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DOAM for Evolutionary Portfolio Optimization: a computational study.

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Abstract

In this work, the ability of the Dynamic Objectives Aggregation Methods to solve the portfolio rebalancing problem is investigated conducting a computational study on a set of instances based on real data. The portfolio model considers a set of realistic constraints and entails the simultaneously optimization of the risk on portfolio, the expected return and the transaction cost.

1 Introduction

The standard problem of portfolio selection consists in allocating wealth among available investments. Let n be the number of available risky assets with expected returns μ_i and variances σ_{ii}; let σ_{ij} be the covariance between the asset i and the asset j and $\Sigma = (\sigma_{ij})$ be the covariance matrix. We denote with x_i the proportion of the capital to be allocated to the asset i. Therefore, the standard problem of portfolio selection can be stated as follows:

$$\min\ x'\Sigma x, \qquad \text{(Min-Risk)} \tag{1}$$

subject to

$$\mu' x = \overline{\mu}, \tag{2}$$

$$x'\underline{1} = 1, \tag{3}$$

$$x_i \geq 0, \qquad i = 1, \ldots, n; \tag{4}$$

The solution of the previous problem is the portfolio with minimum risk among those with a fixed expected return $\overline{\mu}$. Equations (3) and (4) represent

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the balance constraint and the non-negative constraint, respectively; the latter is left out when short sales are allowed. Mean-variance approach allows to trace out the efficient frontier, a set of portfolios that offer the minimum risk level for a given level of reward. The shape of the efficient set differs according to the assumptions in regard to the ability of the investor to sell security short as well as the ability to lend and to borrow funds. The scenario of the classical mean-variance model is an ideal market: no transaction costs, no holding constraints, no limit on portfolio cardinality, no regulatory requirements are present. Since any realistic portfolio problem has to take into account these practical issues, it is necessary to consider a model including costs and constraints. The introduction of such constraints, particularly cardinality range, raises the computational complexity of the portfolio model which turns out to be $\mathcal{NP}$-hard.
In this work, a computational and comparative study on the application of DOAMs on multi-objective rebalancing problem is proposed. Section 2 presents the portfolio rebalancing problem and describes the transaction costs and the multi-objective rebalancing model. In section 3 the Dynamic Objective Aggregation Methods are briefly introduced and the experimental setting is described; section 4 presents the analysis of results, while in the last section some comments are drawn.

2 Portfolio Rebalancing problem

Let us consider now the revision of the current portfolio x^0; let the n vector $x^0 = (x_1^0, \ldots, x_n^0)$ be the current portfolio and let $x = (x_1, \ldots, x_n)$ be the portfolio after rebalance; x is a vector of fraction of the capital invested in each asset, therefore the vector of amount of money actually invested is $\mathcal{C}x$, where $\mathcal{C}$ stands for the available capital or, in rebalancing problem, the value of the current portfolio x^0. In any realistic portfolio rebalancing the investors have to face transaction costs; let $T_{x_0}^x$ be the transaction cost associated with the rebalance from x^0 to x; $T_{x_0}^x$ is a function of the vector of the trading volumes $v = |x - x^0|\mathcal{C}$:

$$v_i = \begin{cases} (x_i - x_i^0)\mathcal{C} & \text{if the exposure to the } i \text{ asset is increased by purchases,} \\ (x_i^0 - x_i)\mathcal{C} & \text{if the exposure to the } i \text{ asset is reduced by sales.} \end{cases}$$

In our model we assume that the transaction costs both for purchases and sales are equal. Furthermore, it is assumed that the cost function is a separable function:

$$T_{x_0}^x(v) = \sum_{i=1}^n t_i(v_i).$$

We are considering small size of trade; in particular, from trading online transaction costs we derive two type of transaction function according to the type of securities, t_i^1 and t_i^2:

$$t_i^1(v_i) = \begin{cases} 0 & \text{if } x_i = x_i^0 \\ \max\{\, \mathrm{C}^1_{\min},\, c_i^{1r} v_i \,\} & \text{otherwise} \end{cases} \tag{5}$$

$$t_i^2(v_i) = \begin{cases} 0 & \text{if } x_i = x_i^0 \\ \min\Big(\ \max\{\, \mathrm{C}^2_{\min},\, c_i^{2r} v_i \,\}, \mathrm{C}^2_{\max}\Big) & \text{otherwise} \end{cases} \tag{6}$$

where $\mathrm{C}^1_{\min}$ and $\mathrm{C}^2_{\min}$ denotes the minimum costs, $\mathrm{C}^2_{\max}$ the maximum costs, c_i^{2r} and c_i^{1r} stand for the commission rates for the asset i. The first trading function entails a fixed cost until a given level of amount traded v^1; beyond v^1 the costs increase linearly with the volume transacted. In the second function, there is an upper bound on the transaction costs, as well: beyond an upper level v^2 the transaction costs are fixed. The model proposed is the

