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THE ANALYSIS OF SINGLE AND RELATED TIME SERIES INTO COMPONENTS: PROPOSALS FOR IMPROVING X–11

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ABSTRACT

Proposals are made for more comprehensive historic and current analysis of the periodic, systematic, and event-conditioned components in economic time series. The effects of unusual events U are distinguished from regular seasonality S and other periodic calendar effects, the trend-cycle C and residual irregularity R.

Improvements to the X–11 program are illustrated to segment the data into regular intervals of monotonic trends and unusual segments, enabling better estimates of regular seasonality for historic and current analysis and forecasting.

A trimonthly weighted–**[1, 2, 1]** moving average of the seasonally adjusted data A (called stream L) is used for short-term analysis of trends, to overcome negative serial correlations in I (and thereby in the month-to-month changes in A), and the deficiencies of the Henderson and MCD moving averages.

Comparisons are made between the component and ARIMA approaches.

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The opinions expressed are the sole responsibility of the author.

PURPOSES AND METHODS OF TIME SERIES ANALYSIS

The increasing needs of Government and business have brought about a considerable expansion of the collection and analysis of economic and other time series. These are primarily monthly series; in some fields, greater depth of analysis may be achieved by the use of daily and weekly series [1], but, in other fields, quarterly data are the most frequent available.

Most monthly (and quarterly) series are affected by seasonality and many by other calendar periodicities (e.g., trading-day effects and festival-date variations). There are usually systematic changes in the series, known as the trend cycle, in which economic analysts are primarily interested, and irregularities of various types which introduce noise distorting the signal (message) of the systematic and periodic fluctuations.

Economic statisticians and others study the behavior of specific time series from the monthly data available for past years and recent months in order to better understand historic, current, and future developments, to formulate policies, and to estimate and evaluate their effects. The analysis may be conducted—

1. In the time domain, studying the behavior of the series month after month, in terms of the components of which we may regard the series as composed. These components are not directly observable (unless the series has been artificially simulated, see [20]), but their characteristics are widely recognized, despite difficulties in definition. (See, e.g., [21].)
2. In the frequency domain, spectral analysis and related techniques. (See, e.g., [25].)
3. Using autoregressive integrated moving average (ARIMA) techniques—Box-Jenkins [11] and similar methods. (See [16; 31].)

The last approach views the series as a finite realization of a specific stochastic process $\{Y_t\}$ that may be adequately depicted by a single mathematical model. Filters can then be estimated (e.g., sinusoidal, autoregressive, moving average) that transform the original series into white noise (a random and uncorrelated series, whose spectrum shows equal power at all frequencies). Parzen [29] and others treat the estimation of these filters as the main purpose of univariate time series analysis, with extensions to multivariate analysis. The random shocks or innovations accompanying the process are analysed together with the periodic and autoregressive relationships.

While the search for the underlying (infinite) process may be appropriate for various physical and industrial series, we find that many economic time series have a more complex structure, which we term "event conditioned." The fundamental intra-annual and secular economic processes cause the basic seasonal and trend-cycle patterns, which may change gradually over specific time segments, but favorable and unfavorable events may cause other significant changes in the series.

The author has studied hundreds of Israel economic series and also many international tourism series [6] and some U.S. and other economic series, using the U.S. Bureau of the Census X–9 and X–11 programs with various modifications [1; 2; 4; 5; 8]. Most Israel series show the effects of strong trends and seasonality, some medium-span cycles, and the effects of a number of unusual events—notably, three wars (in 1956, 1967, and 1973). The effects of unusual events have also been noted in many U.S. series and in those of other countries (e.g., the 1973 fuel crisis). Experience in current analysis of these series [1] and in forecasting [3; 22] and some study of the problems encountered in other countries and other methods used [11; 17; 26; 33] have indicated the need for a more comprehensive approach to time series analysis [1; 7].

This approach pays attention to a feature of empiric time series neglected in most theoretical work, namely that the data refer to specific months, in some of which, events have occurred that significantly affect the specific and related series and their future behavior. Computer programs can automatically treat all the numeric and qualitative data supplied as input, according to predetermined algorithms, but the analyst should view the data and the potentialities of computer analysis in a more comprehensive manner, using case-study techniques, too. In many cases, it is desirable to view the historic series as comprised of a number of consecutive segments (intervals), in most of which the behavior of all the components is describable by a simple model, but with the possibility of some unusual segments or of changes in behavior between successive regular segments.

We may regard comprehensive time series analysis as including the following stages (see fig. 1):

1. Historic Analysis

   Inputs into the Historic Analysis

   - A specific series over a defined time interval (era) (e.g., tourist arrivals to Israel, for the 240 months from January 1956 to December 1975).
   - The subseries of which this series is composed (e.g., subaggregates, by region or economic branch [18]).
   - Microtime analysis of the series (or of relevant subseries)—weekly or daily data (if available, even if only for the latest years).
   - Related series (preferably over all the months analysed, e.g., weather).
   - A diary of the events which may have affected the series over the era analysed.
   - The analysts' knowledge of the characteristics

Figure 1. COMPREHENSIVE ANALYSIS FOR MONTHLY ECONOMIC SERIES INTO COMPONENTS

Item No.	Type of analysis	Inputs	Processing	Outputs
1	Historic	Monthly series Y_{ij} Subseries $Y^{(h)}$ Microtime series Y_d Related series Z Diary of events Characteristics of the series Model (multiplicative, additive, etc.)	Computer program, e.g., X–11 plus supplements — Analysts	Historic estimates for components— $S, F, D; A; C, I; L$ Segmentation into event-conditioned periods r, u, s Forecasts of $S, F, D, \sigma (I)$
2	Current	New data $Y, Y^{(h)}, Y_d, Z$ Current events	Forecasts of S, F, D, possibly of Y, for manual or computerized adjustment — Analysts	Current seasonally adjusted data $A, A^{(h)}$ Current short-term trends $L, L^{(h)}$ Interpretation of the changes in Y, A, L from month-to-month and over longer spans
3	Updating of Factors	Restudy series (usually annually)	Analysts — Analysis program	Revision of S, F, D, A, L (historic and forecasts) if justified by data and/or events
4	Forecasts	Assumptions as to future policies and events and their effects on the components for basic, sub-, and related series	Computer forecasts of S, F, D — Forecasts of C, U — Analysts	Forecasts of $Y, Y^{(h)}$ (point or alternatives)

of the series and of its constituent subseries and of the periodic and other changes in the data over the period analysed. For example, reasons for the fluctuations of the real-world activity (or the stock, e.g., unemployment) and of its statistical measurement—over the days of each month and months of the years,[1] special conditions relating to the collection and processing that might have caused discontinuities in the trend-cycle or in the seasonality, etc.

The analysts' choice of an appropriate model (or possibly experimentation with several models).

Outputs

Estimates of the behavior of all the components of the series (and of sub- or related series) over the era analysed, preferably segmented into specific homogenous intervals, distinguishing the effects of major unusual events.

Forecasts of the seasonal and other periodic factors and of the variability of the irregularity (e.g., $\sigma(I)$) for the next 12 to 24 months to enable current analysis of the development of the series and forecasting.

2. Current Analysis—reporting or control (See [1, ch. II. 4; 31].)

Data continue to flow each month for the series (and sub- and related series), possibly with different time lags until the data are available.

Current analysis is conducted on these data, utilising seasonal adjustment (and adjustments for other relevant factors) to indicate the prevailing changes in the trend-cycle and also changes which might be occurring in the seasonal and other components. The understanding of the data computed as seasonally adjusted data A or current trend will be improved by applying the analysts' knowledge of current events and conditions and any similarities (or differences) to historic effects.

3. Updating the Analysis and Factors

A revised historic analysis should be performed, from time to time, usually after another complete year's data are available (more frequent analysis may introduce undesirably frequent changes in the estimates of seasonality and of seasonally adjusted data). Even after one year, care should be taken before changing the estimates of the seasonal and related components, since stability of the analysis is important; many events and conditions may introduce spurious changes in the estimates.

4. Forecasting

Automatic prediction may be made for future values of the series under the assumption of continuity of prevailing conditions, using an appropriate model for the historic behavior of the series (and related series).

Effective forcasting must however take into account a deeper analysis of past, present and expected future conditions and events according to defined scenarios, using the judgement of one or several analysts (Delphi techniques) [3; 22].

The comprehensive time series analysis is intended to assist the application of the analysts' knowledge and skills together with the best available automatic techniques, rather than to process the data through standard programs with insufficient professional scrutiny. Revision of the assumptions, results, and forecasts may be required every few years.[2]

Organisations having to analyse thousands of series each month face severe problems. The author proposes the training of staff and the development of computer programs (preferably interactive, via on-line terminals, [32]) to conduct comprehensive analysis of the major series at least every year or two, then apply the principles to the family of sub- and related series (e.g., unemployment, by region or age, or construction, by type). This analysis will provide rewarding professional work, improving the series and the methods of analysis and providing the data users with a more complete basis for their further analysis and policy decisions.

No computer program alone can be expected to provide the insight that can be achieved by interaction between the analyst (or better, several analysts) and

[1] Some data that nominally relate to calendar months may be based on four or five reports for complete weeks. Some weekend activity may be included in Monday's data, regardless of its calendar date. Sampling and other errors may be of importance, which may affect all or some of the monthly data and the correlations between data for different months (total or subseries: See the section, "Analysis of Selected U.S. Economic Series, Distinguishing Regular and Unusual Segments").

[2] Comprehensive approaches are applied in other fields of statistics. For example, a statistician should not analyse the data collected in a sample survey without full appreciation of the population studied, the sampling and estimating procedures, questionnaires, coding and editing procedures, nonresponse, etc., and of similar past or current surveys. Similarly, the data arising from an experiment should be analysed after studying the experimental design, measurement procedures, etc. Considerable judgment is used in the compilation of monthly price indexes. Autoanalysis of time series data is all too common, not benefiting from the study of other relevant data and the background of the series and events affecting it. Detailed case studies, indicating the results and interpretations of alternative approaches, are desirable, as in Operations Research and in other sciences. In medicine, there are similar techniques for preliminary diagnosis on the basis of numeric and qualitative historic data (including sets of ECG time series), current analysis under selected policies (treatment, etc.), and forecasting (prognosis), then presenting the results and background as a case study.

computer operations. (See [15].) Millions of dollars are spent on the collection and processing of the major economic series (national and for specific enterprises), and the policy decisions based on them are of major importance to modern economies, justifying more professional effort in analysis.

There is often a suboptimal imbalance between the considerable professional effort devoted to designing a series and the relevant samples and procedures, collecting, editing, and processing the data and the relatively little (and often hasty) analysis of the reasons for the fluctuations.

COMPONENTS IN THE TIME DOMAIN: PERIODIC, SYSTEMATIC, AND EVENT-CONDITIONED EFFECTS

The conventional approach to the analysis of monthly time series assumes that the original data[3] Y are a function of three principal components that operate independently each month:

C trend cycle
S seasonality
I irregularity

Two functions are commonly used for the interrelationship—

Multiplicative

$$Y = C.S.I \qquad (1)$$

used especially for series whose trend varies considerably and for which the absolute seasonal variations are approximately proportional to the trend cycle.

Additive

$$Y = C + S + I \qquad (2)$$

used for series without strong trends, with zero data in specific months, or in other cases where absolute seasonality does not depend mainly on the level of the trend cycle. For some series (e.g., unemployment) some of the subseries appear to behave additively and some multiplicatively, though it is difficult to explain the rationale for the use of a multiplicative, additive, or mixed model.[4] It is usually assumed that C is continuous and that there may be gradual changes in S for each month over the years.

Deeper analysis requires taking into account more components (see appendix):

Periodic.
  Seasonality S (constant or moving M).
 Other calendar effects.
  Trading-day variation D.
  Festival-date variation F.

Extraseasonal annual effects—e.g., nonaverage weather variation W.

Systematic components—the trend-cycle C, comprised of the secular trend T and cycles Cy (business, multiannual, and others). The fundamental characteristic of the trend is its monotonicity over specific periods [30].

Perturbations due to unusual events U.

Other Irregularities and residual fluctuations R (possibly after analysing the effects of related variables Z).

For a given series, these components may behave differently over consecutive segments, possibly even showing discontinuities.

The possible effects of an unusual unfavorable event on the level of a monthly series (excluding periodic effects) are illustrated in figure 2.

Following the basic conditions **a**, there may be a segment **b** during which the event is anticipated (e.g., the approach of a war or devaluation) and during which there may be advances of activity or postponements—increasing (or reducing) the level of the series accordingly. The crisis **c** may be brief (or last for several months) and will usually be accompanied by a considerable drop in the level, followed by a further period of depressed level **d**. The recovery of the series **e** is usually a rapid surge and may comprise a segment of compensation **f** for activity lost (e.g., the arrival of tourists who postponed their visits during the crisis). At that time, there may be an unsettled segment **g** before new stable conditions **h** are achieved that may comprise changes in seasonality as well as in the level and/or rate of growth of the trend.

On the other hand, the unusual event may be of a positive nature (e.g., an exposition that attracts tourists) and the possible effects may then include segments of anticipation, the event itself, and postevent high (or low) activity before new stable conditions are achieved [2].

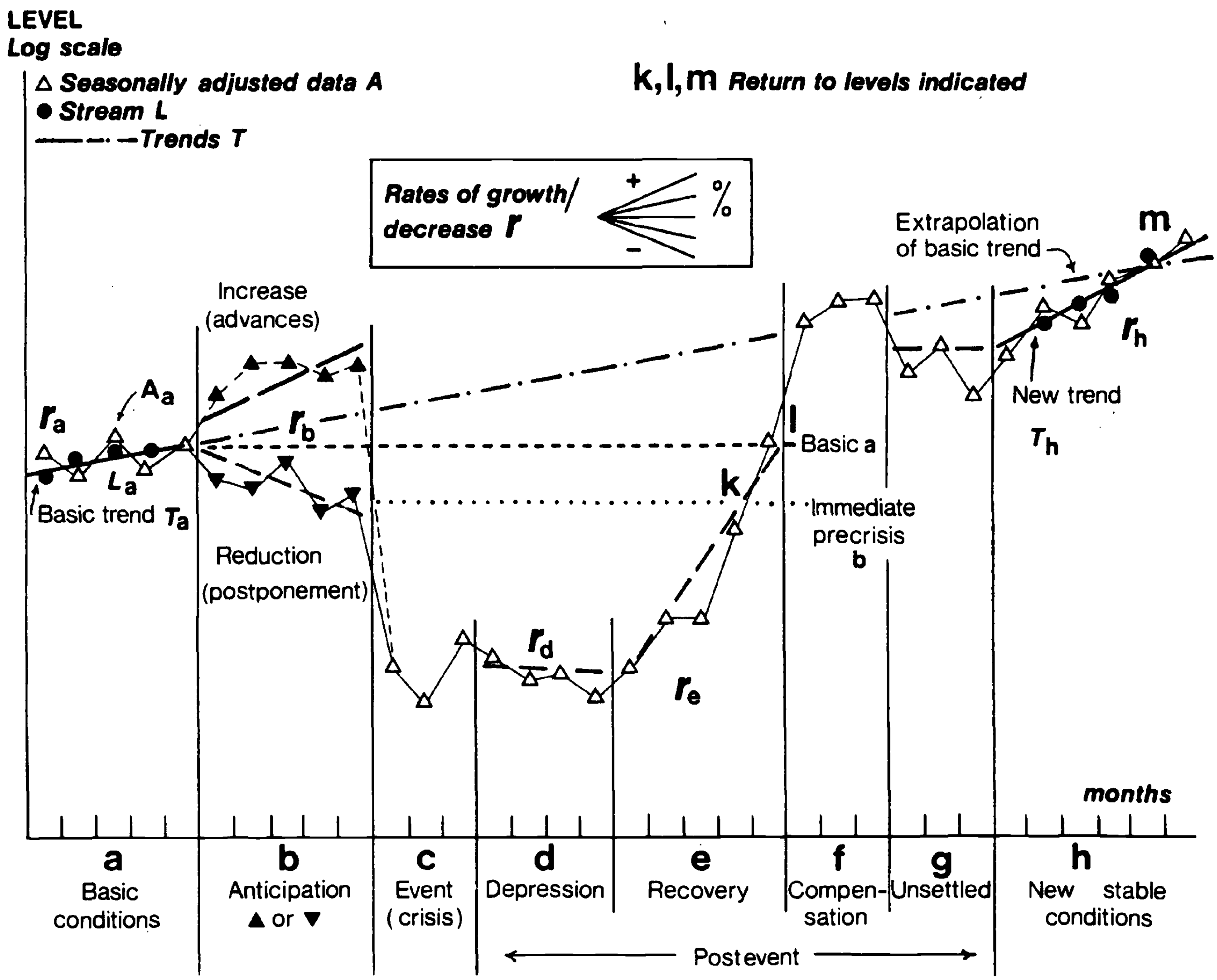
In such cases and in others, e.g., turning points of a business cycle, considerable changes in trend or in seasonality, due to changed policies, etc., the fundamental assumption of continued independent operation of the components may not be applicable.

Many series show some negative serial correlations between consecutive-months' data, and the moving-average filters (as in X–11) may also introduce negative correlations, resulting in a saw-tooth growth (or decline) of the seasonally adjusted data A in regular segments, rather than a smooth trend cycle perturbed

[3] The letter "Y" is used to designate the original data rather than "O", because of the latter's inconvenient resemblance to zero. It would be desirable to standardise all the terminology and symbols for time series analysis. (See the app.)

[4] In ARIMA techniques, a similar decision must be taken whether to use a logarithmic transformation (corresponding to the multiplicative model) or another transformation of the original data, to utilise the additive model.

Figure 2. POSSIBLE EFFECTS OF AN UNUSUAL EVENT ON THE LEVEL OF A MONTHLY SERIES



by random irregularities only.[5] The use of MCD and Henderson moving averages to estimate C often distort short-term trends by averaging levels from different segments. We, therefore, propose the use of—

Bimonthly moving averages (flow B).

Trimonthly weighted averages, with weights [1, 2, 1] (stream L).

that appear to offer many advantages in historic and current analysis of trend cycles. (See app. and table 1f.)

It is, therefore, desirable to apply the seasonal analysis and adjustment in a manner that permits the analysis of all the appropriate components for the series (and for related or subseries), distinguishing all the segments in which their behavior may differ. This approach enables deeper understanding of the series and perhaps a more realistic view of the accuracy with which estimates may be made of the various components throughout the regular segments.

This approach may often enable improved historic and current analysis. Forecasting may also be improved by taking into account the behavior of the components over the variety of past segments, together with the analysts' judgement as to future scenarios.

PROPOSALS FOR IMPROVING THE X-11 AND SIMILAR PROGRAMS

The X-11 program provides a very convenient, automatic method of analysing a wide variety of series into three or more components, offering some options to the analyst [35]. Despite some theoretical and empirical deficiencies, it has proved robust and applicable to numerous economic series in many countries [20].

A number of improvements are proposed to the X-11 program for univariate series, based on the above approach and some other variants and proposals. (See [27].) Most of these improvements are suitable for other similar programs, too. We also propose some procedures for the parallel multivariate analysis of several related series. The application of these improvements is illustrated for some Israel and U.S. series in the following section.[6]

For convenience, the main stages of the standard X-11 multiplicative program are summarised first. (For details, see [35].) We use the following notation for the moving averages used to smooth irregularities:[7]

[m] = moving average over m successive months, uniformly weighted (used with $m=12$ then centred for preliminary smoothing of seasonality in Y, and with $2\leq m\leq 6$ on A for short-term trends).

[H] = the Henderson moving average, of span 9, 13, or 23 for successive months.

[*i*] = weighted moving average over the years i for each month, to smooth SI factors (usual span, 7 years).

[5] The negative serial correlations may be observed in the residual irregular factors R (I computed by the X-11 program regarding the regular segments) and also in the month-to-month percent changes a in A. On the other hand, there may be strong positive serial correlations in the irregularities I over unusual segments, so that the calculations of the average duration of run or serial correlations for all the irregular factors I over the total era analysed may not show a significant departure from randomness. (See [1, pp. 77-78].)

[6] These proposals do not take into account other approaches to time series analysis, e.g., spectral and cross-spectral analysis, ARIMA methods, more detailed econometric analysis of the relationships between series or major changes in the stages of the basic X-11 program, that have been discussed in other papers at the conference [12; 13; 14; 16; 19; 28; 29; 31].

[7] These moving averages assume continuity and create considerable problems in estimating for the first and last years and forecasting; the new approach bypasses these.

Principal Stages of the X-11 Multiplicative Program: Outline

Stage	Symbols	Step of X-11
1. Original series	$Y=CPSI$	A1
2. Prior adjustments (for festival-date variation F, trading-day effects D)[8]	P	A2
3. Prior-adjusted original data	$Y^P=Y/P$	B1
4. Preliminary estimate of trend cycle	$C^1=[2][12]Y^P$	B2
5. Preliminary estimate of SI ratios	$(SI)^1=Y^P/C^1$	B3, 7
6. Estimate $\sigma(I)$, distinguish extreme SI ratios outside $k\sigma(I)$ control limits, estimate weights w for outlying months, impute replacement ratios	SI^w	B4, 9
7. Smooth SI, SI^w by [*i*] to preliminary S	S^1	B5, 10
8. Calculate preliminary seasonally adjusted data	$A^1=Y^P/S^1$	B6, 11
9. Calculate modified s.a.d. (for outliers)	A^{1m}	
10. Smooth to estimate improved trend cycle	$C^2=[\mathbf{H}]A^1$, A^{1m}	B7

Table 1. ANALYSIS OF TOURIST ARRIVALS BY AIR FOR ISRAEL, BY THE X–11 PROGRAM AND SUPPLEMENTARY STAGES: 1956 TO 1975 AND 1976

a. ORIGINAL SERIES *Y* AND ARIMA FORECASTS

Year		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
1956		924	1384	2547	2646	2448	1822	2514	2019	1876	1962	864	1063	22069
1957		1151	1293	1767	2911	2660	2103	3177	2416	3317	2389	1349	1491	26024
1958		1466	2014	4524	8003	4096	3772	4819	3288	2599	2610	1929	2074	41194
1959		1526	2218	4922	6832	6632	4101	6495	3903	4496	3935	2842	3012	50914
1960		2143	3744	6709	9896	6971	5370	9480	6320	5022	6420	4309	5487	71871
1961		3655	6327	9288	9834	8084	7369	14310	9701	7041	7145	4788	5015	92557
1962		3972	5987	9167	13001	10731	8405	14944	8565	7584	7226	5167	6818	101567
1963		4268	6790	10646	16803	11010	9732	18454	11690	9090	10919	6178	9169	124749
1964		5670	7989	16205	14599	12926	11759	19460	13235	10331	10727	7729	10265	140895
1965		5720	8142	12089	21055	17780	13880	22209	17290	12741	12336	7093	12695	162990
1966		7159	11689	17984	22199	16417	16729	28072	17443	13692	15919	7319	11027	185649
1967		7295	10311	17752	19817	15222	9868	30427	22790	18147	22896	12841	24220	211586
1968		12641	18402	27585	47020	27869	30269	57170	39132	28527	31584	19125	27200	366524
1969		14284	16538	34474	32496	25236	31978	60411	40099	24170	26821	17412	29200	353119
1970		17682	22987	35850	32338	29115	34456	64519	46843	25907	21765	17023	33370	381855
1971		20308	26414	44344	57840	39998	51294	87832	64013	43094	53703	31277	46208	566325
1972		29482	39182	72632	59304	58557	54678	83326	55635	47129	58466	30179	38543	627113
1973		27648	35019	48774	72864	54622	53095	84695	58486	51665	16991	20599	37058	561516
1974		24304	34303	52551	59562	41172	41899	67668	52789	37486	47071	29685	37633	526123
1975		21844	27238	49300	39941	36983	40420	65322	51421	41860	50924	31195	51948	508396
1976	a	28079	37503	62344	78792	55087	51165	80145	64743	50699	51261	44111	57669	671597
	b	29782	39096	56356	77084	54455	56871	94259	69578	53329	52549	35401	54378	673138
ARIMA	c	29319	36277	51812	70553	49786	52265	85488	63940	49556	58588	35521	53072	636177

b. ADJUSTMENTS FOR VARIATIONS IN FESTIVAL DATES *F*

(Original series adjusted for festival-date variation Y^P)

	Y^P		F percent		
Year	Mar.	Apr.	Mar.	Apr.	Total
1956	2148	3045	118.574	86.897	22069
1957	2028	2650	87.113	109.865	26024
1958	4174	8353	108.385	95.810	41194
1959	5004	6750	98.359	101.216	50914
1960	7289	9316	92.044	106.225	71871
1961	8220	10902	112.999	90.200	92557
1962	10870	11298	84.332	115.075	101567
1963	11029	16420	96.524	102.335	124749
1964	13623	17181	118.952	84.973	140895
1965	13941	19203	86.716	109.644	162990
1966	17423	22760	103.222	97.534	185649
1967	17883	19686	99.266	100.666	211586
1968	29669	44936	92.975	104.638	366524
1969	32135	34835	107.278	93.286	353119
1970	35850	32338	100.000	100.000	381855
1971	45058	57126	98.416	101.249	566325
1972	64558	67598	112.891	87.731	627113
1973	57270	64368	85.165	113.199	561516
1974	53334	58779	98.532	101.332	526123
1975	41820	47421	117.885	84.227	508396
1976[a]	69197	71940	90.097	109.525	671597

Note: See footnotes at end of table 1i.

