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9 Caps on Adjustable Rate Mortgages: Valuation, Insurance, and Hedging

Eduardo S. Schwartz and Walter N. Torous

9.1 Introduction

In April of 1981, federally chartered thrift institutions were permitted to originate adjustable rate mortgages. Prior to this, thrifts primarily originated long-term fixed rate mortgages. As a result, since thrifts are primarily financed with short-term deposits, a gap between their asset and liability maturities arose, thereby exposing thrifts to considerable interest rate risk. Adjustable rate mortgages reduce a thrift's gap, and hence its interest rate risk, since coupon rates vary with the thrift's cost of funds.

As expected, the origination of adjustable rate mortgages became prevalent with thrift institutions. After falling from 75 percent of total thrift originations in August 1984 to 21 percent of originations in June 1986, adjustable rate mortgages rose to an average of 71 percent of originations during 1988. Currently, approximately $300 billion of adjustable rate mortgages are outstanding. However, as the ongoing thrift crisis would indicate, adjustable rate mortgages did not eliminate thrifts' interest rate risk exposure.

Adjustable rate mortgage originators remain exposed to interest rate risk since the many contractual features of a typical adjustable rate mortgage result in an imperfect adjustment of its coupon rate to changes in the thrift's cost of funds. For example, Section 3806 of the Alternative Mortgage Transaction Parity Act of 1982 states that "adjustable rate mortgage loans originated by a

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creditor shall include a limitation on the maximum interest rate that may apply during the term of the mortgage loan" (p. 4361).[1] The presence of a lifetime cap, as well as other contractual features, prevents a timely and full adjustment in the adjustable rate mortgage's coupon rate to changes in a thrift's cost of funds.

The purpose of this paper is to investigate the interest rate risk exposure of thrifts originating adjustable rate mortgages arising from the various contractual features of these mortgage instruments. In particular, we consider how the resultant interest rate risk exposure varies with the adjustable rate mortgage's lifetime cap, periodic cap, adjustment period, teaser rate, and margin. In addition to quantifying this interest rate risk exposure, we also investigate various means of minimizing it. Given the potentially considerable interest rate risk exposure due to the inclusion of lifetime cap provisions, we value lifetime cap insurance which insures against the adjustable rate mortgage's coupon rate exceeding its lifetime cap. We also discuss dynamic hedging strategies which, in lieu of purchasing insurance, can be pursued by thrifts to minimize the interest rate risk associated with the origination of adjustable rate mortgages.

We investigate these various issues within a two-factor adjustable rate mortgage valuation framework which explicitly takes into account the essential contractual features of the adjustable rate mortgage as well as the prepayment behavior of borrowers. Other studies which have investigated the valuation of adjustable rate mortgages include Buser, Hendershott, and Sanders (1985) and Kau et al. (1985). By comparison, both of these studies assume a one-factor framework and ignore the prepayment behavior of borrowers.

In this paper we abstract from the possibility of default by either the borrower or the originating thrift institution. The borrower's default risk can be taken into account by charging an appropriate default insurance premium (see Schwartz and Torous 1990). For our purposes here, this insurance premium can be assumed to be included in the adjustable rate mortgage's servicing fee. By contrast, we do not deal with the potential strategic behavior of an originating thrift institution in financial distress.

The plan of this paper is as follows. In section 9.2 we detail the various contractual features of adjustable rate mortgages. An understanding of these contractual features is necessary to analyze why an adjustable rate mortgage's coupon rate is typically not perfectly indexed to a thrift's cost of funds. Section 9.3 details our two-factor adjustable rate mortgage valuation model. Following Brennan and Schwartz (1982) and Schwartz and Torous (1989), we take the two factors to be the instantaneously riskless rate of interest and the yield on a default-free consol bond. Monte Carlo simulation techniques are used to value adjustable rate mortgages and various cap options embedded in them, as well as to determine the fair fee to charge for lifetime cap insurance. We present these valuation results in section 9.4. In section 9.5 we discuss various techniques to dynamically hedge adjustable rate mortgages' interest rate risk. Section 9.6 provides our conclusions and suggests policy implications arising from this research.

9.2 Contractual Features of Adjustable Rate Mortgages

An ideal adjustable rate mortgage (ARM) would have a coupon rate which adjusts to perfectly reflect changes in the originating thrift's cost of funds. The thrift's assets and liabilities would then be perfectly matched, thereby eliminating interest rate risk. However, in practice the various contractual features of an ARM typically prevent its coupon rate from perfectly responding to changes in a thrift's cost of funds. In this section, we summarize the essential contractual features of ARMs, emphasizing how these features may prevent the ARM coupon rate from being perfectly indexed to the originating thrift's cost of funds and, as such, exposing the thrift to interest rate risk. For further details regarding the various contractual features of ARMs, see Bartholomew, Berk, and Roll (1986).

9.2.1 Index

The ARM's coupon rate varies directly with its contractually specified index. The two most widely used indices are a cost of funds index (COFI) and a constant maturity (1 year or 5 year) Treasury yield index. The former represents a weighted average of the actual book cost of funds of thrifts located in the Federal Home Loan Bank's eleventh district (Arizona, California, and Nevada), while the latter is constructed from the current yields of 1-year or 5-year Treasury securities.

To the extent that a COFI reflects the book rather than market cost of a thrift's funds, changes in this index will not perfectly reflect changes in market interest rate conditions. Similarly, while levels of the Treasury yield indices move extremely closely with levels of corresponding market Treasury rates, the empirical analysis of Roll (1987) concludes that the method of constructing these indices significantly reduces the correlation between changes in these indices and changes in corresponding market rates. As a result, changes in the coupon rate of an ARM based on a Treasury yield index will also not perfectly reflect changes in market interest rate conditions.

9.2.2 Margin

The ARM's coupon rate equals the prevailing level of the index plus the contractually specified margin, subject to initial discounts and restrictions to be discussed shortly. The margin on ARMs have remained relatively level over the recent past. For example, from January 1986 through April 1989, the margin over index for ARMs based on the 1-year Treasury yield index averaged between 200 and 300 basis points (see Gordon, Luytjes, and Feid 1989). The size of the margin reflects the value of the various options embedded in the ARM, including the option to prepay, as well as the costs of servicing the underlying loan.

