Long-Term Growth Driven by a Sequence of General Purpose Technologies

D. Schiess and R. Wehrli

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Long-Term Growth Driven by a Sequence of General Purpose Technologies*

Daniel Schiess (ETH Zurich)† and Roger Wehrli (ETH Zurich)

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We present a Schumpterian model of endogenous growth with General Purpose Technologies (GPTs) that captures two important historical stylized facts: First, from the beginning of mankind until today GPTs are arriving at an increasing frequency and, second, all GPTs heavily depended on previous technologies. In our model, the arrival of GPTs is endogenous and arises stochastically depending on the currently available applied knowledge stock. This way of endogenizing the arrival of new GPTs allows for a model which is more in tune with the historical reality than the existing GPT models.

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JEL classification: O11, O33, O41

1 General Purpose Technologies and Long-Term Growth

A small number of ground-breaking inventions arriving in ever decreasing time intervals can be identified as important drivers of long-term economic growth: In early human history millennia lay between transforming innovations such as the domestication of plants and animals or the Bronze Age and the Iron Age. Later, the era of the industrial revolution witnessed the rise of the steam engine and production in large-scale factories, followed by the birth of railways and the steam ship in the course of only one century. Finally, current levels of welfare would hardly be possible without the introduction of personal computers and the rapid spread of the Internet usage within a few decades.

The view that such breakthrough technologies or "General Purpose Technologies", or GPTs, can be seen as true "engines of growth" has been shaped by Bresnahan and Trajtenberg (1995): They characterize GPTs as being radical innovations in the sense that they are " . . . characterized by the potential for pervasive use in a wide range of sectors and by their technological dynamism. As a GPT evolves and advances it spreads throughout the economy, bringing about and fostering generalized productivity gains." Furthermore they emphasize that GPTs are "enabling technologies", which give rise to new opportunities instead of offering complete, final solutions. As a matter of illustration, they present a static model where a monopolistic owner of the GPT interacts with corresponding application sectors.[1]

In the course of this paper we present a model where long-run growth is driven by a sequence of GPTs. Our model captures two stylized facts, which looking at the evolution of GPTs in history, immediately spring to mind: First, the time interval between the arrival of new GPTs has become ever shorter. This point can of course, at least partly, be seen as a result of the ever increasing pool of accumulated knowledge. Second, new GPTs are usually based on previously invented technologies and existing knowledge. Taking the Internet as an example, its invention would not have been possible without a multitude of previous inventions ranging from the computer to electricity. We achieve in modelling these stylized facts by extending our own model of Schumpeterian growth and GPTs, as presented in Schiess and Wehrli (2008). While in our previous model long term growth is driven by exogenously arriving new GPTs, we now endogenize the arrival of new GPTs. Specifically, the probability that a new GPT arrives depends on the amount of previously accumulated applied knowledge. This allows us to model long-term growth as driven by a sequence of GPTs which, due to the rising stock of applied knowledge, arrive at ever shorter time intervals. Furthermore, we model economic cycles within the lifetime of a single GPT, assuming that the economic impact of such a new technology decreases

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[1]In this model the assumption that the GPT is provided by a monopolist can, not surprisingly, result in too little research being performed in the sectors applying this technology.

over time.

Before we elaborate on the empirical regularities in the arrival of new GPTs which we address in our model, we turn to a brief survey of the previous literature on GPTs.

1.1 Previous Literature: Short-Term Cycles Caused by GPTs

While GPTs are widely seen as a main driving force of long-term economic growth, a majority of the literature on GPTs is concerned only with the lifetime of a single GPT, with a strong focus on the first phase after its introduction.[2]. This tendency is without doubt motivated by the goal of finding an explanation for the Solow paradox, according to which initially the already ubiquitous computers did not have any impact on productivity statistics. The phenomenon of a new GPT taking decades before having a major impact on economic aggregates is stated for example by David (1990), taking the example of electricity and the computer. Basu, Fernald and Kimball (2006) show in an empirical study, that while technological improvements are beneficial in the long run, they can have contractionary effects in the short run.

Directly referring to the Solow Paradox in the title of their paper, Jacobs and Nahuis (2002) present a model whereby the introduction of a new GPT leads to a decrease in output in the short run. This is caused by high-skill workers being drawn away from output production due to the sudden increase in research productivity induced by the new GPT.

Helpman and Trajtenberg (1998a) explain how the arrival of a new GPT can induce economic cycles as follows: A new, exogenously arriving GPT cannot be put to a productive use in the final goods sector immediately. In order to achieve this, a sufficient number of complementary components needs to be invented first (for example software in the case of computers). As this process requires resources to be moved from the final goods sector to the R&D sector, this results in a temporary decline in measured output. As soon as a sufficient number of components has been developed, the new GPT can be used in the production of the final good, leading to output growth picking up speed again. In Helpman and Trajtenberg (1998b) they additionally model how the diffusion of a new GPT to the various sectors of the economy can prevent it from having an immediate impact on output growth.