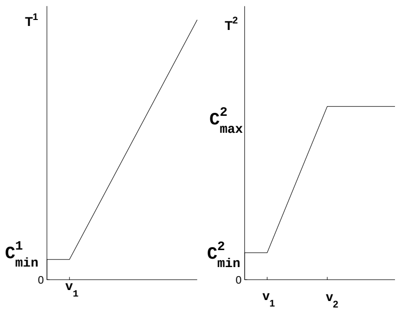

Figure 1: Transaction cost function

following:

$$\begin{aligned} \min \quad & x'\Sigma x, \\ \max \quad & x'\mu, \\ \min \quad & \sum_{i=1}^{n} t_i(|x_i - x_i^0|\mathcal{C}) \\ & x'\underline{1} = 1, \\ & K_1 \leq \sum_{i=1}^{n} Z_i \leq K_2, \\ & l_i Z_i \leq x_i \leq u_i Z_i, \qquad i \in \{1, \ldots, n\} \\ & Z_i \in \{0, 1\}, \qquad i \in \{1, \ldots, n\}. \end{aligned} \tag{7}$$

t_i denotes the cost function of the asset i; K_1 and K_2 are the minimum and maximum number of assets that must be in portfolio; l_i and u_i are lower and upper bound on the holdings in each asset, respectively.

3 DOAMs' Configuration

To solve the portfolio rebalancing problem as a multi-criteria optimization problem, a dynamic scalarization method based on different aggregate functions in an evolutionary optimization scheme is used. The Dynamic Objective Aggregation Methods are based on the standard genetic algorithm included in the Matlab's Genetic Algorithm and Direct Search Toolbox [7]. These algorithms with different rules of weights changing have been first tested on benchmark problems from the literature and compared with a widely used standard multi-objective algorithm: NSGA-II [3]. Computational results of this preliminary campaign of experiments are reported in [4]. The algorithms achieving better results are employed in a second campaign of experiments devoted to tackle the multi-objective portfolio rebalancing problem; therefore, we investigate the ability of the heuristic DOAMs to solve the portfolio rebalancing model. Among the 24 DOAMs tested in the preliminary study [4], obtained combining the weights generation rules and the four strategies considered for the variation of the exponents, the best 6 algorithms are chosen, namely: chaotic, sinusoidal and triangular weights generators are combined with exponents fixed to one (*Chaos-Gen, Sin-Gen, Trian-Gen*) and with the adaptive scheme (*Chaos+Exp, Sin+Exp, Trian+Exp*). In the adaptive scheme the exponent value is incremented when there is no improvement in the optimization process for a given number of iterations, which has been fixed to $\Delta = 0.05\,N$, being N the maximum number of generations that can be produced.
In the preliminary computational study [4], we used the DOAMs for two-objective problems; as the model (7) has three objectives, the aggregate function has the following expression

$$F(x,k) = w_1(k) f_1^t(x) + w_2(k) f_2^t(x) + w_3(k) f_3^t(x).$$

The weights w_k are dynamically modified according to a function $R(k)$ of the generation number k:

$$w_1(k) = R(k), \quad w_2(k) = (1 - w_1)w_1, \quad w_3(k) = 1 - w_1 - w_2.$$

A periodical changing of the weights can be obtained according to a sin or triangle wave; the sinusoidal rule is the following:

$$R(k) = |\sin(2\pi k/F)|, \tag{8}$$

where F is the frequency and it has been fixed to $F = 200$. Whereas, a chaotic variation law to the weights is employed as follows:

$$w_1(k+1) = \mu w_1(k)(1 - w_1(k)). \tag{9}$$

When $\mu = 4$ and $w_1(0) \notin \{0, 0.25, 0.5, 0.75, 1\}$, the previous equation shows chaotic behaviour.