9.2.3 Adjustment Period

An ARM's coupon rate is not continuously adjusted to changes in the level of the underlying index, but rather is adjusted periodically at a contractually stipulated frequency. An ARM's adjustment period is the minimum period of time over which its coupon rate cannot be changed. Typical adjustment periods are six months or one year. Clearly, the higher an ARM's adjustment frequency, the more responsive its coupon rate will be to changes in current market conditions. The fact that ARMs are not continuously adjusted prevents ARM coupon rates from perfectly responding to changes in thrifts' cost of funds.

9.2.4 Teaser Rate

At origination the ARM coupon rate is frequently set below its fully indexed level, index plus margin, so as to provide an initial inducement to the borrower. This initial coupon rate is referred to as the ARM's teaser rate. Typically the teaser rate is in effect for the initial adjustment period. The last few years have seen thrifts more often utilizing teaser rates, offering increasingly larger discounts from the fully indexed loan rate. For example, the size of this discount on ARMS based on a 1-year Treasury index rose sharply from an average of 0.3 percent in late 1986 to an average of 3 percent in late 1988, further increasing to an average of 3.5 percent in early 1989 (see Gordon, Luytjes, and Feid 1989). It has been suggested that the size of this discount has now grown to where new ARM originations are no longer profitable, thereby contributing to the ongoing thrift crisis.

9.2.5 Lifetime Cap

An ARM's lifetime cap contractually stipulates an upper bound which its coupon rate cannot exceed. In other words, if at adjustment the fully indexed loan rate exceeds the lifetime cap, then the ARM's coupon rate remains at the lifetime cap. The ARM's coupon rate is fixed at the lifetime cap until the fully indexed loan rate falls below the lifetime cap at a subsequent adjustment. Section 3806 of the Alternative Mortgage Transaction Parity Act of 1982 requires that all ARMs be subject to a lifetime cap provision. Typically a 5 or 6 percent maximum change in an ARM's coupon rate relative to its teaser rate is permitted over the life of a loan based on a 1-year Treasury index. Lifetime caps subject the ARM originator to potentially significant interest rate risk as the ARM coupon rate is contractually prevented from fully responding to significant increases in the thrift's cost of funds.

9.2.6 Lifetime Floor

An ARM's lifetime floor contractually stipulates a lower bound below which the ARM's coupon rate cannot fall. That is, the ARM coupon remains at the floor if the fully indexed loan rate falls below the floor. As opposed to the lifetime cap, which is beneficial to the borrower, the inclusion of a lifetime floor is generally viewed as being advantageous to the originator. However, it should be noted that the lifetime floor will typically not be binding since most interest-sensitive borrowers will prepay at sufficiently low interest rates in order to lock in low fixed refinancing rates.

9.2.7 Periodic Cap

The periodic cap limits the amount by which the ARM coupon rate can change, in either direction, over any adjustment period. In other words, if the underlying index increases or decreases by more than the periodic cap, the ARM coupon rate changes only by the magnitude of the periodic cap.[2] Typically, ARMs based on a 1-year Treasury index feature periodic caps of 2 percent per year. Periodic caps effect lags in adjustments of the ARM coupon rate to changes in a thrift's cost of funds, though in the case of falling interest rates the consequence would be beneficial to the originator.

While the above discussion has concentrated on each contractual feature of an ARM in isolation, it should be emphasized that these various features may potentially interact with one another. For example, the fact that the teaser rate is usually set well below the fully indexed loan rate implies that the periodic cap will almost certainly be binding at the ARM's first adjustment. It is important to jointly model all of these institutional features to fully capture these interactions and to therefore properly value ARMs.

9.3 A Two-Factor ARM Valuation Model

In this section we develop a two-factor model to value ARMs which takes into account their previously described institutional features. The model allows us to examine the pricing of lifetime cap insurance, other options embedded in ARMs, and the dynamic hedging of these mortgage instruments. Our analysis is couched in perfect frictionless markets and, therefore, we ignore transaction costs, such as points charged at the ARM's origination.

The point of departure of our ARM valuation model is Brennan and Schwartz's (1982) two-factor model of the term structure of default-free interest rates. This model assumes that all relevant information about the term structure can be summarized by the instantaneous riskless rate of interest (the "short rate") and the continuously compounded yield on a default-free consol bond (the "long rate"). Given the assumed dynamics of the short and long rates, an absence of arbitrage opportunities yields the fundamental ARM valuation equation which must be solved subject to appropriate boundary and terminal conditions.

To properly value ARMs it is important to model borrowers' prepayment behavior. Despite the fact that ARM coupon rates tend to vary with market interest rate conditions, the empirical analysis of Bartholomew, Berk, and

Roll (1988) documents that ARM borrowers tend to prepay when refinancing rates are low in order to lock in what they believe are low fixed rates. Clearly a failure to properly model this prepayment behavior will result in the systematic mispricing of ARMs. We incorporate prepayment behavior by specifying the borrower's prepayment function, which gives the conditional probability of a borrower prepaying an ARM, as a proportional hazards model (Green and Shoven 1986). The baseline hazard function measures the effect of seasoning, or mortgage age, on prepayment behavior. However, the conditional probability of prepayment does not depend solely upon the ARM's age. Our prepayment function recognizes that prevailing interest rate conditions also influence the prepayment decision of the ARM borrower.