Aghion and Howitt (1998a) argue that an immediate slowdown in output production after the arrival of a new GPT (as implied by Helpman and Trajtenberg, 1998 a & b) can hardly be observed empirically. While they retain a component building phase, they add a template-building phase coming into effect immedi-

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[2]See Wehrli and Saxby (2008) for a more in-depth literature review

ately after the arrival of a new GPT: Each firm must first develop a "template" specific to its sector, before the economy moves to the component building phase. As these templates are designed by specialized labor without any other uses, there is no measured impact on output during this initial phase.

In the model of Eriksson and Lindh (2000), which is based on Helpman and Trajtenberg (1998a), components that were built for the old GPT can partially be used even after the arrival of a new GPT.[3] Furthermore the arrival of a new GPT is endogenized such that an increase in the number of components built for the current GPT leads to an increase in the probability of a new GPT arriving.

1.2 Previous Literature: GPT Models of Long-Term growth

Despite the fact that GPTs are widely seen as a main driving force of economic growth, models on GPT-driven long-term growth are relatively scarce: Aghion and Howitt (1998b) present a Schumpeterian model where long-term growth is driven by a sequence of innovations with an arrival rate which is proportional to the amount of labor devoted to research. While this model does not directly apply the concept of GPTs it is nevertheless a starting point when trying to explain growth driven by an endogenously created series of innovations.

Explicitly modeling GPTs, Carlaw and Lipsey (2006) developed a model with a sequence of GPTs arriving one after the other, with only one being active in any given period. In their model the arrival of a new GPT is governed by a constant random variable, hence the expected time interval between two GPTs remains always the same. Meanwhile, there is a variation in the size of the impact of a new GPT rising from the endogenously created pool of basic knowledge.

Van Zon and Kronenberg (2003) present a model where again long-term growth is driven by the arrival of new GPTs. While the expected time interval between the arrivals of new GPTs is again fixed, their model allows for different GPTs being active simultaneously. In a further refinement of their baseline model, van Zon and Kronenberg (2006) evaluate different tax policy measures in a scenario where long-run growth is driven by GPTs which are based either on carbon fuels or on non-carbon fuels. The probability of arrival of these GPTs is governed by the amount of currently performed basic R&D and they furthermore allow for various GPTs existing simultaneously.

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[3]This assumption mitigates the initial adverse impact of a new GPT in a similar fashion as the introduction of the template building phase by Aghion and Howitt (1998).

1.3 Stylized Fact 1: Decreasing Time Intervals between GPTs

While the question which technologies qualify as a GPT cannot be answered in a conclusive way,[4] the fact that such transforming technologies arrive at an ever faster pace seems to be undeniable. Taking the list of historical GPTs compiled by Carlaw and Lipsey (2006) as shown in Table 1 as a point of reference, it becomes clear that the interval between the arrival of individual GPTs has steadily decreased in the course of history. This general trend is described by Carlaw and Lipsey (2008, p. 131-133) as follows:

> "Although others might expand or contract our list by a few items it illustrates several important points. First, the current ICT revolution is not unique; there have been (GPT-driven) 'new economies' in the past. Second, GPTs have not been common in human experience, averaging between two and three per millennium over the last 10,000 years. Third, the rate of innovation of GPTs had been accelerating over the whole period. We start with millennia between GPTs, then centuries. In the eighteenth century there are two important GPTs, four in the nineteenth century, and seven in the twentieth."

Despite this strong empirical pattern of an acceleration in the arrival rate of new GPTs in the course of history, none of the previously mentioned models considers this fact: On one hand, the models on the impact of new GPTs on the course of a single economic cycle are by definition not concerned with a sequence of GPTs. On the other hand, the presented models on long-term growth driven by a sequence of GPTs either assume either fixed time intervals between the arrival of new GPTs or a stochastic pattern with no long-term trend in either direction.

1.4 Stylized Fact 2: GPTs Based on Current Stock of Applied Knowledge

Sir Isaac Newton is frequently quoted, for example by Scotchmer (1991), to illustrate that even the greatest minds in history depend on already existent knowledge: "If I have seen far, it is by standing on the shoulders of giants." Just as non-radical inventions more often than not build on previously existing knowledge, all GPTs had its origins to a certain extent in already present technologies.

Even a cursory glance at some of the GPTs in the past shows this very clearly: The invention of moveable type printing by Johannes Gutenberg in the sec-

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[4]see Lipsey, Carlaw and Bekar (1998) for further elaborations.

