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**Budgetary Policies and Available Actions:  
A Generalisation of Decision Rules for  
Allocation and Research Decisions**

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# **Budgetary Policies and Available Actions: A Generalisation of Decision Rules for Allocation and Research Decisions**

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## **Abstract**

The allocation problem in health care can be characterised as a mathematical programming problem but attempts to incorporate uncertainty in costs and effect have suffered from important limitations. A two stage stochastic mathematical programming formulation is developed and applied to a numerical example to explore and demonstrate the implications of this more general and comprehensive approach. The solution to the allocation problem for different budgets, budgetary policies, and available actions are then demonstrated. This analysis is used to evaluate different budgetary policies and examine the adequacy of standard decision rules in cost-effectiveness analysis. The research decision is then considered alongside the allocation problem. This more general formulation demonstrates that the value of further research depends on: i) the budgetary policy in place; ii) the realisations revealed during the budget period; iii) remedial actions that may be available; and iv) variability in parameters values.

## Introduction

Standard decision rules in cost effectiveness analysis (CEA) claim to determine the efficient allocation of health care resources in two contexts. Firstly, in the absence of an exogenous budget constraint a higher authority can set a societal willingness to pay for additional health benefits and implement all independent treatments with positive net benefit. In these circumstances the budget for health care is not fixed but implicitly determined by this social value of health. Alternatively, where a higher authority sets a fixed budget for health care the decision maker must select from all available treatments the subset that maximises health benefits subject to the budget constraint. In these circumstances the cost-effectiveness threshold represents an estimate of the cost-effectiveness of the marginal programme which will be displaced or the reciprocal of the shadow price of the budget constraint (Culyer et al., 2007, Johannesson and Weinstein, 1993). If the estimate of the threshold is not consistent with the budget for health care or the productivity of current activities then resources will not be allocated optimally as the threshold will not identify the true opportunity costs (Birch and Gafni, 1992, Birch and Gafni, 1993, Gafni and Birch, 2006).

Mathematical programming (MP) offers a solution to this allocation problem, but requires full information on the costs and health benefits of all competing treatment options within all health care programmes (Earnshaw and Dennett, 2003). Stinnett and Paltiel (1996) provided a general MP framework to accommodate information regarding returns to scale, indivisibilities, programme interdependence, and ethical constraints. Other authors have examined the implications of indivisibility (Earnshaw and Dennett, 2003, Sendi and Al, 2003), returns to scale (Elbasha and Messonnier, 2004), ethical constraints and multiple budgets over time (Epstein et al., 2007). Many have demonstrated that the standard decision rules in CEA may be optimal when there is a single budget constraint and perfect divisibility, or at least any indivisibilities are small relative to the budget (Epstein et al., 2007, Laska et al., 1999, Stinnett and Paltiel, 1996). MP techniques have been applied to some policy problems (Brandeau et al., 2003, Earnshaw et al., 2002, Zaric and Brandeau, 2001).

All the work cited above assumes that costs and effects are known. In general, however, the value of costs and effects are variable and estimates of their expected values are uncertain. The need to characterise uncertainty and its consequences in CEA is well documented (Claxton, 2008) and generally implemented using probabilistic methods (Claxton et al., 2005) with the results summarised using cost-effectiveness acceptability curves (Fenwick et al., 2001) and value of information analysis (Claxton, 1999). In the absence of irreversibility or significant sunk costs (Eckermann and Willan, 2006) the same standard decision rules, based on expected cost and expected health benefit, are argued to apply (Claxton, 1999). It is the decision to acquire additional information which is primarily determined by uncertainty in costs and effects and variability appears irrelevant in most common circumstances.

It is possible to characterise the full allocation problem under uncertainty explicitly by reformulating the MP as a stochastic analysis. To date there are few proposed stochastic mathematical programming (SMP) formulations in the literature (Al et al., 2005, Sendi and Al, 2003, Sendi et al., 2003). These all suffer from important limitations including the use of arbitrary exogenous parameters, the inability to examine alternative budgetary policies, and the failure to inform both the allocation and research decision problem simultaneously. A more general and unified approach is required. A SMP formulation which overcomes many of these problems has recently been proposed (Chalabi et al., 2008). This paper develops and applies this formulation to a stylised numerical example to explore and demonstrate the implications of a more general and comprehensive approach to allocation and research decisions.

The paper is structured as follows. Firstly, the rationale for the proposed SMP formulation is presented. In doing so, previous approaches are critiqued and their limitations discussed. The allocation problem and the associated numerical example are introduced as well as the formulation that enables the limitations of previous methods to be overcome. The solution to the allocation problem for different budgets, budgetary policies, and their available actions are then demonstrated. This analysis is used to evaluate different budgetary policies and examine the adequacy of standard decision rules in CEA. The research decision is then considered alongside the allocation problem. A more general formulation which can inform research and allocation decisions simultaneously and consistently is presented. The final section concludes with a discussion.



## Methodological Background

A risk neutral decision maker would aim to allocate resources to maximise expected health benefits subject to expected costs meeting the budget constraint. However, since costs and benefits are random variables arising from uncertainty and variability, nearly any *ex-ante* decision about the allocation of resources will lead to some probability that the budget is exceeded. If, *ex-post*, the budget is exceeded there will be a loss of benefits as some programmes and treatments are curtailed, or additional funding is diverted from other sectors of the economy. To allocate resources optimally, *ex-ante* decision makers must consider all the potential realisations of costs and health benefits and identify which programmes will be adjusted in order to minimise the expected loss of benefits. Although it is irrelevant to the allocation decision whether the randomness in costs and health benefits arises from uncertainty or variability, the distinction between these causes of randomness is critical to the question of whether more information would be valuable to inform the allocation decision (Chalabi et al., 2008). This is because the acquisition of additional information will reduce the randomness in the health benefits and costs due to uncertainty but will not affect that associated with variability (Frey and Burmaster, 1999).

Stochastic mathematical programming methods have been proposed to allocate resources between health care programmes when costs and effects are uncertain (Al et al., 2005, Sendi and Al, 2003). These methods are largely based on chance constrained formulations where a probability of meeting the budget is specified so that the conditions of the decision rule are satisfied. Other candidate formulations are those of robust optimization (Leung et al., 2007, Mulvey et al., 1995). Al et al (2005) and Sendi and Al (2003) propose maximising health benefits subject to a probability,  $\alpha$ , of meeting the budget constraint ( $\alpha \leq 1$ ):

$$\begin{aligned} \text{Max. } & E_Z(B(Z, X)) \\ \text{s.t. } & P(C(Z, X) \leq \Psi) \geq \alpha \end{aligned} \quad (1)$$

where  $X$  is the set of decision variables (treatment allocation decisions),  $\Psi$  represents the budget,  $E_Z(\cdot)$  denotes the expectation with respect to the random parameters  $Z$ , and  $P(\cdot)$  denotes the probability of meeting the budget constraint.  $B(Z, X)$  and  $C(Z, X)$  represent the benefits and costs, respectively. The inequality on the budget constraint ensures that the budget is met with a probability of at least  $\alpha$ .

This formulation suffers from a number of disadvantages. Firstly, the decision maker must specify an arbitrary parameter  $\alpha$ . It is not clear how the value of  $\alpha$  should be set, or how any particular value could be used to represent actual budgetary rules which may be imposed on a health care system. For example, a budgetary policy that the health care decision maker cannot run a deficit implies  $\alpha$  is set at 1. This 'hard' budget constraint can impose substantial opportunity costs, ultimately leading to corner solutions, i.e. no health care is provided due to the risk of exceeding the constraint, or restrictive provision of health care and large expected budget surpluses. If  $\alpha$  is arbitrarily set at less than 1 to avoid such outcomes, then the budget can be exceeded but the opportunity costs are not explicitly valued within the formulation. A penalty function was included in the original formulation but this was based on an exogenous and ex-ante assessment of the opportunity costs of the additional resources required (Al et al., 2005). Therefore, although the approach avoids the use of some exogenous and ex-ante assessment of a cost-effectiveness threshold, the formulation depends on an arbitrary parameter and an exogenous, ex-ante penalty function. It is unable to characterise or evaluate real budgetary policies and, since it does not distinguish between uncertainty and variability, is unable to consider the value of information for the research decision.

A key requirement of any proposed solution to the allocation problem is to allow the evaluation of a range of actual budgetary rules in which consistent (endogenous) assessment of the opportunity costs of *ex-post* violation of the constraints is taken into account. In addition, the value of acquiring new evidence to inform the allocation problem in light of its current uncertainty needs to be considered simultaneously and consistently.