9.3.1 ARM Cash Flows

We consider a fully amortizing mortgage having an original principal of $P(0)$ and an original term to maturity of T years with a continuously compounded teaser rate of $c(0)$. As a result, the ARM's continuous payout rate over the initial adjustment period is $C(0)$ where

$$C(0) = c(0)P(0)/(1 - \exp(-c(0)T)).$$

The loan's principal outstanding, $P(t)$, during the initial adjustment period is given by

$$P(t) = P(0)(1 - \exp(-c(0)(T - t)))/(1 - \exp(-c(0)T)).$$

At the ARM's ith adjustment, at time t_i, given a coupon rate of $c(t_i)$, the ARM's payout rate, $C(t_i)$, is now given by

$$C(t_i) = c(t_i)P(t_i)/(1 - \exp(-c(t_i)(T - t_i))),$$

while during the ith adjustment period the loan's principal outstanding is given by

$$P(t) = P(t_i)(1 - \exp(-c(t_i)(T - t)))/(1 - \exp(-c(t_i)(T - t_i))).$$

The ARM's coupon rate at adjustment is determined by adding the contractually specified margin, m, to the prevailing level of the underlying index, $x(t)$, subject to the ARM's lifetime and periodic cap constraints. Let $c^*(t_i)$ represent the fully indexed loan rate in the absence of any cap constraints:

$$c^*(t_i) = x(t_i) + m.$$

If $c^*(t_i) > c(t_{i-1})$, then the ARM's coupon rate is given by

$$c(t_i) = \min[\, c^*(t_i), c_L, c(t_{i-1}) + p],$$

where c_L denotes the ARM's lifetime cap and p the ARM's periodic cap. Conversely, if $c^*(t_i) < c(t_{i-1})$, then the ARM's coupon rate is given by

$$c(t_i) = \max[\, c^*(t_i), c_F, c(t_{i-1}) - p],$$

where c_F denotes the ARM's lifetime floor.

These ARM cash flows represent the mortgagor's contractual interest and repayment of principal, and typically include a servicing fee to the originator. However, since we are interested in the market valuation of the ARM, we must explicitly take into account any included servicing fee. We do so by subtracting an exogenously specified servicing fee, s, from these cash flows:

$$C(t) - sP(t).$$

The ARM's value reflects the investor's receipt of these cash flows, subject to the loan's prepayment.

9.3.2 Prepayment Function

We assume that the mortgagor's annualized conditional probability of prepayment depends upon the mortgage's age as well as prevailing interest rate conditions, and is given by the following proportional hazards model:

$$\pi(x, t) = \pi_0(t)\exp(\beta(x(0) - x(t)). \tag{1}$$

The baseline hazard function, $\pi_0(t)$, measures the effect of seasoning on prepayment behavior. We assume that the baseline hazard function is given by 100 percent of Public Securities Association (PSA) experience:

$$\pi_0(t) = \min(0.024t, 0.06).$$

That is, the annualized baseline probability of prepayment is zero at the ARM's origination, increases by 0.002 per month for the first thirty months of the ARM's life, and then remains constant at an annualized rate of 0.06 from the thirtieth month until maturity.

Prevailing interest rate conditions also influence the mortgagor's prepayment decision. To model interest-sensitive ARM prepayments, we include a single covariate measuring the difference between the underlying index's level at origination, $x(0)$, and its prevailing level, $x(t)$. The higher the prevailing level of the index relative to its level at the ARM's origination, the lower the probability of prepayment, conditional on the ARM not having been previously prepaid. We can interpret β as measuring the speed of prepayment. The larger is β, ceteris paribus, the more sensitive are ARM prepayments to prevailing interest rate conditions.

9.3.3 Valuation Equation

The dynamics of the state variables, the short rate r and the long rate l, are assumed to be given by

$$dr = (a_1 + b_1(l - r))dt + \sigma_1 r dz_1 \tag{2}$$

and

$$dl = l(a_2 + b_2 r + c_2 l)dt + \sigma_2 l dz_2, \tag{3}$$

where z_1 and z_2 are standardized Wiener processes. Increments to z_1 and z_2 are assumed instantaneously correlated:

$$dz_1 dz_2 = \rho dt. \tag{4}$$

In the absence of arbitrage opportunities the value of the ARM, $V(r,l,t)$, must satisfy the following second-order partial differential equation:

$$\begin{aligned} &\tfrac{1}{2}r^2\sigma_1^2 V_{rr} + rl\rho\sigma_1\sigma_2 V_{rl} + \tfrac{1}{2}l^2\sigma_2^2 V_{ll} + (a_1 + b_1(l - r) - \lambda\sigma_1 r)V_r \\ &+ l(\sigma_2^2 + l - r)V_l + V_t - (r + \pi)V \\ &+ \pi P(t) + (C(t) - sP(t)) = 0, \end{aligned} \tag{5}$$

where λ is the market price of short-term interest rate risk. Since the ARM is fully amortizing, the following terminal condition must be satisfied:

$$V(r,l,T) = 0. \tag{6}$$

9.3.4 Monte Carlo Simulation Solution Technique

When cash flows are path dependent, the most efficient numerical method for solving the partial differential equation (eq. [5]), subject to the corresponding terminal condition (eq. [6]), is provided by Monte Carlo solution techniques (see Boyle 1977, and Schwartz and Torous 1989). This is particularly so in our valuation framework where the cash flows depend upon past coupon rates.

Monte Carlo simulation methods require that r and l are generated by the following correlated risk-adjusted stochastic processes:

$$dr = (a_1 + b_1(l - r) - \lambda\sigma_1 r)dt + \sigma_1 r dz_1 \tag{2'}$$

and

$$dl = l(\sigma_2^2 + l - r)dt + \sigma_2 l dz_2. \tag{3'}$$

To value an ARM, we generate correlated normal random variables corresponding to r and l at every month during the ARM's life. At each adjustment we set the ARM's coupon rate as previously discussed in section 9.3.1, and consequently each month determine the ARM's cash flows—contractually obligated plus prepayments. The present value of these cash flows provides a realization of the ARM's value. By repeating this procedure, the average of these realizations gives the required solution of the partial differential equation.

9.3.5 Equilibrium Valuation of Cap Options and the Fair Lifetime Cap Insurance Fee

We can use our ARM valuation procedures to determine the equilibrium values of lifetime and periodic cap options embedded in ARMs as well as the fair fee to charge for lifetime cap insurance.

For example, subtracting the value of an ARM with both lifetime and periodic caps from the value of an otherwise identical ARM without the lifetime cap gives the value of the lifetime cap option. Similarly, by subtracting the value of an ARM without the lifetime cap from the value of an otherwise identical ARM without both lifetime and periodic caps gives the value of the periodic cap option.

Lifetime cap insurance insures against the ARM's coupon rate exceeding its lifetime cap. The fair lifetime cap insurance fee is simply that premium which will make an ARM without a lifetime cap have the same value as an otherwise identical ARM with a lifetime cap. The larger the fair lifetime cap insurance fee, the greater the probability that the ARM's lifetime cap will be binding, and therefore the greater the interest rate risk.

