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Volume Author/Editor: Haitovsky, Yoel, George Treyz, and Vincent Su

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Chapter Title: The Decomposition of Forecasting Error: Methodology

Chapter Author: Yoel Haitovsky, George Treyz, Vincent Su

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Chapter pages in book: (p. 135 - 151)

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onstant Adjustment, 1st Q '53–4th Q '64		
67	0.93	6.60
80	−0.60	4.18
nstant Adjustment, 1st Q '65–4th Q '68		
61	−57.74	6.88
43	−53.87	14.56
nstant Adjustment,[a] 1st Q '53–4th Q '64		
10	1.39	6.79
71	0.04	4.75
22	3.82	10.03
nstant Adjustment,[a] 1st Q '65–4th Q '68		
37	−5.52	9.84
10	−6.88	5.53
09	−0.69	13.40

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Part 2

FORECASTS AND ERROR DECOMPOSITION

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4.1 INTRODUCTION

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The Decomposition of Forecasting Error: Methodology

4.1 INTRODUCTION

The forecasting errors of the Wharton and OBE models are traced to their sources via individual structural equations in the next two chapters. Here we explain our procedure, which permits a decomposition of forecasting errors into the following components of error: (a) the part attributable to the structural equation explaining the variable in question; (b) the part attributable to the rest of the system, including the portion due to the error in (a) after its reverberation throughout the system; (c) the error attributable to the forecaster's failure to correct the stochastic equations fully by adjusting for these problems; (d) the error caused by the forecaster's incorrect guesses as to the future values of the exogenous variable in the model; and (e) the error caused by lagged endogenous variables in multiperiod forecasts.

The breakdown of forecasting errors along these lines enables us to answer a number of questions. Which sector or which specification is primarily responsible for the errors in the model? To what extent do some of the errors systematically tend to cancel or intensify each other? How much does the simultaneous manipulation of both the constant adjustments and the exogenous values improve or hurt the forecasts? To what degree can error be attributed to lags in multiperiod forecasts?

In the presentation that follows we feature a decomposition of GNP, since the GNP series may be considered as an overall single measure of the forecasting performance of a model. The forecasting error in GNP is decomposed into the errors originating in the structural equations describing the endogenous determinants of GNP (these include its demand components), the components of disposable income (evaluated from the supply side), and the effect of the price level forecast. This procedure allows us to break down the observed forecast error into the five components listed above. We illustrate our arguments with a simple linear model. Nonlinearities in the solution of the model will be dealt with only in a heuristic way. It will be shown later that the effect of nonlinearity within the range of our interest appears to be inconsequential.

Our procedure uses the values of coefficients as estimated by the model builders. Thus, we trace the effects of observed error in individual equations on forecast error, using the specification of the model and the estimates of the structural parameters used for the forecast in question. Since the values of the true parameters for the equations are unknowable, they cannot be used. Furthermore, the adjustments of the individual equations by the econometric forecasters were based on the estimated parameters of the model. It should be noted that, while our procedure is appropriate for the systems forecasts we analyze, other procedures should be used to estimate what portion of forecast error is attributable to the inherent need for estimating structural parameters derived from a short sample period rather than using the true values of these parameters.[1]

4.2 ILLUSTRATION WITH A SIMPLE LINEAR MODEL[2]

The first step is a condensed version of the model presented in Chapter 3, with the following structural equations:

[1] See, for example, T. M. Brown, "Standard Errors of Forecast of a Complete Econometric Model," *Econometrica*, Vol. 22, April 1954, pp. 178–92; J. W. Hooper and A. Zellner, "The Error of Forecast for Multivariate Regression Models," *Econometrica*, Vol. 29, October 1961, pp. 544–55; George R. Schink, "Small Sample Estimates of the Variance Covariance Matrix of Forecast Error For Large Econometric Models: The Stochastic Simulation Technique," Ph.D. dissertation, University of Pennsylvania, 1971.

[2] For material similar to some of the ideas in this section, see also P. Paulopoulos, *A*

Aggregate consumption:

$$C_t = \alpha + \beta D$$

Aggregate investment:

$$I_t = \gamma Y_t +$$

Net of transfers and retained earnings:

$$D_t = \xi Y_t$$

National income identity:

$$Y_t = C_t +$$

Disposable income identity:

$$DI_t = Y_t + D$$

Government expenditure:

$$G_t = \text{Exo}$$

Tax revenues:

$$T_t = \text{Exo}$$

From these one can derive the reduc

$$C_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{\alpha(1 - \gamma)$$

$$+ \beta(1$$

$$I_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{\alpha\gamma - \beta$$

$$Y_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{\alpha - \beta(T$$

$$D_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{\alpha\xi - \beta$$

Statistical Model for the Greek Economy 19
Company, 1966.