No.	GPT	Date
1	Domestication of plants	9000 - 8000 BC
2	Domestication of animals	8500 - 7500 BC
3	Smelting of ore	8000 - 7000 BC
4	Wheel	4000 - 3000 BC
5	Writing	3400 - 3200 BC
6	Bronze	2800 BC
7	Iron	1200 BC
8	Waterwheel	Early medieval period
9	Three-masted sailing ship	15th century
10	Printing	16th century
11	Steam engine	Late 18th to early 19th century
12	Factory system	Late 18th to early 19th century
13	Railway	Mid 19th century
14	Iron steamship	Mid 19th century
15	Internal combustion engine	Late 19th century
16	Electricity	Late 19th century
17	Motor vehicle	20th century
18	Airplane	20th century
19	Mass production, continuous process, factory	20th century
20	Computer	20th century
21	Lean production	20th century
22	Internet	20th century
23	Biotechnology	20th century
24	Nanotechnology	Sometime in the 21st century

Table 1: Historical GPTs as listed in Lipsey et al. (2005, p. 132)

ond half of the 15th century dramatically changed the way how both secular and religious knowledge was disseminated: Without this invention neither the spread of Protestant Revolution nor the diffusion of scientific knowledge beyond the walls of monasteries and early universities would have been possible at the pace observed in history. Gutenberg made the huge step away from previous methods of reproduction of written information (such as woodblock printing and the production of manuscripts on parchment) by combining a multitude of existing technologies, instead of starting from scratch: The moveable types were derived from stamps used by jewelers to mark their products, while the printing press itself was modeled on the wine press. Paper already existed in his time and while not suited very well for handwritten volumes, turned out to be ideal for this new application. Furthermore ink similar to the one used for handwriting, instead of the ink used for woodblock printing was applied by Gutenberg. Altogether, despite the fact that this achievement took a great extent of ingenuity and inventive spirit, the invention of modern book printing relied heavily on previously existent applied knowledge from various sectors of the economy.

The same reasoning holds true for the steam engine, which was invented by James Watt towards the end of the 18th century. Previously steam has already been used to drive atmospheric engines, such as the Newcomen engine. Again a large part of James Watt's genius laid in combining a number of already

existent technologies: His steam engine still maintained the basic principle of using steam to move a piston within a cylinder, but while previous engines used atmospheric of atmospheric pressure to drive the power stroke, his engine used steam for this vital step. Another major improvement was that, while in earlier machines steam was condensed inside the cylinder, he added a separate condenser to cool down the steam exhausted from the piston. These two modifications to previous steam engines allowed the Watt engine to extract much larger horsepower from a machine of a given size with a significantly larger fuel efficiency. Not only did this invention heavily build on previously existing technologies, it also facilitated the birth of subsequent GPTs such as the iron steam ship, railways and ultimately the internal combustion engine and the automobile.

As a final example, the invention of the computer would have been impossible without another GPT, electricity, being already in existence. It also relied on previous practical inventions: These date back as far as the idea of storing information on punched cards, as pioneered in the form of the Jacquard loom, and ideas from the theoretical foundations on computing laid out by Alan Turing in the 1930s. Similar musings could, to various degrees, be done for all GPTs in history.

The notion that a higher amount of available knowledge leads to a higher productivity of research has also been contemplated by Romer (1990), p.S83f. *"According to this specification, a college-educated engineer working today and one working 100 years ago have the same human capital, which is measured in terms of years of forgone participation in the labor market. The engineer working today is more productive because he or she can take advantage of all the additional knowledge accumulated as design problems were solved during the last 100 years."*

The remainder of this paper is structured as follows: In Section 2 we will present our theoretical model of GPT-driven endogenous growth. The main implications of our model and the results of our simulations will be shown in Section 3, while we conclude and offer some outlook in Section 4.

2 Long-Term Growth Driven by a Sequence of GPTs

We present a Schumpeterian model of long-term growth in a quality-ladder framework which is a discrete-time version of the model presented in Schiess and Wehrli (2008):[5] The productivity of a current GPT, as given by its im-

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[5]The original model, contrary to the present model, focuses on the time frame immediately before the arrival of a new GPT, whereby the economy before and after this transition phase is characterized by steady-state behavior.

pact on the productivity of quality-improving R&D, is subject to obsolescence.[6] The arrival of a new GPT therefore gives a new boost to the efficiency of R&D. Therefore long-term growth is driven by a sequence of GPTs. Our model has a unique combination of features: On one hand, it focuses on the long-term evolution of the economy. On the other hand, we endogenize the arrival of new GPTs, whereby the probability of discovering a new GPT positively depends on the currently available stock of applied knowledge. Together, these characteristics of our model allow for a more realistic representation of some aspects of the historical development of GPTs: Our model describes long-term sustained growth through the arrival of successive GPTs which arrive endogenously, due to the present stock of applied knowledge. Partly as a result of this, we can account for the fact that the time interval between the arrival of new GPTs has constantly decreased over the course of history.

2.1 Model Overview

There are three sectors present in our model: The final goods sector, the intermediate goods sector and consumption. Being a quality ladder model there is a fixed variety of intermediate goods, each with a corresponding current quality level. As the final good is assembled using these intermediate goods, growth in this model is driven by improvement in the quality level of these intermediate goods. The aggregate amount of final goods in turn can be used for either consumption C, as an input Z needed to perform R&D, or for intermediate goods X. An amount of Y_i of final good i is produced using a constant technology A and labor L_i according to the following production function.

$$Y_i = AL_i^{1-\alpha} \sum_{j=1}^{N} (\tilde{X}_{ij})^\alpha \tag{1}$$

where $0 < \alpha < 1$.