9.4 Valuation Results

This section documents the magnitudes as well as sensitivities of periodic cap options, lifetime cap options, and the fair lifetime cap insurance fee to systematic changes in ARM features. These results will provide insights into the determinants of an ARM originator's interest rate risk exposure. Clearly the greater the interest rate risk exposure of an ARM originator, the greater the possibility of the thrift's financial distress.

Since we are interested in systematically varying the features of an ARM, we require a base case against which to compare these results. For this purpose we consider the following 30-year ARM offered by a southern California thrift in December 1989 which was representative of available ARMs:

index = 1-year Treasury index
lifetime cap = 14 percent
lifetime floor = 8 percent
periodic cap = 1 percent
teaser rate = 8 percent
adjustment period = 1 year
margin = 2.75 percent.

Our valuation analysis assumes that the originating thrift requires a fee of 1 percent to service this ARM. Also, interest rate conditions prevailing in December 1989 are summarized by a short rate of $r = 8$ percent and a long rate of $l = 9$ percent.

Notice that our base case ARM is indexed to a short-term riskless rate of interest. While our valuation framework can accommodate a variety of index specifications, we assume that the ARM's coupon rate is perfectly indexed to the short rate r in order to simplify the subsequent analysis. This assumption is consistent with ARMs being indexed to relatively short-term, as opposed to relatively long-term, rates of interest. However, as a result of this simplifying assumption, our subsequent analysis does not investigate the interest rate risk exposure of an ARM originator owing to changes in the ARM's index not being perfectly correlated with changes in market interest rate conditions.

The risk-adjusted interest rate processes (eqs. [2′] and [3′]) characterize the interest rate environment in which we value ARMs. The parameters of the risk-adjusted interest rate processes used here are taken from Schwartz and Torous (1989). There the short rate is approximated by the annualized one-month CD rate, while the long rate is approximated by the annualized running coupon yield on long-term treasury bonds. Given these data over the sample period December 1982 through April 1987, together with an estimate of the market price of short-term interest rate risk of -0.01, the corresponding maximum-likelihood parameter estimates of the risk-adjusted interest rate processes are summarized as follows:

$$\Delta r = (-0.0416 + 1.987(l - r) - (-0.01)(0.189)r)\Delta t + 0.189r\Delta z_1,$$
$$\Delta l = l(0.125^2 + l - r)\Delta t + 0.125l\Delta z_2,$$

with an estimated correlation coefficient of 0.373.

To complete the characterization of our ARM valuation framework, we must explicitly specify the parameters of the prepayment function. As mentioned earlier, we assume that the baseline hazard function is given by 100 percent of PSA experience. Rather than estimate the speed of prepayment parameter β from ARM prepayment data, we determine that particular value of β which results in our model valuing the base-case ARM at par at origination. As a result, we imply an ex ante β estimate of 41.4. Assuming the appropriateness of our valuation framework, the advantage of implying the speed of prepayment parameter is that it provides a forward-looking estimate which reflects anticipated prepayment behavior.

Given that the base-case ARM is valued at par or $100 at origination,[3] the corresponding value of the base-case ARM's lifetime cap option is $1.87, which translates into an annualized fair lifetime cap insurance fee of 31 basis points. The corresponding value of the base-case ARM's periodic cap option is $5.58. The relatively large value of the periodic cap option reflects the fact that with a teaser rate of 8 percent and an initial fully indexed loan rate of 10.75 percent, the periodic cap of 1 percent will most likely be binding at future adjustments of the base-case ARM's coupon rate. By contrast, for a more typical periodic cap of 2 percent and holding all other features fixed, the value of the ARM's lifetime cap option increases to $3.74, which translates into a fair lifetime cap insurance fee of 61 basis points, while the value of the ARM's periodic cap option decreases to $1.81. These changes reflect the fact that the periodic cap is now less likely to be binding at future coupon rate adjustments.

Our subsequent analysis, summarized in table 9.1 and figures 9.1–9.6, examines the sensitivities of cap option values and fair lifetime cap insurance fees of the base-case ARM to systematic changes in the ARMs' contractual features.[4]

9.4.1 Lifetime Cap

Panel A of table 9.1 documents the sensitivities of ARM cap option values to changes in the level of the ARM's lifetime cap. We consider lifetime caps

Table 9.1 **Value of Cap Options as a Function of ARM's Features**

Panel A

Lifetime cap (%)	10	11	12	13	14	15	16	17	18	19	20
Value of lifetime cap option ($)	7.63	5.41	3.82	2.68	1.87	1.28	0.87	0.57	0.37	0.24	0.14
Fair lifetime cap insurance (basis points)	126	89	63	44	31	21	14	9	6	4	2
Value of periodic cap option ($)	5.46	5.50	5.54	5.56	5.58	5.60	5.60	5.61	5.61	5.61	5.62

Panel B

Lifetime floor (%)	5	6	7	8	9
Value of lifetime cap option ($)	1.89	1.89	1.87	1.74	1.25
Fair lifetime cap insurance (basic points)	31	31	31	29	21
Value of periodic cap option ($)	5.58	5.58	5.58	5.58	5.59

Panel C

Periodic cap (%)	.25	.50	.75	1.00	1.25	1.50	1.75	2.00
Value of lifetime cap option ($)	0.00	0.34	1.10	1.87	2.54	3.06	3.46	3.74
Fair lifetime cap insurance (basis points)	0	6	18	31	42	50	57	61
Value of periodic cap option ($)	14.81	10.59	7.63	5.58	4.12	3.09	2.34	1.81

Panel D

Adjustment period (months)	3	6	9	12	15	18	21	24	27	30
Value of lifetime cap option ($)	4.89	3.80	2.72	1.87	1.23	.81	.50	.31	.18	.10
Fair lifetime cap insurance (basis points)	80	62	45	31	20	13	8	5	3	2
Value of periodic cap option ($)	1.09	2.59	4.15	5.58	6.79	7.59	8.36	8.93	9.28	9.56