As the DOAMs are based on the standard genetic algorithm included in the Matlab's Genetic Algorithm and Direct Search Toolbox, some parameters values need to be specified, before the algorithm execution: we adopted a stochastic uniform selection operator, a scattered crossover function with probability 0.7 and a Gaussian mutation function with probability 0.3; the number of best individuals that will survive to the next population has been fixed to 2.
The population size is of 100 individuals; the archive used is made up of 500 individuals.
In our experiments, we consider two different stopping criteria: in the first set of computational tests the stopping criterion is based on the maximum number of generations to be produced and it is fixed to 500; therefore the computational time can be considered as a performance indicator. Since from the first results it seems evident that the DOAMs are faster than the NSGA-II, the computational tests are then repeated considering the execution time as stopping criterion, i.e. a time limit of 600 seconds is adopted.

3.1 Cardinality and Holding Constraints

Since the DOAMs are population based algorithms, at each generation a population of individuals (children) or solutions are produced by genetic operators of selection, crossover and mutation from the previous generation (parents). Not all possible individuals correspond to feasible portfolios, because of the holdings and cardinality constraints; therefore a procedure provided by Chang *et al.*, [2] is used to assure the individuals to be always feasible.
Let us consider n real numbers $s_i, 0 \leq s_i \leq 1$, $i = 1, \ldots, n$; let the vector $(s_1, \ldots, s_n)$ be an individual (child) of the population generated by the algorithm. Considering the set $Q = \{i \text{ s.t. } s_i \neq 0\} \subset \{1, \ldots, n\}$, $K = |Q|$ is the number of non-zero elements of the individual $(s_1, \ldots, s_n)$. If K is greater than K_1 and lower then K_2, then the individual satisfies the cardinality constraint; otherwise a procedure to assure the cardinality constraints are satisfied is used. This procedure is described in the pseudo-code 1:

After we have assured that the number of non-zero s_i is between K_1 and K_2, we use a procedure to assure that the holding and the balance constraints are satisfied too. This procedure is shown in a pseudo-code in Table 2.

3.2 Portfolio Data Set

The comparison of different DOAMs implementations is performed on a set of instances based on a public data set provided by Beasley and available from OR-Library [1].

Table 1: Procedure for the fulfillment of cardinality constraints

A the set of assets that are in the parents, but are not in the child	
P the set of assets i with $i \in Q$,	
while $\|Q\| > K_2$ delete the asset i with the smallest s_i	
while $\|Q\| < K_1$ **do**	% add asset from parent if possible
if $\|A\| \neq 0$ **then**	
add to P a randomly chosen asset j from A	
$A = A \setminus \{j\}$	
else	
add to P a randomly chosen asset $j \notin Q$ and set $s_j = 0$	
end if	
end while	

Table 2: Feasibility recovery procedure

begin	
if $\sum_{i \in Q} l_i > 1$ or $\sum_{i \in Q} u_i < 1$ **then return**	% infeasible
$L := \sum_{i \in Q} s_i$	
$F := 1 - \sum_{i \in Q} l_i$	% F is the free proportion of the portfolio
$x_i := l_i + s_i F/L \, \forall i \in Q$	% x_i satisfies lower holding and balance
	%iterative procedure for upper holding constraint
$R := \emptyset$	% R contains i with $x_i = u_i$
while there exists an $i \in Q \setminus R$ with $x_i > u_i$ **do**	% iterate until feasible
for all $i \in Q \setminus R$ **if** $x_i > u_i$ **then** $R := R \cup \{i\}$	
$L := \sum_{i \in Q \setminus R} s_i$	
$F := 1 - (\sum_{i \in Q \setminus R} l_i + \sum_{i \in R} u_i)$	% F is the free proportion of the portfolio
$x_i := l_i + s_i F/L \, \forall i \in Q \setminus R$	
$x_i = u_i \, \forall i \in R$	
end while	
end	

The financial data sets (means and variance matrix) are constructed using the stocks involved in five capital market indices. The weekly prices from March 1992 to September 1997 are taken into account for the stocks of Hang Seng (Hong Kong), DAX 100 (Germany), FTSE 100 (UK), SP 100 (USA) and Nikkei 225 (Japan). The size of the five tests problems varies from $n = 31$ (Hang Seng) to $n = 225$ (Nikkei).