Table 1. ANALYSIS OF TOURIST ARRIVALS BY AIR FOR ISRAEL, BY THE X–11 PROGRAM AND SUPPLEMENTARY STAGES: 1956 TO 1975 AND 1976—Continued

c. FINAL WEIGHTS FOR IRREGULAR COMPONENT *w* AND STANDARD DEVIATIONS OF *I*

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	S.D.
1956	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	0.0	100.0	8.0
1957	0.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	0.0	100.0	81.7	100.0	8.0
1958	100.0	100.0	100.0	0.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	8.0
1959	100.0	100.0	100.0	100.0	0.0	100.0	100.0	48.6	100.0	100.0	100.0	100.0	6.8
1960	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	28.1	100.0	100.0	100.0	6.3
1961	100.0	0.0	100.0	100.0	86.4	100.0	100.0	39.7	100.0	100.0	100.0	23.8	5.9
1962	100.0	100.0	100.0	0.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	5.2
1963	100.0	100.0	100.0	29.1	100.0	100.0	100.0	100.0	100.0	98.0	100.0	100.0	4.8
1964	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	0.0	100.0	4.8
1965	100.0	100.0	100.0	100.0	0.0	100.0	55.4	100.0	100.0	100.0	100.0	100.0	5.0
1966	100.0	82.1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	0.0	100.0	0.0	5.4
1967	100.0	100.0	73.9	100.0	100.0	0.0	100.0	100.0	100.0	98.9	100.0	100.0	5.6
1968	100.0	100.0	100.0	15.7	100.0	100.0	100.0	100.0	100.0	100.0	11.5	100.0	6.0
1969	100.0	1.6	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	5.8
1970	100.0	100.0	100.0	0.0	100.0	100.0	100.0	7.3	100.0	0.0	100.0	100.0	7.0
1971	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	6.7
1972	100.0	100.0	100.0	100.0	0.0	100.0	100.0	56.0	100.0	11.3	100.0	13.6	6.7
1973	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	0.0	0.0	80.4	100.0	5.6
1974	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	0.0	100.0	5.6
1975	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	5.6

d. SEASONAL-IRREGULAR RATIOS *SI*, BY PERCENT

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Avg.
1956	46.6	71.3	113.4	165.0	136.0	102.4	140.0	109.8	99.0	101.1	44.2	55.1	98.6
1957	61.3	70.8	112.9	146.3	141.7	105.9	150.6	108.1	141.0	96.7	51.6	53.7	103.4
1958	49.7	64.7	128.3	250.3	122.7	115.4	151.9	106.3	85.3	85.5	61.9	63.9	107.2
1959	44.6	61.4	132.6	172.7	164.8	99.3	153.5	90.3	101.8	87.0	61.0	62.3	102.6
1960	42.5	71.2	133.7	166.7	123.0	93.6	161.6	103.4	77.5	93.3	59.8	74.4	100.1
1961	49.6	87.4	115.1	152.2	110.1	96.7	181.3	120.4	87.3	89.2	60.0	62.3	101.0
1962	48.2	70.3	124.2	127.2	121.0	96.3	174.8	102.0	91.0	86.1	60.3	77.3	98.2
1963	47.0	72.8	115.4	168.7	111.3	96.5	179.0	110.5	83.7	98.1	54.4	79.8	101.4
1964	49.0	68.9	117.6	148.8	112.3	102.4	169.2	114.2	88.3	91.0	65.3	86.3	101.1
1965	47.7	66.9	112.0	149.8	134.2	102.0	160.9	125.0	92.2	89.0	50.1	88.0	101.5
1966	48.0	75.8	109.8	141.4	101.9	105.0	180.0	115.0	93.0	110.6	51.3	76.9	100.7
1967	50.2	69.8	119.4	129.4	97.8	60.9	176.4	121.9	89.0	103.7	54.5	97.1	97.5
1968	48.3	67.5	105.0	154.4	93.3	99.2	184.0	124.7	90.8	101.4	62.5	91.1	101.8
1969	49.2	58.0	113.0	121.0	86.2	107.8	202.5	134.1	80.5	88.4	56.3	92.0	99.1
1970	54.3	69.6	108.0	97.8	88.7	106.0	200.8	147.0	81.1	67.1	50.8	95.2	97.2
1971	55.0	67.9	110.4	133.9	89.9	110.2	180.0	125.7	81.8	99.3	57.0	83.9	99.6
1972	53.8	72.2	120.4	129.1	114.0	108.0	165.1	109.2	91.1	111.8	57.8	74.8	100.6
1973	54.6	69.9	114.1	126.9	106.9	104.7	172.4	125.4	117.5	40.4	49.7	88.2	97.6
1974	56.3	77.6	119.9	134.0	96.1	99.5	160.6	123.2	85.8	106.9	68.4	90.0	101.5
1975	54.9	71.6	113.4	129.5	99.3	104.7	159.8	117.3	88.4	100.1	57.6	91.0	99.0
AVGE	50.5	70.3	116.9	147.3	112.6	100.8	170.2	116.7	92.3	92.3	56.7	79.2	

Table 1. ANALYSIS OF TOURIST ARRIVALS BY AIR FOR ISRAEL, BY THE X–11 PROGRAM AND SUPPLEMENTARY STAGES: 1956 TO 1975 AND 1976—Continued

e. SEASONAL FACTORS S AND S^f, BY PERCENT

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Avg.
1956	47.6	67.9	120.5	162.6	133.3	106.8	148.6	106.7	95.9	93.3	58.3	58.3	100.0
1957	47.1	67.8	122.1	163.5	132.6	105.5	149.6	105.9	95.5	92.9	58.6	59.6	100.1
1958	46.8	67.6	123.6	165.4	130.2	109.8	153.4	109.3	94.3	91.7	58.9	62.0	100.1
1959	46.7	67.6	125.1	163.1	126.5	101.9	158.4	104.8	92.1	90.3	59.6	65.6	100.1
1960	46.8	68.1	125.1	161.6	122.6	99.6	164.4	104.7	90.6	89.6	59.7	69.2	100.2
1961	46.9	69.1	124.0	159.9	119.1	97.8	168.7	109.6	89.0	90.1	59.4	73.5	100.2
1962	47.5	69.9	120.7	155.8	116.2	97.3	172.1	108.5	88.4	91.0	57.9	77.1	100.2
1963	48.0	70.4	118.0	152.4	113.4	98.7	173.6	111.2	88.3	91.4	56.3	81.0	100.2
1964	48.4	70.5	115.7	149.0	110.2	100.0	174.0	114.2	88.9	92.7	54.6	84.5	100.2
1965	48.5	70.4	114.2	145.6	106.8	100.9	174.3	116.6	89.7	94.5	53.8	88.1	100.3
1966	48.8	69.9	112.7	140.9	102.3	101.6	177.1	120.2	89.5	95.7	53.5	90.5	100.2
1967	49.5	69.6	111.5	136.4	98.0	102.4	181.8	122.6	88.7	96.1	53.7	92.0	100.1
1968	50.1	69.1	110.7	132.6	94.1	103.4	186.2	124.9	86.7	96.4	54.0	92.2	100.0
1969	51.2	69.0	110.7	130.6	91.9	104.7	187.8	125.5	85.4	96.6	54.6	91.4	99.9
1970	52.2	68.9	111.6	129.4	92.0	106.1	180.5	126.0	84.4	96.0	54.8	90.0	99.8
1971	53.3	70.0	113.1	129.5	93.6	106.5	182.3	125.0	84.6	96.6	54.8	89.0	99.9
1972	54.3	71.1	114.8	129.9	96.0	106.3	176.1	123.8	85.1	97.9	54.9	88.5	99.9
1973	54.9	72.4	116.3	130.8	98.1	109.3	169.7	122.2	86.9	99.9	55.5	88.7	100.0
1974	55.0	72.8	116.7	130.7	99.7	104.7	166.0	121.6	87.2	100.7	55.8	89.1	100.0
1975	55.1	73.1	116.9	130.4	100.6	104.0	164.9	121.5	87.4	101.0	55.6	89.6	100.0

D10A. SEASONAL FACTORS, ONE YEAR AHEAD

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	AVGE
1976	55.2	73.3	116.9	130.2	101.0	103.7	164.4	121.5	87.5	101.1	55.5	89.9	100.0
1976[d]	54.7	71.4	118.4	129.5	98.0	98.8	157.0	120.2	88.6	105.3	55.7	93.1	100.1

f. SEASONALLY ADJUSTED DATA A AND STREAM L, INDICATING SEGMENTS, BY TYPE

(Average rate of growth per month, by percent, and flags for month-to-month changes, unusual events, and extreme months)

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
A1956	1.942	2.037 +	1.782	Q1.873 -	1.836 -	1.706 M	1.691 +	1.892	1.956	N2.103 +	1.482Z+	1.824 +	1956
L		1.949 -	1.868 -	1.841 -	1.813 -	1.735 =	1.745	1.858	1.977 -	D1.911 -	1.723	1.893 +	
A1957	2.444E+	1.907 +	1.662 -	1.621 +	Q2.007 M	1.994	2.123	2.281 +	Q3.475E+	2.571 +	2.304X	2.566 +	1957
L	2.155 -	1.980 +	1.713 =	1.728 +	1.907	2.029	2.130 +	2.540 +	2.950 -	2.730 +	2.420	2.609 +	
A1958	3.131 -	2.981 +	3.378 +	Q5.112E+	3.147 +	3.633 +	3.141 M	3.121 +	2.756	2.845 +	3.275	3.348 -	1958
L	2.936	3.118 +	3.712 +	4.187 +	3.760 -	3.388 -	3.259 -	3.035 -	2.869	2.930	3.186	3.310 M	
A1959	3.269 -	3.280 +	3.999	4.139 +	Q5.244E+	Q4.024	4.101 -	3.724Y+	4.879 +	4.358	4.770 -	4.592 M	1959
L	3.291	3.457 +	3.854 +	4.380	4.663 -	4.348 -	3.987	4.107	4.460	4.591 =	4.623 =	4.634	
A1960	4.581 +	5.500	5.826 M	5.764 -	5.685 -	5.394	5.766	6.034 -	5.544 +	7.164 +	7.223	7.931 -	1960
L	4.813 +	5.352	5.729 =	5.760 -	5.632 -	5.560	5.740	5.844	6.071 +	6.774	7.385	7.716	
A1961	N7.788 +	9.156E+	6.626	Q6.834 M	6.785X+	7.534 +	8.432	9.185V+	7.912 +	7.926	8.062 +	6.819Y+	1961
L	8.166 =	8.181 +	7.310 -	6.770	6.984	7.571 +	8.396	8.678 -	8.234 -	7.956 -	7.717 -	7.517	
A1962	8.370	8.562	9.004 +	7.250Z+	Q9.237 -	8.634 =	8.682 -	7.894	8.576 -	7.942 +	8.929 M	8.946 +	1962
L	8.030	8.625 -	8.455 -	8.185	8.589	8.797 -	8.473 -	8.262 M	8.247	8.347	8.661	8.860	
A1963	8.899	9.638 -	9.351 +	Q10.774V-	9.711	9.881	10.633 -	10.508 -	10.291 +	11.945W-	10.980	11.326	1963
L	9.070	9.381	9.778	10.152 -	10.014 =	10.016	10.409 =	10.485	10.759	11.290 =	11.308 +	11.338	
A1964	11.721 -	11.332	11.773 -	11.529	11.729 =	11.761 -	11.181	11.589 =	11.615 M	11.574 +	14.148E+	12.146 -	1964
L	11.525 =	11.539 -	11.602 =	11.640 =	11.687 M	11.608 -	11.428 +	11.493 =	11.598	12.228	13.004 -	12.558 -	
A1965	11.794 -	11.558	12.204	13.190 +	Q16.647E+	13.758 -	12.739X+	14.829 -	14.210 -	13.049 =	13.110	14.413	1965
L	11.823 M	11.778	12.289 +	13.808	15.060 -	14.225 -	13.516	14.152 M	14.074 -	13.354 +	13.421	14.153	
A1966	14.676 +	16.711W-	15.466	Q16.148 M	16.045	16.459 -	15.854 -	14.512	15.303	16.627E+	N13.670 +	12.163Z+	1966
L	15.119	15.891 =	15.948 =	15.952	16.174 =	16.204 -	15.670 -	15.045	15.436 =	15.557 -	14.037 -	13.211	
A1967	14.807 =	14.819	16.070W+	14.439	Q19.531 +	N9.633Z+	Q16.737 +	18.586	20.463 +	23.836W+	23.926	26.326 -	1967
L	14.154	15.129	15.349 -	15.118 -	13.782 -	12.883 +	B15.423 +	18.593 +	20.837 +	23.015	24.504	25.450	
A1968	25.223	26.613 =	26.805 +	33.890V+	Q29.620 -	29.266	30.696	31.341	32.913 M	32.777	35.390V+	N29.505 -	1968
L	25.846	26.313	28.528	31.091 -	30.599 -	29.712	30.500	31.573	32.486	33.464 M	33.265 -	30.581 +	
A1969	27.923 +	N23.959Y+	29.018 -	J26.663	J27.462 +	30.555	32.165 M	Q31.961 +	N28.308 -	27.779 +	Q31.879 =	31.934	1969
L	27.327 -	26.215	27.164 +	27.452	28.035	30.184	31.711 -	31.099 -	29.089 M	28.936	N30.869	32.405	
A1970	33.862 -	N33.362 -	32.109 +	24.995Z+	Q31.641	32.489	N34.987	37.185Y+	N30.694 +	22.663Z+	31.061 +	37.071	1970
L	33.256 M	33.174 -	30.644 -	28.435	30.191	32.801	34.712 +	34.913 +	30.309 +	26.770 +	30.464 +	35.822	
A1971	38.085 M	37.748	39.825 +	44.097 -	42.744 +	48.171 =	48.186	51.216 M	50.962	55.595	57.125 -	51.907	1971
L	37.747	38.351	40.374	42.691	44.439	46.818	48.940	50.395	52.184	54.819	55.438 -	53.808 +	
A1972	54.293	55.132	56.049 -	52.033 +	N61.021E+	51.431 -	47.310 -	44.954X+	N55.391	59.690W-	54.933 +	43.562V+	1972
L	53.906	55.151 M	54.816 =	55.284	56.376 -	52.798 -	47.751 =	48.152 +	53.856	57.426 -	53.279 -	48.161 =	
A1973	50.347 -	N48.397	49.255 =	49.223 +	Q55.675 -	50.406 M	49.921 -	47.875 +	Q59.750E+	N17.002Z+	37.098X+	41.762	1973
L	48.163	49.099 M	49.033	50.844	52.745 -	51.602 -	49.531	51.355 +	46.094 +	32.713	33.240 +	41.193	
A1974	44.151	47.114 -	Q45.683 -	N44.989 -	Q41.281 -	40.022	N40.769	N43.429 M	Q42.986	46.741 +	Q53.156E+	N42.239 -	1974
L	44.294	46.015 M	45.867 -	44.236 -	41.893 -	40.523	41.247	42.653	44.035	47.406	48.823 -	44.311 +	
A1975	39.609 -	37.246 -	35.785	36.371 =	36.770	38.850	39.611	42.318 +	47.895	50.444 +	56.058	57.970 /	1975
L	39.676 -	37.471 -	36.297 =	36.324	37.190	38.520	40.097	43.035	47.138	51.210	55.132 +		

Note: See footnotes at end of table 1i.

Table 1. ANALYSIS OF TOURIST ARRIVALS BY AIR FOR ISRAEL, BY THE X–11 PROGRAM AND SUPPLEMENTARY STAGES: 1956 TO 1975 AND 1976—Continued

g. *SR* * AND OTHER *(SI)*^{L} RATIOS, BY PERCENT AND SEGMENT[f]

Year	Jan.	Feb.[g]	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Average	Year
SEG01 1956	46.4 R	71.0 R	115.0 R	165.5 R	135.1 R	105.1 R	02 151.0 R	108.7 S	94.9US	03 102.7SU	50.2 U	56.2 U	95.7	1956
SEG 1957	53.4 U	65.3 U	118.5 U	153.4 U	04 139.5US	103.6 S	149.2 S	95.2US	05 112.4SU	87.5 U	55.8 U	57.2 U	99.3	1957
SEG 1958	49.9 U	64.6 U	112.4 U	199.5 U	109.0 U	111.3 U	147.9 U	108.4 U	06 97.3 R	89.1 R	60.6 R	62.7 R	100.5	1958
SEG 1959	46.4 R	64.2 R	129.8 R	157.6 R	07 142.3RU	94.3 U	162.9 U	95.1 U	100.8 U	08 89.6 R	61.5 R	65.0 R	100.2	1959
SEG 1960	44.5 R	70.0 R	127.3 R	161.8 R	123.8 R	96.6 R	165.2 R	108.1 R	82.7 R	94.8 R	58.3 R	71.1 R	100.3	1960
SEG 1961	46.1 R	09 77.3RU	112.4 U	161.1 U	115.8 U	97.3 U	170.5 U	111.8 U	85.5 U	89.8 U	62.0 U	66.7 U	99.6	1961
SEG 1962	49.5 U	69.4 U	128.6 U	138.0 U	124.9 U	95.6 U	176.4 U	10 106.8 R	92.0 R	86.6 R	59.7 R	76.8 R	100.1	1962
SEG 1963	47.1 R	72.4 R	112.8 R	161.7 R	109.9 R	97.2 R	177.3 R	111.5 R	84.5 R	96.7 R	54.6 R	80.9 R	100.6	1963
SEG 1964	11 49.2RR	69.2 R	117.4 R	147.6 R	110.6 R	101.3 R	170.3 R	115.2 R	89.1 R	92.6 R	12 59.4RU	81.7 U	99.9	1964
SEG 1965	48.4 U	69.1 U	113.5 U	139.1 U	118.1 U	97.6 U	164.3 U	122.2 U	90.5 U	13 96.5 R	52.6 R	89.7 R	99.8	1965
SEG 1966	47.4 R	73.6 R	109.3 R	142.7 R	101.5 R	101.8 R	14 179.2RU	115.9 U	88.7 U	102.3 U	52.1 U	83.5 U	99.9	1966
SEG 1967	51.5 U	68.2 U	116.5 U	130.2 U	110.4 U	76.6 U	15 197.3US	122.6 S	87.1 S	16 98.0 R	52.4 R	95.2 R	100.6	1967
SEG 1968	48.9 R	69.9 R	104.0 R	144.7 R	91.1 R	101.9 R	187.4 R	123.9 R	87.8 R	16 94.4 R	54.1 R	17 88.9RU	100.0	1968
SEG 1969	52.3 U	63.1 U	118.3 U	126.9 U	90.0 U	105.9 U	190.5 U	128.9 U	83.1 U	92.7 U	56.4 U	90.1 U	99.9	1969
SEG 1970	53.2 U	69.3 U	117.0 U	113.7 U	96.4 U	105.0 U	185.9 U	134.2 U	85.5 U	81.3 U	55.9 U	18 89.9 R	99.2	1970
SEG 1971	53.8 R	68.9 R	111.6 R	133.8 R	90.0 R	109.6 R	179.5 R	127.0 R	82.6 R	98.0 R	53.9 R	19 85.9RU	99.8	1971
SEG 1972	54.7 U	71.0 U	117.4 U	122.3 U	103.9UU	20 103.6UU	174.5 U	115.5 U	87.5 U	101.8 U	56.6 U	80.1 U	99.1	1972
SEG 1973	57.4 U	71.3 U	116.8 U	126.6 U	103.6 U	102.9 U	171.0 U	113.9 U	112.1 U	51.9 U	21 62.0US	90.0 S	98.3	1973
SEG 1974	54.9 S	74.5US	22 116.3SU	132.9 U	98.3 U	103.4 U	23 164.1US	123.8 S	85.1 S	99.3US	24 60.8SU	84.9 U	99.9	1974
SEG 1975	55.1 U	72.7 U	115.2 U	25 131.9 R	99.4 R	104.9.R	162.9 R	119.5 R	88.8 R	99.4 R	56.6 R	87.9 R	92.1	1975
AV	47.7	69.9	115.9	149.7	107.7	102.3	170.5	116.0	88.1	94.1	56.4	79.9	OF REGULAR MONTHS	
NO.	9	8	8	9	8	8	7	7	8	11	10	9	102	

Note: See footnotes at end of table 1i.

Table 1. ANALYSIS OF TOURIST ARRIVALS BY AIR FOR ISRAEL, BY THE X–11 PROGRAM AND SUPPLEMENTARY STAGES: 1956 TO 1975 AND 1975—Continued

h. REVISED SEASONAL FACTORS S^*, S^{**}, AND FORECASTS S^f FROM LOG PARABOLIC REGRESSIONS OF SR^* RATIOS, BY PERCENT

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total[h]
1956	46.3	68.3	122.7	165.8	140.4	102.1	149.1	90.1	96.9	87.7	66.2	53.1	1189.6
1957	46.1	68.7	122.1	164.8	134.1	101.5	154.1	93.9	94.7	88.5	64.0	57.5	1190.2
1958	45.9	69.1	121.5	162.7	128.5	101.1	158.7	97.5	93.2	89.4	62.1	61.9	1191.6
1959	45.9	69.4	120.7	160.7	123.4	100.7	163.0	101.1	91.6	90.3	60.3	66.2	1193.4
1960	46.0	69.7	119.9	158.7	118.9	100.4	166.9	104.5	90.3	91.1	58.8	70.4	1195.5
1961	46.1	69.9	119.0	155.7	114.8	100.3	170.4	107.6	89.1	91.9	57.5	74.4	1197.8
1962	46.4	70.1	118.0	154.7	111.2	100.2	173.4	110.6	88.1	92.6	56.4	78.2	1199.9
1963	46.7	70.3	116.9	152.7	108.1	100.2	175.9	113.3	87.2	93.3	55.5	81.7	1201.9
1964	47.1	70.4	115.8	150.8	105.3	100.3	177.9	115.7	86.5	94.0	54.7	84.9	1203.5
1965	47.7	70.4	114.6	148.8	102.8	100.5	179.5	117.9	85.9	94.7	54.1	87.6	1204.6
1966	48.8	70.5	113.3	145.9	100.8	100.9	180.4	119.8	85.5	95.3	53.7	89.9	1205.2
1967	49.1	70.4	112.0	144.9	99.0	101.3	180.9	121.3	85.2	95.9	53.4	91.7	1205.1
1968	49.9	70.4	110.5	143.0	97.5	101.8	180.8	122.5	85.1	96.5	53.2	93.0	1204.3
1969	50.9	70.2	109.1	141.1	96.4	102.4	180.1	123.4	85.1	97.0	53.2	93.8	1202.7
1970	52.0	70.1	107.5	139.2	95.5	103.1	178.9	123.9	85.2	97.5	53.4	94.0	1200.4
1971	53.2	69.9	106.0	137.3	94.8	103.9	177.2	124.1	85.5	98.0	53.7	93.7	1197.3
1972	54.6 i	69.6	104.5 j	135.5	94.5	104.9	175.0	123.9	86.0	98.4	54.2	92.8	1193.5
1973	56.1	69.3	102.6	133.6	94.4	105.9	172.2	123.3	86.5	98.8	54.8	91.4	1189.0
1974	57.8	69.0	100.9	131.8	94.6	107.0	169.0	122.4	87.3	99.1	55.6	89.4	1184.0
1975	59.7	68.6	99.1	129.9	95.1	108.3	165.4	121.2	88.1	99.4	56.5	87.0	1178.4
1976 S′	61.7	68.1	97.3	128.1	95.8	109.7 k	161.4	119.7	89.2	99.7	57.7	84.2	1172.5
1977	63.9	67.7	95.5	126.3	96.8	111.2	156.9	117.8	90.4	99.9	59.0	80.9	1166.3

Note: See footnotes at end of table 1i.

Table 1. ANALYSIS OF TOURIST ARRIVALS BY AIR FOR ISRAEL, BY THE X–11 PROGRAM AND SUPPLEMENTARY STAGES: 1956 TO 1975 AND 1976—Continued

i. REVISED SEASONALLY ADJUSTED DATA A^* AND A^{**}

(In thousands)

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
1956	2.0	2.0	1.8	1.8	1.7	1.8	1.7	2.2	1.9	2.2	1.3	2.0	22.5
1957	2.5	1.9	1.7	1.6	2.0	2.1	2.1	2.6	3.5	2.7	2.1	2.6	27.2
1958	3.2	2.9	3.4	5.1	3.2	3.7	3.0	3.4	2.8	2.9	3.1	3.4	40.2
1959	3.3	3.2	4.1	4.2	5.4	4.1	4.0	3.9	4.9	4.4	4.7	4.5	50.7
1960	4.7	5.4	6.1	5.9	5.9	5.3	5.7	6.1	5.6	7.0	7.3	7.8	72.7
1961	7.9	9.0	6.9	7.0	7.0	7.3	8.4	9.0	7.9	7.8	8.3	6.7	93.4
1962	8.6	8.5	9.2	7.3	9.6	8.4	8.6	7.7	8.6	7.8	9.2	8.7	102.3
1963	9.1	9.7	9.4	10.8	10.2	9.7	10.5	10.3	10.4	11.7	11.1	11.2	124.2
1964	12.0	11.3	11.8	11.4	12.3	11.7	10.9	11.4	11.9	11.4	14.1	12.1	142.5
1965	12.0	11.6	12.2	12.9	17.3	13.8	12.4	14.7	14.8	13.0	13.0	14.5	162.1
1966	14.8	16.6	15.4	15.5	16.3	16.6	15.6	14.6	16.0	16.7	13.6	12.3	183.9
1967	14.9	14.6	16.0	13.6	15.4	9.7	16.8	18.8	21.3	23.9	24.1	26.4	215.4
1968	25.8	26.2	26.8	31.4	28.6	29.7	31.6	31.9	33.5	32.7	35.9	29.2	363.0
1969	28.1	23.5	29.5	24.7	26.2	31.2	33.5	32.5	28.4	27.6	32.7	31.1	349.1
1970	34.0	32.8	33.3	23.2	30.5	33.4	36.1	37.8	30.4	22.3	31.9	35.5	381.2
1971	38.1 k	37.8	42.5 k	41.6	42.2	49.4	49.6	51.6	50.4	54.8	58.2	49.3	565.5
1972	54.0 55.4	56.3	61.7 60.8	49.9	62.0	52.1	47.6	44.9	54.8	59.4	55.7	41.5	640.0
1973	49.2 52.0	50.5	55.8 54.0	48.2	57.8	50.1	49.2	47.4	59.7	17.2	37.6	40.6	563.4
1974	42.0 45.7	49.7	52.9 50.3	44.6	43.5	39.1	40.0	43.1	43.0	47.5	53.4	42.1	541.0
1975	36.6 41.1	39.7	42.2 39.5	36.5	38.9	37.3	39.5	42.4	47.5	51.2	55.2	59.7	526.7

[a]The new original data for January to December 1976 are shown with the festival-date adjustments and festival-adjusted data, in addition to the 1956-75 data used in the basic analysis presented here, in order to stress the problems of current adjustment and updating and to facilitate experimentation with other approaches and programs.

[b]The ARIMA forecasts based on the $(0,1,1) \times (0,1,1)_{12}$ model for 1956 to 1975 (after festival-date adjustment) are shown. The parameters were 0.54604 and 0.69831. Note that the annual total forecasted was very close to the actual 1976 total, despite the fact that the forecasts for March, October, and November were considerably lower than the actual data, while the forecasts for July and August were higher than the actual data (with relatively low summer levels).

[c]ARIMA forecasts based on 1956 to 1975, after modifying the data for October 1973 to January 1974 (which were badly affected by the Yom Kippur War [31]) by means of ARIMA forecasts for these months based on the data for January 1956 to September 1973. The parameters were 0.46274 and 0.61758. The forecasts were closer to actuals for July, August, and October but too low for March and April and for the 1976 total.

Both sets of ARIMA forecasts anticipated the November-December 1976 boom but not its full extent, which was due to the expansion of charter flights from end-October: These are expected to continue throughout 1977, raising the off-season *S* relatively to the peak season. (Research is continuing on this matter.)

[d]Seasonal factors calculated for 1976 from the standard X-11 analysis of 1957-76 data. Note the considerable increase for November and December as compared with the X-11 forecasts based on 1956 to 1975. Detailed analysis of *S* for December 1957 to 1976 is shown in table 3. Detailed analysis of the revisions in the s.a.d. *A* for January 1975 through December 1976 is shown in table 5.

[e]See the subsection on additional stages proposed for the improved analysis of univariate series, stages 17, 18, 23, and 31a.

[f]The regular segments are underlined for convenient study. [5,1,-1] estimates of L' are used at the tails of regular segments to give SR^* ratios (indicated by *—*). (See also table 3.)

[g]The original data *Y*, s.a.d. *A*, stream *L*, and SR^* ratios for February could be standardised to a 28¼-day month because of leap years: trading-day adjustments are also being considered.

[h]The annual totals are not constrained to 1,200 percent.

[i]For January, the last regular ratio was for 1971 and the s and u ratios for 1974 and 1975 (after some expansion of winter tourism) do not seem to justify continuing the rapidly increasing extrapolation after S^* (1971) = 53.2 percent. A similar cautious extrapolation is often made by the X-11 moving average. The 1976 datum justifies the use of S^* (1971) until 1976.

[j]For March and April, there are inaccuracies in the festival-date adjustments, made according to the median date of Passover and Easter each year. The activity is also affected by Independence Day (especially in the years with parades). The March 1968 SR^* and *SI* ratios were apparently low, because the 20th Anniversary Parade on May 2 attracted more tourists during April. The last regular SR^* ratio for March 1971 (111.6 percent) was considerably above the rapidly descending regression estimate S^* (1971) = 106.0 percent, which is used for the following years (especially since the ratios for 1972 to 1975 are also above S^*). An even higher S^* would appear suitable for 1976.

[k]The regression for June is considerably above the last regular factor (for 1975) and the previous three u factors, and S^* (1975) is, therefore, proposed for 1976, especially since tourism from the United States (relatively high in June, see table 4) is increasing since the drop from 1972. Note that these truncation considerations might be automated, with scope for interaction by the analyst. A lower factor (as given by X-11) would also be appropriate.

[l]The s.a.d.'s in the printout are calculated according to the extrapolated S^*, while revised s.a.d. A^{**}, according to the truncated regressions S^{**}, are also indicated.