To overcome the limitations of previous approaches, recently a more general formulation which addresses these key requirements has been proposed (Chalabi et al., 2008). This formulation avoids the use of arbitrary, exogenous parameters and allows the characterisation of actual budgetary policies, including a strict budgetary rule where deficits are not possible and the constraints must always be met. The opportunity costs (the health forgone due to curtailing some programmes and treatments) of violating the budget constraint are incorporated directly. Since uncertainty and variability are distinguished, the value of acquiring information to inform the whole allocation problem can also be considered. This formulation demonstrates that the allocation and research decision problem depends on a number of considerations: i) the size of the overall budget; ii) budgetary policy in place; iii) the information that is revealed and its timing; iv) the subsequent actions available to decision makers; as well as the costs and effects associated with the treatments, populations and programmes which constitute the allocation problem.

## The Allocation Problem

Without loss of generality an allocation problem, which consists of three mutually exclusive treatment options ( $j = 1, \dots, 3$ ) in each of three separate population groups ( $i = 1, \dots, 3$ ) within three independent health care programmes ( $k = 1, \dots, 3$ ) is considered. For simplicity, binary health outcomes ( $a, b$ ) are assumed for each treatment across the health care programmes, and one of the treatments in each of the programmes has zero-cost (i.e., a no treatment option,  $j = 1$ ). The treatment decision for population  $i$  in health care programme  $k$  can be described by a simple decision tree (Chalabi et al., 2008) with health outcomes ( $u_{ijk,a}, u_{ijk,b}$ ), the costs associated with the treatment pathways ( $c_{ijk,a}, c_{ijk,b}$ ) and the probabilities of the binary outcomes ( $p_{ijk,a}, p_{ijk,b}$ ). For each of the health care programmes and populations, treatment  $j = 3$  is more effective and more costly than treatment  $j = 2$  which in turn is more effective and more costly than no treatment  $j = 1$ . For simplicity and to examine the performance of standard decision rules in their most favourable circumstances, costs and benefits are assumed to occur within the single budgetary period. In addition we assume constant returns to scale, divisibility of treatments within populations and a single budget constraint.

Uncertainty in the allocation problem is characterised in terms of the probability parameters, whereas variability is captured in terms of the number of patients in the health outcome and cost states conditional on the probabilities (Chalabi et al., 2008). The set of decision variables is defined by  $X = (x_{ijk} \text{ for } i, j, k = 1, \dots, 3)$  where  $x_{ijk}$  denotes the proportion of population group  $i$  in health care programme  $k$  that is allocated to treatment  $j$ .

The total expected health benefits and the total expected costs with respect to the uncertain and variable parameters  $Z$  are given by

$$E_Z(B(Z, X)) = \sum_{k=1}^3 \sum_{i=1}^3 \sum_{j=1}^3 \left( n_{ik} \times x_{ijk} \times (p_{ijk,a} \times u_{ijk,a} + (1 - p_{ijk,a}) \times u_{ijk,b}) \right) \quad (2)$$

and

$$E_Z(C(Z, X)) = \sum_{k=1}^3 \sum_{i=1}^3 \sum_{j=1}^3 \left( n_{ik} \times x_{ijk} \times (p_{ijk,a} \times c_{ijk,a} + (1 - p_{ijk,a}) \times c_{ijk,b}) \right) \quad (3)$$

respectively, where  $n_{ik}$  is the total number of patients in population group  $i$  and health care programme  $k$  (Chalabi et al., 2008).

The objective of the SMP is to determine the optimal mix of treatments  $x_{ijk}$  that will maximise the health benefits (2) subject to the costs (3) meeting the overall budget constraint and any additional

constraints imposed on the system. To demonstrate the implications of the SMP formulation of this allocation problem it is applied to a numerical example given in Appendix 1.<sup>1</sup>

## The Allocation Decision

The decision maker wishes to allocate resources optimally among the various competing health care interventions, populations and programmes. A higher authority sets the overall budget available and imposes rules on how the budget can be spent. A hard budget constraint may be enforced by the authority, where the health care decision maker cannot exceed the budget under any circumstances. This type of policy will impose substantial opportunity costs (a corner solution where no health care is provided is possible) unless the decision maker can take remedial action to avoid a deficit as information about actual costs and effects is revealed during the budgetary period.

Alternatively, a less restrictive policy (soft budget constraint) may be implemented, i.e., as long as the health care decision maker plans to meet the budget at expectation, any deficit will be indemnified (deficits are likely as costs will be variable and their expectation uncertain). The standard decision rules in CEA are effectively based on this type of budgetary policy since decisions based on expected net benefits mean that surpluses or deficits are valued at the cost-effectiveness threshold. However, this type of budgetary policy is uncommon in budget constrained health care systems and for good reason. There will be substantial asymmetry of information between the higher authority (principal) and the health care decision maker (the agent). It will be impossible, or at least very costly for the principal, to effectively monitor the agent and establish that their ex-ante plans do meet the budget at expectation. In these circumstances the agent is likely to plan to exceed the budget knowing that the deficit will be indemnified and their extravagant plans will be undetected. Below, we explore alternative budgetary policies (soft and hard constraints) and consider the impact of information and actions available to the health care decision maker.

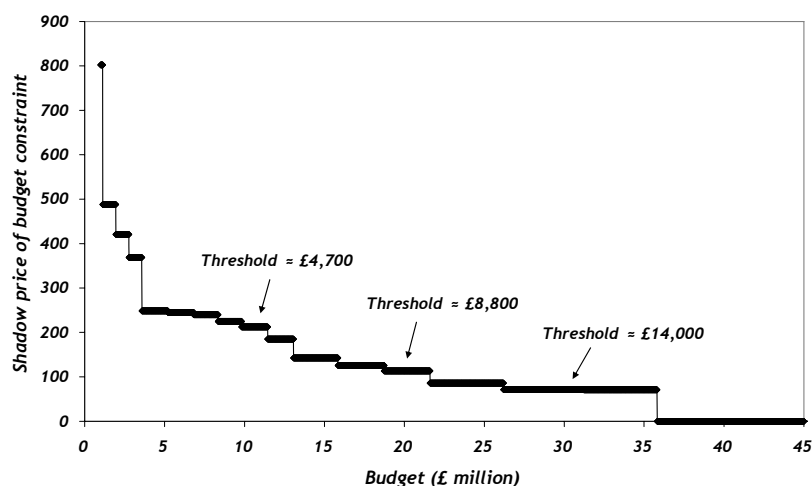
### Soft budget constraint

If a soft budget constraint is implemented the allocation problem can be formulated as follows:

$$\begin{aligned} \text{Max. } & E_Z(B(Z, X)) \\ \text{s.t. } & E_Z(C(Z, X)) \leq \Psi \\ & \sum_{j=1}^3 x_{ijk} = 1 \text{ for } i, k = 1, \dots, 3 \end{aligned} \quad (4)$$

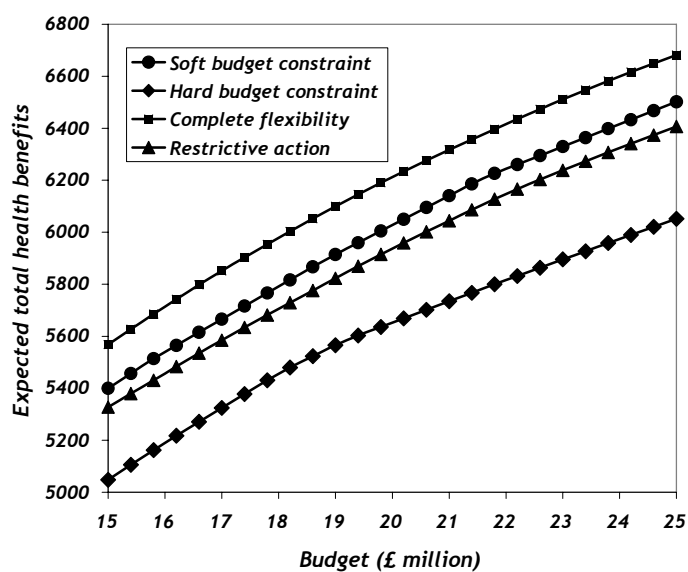
where  $\Psi$  is the overall budget,  $E_Z(B(Z, X))$  is the total expected health benefits, and  $E_Z(C(Z, X))$  is the total expected costs. The constraint  $\sum_{j=1}^3 x_{ijk} = 1$  ensures that every individual in each of the population groups and programmes is allocated to one of the three treatment options.

