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Collective economic decisions and the discursive dilemma

by

Carl Andreas Claussen and Øistein Røisland

❀NB❀ NORGES BANK

Collective Economic Decisions and the Discursive Dilemma*

Carl Andreas Claussen† and Øistein Røisland‡

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Abstract

Most economic decisions involve judgments. When decisions are taken collectively, various judgment aggregation problems may occur. Here we consider an aggregation problem called the 'discursive dilemma', which is characterized by an inconsistency between the aggregate judgment on the *premises* for a conclusion and the aggregate judgment on the conclusion itself. It thus matter for the decision whether the group uses a premise- or a conclusion-based decision-making procedure. The current literature, primarily within jurisprudence, philosophy, and social choice, consider aggregation of qualitative judgments on propositions. Most economic decisions, however, involve *quantitative* judgments on economic variables. We develop a framework that is suitable for analyzing the relevance of the discursive dilemma for economic decisions. Assuming that decisions are reached either through majority voting or by averaging, we find that the dilemma cannot be ruled out, except under some restrictive assumptions about the relationship between the premise-variables and the conclusion.

Keywords: Collective economic decisions, Judgement aggregation, Inconsistency

JEL Classification: D71, E60

†Norges Bank, (Central Bank of Norway), Norges Bank. P.O. Box 1179, Sentrum, 0107 Oslo, Norway (*Phone*: +47–22316104, *fax*: +47–22333568, *Email*: carl-andreas.claussen@norges-bank.no)

‡Norges Bank, (Central Bank of Norway), Norges Bank. P.O. Box 1179, Sentrum, 0107 Oslo, Norway (*Phone*: +47–22316739, *fax*: +47–22333568, *Email*: oistein.roisland@norges-bank.no)

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1   Introduction

Many economic decisions are made by groups rather than individuals. Governments decide fiscal policies, monetary policy committees set interest rates, corporate boards make investment decisions, and families choose their mortgage. Like most other economic decisions, collective decisions are often based on imperfect information and must rely on judgments. For example, interest rate decisions rely on judgments about inflationary pressures and financial fragility, corporate investment decisions rely on judgments of future cash flows and cost of capital, and so on.

Aggregating individual judgments to a 'group judgment' is not straight forward. Recent research, primarily within jurisprudence, philosophy, and social choice, shows that group judgments may be subject to a 'discursive dilemma', see e.g. Dietrich (2003) and List (2004a).[1] The dilemma can be illustrated by the following (fictitious) example: Suppose that George Bush, Colin Powell, and Donald Rumsfeld came together some day in 2002 to decide whether the US should invade Iraq. They agreed that the premises for an invasion are that the following two propositions were judged true: (i) Iraq hides weapons of mass destruction, and (ii) the war can be won with 'acceptable' military losses. This logical link between the judgments on (i) and (ii) and the conclusion is denoted the *rule of inference.* Suppose the individual judgments were as in Table 1.

Table 1

	Weapons of mass destruction?	Acceptable losses?	Invasion?
Bush	Yes	No	No
Powell	No	Yes	No
Rumsfeld	Yes	Yes	Yes
Majority	Yes	Yes	No

As the bottom row shows, the group's aggregate conclusion (No) is inconsistent with its aggregate judgments on the propositions (Yes, Yes) and the rule of inference. This inconsistency makes the group's decision depend, not only on the policymakers' judgments and aggregation method (majority, consensus etc.), but also on its decision procedure. A premise-based decision procedure is a procedure where the policymakers vote on (i) and (ii) separately, and then let the rule of inference dictate the conclusion. If the group used this procedure there would be an invasion. If they instead used a conclusion-based procedure, and voted directly on the conclusion, there would not be an invasion.

The existing literature on the discursive dilemma only looks at binary judgment aggregation. The aggregation is binary because the premises and

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[1]It is also known as the 'doctrinal paradox'.

the conclusion are yes/no judgments on propositions, as in the example above. Such aggregation of yes/no judgments is relevant for many types of decisions in groups. However, most *economic* decisions are not binary. Rather, the typical economic decision-making problem is to find the correct or optimal level of a continuous variable. Furthermore, the premises for the conclusion are typically judgments on continuous variables. Generally, the rule of inference for many economic decisions may be written as

$$c = f(p_1, p_2, ..., p_k), \tag{1}$$

where c is a continuous conclusion variable (e.g., the interest rate, the tax rate, the level of investments, etc.), $p_1, p_2, ..., p_k$ are continuous premise variables, i.e., the information set on which the economic decision is based, and $f(\cdot)$ is some function. The following example illustrates that there are discursive dilemmas also in this case. Consider a group of three policymakers who decide on the size of a policy variable c, the 'conclusion variable'. They all agree that c should depend on the judgment on one premise variable p, and the 'rule of inference' $f(p) = p^2$. Suppose the individual judgments are as in Table 2, and that the aggregation method is majority voting, where the outcome of a vote on a variable is the median judgment on that variable.

