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Strategic Investor Behaviour and the Volume-Volatility Relation in Equity Markets

by

Randi Næs and Johannes A. Skjeltorp

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Strategic Investor Behaviour and the Volume-Volatility Relation in Equity Markets*

Randi Næs
Norges Bank, Bankplassen 2, 0107 Oslo, Norway and
Norwegian School of Economics and Business Administration (NHH)

and

Johannes A. Skjeltorp†
Norges Bank, Bankplassen 2, 0107 Oslo, Norway and
Norwegian School of Management (BI)

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Abstract

We examine the volume-volatility relation using detailed data from a limit order driven equity market. Estimates of the intraday slope of the demand and supply schedules of the order book are found to capture regularities in spreads, trade size and submission strategies which are believed to be related to asymmetric information. On a daily level, the order book slope should also captures differences in dispersion of beliefs about stock values. The relationship between our daily slope measure and the contemporaneous volatility across companies and time supports models where strategic trading and dispersion of beliefs increase both volume and volatility.

Keywords: Market Microstructure, Equity Trading, Asymmetric Information

JEL Codes: G10, G20

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†Corresponding author. E-mail: johannes-a.skjeltorp@norges-bank.no Phone:(+47)22316740 Fax:(+47)22424062

1 Introduction

A variety of studies document that there is a positive correlation between price volatility and trading volume for most types of financial contracts including stocks, Treasury bills, currencies and various futures contracts. Theoretically this can be explained by new information on asset values acting as the driving force on both market prices and trading volume. However, for many types of financial contracts, movements in prices seem much "too large" to be attributed to movements in the fundamental values of the underlying securities.[1] A suggested explanation of this puzzle is that prices are not merely driven by changes in systematic risk factors and asset payoffs but also by changes in the expectations of heterogeneous agents, possibly facing asymmetric information. We contribute to this discussion by studying the relationship between the volume-volatility relation and detailed characteristics of the order book, at the intra-day level as well as in a daily cross-sectional time series setting.

The main theoretical explanations for the volume-volatility relation are summarized in two hypotheses. "The mixture of distribution hypothesis" states that the daily price change and the trading volume are both mixtures of independent normals with the same mixing variable. Harris (1986) links the hypothesis to asset pricing theory, and suggests that the mixing variable is the process that directs the rate of flow of information from systematic risk factors into prices and trading volume. "The dispersion of beliefs hypothesis" states that the volume-volatility relation will be stronger the greater the dispersion of beliefs about security values among investors. The dispersion of beliefs hypothesis is based on an assumption of asymmetric information and strategic investor behavior. Uninformed traders cannot distinguish informed trades from liquidity trades, and by reacting to trades with no information content, they increase both volatility and volume relative to equilibrium values. Intuitively, models based on the two hypotheses should complement rather than substitute each other.

A problem in empirical studies of the volume-volatility relation is that it is hard to test the theoretical explanations in a standard way. The mixture of distribution hypothesis merely states that when trades reflect information, prices will adjust to new equilibria over time. It is hard to set up an alternative hypothesis against which this statement can be tested. Nevertheless, a reconciliation of the general ideas behind the mixture of distribution hypothesis with empirical predictions from market microstructure models may provide a deeper understanding of the price

[1] A standard reference for the stock market is Shiller (1981).

discovery dynamics in financial markets.

In this paper, we first establish that the the standard volume-volatility relation exists also in a pure limit order driven market.[2] We then try to disentangle how the volume-volatility relation emerges by investigating the order placements of investors. In order to do this, we divide the order placement strategies into four groups depending on their aggressiveness. An interesting finding is that within the group of aggressive orders, the most aggressive ones are submitted at the beginning of the trading day. Orders in the passive group, on the other hand, are relatively more passive in the beginning of the trading day. This is systematic across sub-periods, market caps and tick-sizes. Assuming that the aggressive/passive orders are mainly submitted by informed/uninformed investors, we interpret this as reflecting that informational asymmetries are more pronounced at the beginning of the trading day, that there is competition among informed traders, and that uninformed traders require a compensation for the higher risk of trading with informed traders at the beginning of the day. This explanation is also supported by a decrease in various spread measures during the trading day.

A central part of our study centers around the shape of the order book which we argue constitutes a reasonable proxy for the dispersion of beliefs about asset prices. To capture the shape, we use the average elasticity (or slope) of the supply and demand schedules in the limit order book. The higher the elasticity (steeper the slope)[3], the less dispersed are the bid and ask prices in the order book. Hence, if the slopes of the supply and demand schedules in the order book are steep, we interpret this as an indication that there is a high degree of agreement among investors about the fair value of the security. This because orders are submitted close to the prevailing midpoint price. Similarly, if the the slopes are gentle, we interpret this as an indication that the investors disagree about the value of the security. We notice that, in case of asymmetric information, order placement strategies will also reflect traders' fear of being ripped off by someone with superior information. In addition, high volatility may induce investors to submit orders further away from the midpoint to reduce the probability of incurring a loss when reversing their position. On the other hand, the high volatility may itself reflect that there is a high degree of uncertainty related to the valuation

[2]Our analysis also indicates that the number of transactions (as a component of volume) is a good proxy for the mixing variable assuming a mixture of distribution explanation of the relation. This part of the analysis is largely based on Jones et al. (1994). Using daily data of Nasdaq-NMS securities over the 1986-1991 period, they find that the average size of trades has no information content on volatility beyond that contained in the number of transactions. This finding is interpreted as supportive of a mixture of distribution hypothesis where the number of trades is a proxy for the mixing variable (daily information arrivals).

[3]This is in the case of direct demand and supply curves (prices on the x-axis and accumulated volume on the y-axis). In the case of inverted demand and supply curves, the relationship would be opposite.

of the stock.

By investigating how the slope of the order book depends on the types and the aggressiveness of submitted orders, we show that the slope may capture both uninformed traders' fear of being ripped off by informed traders and the dispersion of beliefs among investors about asset valuation. The first component seems more pronounced at an intraday level, while the latter component becomes important across firms and time. We find that there is a systematic negative relation between the average slope of the order book and the price volatility on an intraday level as well as in a daily time series cross section analysis.

Section 2 surveys the literature on the volume-volatility relation and motivates our study. Section 3 describes our data sample and provides results on the basic volume-volatility relation in the Norwegian equity market. Section 4 provides some summary statistics for the order book and discusses how to use this information to proxy for the heterogeneity of investors. Section 5 examines in detail the order flow and order book on a intra-daily basis. Section 6 provides estimation results on the relationship between the shape of the order book and the volume-volatility relation. Section 7 concludes the paper.

2 Literature

The early research into the volume-volatility relation is reviewed in Karpoff (1987). The main theoretical explanation from this period is known as the "mixture of distribution hypothesis" (hereafter the MDH). According to the MDH, there is a positive correlation between daily price changes and trading volume because both variables are mixtures of independent normals with the same mixing variable. Originally, the MDH was suggested by Clark (1973) as an alternative explanation for the observed leptokurtosis in the distribution of log price changes.[4] The basic idea underlying the hypothesis is that prices and trading volume are driven by a time-varying arrival rate of information.[5] Let $\Delta p_{i,t}$ and $v_{i,t}$ be respectively the intraday price change and volume of trade resulting from information event number i on date t, and let n_t be the total number of information events

[4]Mandelbrot (1963) and Fama (1963) showed that the return distributions of commodity and stock prices were leptokurtic, and well approximated by symmetric stable distributions with characteristic exponents between 1 and 2 (the normal distribution has a characteristic exponent equal to 2). An examination of the stable distributions hypothesis for the Norwegian market is provided in Skjeltorp (2000) who shows that a characteristic exponent between 1.6 and 1.7 best characterizes the Norwegian data.

[5]Copeland (1976, 1977)'s "sequential arrival of information" model which is later extended by Jennings et al. (1981) and Jennings and Barry (1983) also predicts a positive relationship between volume and absolute price changes. The main feature of the model is that information is disseminated to only one traders at a time, and the main criticism of the models is that traders cannot learn from the market prices as other traders become informed.

during day t. Assume that (i) the number of events each day, n_t, varies across days, and that (ii) the intraday price changes, Δp, and trading volumes, ν, are jointly independently and identically distributed with finite variances.[6] The daily price change and trading volume are equal to the sum of respectively the intraday price changes and trading volumes, i.e.

$$\Delta P_t = \sum_{i=1}^{n_t} \Delta p_{i,t} \quad \text{and} \quad V_t = \sum_{i=1}^{n_t} \nu_{i,t} \tag{1}$$

where ΔP_t is the daily price change and V_t is the daily trading volume. Given equation (1), and provided that n_t is large, the joint distribution of the daily price change and volume of trade will be approximately bivariate normal conditional on n_t.[7] The volume-volatility relation arises because both price changes and trading volume are likely to be large when the number of information events is large and small when the number of information events is small.[8]

The MDH simply states that price changes and trading volume are directed by the flow of new information. It does not say anything about what type of information or how this information is revealed to investors. Hence, an important limitation of the hypothesis is that it does not address the role of economic agents or market structure for prices and trading volume. Later theoretical work on the volume-volatility relation centers around these issues. Harris (1986) links the MDH to asset pricing theory by suggesting that the mixing variable directs the rate of flow of information from systematic risk factors. A problem with this interpretation is that the movements in prices for many types of financial contracts seem much "too large" to be attributed to movements in the fundamental values of the underlying securities only.[9] This fact suggests that prices are not merely driven by changes in systematic risk factors and asset payoffs but also by changes in the expectations of heterogeneous agents. Figure 1 illustrates the information structure in a standard asset market for the two main types of such models. Panel (a) in the figure describes a "differences in opinion" model, while panel (b) describes a market microstructure model with asymmetric information.
In the "differences of opinion" model, investors are assumed to act differently on the same news, i.e. trading is induced by differences of opinion about publicly available information. Beliefs are updated using Bayes rule. All traders are rational, but they view others as having irrational models.

[6]Our explanation of the MDH is largely based on Harris (1987).

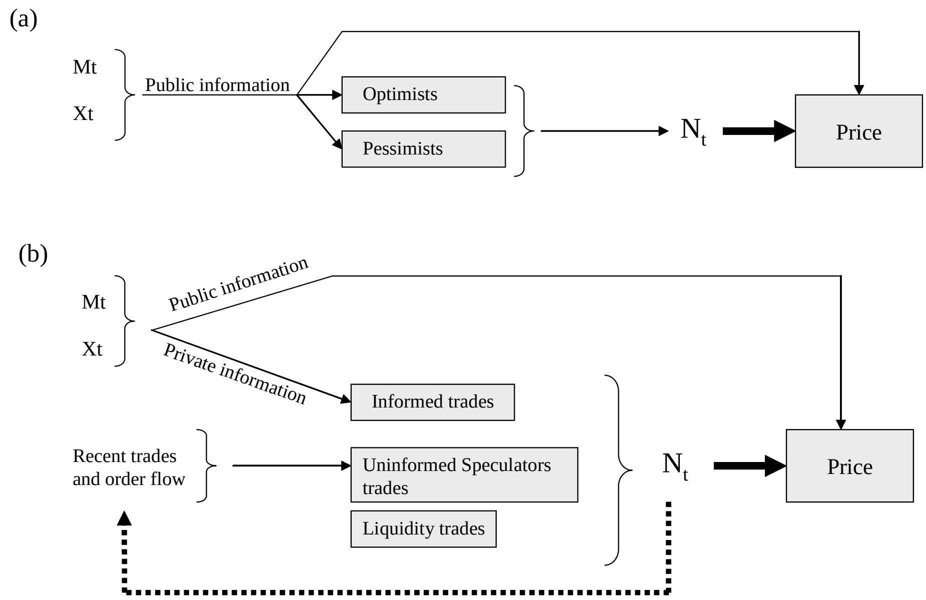
[7]See Harris (1987), page 129.

[8]The variation in the daily number of information events implies that the expectation of the unconditional distribution is a weighted average (or "a mixture") of the conditional distributions.

[9]A standard reference for the stock market is Shiller (1981).

