modeling the statistical nature of the disturbance term remains an empirical question, and the results of this study suggest that there is an additional issue that deserves future consideration in specifying the functional structure of models purporting to explain prices, quantities, or other nonnegatively constrained random variables—namely, the effect of inherent lower bounds on the support of the error distribution of the model.

In a seeming routine hedonic analysis of land values, the routine assumption that any truncation effect induced by the nonnegativity of land values can be ignored in the specification of the model was rejected for a logistic disturbance distribution in half of the cases analyzed by Xu, Mittelhammer, and Barkley (1991, 1993), and was rejected in this study for both of the models analyzed. One wonders how many other researchers, by routinely dismissing the truncation effect without analysis, have introduced a model misspecification that has detrimentally affected both dependent variable predictions and assessments of explanatory variable impacts.

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Notes

[1] Under the null hypothesis of no truncation effect, the conditional expectation of Y becomes simply $g(x; \beta)$ ($= x\beta$ in linear models), which involves neither truncation terms nor the τ parameter. The statistical test effectively examines whether the truncation terms and the τ parameter are irrelevant in the specification of the conditional expectation of Y. The test is akin to testing hypotheses concerning the functional form parameter, λ, in the Box–Cox model.

[2] An animal unit (AU) is considered to be one mature cow with calf, or the equivalent. An animal unit month (AUM) is the amount of forage required by an AU for one month, and an animal unit year (AUY) is the forage required for an AU for one year.

[3] Details concerning the results of estimating the multiplicative heteroskedasticity model, as well as the results pertaining to the subsequent J-tests contrasting the heteroskedastic and normal truncated models, will be provided by the authors to interested readers upon request.

[4] See Davidson and MacKinnon for additional details concerning the implementation of the J-test. Note that

the J-test was also initially used in an attempt to perform the earlier nonnested tests involving the logistic and normal truncated models, but problems of convergence of the nonlinear least squares algorithms resulted in the use of the P-tests reported previously.

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Appendix A: Derivation of $E(Y \mid Y \geq 0)$

Logistic Distribution

The logistic density function for μ when $E(\mu) = 0$ and $\text{Var}(\mu) = \sigma^2$ is defined by

$$h(\mu) = \frac{\exp(-\mu/\tau)}{\tau[1 + \exp(-\mu/\tau)]^2},$$

where $\tau = \sigma\sqrt{3}/\pi$ is a scale parameter. Given that $Y = g(X) + \mu$ (suppressing the parameter vector β), the density function for Y is then

$$f(Y) = \frac{\exp[-Y - g(X))/\tau]}{\tau[1 + \exp(-(Y - g(X))/\tau)]^2}.$$

By definition, the truncated density for Y when $Y \geq 0$ is given by

$$f(Y \mid Y \geq 0) = \frac{f(Y)}{P(Y \geq 0)} \qquad \text{for } Y \geq 0.$$

Given that the cumulative distribution function for Y is given by

$$F(Y) = \frac{1}{1 + \exp(-(Y - g(X))/\tau)},$$

it follows that

$$P(Y \geq 0) = 1 - F(0) = \frac{\exp(g(X)/\tau)}{1 + \exp(g(X)/\tau)} = \frac{1}{1 + \exp(-g(X)/\tau)}.$$

The derivation of $E(Y \mid Y \geq 0)$ then proceeds as follows:

$$E(Y \mid Y \geq 0)$$

$$= \int_0^\infty Yf(Y \mid Y \geq 0)\, dY$$

$$= P(Y \geq 0)^{-1} \int_0^\infty Y \frac{\exp[-(Y - g(X))/\tau]}{\tau\{1 + \exp[-(Y - g(X))/\tau]\}^2}\, dY$$

$$= P(Y \geq 0)^{-1} \int_{-g(x)/\tau}^{\infty} (g(X) + Z\tau) \frac{\exp[-Z]}{\{1 + \exp[-Z]\}^2} \, dZ \qquad [\text{let } Z = (Y - g(X))/\tau]$$