The number of varieties of intermediate goods is given by the fixed number N. The fact that each of these varieties of intermediate goods have a specific quality level is reflected in the following equation.

$$\tilde{X}_{ij} = q^{\kappa_j} X_{ij} \tag{2}$$

X_{ij} is the physical amount of an intermediate good, while $q^{\kappa_j} > 1$ is the highest quality in which this intermediate good is currently available. The current level of this intermediate good is given by κ_j.

The final goods firms, operating under perfect competition, only demand the highest quality available in each sector according to the following aggregate

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[6]Thereby we account, to a certain extent, for the cyclical behavior of the economy within the lifetime of a single GPT.

demand equation:[7]

$$X_j = L(A\alpha q^{\alpha\kappa_j}/P_j)^{1/(1-\alpha)} \tag{3}$$

Intermediate goods firms hold a monopoly on their goods, selling them to the final goods sector with a monopoly markup, whereby they set the price as $P_j = \frac{1}{\alpha}$. Faced with the aggregate demand function (3), intermediate goods firms provide the following amount of an intermediate good X_j:

$$X_j = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}q^{\kappa_j\frac{\alpha}{1-\alpha}} \tag{4}$$

Aggregating across all final goods firms and assuming a fixed aggregate amount of labor L we get the following amount of aggregate amount of intermediate goods X and of final goods Y:

$$X = A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}LQ \tag{5}$$

$$Y = A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}LQ \tag{6}$$

The aggregate quality index Q is given by the following equation:

$$Q \equiv \sum_{j=1}^{N} q^{\kappa_j\alpha/(1-\alpha)} \tag{7}$$

By adding up the highest available quality in each intermediate goods sector Q gives an accurate representation of the current technological level of the economy. For this reason Q will later be used as an indicator of the currently available applied knowledge stock.

Intermediate goods firms can engage in in-house R&D in order to invent a type of intermediate good of an even higher quality. If such a firm succeeds in inventing a new quality in a certain sector, it will hold the monopoly on this good, displacing the previous monopolist. In a Schumpeterian fashion,

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[7]See Schiess and Wehrli (2008) for a more detailed description on the derivation of the equations in this subsection.

it will hold this monopoly (therefore reaping monopoly profits by selling the intermediate good to the final good sector according to the demand function in each period) until it is displaced by yet another successful intermediate goods firm. This process of continuous inventions leads to an increase in the quality of available goods, thereby driving economic growth.

Consumers, representing the third sector of our model, derive utility from consumption as given by:

$$U = \int_0^\infty u(c(t))e^{-\rho t} dt \tag{8}$$

and optimize their consumption path as determined by the following Euler-equation:

$$\frac{\dot{C}}{C} = (1/\theta)(r - \rho) \tag{9}$$

Ultimately the consumption sector is used to close the model, once equilibrium production and R&D expenditures are determined.

2.2 Determinants of R&D Expenditures

An intermediate goods firm can make R&D expenditures $Z(\kappa_j, t)$ in an intermediate goods sector with the current quality level κ_j in order to attempt to invent a good with an even higher quality $\kappa_j + 1$. To find the optimal amount of $Z(\kappa_j, t)$ an individual intermediate goods firm has to consider two main determinants: First it needs to consider the probability of having a research success in relations to its R&D expenditures. Second, it needs to build expectations on the payoff in case of having a research success.

2.2.1 Probability of a Research Success

The probability of a research success is given by the following equation:

$$p(\kappa_j, t) = Z(\kappa_j, t)\phi(\kappa_j)B(t) \tag{10}$$

This expression captures a multitude of effects: Of course a higher amount of R&D expenditures $Z(\kappa_j, t)$ directly increases the probability of having a research success. Furthermore, the currently active GPT positively influences the probability of having a research success through the parameter $B(t)$. This is modeled according to the notion that GPTs do not directly influence the efficiency of final goods production, but rather allow for an increase in R&D efficiency (see for example Jacobs and Nahuis, 2002). The evolution of $B(t)$ over the course of a single GPT and over a succession of GPTs will be presented

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at a later stage. Finally $\phi(\kappa_j)$ is a parameter, which captures the difficulty of doing R&D especially in relation of the quality the R&D firm wants to improve upon. This difficulty parameter $\phi(\kappa_j)$ is given by:

$$\phi(\kappa_j) = \left(\frac{1 - p(\kappa_j)}{\zeta}\right) q^{-(\kappa_j+1)\alpha/(1-\alpha)} \tag{11}$$

The difficulty of performing R&D in a sector with the current quality level κ_j, is therefore determined by three factors: First, there is a constant cost parameter ζ. Second, there is an increase of difficulty of R&D, which rises with the quality of the variety of the intermediate good it wants to improve upon, given by the expression $q^{-(\kappa_j+1)\alpha/(1-\alpha)}$. Third $\phi(\kappa_j)$ contains decreasing returns to current R&D, as captured by the negative impact of $p(\kappa_j)$.

