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R&D Cooperation in Real Option Game Analysis.

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Abstract

Cooperative investments in R&D are a significant driving force of the modern economy. As it well-known, the R&D investments are uncertain and the strategic alliances create synergies and additional information that increase the success probabilities about R&D projects.
The theory of real option games takes into account both the flexibility value of an investment opportunity and the strategic considerations. In particular way, while the non-cooperative options are exercised in the interest of the option holders' payoffs, the cooperative ones are exercised in order to maximize the total partnership value.
In our model we develop an interaction between two firms that invest in R&D and we show the effects of cooperative synergies on several equilibriums. Moreover, we consider that the R&D investments are characterized by positive network externalities that induce more benefits in case of reciprocal R&D success.

Keywords: Real Exchange Options; Cooperation games; Information Revelation; R&D investments.

JEL Classification: G13; C71; D80; O32.

1 Introduction

In recent years, the real option theory has been widely used in evaluating investment decisions in a dynamic environment. The market developments are complex and so the conventional NPV (Net Present Value) rule undervalues the value of a project because this method fails to take into account the market uncertainty, irreversibility of investment and ability to delay entry. So, a firm with an opportunity to invest is holding an "option" like to financial options. As it is well accepted, the real option theory becomes very important as it allows to capture the value of managerial flexibility to grow, delay or abandon projects.
Several models, such as [Shackleton and Wojakowski (2003)], [Trigeorgis (1991)] and [Lee (1997)] and so on, are based on the assumption that the option exercise price, and so the investement cost, is fixed. But, particularly for the R&D investments, it is reasonable to consider that the evolution of the investment cost is uncertain. So,

the R&D invesment opportunity corresponds to an exchange option and not to simple call option: it's the exchange of an uncertain investment cost for an uncertain gross project value. The most importants models that value exchange options are given in [Margrabe (1978)], [McDonald and Siegel (1985)], [Carr(1988)], [Carr(1995)] and [Armada et al.(2007)]. In particular way, [McDonald and Siegel (1985)] value a simple European exchange option while [Carr(1988)] develops a model to value a compound European exchange option. Both models consider that assets distribute "dividends" that, in real options context, are the opportunity costs if an investment project is postponed ([Majd and Pindyck (1987)]).
In addition, the real option approach, combined with game theory, allows to consider the strategic interactions among real option holders and also the market dynamics. The financial options literature does not consider the strategic policies because the option exercise does not influence the characteristic of the underlying security or the options themselves. Differently, real investment opportunities are not held by one firm in isolation and so, the optimal strategic exercise can be derived considering the interactions across option holders.
In this paper we analyse a cooperation between two firms that invest in R&D. In particular way, following [Dias and Teixeira(2004)] and [Dias (2004)] models, we assume that the R&D investments generate an "information revelation" about their success and so, by delaying an investment decision, new information can be revealed that might affect the profitability of the R&D projects. So by the alliance between two players, we show as the information is wholly revealed and captured by two firms to improve their R&D success probabilities. The mutual information gain implies positive network externalities (as it is shown in [Kong and Kwok (2007)] and [Huisman (2001)]) which lead more benefits in case of reciprocal R&D success. Therefore, the externalities can involve different entry decisions and so to induce the cooperation between two firms in order to maximixe the partnership return. Accordingly to positive network externalities, we introduce the growth market coefficients depending by the success or failure of two players.
Moreover, we consider that the R&D investment is realized in a two stage manner, with the commencement of second phase being dependent on the successful completion of the first one. This is known as sequential investment in which each stage provides information for the next thus creating an opportunity (option) for subsequent investment.
This article is suitable to model joint ventures of car producers, alliance between pharmaceutical and oil companies and other cooperation kinds that involve a reduction of R&D risk. For istance, [Kogut(1991)], [Chi(2000)] demonstrated the power of viewing joint ventures as real options to expand in response to future technological and market developments. We differentiate from [Dias and Teixeira(2004)], [Kogut(1991)] and [Chi(2000)] because we use exchange options to value the R&D opportunities at initial time and so to determine the best cooperative strategies.
The paper is organized as follows. Section 2 reviews the Simple and Compound european exchange option pricing models and Section 3 introduces the basic model and derives also the final payoffs of two firms in a non cooperative framework. Section 4 analyses the cooperation between two firms and we show how both firms can split the surplus of cooperation and, in Section 5, we present two numerical examples for the cooperative R&D game. Finally, Section 6 concludes.