Panel E

Teaser rate (%)	6	7	8	9	10	11
Value of lifetime cap option($)	1.51	1.70	1.87	2.00	2.08	2.14
Fair lifetime cap insurance (basis points)	25	28	31	33	34	35
Value of periodic cap option ($)	10.16	7.54	5.58	4.37	3.72	3.03

(continued)

Table 9.1 (continued)

Panel F

Margin (%)	1.25	1.75	2.25	2.75	3.25	3.75
Value of lifetime cap option ($)	0.84	1.22	1.56	1.87	2.19	2.52
Fair lifetime cap insurance (basis points)	14	20	26	31	36	41
Value of periodic cap option ($)	0.33	1.92	3.55	5.21	6.92	8.67

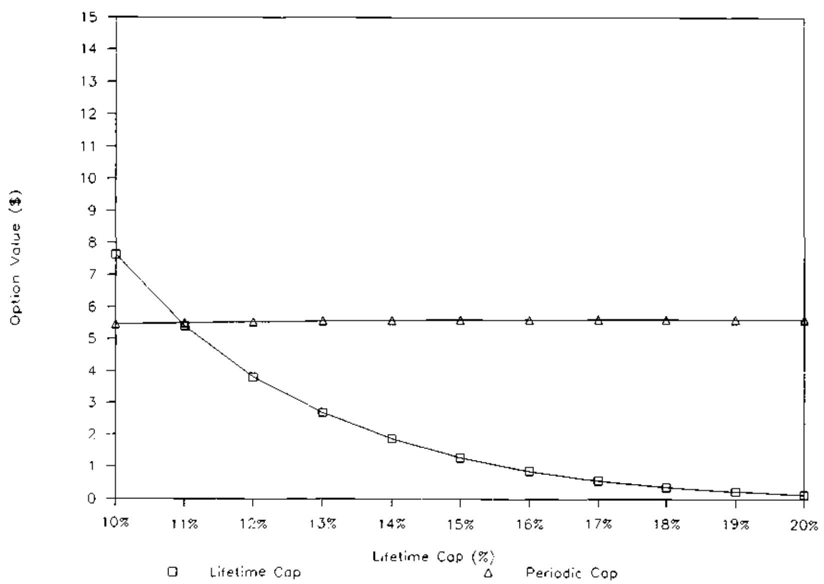


Fig. 9.1 Value of cap options for different lifetime caps

ranging from 10 to 20 percent in increments of 1 percent. These results are graphically presented in figure 9.1.

Notice that the value of the lifetime cap option and the corresponding fair lifetime cap insurance fee decrease with increases in the lifetime cap. For example, when the lifetime cap is 12 percent the fair lifetime cap insurance fee is 63 basis points, while for a lifetime cap of 20 percent the fair lifetime cap insurance fee is reduced to only 2 basis points. This result follows simply from the fact that the higher the level of the lifetime cap, the lower the probability of this cap becoming binding.

By contrast, the value of the periodic cap option increases slightly as the level of the lifetime cap increases. When the lifetime cap is 12 percent the periodic cap option is valued at $5.54, while for a lifetime cap of 20 percent the periodic cap option increases in value to $5.62. Intuitively, as the level of the lifetime cap increases and, as such, the lifetime cap becomes less binding, the probability that the periodic cap will become binding increases.

9.4.2 Lifetime Floor

ARM cap option values are extremely insensitive to changes in the level of the ARM's lifetime floor. We consider lifetime floors ranging from 5 to 9 percent in increments of 1 percent and tabulate the resultant ARM cap option values in panel B of table 9.1. These results are graphically summarized in figure 9.2. The reason for the insensitivity of ARM cap option values to changes in the lifetime floor is that the states of the world in which the lifetime floor becomes binding are precisely those states of the world in which there exists a financial incentive for borrowers to prepay. These results indicate that the lifetime floor option, which is viewed as being beneficial to the ARM originator, is practically worthless. As a result, the lifetime cap option embedded in ARMs typically derives value only from its upper cap.

9.4.3 Periodic Cap

We tabulate the sensitivities of ARM cap option values to changes in the level of the periodic cap in panel C of table 9.1. The periodic cap ranges from 0.25 to 2.00 percent in increments of 0.25 percent, while maintaining a 12-month adjustment period. These results are graphically depicted in figure 9.3.