Figure 1: **The Information Structure**

The figure illustrates the assumed information structure in a "differences in opinion" model (panel a) and a market microstructure model (panel b). From the fundamental asset pricing equation, $P_{i,t} = E_t[\sum_{j=0}^{\infty} M_{t+j} X_{i,t+j}]$, we know that relevant information about the price, P, of an asset, i, may come from either news about the stochastic discount factor, M_i, or news about the payoff, X_i. In the "differences in opinion" model in panel a, all news arrives as public information. Some types of information is immediately incorporated into the asset price. For other types of information, traders disagree on the effects on the valuation of the underlying assets. Trading occurs whenever the cumulative information for a particular type of trader switches from favourable to unfavourable. In the market microstructure model in panel b, new information arrives as either public or private information. Public information is immediately incorporated into the asset price. Informed traders trade on the basis of private information. Uninformed investors are either liquidity traders or speculators. The uninformed investors are trying to infer the private information from the trades, N_t. However, they are not able to separate informed and uninformed trades.



Harris and Raviv (1993) explain the volume-volatility relation by a model of this kind. Two groups of risk neutral speculators receive the same information but disagree on the extent to which it is important. As long as one of the groups remain more optimistic than the other, there is no trading. Trading occurs only and whenever the cumulative information for one of the trader groups switches from favorable to unfavorable.

In the standard asset pricing models, prices adjust immediately as a result of new information, and the trading process itself does not convey incremental information which is relevant to price determination. This is a plausible assumption for some kind of news. Other types of news are likely to be dispersed and not immediately available to all investors in aggregated form. Evidence of the existence of dispersed news is given in French and Roll (1986) who document empirically that asset prices are much more volatile during exchange trading hours than during non-trading hours. This phenomenon cannot be reconciled with a standard asset pricing model unless there is a systematic tendency for price relevant information to arrive during normal business hours only. Modelling dispersed information is the essential feature in the market microstructure models illustrated in panel (b). In these models, there is a group of investors who trade on the basis of private information. The market maker and the uninformed investors can only infer this information from trades and order flows. The room for strategic behavior among agents differ in different models.[10] Shalen (1993) use a market microstructure model to study the volume-volatility relation. In the model, both trading volume and price volatility increases with the dispersion of traders' expectations about fundamental values. This is called the "dispersion of beliefs hypothesis"(hereafter the DBH). The dispersion of beliefs about the value of a security is assumed to be wider the larger the share of the traders in the security that consists of uninformed investors. Uninformed traders cannot distinguish informed trades from hedgers' liquidity trades. Instead they react as if all trades were informative, and thus they increases both volatility and volume relative to equilibrium values.

Harris (1986) finds both empirical support for the MDH based on cross sectional tests of common stocks continuously traded on the NYSE or one of the regional exchanges in the period 1976-1977. The critical assumption behind the tests is that the distribution of the mixing variable is not identical for all securities. Assuming that transactions take place at a uniform rate in event time, Harris (1987) find both theoretical motivation and empirical support for the use of the daily number of transactions as a proxy for the time-varying unobserved information evolution rate.[11] However, since the arrival rate of new information is unobservable, we do not know whether parts of the volume-volatility relation is a result of the actions of heterogeneous traders. If trading is self-generating, the daily number of transactions would be the *true* mixing variable rather than a proxy for the unobserved information evolution rate.

The problem caused by a lack of ability to interpret the mixing variable can be illustrated by looking at the study of Jones et al. (1994). Using a simple regression approach on daily data of Nasdaq-NMS securities over the 1986-1991 period, they find that both volatility and trading volume are positively correlated with the number of daily transactions. However, the average size

[10] In Kyle (1985), informed investors attempt to camouflage their trades by spreading them over time. Kyle's model implies that larger volumes support more informed traders. In Admati and Pfleiderer (1988), a certain amount of the uninformed investors are allowed to act strategically by having the discretion to time their trading. This is shown to imply that within-day trading becomes concentrated. Hence, price changes and transactions are bunched in time, and the effect of volume on price movements will depend on recent volume levels.

[11] Harris (1987) derives and tests several implications of the MDH for transactions data on a sample of 50 NYSE stocks that traded between December 1, 1981 and January 31, 1983. The results from the tests are supportive of the MDH.

of trades contain no additional information about volatility beyond that contained in the number of transactions. If the number of transactions is a good proxy for the mixing variable, this result is supportive of a pure MDH; "..volatility and volume are positively correlated only because both are positively related to the number of daily information arrivals (the mixing variable)." However, trade size would also be unimportant if informed traders camouflage their information, for example by splitting their orders into medium size trades as suggested by the "stealth trading hypothesis" of Barclay and Warner (1993). In this case, the number of daily transactions would be the true mixing variable and the results in Jones et al. (1994) would also support an explanation of the volume-volatility relation based on heterogeneous traders.[12]

Daigler and Wiley (1999) perform an indirect test of the DBH. Using the argument that there is a greater dispersion of beliefs among uninformed traders than among informed traders, and facilitating the possibility of distinguishing traders with different types of information in the futures markets, they test whether uninformed traders contribute to volatility. The results of their study supports the DB-hypothesis: "..uninformed traders who cannot differentiate liquidity demand from fundamental value increase volatility."[13]

3 The Norwegian equity market

3.1 The Oslo Stock Exchange

Our data set is from the the Oslo Stock Exchange (OSE) in Norway.[14] Norway is a member of the European Economic Area, and its equity market is among the 30 largest world equity markets by market capitalization.[15] Table 1 report some general statistics for all the companies listed on the OSE. At the end of 2001, 190 firms were listed on the exchange with a total market value of about 657 mill NOK. 29 percent of this value was owned by foreign investors. The OSE is the

[12] In addition, in order driven markets, a large order is often automatically executed against many smaller orders by the automatic matching system. Thus, even though the original order is large, it may show up as many small trades as it is matched against several smaller orders rendering the average daily trade size unimportant in explaining volatility.

[13] In a similar study, Bessembinder and Seguin (1993) examine the volume-volatility relation and the contribution to volatility from market depth (proxied by open interest) in eight physical and financial futures markets in the 1982-1990 period. Unexpected volume is found to have a larger effect on volatility than expected volume, and large open interest is found to mitigate volatility.

[14] We obtained the data directly from the exchange's surveillance system. The SMARTS© system is the core of the exchange's surveillance operations. Through access to the SMARTS© database, we obtained all the information on orders and trades in the market

[15] Source is FIBV (International Federation of Stock exchanges). Notable Norwegian listings include Norsk Hydro, Telenor, and Statoil.

only regulated market place for securities trading in Norway. Since January 1999, it has operated as a fully computerized centralized limit order book system similar to the public limit order book systems in e.g. Paris, Toronto, Stockholm and Hong Kong.

The OSE allows the use of limit orders, market orders, and various customary order specifications.[16] As is normal in most electronic order driven markets, the order handling rule follows a strict price-time priority.[17] All orders are submitted at prices constrained by the minimum tick size for the respective stocks which is determined by the price level of the stock.[18]

The trading day at the OSE comprises of two sessions; the "pre-trade" session starting at 9:30 and ending with an opening auction at 10:00, and the "continuous trading" session from 10:00 until the trading closes at 16:00.[19] During the pre-trade session, brokers can register trades that were executed after the close on the previous day as well as new orders. The pre-trade session is ended with an opening auction, when all the orders registered in the order book are automatically matched if the prices are crossing or equal. The quoted opening price is thus the price that clears the market. During the continuous trading session, electronic matching of orders with crossing or equal price generates transactions. Orders without a limit price (market orders) have automatic price priority and are immediately executed at the best available prices. At the OSE, market orders are allowed to "walk the book" until it is fully executed. Any remaining part left of the market order is removed from the order book. This is different from the treatment of market orders at e.g. the Paris Bourse, where any remaining part of an unfilled order is automatically converted to a limit order at the current quote.[20]

3.2 The data sample

The data set consists of every order and trade that occurred at the OSE in the period from February 1999 through June 2001.

[16] Participants can also submit hidden orders. When an order is submitted as a hidden order, only a specified fraction of the underlying order is visible to the market.

[17] When a visible part of a hidden order is executed, the next part of the hidden order looses its time priority and is placed at the back of the queue at the respective price level.

[18] For prices lower than NOK 9.99 (Norwegian kroner) the tick size is NOK 0.01, between NOK 10 and NOK 49.9 the tick size is NOK 0.1, between NOK 50 and NOK 999.5 the tick size is NOK 0.5 and for prices above NOK 1000 the tick size is NOK 1.

[19] Prior to February 14 2003, the closing price of the market was the price of the last official transaction. From 14 February 2003, the OSE has started to close the market through a closing auction similar to the opening auction to improve the quality of the official closing price. The closing auction is conducted from 16:00 until 16:10. This change does not affect our sample, since our sample stops in July 2001.

[20] The difference implies that market orders at OSE are more aggressive than market orders at the Paris Bourse. At the Paris Bourse, market orders are essentially marketable limit orders.

Table 1: **Oslo Stock Exchange (OSE) - general statistics**
All numbers in the table are official statistics obtained from the OSE annual reports.

	1999	2000	2001
Number of registered firms	195	192	190
Market capitalization (mill. NOK)	531.65	618.36	656.69
NOK/USD exchange rate[a]	7.81	8.81	8.99
Turnover velocity[b]	88.6	96.7	85.9
Total dividends (mill. NOK)	11427	9365	10444
Ownership structure			
Number of shareholders (individuals)	351062	394304	426739
-norwegians	350485	393645	426183
-foreigners	577	659	556
Foreign ownership (% share capital)	25.43 %	31.46 %	25.49 %
Foreign ownership (% market capitalization)	32.01 %	35.66 %	29.04 %
Market development			
Market index level (TOTX)	1153.74	1366.05	933.22
OSE benchmark index	189.76	195.79	167.18
OSE benchmark index return (%)	48.45	3.18	-14.61

[a] Average midpoint rate for the respective year. [b] Turnover velocity: Average of annualized turnover per month divided by market value at the end of each month. Only capital registered in the VPS.

The trade data contain, for all trades, quantity transacted, a time stamp, brokerage house ID on each side, and an ID for the house initiated the trade as well as whether the house was the buyer or a seller in the transaction. Every trade is linked to the underlying orders through the order ID. Thus, if a large order is executed against many smaller orders resulting in several smaller trades, we can trace each executed part back to the initial order. There are also additional flags attached to each trade that identify special features of the trade such as whether it was an odd-lot trade, an off-exchange trade, a cross (within the same or different brokerage houses), and whether a trade results from a market order or a limit order. The order book data contain all entered orders and all deletions and amendments of orders already in the order book. The order book is described in more detailed in section 4.

In table 2 we provide some descriptive statistics of the trade data throughout our sample period. A large part of the listed firms are traded quite infrequently. Since we examine intraday data, including infrequently traded firms would introduce a large amount of noise into our analysis. We therefore filter the firms based on trading activity through the sample period. The first filtering criterion is that the firm must have been traded in at least 400 out of 597 days, or about 70% of the trading days, and the second criterion is that the firm must have an average of 5 trades per day to be included in our sample. Once the first criterion is applied, the second criterion only removes a few companies from our sample. After the filtering we are left with 108 firms. Note that there were 195, 192 and 190 listed firms at the end of 1999, 2000 and 2001 respectively. The numbers in table 2 are daily cross-sectional averages across the filtered firms.

The table shows that there has been increasing trading activity in the sample period with the total number of trades having tripled and the volume in Norwegian kroner (NOK) having doubled. Further, the average number of daily trades across firms has more than doubled from 32 in the first half of 1999 to 79 in the first half of 2001.[21] The increase in activity has also been accompanied by a decrease in the average percentage spread. To give a better picture of the diversity of the sample, we divide the sample into four portfolios based on their market capitalization value.[22] The general picture is that the number of trades, the trading volume (both in shares and NOK), the prices and the quoted spread increase across firm size portfolios, while the average daily volatility[23], the average trade size and the quoted percentage spread decrease.