2 Exchange Options Methodology

In this section we present the final results of the principal models to value European exchange options.

2.1 Simple european exchange option (SEEO)

[McDonald and Siegel (1985)]'s model gives the value of a SEEO to exchange asset D for asset V at time T. The asset given up is termed the delivery asset while the asset received is the optioned asset. Denoting with $s(V, D, T-t)$ the value of SEEO at time t, the final payoff at the option's maturity date T is $s(V, D, 0) = \max[0, V_T - D_T]$. So, assuming that V and D follow a geometric Brownian motion process given by:

$$\frac{dV}{V} = (\mu_v - \delta_v)dt + \sigma_v dZ_v \tag{1a}$$

$$\frac{dD}{D} = (\mu_d - \delta_d)dt + \sigma_d dZ_d \tag{1b}$$

$$cov\left(\frac{dV}{V}, \frac{dD}{D}\right) = \rho_{vd}\sigma_v\sigma_d\, dt \tag{1c}$$

where:

- V and D are the Gross Project Value and the Investment Cost, respectively;
- μ_v and μ_d are the equilibrium expected rate of return on asset V, and the expected growth rate of the investment cost;
- δ_v and δ_d are the "dividend-yields" of V and D, respectively;
- Z_v and Z_d are the brownian standard motions of asset V and D;
- σ_v and σ_d are the volatility of V and D, respectively;
- ρ_{vd} is the correlation between changes in V and D.

[McDonald and Siegel (1985)] show that the value of a SEEO on dividend-paying assets, when the valuation date $t = 0$, is given by:

$$s(V, D, T) = Ve^{-\delta_v T} N(d_1(P, T)) - De^{-\delta_d T} N(d_2(P, T)) \tag{2}$$

where:

- $P = \frac{V}{D}$; $\quad \sigma = \sqrt{\sigma_v^2 - 2\rho_{v,d}\sigma_v\sigma_d + \sigma_d^2}$; $\quad \delta = \delta_v - \delta_d$;
- $d_1(P, T) = \frac{\log P + \left(\frac{\sigma^2}{2} - \delta\right) T}{\sigma\sqrt{T}}$; $\quad d_2 = d_1 - \sigma\sqrt{T}$;
- $N(d)$ is the cumulative standard normal distribution.

2.2 Compound european exchange option (CEEO)

If the underlying asset is another option, the option is called compound. [Carr(1988)] develops a model to value the CEEO $c(s, \varphi D, t_1)$ whose final payoff at maturity date t_1 is:

$$c(s, \varphi D, 0) = \max[0, s - \varphi D]$$

The CEEO value, considering the valuation date $t = 0$, is given by:

$$\begin{aligned} c(s(V,D,T),\varphi D,t_1) &= Ve^{-\delta_v T} N_2\left(d_1\left(\frac{P}{P^*},t_1\right), d_1\left(P,T\right);\rho\right) \\ &- De^{-\delta_d T} N_2\left(d_2\left(\frac{P}{P^*},t_1\right), d_2\left(P,T\right);\rho\right) \\ &- \varphi De^{-\delta_d t_1} N_1\left(d_2\left(\frac{P}{P^*},t_1\right)\right) \end{aligned} \quad (3)$$

where:

- φ is the exchange ratio of CEEO;
- t_1 is the expiration date of the CEEO;
- T is the expiration date of the SEEO, where $T > t_1$
- $\tau = T - t_1$ is the time to maturity of the SEEO and $\rho = \sqrt{\frac{t_1}{T}}$;
- $d_1\left(\frac{P}{P^*},t_1\right) = \frac{\log\left(\frac{P}{P^*}\right) + \left(-\delta + \frac{\sigma^2}{2}\right)t_1}{\sigma\sqrt{t_1}}$; $d_2\left(\frac{P}{P^*},t_1\right) = d_1\left(\frac{P}{P^*},t_1\right) - \sigma\sqrt{t_1}$;
- P^* is the critical price ratio that solves the following equation:

$$P^* e^{-\delta_v \tau} N(d_1(P^*,\tau)) - e^{-\delta_d \tau} N(d_2(P^*,\tau)) = \varphi. \quad (4)$$

- $N_2(a,b,\rho)$ is the standard bivariate normal distribution function evaluated at a and b with correlation coefficient ρ.