Table 2

	p	$c = p^2$	c
Individual 1	-1	Agree	1
Individual 2	0	Agree	0
Individual 3	1	Agree	1
Median	0	Agree	1

As the first three rows show, the individual conclusions are consistent with the judgments on the premise variable and the rule of inference. However, the aggregate judgments are not consistent, since $0^2 \neq 1$ (bottom row of Table 2). Furthermore, and as a consequence of this inconsistency, a direct vote on the conclusion gives $c = 1$, while separate vote on p and the rule of inference gives $c = 0$.

Any finite set of judgments on variables can be translated into judgments on a set of propositions.[2] Thus, both the example above – and any other finite set of judgments – can be translated into the binary model. However, as we show in the paper, for decisions that can be represented by (1), the existence of the discursive dilemma depends crucially on the functional form of $f(\cdot)$. The binary framework does not make the functional form of $f(\cdot)$

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[2] Any ordering $\succ$ on a set of mutually exclusive judgments, $\{p', p'', p'''\}$ can always be expressed as a set of propositions of the type $\boldsymbol{\rho} = \{p' \succ p'', p'' \succ p''', p' \succ p'''\}$, where '$\succ$' means "closer to truth".

explicit.[3] The binary framework is therefore not well suited for analyzing the discursive dilemma when the premises and the conclusion are judgments on variables. Furthermore, economic decisions are usually modelled as in equation (1). Having to translate these models into a binary framework is both cumbersome and inefficient. For these reasons, we develop a model where conditions for the existence of the discursive dilemma can be analyzed directly. Assuming that decisions are reached either through majority voting or by averaging, we find that the dilemma cannot be ruled out, except under some restrictive assumptions about the rule of inference. Even if we for mathematical convenience focus on continuous variables, the results hold also when the variables are many-valued but discrete.

Although our approach is new, it builds on the literature on binary judgment aggregation. This literature focus on the possibility of consistent judgment aggregation under various conditions. List and Pettit (2002) and List and Pettit (2004) developed a first model of judgment aggregation based on propositional logic and proved an impossibility result. This has later been followed by several stronger impossibility results (Pauly and van Hees (2003); Dietrich (2003); Gärdenfors (2004); van Hees (2004); Nehring and Puppe (2005); Dietrich and List (2005a)) and possibility results (Bovens and Rabinowicz (2004); Dietrich (2003); List (2003), List (2004a), List (2004b); Pigozzi (2004)). Generalizing this approach, Dietrich (2004) has developed a model of judgment aggregation in general logics, which allows the representation of a larger class of aggregation problems.[4]

The impossibility results of the existing literature states that there is no non-dictatorial aggregation method that generally produces consistent collective judgments on interconnected propositions and satisfies some minimal conditions. Since, as mentioned above, a finite set of judgments on continuous variables can be translated into judgments on a set of interconnected propositions, the impossibility results apply also to the continuous variables case. However, with judgments on variables that can take more than two values, inconsistent collective judgments is a necessary, but not a sufficient condition for a discursive dilemma (see Section 4). What is interesting for economic policy is the cases when the inconsistent collective judgments imply that the decision depends on the decision procedure. Our framework enables us to focus on the discursive dilemma only, and not inconsistent collective judgments in general.

It should be noted that judgment aggregation, as studied here, is different from the more traditional discipline of social choice, which was sparked off by Arrow's seminal work (Arrow (1951/1963)). Traditional social choice concerns the problem of aggregating individual preference orderings over

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[3]In the binary framework the functional form is hidden in a set of propositions of the type 'if $p = p'$ then $c = c'$'.

[4]Hylland and Zeckhauser (1979) give an impossibility result for the aggregation of probablility assesments and utilities.

several alternatives into an aggregate preference ordering over these alternatives. Applied to our judgment aggregation setting, this concerns the problem of aggregating individual orderings on alternative judgments on *one* variable into a corresponding aggregate ordering over the judgments on this variable. In table 2, for example, traditional social choice would concern the problem of aggregating the three individuals' orderings over the three judgments on one variable (p or c) into a collective judgment on the *same* variable. In contrast, the type of judgment aggregation we study concerns the consistency between judgments on *different* variables, i.e. between judgments on the premise variables and the judgments on the conclusion (decision) variable. List and Pettit (2004) discuss the relation between Arrows impossibility theorem and the impossibility results on judgment aggregation. Dietrich and List (2005b) prove that that Arrow's theorem is a direct corollary of a more general result on judgment aggregation.

In Section 2, we introduce the analytical framework and present the general results. We present some applications of our results to specific economic decisions in Section 3, and provide a discussion of the assumptions. Section 4. Section 5 concludes.