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78–92; J. W. Hooper and A. Zellner, "The Error
," *Econometrica*, Vol. 29, October 1961, pp.
timates of the Variance Covariance Matrix of
The Stochastic Simulation Technique," Ph.D.

as in this section, see also P. Paulopoulos, *A*

Aggregate consumption:

$$C_t = \alpha + \beta DI_t + U_t \tag{4.1}$$

Aggregate investment:

$$I_t = \gamma Y_t + W_t \tag{4.2}$$

Net of transfers and retained earnings:

$$D_t = \xi Y_t + V_t \tag{4.3}$$

National income identity:

$$Y_t = C_t + I_t + G_t \tag{4.4}$$

Disposable income identity:

$$DI_t = Y_t + D_t - T_t \tag{4.5}$$

Government expenditure:

$$G_t = \text{Exogenous} \tag{4.6}$$

Tax revenues:

$$T_t = \text{Exogenous} \tag{4.7}$$

From these one can derive the reduced form:

$$C_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} [\alpha(1 - \gamma) - \beta(1 - \gamma)(T_t + V_t) + \beta(1 - \xi)(G_t + W_t) + (1 - \gamma)U_t] \tag{4.8}$$

$$I_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{\alpha\gamma - \beta\gamma(T_t + V_t) + \gamma G_t + [1 - \beta(1 - \xi)]W_t + \gamma U_t\} \tag{4.9}$$

$$Y_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} [\alpha - \beta(T_t + V_t) + G_t + W_t + U_t] \tag{4.10}$$

$$D_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} [\alpha\xi - \beta\xi T_t + (1 - \gamma - \beta)V_t + \xi G_t + \xi W_t + \xi U_t] \tag{4.11}$$

Statistical Model for the Greek Economy 1949–1959, Amsterdam, North-Holland Publishing Company, 1966.

Let us compare four types of forecasts, which will differ from each other in two ways: (a) with respect to assumptions regarding knowledge of the values of the exogenous variables essential for the forecasts; and (b) with respect to the ad hoc adjustments made on the structural equations in specific forecasts.

In the first case we shall distinguish between ex post forecasts, where it is assumed that the exogenous values are known, and ex ante forecasts, where the forecaster provides his best guesses about future exogenous values. In the second case, we will show different adjustments made on the structural equations. These usually take the form of an additive disturbance inserted into the single equations in order to account for either the development of exogenous factors *not* included in the model in accordance with the forecaster's judgments, or the patterns in the residuals of the particular equation and any shifts the forecaster may observe in the latest observable periods. This adjustment is usually termed "constant adjustment" because it is accomplished by changing the constant term (intercept) in the structural equations. Thus, if we consider two types of constant adjustments, we have four combinations of both types of forecasts:

1. Ex post plus constant adjustment type I;
2. Ex post plus constant adjustment type II;
3. Ex ante plus constant adjustment type I; and
4. Ex ante plus constant adjustment type II.

Let the ex post exogenous values be denoted by the superscript p, the ex ante guesses of the exogenous values by the superscript a, and their difference by δ. Thus,

$$T^a - T^p = \delta T, \qquad T^p - T^a = -\delta T,$$

$$G^a - G^p = \delta G, \qquad G^p - G^a = -\delta T,$$

and, similarly,

$$\delta W = W^{I} - W^{II}, \qquad \delta U = U^{I} - U^{II}, \qquad \delta V = V^{I} - V^{II}$$

for the constant adjustments types I and II.

Thus, the difference between these forecasts is obtained, with the appropriate change of the sign, by

$$\delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(1 - \gamma)(\delta T + \delta V) + \beta(1 - \xi)(\delta G + \delta W) + (1 - \gamma)\delta U] \qquad (4.12)$$

$$\delta I = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{-\beta\gamma(\delta T +$$

$$+ [1$$

$$\delta Y = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(\delta T + \delta$$

and

$$\delta RE = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta\xi\delta T$$

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$$\delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-$$

where δT and δG are the error
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Similarly, if we want to investi

$$\delta I = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{-\beta\gamma(\delta T + \delta V) + \gamma\delta G + [1 - \beta(1 - \xi)]\delta W + \gamma\delta U\} \tag{4.13}$$

$$\delta Y = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(\delta T + \delta V) + \delta G + \delta W + \delta U] \tag{4.14}$$

and

$$\delta RE = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta\xi\delta T + (1 - \gamma - \beta)\delta V + \xi\delta G + \xi\delta W + \xi\delta U]. \tag{4.15}$$