<sup>1</sup> The allocation problem described in Appendix 1 was solved using the linear program package *lpSolve* in R. (<http://www.r-project.org/>) The R project for statistical computing. This returned the optimal values of the decision variables, the shadow price of the constraints, and the expected health benefits reported in subsequent figures.



**Figure 1: The shadow price (health benefits per additional £1 million)**

The relationship between the shadow price of the budget constraint (the gain in health benefits as the budget grows per additional £1 million) and the overall budget is illustrated in Figure 1. The shadow price falls with the budget in a series of steps. Along each flat portion the same set of optimal treatments are selected but with different proportions of the population receiving them (a mixed programme within a population). At the points of discontinuity, the optimal allocation takes integer-values and there are no mixed programmes. At a budget of £35.8m all the most effective treatments in all populations and programmes are funded in full and further budget increases do not result in any additional health benefits. Hence, the shadow price falls to zero at £35.8m. The total expected health benefits from a soft budgetary policy are illustrated in Figure 2 and increase with budget. Total expected health benefits reach a maximum of 7289 at a budget of £35.8m.



**Figure 2: Expected total health benefits for budgetary policies and available actions**

The solution to (4) yields the same set of decision variables as solving for each of the independent health care programmes separately using the standard CE decision rules with the correct threshold. Since the threshold is the reciprocal of the shadow price it increases with the size of the budget, as shown in Figure 1. Therefore, standard decisions rules in CEA can be optimal but only in very restricted circumstances: i) a single soft budget constraint; and ii) no asymmetry of information between principal and agent, or at least the principal can detect and punish inappropriate plans. These conditions are in addition to those assumptions more commonly discussed: i) perfect divisibility; ii) a threshold equal to the reciprocal of the shadow price of the budget constraint and iii)

no other binding constraints. However, for the reasons discussed above soft budgetary policies are uncommon and will tend to be either costly to effectively monitor or will be exploited by the agents.

### Hard budget constraint

Alternatively a higher authority may implement a hard budgetary policy. To satisfy this requirement all possible random realisations of the uncertain and variable parameters must be anticipated so that decision variables can be chosen ex-ante which will satisfy the constraint under all random realisations. The allocation problem under this hard budget constraint can be formulated as follows:

$$\begin{aligned}
 & \text{Max. } E_Z (B(Z, X)) \\
 & \text{s.t. } C(Z_1, X) \leq \Psi \\
 & \quad C(Z_2, X) \leq \Psi \\
 & \quad C(Z_3, X) \leq \Psi \\
 & \quad \dots \\
 & \quad C(Z_N, X) \leq \Psi \\
 & \quad \sum_{j=1}^3 x_{ijk} = 1 \text{ for } i, k = 1, \dots, 3
 \end{aligned} \tag{5}$$

where  $Z_1, \dots, Z_N$  are  $N$  random realisations of the uncertain and variable parameters (theoretically infinite but set to a large number in the simulation), and  $C(Z_1, X)$  are the costs associated with random realisation  $Z_1$ . The decision maker maximises the expected health benefits subject to the costs from each random realisation meeting the budget constraint. This type of budgetary policy will impose substantial opportunity costs since provision of health care is restricted to avoid the risk of violation of the constraint. It is equivalent to  $\alpha=1$  in the chance constrained formulations proposed by Al *et al* and Sendi and Al in formulation (1).

The expected health benefits when a hard constraint is imposed are illustrated in Figure 2 for a range of budgets. The expected health benefits are lower than with a soft budgetary policy, The difference represents the opportunity cost of imposing a hard constraint and is modest (e.g., 6.9% of health benefits at a budget of £25m) because the only random parameters are probabilities bounded by 0 and 1. In this example the expected health benefits reach the maximum of 7289 when the budget exceeds £46.1m rather than £35.8m, i.e., additional budget is required before decision makers can take the risk of implementing all the most effective treatments. If other variable and uncertain parameters were introduced which are unbounded (i.e. described by distributions which are defined over an infinite range) then a corner solution is possible, i.e. no health care is provided due to the risk of exceeding the budget. In these circumstances there will be no finite budget which will allow decision makers to take such risks.

The analysis of the hard budgetary policy above represents an extreme characterisation of what information will be revealed during the budgetary period and what remedial actions might be available to the decision maker. In this case, once the decision variables are chosen, then irrespective of what information might be revealed (e.g., actual expenditure), there are no actions the decision maker can take to avoid a deficit.

### Available actions

Although decision variables must be chosen ex-ante, as in formulations (4) and (5), a more realistic characterisation of the allocation problem would allow realisations of health outcomes and costs to be revealed over time within the budget period. Such information provides an indication of whether the ex-ante allocation is likely to lead to a deficit at the end of the budget period. The decision maker may be able to take remedial action to revise the initial ex-ante plans to avoid a deficit. One simple way to represent the essential dynamics of the allocation problem is to incorporate a two-stage formulation

where the realisations of costs and outcomes are revealed but the actions available to decision makers may be restricted.

The decision maker initially makes plans to allocate resources to maximise expected health benefits subject to expected costs meeting the budget constraint. For certain random realisations, the initial plans will result in a deficit. When these realisations are revealed the decision maker must choose to curtail some of the initial treatment decisions to avoid a deficit occurring, i.e. certain treatments for certain groups of the population will be cancelled with the result that no treatment or a less costly and less effective treatment is only available to this group. Since the objective is to maximise health benefits, the decision maker will wish to cancel those activities which have the smallest impact on health. This can be formulated in two-stages as follows:

$$\begin{array}{l}
 \text{Max. } E_Z (B(Z, X)) \\
 \text{s.t. } E_Z (C(Z, X)) \leq \Psi \\
 \sum_{j=1}^3 x_{ijk} = 1 \text{ for } i, k = 1, \dots, 3
 \end{array}
 \left. \vphantom{\begin{array}{l} \text{Max. } E_Z (B(Z, X)) \\ \text{s.t. } E_Z (C(Z, X)) \leq \Psi \\ \sum_{j=1}^3 x_{ijk} = 1 \text{ for } i, k = 1, \dots, 3 \end{array}} \right\} \text{1st stage}$$
  

$$\text{for all } Z \left\{ \begin{array}{l}
 \text{Max. } B(Z, Y) \\
 \text{s.t. } C(Z, Y) \leq \Psi \\
 y_{ijk} \leq x_{ijk}^* \text{ for } i, k = 1, \dots, 3, j = 2, 3 \text{ (action constraint)} \\
 \sum_{j=1}^3 y_{ijk} = 1 \text{ for } i, k = 1, \dots, 3
 \end{array} \right. \left. \vphantom{\begin{array}{l} \text{Max. } B(Z, Y) \\ \text{s.t. } C(Z, Y) \leq \Psi \\ y_{ijk} \leq x_{ijk}^* \text{ for } i, k = 1, \dots, 3, j = 2, 3 \text{ (action constraint)} \\ \sum_{j=1}^3 y_{ijk} = 1 \text{ for } i, k = 1, \dots, 3 \end{array}} \right\} \text{2nd stage} \quad (6)$$

$$\text{Expected health benefits} = E_Z (B(Z, Y^*(Z)))$$

The first stage is equivalent to the soft budget constraint formulation (4). The decision maker meets the budget on expectation with respect to the set of random realisations  $Z$ . The resulting output is an ex-ante optimal allocation vector  $X^* = (x_{ijk}^* \text{ for } i, j, k = 1, \dots, 3)$ , which results in expected health benefits given by  $B^* = E_Z (B(Z, X^*))$ . However, the random realisations  $Z$  are revealed during the budget period. When a realisation occurs which means a deficit will occur, the 2<sup>nd</sup> stage allows remedial action. The remedial action has two main objectives: to ensure that for each random realisation of  $Z$ , i) the budget is strictly met, and ii) maximum benefits are achieved within the budget. Therefore, the 2<sup>nd</sup> stage reallocates the remaining resources such that the budget is satisfied strictly for each random realisation  $Z$ , while maximising health benefits. The resulting output is an optimal allocation vector  $Y^* = (y_{ijk}^* \text{ for } i, j, k = 1, \dots, 3)$  for each random realisation  $Z$ .

The reallocation in the second stage depends on what actions are available to the decision maker. Possible ways to characterise the actions which might be available include:

### **No available actions**

This is equivalent to the hard budget constraint discussed above. Since the decision maker is not permitted to run a deficit and no remedial actions are permitted they must set all decision variables ex-ante to meet the budget for every random realisation of the parameters. This is equivalent to formulation (5).