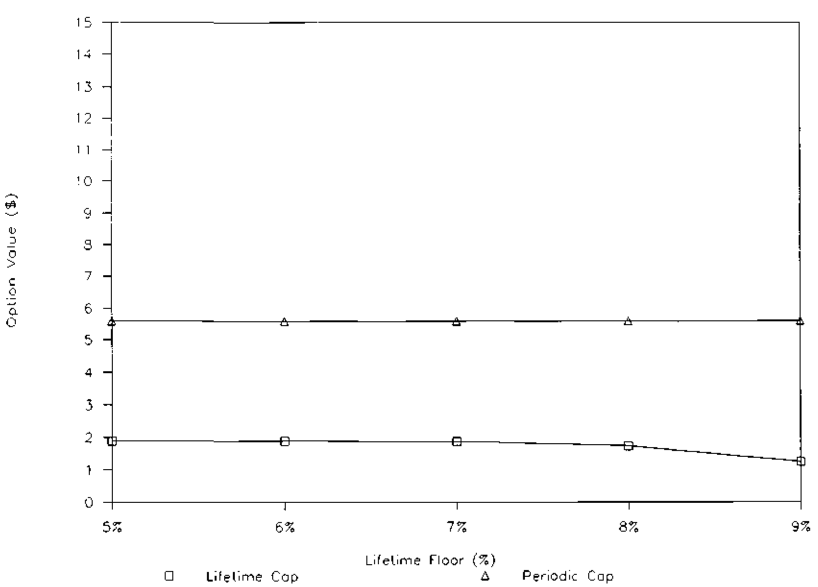


Fig. 9.2 Value of cap options for different lifetime floors

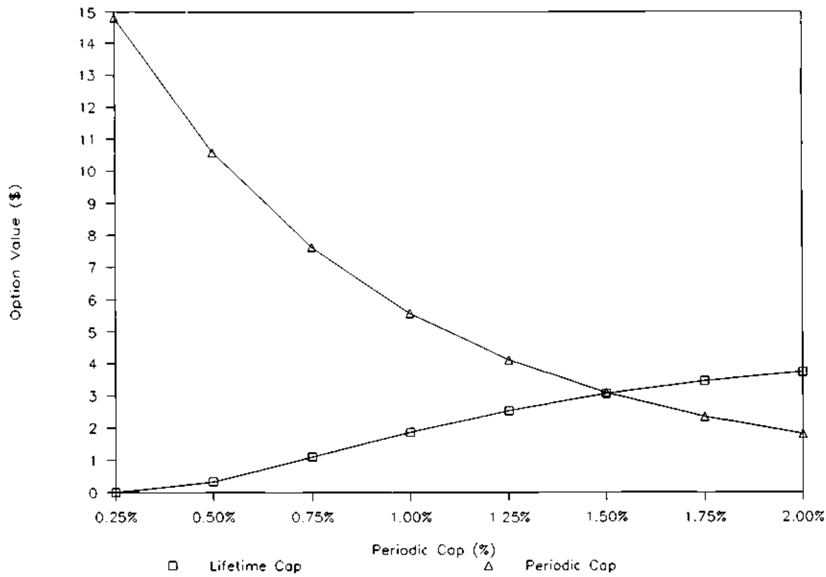


Fig. 9.3 Value of cap options for different periodic caps

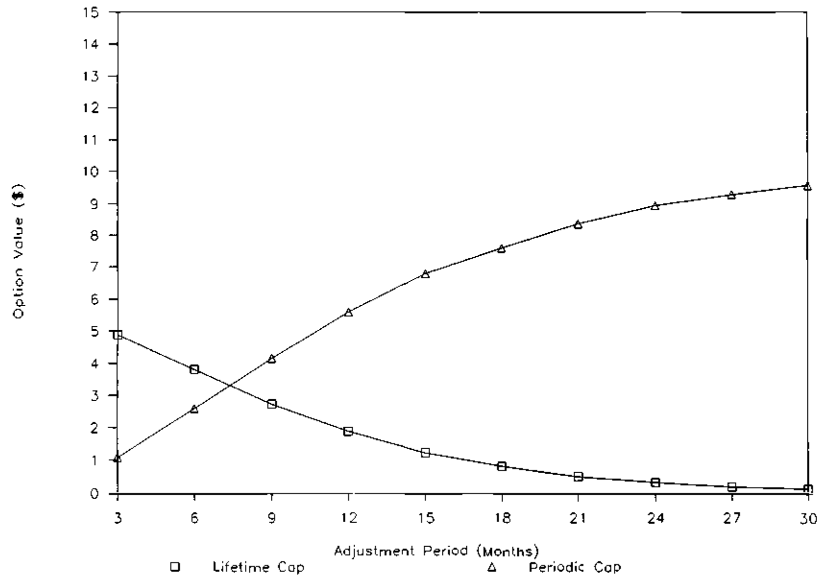
The value of the lifetime cap option and the corresponding fair lifetime cap insurance fee increase with increases in the level of the periodic cap. For a periodic cap of 0.25 percent the fair lifetime cap insurance fee is negligible, but the fee increases to 71 basis points for a periodic cap of 2.00 percent. The smaller the periodic cap, the less binding a given lifetime cap. For example, for a periodic cap of 0.25 percent, it would take a minimum of 24 years for a lifetime cap of 14 percent to be reached from a teaser rate of 8 percent, and hence the value of the lifetime cap option is negligible.

Conversely, the value of the periodic cap option decreases with increases in the ARM's periodic cap. For a periodic cap of 0.25 percent, the periodic cap option is valued at $14.81, while for a periodic cap of 2.00 percent the periodic cap option is valued at $1.81. The smaller the periodic cap, the more likely it will be binding, and hence the more valuable the periodic cap option.

9.4.4 Adjustment Period

We vary the ARM's adjustment period from 3 months to 30 months, in increments of 3 months, and document the resultant ARM cap option values in panel D of table 9.1. Figure 9.4 graphically summarizes these results.

The value of the lifetime cap option and the corresponding fair lifetime cap insurance fee decrease with the lengthening of the ARM's adjustment period. For example, for an adjustment period of 3 months the fair lifetime cap insurance fee is 80 basis points, while for an adjustment period of 30 months the fair insurance fee is only 2 basis points. For a fixed periodic cap, the longer



Fig, 9,4 Value of cap options for different adjustment periods

the adjustment period, the longer, on average, the time required to reach the ARM's lifetime cap, and hence the less valuable the lifetime cap option. If the adjustment period is 30 months, it would take a minimum of 15 years to reach a lifetime cap of 14 percent starting from a teaser rate of 8 percent. In the limit, as the adjustment period approaches the ARM's original term to maturity, the ARM becomes a fixed rate mortgage and the lifetime cap option becomes worthless.

By contrast, the value of the periodic cap option increases with the lengthening of the ARM's adjustment period. For an adjustment period of 3 months the periodic cap option is valued at $1.09, while for an adjustment period of 30 months the periodic cap option is valued at $9.56. Intuitively, the longer the adjustment period, the more likely the periodic cap will be binding.

9.4.5 Teaser Rate

Panel E of table 9.1 reports ARM cap option values as the teaser rate is varied from 6 to 11 percent in increments of 1 percent. These results are graphically presented in figure 9.5.

Increases in the teaser rate result in increases in the value of the lifetime cap option and the corresponding fair lifetime cap insurance fee. For example, for a teaser rate of 6 percent the fair lifetime cap insurance fee is 25 basis points, and the fee increases to 35 basis points for a teaser rate of 11 percent. The higher the teaser rate, the closer the ARM's initial fully indexed loan rate is to the lifetime cap and, as such, the more valuable the lifetime cap option.