The system of the last four equations provides the framework for tracing the forecasting errors of the different forecasting methods in linear systems. In particular, it can be used to explain the differences between the ex ante and ex post single-period forecasts (in systems without lags) for different constant adjustments. For instance, if we want to calculate the difference between ex post with constant adjustments type I and ex ante with constant adjustments type II, we need to insert $-\delta T$ and $-\delta G$ (the ex post-ex ante discrepancies) and δU, δW, and δV (the discrepancies in constant adjustments) and then, knowing the values of the estimated parameters β, γ, ξ, compute δC, δI, δY and δRE. We can also investigate the *pure* effect of either ex post versus ex ante forecast, or differences in the constant adjustments, by setting $\delta U = \delta W = \delta V = 0$ for the former and $\delta T = \delta G = 0$ for the latter. The special feature of linear systems is that the total effect of the ex post versus ex ante discrepancies and the constant adjustment differences is made up of the sum of the pure effects. For instance, the forecasting error in C attributable solely to the ex ante-ex post discrepancies in the two exogenous variables in the system, G and T, is given by

$$\delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(1 - \xi)\delta T + \beta(1 - \xi)\delta G], \tag{4.12a}$$

where δT and δG are the errors the forecaster made in guessing, respectively, the tax revenues T and government expenditures G. Similarly, if we want to investigate the additional forecasting error attributable to the "no constant adjustments" beyond that of *AR* constant adjustment[3] in ex post forecasts, we merely interpret type I and II "constant adjustments" above as *AR* and "no constant adjustments," respectively, and set $\delta T = \delta G = 0$, obtaining

$$\delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} [- \beta(1 - \gamma)\delta V + \beta(1 - \xi)\delta W + (1 - \gamma)\delta U]. \tag{4.12b}$$

The combined effect of the forecasting error attributable to both ex ante-ex post discrepancies and to the two types of constant adjustments is obtained by adding equations (4.12a) and (4.12b). If, on the other hand, the model were nonlinear, the two additive effects would account only approximately for the combined effect, depending upon the degree of nonlinearity.

A special kind of constant adjustment, an artificial one designed to facilitate the forecasting error analysis, is obtained by computing for a particular t:

$$U_t^* = C_t - \alpha - \beta(Y_t - D_t - T_t) \tag{4.16}$$

$$W_t^* = I - \gamma T_t \tag{4.17}$$

and

$$V_t^* = D_t - \xi Y_t. \tag{4.18}$$

The residuals on the left hand side of equations (4.16)–(4.18) are called "structural equation residuals." They are obtained by substituting the realized values for the endogenous variables appearing on the right hand side of structural equations (4.1)–(4.3), rather than the model solution values used in both ex post *and* ex ante forecasts. The concept of "structural equation residuals" (henceforth denoted by *SER*) has the advantage of isolating forecasting errors due to the specification of the equation under investigation from the error due to the simultaneity of the model. Furthermore, the *SER*s can be incorporated easily into the general framework developed above. For this, an alternative interpretation for the *SER*s should be adopted—namely, that if the *SER*s were simultaneously employed as constant adjustments for all stochastic equations in the model, the ex post forecasting errors would vanish. Thus, δC, δI, δD, and δY now become the forecasting errors of the forecasts with the constant

[3] See Chapter 1, p. 9.

adjustments under investigation. That viewed here as the sum of the discrep *SER*s (U_t^*, W_t^*, V_t^*) and the constant forecast, plus the discrepancies betwee of the exogenous values, all weighted

4.3 A NONLINEAR EXAMPLE

The additive feature just describe these systems there are additional exogenous variable discrepancies and as well as interactions within these tw

One simple example of a nonline problems involved. Despite its simplici that are usually present in macroecon in the endogenous variables are intro in the consumption function

$$C_t = \alpha^* + \beta^* DI_t$$

where $P_t = \mu'[Y_t/(Y_t - YMAX_t)]$ is maximum attainable gross national assumed to be exogenous in our sim inversely related to the percentage th attainable level at a particular time

If we solve now for real GNP

$$Y_t^2[1 - \beta^*(1 - \xi) - \gamma] - Y[\alpha^* -$$

$$+ \mu^* + U_t +$$

where $\mu^* = \mu/\mu'$, and, thus,[4]

$$Y_t = \frac{1}{1 - \beta^*(1 - \xi) - \gamma} \frac{1}{2}\Big(\alpha^*$$

$$+ \mu^* + U_t + W_t + G_t$$

$$+ \sqrt{\{[\bar{\alpha}^* - \beta^*(V_t + T_t) +}$$

$$- 4[1$$

[4] Another solution exists, too, with a n only the one presented here will be economi

adjustments under investigation. That is, the forecasting errors are viewed here as the sum of the discrepancies between the "observed" *SERs* (U_t^*, W_t^*, V_t^*) and the constant adjustments actually used in the forecast, plus the discrepancies between the actual and guessed values of the exogenous values, all weighted by their respective multipliers.