2.2.2 Endogenous Arrival of new GPTs

Apart from the difficulty parameter $\phi(\kappa_j)$ and the amount of R&D performed in a sector $Z(\kappa_j)$, the behavior of the GPT level as given by $B(t)$ needs to be determined in order to have a full description of the factors influencing the probability of having a research success. In order to capture some of the stylized facts on the course of an individual GPT and the arrival of new GPTs in history, we model $B(t)$ as follows: At the moment, when a new GPT arrives, $B(t)$ starts off with a level of $\overline{B}$ and subsequently depreciates with a rate d in every period. This process continues until a new GPT arrives, so that $B(t)$ jumps back to $\overline{B}$. Thereby the arrival of new GPT is governed by an endogenous poisson process. Accordingly the current $B(t)$ is given by the following set of equations:

$$B(t) = \begin{cases} (1 - d)B(t - 1) & if & a(t - 1) = 0 \\ \overline{B} & if & a(t - 1) \geq 1 \end{cases} \tag{12}$$

$a(t)$ is drawn by an endogenous Poisson process with the mean $\lambda(t)$ of the Poisson distribution given by:

$$\lambda(t) = \frac{\varphi Q(t - 1)}{B(t - 1)} \tag{13}$$

where $0 < \varphi < 1$.

This specification of the Poisson distribution captures several stylized facts on the long-term evolution of GPTs. The fact that the arrival of new GPTs is commonly facilitated by the amount of currently available applied knowledge is accounted for by the inclusion of the aggregate quality index Q in this equation. Therefore a larger amount of applied knowledge increases the probability of a new GPT arriving. Contrarily, the fact that $B(t - 1)$ negatively affects the

probability of arrival of a new GPT, leads to an initial drop in λ upon the arrival of a new GPT. Afterwards, as $B(t)$ is constantly decreasing, λ rises again. This behavior is consistent with features of other GPT models, where the arrival of a new GPT initially binds significant R&D resources for exploring the possibilities offered by the new GPT. In the example in Carlaw and Lipsey (2006), the arrival of a new GPT leads to a drop in basic research (which in their model is aimed at inventing new GPTs) which picks up speed again over the lifetime of the GPT.

While of course both Q and B constantly change, the behavior of λ over time can be nicely illustrated by deriving the expected time until the arrival of a new GPT. This is given by the waiting time distribution of the above defined Poisson process with rate λ. The time T one must wait until the next GPT arrives is given by the following cumulative distribution:[8]

$$F(t) = P(T \leq t) = 1 - e^{-\lambda t} \tag{14}$$

By differentiating equation (14) we get a density function with mean:

$$E(T) = \frac{1}{\lambda} \tag{15}$$

And finally together with equation (13):

$$E(T) = \frac{B(t-1)}{\varphi Q(t-1)} \tag{16}$$

As can be easily seen the higher the quality index Q, the shorter is the expected waiting time for a new GPT. On the other hand there is also a cyclical component during the lifetime of an individual GPT: While initially, there is an increase in the expected waiting time, it gets gradually lower in the course of a GPT, as the current $B(t)$ gradually drops from the initial $\overline{B}$.

2.2.3 Expected Profits

Another main determinant of the amount of R&D performed is the expected profit in case of a research success. We will first present how R&D firms build their expectations on the expected profit in case of a research success $E[V(\kappa_{j+1})]$. In determining the amount of information the R&D firms have, we basically follow the line of reasoning of Carlaw and Lipsey (2006): They argue that it is impossible for agents to predict either the development of a single GPT or the exact time of arrival of a new GPT. Due to this uncertainty, the assumption of perfect foresight taken in the majority of growth models cannot

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[8]This equation can be derived by using the fact that $P(T > t) = e^{-\lambda t}$ following from the Poisson distribution and the use of the complement rule.

be sustained. They therefore propose two possible ways how agents maximize their utility in each period recursively: One approach would be that agents are not forward looking at all, therefore only considering the marginal products of the current period and maximizing only their current profit. Another possibility is that, while agents are forward looking, they simply assume that marginal products across all sectors remain constant at the level currently observed and maximize accordingly over an infinite time horizon. While Carlaw and Lipsey (2006) choose the first approach for their own model, we use the second approach: Individuals maximize in every period their outcome based on the values of this specific period since we are assuming that they are forward looking but cannot foresee future changes, i.e. they believe that the current marginal productivity of research does not change over time. In the following period the individuals realize that the productivity has changed and readjust their decision. Concretely applied to the R&D sector in our model this basically means that R&D firms assume that current equilibrium levels of the interest rate r and of the probability of having a research success p apply for the whole future.[9] The profit flow per period an innovator possessing the leading-edge technology is given by:

$$\pi(\kappa_j) = \overline{\pi} q^{\frac{\kappa_j \alpha}{1-\alpha}} \tag{17}$$

where $\overline{\pi}$ is the basic profit flow, which is constant over time due to the fact that A and L in final goods production are constant:

$$\overline{\pi} = \left(\frac{1-\alpha}{\alpha}\right) A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L \tag{18}$$

Due to the higher demand for goods of a higher quality level, as can be seen in the demand function (3), the profits of inventors $\pi(\kappa_j)$ increase with the quality level κ_j of the variety of an intermediate good.