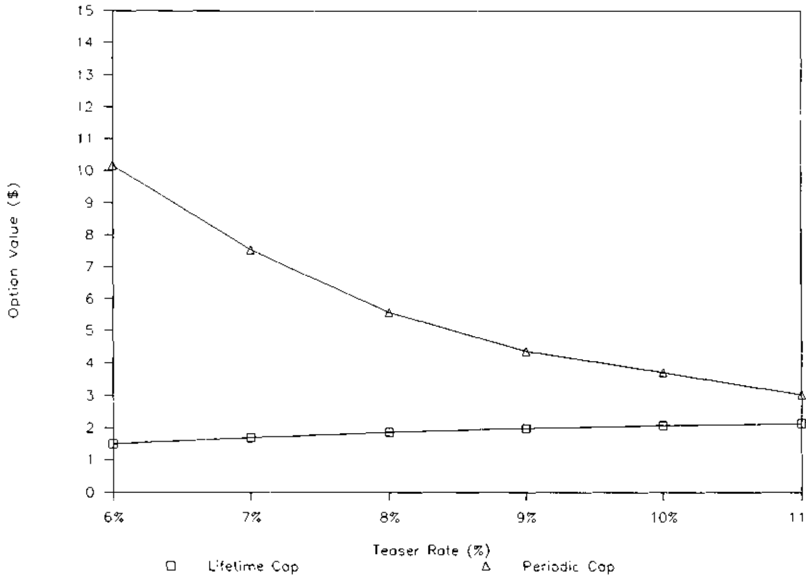


Fig. 9.5 Value of cap options for different teaser rates

However, the value of the periodic cap option decreases with increases in the teaser rate. For a teaser rate of 6 percent the periodic cap option is valued at $10.16, while for a teaser rate of 11 percent the periodic cap option is valued at $3.03. The lower the teaser rate the more valuable is the periodic cap option, since the initial adjustment in the ARM's coupon to the corresponding fully indexed loan rate will be larger and hence the periodic cap will be more likely to be binding.

9.4.6 Margin

Finally, panel F of table 9.1 examines the sensitivities of ARM cap option values to the level of the ARM's margin. The margin is varied from 1.25 to 3.75 percent in increments of 0.25 percent. The results are graphically depicted in figure 9.6.

As expected, the value of the lifetime cap option and the corresponding fair lifetime cap insurance fee increase with increases in the margin. For a margin of 1.25 percent the fair lifetime cap insurance fee is 14 basis points, while for a margin of 3.75 percent the fair fee is 41 basis points. The larger the margin, the greater the possibility that a given lifetime cap will be binding.

Similarly, the value of the periodic cap option increases with increases in the ARM's margin. For a margin of 1.25 percent the periodic cap option is valued at $3.94, while for a margin of 3.75 percent the periodic cap option is valued at $7.60. This result follows from the fact that the larger the margin, the larger will be the initial adjustment in the ARM's coupon rate to the corresponding fully indexed loan rate.

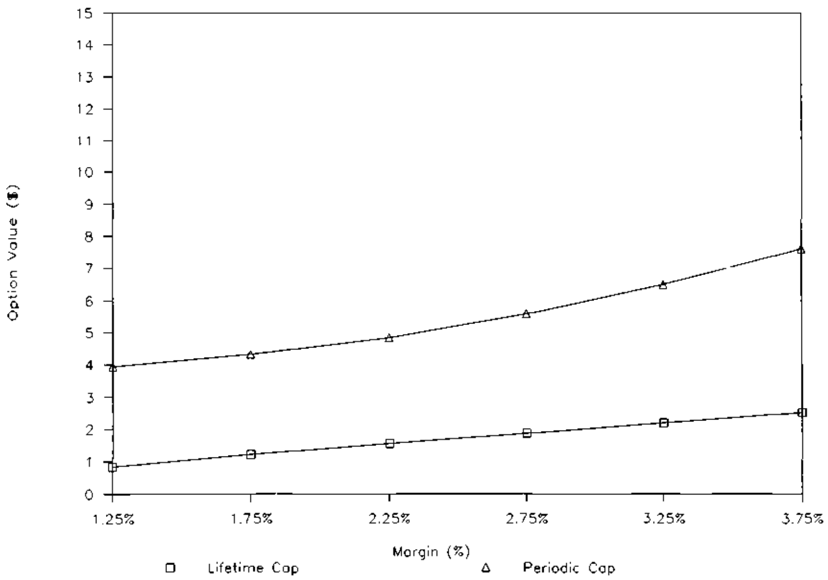


Fig. 9.6 Value of cap options for different margins

9.5 Hedging ARM Interest Rate Risk

Our preceding analysis has established that the various contractual features of ARMs subject originating thrifts to potentially substantial interest rate risk exposure. For example. the presence of a lifetime cap provision contractually prevents an ARM's coupon rate from fully responding to significant increases in a thrift's cost of funds. The thrift's resultant interest rate exposure. as measured by the corresponding fair lifetime cap insurance fee. can be considerable. We now turn our attention to dynamic hedging strategies which will allow the originating thrift to minimize the interest rate risk exposure arising from the various contractual features of ARMs. Alternatively. these dynamic hedging strategies can be implemented by lifetime cap insurers to hedge their resultant interest rate risk.

Our ARM valuation equation (eq. [5]) is based on dynamic hedging arguments. In other words. assuming that the two factors. r and l, determine the value of all default-free interest-rate-dependent claims. it is always possible to form a portfolio of three interest-rate-dependent claims that is insensitive to instantaneous changes in these factors. Therefore. in the absence of arbitrage opportunities. the instantaneous return to this hedge portfolio must equal the prevailing instantaneous riskless rate of interest. These arbitrage arguments yield the partial differential equation (eq. [5]) and also form the basis for hedging interest rate risk inherent in all interest-rate-contingent claims.

We note at the outset that we are able to use our valuation framework to fully hedge the interest rate risk associated with the origination of ARMs because of the following two simplifying assumptions of the model. First, we assume that the ARM's index is one of the model's state variables. Second, we assume a prepayment function which depends only on the prevailing value of the state variable and the mortgage's age, and neglect other demographic and socioeconomic factors which influence borrowers' prepayment decisions. Both of these assumptions do not strictly hold in practice, and therefore will adversely affect our ARM hedging performance. To fully assess these implications would require further empirical analysis.