4.3 A NONLINEAR EXAMPLE

The additive feature just described is lost in nonlinear systems. In these systems there are additional terms of interaction between exogenous variable discrepancies and constant adjustment differences, as well as interactions within these two groups.

One simple example of a nonlinear model will serve to illustrate the problems involved. Despite its simplicity it is typical of the nonlinearities that are usually present in macroeconometric models. The nonlinearities in the endogenous variables are introduced via a "money illusion" effect in the consumption function

$$C_t = \alpha^* + \beta^* DI_t + \mu(1/P_t) + U_t, \qquad (4.19)$$

where $P_t = \mu'[Y_t/(Y_t - YMAX_t)]$ is the price level and *YMAX* is the maximum attainable gross national product in real terms, which is assumed to be exogenous in our simple model. That is, the price level is inversely related to the percentage that real GNP falls short of its highest attainable level at a particular time period.

If we solve now for real GNP we obtain:

$$Y_t^2[1 - \beta^*(1 - \xi) - \gamma] - Y[\alpha^* - \beta^*(V_t + T_t) + \mu^* + U_t + W_t + G_t] + \mu^* YMAX_t = 0, \qquad (4.20)$$

where $\mu^* = \mu/\mu'$, and, thus,[4]

$$Y_t = \frac{1}{1 - \beta^*(1 - \xi) - \gamma}\,\frac{1}{2}\left(\alpha^* - \beta^*(V_t + T_t) + \mu^* + U_t + W_t + G_t + \sqrt{[\alpha^* - \beta^*(V_t + T_t) + \mu^* + U_t + W_t + G_t]^2 - 4[1 - \beta^*(1 - \xi) - \gamma]\mu^* YMAX_t}\right). \qquad (4.21)$$

[4] Another solution exists, too, with a negative sign preceding the square root, but usually only the one presented here will be economically feasible.

The differences operators can be applied to the nonlinear model. Ignoring second order differencing and using the above relationship (4.21), we arrive at an expression of the discrepancy between two predicted values of real GNP with two different constant adjustments and different guesses about the values of the exogenous variables (*YMAX* was treated here as a definition in which no error can occur):

$$\delta Y \approx \frac{-\beta^*(\delta V + \delta T) + \delta U + \delta W + \delta G}{2[1 - \beta^*(1 - \xi) - \gamma]\{1 - 1/(1 + \sqrt{1 - D})\}} \qquad (4.22)$$

$$\text{where } D = \frac{4[1 - \beta^*(1 - \xi) - \gamma]\mu^* YMAX_t}{[\alpha^* + \mu^* - \beta^*(V_t + T_t) + W_t + G_t + U_t]^2}.$$

Notice that by setting $\mu^* = 0$ (or $YMAX_t = 0$) we eliminate the nonlinearity in the model. Indeed, equation (4.22) can be reduced to the expression for the corresponding linear system, i.e., to equation (4.14). More important, it shows that, although the multiplier, being a function of the random variables *V, T, W, G, U,* and *UMAX,* is itself a random variable, it will hardly change with small variations in those variables. The next two chapters will demonstrate that the ex post-ex ante discrepancies varied only slightly when different constant adjustments were made. This last point is important because it allows us to decompose the total forecasting error into its additive components, and to ignore the nonlinear effect of the slight difference in "initial conditions" for alternative forecasts of the same equation.