Stage	Symbols	Step of X–11
11. Estimate irregular factors	$I^2 = A^1/C^2$	B13
12. Iterate stages 5 to 11 three times (with minor changes)		B7–13, C1–19, D1–7
13. Calculate final components and other tables—	SI	D8
	SI^w	D9
	$S = [i]\ SI, SI^w$	D10
Forecasted seasonality	S^f	D10A
	A (A^m: not printed)	D11
	$C = [\mathbf{H}]A.A^m$	D12
	$I = A/C$	D13
M.C.D. short-term trend	$C^{(M)} = [\mathbf{m}]A$	F1
14. Compute and print summary measures		F2
15. Print chart of A and C, by months, over complete era		G1
16. Print charts of SI, SI^w, and S, by years, for each month		G2

Additional Stages Proposed for the Improved Analysis of Univariate Series

17. Print the s.a.d. A (step D11) together with letters **E, V, W, X, Y,** and **Z** as flags to indicate the months finally determined as extreme (in step C17 of X–11)—

Very high extreme	E	Very low extreme	Z ($w=0$)
Rather high	V	Rather low	Y ($0<w\leq 50\%$)
Somewhat high	W	Somewhat low	X ($50\%<w<100\%$)[9]

18. Print signs to indicate monotonicity of the s.a.d. A and the magnitude of the month-to-month changes a—

A considerable increase	$a>k_1$	**+**
A usual increase	$k_2<a\leq k_1$	blank
Equal or a slight increase	$0\leq a<k_2$	**=**
A slight decrease	$0>a\geq k_3$	**M**
A usual decrease	$k_4\leq a<k_3$	**-**
A considerable decrease	$a<k_4$	*

It would also be possible to print positive and negative changes on separate lines.[10] Any other flags inputted (stage 31a) should also be printed out.

19. Compute and print the stream L as the $[\mathbf{1,2,1}]$ moving average of the s.a.d. A to serve as a better approximation to the short-term trend cycle, with flags as above and similar signs to indicate month-to-month changes in the stream. This table may best be interlined with the s.a.d. A. (See table 1f.)

20. Print $(SI)^L$ ratios computed as Y^P/L, achieving improvements in the continuity of these ratios in the regular segments (over the years), as compared to the SI, SI^w ratios, due to the use of the stream L rather than the Henderson moving-average $C^{(H)}$. (See table 3.)

[8] The standard X–11 program has options for regression analysis of D and the Israel CBS variant has a festival-date option. (See [1] and the proposed stages 31 (e) and (f).) The proposals are indicated as additional stages to the standard X–11 program in order to enable improved analysis even before other major changes are developed into computer packages. Stages 19 to 28 are available as the X–76 SIL program, and further stages are being considered.

[9] The classification of the near-extreme months, with partial weights into the latter two groups according to whether $w\gtreqless 50$ percent, is arbitrary but convenient, as shown in the following examples.

[10] These signs are convenient for studying the fluctuations in the seasonally adjusted level (see table 1f), rather than using the separate percent-change tables (e.g., step E6 of X–11). In the proposed standard program, $k_1=-k_4=10$ percent, $k_2=-k_3=1$ percent. These parameters might be related to the variability of the s.a.d. over the regular segments, automatically or by choice of the analyst.

Table 2. ANALYSIS OF A SERIES FOR TOURIST ARRIVALS BY AIR FOR ISRAEL INTO REGULAR AND UNUSUAL SEGMENTS: 1956 TO 1976[a]

Segment[b]	Principal Events[c]	Dates[d]		No. of Months by Type[e]								Levels		
		From	To	Total	Full Weight	Unusually High			Unusually Low			Regular Montonic[f]		
						E	V	W	X	Y	Z	From	To	Growth rate, % per month
												Unusual		Segment-Range
												Low	High	U Ratio
1 r	Continuation of decline (from summer 1955[g])	Jan'56	Jul'56	7	7							1 942	1 691	-2.2
2 s	Improvement in trend	Jul'56	Sep'56	(2)	2							1 691	1 956	(+5.9)
3 u	Sinai campaign (Oct.24-Nov.3), followed by uneven recovery	Oct'56	Apr'57	7	5	1					1	1 482	2 444	U 1.65
4 s	Independence Day Parade (P9, May 5), recovery, but continued U.S. embargo on Americans' visits	May'57	Aug'57	(4)	4							2 007	2 281	(+4.3)
5 u	Renewal of American tourism (Sept. 1), uneven growth culminating in Decennial Parade (Apr.25), followed by drop (partial compensation)	Sep'57	Aug'58	12	9	2			1			2 304	5 112	U 2.22
6 r	Rapid growth	Sep'58	Apr'59	8	8							2 756	4 139	+5.6
7 u	11th Anniversary Parade (May 13), uneven growth (premium to tourists' currency, June)	May'59	Sep'59	5	3	1				1		3 724	5 244	U 1.41
8 r	Renewal of growth	Oct'59	Jan'61	16	15					1[h]		4 358	7 788	+4.0
9 u	Eichman - announcement of capture (Jan.) and trial (Apr-Aug'61); uneven growth, economic problems (devaluation Feb'62)	Feb'61	Jul'62	18	13	1	1		1	1	1	6 626	9 237	U 1.39
10 r	Renewed growth: charter flights from Scandinavia (from Apr'63); visit of Pope Pius VI (Jan 4, '64)	Aug'62	Jan'64	18	16		1[h]	1[h]				7 894	11 721	+2.1
11 r	Stability, slight decline in trend	Jan'64	Oct'64	9	9							11 721	11 574	-0.1
12 u	Increased Scandinavian tourism (group fares); uneven growth (opening Israel Museum, conferences), some terrorist attacks	Nov'64	Sep'65	11	8	2			1			11 558	16 647	U 1.44
13 r	Renewed growth (from lower level)	Oct'65	Jun'66	9	9							13 049	16 459	+3.1
14 u	Renewed tension and terrorist attacks: Six Day War (June 5-10, '67)	Jul'66	Jun'67	12	8	1		1			2	9 633	16 627	U 1.73
15 s	Rapid post-war recovery: (arrivals include some tourism previously entering by land)	Jul'67	Oct'67	(4)	3			1[h]				16 737	23 836	(+11.3)

Note: See footnotes at end of table 3.

Table 2. ANALYSIS OF A SERIES FOR TOURIST ARRIVALS BY AIR FOR ISRAEL INTO REGULAR AND UNUSUAL SEGMENTS: 1956 TO 1976—Continued

Segment[b]	Principal Events[c]	Dates[d] From	Dates[d] To	No. of Months by Type[e] Total	Full Weight	Unusually High E	Unusually High V	Unusually High W	Unusually Low X	Unusually Low Y	Unusually Low Z	Levels: Regular Montonic[f] From / Unusual Low	To / Unusual High	Growth rate, % per month / Segment-Range U Ratio
16 r	Regular growth, post-war conditions, 20th Anniversary Parade (Apr'68)	Oct'67	Nov'68	13	11		2[hi]	1[i]				23 836	35 390	+3.1
17 u	Terrorist attacks in Israel and on air routes[j], some cases of cholera (Aug-Oct'70)	Dec'68	Nov'70	24	20					2	2	22 663	37 185	U 1.64
18 r	Renewed growth following cease-fire with Egypt (Aug & Nov'70) and Jordanian crushing of terrorists (Sep'70)	Dec'70	Nov'71	12	12							37 071	57 125	+4.0
19 u	Disturbed growth (from lower level) following new U.S. economic policy and international currency changes (incl. Israel devaluation Aug'71)	Dec'71	May'72	6	5	1						51 907	61 021	U 1.18
20 u	Lower level and perturbations, following massacres at Lod airport (May'72) and Munich Olympics (Sep'72); increases due to 25th Anniversary Parade (May '73) and increased conference tourism (Sep'73) Yom Kippur War (Oct 6-24, '73)	Jun'72	Oct'73	17	12	1	1	1	1		1	17 002	59 750	U 3.51
21 s	Immediate post-war recovery: Geneva Peace conference	Nov'73	Feb'74	(4)	3				1			37 098	47 114	(+5.9)
22 u	Decrease due to incidents in North and terrorist attacks	Mar'74	Jun'74	4	4							40 022	45 683	U 1.14
23 s	Recovery following Disengagement Agreement with Syria (May'74), disturbed by terrorism and Cyprus war (from July'74)	Jul'74	Oct'74	(4)	4							40 769	46 741	(+4.7)
24 u	Conferences with high participation, devaluation of Israel lira (Nov), terrorist attacks, El Al strikes	Nov'74	Mar'75	5	4	1						35 785	53 156	U 1.48

Note: See footnotes at end of table 3.

Table 2. **ANALYSIS OF A SERIES FOR TOURIST ARRIVALS BY AIR FOR ISRAEL INTO REGULAR AND UNUSUAL SEGMENTS: 1956 TO 1976**—Continued

Segment[b]	Principal Events[c]	Dates[d] From	Dates[d] To	No. of Months by Type[e] Total	Full Weight	Unusually High E	Unusually High V	Unusually High W	Unusually Low X	Unusually Low Y	Unusually Low Z	Levels: Regular Montonic[f] From / Unusual Low	To / Unusual High	Growth rate, % per month / Segment-Range U Ratio
25 r	Recovery in trend following improvements in security situation and in international economy	Apr'75	Dec'75	9	9							36 371	57 970	+6.8
	TOTAL 1956–1975			240	203	11	5	4	5	5	7	1 942	57 970	+1.4
Summary of Segments														
	9 Regular			101	96	-	3 h	1h	-	1 h	-			
	5 Short Monotonic			18	16	-	-	1 h	1	-	-			
	11 Unusual			121	91	11	2	2	4	4	7			
Current Analysis - 1976[k]														
26	Reduced level	Jan'76	Feb'76	2								50 900	51 200	
	Improved level	Mar'76	Apr'76	2								58 400	56 000	
	Previous level	May'76	Aug'76	4								54 500	53 300	
	Increase	Sep'76	Dec'76	4								57 900	64 000	

Note: See footnotes at end of table 3.

Table 3. TOURIST ARRIVALS BY AIR FOR ISRAEL: 1956 TO 1976

(SI ratios and estimates of S for December,[a] by percent)

Year[c]	Based on 1956 to 1974[b]		Based on 1956 to 1975				Based on 1957-1976[g]	Based on stream 1956 to 1975	Log-parabolic regression
	SI ratio	Smoothed S^d	SI ratio	Weight w^e	Replacement SI^w [f]	Smoothed S	S	$(SI)^L = Y^P/L$	S^*
1956 u	55.1	58.3	55.1			58.3	x	56.2	53.1
1957 u	53.6	59.7	53.7			59.6	66.2	57.2	57.5
1958	63.9	61.9	63.9			62.0	67.0	62.7	61.9
1959	62.4	65.5	62.3			65.6	68.7	65.0	66.2
1960	74.4	69.1	74.4			69.2	70.6	71.1	70.4
1961 u (	62.7	73.5		35.2	68.9				
(			62.3	23.8	69.7	73.5	74.0	66.7	74.4
1962	77.5	77.0	77.3			77.1	77.4	76.8	78.2
1963	79.8	80.9	79.8			81.0	81.1	80.9	81.7
1964 u	86.5	84.4	86.3			84.5	84.6	81.7	84.9
1965	87.7	88.0	88.0			88.1	88.1	89.7	87.6
1966 u (	77.2	90.4		3.0	89.1				
(			76.9	0.0	89.5	90.5	90.5	83.5	89.9
1967	97.4	91.9	97.1			92.0	92.0	95.2	91.7
1968 u	90.7	92.1	91.1			92.2	92.3	88.9	93.0
1969 u	92.1	90.7	92.0			91.4	91.7	90.1	93.8
1970	95.0	88.5	95.2			90.0	90.5	89.9[j]	94.0
1971 u	83.4	85.7	83.9			89.0	89.7	85.9	93.7
1972 u (	75.8	83.4		72.3	78.4				
(			74.8	13.6	87.0	88.5	89.6	80.1	92.8
1973 s	85.0	81.4	88.2			88.7	90.4	90.0	91.4
1974 u	74.6[h]	80.3	90.0[h]			89.1	91.6	84.9	89.4
1975		79.8[i]	91.0			89.6	92.7	87.9[k]	87.0
1976						89.9[i]	93.1		84.2

[a]December forms a local peak for this series, relative to November and January, because of Christmas tourism from many countries (see table 4): its seasonal factor (the total of all countries) was always less than 100 percent. The estimates of its S are especially important to estimate the latest level and forecast the next 12 months.

[b]The October 1973 War disturbed the last 3 months of 1973, and seasonal factors estimated from the 1956 to 1972 X-11 processing (89.1 percent for 1972 and 1973) were also used for 1974. The analysis for 1956-74 shows a considerable drop in the seasonal factor estimated for December from 1969 to 1974; this was used for analysis of the postwar changes and current analysis of the 1975 data until the 1956-75 analysis was made (in January 1976).

[c]u indicates that December is part of an unusual segment in this year, s part of a short monotonic segment. (See tables 1 and 2.)

[d]The standard X-11 [3] x [5]-moving average of the SI ratios, with appropriate weights for the 7-tail years. (See [25; 35].

[e]The X-11 weight for the irregular component (step C17), based on 1.5 and 2.5 $\sigma(I)$ limits, is shown here for the two analyses. Note that $\sigma(I)$ for 1972 was estimated as 6.2 percent in the 1956-74 analysis and 6.7 percent in the 1956-75 analysis (but dropping to 5.6 percent for 1973 to 1975).

[f]Based on imputation of modified original data, taking w into account (step D9 of the two analyses). (See [35].)

[g]In the 1957-76 X-11 analysis, December 1957 was assigned $w = 0$, raising S from 1957 to 1962. December 1961 received $w = 17.5$ percent, December 1966 and 1972 $w = 0$, and December 1975 $w = 65.7$ percent, raising the estimates of S for 1968-76.

[h]November 1974 was unusually high (6,800 arrivals for conferences), following a short monotonic segment No. 23. This raised the estimate of $C^{(6)}$ (step D7) for December 1974 and thereby lowered $SI = Y^P/C^{(6)}$ unduly in the 1956-74 analysis. The drop in the level from December 1974 to March 1975 produced a lower $C^{(6)}$ and a more appropriate SI in the 1956-75 analysis.

[i]One-year ahead forecast (step D10A). The use of factors based on 1956-74 gave a 19-percent jump in A from 54,700 in November 1975 to 65,100 in December 1975, corrected to a 3-percent rise from 56,058 to 57,970 in the 1956-75 analysis or to a 12-percent rise from 55,200 to 61,600 in the revised analysis. (See tables 1h and 5.)

[j]November 1970 was the end of unusual segment No. 17, with seasonally adjusted level 31,061, followed by a jump to December (37,071) and then almost monotonic steady growth, with January 1971, 38,085. The use of the stream L, based on these 3 months (35,822), therefore, gave a slightly high estimate for $(SI)^L$ of 93.2 percent, compared with the present use of the [5,1,-1] average L' of December 1970 to February 1971 equals 37,138.

[k]Based on the [-1,1,5] average L' of October, December 1975 = 59,093. The later use of A for January 1976 (50,868, based on S^f forecast from the 1956-75 analysis) would give L = 55,725, $(SI)^L$ = 93.2 percent. The use of the log-parabola time-conditioned moving seasonality appears to be justified and gave an appropriate factor for December 1976. (See table 5.) The causes of moving seasonality are complex, including changes in the relative growth of different countries' winter tourism (see table 4) and the expansion of charter flights in 1975-76 (including flights to Elat).

21. Print a chart of the s.a.d. A and the stream L for each month over the years (similar to the X–11 chart G1), flagging the points and months with appropriate letters and signs. (See fig. 3.[11]). A plot of the original data Y, together with the s.a.d. A, may also be desirable.
22. Print, for each month, a chart of the $(SI)^L$ ratios over the years (similar to X–11 chart G2, but preferably using a logarithmic scale for the multiplicative model). (See fig. 4.) This chart might also show the previous SI ratios and the appropriate flags.
23. The analyst should then study the behavior of the series (and of sub- or related series, if they are analysed in parallel). He may indicate the segmentation of the complete series analysed into—

    r regular segments (distinguishing major changes in the rate of growth[12])
    s short monotonic segments
    u unusual segments

    numbering them from the start of the series.
    This will enable the exclusion of inappropriate $(SI)^L$ ratios from the estimation of the regular seasonality and irregularity, which should be based only on regular SR ratios.
24. The input of this segmentation will permit a further automatic iteration (possibly online to the analyst) in which the tables will show the start of each segment and its type—**r**, **s**, and **u** (table 1g): The flags may also be shown.
25. Prepare revised estimates of the trend-cycle C^* to calculate the SR^* ratios separately for each regular segment, excluding the influence of data in other segments:[13]

    (r) In regular segments the trend T^* may be calculated by the following methods, enabling the calculations of SR^* ratios as Y^P/T^*—

    a. As the stream L, even if not strictly monotonic. For the first and last month of the segment, the L are affected by the adjacent segments, so that an estimate L' may be used, e.g., an asymmetric weighted average [**2, 1**] over the first 2 months or [**5, 1, -1**] to interpolate a line over the first 3 months, similarly for the last months.[14]
    b. As a simple function fitted to the s.a.d. A (or to stream L), e.g., for series showing basically exponential growth, a second order function $T^* = \exp(c_0 + c_1t + c_2t^2)$. It will be necessary to guarantee continúity between the **r** and **s** segments unless there is an obvious discontinuity **B** in the trend cycle. The spline approach may be suitable [9].
    c. As a moving average of more than three terms or other empiric smoothing.
    d. Fitting a monotonic trend T [30].

    (s) In short monotonic segments (2 to 5 months) with exceptionally high rates of growth (or decrease) the regular seasonal pattern may be disturbed. The $(SI)^L$ ratios $= Y^P/L$ may nevertheless be useful for study.

    (u) In unusual segments, the unusual effects component U prevents the calculation of SR^* ratios. The use of the stream L gives SU ratios $= Y^P/L$ which are useful for studying the effects of the unusual event over the months concerned.
26. Improved estimates and forecasts of the "regular seasonality" S^* may be made from these SR^* ratios, for each regular month over the years. Methods of estimating S^* include—

    a. Constant multiplicative factors calculated as the mean of SR^*, if these ratios do not in-

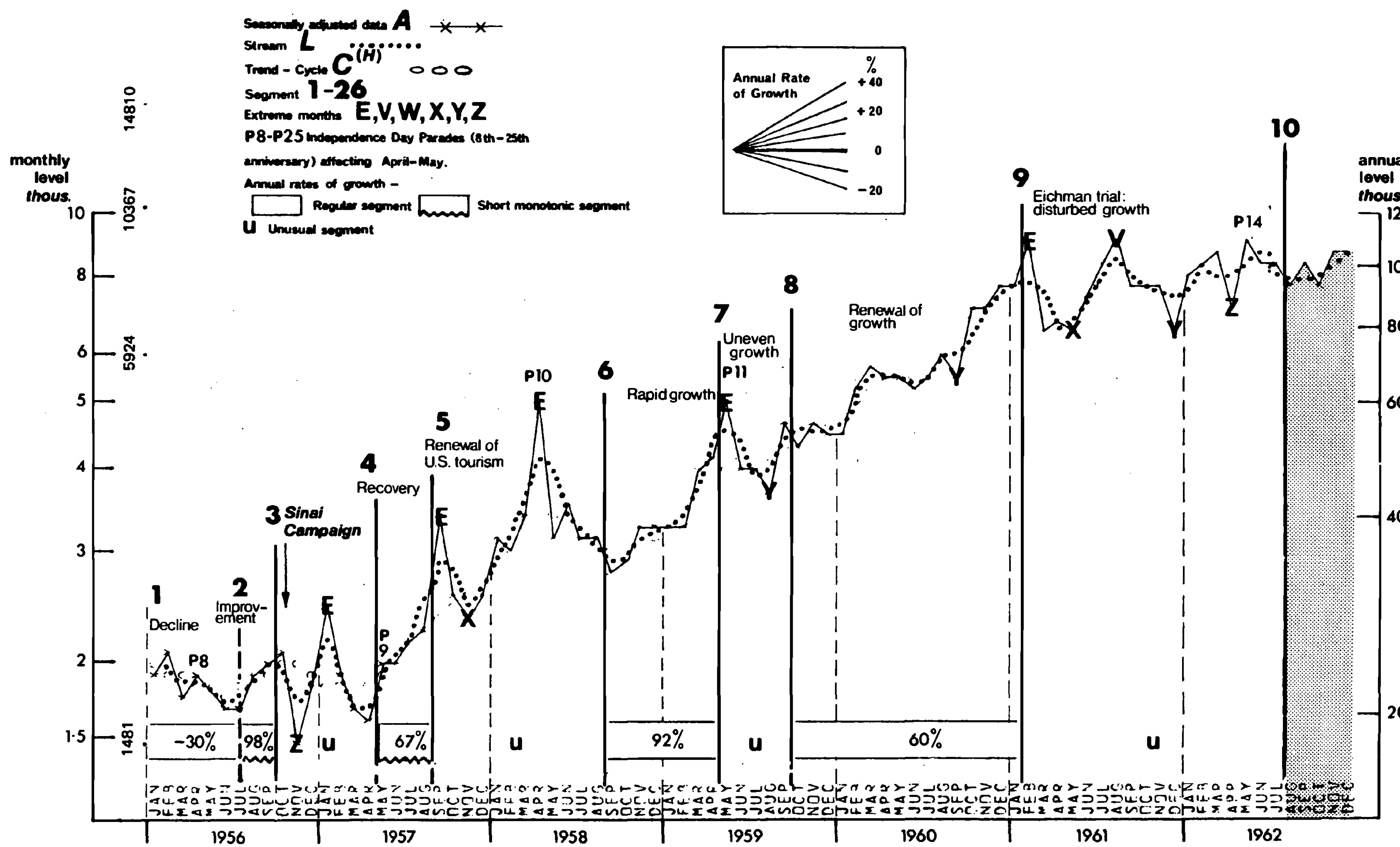
[11] Some technical improvements are proposed in the G1 chart:
a. Clearer identification of the years, e.g., printing of the year at each January only, with a partial vertical rule
b. The possibility of choosing a specific scale for the complete era or for segments (to magnify a specific segment or to enable easy comparison with other charts), rather than the restriction to the X–11 choice of a specific logarithmic scale (in the multiplicative program), depending on the range of the data.
c. Indication of the magnitude of the month-to-month changes a in A (and l in L) as in stages 19 and 20, on two lines above the scale of months.

[12] An example of a major change in the regular rate of growth (other than a cyclical turning point) is shown for tourist arrivals, by air, to Israel in January 1964: segment 10 of growth at 28 percent per annum was followed by a stable segment (1 percent per annum decrease; see figure 3 and table 1f).

[13] Even if the complete era analysed is regular, the Henderson 13-term moving-average $C^{(H)}$ has serious deficiences for estimating C at the ends of the series (because of its weighting structure), providing poor estimates of SI for the crucial last year analysed [16]. The relatively short length of the regular segments in many series stresses the inappropriateness of the Henderson 13-term (or even 9-term) moving average for estimating the historic trend-cycle, since it would span data from nonregular segments. $C^{(H)}$ may also be too low at a turning-point peak (e.g., U.S. unemployment for men 16–19 years old, June 1975, see fig. 6) or too high at a trough.

[14] Manual [5, 1, -1] estimates of L' increased the number of SR^* ratios in table 1g by 14; future programs will provide for their automatic calculation at r tails.

Figure 3. ANALYSIS OF A SERIES FOR TOURIST ARRIVALS BY AIR FOR ISRAEL INTO REGULAR AND UNUSUAL SEGMENTS, BY X–11 AND SUPPLEMENTARY STAGES: 1956 TO 1976



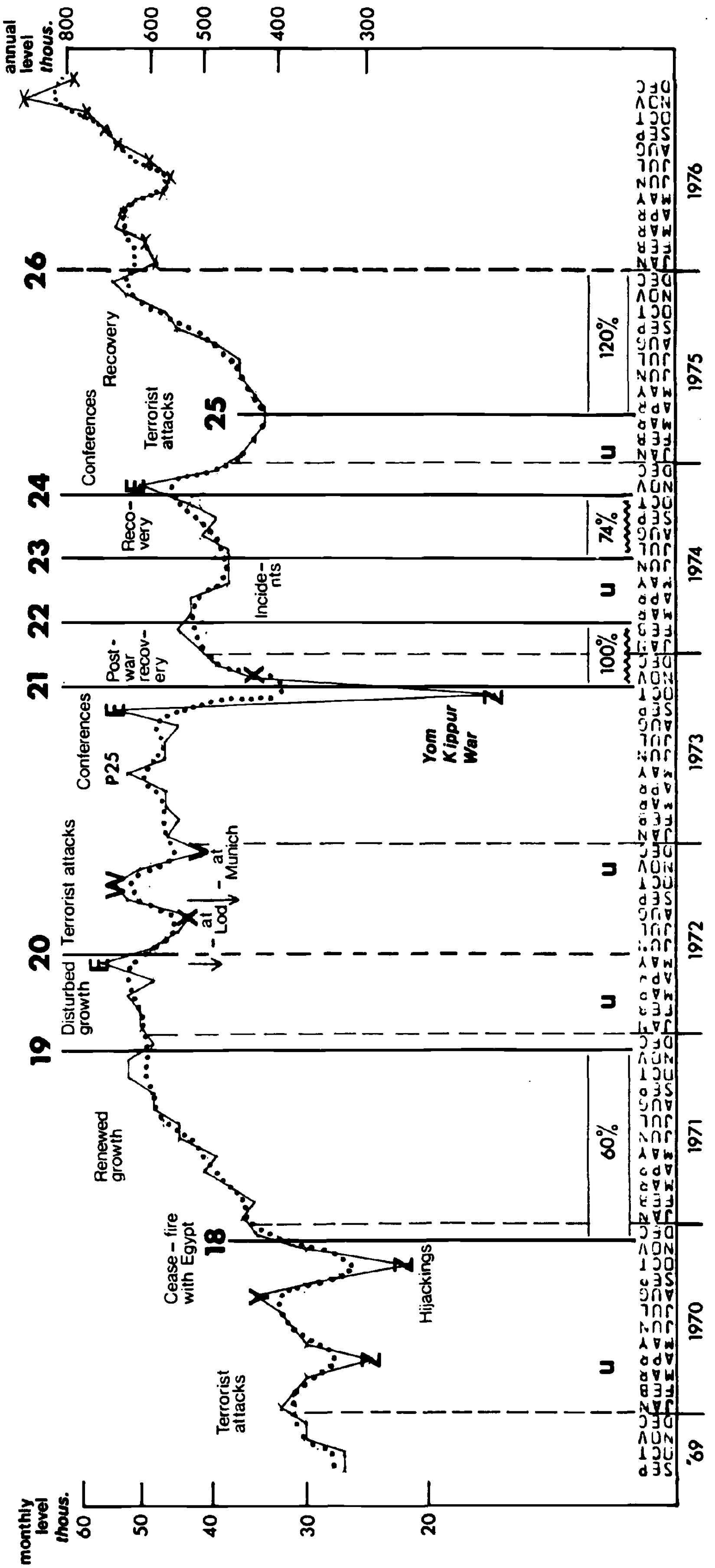


Figure 3. ANALYSIS OF A SERIES FOR TOURIST ARRIVALS BY AIR FOR ISRAEL INTO REGULAR AND UNUSUAL SEGMENTS, BY X–11 AND SUPPLEMENTARY STAGES: 1956 TO 1976—Continued