The incumbent monopolist can reap the monopoly profit $\pi(\kappa_j)$ in every period until it is displaced by an intermediate goods firm inventing an even higher quality. The current probability, that the monopolist with a good of quality κ_j remains in the market, is therefore given by $1 - p(\kappa_j)$. Additionally, future profits are discounted by the current interest rate r. As intermediate goods firms assume that both the interest rate and the probability of a research success remain constant over time, the expected profit of a monopolist is given by:

$$\overline{\pi} q^{\frac{\kappa_j \alpha}{1-\alpha}} + \left(\frac{1-p(\kappa_j)}{1+r}\right) \overline{\pi} q^{\frac{\kappa_j \alpha}{1-\alpha}} + \left(\frac{1-p(\kappa_j)}{1+r}\right)^2 \overline{\pi} q^{\frac{\kappa_j \alpha}{1-\alpha}} + ... + \left(\frac{1-p(\kappa_j)}{1+r}\right)^n \overline{\pi} q^{\frac{\kappa_j \alpha}{1-\alpha}} \tag{19}$$

Since this is a geometric series, by taking the limit n $\rightarrow \infty$ this simplifies to:

$$E[V(\kappa_j)] = \frac{\overline{\pi} q^{\frac{\kappa_j \alpha}{1-\alpha}}}{r + p(\kappa_j)} \tag{20}$$

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[9]Therefore they implicitly assume that no new GPT arrives over the whole lifetime of their products, i.e. that $a(t)$ is always 0.

The interpretation of this expression is straightforward: The expected profit from a successful innovation rises with its quality level, due to the higher demand for higher quality intermediate goods. Contrarily, both a higher discount rate and a higher probability of being displaced by a future competitor lower the expected profit.

2.3 Model Equilibrium

Assuming free entry in the intermediate goods sector we can now find the optimal amount of R&D expenditures $Z(\kappa_j)$, by equalizing expected payoff of such an investment with the according probability of having a research success $p(\kappa_j)$ multiplied by the expected monopoly profit associated with inventing an intermediate good of higher quality, $E[V(\kappa_{j+1})] * p(\kappa_j)$. Therefore the free entry condition is given by:

$$Z(\kappa_j)(\phi(\kappa_j)B(t)E[V(\kappa_{j+1})] - 1) = 0 \tag{21}$$

In order to find the equilibrium aggregate research expenditures we can now insert the expected profit from having a research success given by equation (20) together with the difficulty of performing R&D (11) into this free entry condition. This allows us to easily solve for the probability of having a research success, which is given by:

$$p = \frac{\overline{\pi}B_m - r\zeta}{\zeta + \overline{\pi}B_m} \tag{22}$$

As this equation is independent of the quality level in a single sector, research expenditures are uniformly distributed among all intermediate goods sectors. By inserting this equation into the probability of having a research success as given by equation (10) we can derive the R&D expenditures in a single sector j and then aggregate across all intermediate goods sectors. This results in the following amount of aggregate R&D expenditures:

$$Z(t) = \sum_{j=1}^{N} Z(\kappa_j, t) = \frac{q^{\frac{\alpha}{1-\alpha}}Q(\overline{\pi}B(t) - r\zeta)}{1 + r} \tag{23}$$

As in more precisely elaborated in our baseline quality-ladder model (Schiess and Wehrli, 2008) output growth is proportional to the expected proportional

change of the quality index Q, we can use equation (22) together with the Euler equation (9) to derive the equilibrium growth rate $\gamma(t)$ and the interest rate $r(t)$ at a moment in time:

$$\gamma(t) = \frac{(q^{\frac{\alpha}{1-\alpha}} - 1)(\overline{\pi}B(t) - \rho\zeta)}{\theta\zeta(q^{\frac{\alpha}{1-\alpha}} - 1) + \zeta + \overline{\pi}B(t)} \tag{24}$$

$$r(t) = \frac{\theta\overline{\pi}B(t)(q^{\frac{\alpha}{1-\alpha}} - 1) + \rho(\zeta + \overline{\pi}B(t))}{\theta\zeta(q^{\frac{\alpha}{1-\alpha}} - 1) + \zeta + \overline{\pi}B(t)} \tag{25}$$

Finally, by inserting equation (25) into equation (22) we can derive the following expression for $p(t)$:

$$p(t) = \frac{\overline{\pi}B(t) - \rho\zeta}{\theta\zeta(q^{\frac{\alpha}{1-\alpha}} - 1) + \zeta + \overline{\pi}B(t)} \tag{26}$$

While these equations allow us to determine γ, r and p at a point of time, long-term economic dynamics are driven by a sequence of GPTs, the arrival of which is stochastic, despite the fact that the probability of such an event is endogenous to the model. These long-term dynamics are described in the next section

3 Model Behavior and Numerical Simulations

The qualitative dynamics of our model can already be seen from the previous equations without resorting to computational methods. New GPTs lead to economic cycles as follows: The arrival of a new GPT, by immediately raising the efficiency of R&D, leads to an increase in R&D activities which in turn leads to a larger growth rate. As time passes, due to decreasing returns within a single GPT as characterized by a decrease in $B(t)$, R&D becomes gradually less attractive. This leads to a decrease in growth rates in the economy over the lifetime of a single GPT. On the other hand, the probability of yet another GPT arriving rises over the lifetime of a single GPT. The arrival of a new GPT, revives the growth process, by leading to another increase in R&D activities. In the long term newly arriving GPTs therefore sustain growth. Furthermore, as the available amount of applied knowledge measured by a constant increase in Q increases, the frequency of arrival of new GPTs increases as well.