We extended the Beasley's original instances introducing our more realistic aspects in both objectives and constraints. We use the two transaction costs structures defined by equations (5), (6). The data used for the parameters characterizing the commission costs are realistic values obtained from available trading online data:

$$\mathrm{C}^1_{\min} = 15, \quad c_i^{1r} = 0.30\%,$$
$$\mathrm{C}^2_{\min} = 2.5, \quad c_i^{2r} = 0.20\%, \quad \mathrm{C}^2_{\max} = 20.$$

Two different configurations of constraints are considered:

$$\text{Configuration 1}: \quad K_1 = 9,\, K_2 = 11, K_0 = 10, l_i = 0.05,\, u_i = 0.75,$$
$$\text{Configuration 2}: \quad K_1 = 18,\, K_2 = 22, K_0 = 20, l_i = 0.02,\, u_i = 0.75.$$

Combining the two transaction cost functions with the two constraints' configurations, 4 overall different formulations of the problems are considered. In the conducted experiments we assume the invested capital $\mathcal{C} = 100000$, while the initial portfolios are generated randomly.

4 Computational Results

To compare the different DOAMs on portfolio instances, we use four performance indexes; we consider the hyperarea ratio (HR), and the number of non-dominated elements setting up the efficient frontier (ND); we report the fractional contribution (FC) defined as the percentage of non dominated points contributed by an algorithm on the total efficient frontier obtained unifying all the efficient sets produced by all the algorithms on the same instance. Precisely, the total efficient frontier is obtained unifying all the efficient frontier and executing a dominance analysis: dominated points or eventually double points are removed. The fractional contribution is calculated as the number of points achieved by an algorithm that are present in the total front, out of the number of solutions in total frontier. As last indicator, we report the spacing (S).
Since the first experiments are made using the number of iterations as stopping criterion, we can consider the computational time (T) as performance indicator. As in the second set of experiments a time limit of 600 seconds is adopted, the number of generations (G) is also reported as performance index.
For each experiment three different runs have been executed initializing algorithms with random populations; therefore the values of performance indicators have to be considered as mean values. Tables 3 - 7 contain the average results on 3 runs for each of the 4 algorithms configurations described; the computational results reported are obtained for the five portfolio problems with the stopping criterion of 500 iterations. Tables 8 - 12 contain the average results on 3 runs for each algorithms configurations obtained with a time limit of 600 seconds.
From the first campaign of experiments, as global observation, we can say that all the DOAMs present a promising behavior, but no dynamic objectives algorithm perform always better than others. Furthermore, it can be observed that the DOAMs are, on the whole, faster than the NSGA-II for every instance and for every problem. While the fractional contribution FC of the NSGA-II is generally greater than that one of the DOAMS, the values of the main performance index, i.e. the hyperarea ratio, do not present relevant differences on the average.
Similar arguments can be used also in the experiments made with a fixed computation time. It can be observed that, although the NSGA-II presents again much more high percentages of fractional contributes and higher num-

ber of nondominated points, the hyperarea values of the DOAMs are comparable with that one of NSGA-II, and in several cases they are even better. In particular, in the $20,8\%$ and $23,3\%$ of cases DOAMs outperform NSGA-II in the first and second group of experiments, respectively. The comparative evaluation on this computational campaign of experiments restricted to DOAMs shows Trian-Gen and Sin-Gen and their t-power counterparts as better strategies.

5 Concluding Considerations

We have provided a reliable and general, yet improvable, algorithmic instrument to solve realistic multi-objective optimization problems. The multi-objective approach allows us to solve re-weighting portfolio problem also minimizing transaction costs. The proposed resolution methods based on evolutionary schemes and working with populations of solutions result - after different campaigns of experiments - to be a suitable instrument to solve multi-objective problems. Eventually other objectives can be taken into account in the portfolio problem without the algorithmic approach is changed. Future researches may be done both on the portfolio model and on the algorithmic methods. Since we have considered only the commissions' component of the transaction costs, the market impact cost could be considered. From the algorithmic point of view, in order to exploit the speed of DOAMs, an algorithm combining NSGA-II (or another standard population-based multi-objective algorithm) with a DOAM as local search algorithm deserves to be experimentally evaluated.

Moreover, other DOAMs with different aggregating functions can be considered: in addition to the t-power transformation, e.g. Lin *et al.* in [5] investigated the exponential transformation of the objective functions and its capability to convexify the efficient frontier.

The analysis of the impact of algorithm's parameters on the achievable computational performance is another topic for further research. It could be devoted to point out suitable procedures for the fine tuning of these parameters.

References

[1] Beasley J. E.: OR-Library: distributing test problems by electronic mail. Journal of the Operation Research Society, 41, pp. 1069-72, 1990.