### **Complete flexibility of action**

The decision maker can completely revise their plans when information is revealed. In this case there is no action constraint in the second stage of (6). Since here for simplicity all realisations are revealed

the best allocation for each realisation can be made. This is equivalent to having perfect knowledge of uncertain and variable parameters ex-ante. It represents the best that could be achieved from a hard budget constraint and set of treatments currently available.

### **Restrictive actions**

The decision maker may only have limited opportunities to revise initial decisions. One restrictive action is to increase the proportion of patients in some of the population groups and programmes receiving the no treatment option ( $j=1$ ). This can be represented by the action constraint ( $y_{ijk} \leq x_{ijk}^*$  for  $i, k = 1, \dots, 3, j = 2, 3$ ) in (6). It ensures that the budget is met for every possible random realisation of the uncertain and variable parameters by reducing the proportion of patients receiving the medium ( $j=2$ ) and highest ( $j=3$ ) cost treatment options. This might represent a situation where it is not possible to implement new services and the only available actions are to curtail certain plans, e.g., the cancellation of elective surgery or ward closures to stay within budget. Now, the realisation of a budget deficit imposes some opportunity costs as some groups of patients will not receive the more effective and costly health care that was initially planned and will be switched from  $j=3$  or  $j=2$  to  $j=1$  (no treatment).<sup>2</sup>

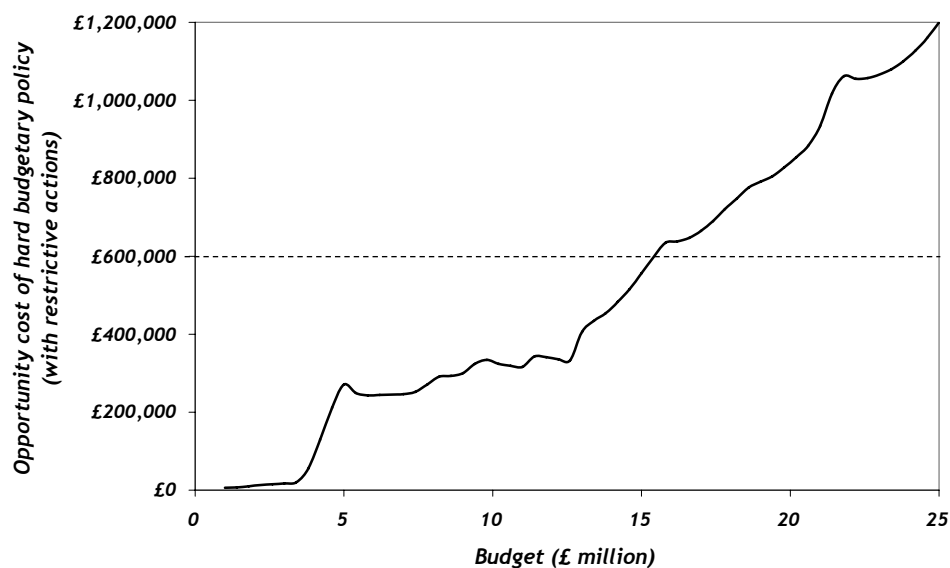
### **Evaluating budgetary policies**

The expected total health benefit of the different budgetary policies and available actions is illustrated in Figure 2 for a range of budgets. Complete flexibility of action when all realisations are revealed (perfect knowledge of random parameters) provides the highest expected benefit and represents the best that could possibly be achieved from a hard budgetary policy.<sup>3</sup> Previously the hard budget constraint with no available actions (5) imposed significant opportunity costs. However, once some restrictive actions are available expected health benefit is greater so opportunity cost is reduced. The decision maker can take the risk of providing effective health care in the knowledge that remedial action is possible if a deficit is realised and corner solutions will be avoided even if random parameters are unbounded. The maximum health benefit of 7289 is achieved at a budget of £42.7m, i.e., lower than £46.1m with the hard constraint in (5) but still higher than £35.8m with the soft constraint in (4). With complete flexibility of action the maximum health benefit is achieved at a budget of £41.6m, i.e., lower than restrictive action but higher than the soft constraint. In this example, all policies and actions including complete flexibility will achieve the same maximum expected health benefits when the budget is sufficiently large (greater than £46.1m); the ex-ante decisions will be the same as ex-post and no remedial actions will be necessary.<sup>4</sup>

<sup>2</sup> There are other restrictive action constraints that are possible, e.g., that services can be substituted as well as simply cut so that patients can be switched from  $j=3$  to  $j=2$  as well as from  $j=3$  to  $j=1$  and from  $j=2$  to  $j=1$ . This less restrictive constraint would reduce the opportunity costs of the hard budgetary policy.

<sup>3</sup> A hard constraint with perfect knowledge can provide better expected health benefits than the soft budgetary policy in (4). However, at higher budgets, when all or most population groups are allocated to the most effective and costly care ( $j=3$ ), any uncertainty or variability will simply mean that a deficit must be covered by the higher authority with a soft constraint (all will continue to receive  $j=3$  but more will be spent on health care). However, perfect knowledge with a hard constraint will mean that some initially allocated to  $j=3$  will be switched to  $j=2$  or  $j=1$  to stay within budget and expected health outcomes will be lower. Perfect knowledge with a soft constraint would provide the best possible outcomes for a given budget. This is not reported but the expected health benefits with a soft constraint and perfect information about uncertain parameters is presented later. Allowing the restrictive actions above with a soft budgetary policy will not change expected health benefit since there is no need to curtail plans to provide effective care as any deficit will be covered. See footnote 4.

<sup>4</sup> In this example the effectiveness of  $j=3 > j=2 > j=1$  and this ranking is not uncertain. There is uncertainty about the magnitude of differences in effectiveness and cost. In other circumstances, when which treatment is most effective is also uncertain then complete flexibility (perfect knowledge) would provide a higher maximum net health benefit.



**Figure 3: Opportunity costs of a hard budgetary policy with restrictive actions**

There remains an opportunity cost of the hard budgetary policy even when restricted actions are possible (the difference in expected health benefit between the soft constraint and restrictive action in Figure 2). These opportunity costs can be expressed as the additional budget the higher authority must provide to achieve the same expected health benefit as with a soft budgetary policy. This is illustrated in Figure 3. It is now possible to consider whether the higher authority should adopt a hard or soft budgetary policy. A soft budgetary policy maybe worthwhile if the cost of monitoring the plans of health care decision makers is less than the opportunity costs of implementing a hard policy.<sup>5</sup> For example, in Figure 3, if the cost of effectively monitoring ex-ante plans is £600,000, then a soft budgetary policy would be worthwhile at budgets above £15.4m. If remedial actions were not available, then opportunity costs would be much greater (£2.3m and £4.8m at a budget of £15m and £25m, respectively) and a soft budgetary policy would be worthwhile at lower budgets or with higher monitoring costs. Similarly if only some realisations are revealed, or only revealed when some proportion of the population has already received ex-ante treatment choices, then these opportunity costs will also be higher.<sup>6</sup> This demonstrates that the choice of budgetary policy for a particular set of treatments, populations and programmes will depend on: i) the overall budget; ii) the variability in random parameters; iii) what realisations are revealed during the budget period; iv) what remedial actions are available to decision makers; and v) the costs of effectively monitoring ex-ante plans.