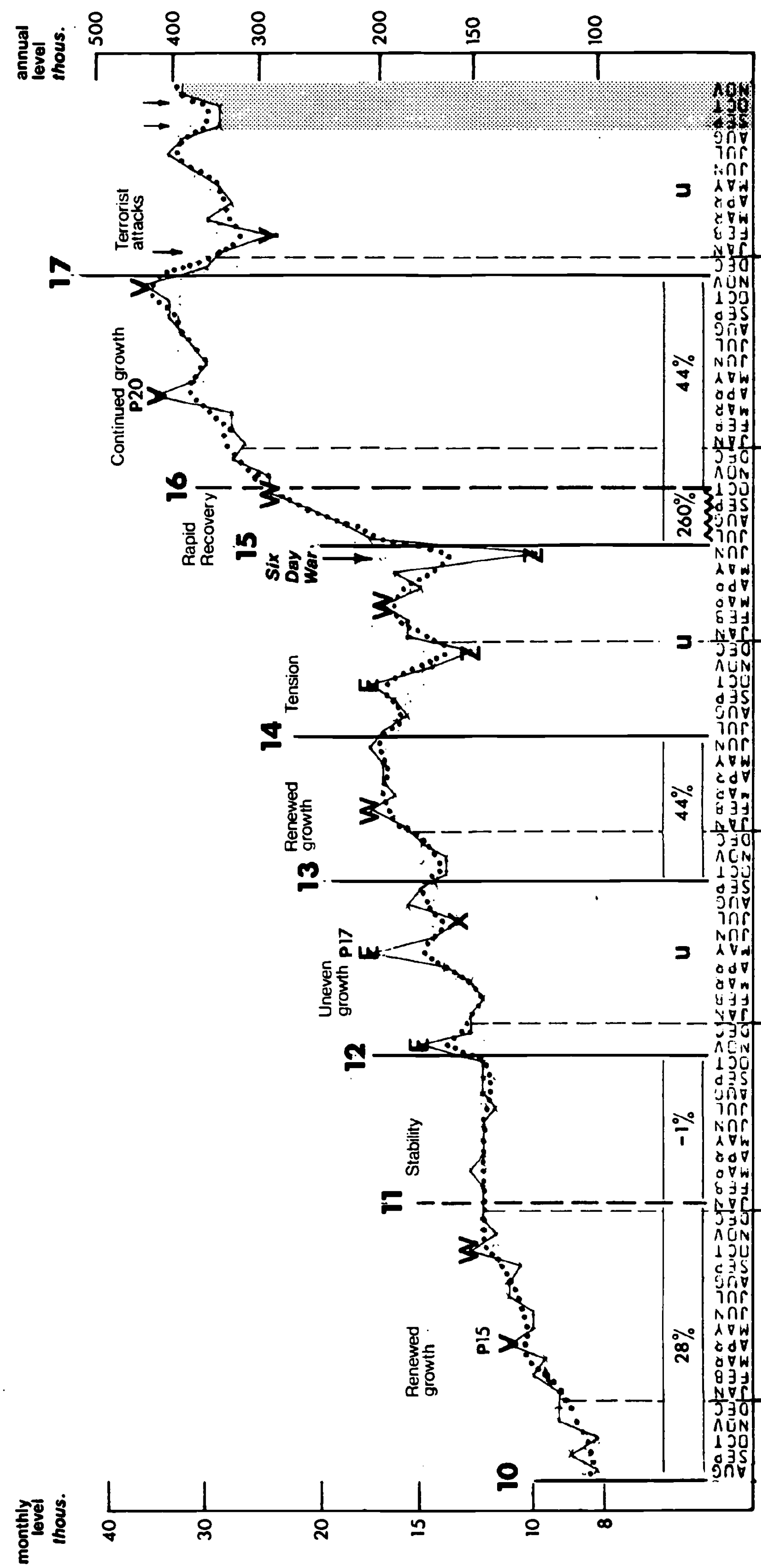
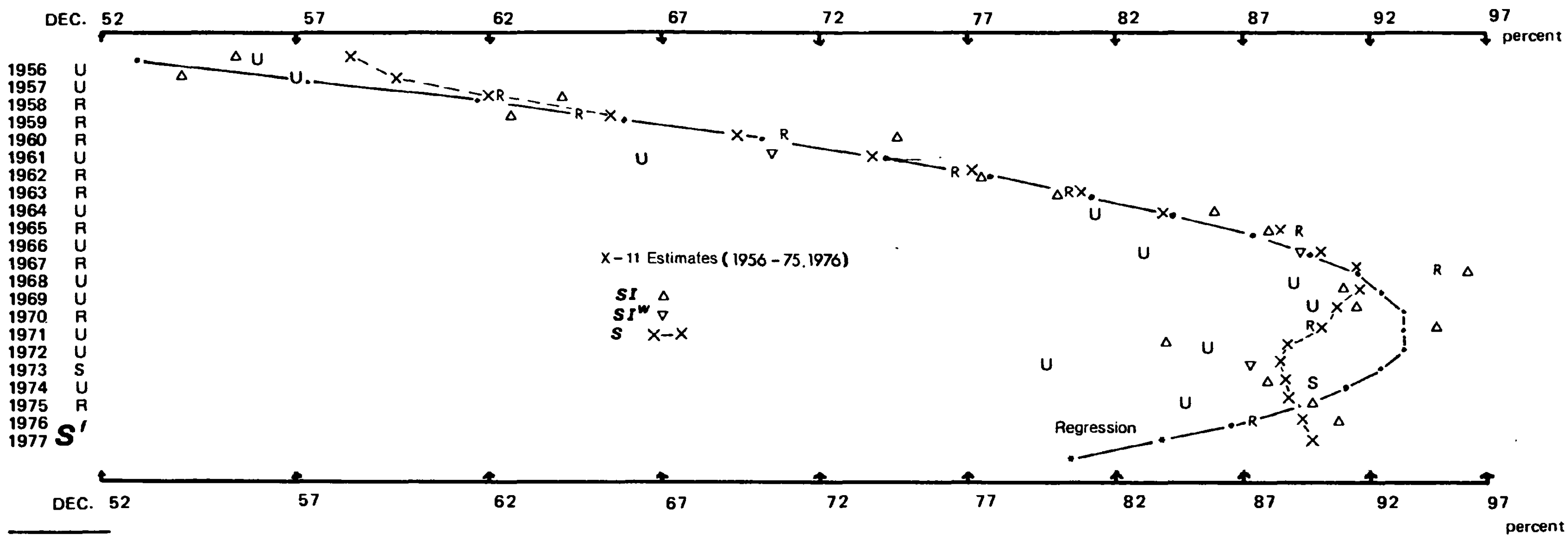


Figure 3. ANALYSIS OF A SERIES FOR TOURIST ARRIVALS BY AIR FOR ISRAEL INTO REGULAR AND UNUSUAL SEGMENTS, BY X–11 AND SUPPLEMENTARY STAGES: 1956 TO 1976—Continued

Figure 4. SR^* AND OTHER $(SI)^L$ RATIOS FOR TOURIST ARRIVALS BY AIR FOR ISRAEL, BY TYPE OF SEGMENT, AND THE LOG PARABOLIC REGRESSION[a] FOR S^* AND S^f: DECEMBER 1956 TO 1975 AND 1976 TO 1977



·Note: The regression equation is $S^* = \exp\,(3.8876 + 0.881\,i - 0.0030\,i^2)$, where i is the year, i (1956) = 1.

dicate significant changes over the years, for each month's seasonality.

b. Varying over the years if necessary, e.g.—

(1) As a quadratic exponential function of the years, enabling extrapolation of S^f for 2 further years if a consistent pattern is observed up to the last year analysed (table 1h and fig. 4).

(2) As a step function (or three lines) if there was a clear discontinuity **J** in the seasonality.

c. Continuing the seasonal factors from the previous analysis after updating with an additional year's data, if no significant changes have been found; X–11 often changes the seasonal factors unnecessarily on updating the series, introducing undesirable changes in the s.a.d. A (especially for the last year).

d. Moving averages are possible but are not proposed for estimating and forecasting the regular seasonality S^* if there are discontinuities or other irregularities; the X–11 imputations of SI^w for extreme months and moving averages S are often unsatisfactory, especially for the last year and forecasting.

We stress understanding the causation of the past and present changes in seasonality in order to better estimate S^* and forecast S^f.

27. The new estimates S^* should be charted together with the SR^* ratios and other $(SI)^L$ ratios (as in X–11 chart G2; see fig. 4).

28. The revised seasonal factors S^* enable the revision of the s.a.d. A^*, stream L^*, and trend-cycle C^*, by segments, providing residual irregular factors R^* (in **r** and **s** segments) and unusual factors U^*, calculated as Y^p/C^*S^*. These should be presented in separate tables for each type of segment.

29. The revised s.a.d. A^* and stream L^* may be charted and permit further iterations, possibly after improving the segmentation. X–11 uses automatic criteria which are applied over the four iterations to identify extreme months' data and to impute modified SI^w, A^m, Y^m in their place (there are options regarding the control limits and treating events such as strikes). The modified A^m are used in the calculation of the Henderson $C^{(H)}$, frequently giving rise to inappropriate estimates of $C^{(H)}$ and SI for months in adjoining regular segments (up to 14 months away). Judgmental estimates of prior correction factors for extreme months or segments are often made when using X–11. The proposed approach estimates regular seasonality from the regular segments' SR^* ratios alone, without needing to make any estimates for the other segments other than their duration. Though this approach reduces the number of years' data available to estimate S for each month (see table 1g), it may provide better estimates and forecasts of regular seasonality S^* that are also more stable when the series is updated.

The independent estimation of S^*_{ij} for each of the months j does not constrain $\Sigma_{i=1}^{12} S^*_{ij} = 1{,}200$ percent for each year; the total of S^*_{ij} and of the s.a.d. A^*_{ij} for each year should be inspected to ensure that the revised adjustment does not introduce serious biases (even X–11's centered seasonal factors may bias the s.a.d., e.g., if there is strong December seasonality and a monotonic trend, as in many sales series).

30. Forecasts—to assist the analyst, a variety of forecasts for the following 24 (or more) months may be printed for consideration, using the forecasted seasonality S^f (and other prior factors D^f, F^f, if analysed) to forecast $Y^f = C^f S^f$. The forecasts for the trend-cycle C^f may be prepared—

a. As extrapolations, using, as base C_0, the latest stream level L and using a variety of exponential growth rates (e.g., from −30 percent to +30 percent per annum, by unit percentages, possibly detailing further, e.g., up to 6.0 percent, by tenths of a percentage).

b. Alternative judgmental estimates of the basic trend-cycle level C_0 for extrapolating.

c. Other growth functions, e.g., linear or a gradual change of growth rates.

d. Other variants of forecasts may be prepared, using suitable control cards, e.g., using estimates U for foreseen unusual events or envisaging a cyclic turning point, based on leading indicators or previous cycles.

Annual totals should be included in such printouts. The printout of a variety of such forecasts for a specific series and for related series (see stage 33) should be of considerable assistance to analysts, and they may be used for control of current developments.

31. In addition to this new approach, a number of technical improvements are proposed to the X–11:

a. Provide for the flagging of unusual months' data (resulting from unusual events or discontinuities in the series, known before the analysis), with signs or letters, by means of additional input cards, e.g.—

B A break in the trend cycle.

D A disturbed month, e.g., by an event affecting only part of the month, which may not be identified by X–11 as extreme or may be given a misleading partial weight. (See, for example, October 1956 in table 1, affected by the Sinai Campaign, starting October 24, but given full weight in the computer analysis.)

N Occurrence of an event expected to lower the level (in the same month or in the following month).

Q Occurrence of an event expected to raise the level.

J A discontinuity in the seasonality.

These flags would be printed in all appropriate tables and graphs. (See table 1f).

b. Input annual totals (or averages) on separate cards to enable verification of the monthly input data; differences should be printed out (slight differences should not abort the continuation of the processing).

c. If the number of digits in the data permits, a space should be left between thousands and the last three digits in all printed tables, for the convenience of analysts and for reproduction.

d. Print the 12-month moving totals of Y (and/or averages per month, at choice) to enable easy analysis of fiscal years, agricultural years, etc., as well as calendar years. Cumulative totals from each January may also be useful.

e. Provide for automatic calculation of the festival-date variation F, based on 2 or 3 successive months' data, according to an appropriate method, presenting the calculations, graphs, and forecasts for at least 2 years ahead.[15] Additional input cards would indicate the appropriate dates, possibly for two festivals whose dates are close (e.g., Easter and Whitsun) or in different seasons. It is desirable to exclude extreme months (and others in unusual segments) from the regression; this may be done automatically after stage 24.

f. Provide for improved automatic calculation of the trading-day variations D, taking into account more than seven types of day (e.g., public holidays and half holidays, trading days at the beginning and end of the months) or variations over the seasons or over the years.

g. Provide for the prior adjustment of related variables' effects Z, including nonaverage weather effects W.

h. Print percent changes for each month, compared with the corresponding month in the previous year, $d_{12}Y$, and possibly comparisons with the corresponding month in the previous 5 years and comparisons for cumulative totals (from January of each year). These conventional comparisons are a poor measure of current changes in C [1] but are useful in studying changes in S, F, or D. If prior adjustments P $(=FD)$ are applied, print also $d_{12}Y^P$.

i. In the multiplicative program, compute and print absolute seasonality $S^{(A)}=C(S-1)$ and absolute irregularity $I^{(A)}=C(I-1)$ [27].

j. Print annual average rates of growth between all pairs of complete years as a growth triangle [34].

k. X–11 operates on 12- or 4-month data for each period (usually monthly or quarterly data of each calendar year). Some series should be regarded as comprised of 5 to 11 data each year, for example, by aggregating 2 or 3 successive months among which there is little activity or considerable transfers of activity (depending on festival dates or weather, etc.); X–11 could be modified to process series of such structure.

l. If there is high irregularity due to month-to-month transfers of activity (due to 4- or 5-week reporting, festival date changes, etc.), it may be desirable to compute seasonally adjusted data for successive pairs of months $A'(t,\ t+1) = (Y_t + Y_{t+1})/(S_t + S_{t+1})$ and then percent changes between alternate A'.[16]

m. Intrinsic seasonal factors S' should be calculated (standardised for the length-of-month for the convenience of the analyst) for activity series for which no trading-day adjustments are made [1].

n. The seasonal peak and trough months may be indicated by **P** and **T** for each year on the tables of seasonal factors S and S' to assist the study of seasonal patterns: Similarly, **p** secondary and **t** for peaks and troughs, **L** for local peaks and troughs. (See table 4.)

[15] The OECD, Burman and Israel CBS methods were described in [8]. The percentage method assumes that the major effect of a change in the festival date is to move a hump (and/or hole) of activity (or stock) proportional to C between 2 (or 3) consecutive months in accordance with some function of the date, while preserving the annual total. Regressions are calculated of the proportion p that each of the 2 (or 3) months' original data Y form of the total over the months affected [1].

[16] For example, for the tourist arrivals series (table 1) A' (1971, March+April)$=(Y_3+Y_4)/(S_3+S_4)$ 1971$=$(44,344+57,840)/(113.1 percent+129.5 percent)$=$102.184/242.6 percent$=$42,120; A' (1971, May+June)$=$45,263. This operation smooths the month-to-month irregularities, caused partly by problems in the festival-date adjustment (table 1f).

the seasonal range and seasonal ratio between S_P and S_T should also be calculated as convenient means of comparing seasonal intensity. Differences between S for each month between the first and last years and in the corresponding seasonal ranges and ratios may also be calculated. The maximal annual utilisation constrained by seasonality (MUS factor $=100/S'_P$) and seasonal underutilisation factor (SUF = 100–MUS) may also be calculated [1; 6].

o. Further summary measures may be prepared, e.g., contributions of the components to variance in each segment and for the complete series, measures of monotonicity over regular segments [30], significance tests.

p. Provide for a brief summary of the most important tables for convenient study and reproduction, e.g., the original data Y, seasonal factors S, s.a.d. A (see [1]), in addition to the detailed printout (full or all necessary tables, as specified [35]).

q. Provide for the output to magnetic tape or punch cards of any table or tables desired for further calculations or for presentation as published tables, for plotting by computer, for insertion in a data bank, etc. (by simple control cards).

r. It is desirable to provide simple textual analysis of seasonality and trends by segment, by computer subroutines.

Additional Stages Proposed for Parallel Analysis of Series

The presentation in parallel of the analysis of related series offers considerable advantages to the analyst, rather than his having to cope with many separate printouts. (See, for example, table 4 and [36].)

Constituent subseries may be related to the total series Y—

Additively,

$$Y=\sum_{h=1}^{H} a_h Y^{(h)}$$

Multiplicatively,

$$Y=\prod_{h=1}^{H} Y^{(h)}$$

(e.g., value = quantity × price)

By division,

$$Y=aY^{(1)}/Y^{(2)}, \text{ etc.}$$

The necessary function and parameters should be inputted on a control card together with the data for each series (desirably, but not necessarily over the complete era analysed).

Related series can also be analysed in parallel even if there is no functional relationship between them. A specific series could be analysed in parallel by two programs, e.g., multiplicative and additive (e.g., unemployment series).

The following additional stages are proposed:

32. Printout of the original data—

  a. Separate tables for each series in sequence.
  b. For each year, parallel presentation of Y, $Y^{(h)}$, and/or $a_h Y^{(h)}$ (for additive subseries).
  c. For each year, parallel presentation for each month of the percentage each subseries constitutes, $p_h=100.a_h Y^{(h)}/Y$; similarly for the annual totals.
  d. Parallel presentation of the percentage distribution for each series, by months, throughout each year $Y_{ij}^{(h)}/Y_i^{(h)}$.

33. Parallel presentation for each year of the following, for subseries and total—

  a. The SI ratios, flagging extreme months (as in stage 17).
  b. The seasonal factors S.
  c. The s.a.d. A and stream L, flagged (as in stage 19).
  d. The irregular factors I, distinguishing R, U.
  e. The forecasts Y^f.

34. Comparison should be made between the direct adjustment of the total series and the aggregated adjustment of the subseries, e.g., for seasonality S (see table 4); the s.a.d. A (see table 5), indicating the percentage constituted in each month, by each $A^{(h)}$; and forecasts. The differences and/or ratios should be printed out.[17]

35. Parallel presentation of charts G1—

  a. For the s.a.d. A, flagged for each subseries and total. (See fig. 5.)
  b. For the stream L, also flagged; if there are more than two subseries, it may be necessary to prepare several charts, preferably all to the same scale.

36. Parallel presentation of the segmentation into regular monotonic and unusual segments over the era analysed. (See table 6.)

[17] It is usually best to adjust the subseries and sum (or multiply) to obtain the total adjusted series, maintaining consistency, unless there are some nonseasonal or highly irregular subseries or compensating changes in the components between two or more subseries. The additional stages are intended to increase the understanding of the components and changes and thereby improve the adjustment procedures and the comprehensive analysis of the phenomena, possibly encouraging the compiling and studying of further subseries.

Table 4. TOURIST ARRIVALS BY AIR FOR ISRAEL: 1968 TO 1975

(Seasonal factors for total and 13 subseries, by country of residence)

Series	Causes of Season-ality, Festivals a b	Year	Seasonal Factors S [a] - % Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual Total (thous.)	Growth ratio 1975/1968	Relative weight %	% aged up to 29[c]	R I1 %[d]
T Total	ACF	1975[e]	55T	73	117	130p	101	104	165P	122p	87	101L	56T	90L	508.5	1.39		32.5	7.0
	JNS	[f]	55	74	118	131	103	105	166	122	88	95	56	89					6.0
	W																		
		1968[e]	50T	69	111	133p	94t	103	186P	125p	87	96L	54t	92L	366.5			34.2	
		[f]	51	65	109	131	90	106	194	131	83	93	56	90					
		Diff.[f]	+5	+9	+9	+1	+13	-1	-29	-9	+4	+2	0	-2				-1.7	
H Aggregate of 13 Sub-Series		1975[f]	56	74	116	137	100	100	163	125	90	95	56	88			100.0		
		1968[f]	51	65	109	133	91	106	191	129	85	92	56	90			100.0		
		Diff.	+5	+9	+7	+3	+9	-6	-28	-4	+5	+3	0	-2					
By Country of Residence[g]																			
1 U.S.A.	ACF	1975	54T	83	105	105	110	136p	178P	92	75t	108L	66	87L	143.9	0.97	28.3	28.1	13.0
	JN	1968	48T	66	104	104	89t	131p	234P	107	71t	97L	59	90L	148.3		40.4	31.1	
		Diff.	+7	+17	0	+1	+22	+5	-56	-14	+5	+10	+7	-2			-12.1	-3.0	
2 France	CEJN	1975	38T	70	103p	108p	69t	84	245P	264P	64	51	40T	66L	66.2	2.08	13.0	38.9	7.6
3 U.K.	CFJN	1975	48T	53	106	160P	114	81	154P	135p	110	99	53t	86L	46.1	1.12	9.1	38.8	6.4
4 Germany F.R.	CEN	1975	41T	68	206P	175p	86	65t	102L	95	98	128p	48t	87L	36.2	3.15	7.1	33.4	11.4
5 Scandinavia[h]	CW	1975	51T	96	174p	191P	104	100	98	70t	67t	101L	63t	86L	27.5	1.39	5.4	35.7	15.9
6 South America	AHJS	1975	156p	119	72t	178P	97	76t	100L	79t	109p	102	42T	69	24.8	2.07	4.9	26.3	11.2
7 Canada	AEJN	1975	47T	93	116p	107	141p	108	168P	93	88	91	60t	87L	22.6	1.64	4.4	28.6	12.5
8 Netherlands	CJN	1975	33T	65	121p	160P	107	90	184P	77t	114p	115p	58t	74	16.7	1.38	3.3	39.1	6.8
9 Switzerland	EN	1975	43T	66	134p	183P	109	60t	138p	69t	106	152p	56t	83L	16.5	1.51	3.2	44.4	8.9
10 South Africa	HJS	1975	106	45T	71	122p	110	95	83	68t	128p	96	68t	206P	16.5	1.83	3.2	31.5	9.1
11 Italy	CEN	1975	42T	78	126p	115	84t	83t	126p	160P	158P	71	48t	109L	16.3	2.06	3.2	22.8	15.6
12 Australia & New Zealand	HJS	1975	102L	68t	79	136p	121p	92	102	106L	83	81	55T	175P	10.1	1.74	2.0	41.5	11.7
13 Other[i]	FN	1975	53T	54	116	161p	90	95	175P	149p	96	78	55t	79	65.1	1.54	12.9	49.0	14.5

[a]The seasonal factors are indicated for the calendar months (28 to 31 days). Festival-date adjustments are made for series indicated by —

E Date of Easter. Easter Sunday varied from March 29 to April 22 (variation possible from March 26 to April 25).
H Date of Passover (first day varied from March 27 to April 21).
F Combined effect of both festivals. The average date was used for adjustment. (See table 1b.)
S for March and April correspond to the median dates of the festivals.

For convenience, the factors are rounded, and the peak and trough months are indicated for each series, based on the intrinsic seasonal factors *S'* (standardised for length of month)—

- P Principal peak.
- T Principal trough.
- L Local peak.
- p Secondary peak.
- t Secondary trough.

The increases or decreases of the seasonal factors from 1968 to 1975 (rounded independently) are shown for the total series and the United States.

[b]Causes of seasonality include—

- A Seasonal variations of air fares (e.g., winter fares from America, November-March; shoulder fares April-May and September-October; and peak fares, June-August).
- C Christians coming for Easter and Christmas (December 25 for Catholics and Protestants, and Greek Orthodox in January).
- J Jews coming for Passover and High Holydays (September and October).
- N Summer vacations, Northern Hemisphere countries.
- S Summer vacations, Southern Hemisphere (December-January).
- W Winter vacations (e.g., from Scandinavia).

Other causes include seasonal discounts in many hotels (there are different periods and varied reductions, by resort and grade, not coordinated with airfare reductions) and inclusive tours (seasonal patterns of the different age and motivation groups). In 1968 and 1973, the Independence Day parades (May 2 and 5, respectively) also attracted tourists.

[c]Young people 24 to 29 years old form a major contribuent of summer tourism (from the Northern Hemisphere), and changes in their proportion explain part of the changes in the seasonal factors for July and August. The proportions are computed for all tourists (including small numbers, by sea and land), and, since 1969, distinguish ages 0-14, 15-19, 20-24, and 25-29 (See [6; 23].

[d]The relative contribution of irregularity to changes from month to month are higher for all subseries than for the total. $\sigma(I)$ is of a similar order of magnitude. The relative contributions of seasonality were 83 to 93 percent, including festival adjustments.

[e]According to the analysis from 1956 to 1975. (See table 1a.)

[f]According to the analysis from 1968 to 1975.

[g]Countries of residence are presented in order of the numbers of tourists, by air, in 1975.

[h]Scandinavia includes Denmark, Finland, Norway, and Sweden. The analysis was from 1969 to 1975, since 1968 was the initial year of expanding charter tourism from Scandinavia.

[i]Principally Belgium, Iran, Mexico, Austria, Turkey, Japan, Rumania.

Table 5. TOURIST ARRIVALS BY AIR FOR ISRAEL: JANUARY 1975 TO DECEMBER 1976

(Seasonal adjustment and analysis, by direct method and subseries)

Month	Seasonally adjusted data: Total, X-11					Stream, total series	Percent change on previous month: In stream L	In seasonally adjusted data		Diffusion index – No. of subseries increasing on previous month[f]
			Total 13 sub-series[c]	Excl. confer-ences[d]	Revi-sed[e]			Total	By sub series	
	A^a	A^b	$A^{(H)}$ Thousands	$A^{(E)}$	A^{**}	L^a	l^a	a^a	$a^{(H)}$	
1975										
Jan.	39.6	39.9	38.1	37.2	41.1	39.7	-10.4	- 6.2	-11.9	3
Feb.	37.2	38.1	36.6	34.7	39.7	37.5	-5.6	- 6.0	- 3.9	6
Mar.	35.8	35.3	35.7	35.0	39.5	36.3	-3.1	- 3.9	- 2.4	7
Apr.	36.4	36.6	35.3	34.6	36.5	36.3	+0.1	+ 1.6	- 1.2	6
May	36.8	37.7	37.1	33.5	38.9	37.2	+2.4	+ 1.1	+ 5.1	7
June	38.8	40.5	40.8	36.2	37.3	38.5	+3.6	+ 5.7	+10.0	11
July	39.6	41.3	40.2	37.5	39.5	40.1	+4.1	+ 2.0	- 1.6	4
Aug.	42.3	42.8	40.3	41.2	42.4	43.0	+7.3	+ 6.8	+ 0.3	6
Sept.	47.9	47.2	46.9	47.0	47.5	47.1	+9.5	+13.2	+16.4	10
Oct.	50.4	48.7	53.1	50.2	51.2	51.2	+8.6	+ 5.3	+13.1	12
Nov.	56.1	48.1	55.4	46.8	55.2	55.1	+7.7	+11.1	+ 4.3	9
Dec.	58.0	56.1	59.8	55.5	59.7	55.7	+1.1	+ 3.4	+ 8.0	10
1976										
Jan.	50.9	51.3	52.8	44.8	52.8	52.7	-5.3	-12.3	-11.7	4
Feb.	51.2	52.5	47.1	46.7	55.1	52.9	+0.3	+ 0.6	-10.7	4
Mar.	58.4	58.5	54.1	61.3	65.3	56.0	+5.8	+14.2	+14.7	6
Apr.	56.0	55.6	54.6	50.2	61.5	56.2	+0.4	- 2.7	+ 0.9	8
May	54.5	56.2	55.6	50.6	57.9	53.6	-4.6	- 4.4	+ 1.8	9
June	49.3	51.8	51.5		47.2	50.5	-5.8	- 9.6	- 7.4	7
July	48.8	51.1	48.3		51.1	50.0	-1.0	- 1.2	- 6.3	3
Aug.	53.3	53.9	50.6		54.1	53.3	+6.6	+ 9.3	+ 4.8	7
Sept.	57.9	57.2	57.5		56.8	57.4	+7.7	+ 8.7	+13.7	11
Oct.	60.6	58.2	64.1		61.4	64.6	+12.5	+ 4.6	+11.6	11
Nov.	79.5	67.1	78.2		76.4	70.9	+9.8	+31.2	+21.9	9
Dec.	64.1	61.9	64.7		68.5	67.9[g]	-4.2	-19.3	-17.2	4

[a] Based on S for 1975 and S^f for 1976 from the 1956-75 analysis. (See table 1a.)

[b] Based on the 1957-76 analysis.

[c] Based on the 1968-75 analyses. (See table 4.)

[d] The data from June 1976 were not available. Note that $A^{(E)}$ for March 1976 was above A and $A^{(H)}$.

[e] Based on table 1i and S^{**} in table 1h.

[f] In terms of the s.a.d. $A^{(h)}$, out of the 13 subseries. Note that the subseries are more irregular than the total and at least three subseries increased during every month of 1975-76.

[g] Estimated using [-1,1,5] weights for the latest month currently adjusted. Note that changes in L may be studied better over spans of 2 months.