To dynamically hedge an ARM's interest rate risk requires offsetting positions in other interest-rate-sensitive securities. In particular, the ARM's sensitivities to changes in interest rates must be offset by the corresponding sensitivities of these other securities. For illustrative purposes, we assume these interest-rate-sensitive securities are default-free coupon bonds of varying maturities, although in practice, to minimize transaction costs, this dynamic hedging strategy would most likely be implemented with interest rate futures.

Table 9.2 documents the sensitivities of the base-case ARM to changes in r and l,[5] as well as the corresponding sensitivities of 9% continuously compounded coupon, non-amortizing, non-callable, default-free bonds of various maturities. For example, the base-case ARM's value decreases by 0.30 percent for a 1 percent increase in r from 8 to 9 percent, while a 1 percent increase in l from 9 to 10 percent will decrease its value by 3.66 percent. By contrast, the sensitivities of a 1-year 9% coupon bond with respect to these changes in r and l are -0.42 percent and -0.54 percent, respectively.

As expected, lengthening the maturity of the default-free bond decreases its sensitivity to the short rate and increases its sensitivity to the long rate. Comparing these sensitivities, notice that the interest rate sensitivities of the base-case ARM are similar to the corresponding sensitivities of the 5-year coupon bond. Despite the fact that the ARM's coupon rate is indexed to the short rate of interest, its numerous institutional features together with the posited prepayment behavior make the ARM's interest rate sensitivities more similar to a 5-year default-free bond as opposed to a default-free bond of shorter term to maturity.

These sensitivities allow us to formulate a dynamic portfolio strategy in the

Table 9.2 Sensitivies of ARM and Various Default-free Coupon Bonds to Changes in Short and Long Rates

	Sensitivity to r	Sensitivity to l
ARM	−.30	−3.66
1-year 9% bond	−.42	−.54
5-year 9% bond	−.32	−3.99
10-year 9% bond	−.18	−7.18
20-year 9% bond	−.05	−10.01
30-year 9% bond	−.02	−10.63

coupon bonds to hedge the interest rate risk incurred by the originator of the base-case ARM. Since the ARM represents an asset to the originating thrift, the structure of its liability portfolio should be such that its sensitivities to the two factors exactly offset the ARM's corresponding sensitivities.

To be more precise, let

$$\beta_r^j = V_r^j / V^j$$

and

$$\beta_l^j = V_l^j / V^j$$

be the sensitivities of asset j with respect to r and l, respectively. Assume that we want to hedge perfectly an investment of X_3 dollars in asset 3 (for example, ARMs) using two other interest-rate-dependent assets, X_1 dollars in asset 1 and X_2 dollars in asset 2 (for example, default-free coupon bonds of differing maturities), then X_1 and X_2 satisfy the following system of equations:

$$X_1\beta_r^1 + X_2\beta_r^2 = X_3\beta_r^3$$

and

$$X_1\beta_l^1 + X_2\beta_l^2 = X_3\beta_l^3.$$

To complete the perfect hedge, the difference $X_3 - (X_1 + X_2)$ must be invested in the instantaneously risk-free asset.

For example, the base-case ARM can be hedged initially by borrowing 51 percent of its value in a 9% 1-year default-free bond, 47 percent in a 9% 10-year default-free bond, and the remaining 2 percent at the prevailing instantaneous risk-free rate. Alternatively, the base-case ARM can be hedged initially by borrowing 91 percent of its value in a 9% 5-year default-free bond, 2 percent in a 9% 1-year default-free bond, and the remaining 7 percent at the prevailing instantaneous risk free rate. Finally, we can initially hedge the base-case ARM by borrowing 95 percent of its value in a 9% 5-year default-free bond, 7 percent at the prevailing instantaneous risk-free rate, and lending 2 percent of its value in a 9% 10-year default-free bond. Of course, as time evolves and the levels of the state variables change, the sensitivities of the base-case ARM and the default-free coupon bonds also change. This implies that the hedge portfolio in the liabilities must be dynamically adjusted according to these revised sensitivities in order to continue to offset the ARM's interest rate sensitivities.

9.6 Conclusions and Policy Implications

This paper develops a two-factor model to value adjustable rate mortgages which integrates their essential contractual features with borrowers' prepayment behavior into a partial equilibrium framework. We value the periodic and lifetime cap options embedded in ARMs, and determine the fair fee to charge for insuring the lifetime cap. We investigate the sensitivities of these

ARM cap option values and the fair lifetime cap insurance fee to systematic changes in the ARM's contractual features. Also, we discuss dynamic hedging strategies which can be used to minimize the interest rate risk exposure associated with the origination of ARMs.

An important characteristic of ARMs currently available is the great diversity in their contractual features. ARMs differ in their underlying index, margin, adjustment period, and teaser rate, as well as their lifetime and periodic cap provisions. Our analysis indicates that all these contractual features must be jointly modeled to take into account properly their interaction in determining the value of ARMs.

ARM originators are subject to potentially considerable interest rate risk exposure. Clearly, the greater the interest rate risk exposure of an ARM originator, the greater the possibility of the thrift's financial distress. Two ways of dealing with this interest rate risk are presented. First, the ARM originator can use dynamic hedging techniques to reduce this risk. Second, the ARM originator can purchase lifetime cap insurance. To be properly valued, this lifetime cap insurance should vary with the contractual features of the ARM, as the originator's corresponding interest rate risk exposure and the possibility of financial distress also varies with these contractual features.

Notes

1. See Wallace and Wang (1989).
2. In this paper we consider periodic rate caps as opposed to periodic payment caps. A periodic payment cap imposes a limit on the changes in an ARM's monthly payment; its effects are similar to those of a periodic rate cap, but can lead to negative amortization. For more details, see Bartholomew, Berk, and Roll (1986).
3. To obtain the estimate of a security's value, we replicate our Monte Carlo procedure one thousand times. Standard deviations of all values reported in this paper are on the order of 0.25 percent.
4. Our sensitivity analysis assumes that prepayment behavior does not vary with the contractual features of the ARM. A more general analysis would allow for interaction between prepayments and these contractual features.
5. To numerically compute partial derivatives, we perturbate the initial values of the state variables and use Monte Carlo simulation methods to compute the resultant security value. The difference in security values divided by the magnitude of the perturbation in the respective state variable approximates the partial derivative or dollar sensitivity.

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