To illustrate this, we simulate our model such that we define one period as being equivalent to one year. Simulations are also used by van Zon et al. (2003 and 2006) and Carlaw and Lipsey (2006) to show the long-term economic development driven by a sequence of GPTs. Unlike these models, our framework can account for the observed increase in the frequency at which new GPTs arrive.[10]

We calibrate our model to cover the historical arrival of new GPTs over the past two centuries, i.e. the years from 1809 until 2008 AD. We thereby again refer to the list of GPTs by Lipsey at all shown in Table 1. According to their list in the 19th century four GPTs appeared (in the order of their appearance: railway; iron steamship; internal combustion engine; electricity) while in the 20th century seven GPTs did arise (in the order of their appearance: motor vehicle; airplane; mass production, continuous process, factory; computer, lean production, internet, biotechnology). Therefore the targeted number of GPTs arriving during the 200 years covered by our simulation is taken as approximately 11, while the aimed growth rate in the final period of our simulation corresponds to the average growth rate of the OECD economy in 2008, which is at 2.9% (OECD, 2008).

The parameter values used in our simulations are shown in Table 2. The initial value of Q, i.e. $Q(1)$ is set to 1. The values for α, θ and ϕ are chosen to be within the value range usually found in the literature. Furthermore $B(1)$ is assumed to be 1.3 with a $\overline{B}$ of 2. The other parameters are used to calibrate the model, in order to achieved the previously stated properties. To do so we have chosen the values so that in the average of one thousand simulation runs, the desired growth rate and arrival rate of GPTs is achieved.[11]

α	0.3
θ	0.2
ρ	0.03
ζ	5
d	0.05
A	15
L	2
$B(1)$	1.3
$\overline{B}$	2
$Q(1)$	1
q	1.1

Table 2: Parameters used for numerical solution

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[10]Carlaw and Lipsey (2006) calibrated their model to an constant expected time interval of 30 years between GPTs, while Van Zon et al. (2003 and 2006) make simulations taking arbitrary parameter values.

[11]This approach is also widely used in the simulation of business cycle models, as for example in Jaimovich and Rebelo (2006).
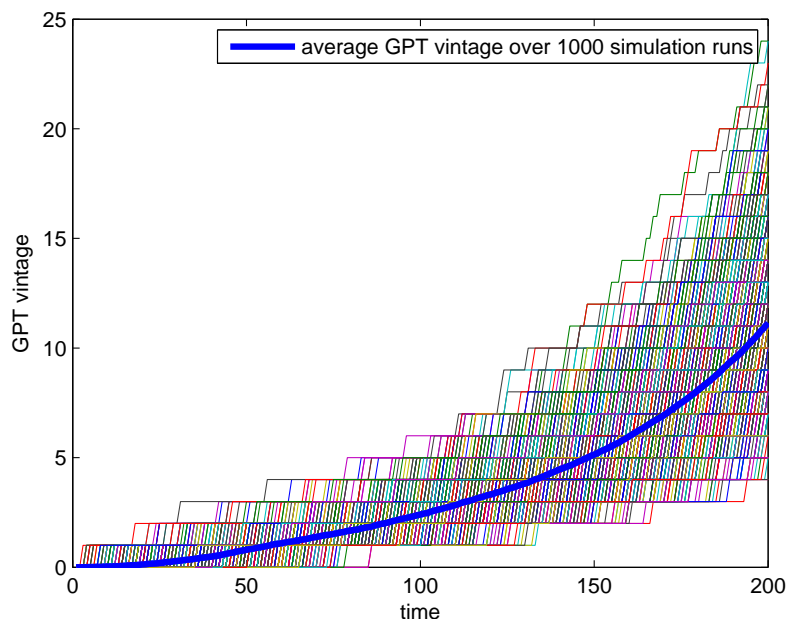
Figure 1: GPT vintage in the 1000 simulation runs

The number of GPT generations arriving over the 200 years resulting from a total of 1000 simulation runs, are shown in Figure 1. The thin lines show the results of individual simulation runs, while the bold line represents the average vintage of GPTs present taken over all simulations. It becomes obvious that our model can be calibrated in a way that can realistically depict both the absolute number of GPTs invented in the past 200 years and their approximate arrival rate. The acceleration of the arrival of new GPTs is also mirrored by our model. Interestingly, while we can chose the parameters so that in terms of expected values, the historical development of the various GPTs can be approximated, there is a wide spread of other scenarios arising using the very same values. The number of GPTs invented during the time frame of our simulation ranges from 4 to 24. This clearly shows that, given that long-term growth is driven by the arrival of subsequent GPTs, the outcome is still highly stochastic.