2 Analytical framework

2.1 Model

Consider a group where N denotes the set of members and where $|N| = n$ is odd, finite and greater than 1. The group, which could be a government, a monetary policy committee, a corporate board, an expert panel, etc., has to make a conclusion on the size of a policy parameter $c \in \mathbb{R}$. The policy parameter could be the level of a tax or a tariff, the interest rate, the optimal size of a plant, etc.

The members of the group agree that their conclusion should depend on the judgments on k premise variables $p_1, p_2, ..., p_k$. Each member $i \in N$ has a separate judgment p_{ij} on each premise variable p_j where $j \in J$, $J = (1, 2, ..., k)$. The set of possible judgments on all premise variables is a Cartesian product of possible judgments for each premise variable. Formally,

Assumption 1 The *set of possible judgments on the premise variables* is $Q = \prod_{j \in J} \left[p_j^-, p_j^+ \right]$ where $J = (1, 2, ..., k)$ and $p_j^- < p_j^+$ for all $j \in J$, $p_j^-, p_j^+ \in \mathbb{R}$.

Individual i's vector of judgments is denoted $\mathbf{p}_i = (p_{i1}, p_{i2}, \ldots, p_{ik})$, where $\mathbf{p}_i \in Q$. The sets of premise and conclusion judgments for the whole group are denoted $P = \{\mathbf{p}_i\}_{i \in N}$ and $C = \{c_i\}_{i \in N}$, respectively. We think of P and C as the judgments that exist after the members of the group have shared the information they possess.

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A 'rule of inference' establishes the logical link between judgments on the premise variables and the conclusion. The rule may, for example, be an explicit formula like $c = p^2$ in Table 2, or the Taylor rule in monetary policy (c.f. Sect. 3). It can also be a more complicated economic model, or an approximation of essential facets of the group's thinking about how premises and the conclusion are logically linked.

Definition 1 *A rule of inference $f(\mathbf{p})$ is a continuously differentiable function that for each set of judgments $\mathbf{p} = (p_1, p_2, ..., p_k) \in Q$ and for each $i \in N$ specifies a conclusion c:*

$$c = f : Q \to \mathbb{R}$$

We abstract from judgment aggregation problems that arise because the individuals have different rules of inference. Hence,

Assumption 2 The individuals have the same rule of inference $c = f(\mathbf{p})$

In line with most of the literature on binary judgment aggregation, we abstract from strategic behavior. The individuals are assumed to report their true judgments.

Assumption 3 *Sincere behavior.* All members of N always report their true judgments and reveal all relevant information they possess.[5]

Denote the vector of the group's aggregate judgments $\mathbf{p}^A = \left(p_1^A, p_2^A, ..., p_k^A\right)$ and the aggregate judgment on the conclusion c^A. Then, if the group aggregates the conclusion directly, for example by voting directly on the conclusion, the aggregate conclusion is c^A. Call such a decision procedure a conclusion-based decision-making procedure (CBP). If the group aggregates the judgments on the premise variables and uses the rule of inference to generate a conclusion, the aggregate conclusion (decision) is $f\left(\mathbf{p}^A\right)$. Call such a decision procedure a premise-based decision procedure (PBP). We say there is a 'discursive dilemma' if the CBP gives a different decision (conclusion) than the PBP. Hence,

Definition 2 *There is a discursive dilemma if $c^A \neq f\left(\mathbf{p}^A\right)$*

Generally, groups may aggregate their judgments in many ways. The existing literature on the discursive dilemma focusses on voting. Recently, the literature on monetary policy committees has also considered 'averaging'

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[5]Assumption 3 and our interpretation of P and C as the set of judgments that exists after the members of the group have shared all relevant information, imply that for $\mathbf{p}_i \neq \mathbf{p}_j$ for some $i, j \in N$, there have to be (i) some imperfections in the information transmission within the group, or (ii) differences between the individuals that make them form different judgments for the same set of information.

as an aggregation procedure, where the group's aggregate judgment is the average of the individual judgments, see Munnich et.al. (1999), Blinder and Morgan (2000), and Gerlach-Kristen (2003), and therefore we analyze this type of judgment aggregation in addition to majority voting. Note also that under certain assumptions, decisions based on consensus can be expressed as an average of the initial judgments, see DeGroot (1974), Chatterjee and Seneta (1977), and Berger (1981).

To model majority voting, we assume that the individuals' ordering on the judgments on each variable p_j, $j \in J$, and the ordering on the conclusion c, are single-peaked. Denote the median judgments on premise j for p_j^m, and the median judgment on the conclusion c^m. With single-peaked orderings, p_j^m will beat any other judgment in a pair-wise vote over the judgments on p_j. Similarly c^m will beat any other alternative in a pair-wise vote over the judgments on c. Hence, if majority voting is used to aggregate judgments, the aggregate judgments are given by $p^m = (p_1^m, p_2^m, ..., p_k^m)$ and c^m.