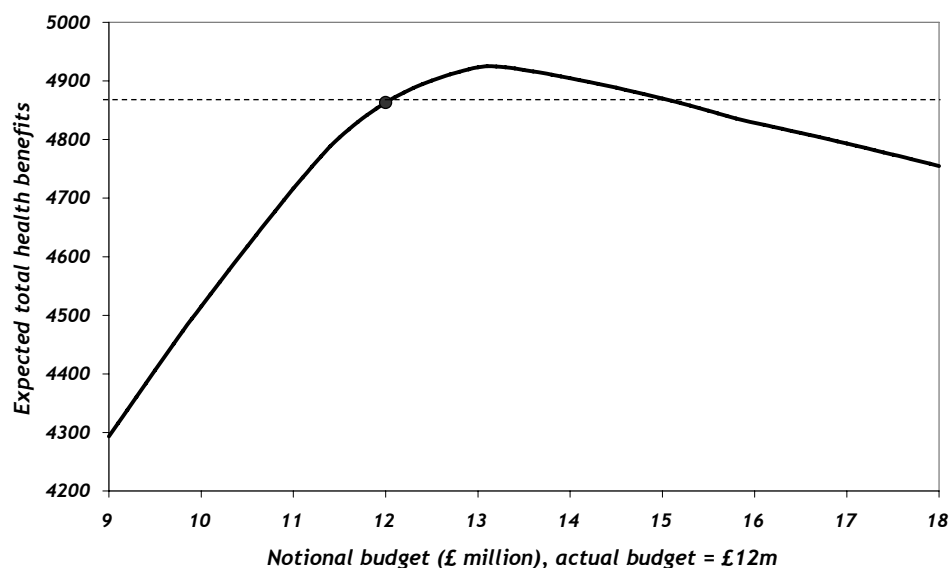
### Decision rules in CEA

It has been established that standard CE decision rules are consistent with a soft budget constraint. All other characterisations of the constraint and available actions reported in Figure 2 lead to different sets of decision variables, non of which can be consistent with standard decision rules. It is well established that standard decision rules are not optimal when there is more than one binding constraint. Each realisation of the random parameters generates another potentially binding constraint when the budget is truly fixed. Therefore standard rules based on expected cost-effectiveness will not be optimal if any realisation becomes binding.

<sup>5</sup> The higher authority can be regarded as risk neutral with respect to any deficits they must indemnify as they will be able to spread and pool risk over the national tax base.

<sup>6</sup> In these circumstances a greater proportion of the population will need to be switched to  $j=1$  and from more valuable treatments and populations. Now corner solutions remain a possibility if realised costs are only revealed late in the budget period when allocating all remaining population to  $j=1$  would still not meet the budget constraint.





**Figure 4: Expected total health benefits for a range of notional budgets when the actual budget is £12m**

The formulation of the two stage problem above, however, still requires the decision maker to plan to meet the budget at expectation in the 1<sup>st</sup> stage. This is somewhat myopic as they may anticipate the need to take remedial actions when making ex-ante decisions. Better expected outcomes maybe obtained when ex-ante decisions are not based on meeting the budget on expectation. This will depend on the reduction in health benefits and the cost savings, which result from switching from the higher to lower cost treatment options to stay within budget. The expected health benefit for a range of 'notional' budgets when the actual hard budget constraint is set at £12m is reported in Figure 4. The optimal set of decision variables are initially obtained by meeting the notional budget at expectation in the 1<sup>st</sup> stage. In the 2<sup>nd</sup> stage remedial action is taken to stay within the actual hard budget of £12m. For notional budgets between £12m and £15m the total health benefit exceeds that obtained when planning to meet the actual budget and reaches a maximum at a notional budget of £13.1m. Therefore, it is better to plan to exceed the hard budget constraint given that the initial decisions will be revised later if a deficit is realised.

Clearly, standard decision rules based on expected cost effectiveness will not be optimal, even ex-ante. However, there are no other simple ex-ante rules for two reasons. Firstly, the amount of 'notional overspend' will depend on the nature of the allocation problem, the information revealed, possible remedial actions available as well as the actual budget. Secondly, it is also possible to find a set of decision variables which improve overall expected outcomes (2<sup>nd</sup> stage) which do not maximise ex-ante expected health benefit (1<sup>st</sup> stage).<sup>7</sup> Therefore, truly optimal ex-ante decisions are unlikely to be consistent with either meeting the budget at expectation or maximising ex-ante expected health benefits. Through this simple but more general formulation of the allocation problem, we have demonstrated that standard decision rules are only optimal in very special circumstances and that there are no simple rules which lead to optimal ex-ante decisions.

<sup>7</sup> For each random realisation of the parameters, there exists an optimal allocation vector  $\tilde{X}$ . These allocation vectors,  $\tilde{X}_1, \dots, \tilde{X}_N$ , provide a sample of all the candidate ex-ante allocations which could be chosen. Implementing each in turn enables a search for those sets of decision variables which improve overall expected benefits. In this case, of 4900 sampled allocations, 470 improved expected health outcome after the 2<sup>nd</sup> stage. All predicted expected costs at the 1<sup>st</sup> stage were higher than the budget of £12m, confirming that a 'notional overspend' maybe optimal. In addition, the allocation which maximised overall health did not provide the highest ex-ante health benefit, demonstrating that it may be better not to maximise ex-ante expected health outcome even with a 'notional overspend'.

## The Research Decision

Allocation decisions based on expected cost-effectiveness will be appropriate only if the budget constraint is regarded as soft as described above. Even in these uncommon circumstances allocation decisions may not simply be based on little - or poor quality - evidence, since the question of whether further research to support allocation between treatments, populations and programmes should be made simultaneously. Therefore there are two conceptually distinct but simultaneous questions that must be addressed within any health care system. Firstly, which technologies should be adopted given the existing evidence and secondly, is additional evidence required to support this allocation decision. To address the second question a means to establish the value of additional evidence or equivalently the expected costs of uncertainty is required.

### Expected value of information

Bayesian decision theory and expected value of information (EVI) analysis provides such an analytic framework (Pratt et al., 1995, Raiffa and Schlaifer, 1959). These methods are used increasingly in operations research, decision analysis, risk analysis and health economics (Artstein, 1999, Azondekon and Martel, 1999). To date all EVI analysis within health care has been based on the standard decision rules used in CEA and therefore addresses the value of information associated with the choice between mutually exclusive treatments for a population group within a particular programme (Ades et al., 2004, Claxton, 1999, Claxton and Sculpher, 2006). For example, if there are  $j$  alternative mutually exclusive treatments in programme  $k$  for population  $i$ , with uncertain parameters  $\theta$ , the optimal decision with current information would be to choose the treatment that generates the maximum expected net health or monetary benefit (NB), i.e.,  $\max_j E_\theta(\text{NB}(j, \theta))$ . If these uncertainties could be resolved (with perfect information) the decision-maker could select the treatment that maximises the net benefit for a particular value of  $\theta$ . However, the true values of  $\theta$  are unknown, so the expected value of a decision taken with perfect information is found by averaging these maximum net benefits over the joint distribution of  $\theta$ , i.e.,  $E_\theta(\max_j \text{NB}(j, \theta))$ . The expected value of perfect information ( $\text{EVPI}_{ik}$ ) for an individual within the relevant population is simply the difference between the expected value of the decision made with perfect information about the uncertain parameters  $\theta$ , and the decision made on the basis of existing evidence:

$$\text{EVPI}_{ik} = E_\theta(\max_j \text{NB}(j, \theta)) - \max_j E_\theta(\text{NB}(j, \theta)) \quad (7)$$

The expected value of perfect information (EVPI) is also the expected opportunity loss and represents an upper bound on the value of evidence that may be generated by conducting research. When expressed in monetary terms for the relevant population, it provides a necessary condition for the research decision (if the costs of the proposed research exceed the EVPI then the research will not be worthwhile). Since the decisions within populations and across programmes are independent (assuming no exchangeability of evidence), the value of information for the whole allocation problem is simply the sum of EVPIs across all these independent decisions:

$$\text{EVPI} = \sum_{k=1}^3 \sum_{i=1}^3 n_{ik} \times \text{EVPI}_{ik} \quad (8)$$

where  $n_{ik}$  is the size of population  $i$  within programme  $k$ .

This approach to informing the research decision suffers from a number of limitations, even if a soft budgetary policy was in place. With a single budget for health care, from which services need to be provided and research conducted, it is not clear where opportunity costs of the resources needed for research actually fall. The use of a CE threshold, even if correctly assessed, will only be appropriate for marginal changes relative to the budget. In essence it assumes that health can continue to be bought at a constant rate. The budget is in fact unlimited at that rate or equivalently unlimited resources for research are available from some other source as long as the research generates health at the rate given by the threshold. This means that in practice the EVPI and therefore the

amount that might be spent on research can be very large and constitute a very large proportion of the total budget. The actual opportunity cost will be substantially higher than that implied by the threshold. The real policy question is, how much of the budget would the decision maker be willing to give up to resolve the uncertainties in the allocation problem? This can only be established by solving the allocation and research decision problem simultaneously even if a soft budgetary policy was in place.