Table 6a. REGULAR AND UNUSUAL SEGMENTS IN SELECTED U.S. ECONOMIC SERIES: 1965 TO 1975

Series [a]	Type of Segment [b]	Duration of Segments[c] 1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	Total months	Peak month, S^d
1 RETAIL SALES All stores	Reg.	3+8,0+3,	6 +	+ 6, N +	+ +	2,4+6,8+	+0,D +	35+6,8+0,	3+8,0	+ N	2+7, N +	2, 4+6	101	D: 118.8
	Unus.	1-2, 9	4-5	7-0		3,7	N	4, 7, N-2,	9		D-J, 8-0	3	25	115.0
2 SALES - All Manuf. Industries	Reg.	1+8, 0+6,	8+	7	2+7, N+	9,	D-4+6-9,	D+ 5, 7 +	6,8+	7, 0 +	9,	5+0	104	7: 105.5
	Unus.	9	7	8-	-1, 8-0	0-N	0-N	6	7	8-9	0 -	4, N	27	105.5
2.1 SALES - Durable Manuf. Ind.	Reg.	1+8,	0+ 6, 8+D #	2+7,	2+6, N+	0 -	3+7-9 D +	+	+ +	7-9	3+0	- 3+0	111	6: 109.3
	Unus.	9	7	1, 8	-1, 7-0		0-N			0-2		N	20	107.2
2.2 SALES - Non-Durable Manuf. Ind.	Reg.	1+ +	6,8-N	+ 5#6+0,D +	2#3 +	9, N +	3#6+0,D	+ +	+ +	2,4+	+ 5,	3+ 6#7+N	115	9: 106.4
	Unus.		7	6, N		0	4-5 N			3	6-	-2	16	104.6
3 UNEMPLOYMENT Men 16-19 years	Reg[e]			1+2,4+6	9+D	D+	D	2+5,	2-5,	8+	+7,0+	5-D	71	2: 127.2
	Unus.[e]			3, 7-	-8	1 - N		1, 6 -	-1,6	- 7	8-9		37	125.8
	Unus.[f]			7, N-	-1,7	6,12	6	9	2-	-5	8		14	6:+294
	Unus.[g]								2-	-5		5-8	8	+283

[a]This analysis is based on the seasonal analysis of the series, conducted by the U.S. Bureau of the Census, by the X-11 program: Series 1, analysed from Jan. 1965 to June 1975 (multiplicative); series 2, 2.1, and 2.2, from Jan. 1965 to Nov. 1975 (multiplicative); series 4, from Jan. 1967 to Dec. 1975 (multiplicative and additive).

[b]The regular (monotonic) and unusual segments were identified (provisionally) from the fluctuations in the seasonally adjusted data A and the weights w for the irregular component (step C17 of X-11). The short monotonic segments may be distinguished. (See figs. 5 and 6.)

[c]The months are indicated by 1 to 9 for January to September, O for October, N for November, and D for December. Regular increasing segments are indicated by +, regular decreases —, and several months constituting an unusual segment —. Turning-point months are underlined. (# indicates an apparent break—discontinuity—in the level of the s.a.d.)

[d]The peak month and its seasonal factor S_p are shown for the first and last years analysed of each series to illustrate the intensity of seasonality and some differences between the series.

[e]Multiplicative analysis.

[f]Additive analysis (used currently).

[g]Revised segmentation, taking into account the high sampling errors and possibilities of explaining some of the summer variations as a function Z of the numbers of school leavers.

Table 6b. SELECTED CHRONOLOGY OF EVENTS AFFECTING U.S. ECONOMIC SERIES: 1971 TO 1975

(Based mainly on the Bureau of Economic Analysis monthly chronologies)

Year	Month	Event
1971	August	New economic policy, devaluing dollar.
	December	Further major currency changes.
1972	May	Devaluation of dollar vis-à-vis the Deutsche Mark.
1973	January	Cease-fire in Vietnam; withdrawal of U.S. forces.
	February	Raising of price of gold, in dollars.
	October	Arab-Israeli War: Energy crisis, followed by reduced production and increased prices.
1974	March	Removal of Arab oil embargo to the United States.
	April	Removal of many wage and price controls. Tornadoes.
	May–October	Raises in new car prices.
	June	Floods.
	November	Closing of automobile plants. Coal strike.
1975	January	Rebates off new car prices.
	April	Fall of Saigon.
	May	Layoffs from steel plants. New York City crisis.
	July	Rebates, new car prices.
	August–October	Raises in new car prices.

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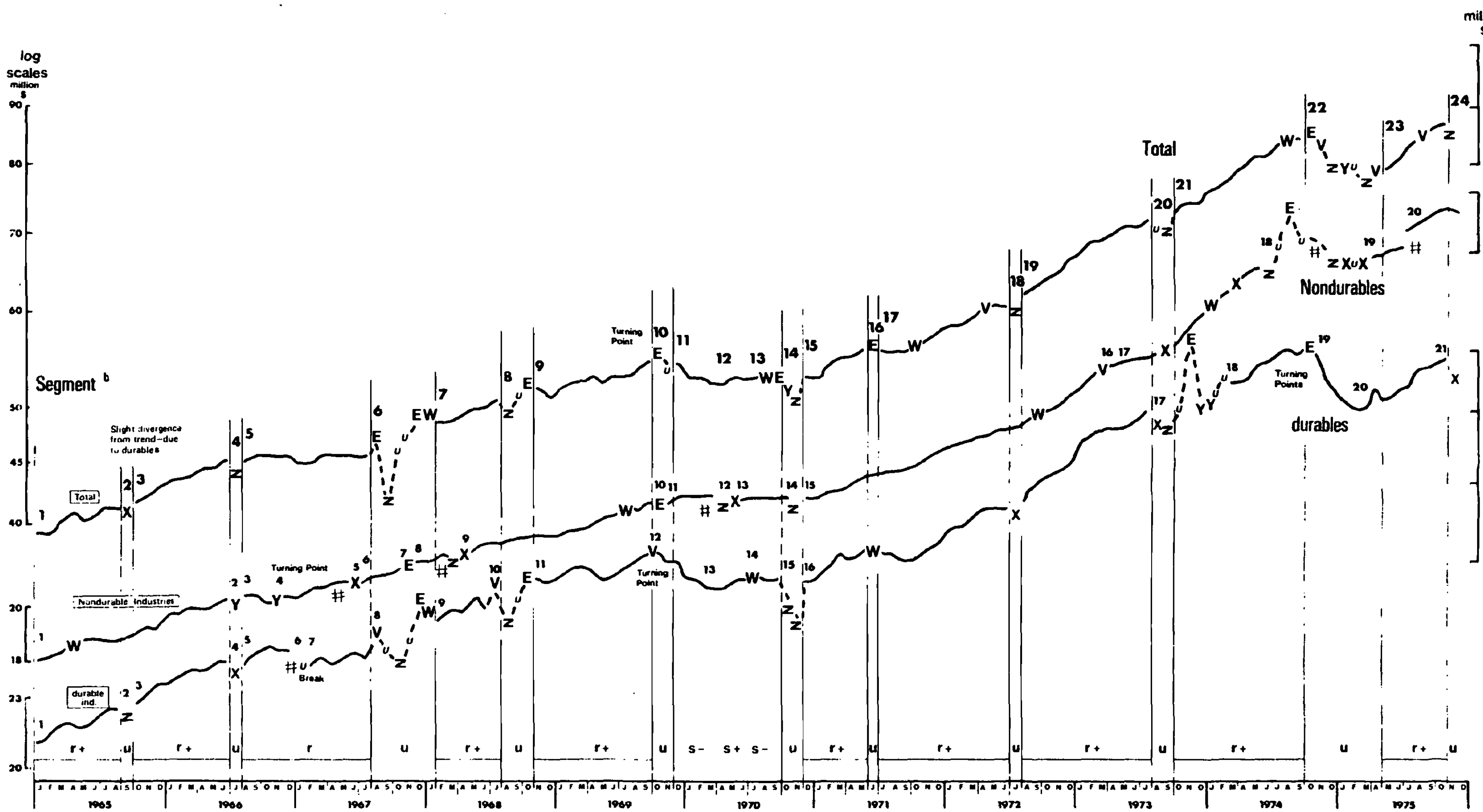
Figure 5. U.S. SALES OF MANUFACTURING INDUSTRIES: TOTAL, DURABLE AND NONDURABLE INDUSTRIES: 1965 TO 1975

Figure 5. U.S. SALES OF MANUFACTURING INDUSTRIES; TOTAL, DURABLE AND NONDURABLE INDUSTRIES: 1965 TO 1975

(Seasonally adjusted data X–11 multiplicative program[a] and segmentation)



a. The X–11 analyses were conducted on the total sales (shipments) and on the total sales of all durable and nondurable industries. The current analysis is conducted on more detailed subseries having s.a.d. that are totaled.

b. The segmentation is based on the extreme months distinguished by X–11 (indicated E, V, W, X, Y, Z) and on fluctuations in the s.a.d., including discontinuities #. The diagram indicates the segmentation for the total series, also numbering the segments of the subseries for comparison. *u* indicates a month in an unusual segment u that was not distinguished by X–11 as extreme.

37. Diffusion indexes of the number of subseries whose s.a.d. increase from month-to-month should be calculated. (See table 5.)
38. Other calculations of the interrelationships of the series (e.g., regressions, cross-spectral analysis) may also be performed.

EXAMPLES OF THE X-11 AND SUPPLEMENTARY ANALYSIS OF SERIES

Analysis of Tourist Arrivals, by Air, to Israel: 1956–76

Tourism series show high and variable seasonality and trends and are greatly affected by unusual events. They are also capable of being analysed into subseries (e.g., by country of origin, destination, age) and show interesting interrelationships between arrivals, departures, person-nights in accommodation (by type and resort), tourism expenditures, etc.

The Israel series on tourist arrivals, by air, has been studied over 15 years [1; 2; 4; 6; 8; 23] and illustrates many points discussed in this paper. In tables 1 to 5 and figures 3 and 4, we present an analysis according to the above approach and some suppplements to X–11. An ARIMA analysis is presented by Roberts [31] and further study is being conducted.

The 240 months from January 1956 through December 1975 may be divided into 25 segments, of which—

1. Nine, totaling 101 months, were regular segments **r**, with rates of growth from –2.2 percent to 6.8 percent per month.
2. Eleven, totaling 121 months, were unusual segments **u**, with assignable causes (in the quality control sense).
3. Five, totaling 18 months, were short monotonic segments **s** of 2 to 4 months, with relatively high rates of change and some disturbances to the regular seasonal pattern.

The analysis into these segments avoids some of the deficiencies of the X–11 program, especially the considerable changes in the identification and weighting of isolated extreme months (that are usually part of unusual segments, though 5 of them may be considered as part of regular segments) and the resulting estimates of seasonality S (see table 3) and the s.a.d. A and the considerable changes that occur after updating (see table 3).

Analysis of a Series together with its Constituent Subseries

A better understanding of this series of tourist arrivals, by air, can be obtained by parallel analysis of the behavior of its subseries.

We first analyse into 13 subseries, by principal countries of residence of the tourist. These series are important for historic and current analysis and for forecasting, by markets, promotion, and planning the supply of transport and other services. X–11 analyses have been made of these series over various periods [1; 6; 8]. Table 4 presents a summary of their seasonal factors for 1975, based on analysis for 1968–75 (following the Six Day War), by comparison with the analysis for the total arrivals series, by air, as analysed for 1968–75, and for the complete era 1956–75 (as in table 1). There are only slight differences in the estimates of S for 1975 (apart from October, for which the 1968–75 analysis provided an unsatisfactory estimate because of the October 1973 war); there are obviously more considerable differences in relation to estimates of S for 1968.

The seasonal patterns of all the subseries are strong and show considerable differences in the timing and amplitude of the peak and trough months, the main causes of which are indicated in the table. The peak seasonal factors for 1975 were up to 264 percent (for France, in August). $S_P^{(h)}$ for two countries only (for the United Kingdom in April, 160 percent and 154 percent in July and Italy, 160 percent in August) were less than S_P for the total series (165 percent in July), indicating that the different strong seasonal patterns counter balance to a considerable extent.

The subseries indicate moving seasonality, especially from the principal source, the United States, for which there were considerable drops in the summer seasonal factors (for July, the peak, and for August). These drops are due partially to the reduction in the proportion of young tourists (noted in the table) and partially to the relative increase of off-season tourism at reduced prices, while the total high-season tourism from the United States dropped in absolute numbers (by 36 percent) from 1968 to 1975.

The seasonal pattern for each year for total tourism may be regarded as the weighted average of the seasonality of all the subseries, as illustrated in the table. It is, therefore, important to note that the growth ratios from 1968 to 1975 varied from 0.97 (i.e., a decrease of 3 percent) for tourist arrivals from the United States, up to 315 percent for tourism from Germany. These changes in the relative weights of the series are the other reason for the moving seasonality of the total series. The seasonally adjusted data aggregated from adjustment of the subseries (that are available about 10 days after the total counts are analysed) are shown in table 5; the differences are generally slight.

One method of reducing irregularity is to deduct any highly irregular subseries. Conference tourism to Israel is nonseasonal but highly irregular (in some countries it is mainly off season), the numbers varying, over the period 1970–75, from zero (October 1973) to 8,052 (March 1974). The data are not yet available monthly, by country of residence, but the deduction of estimates of conference tourists (based on records of the numbers of delegates and including estimates of family accompanying) from total arrivals, by air, reduced some of the irregularities. Current monthly adjustment of seasonality has been made excluding conference tourism. (See table 5). The deduction of this group also improves considerably the weekly analysis of tourist arrivals (based on daily data [1]). Similar deductions of highly irregular subseries, such as the imports and exports of ships and aircraft, were presented in [1]. The s.a.d. for every subseries should obviously be less than for the total but may, on occasion, be higher because of a lower estimate of S, e.g., tourists excluding conference participants in March 1976 (in table 5).

Subseries may also be prepared in greater detail, e.g., by country of residence, according to age group. Study, by age groups, has been made for total tourism, totals from Europe and from North America, indicating that the seasonality differs considerably, by age group (with a peak of 422 percent in July for ages 15–19 from Europe [6]), and this analysis reduces the extent of moving seasonality and indicates the different trends, by market strata.

These subseries are being studied in parallel, using the proposed improvements in X–11 and other multivariate techniques. In some cases, there may be complementary movements between subseries, e.g., between domestic and international tourist nights in accommodation, between currency in circulation and deposits, or employed and unemployed persons.

Analysis of Selected U.S. Economic Series, Distinguishing Regular and Unusual Segments

A similar analysis into segments was made for five typical U.S. series analysed by X–11 and is summarised in table 6.

Retail sales, total— shows 14 monotonic increasing segments from 1965 to June 1975, totaling 101 months, and 15 unusual segments, totaling 25 months. The seasonal analysis is currently conducted on the various subseries, by type of establishment, because of considerable differences in seasonal, trading-day, festival-date, unusual effects, and trend cycles among the sub-series and their s.a.d. then totaled. Further analysis indicated that for many subseries the trading-day factors D explain considerably more variance of Y than seasonality. X–11 can not take into account variations in D over the months and years. Further research is, therefore, proposed on the trading-day factors, e.g., based on daily reports of typical establishments and on analysis of the subseries in parallel.

Sales, all manufacturing industries—shows 10 increasing regular segments and 3 short monotonic segments, totaling 104 months, and 11 unusual segments, totaling 27 months (from January 1965—November 1975). Further analysis into two subseries shows—

1. Sales, durable manufacturing industries—Ten increasing and four decreasing **r** or **s** segments, with apparent discontinuity between December 1966 and January 1967, and seven unusual segments.
2. Sales, nondurable manufacturing industries—Five apparent discontinuities and eight other unusual segments. (See fig. 5.)

Analysis of the SR factors in regular segments indicates considerable moving seasonality, e.g., a reduction in December's factor for all manufacturing industries from 97.3 percent for 1965 to about 94 percent from 1973, apparently due to the reduction in manufacturing because of extended Christmas vacations.

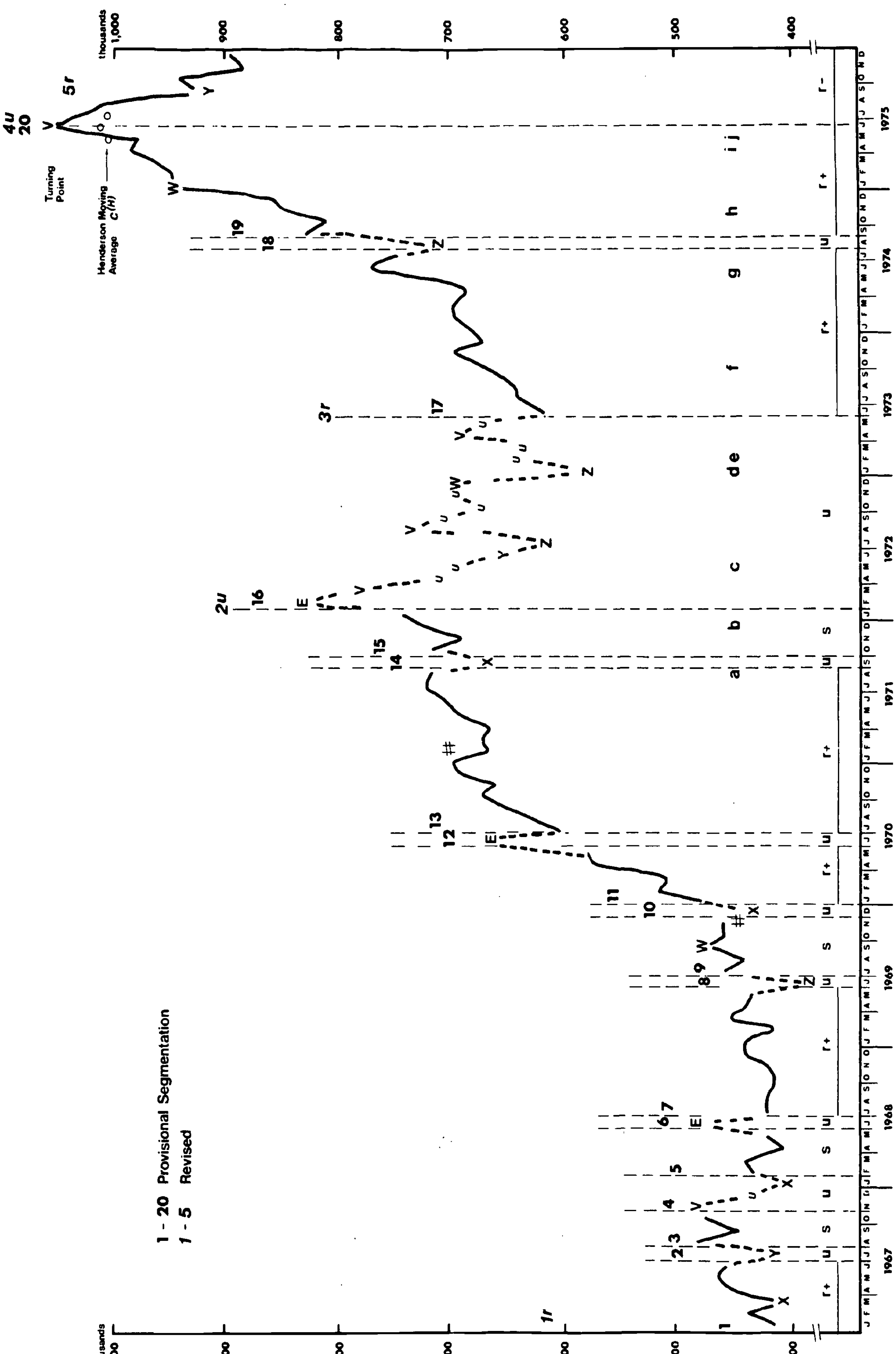
The seasonal analysis is conducted on the subseries, by economic branch, which again have different patterns for C, S, F, D, and U. For some subseries, it was noted that very large projects can give rise to nonseasonal increases in the original data Y, and it is desirable to deduct these or segment them off to obtain the regular seasonality.

Unemployment; men 16–19 years old—The X–11 additive analysis for 1967 to 1975 indicated 7 months with zero weight (**E** or **Z**) and 15 months with partial weights (**V**, **W**, **X**, and **Y** in fig. 6) out of the 108 months, primarily in the summer months. A similar picture arose from multiplicative analysis. The initial segmentation was into 20 segments, with some apparent discontinuities. Discussions and correspondence with John Early (of the BLS) indicated that—

1. This series suffers from relatively high sampling errors that account for many of the extreme months. The sampling and estimating procedures give rise to complex serial correlations.
2. There are very considerable variations over the years in the $S+I$ differences calculated by X–11 ($=Y-C$ in the additive model [35]) for the summer months June through September; a similar picture was obtained using the new $(S+I)^L = Y-L$ differences. These variations are mainly due to the strong differences in the numbers of schoolleavers Z_i each year, most of whom enter the labor force, and may be largely explained by a related variable function $Z_{ij}(z_{ij})$.
3. A new seasonal pattern is developing due to schoolleaving being spread over other months, possibly accounting for the rise in S for January and February. Consideration should, therefore, be given to treating the unemployment of schoolleavers as a separate subseries from other youth unemployment and also to study changes in seasonal work opportunities for youth.

Figure 6. U.S. UNEMPLOYMENT FOR MALES 16 TO 19 YEARS OLD: 1967 TO 1975

(Seasonally adjusted data—X–11 additive program—and segmentation)



a–j. Some events that affected this series from 1971 to 1975 are listed in the chronology of events, table 6b.

4. The trend cycle showed a considerable, but perturbed, drop from the peak of February 1972 to May 1973, then a rapid rise from June 1973 to the turning point of June 1975, followed by a sharp fall. It is doubtful whether additive seasonality is appropriate over all such segments (even after X–11's treatment of extremes), and the Henderson moving average $C^{(H)}$ gives a flattened picture of the cyclical turning point (fig. 6), affecting the $S+I$ differences.
5. After accounting for the schoolleaving effect and unusual segments not due to the sampling errors, it is difficult to discern moving seasonality over these 9 years. The average of the $(S+I)^L$ differences over the regular segments may serve as more appropriate seasonal factors S^f for current analysis in 1976–77 than the X–11 factors; this is being studied.
6. The ARIMA approach will also be experimented with, for comparison.

A COMPARISON OF THE ARIMA AND X–11 COMPONENT APPROACHES

For a series Y represented in the time domain by the multiplicative model

$$Y_t = C_t . S_t . I_t \tag{3}$$

a logarithmic transformation gives the additive model

$$y_t = \log\ Y_t = \log\ C_t + \log\ S_t + \log\ I_t \tag{4}$$

For simplicity, we assume that any necessary prior factors have been applied (e.g., for F, D), since they cannot be treated easily by ARIMA techniques.

For many series, we may assume that over the regular segments—

1. The trend cycle is a quadratic exponential
$$C_t = \exp\ (a + bt + qt^2) \tag{5}$$
possibly with changes in the parameters for some segments.
2. The seasonal factors for each month j move over the years i according to quadratic functions
$$\log S_{ij} = s_j + im_j + i^2 n_j \tag{6}$$
where $\Sigma^{12} s_j = 0$, $\Sigma^{12} m_j = 0$, $|n_j| < |m_j|$, $t = j + 12\ (i-1)$, possibly with some discontinuities after unusual segments.
3. Irregular factors $\log\ I_t = e_t$ are approximately normally distributed in regular segments, but possibly with some negative serial correlations and/or different variances $\sigma(e_j)$ in different months.

The initial detrending is conducted by first differences, together with the initial deseasonalising (differences over 12 months)

$$w_t = D^{12} D y_t \tag{7}$$

$$\begin{aligned} &= a + bt + qt^2 + s_j + im_j + i^2 n_j + e_t - [a + b(t-1) \\ &\quad + q(t-1)^2] - [s_{j-1} + im_{j-1} + i^2 n_{j-1}] - e_{t-1} \\ &\quad - [a + b(t-12) + q(t-12)^2] - [s_j + (i-1) m_j \\ &\quad + (i-1)^2 n_j] - e_{t-12} + [a + b(t-13) \\ &\quad + q(t-13)^2] + [s_{j-1} + (i-1) m_{j-1} \\ &\quad + (i-1)^2 n_{j-1}] + e_{t-13} \\ &= 24q + m_j - m_{j-1} + (2i-1)(n_j - n_{j-1}) \\ &\quad + (e_t - e_{t-1}) - (e_{t-12} - e_{t-13}) \end{aligned} \tag{8}$$

The parsimonious $(0,1,1) \times (0,1,1)_{12}$ model

$$w_t = D^{12} D y_t = c_1 + (1 - c_2 B)(1 - c_3 B^{12}) a_t \tag{9}$$

used in [11; 12; 31], therefore, gives a reasonable approximation to the behavior of the series, but—

1. It is necessary to allow for $c_1 \neq 0$ if the trend cycle shows a significant quadratic form, rather than linear; this was ignored in [11] and [12], giving rise to the upward bias in the forecasts for international airline passengers for 1958–60, based on July 1957. For our demonstration series of tourists to Israel, c_1 was set as 0 in [31] despite the contrary evidence. Second differences Dw_t may be needed.
2. The direction and magnitude of moving seasonality may be important for particular months (e.g., May, December in the demonstration series), giving rise to more or less consistent variations in w_t over the months j rather than the stationarity desired. Roberts treats this as part of the fitted stationary [31]; further differencing D^{24} may be necessary.
3. Any unusual e_u will affect four terms, w_u, w_{u+1}, w_{u+12}, w_{u+13}, unless masked by other unusual effects in related months. It is not easy to analyse such extreme effects, especially when there are many extreme months and unusual segments that cannot be adequately accounted for by a white noise process a_t. Further differencing will spread the unusual events' effects even further.
4. The interpretation of estimates made for c_2, c_3 is not easy, especially when they change between segments or on updating [31].
5. Estimates of seasonal structure and of the historic patterns of changes in seasonality are very difficult [13].
6. It is difficult to apply additional information and judgment regarding future trends and seasonality to specific series, as are often required (especially in regard to tourism).
7. Parsimonious models are elegant to indicate the "wood", but complex series seem to need more complex explanations of the "trees" than can be

supplied by two (or three) parameters in $(0, 1, 1) \times (0, 1, 1)_{12}$ models. The historic and current analysis, based on seasonally adjusted data, seems easier to report in terms of changed monthly levels (fig. 3) than charts of residuals [31] that are affected by several different months' data and events.

FUTURE PROGRESS

Comments, examples, and proposals will be welcomed. Further experimentation is desirable on a variety of empiric and simulated series to develop the most appropriate programs and techniques for seasonal and related analysis and adjustment and to gain experience in their application.

REFERENCES

1. BarOn, Raphael Raymond V. *Analysis of Seasonality and Trends in Statistical Series.* Vol. 1: *Methodology, Causes and Effects of Seasonality;* vol. 2: *Applications in Israel;* and vol. 3: *Tables of the Series Analysed.* Technical Publication No. 39. Jerusalem: Israel Central Bureau of Statistics, 1973.

2. ———. "Analysis of Statistical Series on the Activity of the Tourism Industry in Periods of Crisis and Analysis of Their Recovery Following Crises, in Israel and in the World." Paper presented at the Study Conference on Organisation for Crisis Situations in Tourism, University of Tel Aviv, Leon Racanatti Graduate School for Business Administration, June 4, 1975. (Hebrew, mimeographed.)

3. ———. "Forecasting of Visitors to Thailand." *Thailand Travel Talk*, October and November 1975.

4. ———. "Models for Analyzing Multi-Periodic Transport Series." In *Developments in Operations Research: Proceedings of the Third Annual Israel Conference on Operations Research (in Cooperation with the Operations Research Society of America), Tel Aviv, July 1969.* Vol. 2. Edited by B. Avi-Itzhak. New York: Gordon and Breach Science Publishers, 1971, pp. 389–415.

5. ———. *Seasonality in Israel: Seasonal Analysis and Adjustment of Selected Time Series.* Technical Publication No. 15. Jerusalem: Israel Central Bureau of Statistics, 1963.

6. ———. *Seasonality in Tourism: A Guide to the Analysis of Seasonality and Trends for Policy Making.* Technical Series No. 2. London: The Economist Intelligence Unit, Ltd., 1975.

7. ———. "Seasonality, Trends and Irregularity: A New Approach." *Bulletin of the International Statistical Institute* XLV, bk. 1 (1973): 145–152.