3 The Basic Model Game

In our model we consider two firms (A and B) that have the option to realize their R&D investment at initial time t_0 or to delay the decision at time t_1. As it is know, the R&D investments depends on the resolution of several source of uncertainty that may influence the investment decision of each firm. Assuming by q and p the R&D success probability of firms A and B respectively, we can represent this situation by two Bernoulli distributions Y and X:

$$Y : \begin{cases} 1 & q \\ 0 & 1-q \end{cases} \qquad X : \begin{cases} 1 & p \\ 0 & 1-p \end{cases}$$

The R&D success or failure of one firm generates an information revelation that influences the investment decision of the other firm. So, if firm A's R&D is successful, the firm B's probability p changes in positive information revelation p^+, while p changes in negative information revelation p^- in case of A's failure. Simmetrically, the firm A's R&D success changes in q^+ or in q^- in case of success or failure of firm B at time t_0. Following [Dias (2004)]'s model about the information revelation process, it results that:

$$p^{+} = Prob[X = 1/Y = 1] = p + \sqrt{\frac{1-q}{q}} \cdot \sqrt{p(1-p)} \cdot \rho(X,Y) \tag{5a}$$

$$p^{-} = Prob[X = 1/Y = 0] = p - \sqrt{\frac{q}{1-q}} \cdot \sqrt{p(1-p)} \cdot \rho(X,Y) \tag{5b}$$

$$q^{+} = Prob[Y = 1/X = 1] = q + \sqrt{\frac{1-p}{p}} \cdot \sqrt{q(1-q)} \cdot \rho(Y,X) \tag{5c}$$

$$q^{-} = Prob[Y = 1/X = 0] = q - \sqrt{\frac{p}{1-p}} \cdot \sqrt{q(1-q)} \cdot \rho(Y,X) \tag{5d}$$

where the correlations $\rho(X,Y)$ and $\rho(Y,X)$ are a measure of information revelation from Y to X and from X to Y, respectively. Obviously, the information revelation is considerable when the investment is not realized in the same time. So, if both players invest simultaneously in R&D ore they wait to invest, ther is not information revelation and so $\rho(X,Y) = \rho(Y,X) = 0$ and consequently it results that $p = p^{+} = p^{-}$ and $q = q^{+} = q^{-}$.
The information revelation is a public information process accessible to the other competitors that influences their choices. For istance, it is known that good information about drugs is available in pharmaceutical industry after clinical testing, and so in the first stages of R&D.
The condition to respect to have $0 \leq p^{+} \leq 1$ and $0 \leq p^{-} \leq 1$ according to the positive information revelation that benefits the firm B, namely $\rho(X,Y) \geq 0$ is that:

$$0 \leq \rho(X,Y) \leq \min\left\{\sqrt{\frac{p(1-q)}{q(1-p)}}, \sqrt{\frac{q(1-p)}{p(1-q)}}\right\} \tag{6}$$

The condition (6) must to be respected also for the information revelation process that benefits firm A, namely $\rho(Y,X)$, to have that $0 \leq q^{+} \leq 1$ and $0 \leq q^{-} \leq 1$.
So, with the alliance between A and B, we can assume that information is wholly revealed and we can setting that the cooperative information $\rho_{\max}$ is equal to:

$$\rho_{\max} = \min\left\{\sqrt{\frac{p(1-q)}{q(1-p)}}, \sqrt{\frac{q(1-p)}{p(1-q)}}\right\} \tag{7}$$