More problems with the standard approach to EVI arise with a hard budgetary policy: i) the expected opportunity costs of uncertainty will tend to increase as it leads to either restrictive ex-ante allocations or costly remedial actions; ii) the EVI will therefore depend partly on the realisations that are revealed and the actions available as well as the budgetary policy; iii) the value of resolving the uncertainty associated with parameters will no longer be independent of the variability in these and other random parameters because this will partly determine when realised constraints bind with and without additional information. Resolving uncertainty in one programme may mean that unrelated treatments can be adopted in other independent programmes and/or costly remedial actions elsewhere be avoided, i.e., the value of information within one programme is no longer independent of other independent programmes, populations and treatments. To resolve these problems a more general and unified approach to the allocation and research decision problem is required.

### General approach to EVI

The expected health benefits based on existing evidence under a hard budgetary policy but with some restrictive actions available is provided in (6). Establishing the value of information for this allocation problem requires the sources of variation in the random parameters  $Z$  to be distinguished. The value of these parameters may be variable ( $\Delta$ ) and the estimates of their expected values uncertain ( $\theta$ ). The set of random parameters can therefore be expressed as the union of these two sets ( $Z = \theta \cup \Delta$ ). Information will reduce the randomness due to uncertainty but will not affect the variability in their values.

With this in mind a similar formulation is able to characterise expected health benefits with perfect information where the uncertain parameters are resolved ex-ante,

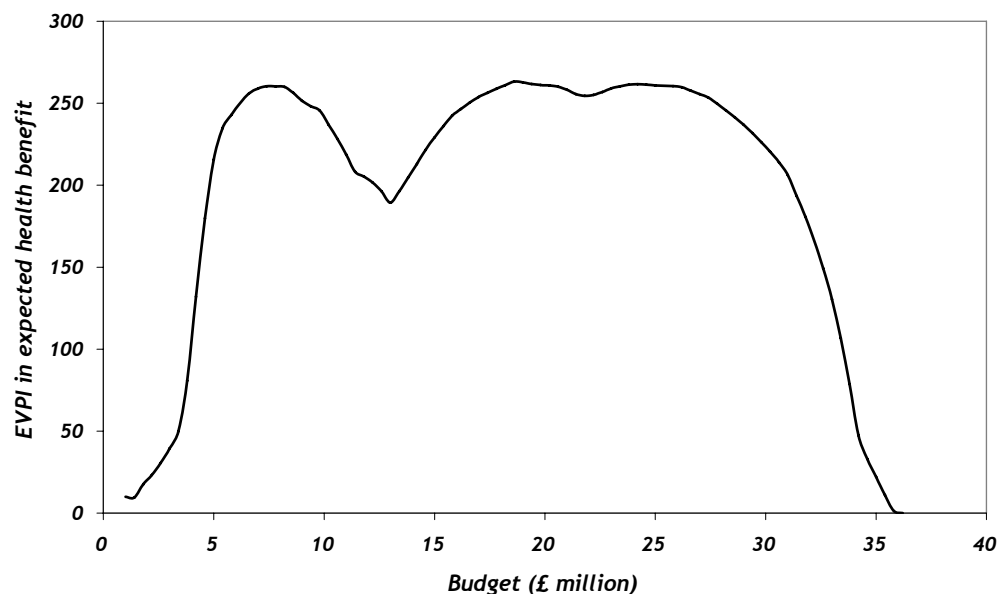
$$\left. \begin{array}{l} \text{for all } \theta \left\{ \begin{array}{l} \text{Max. } E_{\Delta|\theta}(B(\theta, \Delta, X)) \\ \text{s.t. } E_{\Delta|\theta}(C(\theta, \Delta, X)) \leq \Psi \\ \sum_{j=1}^3 x_{ijk} = 1 \text{ for } i, k = 1, \dots, 3 \end{array} \right\} \text{ 1st stage} \\ \text{for all } \Delta \left\{ \begin{array}{l} \text{Max. } B(\theta, \Delta, Y) \\ \text{s.t. } C(\theta, \Delta, Y) \leq \Psi \\ y_{ijk} \leq x_{ijk}^{**} \text{ for } i, k = 1, \dots, 3, j = 2, 3 \\ \sum_{j=1}^3 y_{ijk} = 1 \text{ for } i, k = 1, \dots, 3 \end{array} \right\} \text{ 2nd stage} \end{array} \right\} \quad (9)$$

$$\text{Expected health benefits} = E_{\theta} \left( E_{\Delta|\theta} \left( B(\theta, \Delta, Y^{**}(\Delta)) \right) \right)$$

With perfect information, the expectation in the 1<sup>st</sup> stage is conditional on a particular resolution  $\theta$  of the uncertain parameters, but is carried out over the variability in their values  $\Delta$ . A 2<sup>nd</sup> stage is still required because even for a particular value of  $\theta$  some realisations of the variability could result in a deficit. Therefore, the 2<sup>nd</sup> stage allows remedial actions for every random realisation of the values of the parameters conditional on a particular value of  $\theta$ . However, the true values of  $\theta$  are unknown, so the expected health benefit of an allocation decision taken with perfect information is found by taking

expectation over the joint distribution of  $\theta$ . The EVPI for the allocation problem as a whole is the difference in expected health benefit with perfect (9) and current (6) information:

$$EVPI = E_{\theta} \left( E_{\Delta|\theta} \left( B(\theta, \Delta, Y^{**}(\Delta)) \right) \right) - E_{\theta, \Delta} \left( B(\theta, \Delta, Y^*(\theta, \Delta)) \right) \quad (10)$$



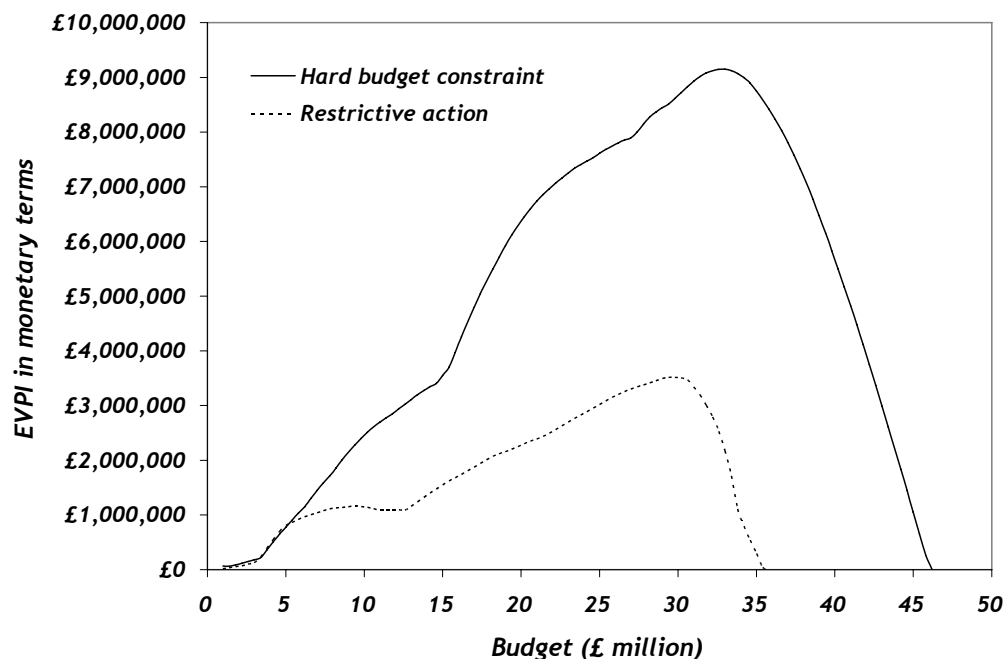
**Figure 5: EVPI in expected health benefit for a hard budgetary policy with restrictive actions**

The EVPI expressed in expected health gains over a range of budgets is illustrated in Figure 5. When the budget is very low very little health care can be provided so there is little opportunity to improve health even when uncertainties are resolved. Equally, when the budget is very large (greater than £35.8m), all the most effective and costly treatment options are able to be implemented in full across all population groups and programmes. Additional information will not produce any gains in health outcome because in this example it is the magnitude of differences in effect and cost between the treatments rather than their ranking which is uncertain.<sup>4</sup> The changes in the slope of the ‘EVPI curve’ illustrated in Figure 5 represent points where the optimal treatments selected switch (decision variables take integer values), i.e., at similar points as the steps in Figure 1. Between these points the same set of treatments are selected but in different proportions within a particular population, i.e. a mixed programme.