4.4 STRUCTURAL EQUATION RESIDUALS VERSUS FORECASTING ERROR

In the second set of tables of Chapters 5 and 6, column I lists the *SER* minus the constant adjustments[5] for the stochastic equations of all endogenous components of GNP and disposable income, respectively. In our notation these values can be properly expressed by δU and δW (the discrepancies between two adjustment procedures for the residuals of GNP's endogenous components) and by δV (the discrepancies in the

[5] No constant adjustment is considered here a special case of constant adjustment, where the constant adjustment is equal to zero.

be applied to the nonlinear model.
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two different constant adjustments
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$$+ \delta U + \delta W + \delta G \over \gamma][1 - 1/(1 + \sqrt{1 - D})\} \qquad (4.22)$$

$$- \gamma]\mu^* YMAX_t \over) + W_t + G_t + U_t]^2$$

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RESIDUALS VERSUS FORE-

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endogenous component of disposable income). Now we assign the *SER* to constant type I, and the constant adjustment under consideration to constant adjustment type II in the definitions of the δ operator. In our tables the values in the first column are subtracted from the corresponding forecasting errors, which are listed in the third column. Thus, in the second column we get

$$\delta C - \delta U = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(1 - \gamma)\delta V + \beta(1 - \xi)\delta W + \beta(1 - \xi)\delta U] \qquad (4.23)$$

and

$$\delta I - \delta W = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta\delta V + \delta W + \delta U], \qquad (4.24)$$

and the sum of both:

$$\delta C + \delta I - \delta U - \delta W = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{-\beta\delta V + [\gamma + \beta(1 - \xi)](\delta W + \delta U)\} \qquad (4.25)$$

$$= \frac{-\beta\delta V + \delta W + \delta U}{1 - \beta(1 - \xi) - \gamma} - \delta W - \delta U. \qquad (4.25a)$$

Equations (4.23) to (4.25) show us how to decompose the ex post forecasting error into (a) the error due to the specifications of the particular equations and (b) that due to the simultaneity of the model. The former is the error attributable directly to the equation in question because it is calculated under the pretense that the "true" values of the variables on the right hand side of the equation are known. The latter is the indirect effect resulting from the reverberation of the *SER*, adjusted by the constant adjustments, throughout the system. The effect of the reverberation throughout the system is given in equations (4.23)–(4.24) by the appropriate multipliers.

For instance, the indirect effect of δU on the consumption error is given by multiplying δU by $\beta(1 - \xi)/[1 - \beta(1 - \xi) - \gamma]$; this is the induced (indirect) effect of U on consumption. When we add up all the errors in the endogenous components of GNP (see equation 4.25a) we get the total (indirect plus direct) effect of the errors in these components less the direct effect, leaving only the indirect effect after their reverberation through the system. Moreover, the induced effect of the errors included in disposable income, $-\delta V$ in our simple example, should be added. Therefore, the direct effects of the errors in the disposable income components are listed in the same set of tables. They are multiplied by the appropriate multipliers and then summed up.

4.5 THE PRICE EFFECT

Next, it is interesting to isolate the price effect from the real value effect on nominal GNP. To this end we use the formula

$$GNP_t = P_t \cdot Y_t, \tag{4.26}$$

and thus

$$\delta GNP = (Y_t + \delta Y)\delta P + P\delta Y. \tag{4.27}$$

We call the first term on the right hand side of (4.27) "error due to price." This error can be further decomposed into the errors in the components of real GNP and their corresponding prices. For this, let us denote the real GNP components by Y_i (i.e., $Y = \sum Y_i$) and their corresponding prices by P_i. We have

$$P = GNP/Y = \sum Y_i P_i / \sum Y_i \tag{4.28}$$

and, applying the δ operator, we get

$$(P + \delta P)\left[\sum (Y_i + \delta Y_i)\right] = \sum (Y_i + \delta Y_i)(P_i + \delta P_i). \tag{4.29}$$

Subtracting (4.28) from (4.29), we get

$$\delta P \sum (Y_i + \delta Y_i) = \sum \delta Y_i (P_i - P) + \sum (Y_i + \delta Y_i)\delta P_i. \tag{4.30}$$

Unless we have peculiar situations in which large discrepancies in δY_i are systematically associated with positive or negative discrepancies between the corresponding prices, P_i and P, the first term on the right hand side of (4.30) can be ignored, and we finally get

$$\delta P \sum (Y_i + \delta Y_i) \approx \sum (Y_i + \delta Y_i)\delta P_i. \tag{4.31}$$

The left hand side is nothing more than the "error due to price." The

expression on the right of (4.31) c
sets of endogenous and exogeno
mentioned above we have further
into the exogenous and endogenou
the ex post-ex ante error decompo