Under averaging, the vector of aggregate judgments on the premises and conclusion is given by $\mathbf{p}^{avg}$ and c^{avg} where $\mathbf{p}^{avg} = (p_1^{avg}, p_2^{avg}, ..., p_k^{avg})$, and $p_j^{avg} = \sum_{i=1}^{n} \frac{1}{n} p_{ij}$ and $c^{avg} = \sum_{i=1}^{n} \frac{1}{n} c_i$.

2.2 Results

2.2.1 Majority voting and $k = 1$

We start by looking at the simpler situation where $k = 1$. Let $\theta(p_i)$ be the numerical position of $p_i \in P$ when the elements of P are arranged in an increasing order. Similarly, let $\theta(c_i)$ be the numerical position of $c_i \in C$ when the elements of C are arranged in increasing order. A necessary condition for a dilemma under majority voting is that $\theta(p_i) \neq \theta(c_i)$ for some q and $s \in N$ where $p_q \neq p_s$. The necessary condition can only be fulfilled if $f(p)$ is non-monotonic on Q. If not, $\theta(c_i)$ is determined entirely by $\theta(p_i)$. Generally, when $k = 1$, the numerical position of an element $c_i \in C$ depends on two factors: (i) the numerical position of $p_i \in P$, and (ii) the functional form of the rule of inference. Thus:

Proposition 1 *If N aggregates judgments by majority voting and $k = 1$, then*

(i) $c^m = f(p^m)$ for all $P \subset Q$ if $f(p)$ is monotonic for $p \in Q$,

(ii) there exists a $P \subset Q$ such that $c^m \neq f(p^m)$ if $f(p)$ is non-monotonic for $p \in Q$.

Proof. As indicated before the proposition. ∎

Proposition 1 states that if the group aggregates judgments by majority voting, a discursive dilemma cannot be ruled out if the rule of inference is non-monotonic on its domain. It can only be ruled out if the rule of inference is monotonic in its domain.

A general proposition for when there will be a dilemma does not exist, since the existence of a dilemma depends both on the functional form of the rule of inference and the particular set of judgments. With assumptions 1–3 there always exist sets of judgments (P) where all elements are in the monotonic parts of a rule. Furthermore, even if the set of judgments covers also the non-monotonic part of the rule, there may still be a set of judgments that generates a linear relationship between the judgments and the conclusion.[6] However, for non-monotonic rules with only one local maximum or minimum we can reach a stronger conclusion. Let $p^{\max} \equiv \max P$ and $p^{\min} \equiv \min P$, and $p^* \equiv \arg\max f(p)$ if $f(p)$ is non-monotonic with one local maximum, and $p^* \equiv \arg\max -f(p)$ if $f(p)$ is non-monotonic with one local minimum. Call the set of judgments P *dispersed* if it has elements in both the increasing and decreasing parts of the rule of inference, i.e.

Definition 3 *The set of judgments P is dispersed if $p^{\min} < p^*$, and $p^{\max} > p^*$.*

We then have the following result.

Corollary 1 *If N aggregates judgments by majority voting, $k = 1$, and $f(p)$ has either one local maximum or one local minimum, then there will be a discursive dilemma iff P is dispersed and*
   (a) $f(p)$ has one local maximum and $f(p^{\max}) < f(p^m)$ and $f(p^{\min}) < f(p^m)$, or
   (b) $f(p)$ has one local minimum and $f(p^{\max}) > f(p^m)$ and $f(p^{\min}) > f(p^m)$.

Proof. See appendix. ∎

Corollary 1 provides sufficient conditions for a dilemma. The corollary says that in order for there to be a dilemma, there must be judgments on both the increasing and decreasing parts of the rule. Furthermore, if the rule of inference has one local maximum, the highest and lowest judgments on the premise variable must imply a lower conclusion than the one that follows from the median judgment. Similarly, if the rule of inference has one local minimum, the highest and lowest judgments on the premise variable must imply a higher conclusion than the one that follows from the median judgment.

We may also analyze the way that the decision is skewed, depending on which decision procedure is used.

Corollary 2 *If N aggregates judgments by majority voting, $k = 1$, and $f(p)$ has either one local maximum or one local minimum, then*
   (a) $c^m \leqslant f(p^m)$ if $f(p)$ is non-monotonic with one local maximum, and
   (b) $c^m \geqslant f(p^m)$ if $f(p)$ is non-monotonic with one local minimum.

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[6]For example, $k = 1$, $n = 3$, $Q = \mathbb{R}$, $f(p) = \sin p$, and $P = (-\pi, 0, \pi)$

Proof. See proof of Corollary 1 in Appendix. ■

Corollary 2 says that the CBP tends to yield a lower (higher) c than the PBP when $f(p)$ is concave (convex).