$$= g(X) + P(Y \geq 0)^{-1} \int_{-g(X)/\tau}^{\infty} Z\tau \frac{\exp[-Z]}{\{1 + \exp[-Z]\}^2} \, dZ$$

$$= g(X) + P(Y \geq 0)^{-1}\tau \left[\frac{\exp(g(X)/\tau)\left(\frac{-g(X)}{\tau}\right) + [1 + \exp(g(X)/\tau)]\ln(1 + \exp(g(X)/\tau))}{1 + \exp(g(X)/\tau)} \right]$$

$$= g(X) - g(X) + P(Y \geq 0)^{-1}\tau \ln(1 + \exp(g(X)/\tau))$$

$$= \left[\frac{\exp(g(X)/\tau}{1 + \exp(g(X)/\tau} \right]^{-1} \tau \ln(1 + \exp(g(X)/\tau))$$

$$= \tau\{1 + \exp[-g(X)/\tau]\}\ln\{1 + \exp[g(X)/\tau]\},$$

where we have used the fact that

$$P(Y \geq 0) = \frac{\exp(g(x)/\tau)}{1 + \exp(g(x)/\tau)}.$$

Normal Distribution

The normal density function for Y, with mean $g(X)$ and variance τ^2, is given by

$$f(Y) = \frac{1}{\sqrt{2\pi\tau^2}} \exp\left[-.5\left(\frac{Y - g(X)}{\tau} \right)^2 \right].$$

The truncated density function for Y is defined by

$$f(Y \mid Y \geq 0) = \frac{f(Y)}{P(Y \geq 0)} \qquad \text{for } Y \geq 0.$$

The term $P(Y \geq 0)$ can be derived as

$$P(Y \geq 0) = \int_0^{\infty} f(Y)\, dY = \int_0^{\infty} \frac{1}{\sqrt{2\pi\tau^2}} \exp\left[-.5\left(\frac{Y - g(X)}{\tau} \right)^2 \right] dY$$

$$= \int_{-g(X)/\tau}^{\infty} \frac{\tau}{\sqrt{2\pi\tau^2}} \exp[-.5(Z)^2]\, dZ \qquad [\text{let } Z = (Y - g(X))/\tau]$$

$$= \int_{-g(X)/\tau}^{\infty} \frac{1}{\sqrt{2\pi}} \exp[-.5(Z)^2]\, dZ = 1 - \Phi[g(X)/\tau].$$

Then the derivation of $E(Y \mid Y \geq 0)$ proceeds as follows:

$$E(Y \mid Y \geq 0)$$

$$= \int_0^{\infty} Yf(Y \mid Y \geq 0)\, dY$$

$$= [\Phi(g(X)/\tau)]^{-1} \int_0^{\infty} Y \frac{1}{\sqrt{2\pi\tau^2}} \exp[-.5((Y - g(X))/\tau)^2]\, dY$$

$$= [\Phi(g(X)/\tau)]^{-1} \int_{-g(X)/\tau}^{\infty} (g(X) + Z\tau) \frac{1}{\sqrt{2\pi}} \exp[-.5Z^2]\, dZ \qquad \left[\text{let } Z = \frac{Y - g(X)}{\tau} \right]$$

$$= g(X) + [\Phi(g(X)/\tau)]^{-1} \int_{-g(X)/\tau}^{\infty} Z\tau \frac{1}{\sqrt{2\pi}} \exp[-.5Z^2]\, dZ$$

$$= g(X) + [\Phi(g(X)/\tau)]^{-1} \int_{.5(-g(X)/\tau)^2}^{\infty} \frac{\tau}{\sqrt{2\pi}} \exp[-V]\, dV \qquad [\text{let } V = .5Z^2]$$

$$= g(X) + [\Phi(g(X)/\tau)]^{-1} \frac{\tau}{\sqrt{2\pi}} [-\exp(-V)] \Big|_{.5(-g(X)/\tau)^2}^{\infty}$$

$$= g(X) + [\Phi(g(X)/\tau)]^{-1}\tau[\phi(g(X)/\tau)].$$

Appendix B: Demonstration that $\lim_{\tau \to 0} E(Y \mid Y \geq 0) = g(X)$ for $g(X) > 0$