We also report the average correlations between the trading volume, the trade size and the number of transactions. The correlation structure in our sample is quite similar to the one documented for the US market in Jones et al. (1994). The correlation between the average trade size and the number of trades is low, and both the average trade size and the number of trades have high positive correlations with share volume. Hence, the two components of share volume seem to contain different information about volume. The same structure is evident when we calculate correlations over sub-periods of half a year. Note, however, that the correlation between share volume and average trade size has decreased from around 62 percent in the first half year of 1999 to around 20 percent in the first half of 2001.

3.3 The volume-volatility relation

To investigate if there is a volume-volatility relation in our data sample, we follow the regression approach in Jones et al. (1994). First, we measure the daily return volatility using the standard procedure in similar empirical studies,[24] by running the following regression for each firm i,

[21] At the same time, the average trade size has gone down from 3429 shares to 2648 shares. This decline is most likely related to the introduction and growth of online trading in the sample period, since these traders generate a lot of trades of small sizes. During our period, the fraction of total trades coming from pure online brokerage houses has increased from 0% to almost 10%.

[22] The firms are assigned to a market capitalization portfolio based on their market capitalization value at the beginning of each year.

[23] The volatility measure is discussed in detail the next section.

[24] See Schwert (1990), Bessembinder and Seguin (1993), Jones et al. (1994), and Daigler and Wiley (1999).

Table 2: **Descriptive statistics and correlations across subperiods and size portfolios**

The table shows some descriptive statistics for the whole sample period as well as for sub-periods of half years. The market cap groups are resorted at the beginning of each year to account for large changes in market cap for some firms across years. Groups 1 consists of the 25% smallest firms while group 4 is the 25% largest firms. The Pearson correlation coefficients between the trading activity variables are also reported. Market values are reported in mill. NOK. The number of trades (N) is the average number of daily trades across all firms. The share volume (V) is the average daily share volume (in 1000 shares) across all firms. The average trade size (AV) is the average number of shares in each trade averaged across all firms for the sample period. The quoted spread is calculated as a percent of the spread midpoint. Effective spread is calculated as the difference between the execution price and the spread midpoint (in per cent of the spread midpoint) multiplied by two.

	Whole	Sub-periods (half years)					Market Capitalization groups			
	sample	1999.1	1999.2	2000.1	2000.2	2001.1	1	2	3	4
Aggregate statistics:										
Number of firms	108	107	108	108	108	104	27	27	27	27
Trades (in thousands)	3724	328	545	946	953	953	390	522	504	2309
Shares traded (mill.)	9585	1339	2300	2027	2072	1847	1707	1922	919	5037
NOK volume (bill.NOK)	648	67	131	152	153	146	21	44	68	516
Cross-sectional averages:										
Market cap (mill.NOK)	5259	4120	4714	5507	6127	5836	354	938	2339	13978
Price	88.4	71.8	82.7	102.7	102.3	81.9	23.34	62.43	105.66	150.73
Daily volatility (%)	2.71 %	2.64 %	2.89 %	2.98 %	2.48 %	2.57 %	3.49 %	2.98 %	2.30 %	2.29 %
Shares traded (thousands)	151	130	167	155	151	153	116	171	78	288
Trades	58	32	40	72	69	79	28	41	41	148
Tradesize (AV) in shares	2890	3429	3365	2453	2551	2648	4859	2684	1549	1912
Quoted spread (NOK)	1.65	1.55	1.62	1.79	1.78	1.50	0.94	1.63	2.11	1.57
Effective spread (NOK)	1.22	1.12	1.14	1.34	1.36	1.13	0.68	1.20	1.59	1.16
Quoted % spread (midpt.)	3.04 %	3.66 %	3.49 %	2.62 %	2.55 %	2.89 %	4.74 %	2.77 %	2.40 %	1.34 %
Effective % spread (midpt.)	2.22 %	2.67 %	2.48 %	1.92 %	1.89 %	2.15 %	3.38 %	2.03 %	1.85 %	0.99 %
Correlations:										
Corr(AV,N)	-0.061	0.045	0.051	-0.091	-0.085	-0.084	-0.116	0.280	0.172	-0.066
Corr(V,N)	0.525	0.660	0.591	0.724	0.568	0.442	0.690	0.480	0.847	0.365
Corr(V,AV)	0.330	0.358	0.438	0.290	0.288	0.201	0.393	0.932	0.504	0.759

$$R_{i,t} = \sum_{k=1}^{5} \alpha_{i,k} D_{k,t} + \sum_{j=1}^{12} \beta_{i,j} R_{i,t-j} + \hat{\varepsilon}_{i,t} \tag{2}$$

where $R_{i,t}$ is the return of security i on day t, and $D_{k,t}$ is a day-of-the-week dummy for day k. To avoid measurement errors due to the bid-ask bounce, we calculate returns from the average of bid-ask prices at the close. The 12 lagged return regressors estimate short-term movements in conditional expected returns. The residual, $\hat{\varepsilon}_{i,t}$, is an estimate of the unexpected return of security i on date t. Next, we estimate the regression equations suggested in Jones et al. (1994) to determine the relative effects of number of trades (N) and trade-size (AV) for volatility,

$$\textbf{Model I: } |\hat{\varepsilon}_{t,i}| = \alpha_i + \alpha_{i,m} M_t + \beta_i AV_{i,t} + \sum_{j=1}^{12} \rho_{i,j} |\hat{\varepsilon}_{i,t-j}| + \eta_{i,t} \tag{3}$$

$$\textbf{Model II: } |\hat{\varepsilon}_{t,i}| = \alpha_i + \alpha_{i,m} M_t + \gamma_i N_{i,t} + \sum_{j=1}^{12} \rho_{i,j} |\hat{\varepsilon}_{i,t-j}| + \eta_{i,t} \tag{4}$$

$$\textbf{Model III: } |\hat{\varepsilon}_{t,i}| = \alpha_i + \alpha_{i,m} M_t + \beta_i AV_{i,t} + \gamma_i N i, t + \sum_{j=1}^{12} \rho_{i,j} |\hat{\varepsilon}_{i,t-j}| + \eta_{i,t} \tag{5}$$

The $\rho_{i,j}$'s measures the persistence in volatility across 12 lags. M_t is a dummy variable that is equal to 1 for Mondays and 0 otherwise, $AV_{i,t}$ is the average trade size (total number of shares traded divided by the number of transactions for stock i on date t), and $N_{i,t}$ is the number of transactions in security i on date t. The regressions are run for each firm and then the parameter estimates are averaged across firms.

The first part of table 3 provides the results from the estimation of regression equations 3-5 using daily returns for all companies in our filtered sample. Overall, our results are very much in line with the results in Jones et al. (1994). The explanatory power of model 2 (with respect to the adjusted R-squared), where volume is measured by the average number of daily trades, is almost the double of the explanatory power of the model 1, where volume is measured by the average trade size. Moreover, the average trade size has little marginal explanatory power when volatility is conditioned on number of transactions in model 3. These results are further supported by the characteristics of the sampling distributions of individual-firm coefficients and t-statistics of the two variables. In model 3, 95.4 percent of the coefficients for the average number of trades are statistically significant, and 99.1 percent of the average number of trades coefficients were greater than zero. Similar numbers for the average trade size were respectively 24.1 percent and 57.4

percent.

As a robustness check we also estimate the equations for sub-periods of half years. The results from the whole sample regression are confirmed in the sub-sample regressions (not shown in the paper). Most notably, the $\hat{\gamma}$ estimates of the effect of trades (N), as well as their distributional properties, are very stable across sub-periods. The $\hat{\beta}$ estimates, however, vary considerably across sub-periods and are less significant than $\hat{\gamma}$ when for model 1 relative to model 3.

Jones et al. (1994) find that trade size has some information content for some of the smaller Nasdaq-NMS firms. This finding is interpreted as supportive of the notion that private-information based trading is important only for the smallest firms on the stock market. To check for similar features in our data sample, we re-estimate the three regression models on the four size portfolios. The results from these estimations are presented in the second part of table 3. Generally, the results from running separate regressions for each size portfolios follow the same pattern as the results from running the regression for the whole sample. However, while Jones et al. (1994) find that trade size has stronger explanatory power for the small firms, we find the opposite result that the explanatory power of trade size is the strongest for the largest firms. On the other hand, only about half of the trade size estimates from the single firm regressions are greater than zero indicating that the effect may not be very systematic cross-sectionally.

4 Characteristics of the order book

In this section we provide some descriptive statistics of the order book and discuss how to use order book information to investigate the heterogeneity of investors in the market.

Our order data are extraordinarily rich. For each order, we have a time stamp, a unique order id, the disclosed/undisclosed quantity as well as flags indicating whether the order was a buy or sell order, whether the order is a new order, a deletion of an order or an amendment to an existing order (price change and/or volume change). In addition, a unique brokerage house ID is attached to each order. Moreover, compared to the Paris Bourse data in Biais et al. (1995), our data are not restricted to include placements, amendments and deletions of orders within the 5 best quotes. We have access to all orders and are able to reconstruct the full order book at any point of time. The descriptive statistics discussed in this section are based on 6 hourly spaced snapshots of the entire order book during each trading day for each listed company during our sample period[25]. The

[25]The order book is rebuilt at 10:30, 11:30, 12:30, 13:30, 14:30 and 15:30 each trading day for each firm. We

Table 3: **Regression results - whole sample and size portfolios**

The table show the results from the estimation of three regression models of the volume/trade size -volatility relation across market capitalization portfolios as suggested in Jones et al. (1994):

$$\textbf{Model I: } |\hat{\epsilon}_{t,i}| = \alpha_i + \alpha_{i,m} M_t + \beta_i AV_{i,t} + \sum_{j=1}^{12} \rho_{i,j} |\hat{\epsilon}_{i,t-j}| + \eta_{i,t}$$

$$\textbf{Model II: } |\hat{\epsilon}_{t,i}| = \alpha_i + \alpha_{i,m} M_t + \gamma_i N_{i,t} + \sum_{j=1}^{12} \rho_{i,j} |\hat{\epsilon}_{i,t-j}| + \eta_{i,t}$$

$$\textbf{Model III: } |\hat{\epsilon}_{t,i}| = \alpha_i + \alpha_{i,m} M_t + \beta_i AV_{i,t} + \gamma_i Ni,t + \sum_{j=1}^{12} \rho_{i,j} |\hat{\epsilon}_{i,t-j}| + \eta_{i,t}$$

Using the Jones et al. (1994)'s notation we have that "$|\epsilon_{t,i}|$ is the absolute value of the return of security i in period t, conditional on its own 12 lags and day-of-week dummies, M_t is a dummy variable that is equal to 1 for Mondays and 0 otherwise, $AV_{i,t}$ is the average trade size, $N_{i,t}$ is the number of transactions for security i on day t, and the coefficients $\rho_{i,t}$ measure the persistence in volatility." Column 3-5 show parameter estimates averaged across all individual firm regression equations, while column 6-9 show the parameter distribution. $\hat{\beta}$ is the average parameter estimate for the average trade size variable (AV), $\hat{\gamma}$ is the average parameter estimate for the number of trades variable (N). In the distribution of estimates column we report, respectively, the percentage of $\hat{\beta}$ and $\hat{\gamma}$ estimates over all single firm regression equations that are significant (has a t-value greater than 2). In the last two columns we report the percentage of parameter estimates that are greater than zero. The first part of the table shows the results from running the regression equations over the whole sample. The second part of the table shows the similar results when we split the sample into four size portfolios.