2.2.2 Majority voting and $k > 1$

If $k > 1$, the numerical position of an element $c_i \in C$ depends on the shape of the rule of inference and the numerical position of the judgments on *two* different premise variables. A simple example where $f(\mathbf{p}) = p_1 + p_2$, $n = 3$, $P = \{(2, 3), (4, 1), (1, 2)\}$ shows that with $k > 1$, a linear rule of inference does not rule out a dilemma. Our second proposition generalizes this insight.

Proposition 2 *If N aggregates judgments by a simple majority rule, and $k > 1$, then there exists a $P \subset Q$ such that $c^m \neq f(\mathbf{p}^m)$.*

Proof. See appendix. ■

Proposition 2 states that if the group aggregates judgments by majority voting, and there is more than one premise variable, then a discursive dilemma cannot be ruled out.

As in the case when $k = 1$, it is not possible to specify a general theorem for when there *will be* a dilemma. Nor do there exist specific functional forms $f(\mathbf{p})$ for which there will never be a dilemma.

2.2.3 Averaging

If the rule of inference is linear, there can never be a dilemma under averaging since then $c^{avg} = f(\mathbf{p}^{avg})$ for any $\mathbf{p}$. If $f(\mathbf{p})$ is strictly concave or convex and $k = 1$ there must be a dilemma if the individuals have different judgments on the premise variable (which also follows from Jensen's inequality). Thus, if $k = 1$ and the rule of inference is non-linear on Q, then there exist sets of judgments with a discursive dilemma. Our third proposition generalizes this result.

Proposition 3 *If N aggregates judgments by averaging, then*

(i) $c^{avg} = f(\mathbf{p}^{avg})$ for all $P \subset Q$ if $f(p)$ is a linear function for all $\mathbf{p} \in Q$,

(ii) there exists a $P \subset Q$ such that $c^{avg} \neq f(\mathbf{p}^{avg})$ if $f(p)$ is a non-linear function for some $\mathbf{p} \in Q$.

Proof. See Appendix. ■

Proposition 3 states that if the group aggregates judgments by averaging, a discursive dilemma cannot be ruled out if the rule of inference is non-linear on its domain. It can be ruled out if the rule of inference is a linear function.

If the rule of inference is *strictly* concave or convex we can make two corollaries. The first regards a situation that may very well prevail.

Corollary 3 *If N aggregates judgments by averaging, $k = 1$, $f(p)$ is strictly concave or convex for $p \in Q$, and $p_i \neq p_s$ and $i, s \in N$, then $c^{avg} \neq f(p^{avg})$.*

Proof. Jensen's inequality ∎

Corollary 3 says that with averaging there will always be a dilemma if at least two individuals have different judgments on the same premise variable, and the rule is strictly concave or convex.

The second corollary regards how the decision will be skewed depending on the decision procedure.

Corollary 4 *If N aggregates judgments by averaging and $k = 1$, then $c^{avg} \leq f(p^{avg})$ when $f(p)$ is strictly concave in Q, and $c^{avg} \geq f(p^{avg})$ when $f(p)$ is strictly convex in Q.*

Proof. Jensen's inequality ∎

The corollary says that the CBP tends to give a lower(higher) c than the PBP when the rule of inference is strictly concave(convex).

3 Applications

3.1 Linear rules of inference

Monetary policy decisions are usually taken by a group, often called a monetary policy committee (MPC), and involve judgments on many variables.

It has become popular to specify interest rate decisions in terms of an 'interest rate rule', for example a Taylor rule (Taylor (1993)). Suppose that all the members of the MPC specify their interest rate proposals according to the following (classic) Taylor rule

$$i_t = r_t^* + \pi^* + a(\pi_t - \pi^*) + by_t, \tag{2}$$

where i_t is the nominal interest rate in period t, r_t^* is the neutral/natural real interest rate, which is assumed to vary over time, π^* is the desired rate of inflation (inflation target), π_t is actual inflation, and y_t is the output gap. In practice, the neutral real interest rate r_t^* and the output gap y_t cannot be observed and are subject to judgment. It is therefore reasonable to assume that the MPC members will, to some extent, disagree on the estimates of these variables. We assume that π_t can be observed perfectly, and with no loss of generality we consider a situation where inflation is equal to the target, i.e. $\pi_t = \pi^* = 2$. Moreover, we set $b = 0.5$ as in Taylor's (1993) classic specification. In order to keep the example as simple as possible, suppose the MCP consists of three members. Members each have their own estimates of r_t^* and y_t, represented in Table 3.

Table 3

	r_t^*	y_t	i_t
Member 1	2.1	2.6	5.4
Member 2	3.0	1.0	5.5
Member 3	2.2	1.2	4.8

In voting, the possibility of a dilemma can never be excluded, c.f. Proposition 2. From Table 3 we see that $i_t^m = 5.4$, while $r_t^{*m} + 2 + 0.5 y_t^m = 4.8$, and the discursive dilemma therefore applies in the example. With averaging, there will never be a dilemma when the rule is linear, c.f. proposition 3. In the example in table 3 we have $i_t^{avg} = 5.2$ and $r_t^{*avg} + 2 + 0.5 y_t^{avg} = 5.2$.