8. ———. *Seasonality and Trends in Israel Tourism.* Technical Publication No. 30 (rev. ed.). Jerusalem: Israel Central Bureau of Statistics, 1969.

9. Bellman, R., and Roth, R. "Curve Fitting by Segmented Straight Lines." *Journal of the American Statistical Association* 64 (September 1964): 1079-1084.

10. Bonin, J. M., Kahl, A. L., Jr., and Waters, J. B. "A Bibliography on Seasonal Estimation Methods and Economic Analysis of Seasonality." University of Georgia, 1969. (Mimeographed.)

11. Box, George E. P., and Jenkins, G. M. *Time Series Analysis: Forecasting and Control.* San Francisco: Holden-Day, Inc., 1971.

12. ——— and Tiao, George C. "Intervention Analysis with Applications to Economic and Environmental Problems." *Journal of the American Statistical Association* 70 (March 1975): 70–79.

13. Brewer, K. R. W. "Component Analysis and Seasonal Adjustment of ARIMA Time Series." Personal Communication, 1976.

14. Burman, J. P. "Moving Seasonal Adjustment of Economic Time Series." *Journal of the Royal Statistical Society*, ser. A, 128 (1965): 534–558, and 129 (1966): 274.

15. Cornfield, J. "A Statistician's Apology," *Journal of the American Statistical Association*, 70 (March 1975): 7–14.

16. Dagum, Estela Bee. "Seasonal Factor Forecasts from ARIMA Models." *Bulletin of the International Statistical Institute*, Warsaw, 1975, XLVI, 3:203–216.

17. De Fontenay, A., "The Composition of a Time Series as an Algebra of Averages." *Bulletin of the International Statistical Institute* XLV, bk. 1 (1973): 395–415.

18. Dunn, D. M., Williams, W. H., and De Chaine, T. L. "Aggregate versus Subaggregate Models in Local Area Planning." *Journal of the American Statistical Association* 71 (March 1976): 68–71.

19. Durbin, James, and Kenny, P. B. "Seasonal Adjustment When the Seasonal Component Behaves Neither Purely Multiplicatively nor Purely Additively."
Included in this report.

20. Fase, M. D. G., Koning, J., and Volgemant, A. F. "An Experimental Look at Seasonal Adjustment." *De Economist* 121 (1973): 441–480.

21. Granger, Clive W. J. "Seasonality: Causation, Interpretation, and Implications."
Included in this report.

22. International Union of Official Travel Organisations. "Forecasting of Tourism Flows," by Raphael Raymond V. BarOn. In *The Measurement of Tourism: A Guide Based on a Seminar Held at Haslemere, England, September 1974.* London: British Tourist Authority, 1975, pp. 28–41.

23. Israel Ministry of Tourism and Central Bureau of Statistics. *Tourism: 1975.* Special Publication No. 523. Jerusalem: Central Bureau of Statistics, 1976.

24. Kallek, Shirley. "An Overview of the Objectives and Framework of Seasonal Adjustment."
Included in this report.

25. Kendall, M. G. *Time Series Analysis.* London: Charles Griffin and Co., Ltd., 1975.

26. Kenny, P. B. "Problems of Seasonal Adjustment." *Statistical News* 29 (1975) : 1–6.

27. Klevinaken, A., and Bonin, J. M. "Thoughts on an 'X–12'," *Proceedings of the Business and Economics Section of the Meeting of the American Statistical Association.* Washington, D.C.: 1970.

28. Kuiper, John. "A Survey and Comparative Analysis of Various Methods of Seasonal Adjustment."
Included in this report.

29. Parzen, Emanuel. *An Approach to Time Series Modeling and Forecasting, Illustrated by Hourly Electricity Demands.* Statistical Science Technical Report No. 37. Buffalo: State University of New York at Buffalo, 1976.

30. Raveh, A. "Finding Periodic Patterns in Time-Series with Monotone Trend: A New Technique." In *Theory Construction and Data Analysis in Behavioral Sciences in Honor of Louis Guttman.* Edited by S. Shye. Jerusalem: Israel Institute of Applied Sociological Research, 1978.

31. Roberts, Harry V. "Comments on 'The Analysis of Single and Related Time Series into Components: Proposals for Improving X–11' by Raphael Raymond V. BarOn."
Included in this report.

32. Tarter, M., and Silvers, A. "Implementation and Applications of Bivariate Gaussian Mixture Decomposition." *Journal of the American Statistical Association* 70 (March 1975) : 47–55.

33. United Nations, Statistical Office. "Newsletter on Applied Time Series Analysis: Seasonal and Related Adjustments." Edited by Raphael Raymond V. BarOn. In *Statistical Notes*, ser. B, 34 (March 7, 1972) : 103–115.

34. U.S. Department of Commerce, Bureau of the Census. *Long-Term Economic Growth: 1860-1965.* Washington, D.C.: Government Printing Office, 1966.

35. ———. *The X–11 Variant of the Census Method II Seasonal Adjustment Program*, by Julius Shiskin, Allan H. Young, and J. C. Musgrave. Technical Paper No. 15. Washington, D.C.: Government Printing Office, 1967.

36. ———, Bureau of Economic Analysis. *Business Conditions Digest.* (Monthly). Washington, D.C.: Government Printing Office.

APPENDIX

Analysis of Monthly Economic Time Series into Time-Domain Components for Periodic, Systematic, and Event-Conditioned Effects: Summary

Y_{ij} (*or* Y_t) designate the historic data for years $i=1, \ldots, n$; months of each year $j=1, \ldots, 12$; $t=12(i-1)+j$ months from start (the first and last years are assumed to have all 12 months' data). Forecasts are required for year $n+1$

Component and its characteristics	Causes	Mathematical models		Typical problems of estimating in historic and current analysis and forecasting
		Time domain (component analysis)	Frequency domain and autoregression	
Y (or O) **Original Series (unadjusted series)** Activity (or stock) fluctuates periodically, systematically and irregularly.	Combined fluctuations of all relevant components. Fluctuations may also be related to changes in constituent subseries $Y^{(h)}$ (possibly in more than one dissection—e.g., additive, by type and/or by region, or multiplicative).	Simple multiplicative $Y=C.S.I.$; Simple additive $Y=C+S+I$; Mixed and complex, e.g. $Y=C(S+M)F.D.W$ $Z.U.R.$ (See [1].) Multiplicative models are applicable if all $Y_t>0$. Components usually assumed independent.	Spectrum $=\Sigma$ spectra of additive components (possibly after logarithmic transformation) with peaks at seasonal and cyclic frequencies. ARIMA (Box-Jenkins) models, e.g., $(p,d,q)\times(P,D,Q)_{12}$ models $\phi_p(B)\Phi_P(B^{12})(1-B)^d$ $(1-B^{12})^D Y_t=\theta_q(B)\theta_Q(B^{12})a_t$, possibly using log Y or other transformation; B is the backshift operator $BY_t=Y_{t-1}$, a_t random noise (shocks, innovations [11]).	Y may suffer from measurement errors (including sampling and response errors) and discontinuities in the phenomena or discontinuities due to changes in the definitions, coverage, methods etc. Components may show interdependence. Forecasts are based on extrapolations or judgmental forecasts of C and S, possibly also of D, F, T, Cy, U, and Z, for basic series Y or by forecasting subseries $Y^{(h)}$. (See, e.g., [3; 22].)

THE MAJOR COMPONENTS

PERIODIC COMPONENTS				
***S* Seasonality** Fluctuations recurring every year with similar intensity in corresponding months. One or more peaks or troughs	Climate, institutional timing (taxes, holidays), individual habits, etc.	Constant multiplicative S_j, $(\Sigma_{j=1}^{12} S_j)/12=1$; constant additive s_j, $\Sigma^{12} s_j=0$; and mixed constant additive and multiplicative	Constant seasonality shows high spectral power at seasonal frequencies, corresponding to periods of 12, 6, 4, 3, 2.4, and 2 months, depending on pattern of S. Autoregression primarily of lag	The choice of model is not always according to clear criteria.

APPENDIX—Continued

Analysis of Monthly Economic Time Series into Time-Domain Components for Periodic, Systematic, and Event-Conditioned Effects: Summary—Continued

Y_{ij} (*or* Y_t) designate the historic data for years $i=1, \ldots, n$; months of each year $j=1, \ldots, 12$; $t=12\ (i-1)\ +j$ months from start (the first and last years are assumed to have all 12 months' data). Forecasts are required for year $n+1$

Component and its characteristics	Causes	Mathematical models		Typical problems of estimating in historic and current analysis and forecasting
		Time domain (component analysis)	Frequency domain and autoregression	
THE MAJOR COMPONENTS—Continued				
each year; usually not sinusoidal or symmetric. Discontinuities are possible after a major change in causes, e.g., after an unusual event.		$\dot{S}_{ij}=s_j+C_{ij}S_j$ (produces moving multiplicative).	12 months, requiring B^{12}.	
***M* Moving Seasonality** may be distinguished relative to basic seasonality $S_{\mathbf{B}j}$ (estimated for specific year **B**).	Seasonal effects may vary over the years because of changes in conditions, preferences, the relative trends of subseries with different seasonal patterns $S^{(h)}$, or relative strength of conditions producing activity (or stock) in specific months [1; 21].	Moving multiplicative $S_j=S_{\mathbf{B}\,j}+M_{ij}$, $\Sigma^{12}S_{\mathbf{B}j}=12$, $\Sigma^{12}\,M_{ij}=0$ (for every i). More complex models are also used [19].	Some spectral power will be transferred to nonseasonal frequencies, too. Moving seasonality complicates autoregression, since $w_{ij}=(1-B)\,(1-B^{12})\,Y_{ij}$ will depend on j.	Moving seasonality is usually estimated by smoothing the SI ratios (calculated from the $Y\div$ estimate of C, taking into account D, F, if necessary), by moving averages over years i for each month j. Outlying SI ratios may have to be identified and more regular $(SI)'$ may be imputed in their place. Estimated S_{ij} for specific i will depend on span of years analysed: tails of moving averages may give unreliable estimates, especially for S_{nj}. The constraints are not appropriate to maintain annual totals after seasonal adjustment, especially under strong trends. SR ratios that are defined only in regular segments may be used for

				regular segments may be used for improved estimates and forecasts (e.g., by regression). In current analysis and forecasting, problems arise of distinguishing whether recent changes are due to causes that are expected to continue to affect S or whether to continue to use previous estimates and ascribe these changes to C, U, R, or G.
S^f Seasonality Forecasted for 12 months (or more) ahead.	Current seasonality and prevailing or expected changes in seasonal patterns (and in relative trends of subseries with different $S^{(h)}$).	In X–11 cautious half-extrapolation $S^f_{n+1,j}=S_{nj}+\frac{1}{2}(S_{nj}-S_{n-1,j})$. (See [35].) Extrapolation of regression functions for S_{ij} on i are possible for each j.	Current analysis and ARIMA forecasting are usually based on Y^f rather than on separate estimates of S^f. (But, see [13].)	Extrapolations from S_{nj}, $S_{n-1,j}$ will suffer from errors in their estimation by moving averages. Prevailing changes in Y may be due to changes in C, G, or U but wrongly ascribed to S. New conditions may change S in specific months differently from past trends in S_{ij}.
S' Intrinsic Seasonality	Effect of length-of-month factor L.M. on activity.	$S'=S\times 365\frac{1}{4}\div$L.M.		Useful to describe seasonal patterns if no significant trading-day effects D are present. Differences between activity in 28– and 29–day Februaries may require prior adjustment, rather than treating all Februaries as 28¼-day average L.M.
$S^{(A)}$ Absolute Seasonality	Combined effects of S and C.	In multiplicative model, $S^{(A)}=C(S-1)=Y/I-C$.	Shows spectral power at seasonal and lower frequencies. ARIMA models do not usually distinguish.	Estimation depends on difficulties in estimating I, C.
***P* Other Calendar Effects**	Differential activity over days of year and, in specific Gregorian months (for which data are usually compiled), in successive years.	Simple additive $Y=C+S+P+I$, $P=D+F$; Simple multiplicative $Y=C.S.P.I.$, $P=D.F.$	These effects complicate spectrum of monthly data or autoregression and are usually adjusted prior to analysis.	If series is analysed into C, S, I without prior adjustments, it is difficult to distinguish P in I from R.

APPENDIX—Continued

Analysis of Monthly Economic Time Series into Time-Domain Components for Periodic, Systematic, and Event-Conditioned Effects: Summary—Continued

Y_{ij} (*or* Y_t) designate the historic data for years $i=1, \ldots, n$; months of each year $j=1, \ldots, 12$; $t=12\ (i-1)\ +j$ months from start (the first and last years are assumed to have all 12 months' data). Forecasts are required for year $n+1$

Component and its characteristics	Causes	Mathematical models		Typical problems of estimating in historic and current analysis and forecasting
		Time domain (component analysis)	Frequency domain and autoregression	
THE MAJOR COMPONENTS—Continued				
***D* Trading-Day Variation** Scaling factors relative to standard month (or day).	Different rates of activity (or stock) $q(d)$ over the 7 days of each week, possibly also periodic in each month (date-of-month effect), or significantly different on special dates (e.g., festivals or dates related to fiscal year) and on several days before and after these special dates.	Seven constant factors $D(d)$ for the days of the week, $\Sigma^7 D(d)=7$, $D_{ij} = \Sigma_{d=1}^{7} n_{ij}(d).D(d)/n_{ij}$, $n_{ij}(d)$ = number of days d in month (4 or 5), n_{ij}=28 to 31. (See [35].)	Q different factors $D_q(d)$ may take into account more than seven types of days (e.g., days at beginning and end of month, festivals and related days).	Factors may vary over the months of each year (especially if there are many festivals or special days) or over the years. D may differ among subseries (e.g., industries with 5, 6, and 7 working days). May introduce negative serial correlation in Y and I if not adjusted for (e.g., because of 5th Saturday or Sunday occurring in nonconsecutive months). Regression methods are used (e.g., of I on structure of regular months), but the assumptions are simplistic (especially constancy over months and years) and are not appropriate for many series. External factors (e.g., calculated from daily series) may be useful in studying historic and current patterns and estimating. Forecasts $D_{n+1,j}$ can be made based on the structure of months and regression functions (or judgment as to the effect of changes).

***F* Festival-Date Variation** Distribution of activity (or stock) among 2 (or 3) adjacent months differs over years because of specific dates, e.g., of festivals. Annual totals may also be affected.	Changes in calendar dates of moveable festivals (e.g., Easter) or of school holidays, introduction of new models of cars, seasonal discounts or surcharges, etc.	Multiplicative factor $F_{ij}=f_j(x_i)$, as function of date x_i of festival in year i (measurable from median date over period analysed or first possible date). Relative proportion of activity p_{ij} out of total for months affected may be a linear or other (e.g., bounded) function of x_i when the festival-date variation moves a hump of activity proportional to C (possibly preceded or followed by less activity), preserving annual totals [1]. I_{ij} may be an independent function of x_i for each j affected.	Disturbs spectrum and autoregression relating to annual periodicities and relationships between successive months. Prior adjustment desirable.	Regression analysis may indicate regular effect of x_i on Y, on p or R in the months affected; months in unusual segments should be excluded. Functional relationships are difficult to determine, especially if there is interaction with the day-of-week of the festival (e.g., a long weekend in some years) or with the level of activity over the months affected. There may be different effects on specific subseries or effects of two or more related festivals over some months. Daily series may assist in establishing the effect, function, and parameters. Forecasts are possible based on x_{n+1}. If the percentage method is used, an estimate $F_{n+1,\,j+1}$ for the first month affected may be made on the basis of forecasted $p'_{n+1,j}$ but needs revision after measuring $Y_{n+1,\,j+1}$ and total activity over the months affected.
EXTRASEASONAL ANNUAL EFFECTS				
***W* Weather Variation** (nonaverage). Changes in seasonality for specific months, advance or retard of seasonal peaks or troughs or change of magnitude of their effects.	Specific climatic conditions c_{ij} for months j vary each year around multiannual average climate $\bar{c}_j$. Varying date of start and end of season (e.g., monsoon, ice, harvest).	Component $W(c_{ij})$ as function of climatic variables, mutiplicative (or additive) to S. Example, effect of degree-days (relative to average for month) on consumption of energy for heating or cooling or on ice cream sales.	Disturbs spectra of major components. Cross-spectral analysis or regression on climatic variables may indicate relationships, especially time lags.	There are problems of choosing the climatic variables (e.g., minimum, average or maximum temperature; rainfall, snow, ice, or their combination), computing weighted averages over the days of the months and over relevant geographic areas (the variance may be high). Regression analysis of Y, A, or R on c is difficult because of the need to exclude unusual periods (extreme weather)

APPENDIX—Continued

Analysis of Monthly Economic Time Series into Time-Domain Components for Periodic, Systematic, and Event-Conditioned Effects: Summary—Continued

Y_{ij} (*or* Y_t) designate the historic data for years $i=1, \ldots, n$; months of each year $j=1, \ldots, 12$; $t=12\ (i-1)\ +j$ months from start (the first and last years are assumed to have all 12 months' data). Forecasts are required for year $n+1$

Component and its characteristics	Causes	Mathematical models: Time domain (component analysis)	Mathematical models: Frequency domain and autoregression	Typical problems of estimating in historic and current analysis and forecasting
THE MAJOR COMPONENTS—Continued				
				and because of interactions, possible time lags and different effects on subseries (e.g., by area, product). This component may be useful for current analysis and, possibly, for forecasting 1 or 2 months ahead (e.g., agricultural production, prices).
SYSTEMATIC COMPONENTS				
***C* Trend Cycle** Smooth changes, not periodic over each year; occasional discontinuities are possible.	Basic demographic, socioeconomic, and nonseasonal causes.	Functional representation (see T, Cy). Empiric (nonfunctional) estimates are made by smoothing A (or A^m), e.g., by moving averages.	High spectral power at periods over 12 months (low frequencies). Y_t shows strong autoregression on Y_{t-1}. Direct ARIMA estimation is difficult [13; 31].	May not usually be estimated satisfactorily by one simple function over long series. Current analysis is important to indicate changes in the rate of growth and cyclical turning points, but it is difficult to distinguish significant changes in C from I or U or from changes in S. Moving averages may distort the underlying C, especially if equal weights are used over 3 or more months, $C^{(M)}$ or long averages $C^{(H)}$ spanning segments

of different characteristics.
Forecasts may be based on com-

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				of different characteristics. Forecasts may be based on components T and Cy or on adaptive ARIMA process.
***T* Trend (secular)** Monotonic growth (or decline) over segments of at least 6 months. Discontinuities are possible due to unusual events or changes in definitions, etc.	Growth of the population supplying and/or demanding the activity, technological and other long-term changes (e.g., in preferences and habits).	Polynomial, exponential, or other functions, defined over segments (intervals) $t=(1, t_1), (t_2, t_3), \ldots,$ but not defined over unusual segments. Empiric (nonfunctional) estimates are usually based on moving averages. May be studied as polytonic sequence of monotonic trends [30].	Estimate of trend may be used to de-trend Y into a stationary series (e.g., prewhitening by first or d differences) $w_t=(1-B)^d Y_t$ (possibly after logarithmic transformation). In spectral analysis, T will appear as long cycles together with Cy.	Difficult to distinguish T from Cy and U. Determination of the functions (including order of polynomial) and segmentation are subjective. The segmentation and functions may differ for subseries having growth rates r that usually differ. Forecasting is possible by extrapolation of latest trend function (for total or by subseries $T^{(h)}$) or as adaptive ARIMA process if conditions are not expected to change significantly. Judgmental estimates of growth parameters (taking into account expected conditions) may be more suitable, either for point estimates or for a range of forecasts (from pessimistic to optimistic).
***Cy* Cycles** Superannual oscillations, with relative highs and lows over periods of more than 12 months, e.g., Cb, G, Co.	Economic, psychological, and other causes that may change over the phases of the cycles (naturally or because of intervention, e.g., countercyclic policies) or having relative effects that may change.	Sinusoidal or polynomial, possibly with different parameters for each cycle or phase.	High spectral power at cyclical period—if there is only one cycle or several of similar (or related) periods. Complex spectrum if the cycles have different periods.	Periods and amplitudes are not necessarily equal; increases and decreases are not always symmetric. Cy may interact with T (e.g., varying rates of growth without clear turning points) and be difficult to distinguish as a separate component, especially if subseries have different cyclical patterns.
***Cb* Business Cycles**	Relative growth and decline of major economic variables.		Spectral analysis is useful if the cycles are of similar period and to study crossspectral relationships with other series.	Diffusion indexes, based on relative changes in subseries, are useful. Forecasts may be based on the current phase (by comparison with previous cycles) and on the timing of cycles in related series—leading, lagging, or coin-

APPENDIX—Continued

Analysis of Monthly Economic Time Series into Time-Domain Components for Periodic, Systematic, and Event-Conditioned Effects: Summary—Continued

Y_{ij} (*or* Y_t) designate the historic data for years $i=1, \ldots, n$; months of each year $j=1, \ldots, 12$; $t=12\ (i-1)\ +j$ months from start (the first and last years are assumed to have all 12 months' data). Forecasts are required for year $n+1$

Component and its characteristics	Causes	Mathematical models: Time domain (component analysis)	Mathematical models: Frequency domain and autoregression	Typical problems of estimating in historic and current analysis and forecasting
THE MAJOR COMPONENTS—Continued				
				cident, with respect to the cyclical high and lows of Y or on econometric models. Changed conditions may reduce the utility of historic relationships.
***G* Multiannual Effects**	Events occurring once every 2 to 5 years (e.g., elections, wage contracts, and related strikes or price increases; Olympic games; expositions, festivals, or large congresses).	Supplementary factor G_{ij} may be used for months affected.	If the period is constant (e.g., 36 months), high spectral power will be shown and autoregression will require B^{36} operations.	Difficult to distinguish this component from Cb, especially if the effect differs significantly each repetition. Problems in constraining totals of S and G over the months of each year since their combined effects will cancel out *vis-a-vis* C only over the multiannual period. Forecasts may be improved by taking G into account.
***Co* Other Cycles** Agricultural, sales, etc. (the Pharoah-Joseph effect).	Good years, 1 or more, followed by 1 or more poor years, not explained by W.		Spectral analysis is useful only if the period is constant.	

PERTURBATIONS				
***U* Effects of Unusual Events** Short-term disturbances of Y, C.	Acts of God (earthquakes, typhoons, etc.) and of man (political, monetary, fiscal, military and other events, strikes, celebrations, etc.). The direct effects, anticipations of the event and its immediate after effects (including possible compensation for lost or extra activity).	Extreme values (outlying months **E** may be identified (e.g., when I (or SI) falls outside control limits of $\pm k\sigma(I)$, usually $k=2.5$). Less extreme month **e** may also be identified (e.g., outside 1.5σ limits) and given partial weights w [35]. Distinction of U as a separate component throughout unusual segments (rather than together with R as I) improves the analysis of C and the calculation of regular seasonality S over regular segments but requires additional stages to X–11. Over regular segments, $U=1$ in the multiplicative model $Y=CSPUR$ or $=0$ in additive models).	Spurious periodicities may be shown in spectra of S and C unless U is distinguished. In ARIMA models, these effects may be treated as random shocks a_t or, better, as interventions [12].	Some unusual months may not be identified as extreme (since their SI fall inside control limits). It is difficult to determine the unusual months and segments in an invariant manner. The identification of extreme months from SI ratios calculated as $Y \div C$ and their weights w, will change (to some extent) according to the specific years analysed because of different estimates of C and σ. Estimates of C that are affected over several months by extreme values may cause regular months to have apparently extreme SI values. Extremes may differ for subseries, causing difficulties in distinguishing in total series. Professional judgment may be essential, based on sudden and unusual changes in A (and in $A^{(h)}$ for subseries) that have assignable causes (as in statistical quality control). U is estimated in X–11 together with R as I if isolated months are affected. Longer unusual segments will generally affect estimates of C, too (despite $C^{(H)}$ being based on $A^{(m)}$), thus, affecting estimates of $SI=Y/C$ and of S (when calculated as moving average of SI and imputed SI^{w}). Very rapid surges or drops in the level of a series around an unusual event are possible plus nonseasonal transfers of activity (e.g., partial compensation for lost or excessive values). Forecasts are possible if the effects of similar historic events have been analysed.

APPENDIX—Continued

Analysis of Monthly Economic Time Series into Time-Domain Components for Periodic, Systematic, and Event-Conditioned Effects: Summary—Continued

Y_{ij} (*or* Y_t) designate the historic data for years $i=1, \ldots, n$; months of each year $j=1, \ldots, 12$; $t=12\ (i-1)\ +j$ months from start (the first and last years are assumed to have all 12 months' data). Forecasts are required for year $n+1$

Components and its characteristics	Causes	Mathematical models		Typical problems of estimating in historic and current analysis and forecasting
		Time domain (component analysis)	Frequency domain and autoregression	
THE MAJOR COMPONENTS—Continued				
IRREGULARITIES				
***I* Irregular Factors** Fluctuations in Y and A not explained by C, S, P, and W.	Irregularities in the activity (or stock) represented by the original series Y or in its measurement (e.g., sampling and response errors, monthly approximations based on 4 or 5 complete weeks' reporting, imputations). Deficiencies in seasonal and related adjustment Y/SP and in estimates for C (e.g., residual S, D, F, W, U).	In multiplicative model estimated as $Y/CSP = A/C$ but may then comprise U. Ideally, normally distributed stochastic variable, independent of other components and with no serial correlation: $E(I)=1$.	Should show approximately uniform spectral power over all frequencies (including the seasonal frequencies), i.e., white noise.	I may not maintain a simple multiplicative relationship to C and S, especially because of U: Then, average of $I \neq 1$. May show positive serial correlation over unusual periods, negative serial correlations over regular periods (because of serial correlations in Y or those introduced into A by seasonal analysis procedures). Irregularities in subseries may be complementary (e.g., because of substitution), independent, or positively correlated (because of common causes).
***R* Residual Irregular Factors** I, after distinguishing U.	Irregularities which can not be explained and adjusted as distinct components—including deficiencies in the model and the estimates of the components	Multiplicative model $Y= CSPUR$ defines R in regular segments as Y/CSP and as I/U in unusual segments only if U is quantified. R may be normally distributed, but	White noise, possibly with negative correlations over spans of 1 month.	R is not defined over unusual segments unless separate estimates are made for U. Forecasts are possible of confidence intervals based on $\sigma_j(R)$ if the variance is approximately constant over the years.

	S, P, W, U, and Z.	negative serial correlations are possible and different variances $\sigma_j(R)$ for specific months (e.g., because of W).		
EFFECTS OF RELATED VARIABLES				
Z Related Variable' Effect	Variations in one or more related series z (e.g., commodity prices affecting quantities marked, or vice versa) may assist explanation of seasonal, trend-cycle and irregular changes in Y.	Factors $Z(z_t)$ or $Z(z_{t-a}, \ldots, z_{t+b})$, $Z(z^{(1)}, z^{(2)}, \ldots)$ may improve the analysis in a similar manner to W.	Cross-spectral analysis may indicate relationships.	Regression of Y (or major components) on z (or its trend, seasonality, etc.). Econometric analysis (including lead or lag relationships) may improve analysis and forecasting.
		APPROXIMATIONS TO THE TREND CYCLE		
A Seasonally Adjusted Data s.a.d.) A nonperiodic series showing systematic changes but subject to irregularities.	Changes in C as perturbed by I (or U and R). Also comprises effects of errors in estimating S, P (and W, Z, if used).	Multiplicative, $A = Y/S.P = C.I = C.U.R$ (for current year, $Y/S'P'$); Additive, $A = Y - S - P = C + I = C + U + R$. W, Z may also be taken into account. If subseries add to total, indirect adjustment is possible $A^H = \Sigma_i^H A^{(h)}$ and usually preferable.	Should show high spectral power at C periodicities (over 12 months) and approximately uniform power at other frequencies (as I), without peaks or troughs at seasonal frequencies.	The utility of the s.a.d. depends on the validity of the estimates of S and P over the n years analysed and of their forecasts for the current year analysed. Estimates and forecasts may vary according to the specific years analysed. A^H based on subseries will give a somewhat different s.a.d. than the direct analysis and adjustment A of the total series, possibly less perturbed if some subseries are complementary (or show substitution of activity). Irregularities cause difficulties in studying the prevailing trend-cycle C and in detecting significant changes in C or S. Annual totals A_t will usually differ from original totals Y_t.