4.6 MULTIPERIOD FORECAST

We have deliberately avoided
now to introduce them. To illustr
errors we resort to our simple linea
and investment equations to inclu

$$C_t = \alpha + \sum_{i=0}^{p} \beta_i \ldots$$

$$I_t = \sum_{j=0}^{q} \gamma_j Y_t \ldots$$

where (say) $q > p$ and $\beta_{p+1} = \ldots$
for Y becomes

$$Y_t = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0}\left\{ \alpha + \ldots \right.$$

$$- \sum_{i=0}^{q} \beta_i (V_{t-i} \ldots$$

and, applying the differencing oper

$$\delta Y = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0}\left\{ \sum_{i=1}^{q} \ldots \right.$$

$$- \sum_{i=0}^{q} \beta_i (\delta V_{t-} \ldots$$

That is, the multiperiod forecastin
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plate the price effect from the real value
end we use the formula

$$P_t = P_t \cdot Y_t. \quad (4.26)$$

$$Y_t + \delta Y)\delta P + P\delta Y. \quad (4.27)$$

t hand side of (4.27) "error due to price."
posed into the errors in the components
ling prices. For this, let us denote the real
$\sum Y_i)$ and their corresponding prices by

$$Y = \sum Y_i P_i / \sum Y_i \quad (4.28)$$

get

$$)] = \sum (Y_i + \delta Y_i)(P_i + \delta P_i). \quad (4.29)$$

ve get

$$Y_i(P_i - P) + \sum (Y_i + \delta Y_i)\delta P_i. \quad (4.30)$$

uations in which large discrepancies in
with positive or negative discrepancies
s, P_i and P, the first term on the right
ed, and we finally get

$$) \approx \sum (Y_i + \delta Y_i)\delta P_i. \quad (4.31)$$

re than the "error due to price." The

expression on the right of (4.31) can be further decomposed into two sets of endogenous and exogenous prices. In the two sets of tables mentioned above we have further decomposed the "error due to price" into the exogenous and endogenous price effects, which serve as part of the ex post-ex ante error decomposition.

4.6 MULTIPERIOD FORECASTS

We have deliberately avoided lags in our simple model, but it is time now to introduce them. To illustrate the effect of lags on forecasting errors we resort to our simple linear models, but modify the consumption and investment equations to include lags:

$$C_t = \alpha + \sum_{i=0}^{p} \beta_i DI_{t-i} + U_t. \quad (4.32)$$

$$I_t = \sum_{j=0}^{q} \gamma_j Y_{t-j} + W_t \quad (4.33)$$

where (say) $q > p$ and $\beta_{p+1} = \ldots = \beta_q = 0$. The reduced form equation for Y becomes

$$Y_t = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \alpha + \sum_{i=1}^{q} Y_{t-i}[\beta_i(1 - \xi) + \gamma_i] - \sum_{i=0}^{q} \beta_i(V_{t-i} + T_{t-i}) + U_t + W_t + G_t \right\} \quad (4.34)$$

and, applying the differencing operator δ, we finally get

$$\delta Y = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \sum_{i=1}^{q} \delta Y_{t-i}[\beta_i(1 - \xi) + \gamma_i] - \sum_{i=0}^{q} \beta_i(\delta V_{t-i} + \delta T_{t-i}) + \delta U + \delta W + \delta G \right\}. \quad (4.35)$$

That is, the multiperiod forecasting error in period t is the sum of (a) the errors made in Y in the earlier periods, weighted by the respective marginal propensities to consume times the leakage in retained earnings and marginal propensities to invest appropriate to each period, (b) the errors in the retained earnings equation and the errors made by the forecasters in guessing the values of the exogenous variable T in their ex ante forecasts, weighted by the marginal propensity to consume, and (c) the contemporaneous errors in consumption, investment, and the exogenous variable government expenditure (in ex ante forecasts)—all multiplied by the multiplier.

Alternatively, equation (4.35) can be rewritten as

$$\delta Y = \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\left\{\sum_{i=1}^{q}\gamma_i(\delta Y)_{t-i} - \beta_0(\delta V + \delta T) + \sum_{i=1}^{q}\beta_i(\delta DI)_{t-i} + \delta U + \delta W + \delta G\right\}. \tag{4.36}$$

This, again, is a function of the earlier errors in real income and disposable income and of the contemporaneous errors in the real income and disposable income components, all weighted by the appropriate marginal propensities and the multiplier.

Our analytical scheme can be used to detect the effect of lags on forecasting errors. This is done by comparing a long-span forecast aimed at a particular period with shorter-span forecasts made later, and aimed at the same period. This comparison is useful for the error decomposition because the *SER*s pertaining to the same period are the same, irrespective of the forecasting span they represent, and thus the remaining error is due to the different lags. In order to put this formally we write

$$\delta Y_{t(s)} = \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\left\{\sum_{i=1}^{r}\delta Y_{t(s-i)}[\beta_i(1-\xi)+\gamma_i] - \sum_{i=0}^{r}\beta_i(\delta V_{t(s-i)} + \delta T_{t(s-i)}) + \delta U_{t(s)} + \delta W_{t(s)} + \delta G_{t(s)}\right\}, \tag{4.37}$$

where $r = min(s,q)$ and the subscript in parentheses denotes the forecasting span, while the subscript preceding it denotes the jump-off period (the latest period for which data were available). Thus, the period for which the forecast was made is given by adding up both subscripts. Notice that by definition $\delta Y_{t(0)} = 0$.