		Parameter estimates			Distribution of estimates			
Model	Firms	$\hat{\beta}$ (AV)	$\hat{\gamma}$ (N)	adj. R^2	% t($\hat{\beta}$)>2	% t($\hat{\gamma}$)>2	% $\hat{\beta}$ >0	% $\hat{\gamma}$ >0
Model I: Tradesize (AV)	108	0.145	-	0.057	26.9 %	-	81.5 %	-
Model II: Trades (N)	108	-	0.031	0.145	-	95.4 %	-	100.0 %
Model III: Both (AV,N)	108	0.053	0.031	0.149	22.2 %	94.4 %	58.3 %	100.0 %
Model I: Tradesize (AV)								
1 (small)	27	0.145	-	0.080	16.2 %	-	78.4 %	-
2	27	0.219	-	0.055	18.2 %	-	77.3 %	-
3	27	0.274	-	0.048	19.0 %	-	64.3 %	-
4 (large)	27	1.021	-	0.038	30.8 %	-	79.5 %	-
Model II: Trades (N)								
1 (small)	27	-	0.052	0.174	-	89.2 %	-	97.3 %
2	27	-	0.028	0.147	-	75.0 %	-	95.5 %
3	27	-	0.036	0.136	-	81.0 %	-	95.2 %
4 (large)	27	-	0.014	0.174	-	79.5 %	-	92.3 %
Model III: Both (AV,N)								
1 (small)	27	0.079	0.053	0.175	10.8 %	86.5 %	64.9 %	97.3 %
2	27	0.076	0.030	0.148	4.5 %	75.0 %	54.5 %	95.5 %
3	27	0.075	0.036	0.140	16.7 %	78.6 %	45.2 %	95.2 %
4 (large)	27	0.237	0.014	0.179	30.8 %	82.1 %	35.9 %	94.9 %

sample period consists of a total of 597 trading days.

4.1 Composition of orders

In a limit order market, participants either demand liquidity through market orders or supply liquidity through limit orders to buy or sell. Whether an investor choose to use a market order or a limit order will depend on his or her motivation for the trade and how pre-committed he or she is to execute it. In the market microstructure literature, one typically makes a distinction between informed investors and uninformed investors (or liquidity traders). An uninformed investor perceives the current market prices as the correct price, and hence, is mainly concerned about minimizing transaction costs given the degree of trade urgency. The more quickly a liquidity trader needs to trade, the more aggressive order he or she has to submit to obtain price and time priority. Liquidity traders who need to buy or sell large amounts of shares will typically try to minimize the price impact from their trades by following different order splitting strategies or by using hidden orders conditional on current market conditions. Informed traders want to exploit their information advantage. Typically, the information concerns asset payoffs.[26] If the information is "short lived", informed traders prefer aggressive order placement strategies to extract the profits of their information as quickly as possible before it becomes common knowledge. Especially if the private valuation is far away from the current market price. If the information is "long lived", it may be more profitable to work the orders into the market through orders of average sizes over time ("stealth trading") or by hidden orders. Hence, both informed and uninformed traders may act to minimize the price impact from their trades to reduce execution costs, but the underlying motivation for their trading is different. After the trades have been executed, only the trades of informed investors should have a permanent price effect contributing to price discovery.

Price formation in a limit order market was first modeled by Glosten (1994). In this model, privately informed investors are assumed to submit market orders while uninformed investors are assumed to submit limit orders. Hence, the choice between market orders and limit orders is not explicitly modelled. Handa and Schwartz (1996) analyze the rationale for limit order trading based on an analytical framework where traders face a trade off between the gains from supplying liquidity

exclude order volume above/below 100 ticks away from the inner quotes. For a stock trading at NOK 100 with a minimum tick size of NOK 0.5 this would mean that orders above NOK 150 and below NOK 50 is excluded from our calculations. Our sample period stretches from February 1999 through May 2001.

[26]The information advantage may also concern the likelihood of an information event or the exposure to informed trading, cf Harris and Hasbrouck (1996) and Foucault et al. (2003b).

and the losses from trading with informed investors. Based on a simple empirical test they find that limit orders seem to be better than market orders for traders with relatively well-balanced portfolios. Harris and Hasbrouck (1996) analyze empirically the tradeoffs involved in the choice between market orders and limit orders by computing several order performance measures for a sample of NYSE SuperDOT orders. Their main findings are that limit orders placed at or better than the best quotes generally perform better than market orders, and that passive limit order traders cannot profitably operate as quasi-dealers in competition with the specialists. Foucault (1999) use a game theoretic model to study price formation and order placement decisions in a limit order market. By using limit orders, traders get better execution prices at the cost of non-execution risk and a winners curse problem. Both Handa and Schwartz (1996) and Foucault (1999) predict a positive relation between the proportion of limit orders in the market and price volatility. More specifically, in Handa and Schwartz (1996), increased volatility makes the gains from supplying liquidity exceed the loss from trading with informed investors, and in Foucault (1999) increased volatility makes market orders more costly relative to limit orders. In addition, Foucault et al. (2003a) model a limit order market where traders are characterized by different impatience and choose between market orders and limit orders in order to minimize their transaction costs. In equilibrium, the less patient traders are likely to demand liquidity while the more patient traders are likely to provide it.

We group the orders in our sample into four types based on their trading aggressiveness. "Market orders" are orders with no limit price. "Aggressive limit orders" are orders that are placed at the opposite quote (marketable limit order) or at a price further away from the best quote on the opposite side.[27] "Quote improving orders" are orders that are placed in between the inner quotes, and "Passive orders" are orders that are placed at the best (same side) quote or further away from the market. Panel A in table 4 shows the composition of orders and the order book activity for our data sample. The numbers in the table are daily cross sectional time series averages of order volumes (in shares), and the number of orders submitted. The numbers are averaged over each of the three years in the sample as well as over market capitalization quartiles.[28]
From the table, we can see that there is a great deal of heterogeneity in order placements among the traders in the market. The use of market orders is modest, however, market orders and aggressive

[27] A marketable limit order is submitted at the opposite quote. If the volume at the opposite quote is greater than the order size, a marketable limit order is essentially a market order.
[28] Each firm is assigned to a market capitalization quartile at the beginning of each year.

Table 4: **Order types, order sizes and order book distribution**

Panel A in the table shows daily cross sectional averages for different groups of orders. Limit orders are classified into three different types based on their aggressiveness. Market orders (MO) constitute a separate group. Passive orders are orders that are submitted at the best (same side) quote or further away from the market. Quote improving orders are orders that are submitted in between the inner quotes prevailing at order submission, and aggressive orders (Aggr.) are orders that are submitted at the opposite quote (marketable limit order) or at a price further away from the market on the opposite side. Panel A also provide statistics on the average order size (in shares) for each order class. The statistics are calculated across all firms as well as market capitalization quartiles with yearly sorting. The numbers in parantheses are each order class' fraction of total orders. Panel B shows the daily average fraction of accumulated volume in the order book (both bid and ask side) averaged across all firms. The statistics is calculated across all firms, across minimum tick sizes and market capitalizations.

PANEL A: Order types and order sizes

		Submitted orders					Order sizes			
	Firms	Total orders	Passive	Quote impr.	Aggr.	MO	Passive	Quote impr.	Aggr.	MO
All firms	108	94	42 (0.44)	15 (0.16)	34 (0.36)	4 (0.04)	6428	7063	5882	1715
				Market capitalization quartiles						
1 (small)	27	45	22 (0.45)	10 (0.21)	14 (0.31)	3 (0.06)	10708	11501	9824	4341
2	27	52	23 (0.43)	10 (0.19)	18 (0.34)	3 (0.05)	6244	7460	5634	1382
3	27	53	22 (0.41)	10 (0.19)	19 (0.37)	3 (0.05)	3437	3900	3038	531
4 (large)	27	224	100 (0.45)	31 (0.14)	87 (0.39)	7 (0.03)	5324	5392	5032	605

PANEL B: Order book volume distribution (normalized)

Minimum tick size	ATQ	+/- 1 tick	+/- 5 tick	+/- 10 tick	+/- 20 tick	+/-50 tick	+/-100 tick
All firms	20.9 %	34.7 %	56.8 %	69.4 %	78.4 %	88.6 %	100.0 %
				Minimum tick size			
0.01	20.2 %	30.8 %	37.8 %	49.0 %	60.1 %	82.2 %	100.0 %
0.1	22.2 %	34.2 %	53.2 %	67.4 %	79.4 %	91.7 %	100.0 %
0.5	22.3 %	39.1 %	65.8 %	78.4 %	88.1 %	95.5 %	100.0 %
1	7.0 %	10.7 %	17.6 %	25.1 %	38.8 %	70.0 %	100.0 %
				Market capitalization quartiles			
1 (small)	19.1 %	29.7 %	45.2 %	56.6 %	68.2 %	84.0 %	100.0 %
2	21.6 %	34.9 %	56.3 %	69.6 %	79.9 %	91.1 %	100.0 %
3	23.6 %	38.3 %	62.7 %	75.5 %	83.8 %	92.6 %	100.0 %
4 (large)	19.3 %	34.6 %	62.9 %	75.9 %	84.3 %	91.0 %	100.0 %

limit orders together constitute around 40 percent of the average daily number of submitted orders. Measured in number of shares, there are quite large variation in the size of the submitted orders across order classifications.[29] Quote improving orders are the largest for the entire sample as well as within market capitalization groups, while market orders are the smallest. This pattern is also systematic across sub periods (not shown in the table). Measured over the whole sample, on average 94 orders are submitted during a trading day for a firm. The activity is considerably higher for the largest firms than for firms in the other groups. The average daily number of orders submitted in this category was 224, while the similar average for the three other groups ranged from 45 to 53. For comparison, Biais et al. (1995) report an average of 160 orders for the Paris Bourse in 1995.

In Panel B in table 4, we show the distribution of volume in the order book averaged across all firms and dates. At each tick level, the fraction of total shares in the order book is averaged over the 6 order book snapshots.[30] The table shows the order book distribution across minimum tick sizes and market capitalization quartiles.[31] Around 35 percent of the order book depth is concentrated at the quotes or plus/minus one tick from the quotes. This is quite stable both across tick sizes and across market cap quartiles. Note that the depth within +/- 5 ticks, which is what Biais et al. (1995) investigate, only includes 56 percent of the total order book depth in our sample. There does not seem to be large differences in order depth across market capitalization quartiles. The largest tick size category is special in that it only contains one, highly volatile and very actively traded, company (as much as 30 percent of the order depth lies between 50 and 100 ticks away from the quotes).

4.2 Heterogeneity of investors

In section 3 we established that there exists a similar volume-volatility relation in the Norwegian equity market as has been found for the US by e.g. Jones et al. (1994) and in the UK by Huang and Masulis (2003). If we interpret the number of trades as a proxy for the mixing variable, our results also support the MDH.

[29] A part of this variation can probably be explained by differences in the price level of the stocks, both over time and over market capitalizations.

[30] For instance, at the ask side of the book for one company/snapshot, we divide the aggregate number of shares at each tick by the total number of shares supplied (offered) at that time/snapshot. We do this for each snapshot, and average across all snapshots on the particular date to obtain the average fraction supplied on each tick for the security. Since we limit the order book to orders within +/- 100 ticks from the bid ask midpoint, the fraction of aggregate volume at +/- 100 ticks is 100%. The limit on 100 +/- ticks result in that we disregard less than 5 percent of our sample.

[31] If a firm trades across two minimum tick sizes on the same day, we remove that company for that day from the sample. The results do not change if we include these observations.

The limit order book reflects aggregated buy and sell interests at various prices. Each ask (bid) price reflects the lowest (highest) price at which different investors are willing to sell (buy) the security, given their motivations for the trade and their beliefs and information about the value of the company at a particular point in time.[32] Hence, the shape of the order book should make a reasonable proxy for the dispersion of beliefs about asset prices.[33] At the same time, if there is private information in the market, order placement strategies will also reflect traders' fear of being ripped off by someone with superior information. Foucault et al. (2003b) model a limit order market where liquidity suppliers have asymmetric information on the "rip off risk", and show that this affects the shape of the order book.[34] Empirical regularities found in intraday transactions data indicate that there are intraday variations in the rip off risk. If so, we should expect the rip off risk to be reflected in intraday variations in the shape of the order book.

Generally, no group of investors is likely to have access to perfect information about either future asset payoffs or the extent of informed trading. Significant differences in the daily average of the shape of the order book across different companies and/or over time should therefore also reflect differences in investors beliefs about underlying asset values. Figure 2, may help to illustrate this point.