Logistic Distribution

Use L'Hopital's rule on $E(Y \mid Y \geq 0)$ as follows:

$$\begin{aligned}\lim_{\tau \to 0} E(Y \mid Y \geq 0) &= \lim_{\tau \to 0} \frac{\partial[(1 + \exp[-g(X)/\tau])\ln(1 + \exp[g(X)/\tau])]/\partial\tau}{\partial(\tau^{-1})/\partial\tau} \\ &= \lim_{\tau \to 0} g(X)\{1 - \exp[-g(X)/\tau]\}\ln\{1 + \exp[g(X)/\tau]\} \\ &= g(X),\end{aligned}$$

where the last equality follows from a second application of L'Hopital's rule.

Normal Distribution

$$\begin{aligned}\lim_{\tau \to 0} E(Y \mid Y \geq 0) &= \lim_{\tau \to 0} [g(X) + [\Phi(g(X)/\tau)]^{-1}\tau[\phi(g(X)/\tau)]] \\ &= g(X),\end{aligned}$$

which follows immediately from the fact that

$$\lim_{\tau \to 0} \phi(g(X)/\tau) = 0 \text{ and } \lim_{\tau \to 0} [\Phi(g(X)/\tau)]^{-1} = 1.$$

Appendix C: Estimating $s^2(t)$ and Choosing t in the Bierens' Test

Let $\Theta = (\beta' \mid \tau)'$. A consistent estimator of the asymptotic variance of $n^{1/2}\hat{M}(t)$ is given by

$$\hat{s}^2(t) = n^{-1}\sum_{j=1}^{n} (Y_j - H(X_j; \hat{\Theta}))^2\left[\exp(t'\xi(X_j)) - \hat{b}(t)\hat{A}^{-1}\frac{\partial H(X_j; \hat{\Theta})}{\partial\Theta'}\right]^2,$$

where

$$\hat{b}(t) = n^{-1}\sum_{j=1}^{n}\left[\frac{\partial H(X_j; \hat{\Theta})}{\partial\Theta'}\exp(t'\xi(X_j))\right]$$

and

$$\hat{A} = n^{-1}\sum_{j=1}^{n}\left[\frac{\partial H(X_j; \hat{\Theta})}{\partial\Theta'}\frac{\partial H(X_j; \hat{\Theta})}{\partial\Theta}\right].$$

Regarding the choice of t, let T represent a hypercube in R^m, where m equals the number of explanatory variables in the hedonic model (i.e., m is the column dimension of X). Following Bierens, define

$$T = \times_{i=1}^{m} [1, 5].$$

Let $t_o, t_1, \ldots, t_v$ be $v + 1$ random choices of $(m \times 1)$ vectors contained in the set T, and define $\hat{t} = \underset{t \in t_1, \ldots, t_v}{\operatorname{argmax}} \hat{W}(t)$. Then t is chosen via the following decision rule:

$$t = \begin{Bmatrix} t_o \\ \hat{t} \end{Bmatrix} \quad \text{if} \quad \begin{Bmatrix} \hat{W}(\hat{t}) - \hat{W}(t_o) \leq \gamma n^{\rho} \\ \hat{W}(\hat{t}) - \hat{W}(t_o) > \gamma n^{\rho} \end{Bmatrix}$$

for positive constants γ and ρ. In this study, $\gamma = \rho = .5$, which was chosen based on Bierens' (p. 1453) Monte Carlo evidence. The value of v is chosen to be equal to the number of data observations available, which in the current application was 452.