We can observe that in the symmetrical case in which both firms have the same success probability $p = q$, then it results $\rho_{\max} = 1$ and so $q^{+} = 1$ and $p^{+} = 1$. This means that, in case of A's R&D success at time t_0, it involves the B's success at time t_1 in the cooperation treatment since the information revelation is fully captured and vice-versa.
Moreover, we assume that R&D investments are characterized by network externalities that induce more benefits in case of reciprocal R&D success. So we denote by:

$$K_{0_S 0_S}, \quad K_{0_S 1_S}, \quad K_{1_S 0_S}, \quad K_{1_S 1_S}$$

the growth market coefficients in case of A and B success. The 0 and 1 mean that the R&D investment is realized at time t_0 or t_1 respectively, while the S denotes the success. The first part denotes the operation of considered firm, while the second part is the situation of the other firm. So, if A and B invest successfully in R&D at time t_0 and t_1 respectively, firm A takes $K_{0_S 1_S}$ while B obtains $K_{1_S 0_S}$. If both firms invest simultaneously with success at time t_0, then they will take $K_{0_S 0_S}$, while if the investments are realized at time t_1 they will have $K_{1_S 1_S}$. In the same way we denote by:

$$K_{0_S 0_F}, \quad K_{0_S 1_F}, \quad K_{1_S 0_F}, \quad K_{1_S 1_F}$$

the market coefficients for the winning firm assuming the failure, denoted by F, by the other player. Moreover, as the unsuccess of one player does not produce network externaility, we can write that:

$$K_{0_S 0_F} = K_{0_S 1_F} \equiv K_{0_S}; \quad K_{1_S 0_F} = K_{1_S 1_F} \equiv K_{1_S}$$

Finally, in case of failure of considered firm, its market coefficient will be equal to zero whether in case of success or failure of other firm. So we have that:

$$K_{0_F 0_S} = 0, \quad K_{0_F 1_S} = 0, \quad K_{1_F 0_S} = 0, \quad K_{1_F 1_S} = 0$$

and

$$K_{0_F 0_F} = 0, \quad K_{0_F 1_F} = 0, \quad K_{1_F 0_F} = 0, \quad K_{1_F 1_F} = 0$$

Now, we can set the relations among the growth market coefficients K using these assumptions:

- Positive Network Externality: as it is shown [Huisman (2001)], the growth market coefficient in case of both R&D success will be bigger than the situation in which only one firm invests successfully, and so:

$$K_{SS} > K_S \tag{8a}$$

- R&D Success Time: the market coefficient increases if the reciprocal R&D success is realized at time t_0 rather than t_1, because there is more time to benefit both network externalities and R&D innovations. In the situation in which only one firm invests successfully, the market coefficient enlarges if the success is realized at time t_0 rather than t_1:

$$K_{0_S 0_S} > K_{1_S 1_S}; \quad K_{0_S} > K_{1_S} \tag{8b}$$

- First Mover's Advantage: the firm that realizes with success the R&D investment at time t_0 will receive an higher market coefficient than other player that postpones successfully the project at time t_1:

$$K_{0_S 1_S} > K_{1_S 0_S}; \tag{8c}$$

To determine the growth market coefficients K, we assume that they depend by a parameter k involving the R&D innovation and by length of R&D benefits until the expiration time T. For the positive network externality, we take into account two times the one firm market coefficient. So, assuming that the initial time $t_0 = 0$, we have that:

$$K_{0_S} = kT \tag{9a}$$
$$K_{0_S 0_S} = 2kT \tag{9b}$$
$$K_{1_S} = k(T - t_1) \tag{9c}$$
$$K_{1_S 1_S} = 2k(T - t_1) \tag{9d}$$

We suppose to fix T, it is obviuos that if t_1 decreasing, then the coefficients $K_{1_S1_S}$ and K_{1_S} increase their value. In fact, if $t_1 = 0$ then there is not delay and $K_{1_S1_S} = K_{0_S0_S}$ and $K_{1_S} = K_{0_S}$. Finally, to determine $K_{0_S1_S}$ and $K_{1_S0_S}$, we assume that:

$$K_{0_S1_S} = 2k(T - t_1) + kt_1 \quad (9e)$$

$$K_{1_S0_S} = 2k(T - t_1) - kt_1 \quad (9f)$$

If one firm invests successfully at time t_0 and the other player at time t_1, we have that the first firm takes the network externality starting from time t_1, namely $K_{1_S1_S}$ plus the first mover's advantatege kt_1 until time t_1. Simmetrically, the market coefficient $K_{1_S0_S}$ for the second firm that postpones its choice will be $K_{1_S1_S}$ minus kt_1. We can observe that if $t_1 = 0$, so if there is not postponement, then $K_{0_S1_S} = K_{1_S0_S} = K_{0_S0_S}$. Finally, to ensure that condition (8a) holds, we need to impose that $t_1 < \frac{T}{3}$. This condition is reasonable with the consideration that the information revelation disappears in time and furthermore, if one firm invests at time t_0, the other firm decision will be made within $t_1 < \frac{T}{3}$ to allow the realization of development phase in $T - t_1$.
First to start, we state as Leader the pionner firm (A or B) that invests in R&D at time t_0 earlier than other one, namely the Follower, that postpones the R&D investment decision at time t_1. We denote by R the R&D investment for the development of a new product, V the overall market value deriving by R&D innovations and D is the total investment cost to realize new goods. We consider that the production investment of each firm is proportional to its market share and it can be realized only at time T, that is the time needed for to develop the new product. Hence, we suppose that the option to enter in the market is like an European exchange option.

3.1 The Leader's Payoff

We analyse the Leader's payoff assuming that firm A (Leader) invests in R&D at time t_0 while firm B (Follower) decides to wait to invest. So, the Leader spends the investment R at time t_0 and obtains, in case of its R&D success with probability q, the development option. In particular way, if also the Follower's R&D invesment is successfully at time t_1, the growth market coefficient will be $K_{0_S1_S}$ and the Leader holds the development option $s(K_{0_S1_S}V, K_{0_S1_S}D, T)$ to invest $K_{0_S1_S}D$ and claims a market value equal to $K_{0_S1_S}V$ as it is illustred in the Fig.(1(a)). So the Leader's payoff is:

$$\begin{aligned} L_A^S(V, D) &= -R + q \cdot s(K_{0_S1_S}V, K_{0_S1_S}D, T) \\ &= -R + qk(2T - t_1)\left(Ve^{-\delta_v T}N(d_1(P, T)) - De^{-\delta_d T}N(d_2(P, T))\right) \end{aligned} \quad (10)$$

The probability to have $K_{0_S1_S}$ depending by the Follower's R&D success that is p^+ since it receives the information revelation from Leader's investment occurred at time t_0. But, if the Follower's R&D fails, the Leader's market coefficient in case of its R&D success is K_{0_S} and it receives the following payoff:

$$\begin{aligned} L_A^F(V, D) &= -R + q \cdot s(K_{0_S}V, K_{0_S}D, T) \\ &= -R + qkT\left(Ve^{-\delta_v T}N(d_1(P, T)) - De^{-\delta_d T}N(d_2(P, T))\right) \end{aligned} \quad (11)$$

as it is shown in the Fig.(1(b)). So, computing the expectation value between Eqs. (10) and (11), the Leader's payoff (firm A) is:

$$L_A(V, D) = p^+ \cdot L_A^S(V, D) + (1 - p^+) \cdot L_A^F(V, D) \quad (12)$$

Simmetrically, assuming that firm B (Leader) invests at time t_0 while firm A (Follower) decides to postpone its decision, the Leader's payoff became:

$$L_B(V,D) = q^+ \cdot L_B^S(V,D) + (1-q^+) \cdot L_B^F(V,D) \tag{13}$$

(a) Leader's payoff in case of Follower's success

(b) Leader's payoff in case of Follower's failure

Figure 1: Leader's payoffs

3.2 The Follower's Payoff

Now we focus on the Follower's payoff assuming that firm B (Follower) decides to postpone its R&D investment decision at time t_1 and firm A (Leader) invests at time t_0. If the Leader's R&D investment is successfully (with a probability q), then the Follower's probability success became p^+ and its growth market coefficient is $K_{1_S 0_S}$. So, after the investment R, the Follower holds with a probability p^+ the development option $s(K_{1_S 0_S}V, K_{1_S 0_S}D, \tau)$ to invest $K_{1_S 0_S}D$ and claims a market value equal to $K_{1_S 0_S}V$. So the Follower's payoff at time t_0 is a CEEO with maturity t_1, exercise price equal to R and the underlying asset is the development option $s(K_{1_S 0_S}V, K_{1_S 0_S}D, \tau)$ as it is represented in the Fig.(2(a)).