The figure shows the average order books for two companies listed on the OSE. The order books are averaged over the five last days in May 2001, and are normalized in the sense that they show the percentage of orders within an increasing/decreasing number of ticks away from the quotes (zero in the figure is the best quote on each side of the market). The upper picture shows the average order book for Norsk Hydro (NHY) while the lower picture shows the average order book for Opticom (OPC). Both companies are among the most liquid at the exchange.[35] Norsk Hydro is a leading

[32]Biais et al. (1995) note that the shape of the order book may reflect the competition among buyers/sellers as well as the correlation in their valuations. If the supply and demand curves are inelastic and volume is concentrated around the inner quotes, this may reflect that the valuations among various investors are correlated on each side of the market relative to the case where the valuations are more dispersed and the order book is more elastic.

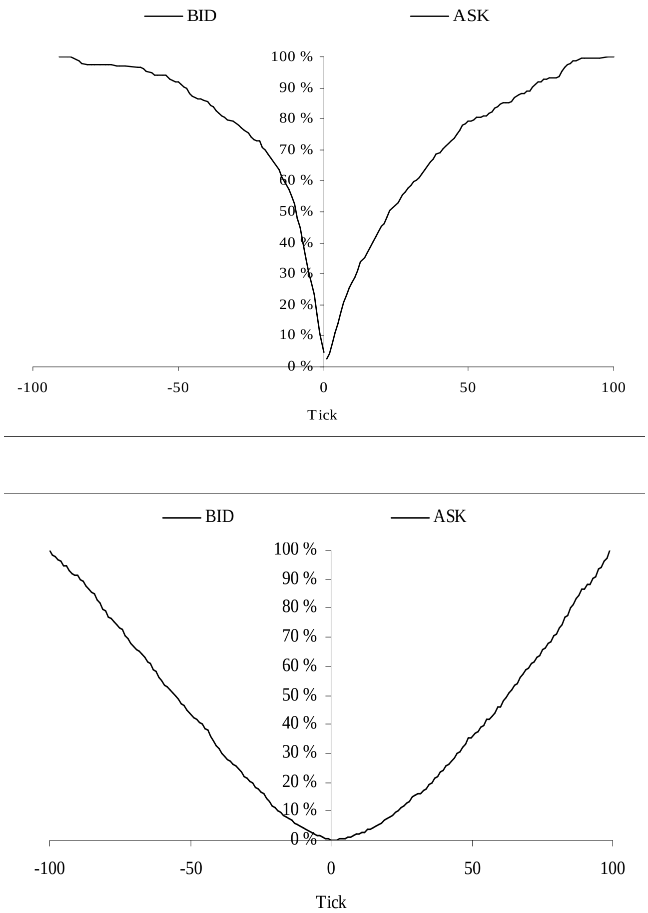
[33]The order book does not necessarily reflect all trading interest. Some orders are used by investors to protect themselves against losses when markets move rapidly. One such type of orders are "stop orders" which are orders that are activated when the market price reaches a specific level. These orders are essentially instructions to the broker who monitors the market on behalf of the investor and submit the (limit or market) order when the price level reaches the stop price, i.e. they are not already activated orders such as limit orders. In addition, Biais et al. (1995) find evidence, at the Paris Bourse, that there is potential liquidity supply outside the book from traders that monitor the book searching for favourable trading opportunities.

[34]When the book is thin, uninformed liquidity traders will be reluctant to add depth because it may be an indication of high rip-off risk. The informed liquidity traders exploit this by bidding less aggressively than in the case where the liquidity traders have symmetric information.

[35]During the period illustrated in the figure, both companies traded in prices around NOK 400-500 and had a tick size of NOK 0.5. For Norsk Hydro the calculated average order book is based on around 2000 orders with a share volume of around 400 000 shares. For Opticom the similar calculations are based on around 4000 orders with a share

Figure 2: **Average order books**

The figure illustrates the order book for two different companies listed on the OSE. The upper picture shows the order book for Norsk Hydro, a large Norwegian blue chip company, and the lower picture shows the order book for Opticom, a Norwegian IT company. The order books are averaged over the last five days of May 2001. The picture shows the percentage of orders within ticks away from the quotes. Zero represents the best quote on each side of the market.



energy, aluminium and fertilizer company, based in Norway. It has 50,000 employees in 60 countries worldwide. The company's operations are well known and there is a a large amount of current available information about the company, including experts analysis. Opticom, on the other hand, is a relatively new IT company which currently has under 100 employees. The company describes its business concept as pioneering research and development in new technology in electronics. The company has no cash flow and very uncertain future income possibilities. Hence, a major difference between the two companies is related to valuation uncertainty. Investors' opinions about the underlying value of the two companies should be much more dispersed for Opticom than for Norsk Hydro. This is exactly what the pictures of the order books of the companies indicate: while on average about 50 percent of the orders for Norsk Hydro has limit prices which lie within 5 ticks from the quoted spread, the similar percentage for Opticom is only about 10 percent.

This difference in the average shape of the order book results from the fact that traders systematically submit orders further away from the midpoint in Opticom than in Norsk Hydro. One plausible reason for this is that investors are more uncertain about the true value of Opticom than Norsk Hydro, and that this higher dispersion of beliefs in Opticom is reflected in orders that are submitted across a wider range of prices than in Norsk Hydro. The difference in the order book shape may also come from pick off risk, inducing investors to submit orders at prices with a compensation for the risk of trading with an informed trader. Probably both effects contribute to explaining the pictures we see in figure 2. However, while it is obvious that there are huge differences in valuation uncertainty between the two companies, it is not so obvious that there should be such a big difference in pick off risk. More importantly, it is mainly orders submitted close to the midpoint that are exposed to pick off risk, not orders submitted further away from the market. Thus, pick off risk should affect the spread and volumes at the inner ticks, not the distribution of orders across the the entire order book. Nontheless, it is important to note that we do not attempt to differentiate between the two effects in this analysis.

To capture the shape of the order book, we propose to use the average elasticity/slopes of the supply and demand schedules in the order book. The more gentle (steeper) the slope, the more widely distributed (concentrated) are the bid and ask prices in the order book. Hence, if the slopes of the supply and demand curves in the order book are gentle, that is, if the volume in the order book is distributed across a wide range of prices, we interpret this as an indication that the investors disagree about the value of the security. Similarly, if the slopes are steep, that is, if a large fraction

volume of around 200 000 shares.

of the order book volume is concentrated around the inner quotes, we interpret this as an indication that there is a high degree of agreement among investors about the fair value of the security.[36] We calculate our slope/elasticity estimate for each company and for each of the generated snapshots of the order book in the following steps:

1. First, for each side of the order book, we accumulate the aggregate number of shares supplied/demanded at each price level, such that at each price level we get the total volume supplied (demanded) at that price or lower (higher).

2. Next, we normalize the accumulated shares at each tick level (on the ask and bid side separately) relative to the total number of shares supplied/demanded at the relevant snapshot. Thus, the percentage of the shares in the order book supplied (demanded) at the highest (lowest) ask (bid) price/tick is 100 per cent.

3. Next, we calculate the "local" elasticity at each price level (explained in equation 10 and equation 11 in the appendix).

4. Next, we average across all price levels to obtain an average elasticity/slope for the bid and ask side for that snapshot.

5. Finally, we take the average of the bid and ask slope to get one slope measure for the snapshot.

We normalize the order book because we want to take into account that there is a close relationship between our slope measure and the liquidity of the underlying stock. Less liquid firms generally have a higher volatility since the order book does not contain enough volume to absorb large trades without moving prices to much. In addition, less liquid stocks generally have a higher spread since investors require a discount when buying and a premium when selling the stock. Thus, a positive relationship between order book elasticity and volatility is expected a priori. By normalizing the order book, we get the fraction of total shares supplied/demanded at each price level. In addition, we calculate slope measures based on two different weighting schemes. In the first we equally weight each local slope, and in the latter, we weight each local slope by its distance (in ticks) from the inner quote. In addition, we also calculate a "non-normalized" version of the slopes by using the absolute volume (in shares) at each order book level.

[36]Disagreement about the value of the firm may be a function of the degree to which the company releases information to the market, the number of analysts covering the stock etc.

Figure 3: **Calculating the demand and supply elasticities**

The figure illustrates how the local slopes/elasticities of the bid and ask side of the order book is calculated for one "snapshot" time on one date for one company. The figure illustrates this for a case where there are only 4 price levels on both side of the book. On the left y-axis we have the fraction of aggregate share volume for the demand (bid) side of the order book at each tick level, and on the right y-axis we have the fraction of aggregate share volume for the supply (ask) side of the order book at each tick level. The solid step-line is the supply (right) and demand (left) curves across the various price levels. On the x-axis, we have the various price levels, where p^M is the bid ask midpoint, and prices greater than p^M are ask prices and prices below p^M are bid prices. The difference between p_1^B (best bid) and p_1^A (best ask) is the quoted spread. In addition, the dotted line-segments connecting each level of the order book has slopes denoted by Δs, which are the normalized local elasticities of the demand and supply curves calculated in eq.10 and eq.11.

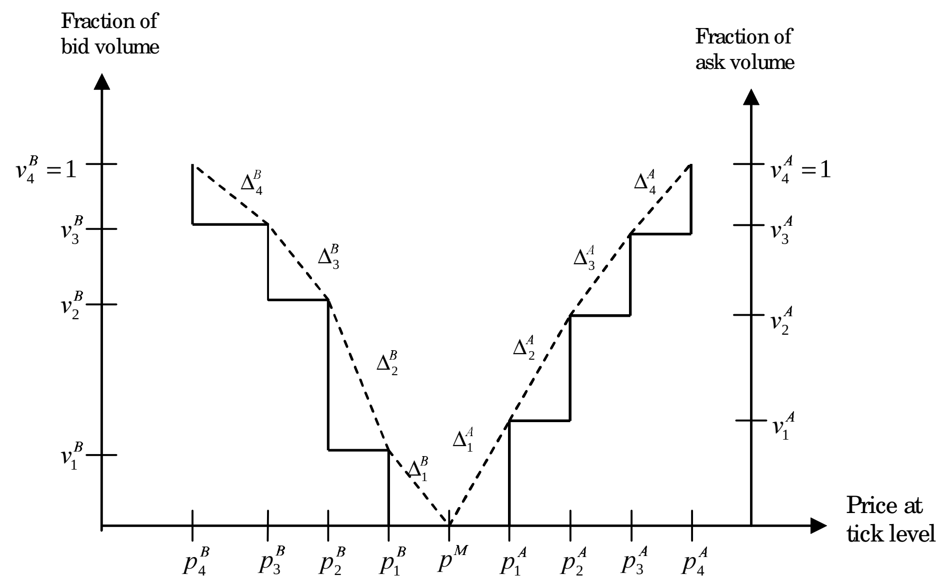


Figure 3 illustrates how the local elasticities, Δ_τ^A and Δ_τ^B, are calculated. For illustrative purposes, the order book in the figure stretches only across 4 price levels on each side. In the figure, p_1^A is the best available ask-price (inner ask quote) with volume fraction of v_1^A supplied at that ask price. The volume fraction at the next tick level (v_2^A) is thus the accumulated volume supplied at price p_1^A and p_2^A relative to the total volume in the order book at each side. The local elasticity of the supply curve at p_1^A would thus be the slope Δ_2^A in the figure. A more specific explanation of the calculation is provided in the appendix.

5 Intraday analysis of the order book

In this section, we present intraday statistics of the limit order book.

Table 4 showed that the use of market orders was quite modest in our data sample. Hence, to describe the composition of orders in the book we focus on the aggressiveness of the limit orders. To facilitate this, we calculate a separate index reflecting limit order aggressiveness. The aggressiveness of an order is measured by the average number of ticks away from the best quote (on the same side) that the order is placed.[37] Thus an index number of zero means that the average order is placed at the quote, a positive index number means that the order is placed above (below) the bid (ask), and a negative number means that the average order is placed below (above) the bid (ask).[38] Formally, for an order of type k, the aggressiveness of a buy order with a limit price p^B is calculated as,

$$\lambda_k^{buy} = (p^B - bid)/ticksize \tag{6}$$

Similarly, a sell order with a limit price p^S is calculated as,

$$\lambda_k^{sell} = (ask - p^S)/ticksize \tag{7}$$

where bid and ask is the best bid quote and best ask quote, respectively, when the order is submitted.