3.2 Non-linear rules of inference

One type of premise variable that typically yields a non-linear rule of inference is the effects of the policy instrument. When deciding the appropriate level of the policy instrument, one has to take into account how the instrument affects the target variable(s). In many situations there will be disagreement about the exact effects of the policy instrument.

The difference between linear and non-linear rules of inference has its counterpart in the difference between additive and multiplicative uncertainty. Differences in individual judgments are caused by uncertainty, and it is natural to relate judgment aggregation problems to the literature on policymaking under uncertainty. We will therefore illustrate the discursive dilemma within the framework of the classic model by Brainard (1967). Suppose that the relationship between the target variable y and the policy instrument z is given by

$$y = \alpha z + x, \tag{3}$$

where x represents exogenous variables that affect the policymakers' target variable. Equation (3) may represent a wide range of policy effects, for example, the monetary policy transmission mechanism, the effect of unemployment benefits on equilibrium unemployment, the effect of tariffs on the trade balance, the effect of fiscal expenditures on GDP, and so on. In many theoretical models, one can log-linearize the reduced form and yield an expression equivalent to (3).

We assume that α cannot be observed by the policymakers and is perceived as stochastic. Committee members each have their own estimate/judgment of α, denoted α_i, $i = 1, .., n$. For simplicity, we assume that each committee member perceives α to be equally uncertain, represented by the variance σ_α^2, which therefore has no subscript for committee member. The policymakers' objective is to set the policy instrument such that the target variable y is brought as close as possible to the target level y^*. The objective function is quadratic and given by

$$U = -(y - y^*)^2. \tag{4}$$

Due to uncertainty about α, the committee seeks to maximize $E(U)$ with respect to z. Member i's policy proposal is based on maximizing $E_i(U)$, where E_i is the expectations operator based on member i's estimate of α. Straight-forward optimization gives the following level for the policy instrument proposed by member i,

$$z_i = \frac{\alpha_i}{\alpha_i^2 + \sigma_\alpha^2}(y^* - x). \tag{5}$$

We will denote $\frac{\alpha_i}{\alpha_i^2 + \sigma_\alpha^2}$ the 'policy response coefficient', as it says how strongly the policy instrument responds to the exogenous variables. Without loss of generality, we normalize $(y^* - x)$ to one, so that the rule of inference can be written as

$$f(\alpha) = \frac{\alpha}{\alpha^2 + \sigma_\alpha^2} \tag{6}$$

Figure 1 shows the shape of the policy response coefficient when $\alpha > 0$.[7] The rule is clearly non-monotonic.
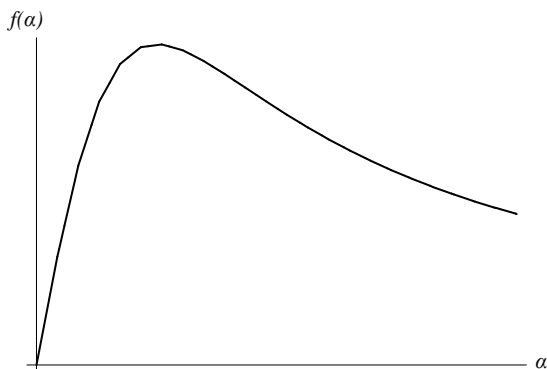

Figure 1

Consider first *voting*. We have from Proposition 1 that with one premise variable, one cannot exclude a dilemma if the rule of inference is non-monotonic. Thus, a discursive dilemma cannot be ruled out if α can take

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[7] The figure for $\alpha < 0$ is the mirror image.

values on both the increasing and decreasing part of $f(\alpha)$. Whether there actually will be a dilemma depends on the distribution of estimates/judgments. According to Corollary 1, a necessary and sufficient condition for the dilemma when the rule of inference is given by (6) is

$$f(\alpha_m) > f(\alpha_{\min}) \text{ and } f(\alpha_m) > f(\alpha_{\max})$$

If the discursive dilemma applies, a conclusion-based decision procedure will always give rise to a more cautious policy response than a premise-based procedure (corollary 2). An extreme case is when the distribution of estimates is such that $f'(\alpha_m) = 0$. In that case the premise-based decision procedure will give a policy response that is based on the most extreme value of the members' response coefficients.

Consider next *averaging*, and note that the rule of inference is single-peaked and globally non-linear. It thus satisfies the conditions in Corollary 3, so that decisions based on averaging will generally yield a discursive dilemma. An important question is whether a premise-based procedure would result in a more or a less cautious policy (in addition to the cautiousness due to multiplicative uncertainty). From Corollary 4, we know that a premise-based procedure would give a weaker policy response if the rule of inference is strictly concave in α_i, while it will give a stronger policy response if it is strictly convex. We know that the rule of inference is strictly concave when $0 < \alpha_i < \sqrt{3}\sigma$ and strictly convex when $\alpha_i > \sqrt{3}\sigma$. The sign of the discursive dilemma is therefore ambiguous. However, the higher the degree of uncertainty relative to the point estimate, the more likely it is that the conclusion-based procedure will yield a weaker policy response than a premise-based procedure.