APPENDIX—Continued

Analysis of Monthly Economic Time Series into Time-Domain Components for Periodic, Systematic, and Event-Conditioned Effects: Summary—Continued

Y_{ij} (*or* Y_t) designate the historic data for years $i=1, \ldots, n$; months of each year $j=1, \ldots, 12$; $t=12(i-1)+j$ months from start (the first and last years are assumed to have all 12 months' data). Forecasts are required for year $n+1$

Components and its characteristics	Causes	Mathematical models: Time domain (component analysis)	Mathematical models: Frequency domain and autoregression	Typical problems of estimating in historic and current analysis and forecasting
APPROXIMATIONS TO THE TREND CYCLE—Continued				
				Forecasts of A^f based on C^f are used with S^f and P^f to forecast Y^f.
A^M **Modified Seasonally Adjusted Data** A modified for extreme months.	Changes in C and R apart from U (if estimable).	In X–11, modified original data Y^m are estimated on the basis of the regular C and S estimated for nonextreme months, then A^m is calculated as Y^m/S, i.e. as A/U if U had been estimated separately.		The X–11 estimate of C may be inaccurate if there are several consecutive unusual months or if trends differ considerably before and after an extreme month or months.
$C^{(M)}$ **M.C.D. Short-Term Trend**	Moving average of A over m months (months for cyclical dominance, $1 \leqq m \leqq 6$, (see [35]). May be ascribed to latest month t or centered to $t-\frac{1}{2}(m-1)$.	$C_t^{(M)}=[\mathbf{m}]A$ $=(A_t+\ldots A_{t-m+1})/m.$ $C_t^{(M)}-C_{t-1}^{(M)}$ $=(A_t-A_{t-m})/m,$ $=(A_t+\ldots A_{t-m+1})/m.$ i.e., the difference of consecutive values of $C^{(M)}$ relates to the change in A between month $t-m$ and t, irrespective of intermediate changes, causing high fluctuations.	High correlations between $C_t^{(M)}, \ldots, C_{t-m+1}^{(M)}$ because of common $A_{t-1}, \ldots, A_{t-m+1}$.	Lags by $m/2$ months after latest datum. May lag (or lead) a turning point by 1 or 2 months. A poor measure of historic or current change over (or near) an unusual segment or when I is high, fluctuating according to I_t-I_{t-m}. M.C.D. is determined from the average behavior of the series, including unusual segments, therefore, $C^{(M)}$ is not always useful for historic or current analysis.

$C^{(H)}$ Henderson Trend Cycle to represent the regular trend-cycle C with minimal irregularity.	Henderson-weighted moving average (of span 13, 9, or 23 terms) smooths A, after imputing A^m for extreme months.	$C^{(H)} = [\mathbf{H}]\ A^m$. Selection of span of moving average may depend on relative irregularity measured in X-11 analysis [35].	Shows spectrum as for C, but modified for extreme months.	Estimates of A and A^m will usually depend on specific years n analysed. Long moving averages may span segments having different trend-cycle characteristics C, especially if there are unusual segments or discontinuities that reduce the accuracy of estimates of C for specific months. Estimates for the first and last 6 months for 13-term [**H**] (or 4 or 11 months for 9- or 23-term [**H**] are unreliable. Not useful for current analysis.
***B* Flow** Bimonthly moving average of A.	Smooths A, especially any negative serial correlations in I (because of R and/or U).	$B_{t-\frac{1}{2}} = \frac{1}{2}(A_t + A_{t-1})$. If A_E is an isolated extreme month, modified flow $B'_{E\pm\frac{1}{2}} = \frac{1}{2}(A_{E+1} + A_{E-1})$ may be used [1].	Shows a smoother spectrum than A but with a high serial correlation for the span of 1 month.	Smoother than A and more sensiive than $C^{(M)}$ for current analysis, but lags A by one-half month.
***L* Stream** Trimonthly weighted moving average of A.	Smooths irregularities better and provides an approximation to the trend cycle for every month.	L_{t-1} $= \frac{1}{4}(A_t + 2A_{t-1} + A_{t-2})$ $= \frac{1}{2}B_{t-\frac{1}{2}} + B_{t-1\frac{1}{2}}$ $= [2]^2 A$.		Smooths A in both regular and unusual segments for historic and current analysis. Lags behind latest data by 1 month. Suffers from inaccuracies in A due to deficiences in estimates of S, P, especially if they relate to consecutive months, and may not always show smooth C monotonic over regular segments. At tails of regular segments, asymmetric weighting needed, e.g., [**5, 1,–1**].

APPENDIX—Continued

Analysis of Monthly Economic Time Series into Time-Domain Components for Periodic, Systematic, and Event-Conditioned Effects: Summary—Continued

Y_{ij} (*or* Y_t) designate the historic data for years $i=1, \ldots, n$; months of each year $j=1, \ldots, 12$; $t=12(i-1)+j$ month from (the first and last years are assumed to have all 12 months' data). Forecasts are required for year $n+1$

Change	Equation	Notes[1]
MEASURES OF CHANGES IN THE COMPONENTS		
y Month-to-month percent change in Y $y_t=d_1Y_t=100\ (Y_t/Y_{t-1}-1)$.	$y=a(1+s/100)+s$; changes in F, D may also be taken into account.	Comprises relative changes in S, I, C, possibly in F, D.
s Month-to-month percent change in S $S_t=d_1S_t=100(S_t/S_{t-1}-1)$.	$s=100(y-a)/(100+a)$.	
a Month-to-month percent change in A $a_t=d_1A_t=100(C_tI_t/C_{t-1}I_{t-1}-1)$.	$a=100(y-s)/(100+s)$.	Comprises relative changes in I and C, errors in estimating S_t, S_{t-1} (possibly in F, D). Study of a over successive months indicates changes in C or U.
b "Bi-s.a.d.", month-to-month percent change based on flow B [1].	$b_{t-1/2}=100(B_{t-1/2}-B_{t-2 1/2})/(B_{t-1/2}+B_{t-2 1/2})$ $=100(A_t+A_{t-1}-A_{t-2}-A_{t-3})/(A_t+A_{t-1}+A_{t-2}+A_{t-3})$.	Smooths negative correlations in A; estimates changes over spans of 4 months on a symmetric basis.
l Month-to-month percent change based on stream L	$l_t=1/2\ 100(L_t-L_{t-1})/L_{t-1}$ $=50(B_{t+1/2}-B_{t-1 1/2})/L_{t-1}$.	Numerator similar to b, but changes are calculated on $2L_{t-1}$ as denominator.
$d_{12}Y$ Percent change in Y compared with corresponding month last year.	$d_{12}Y=100(Y_{ij}/Y_{i-1,j}-1)$.	Comprises relative changes from last year in C and I and possibly in moving S or in F, D.

[1] See [1].

COMMENTS ON "THE ANALYSIS OF SINGLE AND RELATED TIME SERIES INTO COMPONENTS: PROPOSALS FOR IMPROVING X–11" BY RAPHAEL RAYMOND V. BAR ON

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INTRODUCTION

In his paper, BarOn emphasizes two important facts of life in seasonal adjustment. The first is that adjustment is the result of analysis and must not ever be viewed too mechanically, although the availability of high quality mechanical solutions is very necessary. The second is the more specific case of being prepared to identify and account for discontinuities in either the trend cycle or seasonal pattern. Unfortunately, beyond these general propositions, which are certainly worthy of emphasis, the paper actually offers very little to further the adequacy of adjustment.

There is almost no theoretical discussion of the proposed changes contained in the paper. Most of it is simply definitional. Such discussion, as does exist, is rather inadequate. For example, no evidence or argument is given for the proposition that there is seasonal or cyclical discontinuity at business-cycle turns. Nor is any evidence cited on negative serial correlation in X–11 adjusted series, a proposition that is contrary to my own experience.

BarOn supplies a list of proposed changes to the X–11. Unfortunately, many of them are not substantive suggestions and only relate to the computer program, per se. Following are my comments on those with substantive content. The numbers are BarOn's own references.

DISCUSSION

Item 25

Use of an "appropriate" method to provide festival date adjustments is recommended. No discussion of what is an appropriate method is supplied.

Item 29

I assume the proposal here is to compute seasonally adjusted data for some pairs of months jointly, but there is no real discussion of how this is to be done. Even the symbolic representation is doubtful since $Y_t+Y_{t+1}=A_tS_t+A_{t+1}S_{t+1}$ cannot be reduced to that supplied by BarOn.

Item 31

Most of the flags suggested here are cosmetic, which I would find more confusing than useful. One suggestion is symptomatic of a large number of other types of suggestions. He proposes to flag observations by symbols to tell the degree of monotonicity. His set of flags is asymmetrical, and the intervals selected are completely arbitrary rather than being related to the overall variability of the series. He supplies no clue as to what these flags can tell us that the ADR's and existing percent-change tables cannot.

Items 33, 34, and 35

BarOn's use of the short-term stream variable is altogether baffling. Some effort went into the X–11 selection of the Henderson weights, but what the stream supplies other than the obvious "more short term change" is a mystery. His whole view seems to be that economic activity takes place in a series of very short discontinuous happenings, a view that has not generally been espoused and one for which he makes no particular argument. He also seems to miss the implications of sampling variability that will produce unexplained irregulars.

Item 36

The proposed labeling of periods is replete with such nonoperational concepts as "major changes in rate of growth."

Item 38

Several trend estimation procedures are catalogued with no indication of why these should be any better than the Henderson (estimates that will follow a third-degree polynomial). Some of these proposals are very function specific and make much more stringent assumptions than does the X–11; yet, there is no argument for their superiority. Similarly, there are neither data nor closely reasoned arguments for how the X–11's own extreme identification and prior adjustment factors are not adequate for the job of eliminating the effects of extreme cases. The X–11 has a wide battery of choices open to the user that allow for special series problems, while keeping the selection and treatment of extremes under the control of objective, uniform criteria. The anecdotal citations of presumed misclassifications of extremes by X–11 are not clear to me.

Item 39

This item touches on two areas of great concern to practicioners of seasonal adjustment—the selection of the proper length moving average of the SI ratios and the estimation of year-ahead factors. The author, however, only recites the issues without any benefit of either logical or empirical analysis.

Items 51–55

Here, some trivial table formatting points are made for the simultaneous presentation of component series. The fundamental methodological difficulties in adjusting the summary series versus summarizing the adjusted series are not even addressed.

The paper concludes with the anecdotal presentation of tourism data for Israel and five U.S. series. However, there is no evaluation of the effects of BarOn's changes to the X–11 on the adjustment of these series. In fact, there are not even any results that the reader could compare for himself. Even the author's identification of unusual periods remains a mystery. If, as he says, the X–11 weights for extremes were used to identify the unusual periods, then what is the advantage of an extra identification of the same events externally to the X–11?

In summary, this paper is more a prospectus for research rather than one that produces results. It is not even a very complete prospectus, since there is no theoretical argument made for the likely benefits of the work. The bulk of the paper consists of trivial programing changes to the X–11 computer program.

COMMENTS ON "THE ANALYSIS OF SINGLE AND RELATED TIME SERIES INTO COMPONENTS: PROPOSALS FOR IMPROVING X-11" BY RAPHAEL RAYMOND V. BAR ON

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Abstract

BarOn has illustrated one practical approach to data analysis that is guided by traditional decomposition ideas and illuminated by detailed knowledge of the background of the data. I raise the possibility that explicit statistical models, such as the multiplicative seasonal models of Box and Jenkins, might provide better guidance and might discipline the tendency to dwell too much on the details of the data and, thus, miss underlying statistical regularities. Statistical theory, properly applied, suggests how one can serve most practical requirements of forecasting, scientific understanding, and reporting, without resort to seasonal adjustments and numerous ad hoc judgments. A simple alternative decomposition can be based on the ARIMA approach; this yields identifiable components that I have named "fitted nonstationary," "fitted stationary," and "residual." This decomposition may serve some of the aims of the traditional decomposition and avoid some of the disadvantages.

INTRODUCTION

Ever since reading a 1973 draft of the Cleveland and Tiao discussion [1], I have had reservations about the idea of a single approach, such as X-11, to seasonal adjustment of diverse time series. These reservations gradually extended to the idea of seasonal adjustment itself. Hence, I approached BarOn's paper with concern about my ability to evaluate his contributions to a tradition that I found hard to follow. My concern turned out to have been unfounded. My opinions in no way kept me from admiring the professional quality of BarOn's approach and his skills of data analysis nor from getting a better understanding of why seasonal decomposition is of such great interest to economic analysts. I have come to see the practical usefulness of adjustment procedures and to appreciate the robustness of X-11. BarOn's paper has led to a substantial modification in my own initial position and to some proposals of my own for improving BarOn's proposals.

His proposals are not really ones for enhancing X-11 in some fundamental, technical way or replacing it with something basically different, as are the proposals set forth in other papers presented at the conference. Rather, BarOn's paper represents a working philosophy for the application of data analysis to time series. The idea of decomposition and the application of X-11 provide the framework for a microscopic examination of the data, an examination aimed equally at better understanding of the past and an enhanced ability to predict what lies ahead. His rationale is expressed in this key sentence: "While the search for the underlying (infinite) process may be appropriate for many physical and industrial series, we find that many economic time series have a more complex structure that we term 'event conditioned'." Starting from X-11 and using outputs therefrom, BarOn draws on his knowledge of the historical context of a time series in order to divide the overall period of the series into subperiods of distinguishably different behavior, the event-conditioned subperiods. He provides a number of computer displays to aid this process and gives many detailed suggestions on procedures, down to suggestions for formatting of tables, as his steps unfold. Judgmental intervention by the analyst is essential. BarOn also extends his approach to suggested procedures for studying the relationship between an aggregate series and its constituent subseries.

In the paper presented at the conference, BarOn has substantially condensed the descriptions of his methods. As a result of this condensation and of notation that is

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sometimes not self-explanatory nor adequately explained, I sometimes had trouble in following details of his development. But, from his main example, "Analysis of Tourist Arrivals, by Air: Israel, 1956–75," one can obtain an accurate understanding of his approach. The most interesting product is the classification—largely ex post facto—of the 240 months from January 1956 through December 1975 into 25 subperiods of varying length, named "regular periods," "unusual periods," and "short monotonic periods." An overview of the output from his analysis is shown in table 1f and especially in the fascinating figure B, supported by the tabular chronology of table 2.

In order, thus, to divide a series into event-conditioned subperiods, the statistician must get to know his data and their background very well. He cannot be content with casual inspection of a few stereotyped measures, such as F ratios or multiple correlation coefficients, as is common practice in much applied statistical work. I can only admire BarOn's detailed grasp of his material, especially his understanding of the special features of the constituent series of tourist arrivals from individual countries and his ability to relate real-world events to what is happening in his time series. At the same time, however, I fear that his knowledge of the individual trees sometimes gets in the way of an adequate perception of the forest. His implicit guiding model—the decomposition idea—does not serve to discipline his imagination, and, at times, I feel that BarOn, much like the stock market chartist, perceives apparently meaningful patterns in chance configurations. He does not attempt to formulate and fit a parsimonious quantitative model in order to capture the essence of the phenomenon under study.

In the example of air tourist arrivals in Israel during a 20-year period punctuated by three wars and numerous strong political shocks, it is natural to think that unusual events cannot be captured by a parsimonious statistical model. Yet, the experience of many statisticians, shared by me, is that the methods of parametric time series analysis associated with the names of Box and Jenkins give a good account of themselves over a broad range of applications. The simple idiot models often work. They seem to work as well in business and economic applications as in applications from the natural sciences, where BarOn is more inclined to feel the search for the underlying (infinite) process to be appropriate. The burgeoning published and unpublished literature in business and economics suggests that the complex structure of many economic time series may not be a barrier to parsimonious statistical explanation. My own experience and feelings lead me to this paraphrase of Sir Francis Galton's famous statement about the normal distribution: "ARIMA models reign with serenity and in complete self-effacement amidst the wildest confusion. Whenever a time series of chaotic elements is taken in hand and marshalled by the Box-Jenkins approach, an unsuspected and most beautiful form of regularity proves to have been latent all along." That is laying it on a little too thick, of course. There are refractory time series, or segments of series, for which one has substantial reservations about the adequacy of fit of any simple model. If one looks closely enough, there are minor blemishes in the diagnostic checks on the seemingly most successful fits (examples will be mentioned). But, it is certainly a natural first thought to look at the tourist series from the vantage point of Box and Jenkins. The following reconnaissance is designed to see whether the idea of model fitting has any applicability to a series that has been divided into 25 event-conditioned subperiods, not to attain a finely-tuned model. One byproduct of BarOn's paper will be its challenge to statisticians to explore this fascinating series further than BarOn—with his approach—or as I—with Box and Jenkins quickly—have done.

RECONNAISSANCE WITH BOX-JENKINS METHODOLOGY

A quick interactive computer session suggested that the tourist series could be advantageously transformed to logarithms (natural logarithms for convenience of interpretation) and doubly differenced, once consecutively and once at the seasonal lag of 12 months. Although sometimes subject to critical scrutiny, these are all common initial steps in the analysis of economic time series. Thus, if D is used to denote the differencing operator, the random variable of interest is $\tilde{w}_t = D^{12}D \log \tilde{y}_t$, where y_t represents the (unadjusted) number of arrivals in period t. A convenient property of this transformation is the following: If a point forecast of $\tilde{w}_t$ is denoted by $\hat{w}_t$, then $100(w_t - \hat{w}_t)$ is approximately the percentage error of the implied point forecast $\hat{y}_t = \exp(\hat{w}_t)$ of the original series, so long as $100(w_t - \hat{w}_t)$ is less than, e.g., 10 or 20 percent, depending on your tolerance of approximation error.

By the usual graphical and numerical diagnostic checks, the transformed data appeared stationary with the qualification of a large negative outlier in October 1973, a correspondingly large positive outlier in October 1974, and some minor reservations about nonconstant scatter. The Yom Kippur War is an obvious assignable cause of both outliers, since the war occurred in October 1973, and its first anniversary was in October 1974. At this stage, it appeared to be the only assignable cause.

Since I was working with a small computing system that permitted the nonlinear least-squares fitting (with back forecasting) of an ARIMA model with no more than 100 observations, I started with the first 100 w_t's: February 1957—June 1966. (The double differencing of the series removes the first 13 observations from explicit appearance.) Large negative autocorrelations at lags 1 and 12 suggested a multiplicative seasonal model with first-order moving-average terms at lag 1 and at the seasonal lag of 12. The fitted model was

$$D^{12}D \log y_t = -0.0011 + (1-0.67B)(1-0.81B^{12})\epsilon_t$$

where B is the backshift operator, and ϵ_t is the disturbance at time period t, with a standard deviation of residuals of

0.167. The latter can be interpreted roughly to signify a standard error of about 17 percent in a one-period-ahead forecast of tourist arrivals. (This number is, of course, larger than the corresponding number for a seasonally adjusted version of the series, because, as explained by Cleveland and Tiao [1], a large component of unexplained variation is removed by the seasonal adjustment.) The diagnostic checks, including checks for normality, were satisfactory. Note that no war occurred during this period but that the effects of the Sinai Campaign of October 1956 were reflected within the early values of w_t.

Next in the reconnaissance, I fitted the same ARIMA model for the next 100 w_t's, July 1966—September 1973. The fitted model was

$$D^{12}D \log y_t = -0.0008 + (1-0.51B)(1-0.86B^{12})\epsilon_t$$

with a standard deviation of residuals of 0.165. Although the diagnostic checks were not quite as good as for the first 100 w_t's, they were still reasonably satisfactory. The largest negative residual came in June 1967, the month of the Six Day War, but the overall conformity of the residuals to normality was excellent. Note that my purpose here is not to demonstrate that there was no structural change at all between the two periods thus far fitted, but that the structure appears to have been rather similar, in contrast to the frequent shifts of structure implied by the concept of event-conditioned subperiods.

The final stage of my reconnaissance included the final 100 w_t's, which overlapped with 73 of the values just reported and brought the Yom Kippur War of October 1973 into the analysis. Here, the simple ARIMA model was indeed in trouble, but the trouble was localized to the single month of October 1973 for which the residual was about minus five in units of the standard deviation of all residuals for the fit. The fitted model was

$$D^{12}D \log y_t = -0.0021 + (1-0.54B)(1-0.89B^{12})\epsilon_t$$

with a standard deviation of residuals of 0.202, which is larger than the two previous values because of the residual for October 1973. Aside from the one outlying residual, the diagnostic checks held up well. The residual for October 1974, the month for which w_t was an enormous positive outlier, was within two standard deviation units of zero.

Hence, in 20 turbulent years, there was 1 month for which the parsimonious multiplicative ARIMA model did not give a reasonably good account. For that month, some special treatment—interpolation, Winsorization, segmentation of the series, or (as Box suggested orally at the conference) intervention analysis—is in order. But, it took a war of the scope of Yom Kippur to embarrass the model in any substantial way. Recall that relative to Israel's population, the Yom Kippur War was more costly than was the Vietnam War to the United States, and that mobilization, confrontation, and substantial shelling lasted long after October of 1973.

My purpose is not to deny the value of BarOn's close examination of the data but to provide perspective that is easily lost when close examination is not guided and disciplined by an explicit statistical model. Clearly, we want to cope with shocks like the Yom Kippur War when we analyze data, but we should not see every minor perturbation as a signal for ad hoc treatment. Furthermore, the promise of the multiplicative ARIMA-seasonal model signals a message about fruitful approaches to the study of seasonality.

I would like to suggest that BarOn's skills in data analysis could be used to greater advantage if guided by the Box-Jenkins framework. If, for example, recent residuals from a tentative Box-Jenkins fit to a time series, showed some sign that the model structure was undergoing a shift from what had previously seemed satisfactory, I would like to have the benefit of his judgment. Furthermore, I think that his judgment would be aided by a careful examination of residuals and also fitted values from ARIMA models.

I would say more. The Box-Jenkins methodology can be invoked to accomplish the aims of the decomposition approach that BarOn finds helpful, either within the framework of traditional ideas of decomposition (as illustrated by a number of contributions to this conference) or by suggesting an alternative approach to decomposition, which I shall suggest later in this discussion. This alternative is unique, given the identification and fitting of an ARIMA model to the data, and it gives an interesting and novel perspective on what is happening in the data.

In order to develop my proposals, I shall first examine the contributions of the traditional approach to decomposition, represented by X–11, to three major goals: Forecasting, scientific understanding, and reporting. For each of these goals, I shall examine different methods of achieving the benefits usually sought from decomposition. Finally, I shall outline the alternative approach to decomposition.

FORECASTING

Does the decomposition of a series into components, such as S and L in BarOn's notation, aid in forecasting tourist arrivals? To the extent that the fitted ARIMA model corresponds closely to reality, the statistical analysis of the data, based on Box-Jenkins methods, is based on the likelihood function. Whether one's orientation to statistics is the Bayesian or sampling theory, this fact provides the basis for strong claims of optimality for parameter inferences and for the forecasts derived therefrom. Some time series applications, especially in situations of near nonstationarity, may cause us to reexamine these claims, as is suggested by Gonedes and Roberts [2]. But, even when confronted by apparently pathological behavior of estimation methods based on the likelihood function, we are better advised to deal with the problem in terms of study of the model and the likelihood function, including the possibility of misspecification of the model, than to abandon statistical orthodoxy founded on specification and analysis of an explicit model. Orthodoxy promises the possibility of computing forecasts of any quantity that may be of interest. For example, the Bayesian predictive distribution is the basis for predictions of next month's tourist arrivals, next year's arrivals, percentage changes in arrivals from year to year for the next 5 years, and so on. (Some of these possibilities may pose computational problems, especially in obtaining confidence limits, but the apparatus is there to be used.) If some one says that he is interested in "trend" and is willing to define what that means, predictions can be derived for trend. All this follows from analysis of a formal statistical model.

I find it hard to see how the traditional decomposition approach has anything to add. It may (and probably will) turn out that our favored model is one for which X–11 will do reasonably well what is expected of it, as is explained by Cleveland and Tiao [1]. Even so, decomposition, at best, can only introduce an extra step that later will have to be undone. Recall that the nonseasonally adjusted values of a series reflect directly what is happening in the real world. There were 21,844 tourist arrivals in January 1975, as opposed to 39,609 seasonally adjusted arrivals. The hotels of Israel had 21,844, not 39,609, potential customers. Thus, if a forecasting scheme is based on seasonally adjusted values, these forecasts would have to be converted back to the implied forecasts for nonadjusted values—hence, the unnecessary step that has to be undone at the end.

It may be objected that seasonally adjusted data have sometimes acquired a kind of official status for policy decisions, as in estimates of the percentage of unemployment in the labor force, or that economists demand estimates of the cycle trend or seasonal components to guide their thinking. Even here, however, it is not clear that a statistician would want to work with the traditional decomposition methods in making a forecast. It might be more satisfactory to make forecasts of unadjusted data and then, for example, apply the seasonal adjustment procedure to convert to forecasts of seasonally adjusted numbers.

Hence, it appears to me that seasonal adjustments can be only a source of trouble to a statistician interested in forecasting unadjusted values. Unfortunately, the seasonally adjusted version of a time series is often much more readily available than is the unadjusted version. Furthermore, the seasonally adjusted data have the additional disadvantage of being inherently more subject to revision because of the revision of weights as later data come in, and they reflect certain ad hoc judgments by the adjusters, since computer routines, such as X–11, demand some discretion in the treatment of apparent outlying observations and in setting certain parameters.

Let me add an important qualification to my position against the use of adjusted data for forecasting. Practical experience may have surprises for those who, like me, are guided by statistical doctrine in deciding what is likely to work in practice. For example, there is the puzzling finding of Newbold and Granger [4] that composite forecasting methods can sometimes outperform, on predictive tests, methods based on the analysis of a single, apparently adequate, model. Decomposition may have merits that now appear unlikely to me. To illuminate the question, actual predictive tests are needed. Preliminary work reported by Plosser [5] is suggestive of advantages for the use of unadjusted data.

SCIENTIFIC UNDERSTANDING

Seasonal decomposition seemingly helps us to understand what is really going on after removal of irrelevant seasonal sources of variation. In studying the relationship between time series, it seemingly avoids spurious correlation, contributed by common seasonality—hence, the powerful intuitive appeal of seasonal adjustments to economists and other scientists working with time series. But, not all intuitions are persuaded. (See, for example, Laffer and Ranson [3].) I see no guiding principle of statistics to say whose intuition is better. But, surely the route to better scientific understanding is to incorporate the seasonality directly into multivariate models that are formulated in terms of unadjusted data so that the source, transmission, and effects of seasonal variations can be better understood. The work of Plosser [5], which owes its initial impetus to work of Zellner (Zellner and Palm [7; 8]), is pathbreaking in this direction. Other contributers to this conference are also working along these lines.

Scientific understanding is also served by better modeling of univariate time series. The apparent predictive success of univariate models can have direct implications for econometric inferences, as is shown by the papers just cited. It is interesting to note that the multiplicative seasonal model identified for the tourist arrivals in Israel is the same model as that identified by Cleveland and Tiao [1] for a series on international airline passengers from January 1949 through December 1960.

ARIMA models can be used to model the postulated seasonal and cycle-trend components of a time series. But, they may also be used to view the world directly, without any commitment to a decomposition concept. Witness, for example, the revolution in thinking about price behavior on organized markets when economists and students of finance observed that a very simple ARIMA model—random walk with drift—accounts surprisingly well for much of market behavior. In macroeconomics, the notions of "cycle trend" and "turning point," deeply embedded in current thought, can easily divert attention from a clearer view of what is going on, a view obtainable from more explicit stochastic models, such as the ARIMA ones. For example, in connection with an elementary statistics course where I have been using the data for a problem, I have recently noticed that the natural log of annual real GNP for the United States from 1890 to 1941 is well modeled by a random walk with mean change 0.031 and standard deviation 0.072. The shorter postwar period or 1948–1974 is equally well modeled by a random walk with drift; the mean and standard deviation of changes are 0.036 and 0.030. (For a theoretical discussion of the implications, see Ranson [6].)