Now we may decrease the forecasting span by one period and move the jump-off period ahead, since we wish to compare forecasts made for the same period. We obtain

$$\delta Y_{t+1(s-1)} = \frac{1}{1-\beta_0(1-\xi)-\gamma_0} \{ \ldots - \sum_{i=0}^{r-1}\beta_i(\delta V_{t+1(s-1-i)} + \delta T_{t+} \ldots$$

and now subtract (4.38) from (4.37

$$\delta Y_{t(s)} - \delta Y_{t+1(s-1)} = \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\left\{\sum_{i=1}^{r-1}(\delta Y_{t(} \ldots + \delta Y_{t(s-r)}[\beta_r(1-\xi)+\gamma_r] - \sum_{i=0}^{r-1} \ldots + \delta T_{t(s-i)} - \delta T_{t+1(s-1-i)}) - \beta_r \ldots + \delta U_{t(s)} - \delta U_{t+1(s-r)} + \delta W_{t(s} \ldots + \delta G_{t(s)} - \delta G_{t+1(s-1)}\right\}.$$

For instance, if our consumption fu
2) and our investment function (4.
compare a two-quarters-ahead
forecast, equation (4.39) reduces t

$$\delta Y_{t(2)} - \delta Y_{t+1(1)} = \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\left\{\delta Y_{t(} \ldots - \beta_0(\delta V_{t(2)} - \delta V_{t+1(1)} + \delta T_{t(2)} - \delta T_{t+1(1)}) - \beta_1 \ldots + \delta U_{t(2)} - \delta U_{t+1(1)} + \delta W \ldots + \delta G_{t(2)} - \delta G_{t+1(1)}\right\}.$$

The formula can be easil
four-quarter forecast with a firs
later and pertaining to the sam

ginal propensity to consume, and (c) consumption, investment, and the penditure (in ex ante forecasts)—all

can be rewritten as

$$\ldots(\delta Y)_{t-i} - \beta_0(\delta V + \delta T) \qquad (4.36)$$

$$\ldots(\delta DI)_{t-i} + \delta U + \delta W + \delta G \Big\} .$$

earlier errors in real income and mporaneous errors in the real income s, all weighted by the appropriate plier.

used to detect the effect of lags on mparing a long-span forecast aimed r-span forecasts made later, and mparison is useful for the error de- aining to the same period are the ng span they represent, and thus ferent lags. In order to put this for-

$$\ldots \delta Y_{t(s-i)}[\beta_i(1-\xi)+\gamma_i] \qquad (4.37)$$

$$\ldots \delta U_{t(s)} + \delta W_{t(s)} + \delta G_{t(s)} \Big\} ,$$

cript in parentheses denotes the preceding it denotes the jump-off ta were available). Thus, the period given by adding up both subscripts.

ecasting span by one period and nce we wish to compare forecasts

$$\delta Y_{t+1(s-1)} = \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\Bigg\{\sum_{i=1}^{r-1}\delta Y_{t+1(s-1-i)}[\beta_i(1-\xi)+\gamma_i] - \sum_{i=0}^{r-1}\beta_i(\delta V_{t+1(s-1-i)}+\delta T_{t+1(s-1-i)}) + \delta U_{t+1(s-1)} + \delta W_{t+1(s-1)} + \delta G_{t+1(s-1)}\Bigg\} \qquad (4.38)$$

and now subtract (4.38) from (4.37):

$$\begin{aligned}\delta Y_{t(s)} - \delta Y_{t+1(s-1)} &= \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\Bigg\{\sum_{i=1}^{r-1}(\delta Y_{t(s-i)} - \delta Y_{t+1(s-1-i)})[\beta_i(1-\xi)+\gamma_i] \\ &+ \delta Y_{t(s-r)}[\beta_r(1-\xi)+\gamma_r] - \sum_{i=0}^{r-1}\beta_i(\delta V_{t(s-i)} - \delta V_{t+1(s-1-i)} \\ &+ \delta T_{t(s-i)} - \delta T_{t+1(s-1-i)}) - \beta_r(\delta V_{t(s-r)} + \delta T_{t(s-r)}) \\ &+ \delta U_{t(s)} - \delta U_{t+1(s-r)} + \delta W_{t(s)} - \delta W_{t+1(s-1)} \\ &+ \delta G_{t(s)} - \delta G_{t+1(s-1)}\Bigg\}.\end{aligned} \qquad (4.39)$$