The growth cycles triggered by the repeated arrival of new GPTs is shown in Figure 2 for a typical simulation run: The arrival of a new GPT gives boost to growth, as it mitigates the decreasing returns over time within a single GPT generation. The corresponding absolute GDP growth is shown in Figure 3.

The development of R&D expenditures is equally interesting: Just as economic growth, R&D is subject to a cyclical behavior which is subject to the arrival of new GPTs. Furthermore an increase of R&D expenditures can be observed over time.
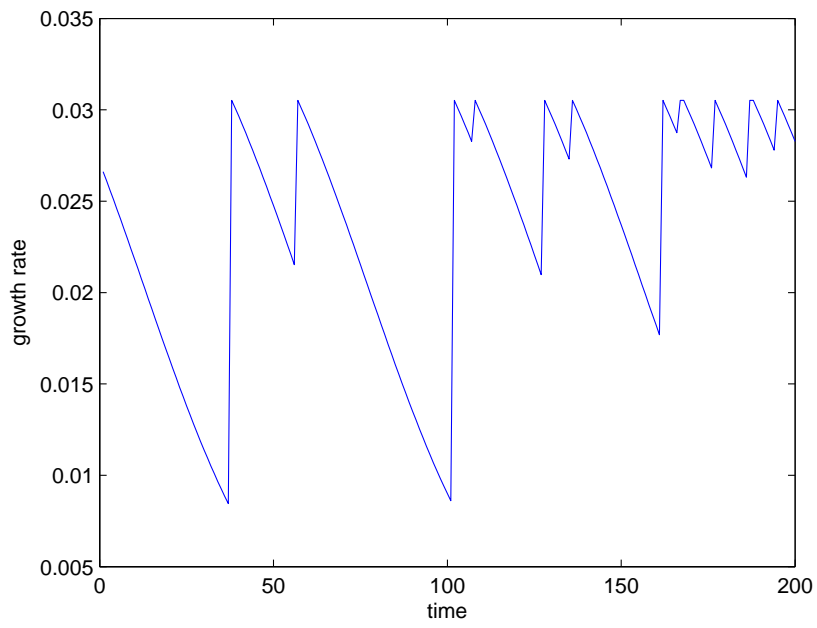
Figure 2: growth rate

Figure 3: GDP

17
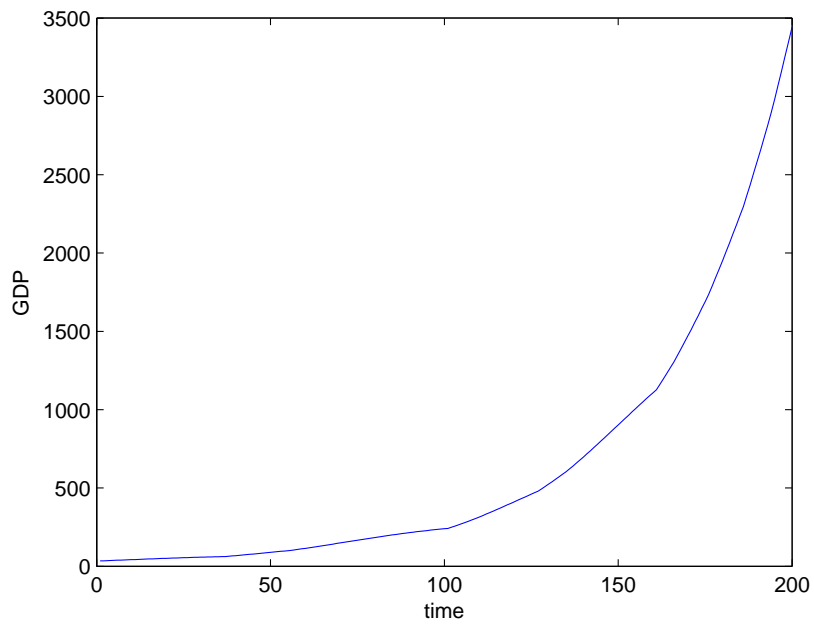
4 Conclusions and Outlook

We have presented a quality ladder model of endogenous Schumpeterian growth where new GPTs arrive endogenously with a probability governed by the amount of applied knowledge available. This model allows for a more realistic description of the historical facts surrounding the emergence of new GPTs and the frequency of their arrival which state that new GPTs arise in an ever higher frequency. Furthermore, not only the arrival of new GPTs is stochastic but also the development of new qualities of intermediate goods in single sectors is subject to uncertainty. While this combination of features is both unique from a modelling perspective and fits stylized facts drawn from economic history, there are of course several possibilities for future extensions. One way to enrich our model would be to introduce a basic research sector, which unlike the R&D in the intermediate goods sector would be devoted to research aiming at the invention of new GPTs. This would furthermore allow for a role of government funded R&D, as private firms have a low incentive to perform such basic R&D due to the fact that returns on new GPTs are hard to appropriate. Finally, a future model could consider that an ever increasing number of GPTs arriving could compete for scarce R&D resources, thereby partially obliterating the beneficial effects of these "engines of growth".

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