Finally, the output of decomposition methods—illustrated by BarOn's figure B—may encourage a false confidence in ex post facto explanations of why things have happened as they have. I have the same feeling of uneasiness as I do when listening to stock market analysts explain today why the market yesterday had such a strong yield rally. Moreover, the theoretical constructs lurking behind BarOn's figure A appear to collide with the fact that ARIMA models are driven by random shocks. (This does not, of course, preclude the possibility that ex post facto one may form some idea of the identity of some of the shocks by examination of residuals nor that attempts to identify shocks may not be useful in the identification of promising leading indicators for a transfer function formulation.)

REPORTING

The strongest case for seasonal adjustments arises from ease of reporting. When seasonally adjusted unemployment rises from 7.5 percent of the labor force in June 1976 to 7.8 percent in July, these two numbers communicate the message that the economy may be recovering less well than had been expected. In my opinion, the communication function is the main reason for the widespread use of seasonal adjustments by statistical agencies of many governments. For this reason, I suspect that these adjustments will be with us for a long time, even if all statisticians and econometricians shared fully the reservations that I have been expressing. From this perspective, I am impressed by the contributions made by X–11 and interested in technical improvements that may be possible.

But even at the level of simple reporting, seasonal adjustments encounter problems, many of which were discussed by Shiskin in his keynote address to this conference. The necessary revisions of the first reports, for example, tend to confuse, although the Canadian approach of using ARIMA methods to obtain better weights may alleviate this problem.

As I have reflected further on the function of seasonally adjusted data in reporting, I have become troubled by more fundamental questions. If statisticians, trying to predict or to understand, are well advised to work with unadjusted data, as I have argued, why are government officials, members of Congress, business and labor leaders, and the general public better served by adjusted data? One answer is that it may be better to get over the correct general idea in a simple, if potentially misleading, way than to confuse laymen by a more accurate presentation that is harder to understand. Professors face this dilemma in their teaching and know that there is no easy resolution.

Furthermore, seasonally adjusted data can confuse as well as enlighten. If a layman should discover that seasonally adjusted and seasonally unadjusted unemployment have just moved in opposite directions or if he is told to ignore a change in the adjusted series because it is a consequence of an anomaly in the method of adjustment, we can hardly blame him if he feels confused or even suspicious.

We may learn something about problems of reporting from business practice where the numbers being reported—sales, profits, etc.—are very important to the audience. I have the impression that seasonally adjusted data are rarely used or emphasized by business firms in their internal work, in spite of the generations of students exposed to seasonal adjustment procedures in elementary statistics texts. Advocates of seasonal adjustment regard this as evidence of technical backwardness. I have been told that one of the original forces making for introduction of seasonal adjustments was the inadequacy of comparison of today's unadjusted number with the corresponding number 1 year ago. In my observation, however, businessmen usually make a slightly more sophisticated comparison that is based on three-number reporting. For example, current sales may be reported and compared with sales last month and with sales 1 year ago. As I shall argue in a moment, this kind of comparison is consistent with what we learn frequently from statistical analysis, namely that the $D^{12}D$ transformation is often a good transformation to attain stationarity, which is, in turn, an important stage to be reached in the process of identification of an ARIMA model. The expression of these comparisons in percentage terms, also common in business practice, is consistent with the frequent usefulness of the log transformation in statistical work.

Note further that seasonal adjustment is not ordinarily practiced in areas of natural science where seasonal influences are strong. Someone has observed that meteorologists don't report seasonally adjusted temperature or rainfall. They give unadjusted data, and they may put these data in perspective by comparison with "normal" or "expected at this time of year". My argument here is only slightly weakened by the fact that, unlike many economic time series, some meteorological series appear to be essentially stationary over long periods of time.

It is worth pursuing the idea of comparing what actually happened with what might reasonably have been expected to have happened. As an example, let me quote a news item from page 4 of *The Wall Street Journal* of August 5, 1976.

> New-car deliveries in July rose 9 percent from a year earlier, but the gain was less than some auto sales experts expected. Sales rose to about 865,000 units from about 793,000 a year before.
>
> . . . Domestic-make dealers delivered 736,780 cars, up 16 percent from 636,666 a year earlier.
>
> However, Detroit analysts had expected stronger U.S.-make sales last month. For example, one analyst who works for a Big Three auto maker had predicted domestic car sales almost 25,000 units higher than the number actually sold last month.

(In fairness, the article later points out that July's seasonally adjusted annual selling rate was 8.7 million units, as compared with 8.9 million in June. Note that the seasonally adjusted sales were down, while the unadjusted sales were up.)

SUGGESTED ALTERNATIVES FOR REPORTING OF SEASONAL TIME SERIES

I have suggested that seasonal adjustments permit simple two-number reporting in which the reader only needs to compare this month's and last month's seasonally adjusted numbers. Here is an alternative two-number report that does not entail seasonally adjusted numbers: Report this month's unadjusted number together with an estimate of what would have been expected in the light of recent and past behavior of the series of interest, as was illustrated by the report on automobile sales. To obtain the expected estimate in a reasonably reproducible way, a univariate ARIMA model could be fitted to, e.g., the most recent 100 observations from the series. (To avoid the appearance of getting into the forecasting business, the reporting agency could include the current observation in the fitting process.) The practicality of such an approach is suggested by the use of the ARIMA refinement of X–11 used in Canada; my suggestion could be adopted as a byproduct of that computation. Thus, in the automobile example, the verbal report might run along these lines: "July domestic sales were 736,780, which is 3.1 percent less than the 760,000 that would have been expected in the light of recent statistical tendencies." Although it would require an additional number to do so, it might be well to include a standard error of the expected number, also a byproduct of the ARIMA analysis, for the benefit of readers who wished to form some judgment about the significance of the departure of actual from expected.

An even simpler application of the same idea would be an extension of current business practice without the need for any statistical fitting of ARIMA models. This would entail the simple comparison of the percentage change from last month to this, with the corresponding change a year ago. Thus, if there was a 5-percent increase from last month to this, and a 2-percent increase in the same period last year, the key number would be 5–2=3 percent. I shall explain the rationale for this in the next section.

Graphical reporting is also important. As the basic graph, I would suggest the sequence plot of values fitted by application of a statistical model, such as a univariate ARIMA model. On this graph, the individual fitted values could be connected by solid line segments, and the actual values could be shown as isolated points. The basic scheme is illustrated in figures 1 and 2, the basis of which is developed in the next section. Optionally, the forecast function (possibly with confidence limits) could be shown as an extension of the fitted-value function. For general audiences, the units would be those of the actual series or of logs thereof, without differencing; this mode is illustrated in figure 1. For more technical reporting, the differenced series could be shown, as is illustrated in figure 2. Thus $\log y_t$ would be the basis of the general report, and w_t would be the basis of the technical report. If no differencing were necessary, the two reports would come to the same thing. Notice that this proposal is an extension of the idea of the process control chart in statistical quality control.

AN ALTERNATIVE DECOMPOSITION

One approach to the application of ARIMA models to seasonal decomposition, illustrated by several papers at this conference, is to model the seasonal and cycle trend separately, by ARIMA models. If, however, a unified ARIMA model can be identified for the time series, an alternative decomposition is immediately available. This decomposition may achieve some of the purposes of the traditional decomposition, and it is interesting intrinsically for its illumination of the ARIMA fitting process.

The Box-Jenkins approach stresses the value of differencing of the data to attain stationarity in many applications in which the original time series appears nonstationary any time series with a fixed seasonal component, for example, is nonstationary. If one models the fixed seasonal by a regression function with the usual dummy variables for seasons, then differencing at the seasonal gap will remove this source of nonstationarity. If the model includes a polynomial trend, then differencing reduces the degree of the polynomial by one. Thus, the nonstationarity attributable to a linear trend is removed. If the autoregressive part of the stochastic component of the model is a random walk, then differencing eliminates this source of nonstationarity, etc.

To see how this differencing step can be exploited, let us suppose, for concreteness, that the $D^{12}D$ transformation of logs is applicable. We can think of $D^{12}D \log \hat{y}_t$ as the result of a first-stage fitting process. If, as is often reasonable, we are willing to specify that the mean of this stationary random variable is 0, the fitting expression can be written as follows:

$$D^{12}D \log \hat{y}_t=0$$

whence

$$\log \hat{y}_t=\log y_{t-1}+D \log y_{t-12} \qquad (1)$$

In figure 1, this fitting process is exemplified for BarOn's series from February 1957 through May 1965. The fitted values are connected by line segments, and the actuals are shown as asterisks about the fitted values. The residuals from the fit are simply $w_t=D^{12}D \log y_t$, and these are also shown. My suggested reporting practice of comparing the most recent percentage change against the percentage change 1 year ago is then essentially a suggestion to report this particular kind of residual. What is surprising to many people is how well this first-stage fitting scheme works, or, alternatively, how much of the

Figure 1. **ACTUALS, FITTED NONSTATIONARY, D12DLOGY**

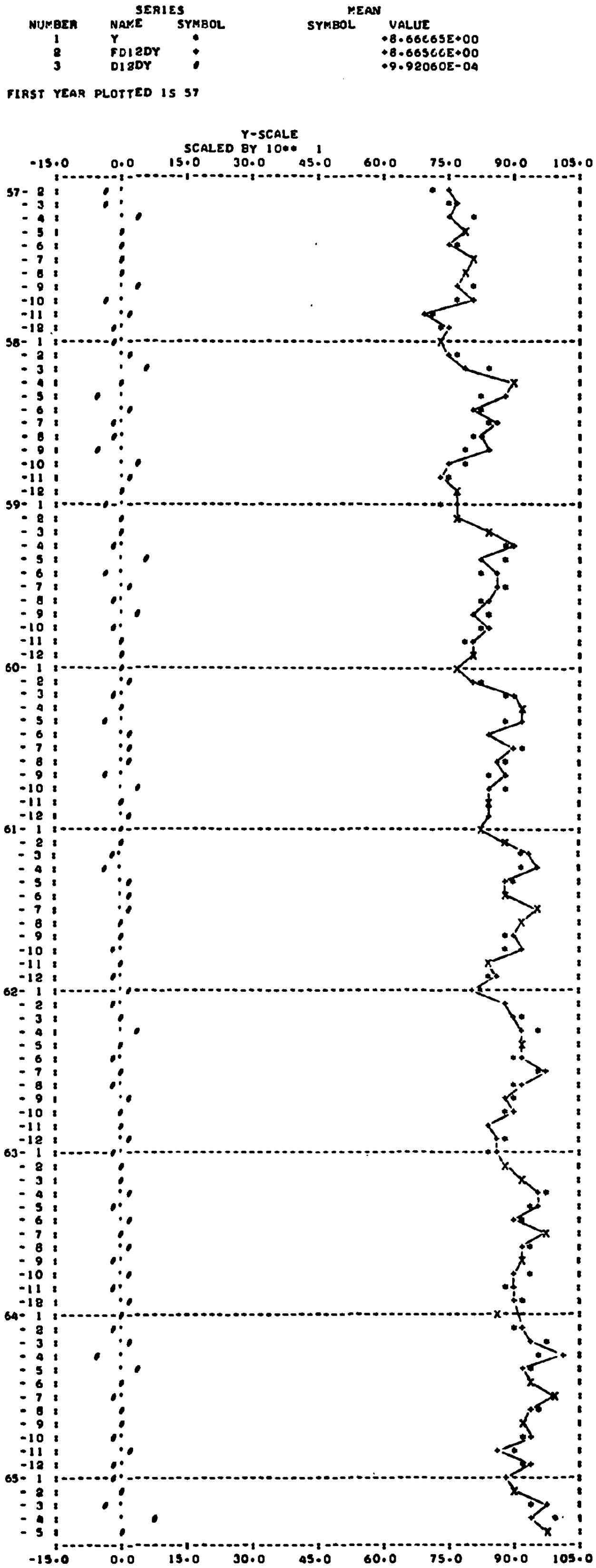


Figure 2. **D12DLOGY AND FITTED STATIONARY**

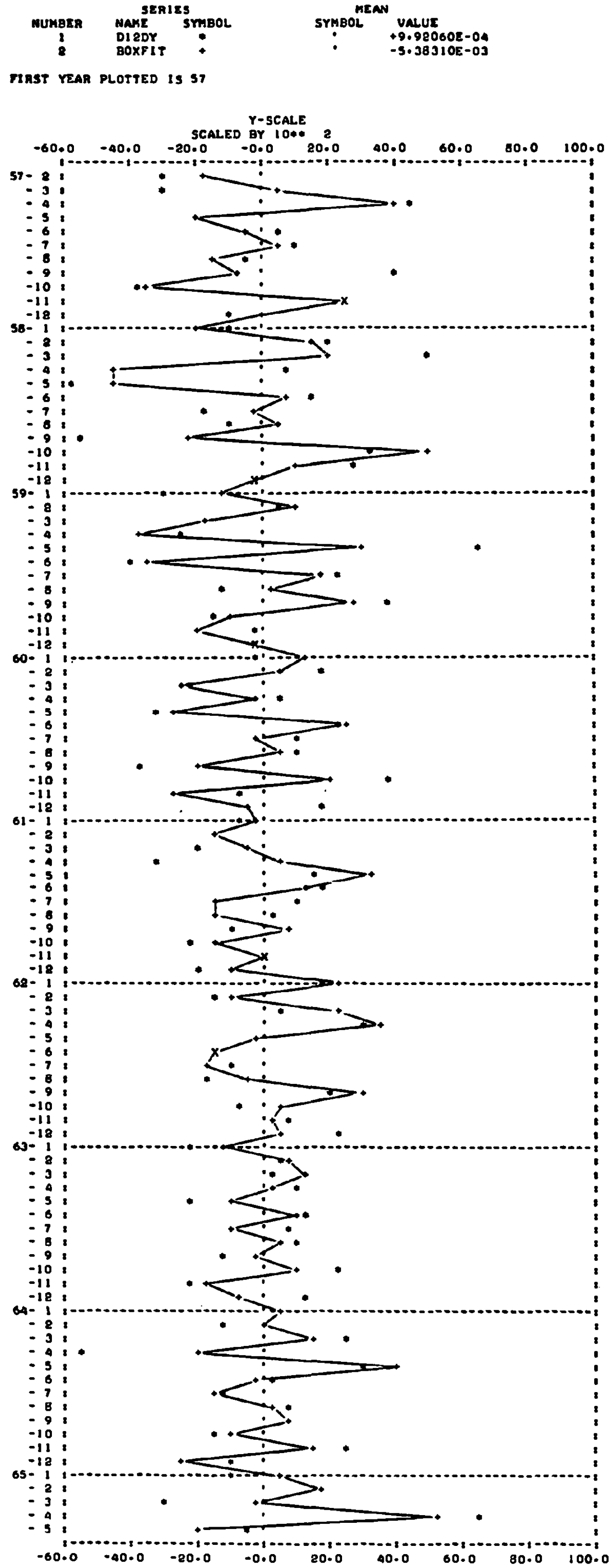
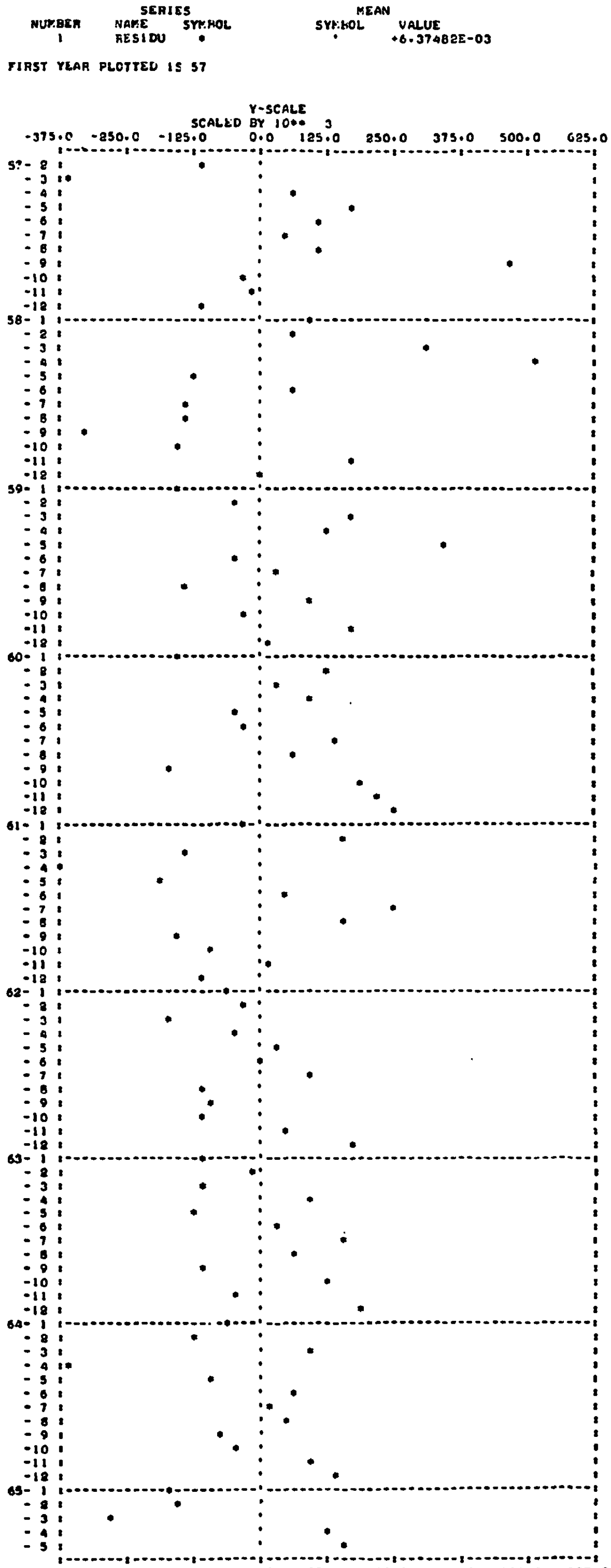


Figure 3. **RESIDUAL**



visual impression of cycle trend and seasonal of the sequence plot of the original series is accounted for by this stage of fitting. The tourist series is only an example, of course, but figure 1 presents a picture that appears to be typical of many important business and economic time series.

I propose that the fitted values defined by formula (1) (or its generalization) be considered the first component of what may be called an ARIMA decomposition, and I suggest that this component be called fitted nonstationary. In terms of traditional decomposition terminology, the fitted nonstationary would include, if present, fixed seasonal and quadratic trend.

The second component is a byproduct of the ARIMA analysis of $w_t = D^{12}D \log y_t$. This nonlinear regression procedure, like any regression procedure, defines fitted values and residuals. I propose that these fitted values be considered the second component of the ARIMA decomposition. A name parallel to that suggested in the previous paragraph would be fitted stationary. In traditional terminology, the fitted stationary would include, if present, the moving seasonal, although it is not clear to me how one would isolate this one subcomponent from other subcomponents of fitted stationary, even for the multiplicative seasonal ARIMA model. For BarOn's first 100 w_t's, the fitted stationary is shown in figure 2 as BOXFIT, while the values being fitted are designated by $D^{12}DY$, which is a contraction of $D^{12}D \log y_t$. (In this fitting, I have constrained the constant of the ARIMA model to be zero, since, otherwise, the fitted nonstationary would not reflect all deterministic components of the original series, and since the suppression of the constant appears justified by the data. The fit, moreover, is virtually identical to the fit reported from my initial reconnaissance.)

Note that the BOXFITS in figure 2 have a seemingly random appearance, corresponding to what appear to be random shifts of the current tendency of $D^{12}D \log \bar{y}_t$. Visual and numerical diagnostic checks confirm that the behavior is nonrandom; for example, there are 59 runs above and below the mean, compared with the random expectation of 50.92. I think that the fitted stationary component may be close to what BarOn has in mind when he divides up a series into event-conditioned subperiods. I would be most interested in his detailed reaction to figure 2.

The final component is, of course, the residuals from the ARIMA fit, and the name "residual" seems adequate. The residuals are shown separately in figure 3 since to superimpose them on figure 2 would present a cluttered picture. These are the things that BarOn (or any statistician) should look at in deciding whether the model is adequate; note that there is some hint of greater variability in the earlier part of the period. The residual component is also useful, in continuing real-time surveillance, for deciding whether a model fitted at any earlier time still seems adequate in the light of the most recent data.

Thus, in summary, we have

$$ACTUAL = FITTED\ NONSTATIONARY + FITTED\ STATIONARY + RESIDUAL$$

Given the model and the method of estimation, the decomposition is unique. It is simply a byproduct of what many statisticians would actually do in a parametric analysis of a time series. All components are identifiable. The most important limitation that I see arises in those borderline cases in which the degree of differencing is not obvious. For the first 100 observations presented here, one could argue with some cogency for single differencing at the seasonal gap. If so, the appearance of the fitted stationary component would be somewhat different than that shown in figure 2.

Whether the decomposition is useful can only be determined by the experience of those who try it out in serious applications. It has the advantage of being tied directly to a statistical approach that is often useful in the analysis of seasonal time series, and, at the least, it appears to be a useful way to visualize what that approach is doing. It does not put seasonal in one component but allocates fixed seasonal to fitted nonstationary and moving seasonal to fitted stationary. Perhaps that has conceptual advantages if we re-examine the appeal of the notion of seasonality.

If the ARIMA model is of the multiplicative seasonal form, as was true of the model I used for BarOn's data, it is tempting to carry the fitted stationary into components corresponding with the long-term and short-term parts of the ARIMA model. This decomposition, however, is not unique, for the same reason that an ordinary regression analysis does not yield the same coefficient for a variable if it comes in at different steps of a stepwise regression. Some experimentation with the idea might, nonetheless, be helpful.

The suggested decomposition into fitted nonstationary, fitted stationary, and residual would work for any ARIMA model, including even the simple special case of a random series. Here, the fitted nonstationary is zero, since no differencing is necessary; the fitted stationary is a constant, the sample mean (estimated process mean); and the residual is the only variable component, representing deviations of the observations from the mean.

REFERENCES

1. Cleveland, W. P., and Tiao, G. C. "Decomposition of Seasonal Time Series: A Model for the Census X–11 Program." *Journal of the American Statistical Association* 71 (September 1976): 581–587.
2. Gonedes, Nicholas J., and Roberts, Harry V. "Statistical Analysis of Random Walks and Near Random Walks." *Journal of Econometrics* 6, 3 (November 1977): 289–308.
3. Laffer, Arthur B., and Ranson, R. David. "A Formal Model of the Economy." *Journal of Business* 44 (July 1971): 247–270.
4. Newbold, P., and Granger, C. W. J. "Experience with Forecasting Univariate Time Series and the Combination of Forecasts." *Journal of the Royal Statistical Society*, ser. A, pt. 2, 137 (1974): 131–165.
5. Plosser, Charles I. "A Time Series Analysis of Seasonality in Econometric Models." Ph.D. dissertation draft, Graduate School of Business, University of Chicago, 1974.
6. Ranson, R. D. "Money, Capital, and the Stochastic Nature of Business Fluctuations." Ph.D. dissertation, Graduate School of Business, University of Chicago, 1976.
7. Zellner, Arnold, and Palm, Franz. "Time Series Analysis and Simultaneous Equation Econometric Models." *Journal of Econometrics* 2 (May 1974): 17–54.
8. ——— and ———. "Time Series and Structural Analysis of Monetary Models of the U.S. Economy." *Sankÿa: The Indian Journal of Statistics*, ser. C, pt. 2, 37 (1975): 12–56.

RESPONSE TO COMMENTS BY JOHN F. EARLY AND HARRY V. ROBERTS

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Many of the problems arising in seasonal adjustment appear, to me, to be due to unclear concepts and insufficient understanding of the X–11 and other automatic programs' capabilities and limitations. The conventional model $Y = C.S.I$ does not take into account the other time-domain components described in my paper with explanations regarding their estimation (to the best of my knowledge, for the first time, in detail). I stress the difficulties of defining and estimating the trend-cycle C in the unusual segments that are found in many series and, therefore, propose estimation of regular seasonality from the behavior of the series in the regular segments only. Even in regular segments, smooth estimates of C, by moving averages spanning into irregular segments, will increase the variability of SI and cause difficulties in estimating and forecasting S.

The comprehensive approach proposed utilises other data in the hands of the analyst, rather than the usual but suboptimal univariate analysis. I agree that there is an element of subjectivity in my approach but believe this desirable in applied statistics to give more meaningful analysis. Study of the data, month-by-month, especially of extreme months, and identification of regular and nonregular segments may well give rise to methods of improving the series or its analysis, e.g.—

1. Explaining many of the monthly variations in unemployment of youth in the United States (and probably in other countries, too) as a function of the numbers of schoolleavers, rather than due to moving regular seasonality.
2. The need to study and adjust appropriate subseries with different seasonal patterns (taking into account other relevant components, including festival-date and trading-day effects); this is analogous to stratifying a population into homogeneous strata before sampling.

X–11's automatic treatment of extreme months gives rise to considerable and undesirable changes in the seasonal factors S, especially for the crucial last year and forecasts for the current year and, thereby, in the seasonally adjusted data A. The Statistics Canada X–11 ARIMA method improves the estimates somewhat, but I suspect that series with high irregularity (such as the U.S. unemployment of youth) will continue to show high irregularity and changes in S on updating, until the analysis is improved by consideration of the related variables' effect Z, of sampling irregularity and its autocorrelations and specific unusual segments.

Ideally, the statistician should study the changes in Y and estimated in SI, S, A, and L over the last year in order to estimate the regular seasonal pattern and that for the coming year, taking into account whether rises (or falls) in specific months are expected to recur next year. The standard X–11 and ARIMA techniques do not encourage this approach.

In the revised paper, I have given further attention to the ARIMA approach, used it for the demonstration series (table 1a) and compared it with the component approach. The ARIMA estimates for this series are very sensitive to the Yom Kippur War and to alternative estimates for that segment, and they are also affected by other major unusual events. The use of ARIMA estimates for 1 or 2 years ahead may well improve the X–11 analysis of many series in an automatic fashion, and direct estimates of components may be possible via ARIMA. (See [13].)

In reply to some of Mr. Early's specific comments—

1. Unemployment series show the difficulties of applying a simple additive or multiplicative model for seasonality during rapid cyclical changes. Specific strata or subseries are liable to change faster than others, resulting in lack of independence of the components C, S, and I for a total series.
2. Negative serial correlations may often be observed in the irregular factors, computed by X–11, over the regular segments, after distinguishing unusual segments, and also in the month-to-month changes a in regular seasonally adjusted data A, e.g.—

Series	Average duration of run, ADR		Serial correlation for 1-month span
	R	a	σ_1 (R)
Israel, tourist arrivals, by air: 1956-75	1.37	1.20	-0.08
U.S. sales, all manufacturing industries: 1965-75	1.47	1.47	-.19
U.S. unemployment, men 16-19 years old: 1967-75	1.43	1.31	-.20

The saw-toothing of the s.a.d. may be seen in many series' regular segments, even when strong trends produce an overall positive serial correlation in A. On the other hand, unusual segments may show strong positive correlations in I and A for several successive months.

3. Festival-date adjustments are now described in further detail in footnote 15.
4. Seasonal adjustment of pairs of months is detailed in footnote 16.
5. The flags enable convenient study of s.a.d., etc. (table 1f) without having to refer simultaneously to two or three tables of the X–11 printout; in my experience, the inconvenience of studying several tables or charts is one of the reasons for insufficient professional analysis of X–11 printouts and especially of the identification and imputation of extremes. The ADR indicates the average duration of run over the entire era analysed but not the detailed month-to-month changes.
6. The stream L is especially useful for analysis of the last year and for current analysis, for which the Henderson 13–term moving average $C^{(H)}$ is not appropriate and the MCD average $C^{(M)}$ has many disadvantages, described in the paper. Other estimates of trend may be suitable for long regular segments.
7. X–11 will often indicate a regular month's data as extreme because of a biased estimate $C^{(H)}$ of the trend cycle, while not identifying many months that were obviously affected by unusual events. Tables 1g and 2 indicate the advantages of identifying unusual segments using a chronology of events and L.

I agree that much further research and experimentation are necessary. Detailed case studies of alternative analyses of suitable series and their updating are useful tools that, I hope, my paper will stimulate.