[2] Chang T.-J., Meade N., Beasley J. E., *Heuristics for cardinality constrained portfolio optimization*, Computers Operation Research 27, pp. 1271-1302, 2000.

[3] Deb K., Pratap A., Agarwal S. and Meyarivan T., *A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II*, in IEEE Transactions on Evolutionary Computation, Vol. 6, No. 2, pp. 182-197, 2002.

[4] Dellino G., Fedele M., Meloni C., *Dynamic Objectives Aggregation in Multi-objective Evolutionary Optimization* Technical Report DEE - Politecnico di Bari.

[5] Li D., Biswal M. P., *Exponential transformation in convexifying a noninferior frontier and exponential generating method*, Journal of Optimization Theory and Applications: Vol.99, No. 1, pp. 183-199, 1998.

[6] Lin, D., Wang S., Yan H., *A multiobjective genetic algorithm for portfolio selection problem*, Proceedings of ICOTA, 2001.

A multiobjective genetic algorithm for portfolio selection. Working Paper, Institute of Systems Science, Academy of Mathematics and Systems Science Chinese Academy of Sciences, Beijing, China.

[7] The MathWorks Inc., *Genetic Algorithm and Direct Search Toolbox*, Natick, Massachussetts, 2004.

Table 3: Average values of T, ND, HR, FC and S for the portfolio problem 1 with a fixed (500) number of generations.

Form.	Perf.Indic.	NSGAII	Chaos-Gen	Sin-Gen	Trian-Gen	Chaos+ Exp	Sin.+ Exp	Trian.+ Exp
I	T(sec.)	1494	429	521	475	470	459	495
	ND	497	201	338	313	205	208	221
	HR(%)	11	7	8	9	8	7	7
	FC(%)	65.22	3.08	13.03	10.93	2.79	2.45	2.97
	S	94.03	158.01	140.98	175.35	77.21	146.71	97.67
II	T(sec.)	1242	423	460	508	453	464	492
	ND	455	143	184	158	96	104	115
	HR(%)	10	8	12	12	11	12	9
	FC(%)	88.77	2.40	3.80	3.50	0.33	0.47	0.61
	S	0.62	2.04	0.95	1.43	2.08	1.95	2.65
III	T(sec.)	1412	469	559	542	531	585	523
	ND	496	417	442	453	424	440	426
	HR(%)	14	17	9	8	9	11	10
	FC(%)	55.19	6.78	8.06	7.69	7.43	7.51	6.36
	S	94.14	198.46	89.50	150.51	106.92	254.01	110.74
IV	T(sec.)	1299	487	525	541	495	520	533
	ND	465	187	213	179	165	181	198
	HR(%)	15	11	11	9	7	12	13
	FC(%)	84.94	2.95	2.66	3.16	2.64	2.28	1.67
	S	0.68	2.30	1.08	1.25	1.94	2.13	1.08

Table 4: Average values of T, ND, HR, FC and S for the portfolio problem 2 with a fixed (500) number of generations.

Form.	Perf.Indic.	NSGAII	Chaos-Gen	Sin-Gen	Trian-Gen	Chaos+ Exp	Sin.+ Exp	Trian.+ Exp
I	T(sec.)	2173	993	1019	1033	1088	1052	1085
	ND	496	154	177	171	168	171	190
	HR(%)	10	3	11	12	5	6	3
	FC(%)	78.82	1.32	5.76	5.76	0.87	2.05	3.91
	S	27.39	108.50	241.20	355.41	162.25	180.05	151.65
II	T(sec.)	2067	991	985	1028	1019	1024	1064
	ND	495	106	110	104	86	793	827
	HR(%)	6	6	7	8	6	6	5
	FC(%)	87.06	2.22	7.39	2.93	0.11	0.11	0.17
	S	0.45	2.30	1.50	2.75	2.54	2.24	2.75
III	T(sec.)	2078	1025	1087	1098	1043	1042	990
	N.pt	498	276	244	249	268	275	227
	HR(%)	12	9	9	11	9	8	11
	FC(%)	60.61	8.12	6.54	5.71	5.67	5.71	4.44
	S	59.39	343.47	337.37	216.04	337.89	196.35	391.83
IV	T(sec.)	1978	1042	1069	1198	1058	1106	1105
	ND	492	123	123	105	121	125	131
	HR(%)	20	5	5	6	6	5	6
	FC(%)	88	1.12	2.14	2.85	1.31	1.32	1.61
	S	0.55	4.25	2.76	3.16	2.42	3.10	3.07

Table 5: Average values of T, ND, HR, FC and S for the portfolio problem 3 with a fixed (500) number of generations.