Table 5 shows intraday statistics for our slope measure (calculated at the end of each time interval), the price volatility (measured as the absolute hourly return between trade prices closest to the end of each time interval), the quoted and the effective spread, the number of trades executed

[37] Our measure is similar to the measure of limit order aggressiveness proposed by Harris and Hasbrouck (1996).
[38] We cannot calculate the aggressiveness for market orders since these orders do not have a price limit.

Table 5: **Intraday statistics**

The table shows intraday statistics for our slope measure, the volatility (the absolute hourly return between trade prices closest to the end of each interval), the quoted spread, the effective spread, the number of trades executed during the time interval, the trade size (in shares), the number of orders submitted during the time interval and the order size (in shares). All numbers are daily averages across all firms in the sample. Note that the first and last time windows are half an hour while the rest of the time windows are hourly. In addition, the slope is calculated at the end of each interval.

	Time window						
	10:00 to 10:30	10:30 to 11:30	11:30 to 12:30	12:30 to 13:30	13:30 to 14:30	14:30 to 15:30	15:30 to 16:00
Slope (end of time-window)	30.51	34.37	35.78	36.34	36.80	36.97	
Volatility (absolute return)	-	1.34 %	0.81 %	0.72 %	0.74 %	0.88 %	0.86 %
Quoted spread	2.36	1.73	1.47	1.37	1.33	1.31	1.39
Effective spread	1.79	1.27	1.05	1.00	0.95	0.95	1.05
Trades	10.38	11.81	9.38	9.05	9.52	10.81	10.40
Trade size (shares)	2314	2653	2759	2774	2834	3027	3123
Orders	15.45	18.16	13.10	12.36	12.47	14.02	11.66
Order size (shares)	6858	6385	5723	5818	5795	6383	6706

during the time interval, the trade and order sizes measured in shares, and the number of orders submitted during the time interval. All numbers are daily averages across all firms in the sample. Notable characteristics of the intraday statistics in table 5 are

- The quoted and the effective spread both have a U-shape, with the highest spread at the beginning of the day.
- The average trade size is smallest at the beginning of the day, and increasing throughout the trading day.
- The average number of orders and trades both have a U shaped, with less orders being placed and trades being executed in the middle of the day, and most orders being placed and trades being executed at the beginning of the day.

These regularities are also systematic across sub periods.[39] Similar systematic intraday regularities have been found in other markets (e.g. US, France, Hong Kong). The main explanation suggested for these findings is that the probability of trading with informed traders is largest at the beginning of the trading day and then diminish during the day. If this explanation is correct, a patient liquidity trader who fears to be picked off by informed investors at the beginning of the day has two main options. If she believes that the probability of trading with informed traders will diminish during

[39]We calculate the statistics across sub periods of years, half years and quarters and find that the results are both qualitatively and quantitatively the same.

the day, she can act strategically and delay her trading, as suggested by Admati and Pfleiderer (1988). Alternatively, she can submit her orders at the beginning of the day and take account of the increased probability of incurring a loss by placing them at prices including a discount (buys) or a premium (sells). This can explain the higher spread at the beginning of the trading day.[40] Assuming that the informed traders are trying not to reveal their information too quickly, we would also expect to see a higher number of small trades at the beginning of the trading day (stealth trading strategies).

Our data sample enables us to investigate the stylized facts documented in the earlier literature and in table 5 in more detail. Table 6 shows intraday changes in order aggressiveness, average number of orders, fraction of order types, and order sizes. Figure 4 illustrate graphically the intraday patterns in order aggressiveness, order size, order book slope, quoted and effective spread, and fraction of order types.

If uninformed investors believe that there is more asymmetric information at the beginning of the trading day, we would expect to see that they place orders at limit prices further away from the midpoint price at the beginning of the trading day, and then, closer to the midpoint prices later in the day, as the market price adjusts to the private information. Moreover, we would expect that the orders placed by better informed investors were most aggressive in the beginning of the day, especially if informed investors are competing to extract profits from the same information. This is exactly what is indicated in our data sample. There are systematic differences in the aggressiveness of different types of orders during the course of the day. “Away from market” orders, which make up a large part of the order book, is placed relatively much further away from the inner quotes at the beginning than at the end of the day. If this type of orders are mainly submitted by uninformed traders, it indicates that they require a higher compensation for trading early in the day relative to later in the day.[41] Furthermore, orders that are more aggressive, and likely to stem from better informed investors or pre-committed liquidity traders, are relatively more aggressive at the beginning of the day than later in the day. At the end of the trading day all types of orders are submitted closer to the inner quotes, indicating that most of the private information is incorporated into the prices.

[40] The increase in spreads towards the end of the day is due to higher liquidity demand and possibly more cancellation of orders just before the close.

[41] Another interpretation is that uninformed traders have not yet processed all publicly available information (e.g. newspapers, new analyses, gossip etc.), and are more passive when submitting their orders before they have been able to read and interpret this information.

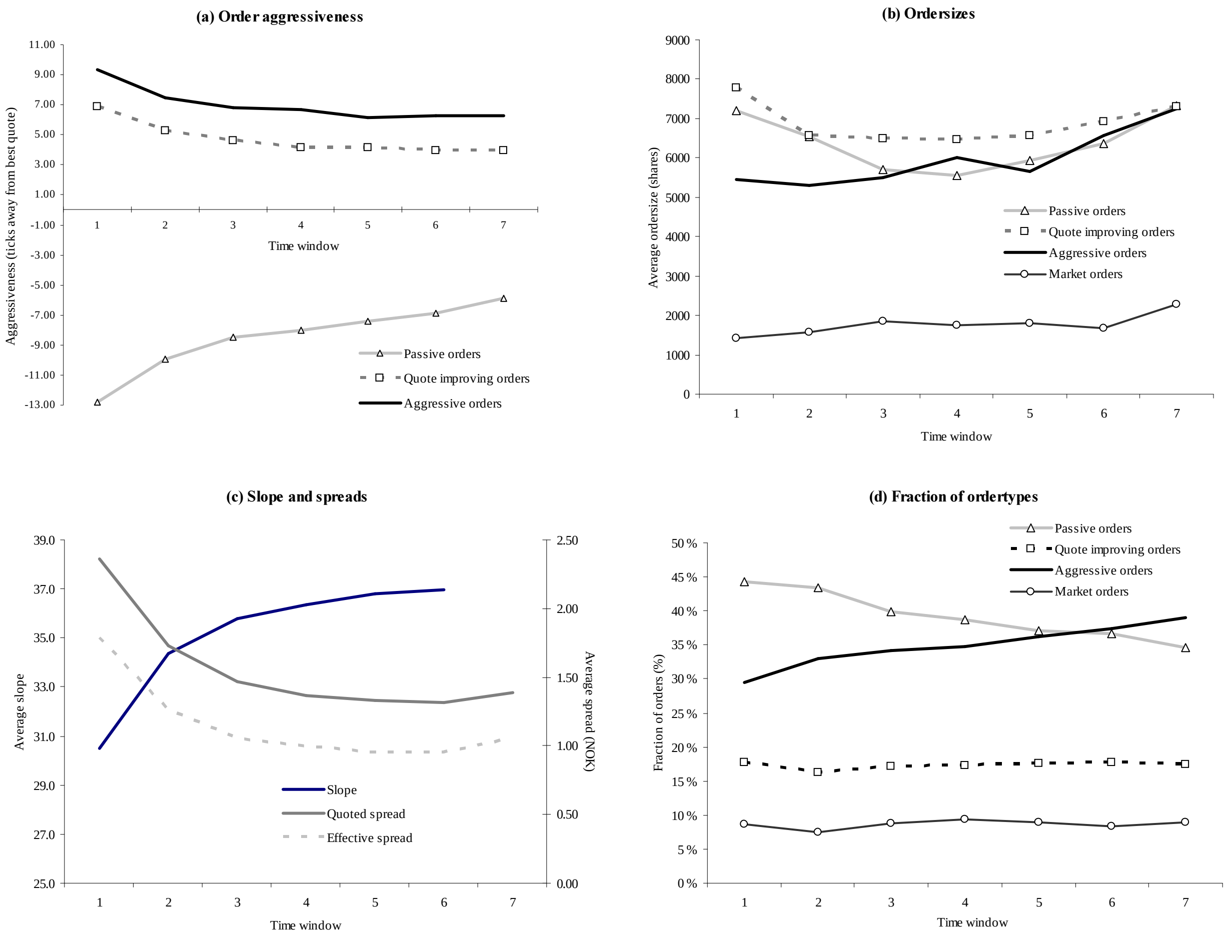
Table 6: **Intraday statistics and aggressiveness of orders**

In the table we decomposes all submitted orders within each time interval into groups based on the aggressiveness of the orders. The least aggressive orders, ”away from market”, are orders placed at or away from the quote on the same side of the book. This would be e.g. a buy order with a price (bid) equal to or lower than the current best bid, or a sell order with a price (ask) equal to or higher than the best ask price. The second type of orders, ”quote improving orders”, are orders that improve the best quotes. This would be e.g. a buy order with a price higher than the current best bid, but lower than the best ask quote. The third type of orders are ”aggressive orders” are orders placed at the opposite quote or higher(buys)/lower(sells). In the table we calculate the average number of orders of each type placed within each time window, the percentage fraction of all orders of each type, and the average order size in shares and NOK. For each type of order we also calculate an aggressiveness index equal to the average number of ticks away from the best quote (on the same side) that an order is submitted. Thus an index number of zero means that the average order is placed at the quote, a positive index number means that the order is placed above/below the bid/ask, and a negative index number means that the average order is placedbelow/above the bid/ask. We do not calculate the aggressiveness for market orders since these by definition do not have any limit price. Note that the first and last time window are half an hour while the rest of the time windows are hourly.

	Time window						
Order type	10:00 to 10:30	10:30 to 11:30	11:30 to 12:30	12:30 to 13:30	13:30 to 14:30	14:30 to 15:30	15:30 to 16:00
				Aggressiveness (avg. ticks away from best quote)			
Passive orders	-12.81	-9.96	-8.45	-8.02	-7.44	-6.90	-5.87
Quote improving orders	6.90	5.30	4.65	4.17	4.16	3.94	3.96
Aggressive orders	9.36	7.46	6.82	6.68	6.17	6.29	6.29
Average aggressiveness (weighted)	-1.69	-1.00	-0.25	-0.06	0.20	0.52	1.11
				Average number of orders			
Passive orders	8.2	9.1	6.4	5.9	5.7	6.2	5.0
Quote improving orders	3.3	3.4	2.7	2.6	2.7	3.0	2.5
Aggressive orders	5.4	6.9	5.5	5.3	5.5	6.3	5.6
Market orders	1.6	1.6	1.4	1.4	1.4	1.4	1.3
				% fraction of orders of type			
Passive orders	44.2 %	43.4 %	39.9 %	38.7 %	37.2 %	36.7 %	34.6 %
Quote improving orders	17.7 %	16.3 %	17.1 %	17.3 %	17.6 %	17.7 %	17.4 %
Aggressive orders	29.4 %	32.9 %	34.2 %	34.7 %	36.2 %	37.3 %	39.0 %
Market orders	8.6 %	7.4 %	8.8 %	9.3 %	9.0 %	8.3 %	8.9 %
				Order size (shares)			
Passive orders	7202	6548	5716	5557	5938	6370	7317
Quote improving orders	7793	6568	6486	6470	6561	6915	7294
Aggressive orders	5461	5301	5498	6008	5649	6569	7239
Market orders	1412	1576	1855	1751	1795	1678	2281
				Order size (1000 NOK)			
Passive orders	275	235	222	221	242	267	346
Quote improving orders	274	253	258	265	274	290	328
Aggressive orders	188	204	204	214	227	376	307
Market orders	36	39	46	42	40	43	69

Figure 4: **Intraday characteristics**

The figures shows cross sectional averages across 7 intraday windows for various measures. The windows and numbers correspond to those in tables 5 and 6. Note that window 1 and 7 are half hour intervals from 10:00 to 10:30 and 15:30 to 16:00 respectively, while windows 2 to 6 are hourly intervals starting every half hour. Figure (a) shows the average aggressiveness of different order types. The first type of orders, "passive orders", are placed at or away from the quote on the same side of the book. This would be e.g. a buy order with a price (bid) equal to or lower than the current best bid, or a sell order with a price (ask) equal to or higher than the best ask price. The second type of orders, "quote improving orders", are orders that improve the best quote (on the same side). This would be e.g. a buy order with a price higher than the current best bid, but lower than the best ask quote. The third type of orders are "aggressive orders" are orders placed at the opposite quote or higher(buys)/lower(sells). Figure (b) shows the average order size within each limit order group and the average order size of market orders. Figure (c) show the average slope on the left axis and the average quoted and effective spreads on the right axis. Note that the slope is calculated from the order book snapshot taken at the end of each window. Figure (d) shows the fraction of each order category which is placed within each window.