4   Discussion

We have assumed that the premises are continuous variables (Assumption 1). This assumption is not necessary for our results, but is convenient as it rules out particular combinations of Q and $f(\mathbf{p})$ for which there will never be a dilemma.[8] It is easy to construct examples with a dilemma even if premises and conclusions are not continuous variables.[9] As long as the premise variables are not perfectly correlated our results also hold true if the domain is more restricted than a Cartesian product (Assumption 1).

Assumption 2, that all individuals have the same rule of inference, may seem very restrictive. However, it does *not* mean that the individuals have to agree on a specific policy rule (e.g., a Taylor rule). $f(\mathbf{p})$ represents what all members of the group can subscribe to. For example, consider the following 'policy rule': $c = \alpha x$, where c is the decision variable (e.g., the

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[8]Example: $p(\mathbf{x}) = ax_1 + b\sin x_2$, and $Q := \{x_1 \in \mathbb{R}, \ x_2 = -2\pi, 0, 2\pi\}$

[9]Example: $p(\mathbf{x}) = x_1 + x_2$, $Q := \{x_1 \in 1, 2, 3; x_2 \in 1, 2, 3\}$.

central bank's key interest rate), x is an economic variable (e.g., the rate of underlying inflation), and α is a parameter that says how much a change in x should affect c. If the individuals disagree on both x and α, the rule of inference has two premise variables; x and α. One may easily generalize this example to show that each individual may have a different policy rule for their decisions – even policy rules with different right-hand side variables and functional forms – but yet it will be possible to formulate a rule of inference that all agree on.

We have also assumed that the individuals report their true judgments (Assumption 3). Our results hinge on this assumption. Suppose, for example, that $k = 1$, $f(p)$ is non-monotonic with one local maximum and $n = 3$. Then, if the decision procedure is majority voting, the individual with the median conclusion judgment (c^m) can always report a false premise judgment so that the conclusion under a premise-based decision procedure will be c^m (c.f. figure 1). If the decision procedure is averaging, each member can report a judgement that affects the average judgement so that it coincides with her own true judgement. Consequently there is no pure strategy Nash equilibrium.[10] However, there are good reasons for making Assumption 3. The first is methodological. In order to analyze strategic behavior, one must first understand the equilibria without strategic behavior. Second, sincere behavior is a reasonable assumption for expert panels and policy committees like (some) MPCs. Such groups are supposed to pool information and judgment, not to aggregate preferences. The members of such groups are supposed not to let their preferences over outcomes influence their behavior.

There already exist impossibility theorems for the aggregation of judgments on interconnected propositions (binary decisions), c.f. the introduction. Since judgments on variables that can take many values can be mirrored in a set of judgments on interconnected propositions, our exercise may therefore seem superfluous. However, with judgments on variables that can take more than two values, inconsistent collective judgments is only a necessary, and not a sufficient condition for a discursive dilemma. A simple example prove this.

Suppose $k = 1$, $N = \{A, B, C\}$ and $f(\mathbf{p}) = p$. Let $\mathbf{P} = (p_A, p_B, p_C)$, and the ordering on this set of judgments be

$A : p_A \succ p_B \succ p_C,$

$B : p_B \succ p_C \succ p_A,$

$C : p_C \succ p_B \succ p_A.$

It follows that $p^m = p_B$, and $c^m = c_B$. Thus, there is no discursive dilemma as we have defined it (c.f. definition 2).

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[10]List (2004c) discusses strategic voting in the aggregation of judgments on interconnected propositions (the binary case), and notes that if all individuals act strategically under majority voting, a (formal) premise-based procedure will give a decision that is identical to that of a conclusion-based procedure.

Now, let the propositions ρ_1, ρ_2 be defined as $\rho_1 : p_B \succ p_A$ and $\rho_2 : p_B \succ p_C$. Let ρ_3 be the proposition that ρ_1 and ρ_2 are true: $(\rho_3 \leftrightarrow \rho_1 \wedge \rho_2)$. We can then summarize the individuals' judgments on these propositions as in the three first rows of Table 4.

Table 4

	ρ_1	ρ_2	ρ_3
Individual A	No	Yes	No
Individual B	Yes	Yes	Yes
Individual C	Yes	No	No
Majority	Yes	Yes	No

The collective judgments are clearly inconsistent (bottom row). Thus, in the example there are inconsistent collective judgments, but no discursive dilemma.

The interesting cases for economic policy, are the cases when the inconsistent collective judgments imply a discursive dilemma. We therefore develop a framework that enables us to focus on the discursive dilemma only, and not inconsistent collective judgments in general.