Form.	Perf.Indic.	NSGAII	Chaos-Gen	Sin-Gen	Trian-Gen	Chaos+ Exp	Sin.+ Exp	Trian.+ Exp
I	T(sec.)	2150	1057	1102	1156	1094	1156	1141
	ND	498	219	287	271	218	197	192
	HR(%)	6	8	7	6	5	5	8
	FC(%)	65.20	2.96	12.21	9.64	1.91	1.45	5.50
	S	35.55	219.53	118.27	149.65	123.68	184.15	332.23
II	T(sec.)	2125	1006	1055	1083	1032	1046	1066
	ND	497	138	143	137	98	104	119
	HR(%)	12	9	13	10	10	11	5
	FC(%)	84.61	2.54	7.11	3.36	0.71	1.14	0.34
	S	0.79	3.12	1.47	2.11	2.70	4.47	3.19
III	T(sec.)	2088	1170	1202	1249	1206	1270	1153
	ND	499	442	399	439	434	446	442
	HR(%)	12	8	10	8	11	10	8
	FC(%)	46.69	8.30	8.28	9.58	8.64	9.03	10.19
	S	69.09	114.66	349.14	199.19	222.33	326.37	198.90
IV	T(sec.)	2044	1081	1120	1160	1083	1254	1086
	ND	497	180	186	163	172	191	169
	HR(%)	14	9	10	7	10	7	6
	FC(%)	76.28	10.50	4.60	2.48	2.00	2.16	1.25
	S	0.48	3.64	2.91	2.47	4.34	2.94	2.39

Table 6: Average values of T, ND, HR, FC and S for the portfolio problem 4 with a fixed (500) number of generations.

Form.	Perf.Indic.	NSGAII	Chaos-Gen	Sin-Gen	Trian-Gen	Chaos+ Exp	Sin.+ Exp	Trian.+ Exp
I	T(sec.)	2316	1233	1266	1293	1248	1281	1255
	ND	498	254	349	335	278	265	282
	HR (%)	6	7	6	7	4	9	5
	FC (%)	72.45	2.90	9.59	10.48	1.21	1.04	1.80
	S	89.85	105.19	97.61	122.26	106.07	137.73	126.58
II	T(sec.)	2347	1123	1288	1212	1151	1179	1200
	ND	499	176	227	215	123	130	139
	HR (%)	18	11	17	15	6	8	7
	FC (%)	76.86	2.62	11.03	8.30	0.62	0.77	0.10
	S	0.49	1.62	1.36	1.54	1.88	1.61	0.58
III	T(sec.)	2269	1493	1363	1401	1451	1393	1365
	ND	499	408	457	431	428	409	500
	HR (%)	8	6	9	6	13	13	9
	FC (%)	57.59	5.83	6.41	7.43	8.08	7.46	7.58
	S	68.97	108.26	157.97	123.16	162.36	172.88	143.67
IV	T(sec.)	2240	1277	1297	1369	1280	1435	1300
	ND	497	228	237	234	244	208	207
	HR (%)	12	6	9	10	9	9	10
	FC (%)	82.58	5.36	6.31	2.91	0.38	0.38	1.00
	S	0.99	2.08	1.56	1.81	1.49	2.13	2.01

Table 7: Average values of T, ND, HR, FC and S for the portfolio problem 5 with a fixed (500) number of generations.

Form.	Perf.Indic.	NSGAII	Chaos-Gen	Sin-Gen	Trian-Gen	Chaos+ Exp	Sin.+ Exp	Trian.+ Exp
	T(sec.)	10014	8171	7935	8190	9102	8314	8361
I	ND	497	108	165	175	120	129	104
	HR(%)	10	8	6	5	7	7	6
	FC(%)	72.53	1.78	7.98	9.18	4.43	4.19	1.63
	S	61.34	324.77	457.75	321.42	585.95	290.02	355.67
	T(sec.)	9384	8253	8422	8386	9167	8530	9227
II	ND	478	48	73	58	42	61	46
	HR(%)	16	9	8	9	8	8	8
	FC(%)	89.24	1.40	5.22	3.17	0.43	0.79	0.06
	S	0.47	4.05	4.77	7.10	7.18	5.18	3.03
	T(sec.)	8728	7949	7292	7803	7874	8337	8495
III	ND	499	143	179	164	167	149	153
	HR(%)	10	2	3	2	2	3	3
	FC(%)	69.20	4.31	11.11	5.30	3.75	3.61	3.09
	S	75.41	864.95	634.14	959.51	832.01	435.95	416.52
	T(sec.)	9063	9281	7448	8352	8658	9438	8701
IV	ND	490	86	59	71	72	81	73
	HR(%)	23	3	4	3	3	3	4
	FC(%)	89.00	0.79	2.81	1.89	1.88	1.63	0.86
	S	0.60	5.23	13.90	11.30	18.68	10.38	3.02