**Table 1: EVPI in monetary terms**

Budget	Current information	Perfect information	EVPI (health gains)
£10,904,000	4641	<b>4863</b>	
£12,000,000	<b>4863</b>	5067	204

The monetary value of information is how much of the budget the decision maker should be willing to give up to resolve the uncertainties in the allocation problem. This is the reduction in budget required with perfect information that will generate the same expected benefits as with current information. It represents the maximum amount of budget that the health care system should be willing to give up for additional evidence and places an upper bound on research expenditure. The conversion of EVPI in health gains to monetary terms is demonstrated in Table 1 for a budget of £12m. Decreasing the budget of £12m to £10.904m with perfect information (i.e. a decrease of £1.096m) gives the same expected benefits as a budget of £12m with current information. Therefore, the EVPI of 204 units of health gain equates to an EVPI of £1.096m.



**Figure 6: EVPI in monetary terms for a hard budgetary policy with no available actions versus restrictive actions**

The EVPI in monetary terms over a range of budgets is illustrated in Figure 6 for a budgetary policy with restrictive actions. The same characteristics are evident. EVPI will be low at very low budgets and also low at very high budgets when all the most effective care can be provided.<sup>4</sup> Also the changes in the slope of the 'EVPI curve' in Figure 6 are consistent with those in Figure 5. However, the implicit monetary value of the health gains offered by additional information increases with the budget. At a budget of £7m, the EVPI in health gains is only just lower than at budget of £25m but the EVPI in monetary terms is substantially lower. Although there is no unique shadow price or threshold (each realisation of random parameters generates a new potentially binding constraint with its own shadow price) and any shadow price or threshold will only be relevant for marginal changes, the relationship between budget and the implied value of health still exists. However, it can only be inferred from the solutions to the allocation problem and will be specific to the non-marginal change as well as the budgetary policy, what realisations are revealed and the remedial actions available.

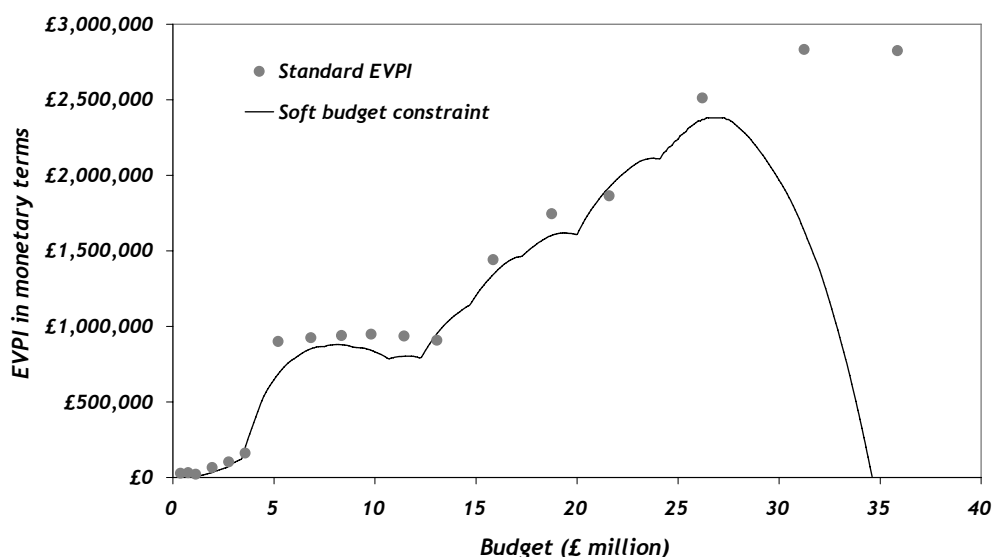
The EVPI will also depend on the budgetary policy and the available actions. For example, the EVPI with a hard budgetary policy with no available actions is the difference between the expected health benefits with perfect information in (11) below and current information in (5).

$$\left. \begin{array}{l}
 \text{Max. } E_{\Delta|\theta} (B(\theta, \Delta, X)) \\
 \text{s.t. } C(\Delta_1(\theta), X) \leq \Psi \\
 \quad C(\Delta_2(\theta), X) \leq \Psi \\
 \quad C(\Delta_3(\theta), X) \leq \Psi \\
 \quad \dots \\
 \quad C(\Delta_N(\theta), X) \leq \Psi \\
 \quad \sum_{j=1}^3 x_{ijk} = 1 \text{ for } i, k = 1, \dots, 3
 \end{array} \right\} \text{for all } \theta$$

(11)

$$\text{Expected health benefits} = E_{\theta} \left( E_{\Delta|\theta} (B(\theta, \Delta, X^*)) \right)$$

This EVPI expressed in monetary terms is also illustrated in Figure 6. The EVPI with no actions available is substantially higher at all budgets and remains positive until a higher total budget. This should be expected, as any realisation of parameters that implies a deficit will impose greater opportunity costs because no remedial actions are available and additional budget is required before decision makers are able to take the risk of providing the most effective care. Therefore, the value of resolving uncertainty will tend to be higher when actions are more restrictive or not all realisations are revealed or revealed later. This also demonstrates that the EVPI depends, in part on the variability in the values that parameters can take. For example, without variability the expected health benefits with perfect information would be the same in (9) as (11) and the expected health benefits with current information in (6) and (5) would change.



**Figure 7: EVPI in monetary terms for a soft budgetary policy compared to standard EVPI**

The EVPI under a soft budgetary policy can also be established in a similar way. The expected health benefit with current information is provided in (4). With perfect information decision variables can be chosen conditional on a particular value of  $\theta$  which meets the budget at expectation over  $\Delta$ . The expected health benefit with perfect information is the expectation over these conditional choices. The monetary EVPI for a soft budgetary policy is illustrated in Figure 7 for a range of budgets. The same characteristics are evident. EVPI will be low at very low budgets and also low at very high budgets when all the most effective care can be provided.<sup>4</sup> The changes in the slope of the 'EVPI

curve' are consistent with those in Figures 5 and 6. The EVPI is, as expected, lower than with a hard budgetary policy with either no actions or restricted actions available.<sup>8</sup>

The standard approach to EVPI described in (7) and (8) is also illustrated in Figure 7 for a range of budgets, using the 'correct' thresholds reported in Figure 1. For budgets less than £25m it does appear similar to the EVPI for a soft budgetary policy, confirming the previous discussion that the standard approach to EVPI is based on the standard decision rules in CEA which in turn are only consistent with a soft constraint. However, as discussed above, there are good reasons why the standard approach will not be entirely consistent even with a soft budgetary policy. For example, standard EVPI is substantially higher at budgets greater than £25m. The reason is that when the budget is sufficiently high, all the most costly and effective care ( $j = 3$ ) is received. In the standard approach the cost effectiveness of this care remains uncertain and for some realisations it may not be regarded as cost-effective when compared to a fixed threshold. In essence this assumes that any costs saved by rejecting  $j = 3$  can buy more health elsewhere at this fixed rate. Of course the reality is very different: health cannot continue to be bought at a fixed rate. Once all population groups are allocated to  $j = 3$  ex-ante, there is no gain from switching them to less effective care irrespective of realised values.<sup>4</sup>

The research decision problem is, how much of the budget should a decision maker be willing to give up to resolve the uncertainties in the allocation problem? This can only be established by solving the allocation and research decision problem simultaneously using a more general and unified approach than is offered by the current approaches to EVI. Indeed, the standard approach to EVPI is a peculiarly special case. A more general formulation demonstrates that the value of information and need for evidence depends on: i) the budgetary policy in place; ii) the realisations revealed during the budget period; iii) remedial actions that may be available; and iv) variability in parameters values.

## Discussion

It is possible to characterise the allocation and research decision problem when costs and effects are both uncertain and variable without relying on exogenous and arbitrary parameters. In doing so it is possible to formulate a more general and unified approach which can simultaneously address allocation and research decisions. This more general approach demonstrates that standard decision rules in CEA are a very special case which requires budget constraints to be soft in addition to assumptions of perfect divisibility, constant returns and all costs and benefits occurring within the budgetary period. If the budget constraint cannot be violated then the optimal allocation will depend on the realisations that are revealed to the decision maker and what remedial actions will be available. In these circumstances truly optimal ex-ante decisions are unlikely to be consistent with either meeting the budget at expectation or maximising ex-ante expected health benefits. Through this simple but more general formulation of the allocation problem, we have demonstrated that standard decision rules are only optimal in very special circumstances and that there are no simple rules which lead to optimal ex-ante decisions. It also becomes possible to evaluate alternative budgetary policies that may be imposed by a higher authority. The choice of budgetary policy for a particular set of treatments, populations and programmes will depend on: i) the overall budget; ii) the variability in random parameters; iii) what realisations are revealed during the budget period; iv) what remedial actions are available to decision makers; and v) the costs of effectively monitoring ex-ante plans.