For instance, if our consumption function (4.32) contains two lags (p = 2) and our investment function (4.33), 3 lags (q = 3), and we wish to compare a two-quarters-ahead forecast with a one-quarter-ahead forecast, equation (4.39) reduces to

$$\begin{aligned}\delta Y_{t(2)} - \delta Y_{t+1(1)} &= \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\Big\{\delta Y_{t(1)}[\beta_1(1-\xi)+\gamma_1] \\ &- \beta_0(\delta V_{t(2)} - \delta V_{t+1(1)} \\ &+ \delta T_{t(2)} - \delta T_{t+1(1)}) - \beta_1(\delta V_{t(1)} + \delta T_{t(1)}) \\ &+ \delta U_{t(2)} - \delta U_{t+1(1)} + \delta W_{t(2)} - \delta W_{t(1)} \\ &+ \delta G_{t(2)} - \delta G_{t+1(1)}\Big\}.\end{aligned} \qquad (4.40)$$

The formula can be easily extended to compare, say, also a four-quarter forecast with a first-quarter forecast made three quarters later and pertaining to the same quarter that the four-quarter forecast was aiming at, i.e.:

$$\delta Y_{t(4)} - \delta Y_{t+3(1)}$$

$$= \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\Big\{[\beta_1(1-\xi)+\gamma_1]\delta Y_{t(3)} + [\beta_2(1-\xi)+\gamma_2]\delta Y_{t(2)} + \gamma_3\delta Y_{t(1)} - \beta_0[\delta(V+T)_{t(4)} - \delta(V+T)_{t+3(1)}] - \beta_1\delta(V+T)_{t(3)} - \beta_2\delta(V+T)_{t(2)} + \delta(U+W+C)_{t(4)} + \delta(U+W+G)_{t+3(1)}\Big\}. \tag{4.41}$$

Thus, as expected, the difference between any two forecasts made at two different points in time, but referring to the same time period, includes the errors in the endogenous and exogenous components of GNP which enter into the lags and, of course, the contemporaneous errors.

However, expressions (4.40) and (4.41) can be simplified in ex post no constant adjustments, since—as was pointed out before—there are no errors in the exogenous variables in ex post forecasts and the *SER*s are the same, irrespective of the forecasting span. Thus, (4.40) and (4.41), respectively, become

$$\delta Y_{t(2)} - \delta Y_{t+1(1)}$$

$$= \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\Big\{\delta Y_{t(1)}[\beta_1(1-\xi)+\gamma_1] - \beta_1\delta V_{t(1)}\Big\} \tag{4.42}$$

$$= \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\Big\{\gamma_1\delta Y_{t(1)} + \beta_1\gamma\delta DI_{t(1)}\Big\}$$

and

$$\delta Y_{t(4)} - \delta Y_{t+3(1)}$$

$$= \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\Big\{[\beta_1(1-\xi)+\gamma_1]\delta Y_{t(3)} + [\beta_2(1-\xi)+\gamma_2]\delta Y_{t(2)} + \gamma_3\delta Y_{t(1)} - \beta_1\delta V_{t(3)} - \beta_2\delta V_{t(2)}\Big\} \tag{4.43}$$

$$= \frac{1}{1-\beta_0(1-\xi)-\gamma_0}\Big\{\beta_1\delta DI_{t(3)} \ldots + \gamma_2\delta Y_{t(2)} + \gamma_3\delta Y_{t(1)}\Big\}.$$

Thus, to isolate the pure effect of lags
effects of different constant adjustme
the forecaster's wrong guesses as to
need only subtract the ex post "no co
in a later period from one pertaining to
This will be demonstrated in the last s

$$= \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \beta_1 \delta DI_{t(3)} + \beta_2 \delta DI_{t(2)} + \gamma_1 \delta Y_{t(3)} + \gamma_2 \delta Y_{t(2)} + \gamma_3 \delta Y_{t(1)} \right\}.$$

Thus, to isolate the pure effect of lags in multiperiod forecasts, with the effects of different constant adjustments in the various forecasts and of the forecaster's wrong guesses as to exogenous values filtered off, one need only subtract the ex post "no constant adjustment" forecast made in a later period from one pertaining to the same period but made earlier. This will be demonstrated in the last set of tables in Chapters 5 and 6.