5 Summing up

In this paper we have developed a model to study an inconsistency that may arise when individual judgments on a set of continuous premise variables and a continuous conclusion variable are aggregated into group judgments on these variables. We have looked at two aggregation methods: majority voting and averaging. We have shown that in both cases the group's conclusion is prone to be inconsistent with the collective judgments on the premise variables. This inconsistency arises even though each individual have consistent judgments. The aggregate inconsistency makes the decision depend on the group's *decision procedure*: a conclusion-based decision procedure, where the group aggregates the conclusion directly, gives another decision than a premise-based decision procedure, where the group first aggregates the judgments on the premise variables and then lets these aggregate judgments dictate the decision. We find that the possibility of an inconsistency depends on the combination of two factors: (i) the functional form of a 'rule of inference', which represents the logical link between the conclusion and the judgments on premise variables, and (ii) the set of possible judgments on the conclusion variable and the premise variables.

Although we are particularly interested in collective *economic* decisions, our findings are relevant for many other collective decisions. A team of doctors deciding how much of a drug to give a patient, juries deciding the duration of a prison sentences, etc. will face aggregation problems. Their decision may depend on the decision procedure. Generally, the results apply

to any collective decision that depends on the judgments on a set of premise variables.

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Appendix. Proofs

Proof of Corollary 1

Let $P^{<m} := \{p \in P \mid p < p^m\}$ and $P^{>m} := \{p \in P \mid p > p^m\}$

Part (a):

Step 1:

Suppose $f(p^{\max}) > f(p^m)$. Then $p^m < p^*$. Consequently $f(p) < f(p^m)$ for $p \in P^{<m}$ and $f(p) > f(p^m)$ for $p \in P^{>m}$ which implies that $c^m = f(p^m)$.

Suppose $f(p^{\min}) > f(p^m)$. Then $p^m > p^*$. Consequently $f(p) < f(p^m)$ for $p \in P^{>m}$ and $f(p) > f(p^m)$ for $p \in P^{<m}$ which implies that $c^m = f(p^m)$.

Suppose $f(p^{\max}) < f(p^m)$ and $f(p^{\min}) < f(p^m)$. Then $f(p) < f(p^m)$ for $p \in P^{<m}$ and $P^{\max}$, or $f(p) < f(p^m)$ for $p \in P^{>m}$ and $P^{\min}$ which imply that $c^m < f(p^m)$.

Step 2:

Suppose $f(p^{\max}) = f(p^m)$. If $f(p^{\max}) = f(p^m)$ because $p^m = p^{\max}$, then $c^m = f(p^m)$. If $f(p^{\max}) = f(p^m)$ and $p^m \neq p^{\max}$, then $f(p) < f(p^m)$ for $p \in P^{<m}$ and $f(p) \geq f(p^m)$ for $p \in P^{>m}$, and consequently $c^m = f(p^m)$. The proof for $f(p^{\min}) = f(p^m) \implies c^m = f(p^m)$ is parallel.

Part (b): Parallels the proof of (a).

Proof of Proposition 2.

Assume that $k = 2$, i.e. $c = f(p_1, p_2)$. Let $(p_1', p_2') \in Q$ be judgments on the premise variables such that $f(p_1', p_2') = c'$, and $(p_1'', p_2'') \in Q$ be judgments on the premise variables such that

$$f(p_1', p_2') < f(p_1', p_2'') \text{ and } f(p_1', p_2') < f(p_1'', p_2'). \tag{7}$$

Suppose $n = 3$, and $\mathbf{p}_1 = (p_{11}', p_{12}')$, $\mathbf{p}_2 = (p_{21}', p_{22}'')$, $\mathbf{p}_3 = (p_{31}'', p_{32}')$. Then $\mathbf{p}^m = (p_1', p_2')$, and it follows from (7) that $c^m > f(\mathbf{p}^m)$ and there is a discursive dilemma. Under assumption 1 judgments that fulfills (7) always exist. Thus proposition 1 for the case when $n = 3$, and $k = 2$.

If $k = 2$ and $n > 3$, there always exist – under assumption 1 – sets of judgments that fulfills criteria similar to (7).

If $k > 2$, there will be a dilemma when the members of the committee agree on all premise variables except for two where the judgments fulfill (7).

Proof of Proposition 3.

Part *(i)*: Property of linear functions.

Part *(ii)*: Let $P' := \{P \in Q \mid p_{ij} = p_{zj} \text{ for } i, z \in N \text{ and } j \in J \setminus \{s\}\}$. Then, since $f(\mathbf{p})$ is a non-linear function for $p_s \in Q$, there exists a set of judgments $P \in P'$ where $p_{is} \neq p_{zs}$ such that $c^{avg} \neq f(\mathbf{p}^{avg})$ (Jensen's inequality).

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