By distinguishing uncertainty and variability as the source of randomness in costs and outcomes it is possible to address the research decision problem simultaneously and consistently. The research decision problem is how much of the budget a decision maker should be willing to give up to resolve uncertainties in the allocation problem? This represents the maximum the health care system should be willing to give up for additional evidence and places an upper bound on research expenditure.

<sup>8</sup> It should be noted that all realisations are revealed in the formulation of the hard constraint in (6) but not in the soft constraint in (4). However, in this case, even if the same realisations were revealed under a soft constraint non of the restrictive actions would be taken because the only possibility is to switch patients to no treatment which is known to be less effective. In other circumstances where either more actions are possible or the ranking of the effectiveness of treatments is also uncertain then explicitly modelling the realisations revealed and the available actions would be necessary under both a soft and hard budgetary. Also see footnotes 3 and 4.

This EVPI for the whole allocation problem will depend on the budgetary policy in place, the realisations revealed and remedial actions available as well as the variability in parameter values. The standard approach to EVPI is a peculiarly special case which requires both a soft constraint and the assumption that health can continue to be purchased at a constant rate. It should be apparent that resolving uncertainty in one programme may mean that unrelated treatments can be adopted in other independent programmes and/or costly remedial actions elsewhere be avoided, i.e., the value of information within one programme is no longer independent of other independent programmes, populations and treatments.

Whilst preserving the essential dynamics and key features of the allocation problem a number of simplifications have been made in developing this formulation and applying it to a simple numerical example. A number of extensions could be considered. These include considering which realisations will be revealed and at what point during the budgetary period. Currently the two stage formulation allows all realisations to be revealed to the decision maker, but restricts the actions that can be taken. However, the realisations of costs falling on the budget may be more likely to be revealed than health outcomes. Also they will tend to be revealed over time during the budgetary period as populations are treated, initially based on ex-ante plans. Even if the decision maker has unrestricted actions available some proportion of the population will have received ex-ante treatment choices and these costs can not be recovered. Any remedial action is only possible for that proportion of the population not yet treated but such decisions must be taken before the remaining costs and effects are revealed. Therefore rather than a two stage formulation a multi stage or dynamic programming formulation would be necessary. This would provide a means to consider the value of investing in information systems which allow realisations to be revealed earlier, avoiding more costly remedial actions later in the budgetary period. Finally, decision makers face more than one budgetary period and face decisions which impact on costs and benefits falling in different periods. Multiple budgetary periods would allow exploration of budgetary rules where decision makers may run deficits in one period if covered by subsequent surpluses and where a current surplus can be carried forward to the next period. This would also allow evaluation of policies where decision makers could borrow against future budgets at some market rate of return and explore the implications of uncertainty in future budgets. However, although extending this analysis in a variety of way would be of value the key insights provided by this simplified formulation would remain.

The analysis has demonstrated that there are no simple ex-ante decision rules in most common circumstances and the value of information cannot be established for one programme independently of the rest of the allocation problem. Optimal allocation and research decisions would require full knowledge of the expected costs and effects of all the available treatments for every population across all health care programmes. In addition it would require knowledge of variability in these values, the uncertainty in their expected values and an understanding of when realisations may be revealed. In a similar way to the problem of second best, these informational requirements means that it is not feasible to identify truly optimal allocation and research decisions for current policy purposes. Nevertheless it is valuable to understand the circumstances in which standard decision rules and analysis may be expected to be a poor guide to optimal ex-ante allocation and research decision, i.e., a useful question is how sub optimal are allocation and research decisions likely to be if based on standard analysis. This is similar to the more familiar question, are first best rules in a second best world reasonable in a situation of information poverty.

With a soft constraint the standard decision rules will be optimal if the common assumptions of divisibility and constant returns also hold. However, if the constraint is hard then technologies will need to be more cost-effective (an ICER substantially below the threshold) before the decision maker should take the risk of an ex-ante decision to adopt them. Greater cost-effectiveness would need to be observed ex-ante when little is revealed during the budgetary period, only revealed later when resources are already committed or if the remedial actions are more restricted. If this is extended to multiple budgetary periods how much more cost-effective a technology will need to be will also depend on the cost of covering deficits from future budgets (both the rate of interest and opportunities forgone in the future period) and the possibility and returns of carrying surpluses forward to future periods. Similarly some general conclusions can be drawn for the research decision. For example, with a hard constraint current estimates of value of information will tend to underestimate the opportunity cost of uncertainty and neglect the impact that resolving uncertainty would have on other unrelated programmes. On the other hand if the constraint is indeed soft, the standard approach may tend to overestimate value by assuming that health can be purchased at a constant rate. Therefore,



estimates of value of information that are very high (relative to the total budget) should be interpreted with caution. They may indicate the relative importance of evidence but not necessarily indicate that a substantial proportion of the total budget should be devoted to such research. The more general formulation presented above shows that standard decision rules and measures of value are necessarily proxies for an uncertain and complex process. The use of such proxies for practical policy purposes is both inevitable and commonplace in all areas of public (and private) choice. Understanding the key characteristics of the process they proxy and the circumstance in which they are expected to perform particularly badly seems valuable and responsible.

### Appendix 1

**Table A1: Data used for 3 hypothetical treatment alternatives ( $j = 1, \dots, 3$ ) in 3 separate population groups ( $i = 1, \dots, 3$ ) within 3 independent health care programmes ( $k = 1, \dots, 3$ ).**

Programme k	Population i	Treatment j	Population size N	Outcome: a			Outcome: b			Expected Utility EU	Expected Cost EC	Incremental CE ratio ICER (£)
				Utility	Cost (£)	Probability <sup>†</sup>	Utility	Cost (£)	Probability <sup>†</sup>			
1	1	1	1000	0.1	0	1.00	0.7	0	0.00	0.1	0	937.50
		2		0.3	450	0.50	0.7	300	0.50	0.5	375	
		3		0.6	6600	0.15	0.95	1100	0.85	0.8975	1925	
	2	1	1000	0.1	0	1.00	0.7	0	0.00	0.1	0	2031.25
		2		0.3	975	0.50	0.7	650	0.50	0.5	812.5	
		3		0.6	12000	0.15	0.95	2000	0.85	0.8975	3500	
	3	1	1000	0.1	0	1.00	0.7	0	0.00	0.1	0	4062.50
		2		0.3	1950	0.50	0.7	1300	0.50	0.5	1625	
		3		0.6	21000	0.15	0.95	3500	0.85	0.8975	6125	
2	1	1	1000	0.1	0	1.00	0.6	0	0.00	0.1	0	1071.43
		2		0.3	450	0.50	0.6	300	0.50	0.45	375	
		3		0.6	6600	0.15	0.85	1100	0.85	0.8125	1925	
	2	1	1000	0.1	0	1.00	0.6	0	0.00	0.1	0	2321.43
		2		0.3	975	0.50	0.6	650	0.50	0.45	812.5	
		3		0.6	12000	0.15	0.85	2000	0.85	0.8125	3500	
	3	1	1000	0.1	0	1.00	0.6	0	0.00	0.1	0	4642.86
		2		0.3	1950	0.50	0.6	1300	0.50	0.45	1625	
		3		0.6	21000	0.15	0.85	3500	0.85	0.8125	6125	
3	1	1	1000	0.1	0	1.00	0.5	0	0.00	0.1	0	1250.00
		2		0.3	450	0.50	0.5	300	0.50	0.4	375	
		3		0.6	6600	0.15	0.75	1100	0.85	0.7275	1925	
	2	1	1000	0.1	0	1.00	0.5	0	0.00	0.1	0	2708.33
		2		0.3	975	0.50	0.5	650	0.50	0.4	812.5	
		3		0.6	12000	0.15	0.75	2000	0.85	0.7275	3500	
	3	1	1000	0.1	0	1.00	0.5	0	0.00	0.1	0	5416.67
		2		0.3	1950	0.50	0.5	1300	0.50	0.4	1625	
		3		0.6	21000	0.15	0.75	3500	0.85	0.7275	6125	

<sup>\*</sup>Probability of outcome *b* given treatments *j*=2 and *j*=3 assumed to follow a beta distribution. The parameters of the beta distribution were obtained by the method of moments. A beta(12,12) and beta(9.99,1.76) was fitted to treatments *j*=2 and *j*=3, respectively, in each population group.

<sup>†</sup>Probability of outcome *a* is 1 minus probability of outcome *b*.

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