Table 8: Average values of G, ND, HR, FC and S with time fixed to 10 minutes for the portfolio problem 1.

Form.	Perf.Indic.	NSGAII	Chaos-Gen	Sin-Gen	Trian-Gen	Chaos+ Exp	Sin.+ Exp	Trian.+ Exp
	G	223	602.33	573.67	572.67	666	678	685.33
	ND	496	248	273	238	260	270	258
I	HR (%)	20	14	12	14	12	12	12
	FC (%)	67.99	5.55	5.14	5.22	4.23	4.64	5.66
	S	25.94	75.45	117.63	135.15	108.89	146.53	325.56
	G	278	604	600.33	611.33	715.33	689.33	721
	ND	394	164	185	162	103	112	113
II	HR (%)	29	27	25	28	24	22	24
	FC (%)	78.37	6.04	7.10	6.16	0.69	0.69	1.16
	S	0.55	1.22	1.25	1.13	1.70	1.34	1.71
	G	211.67	462	467.67	469.33	505.33	496	489.67
	ND	496	430	437	419	468	443	446
II	HR (%)	23	10	10	18	13	12	11
	FC (%)	53.01	6.89	7.52	8.36	7.76	7.36	7.69
	S	61.03	86.27	154.89	201.22	121.19	153.87	130.12
	G	241.33	531.67	525.67	528.33	570	580.33	633.33
	ND	455	181	193	172	187	182	187
II	HR (%)	15	10	11	12	10	11	12
	FC (%)	83.51	2.85	4.68	1.61	1.80	1.91	2.14
	S	0.50	1.84	1.24	1.51	1.67	1.03	2.30

Table 9: Average values of G, ND, HR, FC and S with time fixed to 10 minutes for the portfolio problem 2.

Form.	Perf.Indic.	NSGAII	Chaos-Gen	Sin-Gen	Trian-Gen	Chaos+ Exp	Sin.+ Exp	Trian.+ Exp
	G	157.33	278.67	283.67	281	304	306	308.33
	ND	489	160	156	140	150	137	128
I	HR (%)	10	9	8	8	8	9	10
	FC (%)	82.89	1.63	3.80	4.05	1.99	1.82	2.91
	S	13.67	176.26	55.53	177.18	184.57	265.95	239.83
	G	163	293.67	292.33	287	299	299.33	303.33
	ND	487	77	101	91	84	83	69
II	HR (%)	14	15	18	16	14	16	14
	FC (%)	84.53	3.03	5.78	4.09	1.62	1.86	0.35
	S	0.43	3.04	2.55	3.60	2.10	2.00	3.30
	G	154.67	265	264.67	264	288.67	276	276.33
	ND	496	205	204	211	203	215	196
III	HR (%)	17	9	5	9	9	8	6
	FC (%)	66.59	5.61	5.19	5.95	6.55	6.59	4.94
	S	57.79	302.44	349.65	407.66	263.01	243.07	257.12
	G	162.67	269.33	271.33	273	279.67	276.67	293
	ND	463	135	136	120	131	126	141
IV	HR (%)	25	9	9	7	6	7	8
	FC (%)	83.69	4.27	2.38	2.84	210	1.91	2.43
	S	0.67	4.19	2.60	3.36	3.74	2.48	2.18

Table 10: Average values of G, ND, HR, FC and S with time fixed to 10 minutes for the portfolio problem 3.