The average number of passive orders ("away from market") decreases throughout the day, while the average number of quote improving orders and aggressive orders has a U-shape. The number of submitted market orders decreases throughout the trading day.[42] The intraday pattern in the relative fraction of each order type indicates that more orders are submitted closer to the midpoint at the end of the day. When examining the average order sizes across the different order types, "away from the market" orders are the largest at the open and close, while the most aggressive limit orders and market orders are the smallest and increase in size throughout the day. If informed investors mainly use aggressive limit orders and market orders, this may indicate that they submit smaller orders when their information is the most valuable (stealth trading).

Given that there is more asymmetric information at the beginning of the trading day, the intraday pattern of our slope estimate captures asymmetric information about stock values during the trading day quite well. The slope increases (at a diminishing rate) across the day, with a minimum at the beginning of the day and a maximum at the end of the day. This indicates that the order book is more dispersed in the morning relative to later in the day. In other words, a larger fraction of the order volume is submitted further from the inner quotes just after the open compared to later in the day. Recall that the average slope is calculated from the normalized order book, so that the slope does not just reflect that there are less orders in the order book early in the day.[43] Across time windows, the average slope increases at a diminishing rate and becomes more concentrated and elastic at the end of the day.

6 The Volume-Volatility-Order Book Relation

In section 4, we argued that our estimate of the average daily slope of the order book could be used as a proxy for the dispersion of beliefs among investors about asset values. In this section, we provide some descriptive statistics of the estimated daily slope and investigate whether this variable can explain the contemporaneous market volatility in addition to the variables suggested by Jones et al. (1994).

Panel A in table 7 shows the correlations between the different versions of our daily slope measure which we outlined in section 4. As can be seen from the table, the slope estimates are

[42]Note that the first and last windows in the table are half-hour intervals, while all other windows are hourly intervals.

[43]A lower average slope reflects that the order book is more elastic which implies that a lower fraction of the order volume is close to the inner quotes relative to further out in the book.

Table 7: **Variable correlations**

Panel A shows the correlation between different slope measures. "Volume slope" is the average slope calculated from the order book *without* normalizing the volume at each tick relative to the total share volume in the book. "Weighted volume slope" is the weighted version of "volume slope", where we weight each local slope by the distance in ticks from the best quote. "Normalized slope" is when we at each snapshot normalizes the volume at each tick relative to the total number of shares in the book (on the respective side), and "Weighted normalized slope" is the tick weighted version of the "Normalized slope". Panel B shows the correlations between various activity and liquidity variables as well as our elasticity variable (SLOPE).

PANEL A: Correlation between slope measures

	Volume slope	Weighted volume slope	Normalized slope (SLOPE)
Weighted volume slope	1.00		
Normalized slope (SLOPE)	0.72	0.72	
Weighted normalized slope	0.72	0.72	0.98

PANEL B: Variable correlations

	Trades (N)	Trade size shares (AV)	MCAP	SPREAD	SLOPE	Ordervol. (OV)
Tradesize shares (AV)	-0.02					
MCAP	0.25	-0.04				
SPREAD	-0.20	0.16	-0.17			
SLOPE	0.13	-0.08	0.44	-0.32		
Order volume shares (OV)	0.19	0.16	0.06	-0.06	0.04	
Trade volume shares (V)	0.43	0.33	0.14	-0.13	0.08	0.45

highly correlated. However, the variance of the weighted slope estimate is higher. This is mainly due to the high variation in order book volume across firms and time.

Table 8 provides some distributional statistics for the slope estimate over the whole sample, for the separate years, and for our four size portfolios.[44]

As one would expect, the slope increases with market capitalization. Thus, larger and more liquid stocks have a higher fraction of the order book volume concentrated at or around the best quotes, while smaller firms have more dispersed order books. Another feature to note about the table is that the average/median slope decreases across market capitalizations. A possible reason for this is that large companies are generally easier to price than small companies due to more available information and more frequent analysis by financial experts.

Figure 5 illustrates the relationship between the daily equally weighted slope estimate and the contemporaneous daily volatility, measured as the average daily absolute return over the trading

[44]We report results from the use of the equally weighted normalized slope estimates only. When we weight the local slopes by the distance from the best tick, the results are quantitatively similar.

Table 8: **Distribution of equally weighted slope estimates**

The table shows the distribution of the slope estimates for the case where each local slope is equally weighted and each side of the order book is normalized with respect to the total number of shares on each side. Panel A report the estimates for the entire sample and across minimum tick sizes. Panel B report the estimates across market capitalization portfolios and years. In this case each company is assigned to a market capitalization portfolio each trading day instead of yearly (as used before). This is because the slope estimates can vary widely across days so that a yearly sorting would not capture the differences across market capitalizations to the same degree. Thus, N reflects the number of firm/date observations.

PANEL A

				Distribution of slope estimates							
	N	MCAP	Price	P5	P10	P25	Median	Mean	P75	P90	P95
All firms	51015	7294	145	9.1	11.9	18.3	29.2	37.2	46.7	70.9	91.5
1999	16968	5948	110	9.4	12.6	20.3	33.2	41.4	53.0	79.2	101.3
2000	23853	7737	180	9.6	12.2	18.0	27.6	35.3	43.5	66.3	86.0
2001	10194	8498	122	7.8	10.6	16.5	27.0	34.7	43.4	65.2	85.7

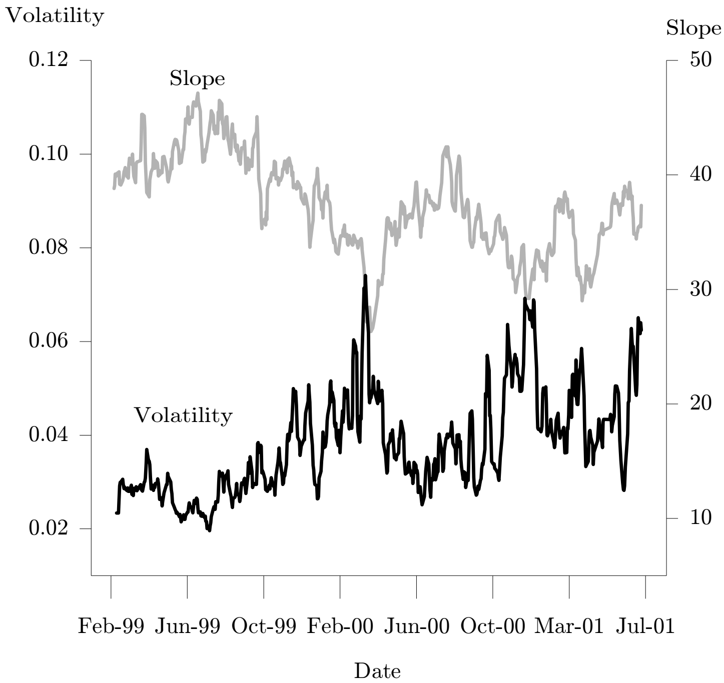
PANEL B

				Distribution of slope estimates							
	N	MCAP	Price	P5	P10	P25	Median	Mean	P75	P90	P95
MCAP Q1 (small)	12532	259	21	5.7	7.4	11.4	17.2	20.9	25.9	38.2	47.9
1999	4163	213	19	5.9	7.5	11.4	17.5	21.2	26.7	39.3	48.2
2000	5864	282	22	6.3	8.2	12.1	17.6	20.9	25.7	36.6	45.9
2001	2505	283	22	4.6	6.0	10.0	15.9	20.5	25.2	40.4	50.9
MCAP Q2	12828	1005	64	10.6	12.9	18.1	26.3	31.8	39.0	56.4	70.9
1999	4264	869	50	11.3	14.0	19.7	28.8	34.0	41.8	59.3	74.4
2000	5999	1035	69	10.7	12.8	17.7	25.1	30.2	36.9	53.1	66.8
2001	2565	1158	76	9.8	12.0	16.9	25.2	31.8	39.2	57.8	73.2
MCAP Q3	12672	2786	121	12.0	15.3	22.2	32.5	39.0	48.1	69.9	87.2
1999	4210	2289	106	15.3	19.1	26.5	38.1	45.2	55.8	78.9	98.1
2000	5934	2914	133	11.6	14.7	21.1	30.4	36.5	44.3	65.0	82.4
2001	2528	3315	121	10.6	13.1	19.5	28.9	34.5	43.0	61.3	75.8
MCAP Q4 (large)	12983	24698	369	18.0	22.1	31.6	47.1	56.5	69.4	101.6	128.8
1999	4331	20016	261	23.0	28.2	38.9	55.7	64.2	79.0	111.6	136.6
2000	6056	26320	491	16.7	21.0	29.2	44.0	53.1	64.6	94.6	120.3
2001	2596	28727	263	15.7	19.9	27.8	41.1	51.5	61.5	96.6	125.9

day. Both variables are averaged over all securities which are traded during the respective days.

Figure 5: **Average slope and volatility**

The figure illustrates the relationship between the estimate of the average daily slope of the order book and the contemporaneous daily price changes. The left axis measures the price change calculated as the average daily absolute return. The right axis measures the slope estimate calculated as the daily equally weighted slope, averaged over all companies that were traded during the trading day.



The figure indicates that there is a negative relationship between the variables at a very aggregate level, i.e. the price volatility seems to be high (low) when the average daily slope of the order book is low (high). To examine more formally whether dispersion of investors' beliefs can explain the contemporaneous volatility across firms and time, we estimate modified versions of the volume-volatility regression equations in section 3.3. Since our proxy for dispersion of beliefs may also proxy for liquidity, it is important that we control for other liquidity measures such as the market capitalization, spread and order book volume. As can be seen in table 7, the SLOPE variable is positively correlated with market capitalization (MCAP) and the spread. However, it is not highly correlated with the order book volume (OV) or any of the other variables. The correlation between SLOPE and N is only about 13 percent.

Our model is estimated as a panel regression with one-way fixed effects. Since not all firms are traded every day,[45] our sample is unbalanced.[46] We estimate 3 models with varying explanatory variables. Model 1 is the same as in the previous analysis, but with the SLOPE variable and the other liquidity variables (market capitalization (MCAP), quoted inner spread (SPR) and order book volume (OV)) as additional variables. In model 2, we do not control for other liquidity variables, and in model 3 we exclude the trading activity (mixing) variables. We also estimate the same regression equations across 3-month sub-periods.

The results from our final estimations are provided in table 9. Generally, the estimated models can thus be written out as,

$$\mid \epsilon_{it} \mid = \sum_{k=1}^{K} \mathbf{X}_{itk}\beta_k + \eta_{it} \tag{8}$$

where $\mid \epsilon_{it} \mid$ is the daily volatility estimate, $\mathbf{X}_{itk}$ is the matrix of explanatory variables (k) across time (t) for each company (i) and $\eta_{i,t} = \nu_i + \varepsilon_i, t$ defines the error structure with ν_i as the non-random fixed, firm specific, effect. Our model can be written as,

$$\begin{aligned} \mid \epsilon_{i,t} \mid = \beta_0 M_{i,t} + \beta_1 N_{i,t} + \beta_2 AV_{i,t}\beta_3 MCAP_{i,t} + \beta_4 SPR_{i,t} + \\ \beta_5 OV_{i,t} + \beta_6 SLOPE_{i,t} + \sum_{j=1}^{12} \rho_{i,j} \mid \hat{\epsilon}_{i,t-j} \mid + \eta_{i,t}. \end{aligned} \tag{9}$$

The first thing to note in Panel A in table 9 is that the dispersion of beliefs variable (SLOPE) is highly significant and negative. Thus, the more dispersed beliefs about the value of the firm, the greater the volatility. This is an indication that a larger dispersion of prices in the order book is linked to more "noise trading".[47] The result could also be related to rip off risk: if some liquidity suppliers are informed about the volatility, as in the Foucault et al. (2003b) model, they may find it optimal to bid less aggressively when they know that the volatility is large.

The partial R^2 indicates that the SLOPE variable is significant compared to the other variables. In model 2, when the liquidity proxies are removed, we see that the partial R^2 for SLOPE increases.[48] This indicates that the slope variable is also proxying for liquidity. However, the parameter estimate

[45]Recall that we filtered out firms with less than 400 trading days throughout our sample period of 597 days.

[46]We also estimate all models on a balanced sample, reducing our sample to 25 firms when filtering out all firms that are not traded every day in the sample. The estimates and tests are quantitatively similar.

[47]A problem could be that a steeper slope implies a less pronounced bid-ask bounce, and thus a lower volatility. However, as outlined in section 3.3, we try to avoid measurement errors due to the bid-ask bounce by calculating returns using the average of bid-ask prices at the close.

[48]The partial R^2 is calculated as the average partial R^2 across single firm regressions.

is stable across the three models. Another interesting thing to note is that the number of trades (N) has a much larger contribution to R^2 than the trade size (AV) which has a significant parameter, but a negligible R^2 contribution. Thus, the Jones et al. (1994) result is also evident in the panel analysis. Furthermore, we run the Hausman specification test to determine whether a random effects model would be more appropriate than the one-way fixed effects model. The Hausman test compares an inefficient but consistent estimator (the fixed effects case) to an efficient estimator (the random effects case).[49] In all of the regressions we reject, at the 1% level, the hypothesis that a random effects model would be more appropriate.

It is also interesting to note the stability and significance of the dispersion proxy also across sub-samples, shown in panel B in table 9. The SLOPE variable is significant at the 1% level within each sub-sample. Also when performing an F-test for fixed effects, we reject the null of no fixed effects at the 1% level within each sub-period regression. We interpret our results as supportive for models where strategic trading and dispersion of beliefs among uninformed investors increase both volatility and volume, and that this can be captured by the characteristics of the limit order book.

[49]Thus, the Hausman test is a test of H_0, that random effects would be consistent and efficient, versus H_1, that random effects would be inconsistent. Rejecting H_0 would suggest that we should use a fixed effects specification.

Table 9: **Panel regression with slope as explanatory variable across whole sample**

The table shows the results from the panel regression with one-way fixed effects (least squares dummy variable estimation) for the whole sample (Panel A) and for sub-periods of 3 month periods (Panel B). The estimated model (model 1) is,

$$| \epsilon_{i,t} |= \beta_0 M_{i,t} + \beta_1 N_{i,t} + \beta_2 AV_{i,t} \beta_3 MCAP_{i,t} + \beta_4 SPR_{i,t} + \beta_5 OV_{i,t} + \beta_6 SLOPE_{i,t} + \sum_{j=1}^{12} \rho_{i,j} \mid \hat{\epsilon}_{i,t-j} \mid + \eta_{i,t}.$$

where $\eta_{i,t} = \nu_i + \varepsilon_i, t$ defines the error structure with ν_i as the non-random fixed, firm specific, effects. $| \epsilon_{i,t} |$ is the absolute return adjusted for day of week effects and autocorrelation in returns. M is a dummy variable for Monday, N is the number of transactions, AV is the average trade size in shares, $MCAP$ is the market capitalization SPR is the relative spread (quoted spread in % of the midpoint price), OV is the total number of shares in the order book (sum of all orders on bid and ask side of the order book) and $SLOPE$ is the average slope of the bid and offer side of the order book. Panel A, shows the parameter estimates for 3 variations of the full model (model 1). In model 2, we do not control for other liquidity variables ($MCAP$, SPR, OV), and in model 3 we exclude the trading activity (N) and trade size (AV) variables. The table shows the associated t-values as well as the adjusted R^2 for each portfolio regression. The autoregressive estimates have been excluded from the table. ** denotes significance at the 1% level. Panel B, shows the sub-period estimates for model 1 for the $SLOPE$, N and AV variables with associated t-values in parenthesis, and the partial R-squared. For each period, the model R-squared, F-test for fixed effects, and number of cross sectional observations (N) and number of time series observations (T) are reported in the last four rows of the table.

PANEL A: Whole sample regression

	MODEL 1		MODEL 2		MODEL 3	
Variables	Estimate	Partial R^2	Estimate	Partial R^2	Estimate	Partial R^2
M	0.023	0.00 %	0.039	0.00 %	-0.010	0.00 %
Trades (N)	0.005**	4.17 %	0.005**	4.17 %	-	-
Trade size shares (AV)	0.020**	0.80 %	0.018**	0.80 %	-	-
MCAP	-0.015**	1.97 %	-	-	-0.001	0.82 %
Spread	0.234**	1.04 %	-	-	0.182**	0.63 %
Avg.slope	-0.007**	2.17 %	-0.009**	3.95 %	-0.008**	2.25 %
Order book volume (shares)	0.026**	0.11 %	0.025**	0.09 %	0.052**	0.85 %
Adj.R^2	21.5 %		20.5 %		18.3 %	
N (cross section obs.)	98		98		98	
T (time series obs.)	572		572		572	
F-test fixed effects	16.34**		14.04**		10.80**	

PANEL B: Sub-period regression

	SLOPE		Trades (N)		Trade size shares (AV)					
Quarter	β_6 (t-value)	part.R^2	β_1 (t-value)	part.R^2	β_2 (t-value)	part.R^2	adj.R^2	F test fixed eff.	N	T
1999.1	-0.007 (-1.4)	2.13 %	0.016 (4.9)	0.79 %	0.074 (1.5)	2.48 %	37.6 %	2.6**	61	14
1999.2	-0.005 (-2.7)	2.65 %	0.013 (11.0)	0.41 %	0.049 (3.0)	4.93 %	26.6 %	4.3**	87	59
1999.3	-0.005 (-3.2)	2.78 %	0.011 (11.2)	0.79 %	0.059 (4.0)	5.06 %	36.7 %	7.7**	96	66
1999.4	-0.006 (-2.8)	3.16 %	0.014 (16.0)	1.94 %	0.033 (2.9)	1.65 %	27.5 %	5.5**	97	64
2000.1	-0.007 (-3.1)	2.17 %	0.013 (26.4)	7.75 %	0.027 (1.7)	0.48 %	31.0 %	8.0**	98	65
2000.2	-0.006 (-3.0)	2.90 %	0.012 (18.9)	10.44 %	0.025 (1.1)	1.11 %	30.7 %	5.2**	98	58
2000.3	-0.007 (-4.3)	1.89 %	0.009 (20.9)	7.87 %	0.006 (0.5)	0.12 %	29.6 %	6.4**	98	65
2000.4	-0.009 (-4.5)	3.30 %	0.008 (16.2)	6.01 %	0.023 (2.1)	0.33 %	21.6 %	4.1**	97	63
2001.1	-0.009 (-4.6)	1.71 %	0.003 (6.3)	7.93 %	0.001 (0.5)	0.59 %	25.6 %	5.2**	93	64
2001.2	-0.008 (-4.6)	1.86 %	0.002 (8.9)	3.47 %	0.030 (2.1)	0.21 %	25.9 %	4.7**	88	54
Average	-0.007 (-3.4)	2.46 %	0.010 (14.1)	4.74 %	0.033 (1.8)	1.70 %	29.3 %			

7 Conclusions

A positive correlation between price volatility and trading volume has been documented in a variety of studies. However, the explanations for the phenomenon is still not well understood. Finding plausible explanations for the relation is important in that it can enhance our understanding of how information is disseminated into market prices. There are two, mainly complementary, explanations. The mixture of distribution hypothesis states that the volume-volatility relation is driven by a directing process that can be interpreted as the flow of information. More specifically, the daily price change and the trading volume are thought to be mixtures of independent normals with the same mixing variable. The dispersion of beliefs hypothesis states that the volume-volatility relation is stronger the greater the dispersion of beliefs about security values is among investors. The explanation behind this statement is based on asymmetric information and strategic investor behavior. Uninformed traders cannot distinguish informed trades from liquidity trades, and by reacting to trades with no information content, they increases both volatility and volume relative to equilibrium values in a situation with symmetric information.

We examine the volume-volatility relation empirically using a very detailed data set from the Oslo Stock Exchange (OSE). We first show that the result in Jones et al. (1994) that average size of trades has little marginal explanatory power when volatility is conditioned on trade frequency also applies in a pure limit order driven market. A unique feature of our data sample is that we can rebuild the whole order book at any time during the trading day. This enables us to investigate the intraday pattern of the order flows and to test a version of the distribution of beliefs hypothesis where we measure dispersion of beliefs by the slope of the demand and supply schedules of the order book. Our intraday analysis supports the notion that there is more asymmetric information early in the trading day, and that this is captured by the slope of the order book. We also find that there is a significant negative relationship between our proxy for the dispersion of beliefs among investors and the contemporaneous market volatility which also is stable across sub-periods.

A Calculating slope measures

To explain the slope calculation more specifically, let N_A and N_B be respectively the total number of ask and bid prices (tick levels) containing orders. Let τ denote the tick level, and let $\tau = 1$ represent the inner quote. Furthermore, let p_1^B and p_1^A be respectively the best bid- and ask prices, and let p^M denote the bid-ask midpoint (which is the average of p_1^B and p_1^A). Let ν_τ^B and ν_τ^A be respectively the percentage of total volume at each tick level on the bid and ask side of the book. Finally, let ω_τ^B and ω_τ^A denote the weight of the local slope calculated at tick level τ for respectively the bid and the ask side of the book. The average elasticity for the supply curve, SE, on day t at snapshot time $s \in [1..6]$ for company i can then be represented as,

$$SE_{i,t}^s = \left\{ \frac{\nu_1^A}{p_1^A/p^M - 1} \omega_{i,1}^A + \sum_{\tau=1}^{N_A} \frac{\nu_{\tau+1}^A - \nu_\tau^A}{p_{\tau+1}^A/p_\tau^A - 1} \omega_{i,\tau}^A \right\} \tag{10}$$

Similarly, the demand curve, DE, can be represented as,

$$DE_{i,t}^s = \left\{ \frac{\nu_1^B}{\mid p_1^B/p^M - 1 \mid} \omega_{i,1}^B + \sum_{\tau=1}^{N_B} \frac{\nu_{\tau-1}^B - \nu_\tau^B}{\mid p_{\tau-1}^B/p_\tau^B - 1 \mid} \omega_{i,\tau}^B \right\} \tag{11}$$

The first term of both equations expresses the slope between the bid-ask midpoint and the best bid and ask prices, while the second term of both equations expresses the sum of the local elasticities for the rest of the order book. The average elasticity in the order book at snapshot s is just the average of $SE_{i,t}^s$ and $DE_{i,t}^s$,

$$SLOPE_{i,t}^s = \frac{SE_{i,t}^s + DE_{i,t}^s}{2} \tag{12}$$

If we based our estimates of daily elasticities on one snapshot only, they could easily be biased due to large trades having temporarily reduced the liquidity of one side of the book or systematic time of day effects. To obtain a less noisy representation of the average daily supply and demand curves for each firm on each date, we therefore average the slopes across the 6 snapshots, i.e.

$$SLOPE_{i,t} = \frac{1}{6} \sum_{s=1}^{6} SLOPE_{i,t}^s \tag{13}$$

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