Form.	Perf.Indic.	NSGAII	Chaos-Gen	Sin-Gen	Trian-Gen	Chaos+ Exp	Sin.+ Exp	Trian.+ Exp
I	G	154.67	271.33	280.67	268	297.67	295	297.67
	ND	493	165	166	211	164	180	170
	HR (%)	22	11	12	14	14	14	16
	FC (%)	70.10	3.83	4.75	6.31	4.69	5.09	4.20
	S	51.01	313.49	322.60	184.93	272.19	183.89	139.44
II	G	158.67	275.33	279.67	284.67	290.33	297.67	296.33
	ND	485	93	126	95	88	93	80
	HR (%)	20	20	19	24	20	22	22
	FC (%)	86.69	2.52	5.73	2.98	0.91	0.67	1.17
	S	0.53	4.67	0.98	5.66	4.98	4.35	3.50
III	G	149.33	242.33	245.67	247.33	258.67	254.67	257.67
	ND	497	365	343	358	364	338	327
	HR (%)	17	13	10	15	8	11	10
	FC (%)	49.70	9.22	8.11	8.74	8.08	7.40	7.71
	S	119.62	329.52	324.19	303.40	236.53	242.83	353.28
IV	G	158.67	264.67	268	266.33	276.33	269.67	291.33
	ND	466	161	143	166	152	161	141
	HR (%)	18	10	14	8	13	11	12
	FC (%)	76.08	8.26	4.06	1.87	1.34	1.85	6.19
	S	0.57	4.37	3.71	3.03	4.08	7.86	4.96

Table 11: Average values of G, ND, HR, FC and S with time fixed to 10 minutes for the portfolio problem 4.

Form.	Perf.Indic.	NSGAII	Chaos-Gen	Sin-Gen	Trian-Gen	Chaos+ Exp	Sin.+ Exp	Trian.+ Exp
I	G	140.67	236.33	245.33	245.67	252.33	247.67	258
	ND	496	248	215	219	224	231	218
	HR (%)	8	8	8	9	12	10	11
	FC (%)	82.80	6.13	2.09	1.15	2.14	2.09	1.65
	S	54.56	128.78	128.21	153.19	135.09	127.00	123.11
II	G	140	256.67	252.33	249.33	256	264.33	259.67
	ND	495	135	158	145	96	119	100
	HR (%)	15	9	18	15	10	22	16
	FC (%)	77.80	6.83	8.37	6.62	0.05	0.16	0.17
	S	0.71	3.10	2.10	1.65	0.84	0.61	0.48
III	G	136.67	220.67	222.67	222.33	215.33	218	224
	ND	496	359	311	357	348	376	377
	HR (%)	6	10	9	8	9	9	9
	FC (%)	60.54	6.72	6.99	5.64	6.19	6.27	6.09
	S	51.22	137.55	166.41	171.04	166.23	177.56	196.87
IV	G	140.33	236.33	236.33	236.67	238.67	249.33	248
	ND	494	201	175	210	226	200	213
	HR (%)	12	9	8	7	8	8	7
	FC (%)	77.45	2.25	3.06	5.43	4.47	4.32	3.68
	S	0.89	1.55	2.17	1.74	2.24	1.54	1.73

Table 12: Average values of G, ND, HR, FC and S with time fixed to 10 minutes for the portfolio problem 5.

Form.	Perf.Indic.	NSGAII	Chaos-Gen	Sin-Gen	Trian-Gen	Chaos+ Exp	Sin.+ Exp	Trian.+ Exp
I	G	31.67	28	28.33	28.33	29.33	28.67	31
	ND	182	47	54	45	55	41	41
	HR (%)	18	7	8	8	6	7	8
	FC (%)	78.22	2.63	3.09	5.62	3.84	2.70	2.76
	S	118.73	428.42	589.26	799.14	473.94	1182.36	594.27
II	G	31.67	28.67	28.67	27.33	30	28.67	29.67
	ND	140	26	29	30	26	35	26
	HR (%)	19	6	6	7	6	6	5
	FC (%)	81.48	1.41	0.23	11.54	1.78	1.62	1.63
	S	1.24	4.83	10.89	5.28	11.14	4.73	7.80
III	G	29.33	30.67	29.67	32	33	32.67	34.33
	ND	146	31	43	51	39	38	39
	HR (%)	26	5	6	7	5	5	7
	FC (%)	76.06	0.33	7.92	5.47	3.83	3.58	3.47
	S	153.33	1585.88	933.33	539.71	1175.15	887.59	1116.28
IV	G	30.67	32.67	29.67	29.33	31.33	32	34
	ND	79	29	20	33	29	38	38
	HR (%)	26	7	4	4	6	3	3
	FC (%)	75.88	8.47	4.57	2.22	5.26	4.96	2.16
	S	2.61	12.67	16.03	17.02	9.43	13.27	6.28