The CEEO payoff at expiration date t_1 with positive information revelation is:

$$c(p^+ s(K_{1_S 0_S}V, K_{1_S 0_S}D, \tau), R, 0) = \max[p^+ s(K_{1_S 0_S}V, K_{1_S 0_S}D, \tau) - R, 0]$$

According to [Carr(1988)]'s model, we assume that $R = \varphi D$ is a proportion φ of asset D. Hence, denoting by $c(p^+)$ the CEEO at time t_0, namely:

$$c(p^+) \equiv c(p^+ s(K_{1_S 0_S}V, K_{1_S 0_S}D, \tau), \varphi D, t_1)$$

we can write, using the Eq. (3), the value of CEEO with positive information:

$$\begin{aligned} c(p^+) &= p^+ k(2T - 3t_1) V e^{-\delta_v T} N_2 \left(d_1 \left(\frac{P}{P^*_{upB}}, t_1 \right), d_1(P,T); \rho \right) \\ &\quad - p^+ k(2T - 3t_1) D e^{-\delta_d T} N_2 \left(d_2 \left(\frac{P}{P^*_{upB}}, t_1 \right), d_2(P,T); \rho \right) \\ &\quad - \varphi D e^{-\delta_d t_1} N_1 \left(d_2 \left(\frac{P}{P^*_{upB}}, t_1 \right) \right) \end{aligned} \tag{14}$$

where P^*_{upB} is the critical value that makes the underlying asset of $c(p^+)$ equal to exercise value. Hence P^*_{upB} solves the following equation:

$$p^+ s(K_{1_S 0_S}V, K_{1_S 0_S}D, \tau) = \varphi D$$

and assuming the asset $K_{1_S 0_S} D$ as numeraire we can rewrite the above equation as:

$$P^*_{upB}\, e^{-\delta_v \tau} N(d_1(P^*_{upB}, \tau)) - e^{-\delta_d \tau} N(d_2(P^*_{upB}, \tau)) = \frac{\varphi}{p^+ \cdot (2T - 3t_1)} \tag{15}$$

Alternatively, in case of Leader's failure, the Follower's R&D success probability changes in p^- and its market coefficient is K_{1_S}. So, the Follower's payoff at time t_0 is a CEEO with maturity t_1, exercise price equal to R and underlying asset is the development option $s(K_{1_S}V, K_{1_S}D, \tau)$ as it is shown in the Fig.(2(b)). Hence, the CEEO payoff at expiration date t_1 with negative information revelation is:

$$c(p^- s(K_{1_S}V, K_{1_S}D, \tau), R, 0) = \max[p^- s(K_{1_S}V, K_{1_S}D, \tau) - R, 0].$$

Denoting with $c(p^-)$ the CEEO at time t_0 with negative information, i.e.:

$$c(p^-) \equiv c(p^- s(K_{1_S}V, K_{1_S}D, \tau), \varphi D, t_1)$$

we can write, using the Eq. (3), the value of CEEO with negative information:

$$\begin{aligned} c(p^-) \;\; = \;\; & p^- k(T - t_1) V e^{-\delta_v T} N_2 \left(d_1 \left(\frac{P}{P^*_{dwB}}, t_1 \right), d_1 (P, T); \rho \right) \\ & - p^- k(T - t_1) D e^{-\delta_d T} N_2 \left(d_2 \left(\frac{P}{P^*_{dwB}}, t_1 \right), d_2 (P, T); \rho \right) \\ & - \varphi D e^{-\delta_d t_1} N_1 \left(d_2 \left(\frac{P}{P^*_{dwB}}, t_1 \right) \right) \end{aligned} \tag{16}$$

where P^*_{dwB} is the critical price that solves the following equation:

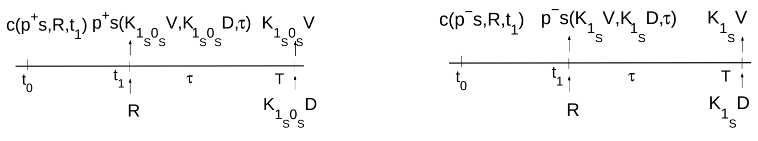
$$P^*_{dwB}\, e^{-\delta_v \tau} N(d_1(P^*_{dwB}, \tau)) - e^{-\delta_d \tau} N(d_2(P^*_{dwB}, \tau)) = \frac{\varphi}{p^- \cdot k(T - t_1)}. \tag{17}$$

The Follower obtains the CEEO $c(p^+)$ in case of Leader's success with a probability q or the CEEO $c(p^-)$ in case of Leader's failure with a probability $(1 - q)$. Hence, the Follower's payoff at time t_0 is the expectation value:

$$F_B(V, D) = q\, c(p^+) + (1 - q)\, c(p^-) \tag{18}$$

Similary, if we consider that firm B (Leader) invests in R&D at time t_0 and firm A (Follower) decides to wait to invest we have that:

$$F_A(V, D) = p\, c(q^+) + (1 - p)\, c(q^-) \tag{19}$$



(a) Follower's payoff in case of Leader's success

(b) Follower's payoff in case of Leader's failure

Figure 2: Follower's payoffs

3.3 The A and B payoffs in case of simultaneous investment

In this case, we analyse the situation in which both firms invest in R&D at time t_0. We can assume that there is not information revelation since the investment is simultaneous but both players can benefice of network externalities. First of all, we determine the firm's A payoff. Assuming the firm B's R&D success, A receives the development option with a growth market coefficient $K_{0_S 0_S}$ in case of its R&D success. So, after the investment R at time t_0, player A receives the development option $s(K_{0_S 0_S}V, K_{0_S 0_S}D, T)$ with a probability q:

$$\begin{aligned} S_A^S(V,D) &= -R + q \cdot s\left(K_{0_S 0_S}V, K_{0_S 0_S}D, T\right) \\ &= -R + q2kT\left(Ve^{-\delta_v T}N(d_1(P,T)) - De^{-\delta_d T}N(d_2(P,T))\right) \end{aligned} \quad (20)$$

But, assuming the firm B failure, A receives the development option with a growth market coefficient K_{0_S} in case of its success:

$$\begin{aligned} S_A^F(V,D) &= -R + q \cdot s\left(K_{0_S}V, K_{0_S}D, T\right) \\ &= -R + qkT\left(Ve^{-\delta_v T}N(d_1(P,T)) - De^{-\delta_d T}N(d_2(P,T))\right) \end{aligned} \quad (21)$$

So, recalling that firm B's probability success is equal to p, the firm's A payoff in case of simultaneous investment will be the expectation value between Eqs. (20) and (21):

$$S_A(V,D) = p \cdot S_A^S(V,D) + (1-p) \cdot S_A^F(V,D) \quad (22)$$

Simmetrically, the firm's B payoff will be:

$$S_B(V,D) = q \cdot S_B^S(V,D) + (1-q) \cdot S_B^F(V,D) \quad (23)$$

3.4 The A and B payoffs when both firms wait to invest

Finally, we suppose that both firms decide to delay their R&D investment decision at time t_1 and we can setting that there is not information revelation. First of all, we analyse the situation of firm A. Assuming the R&D success of firm B, then the growth market coefficient of player A will be $K_{1_S 1_S}$. So, after the investment R at time t_1, firm A holds with a probability q the development option $s(K_{1_S 1_S}V, K_{1_S 1_S}D, \tau)$ to invest $K_{1_S 1_S}D$ and claims a market value equal to $K_{1_S 1_S}V$. Then the firm's A payoff at time t_0 is a CEEO with maturity t_1, the exercise price equal to R and the underlying asset is the development option $s(K_{1_S 1_S}V, K_{1_S 1_S}D, \tau)$ with a probability q. Thus, according to [Carr(1988)]'s model, and assuming that R is a proportion φ of asset D, the CEEO in case of firm's B success is:

$$W_A^S(V,D) = c\left(q \cdot s(K_{1_S 1_S}V, K_{1_S 1_S}D, \tau), \varphi D, t_1\right) \quad (24)$$

and specifically:

$$\begin{aligned} W_A^S(V,D) &= q2k(T-t_1)Ve^{-\delta_v T}N_2\left(d_1\left(\frac{P}{P^*_{wsA}}, t_1\right), d_1(P,T); \rho\right) \\ &\quad - q2k(T-t_1)De^{-\delta_d T}N_2\left(d_2\left(\frac{P}{P^*_{wsA}}, t_1\right), d_2(P,T); \rho\right) \\ &\quad - \varphi De^{-\delta_d t_1}N_1\left(d_2\left(\frac{P}{P^*_{wsA}}, t_1\right)\right) \end{aligned} \quad (25)$$

where P^*_{wsA} is the critical value that solves the following equation:

$$q \cdot s\left(K_{1_S 1_S} V, K_{1_S 1_S} D, \tau\right) = \varphi D$$

and assuming the asset $K_{1_S 1_S}\, D$ as numeraire we can rewrite the above equation as:

$$P^*_{wsA} e^{-\delta_v \tau} N(d_1(P^*_{wsA}, \tau)) - e^{-\delta_d \tau} N(d_2(P^*_{wsA}, \tau)) = \frac{\varphi}{q \cdot 2k(T - t_1)} \tag{26}$$

But, in case of firm's B failure, the firm A growth market coefficient will be K_{1_S}. So, after the investment R at time t_1, firm A obtains with a probability q the development option $s(K_{1_S} V, K_{1_S} D, \tau)$. Thus, using [Carr(1988)]'s model, the firm' A payoff at time t_0 is a CEEO where the underlying asset is $s(K_{1_S} V, K_{1_S} D, \tau)$ with a probability q:

$$W^F_A(V, D) = c\left(q \cdot s(K_{1_S} V, K_{1_S} D, \tau), \varphi D, t_1\right) \tag{27}$$

and specifically:

$$\begin{aligned} W^F_A(V, D) \quad = \quad & qk(T - t_1) V e^{-\delta_v T} N_2\left(d_1\left(\frac{P}{P^*_{wfA}}, t_1\right), d_1\left(P, T\right); \rho\right) \\ & - qk(T - t_1) D e^{-\delta_d T} N_2\left(d_2\left(\frac{P}{P^*_{wfA}}, t_1\right), d_2\left(P, T\right); \rho\right) \\ & - \varphi D e^{-\delta_d t_1} N_1\left(d_2\left(\frac{P}{P^*_{wfA}}, t_1\right)\right) \end{aligned} \tag{28}$$

where, as seen before, P^*_{wfA} is the critical value that solves the following equation:

$$q \cdot s\left(K_{1_S} V, D_{1_S} D, \tau\right) = \varphi D$$

and assuming the asset $K_{1_S}\, D$ as numeraire we can rewrite the above equation as:

$$P^*_{wfA} e^{-\delta_v \tau} N(d_1(P^*_{wfA}, \tau)) - e^{-\delta_d \tau} N(d_2(P^*_{wfA}, \tau)) = \frac{\varphi}{q \cdot k(T - t_1)} \tag{29}$$

Hence, recalling that the firm B success is equal to p, we can compute the firm A payoff as the expectation value between Eqs. (24) and (27):

$$W_A(V, D) = p\, W^S_A(V, D) + (1 - p)\, W^F_A(V, D) \tag{30}$$

Similary, the firm B payoff is:

$$W_B(V, D) = q\, W^S_B(V, D) + (1 - q)\, W^F_B(V, D) \tag{31}$$

3.5 Non cooperative Critical market values

Now, to determine the non cooperative Nash equilibriums denoted by $v(A)$ and $v(B)$, we analyse the relations among the strategic payoffs according to several expected market values V at time t_0 and considering fixed the invest cost D at time t_0. Therefore, we are able to determine the critical market values that delimite the several Nash equilibriums.

First of all, we analyse the relation between the Leader and the Waiting strategy considering only the variable V and, to simplify the notation, we do not considering the dividends to compute the derivatives. We can observe that:

- $L_i(0) = -R; \quad W_i(0) = 0;$

- $\frac{\partial L_A}{\partial V} = qN(d_1(P,T))k[p^+(2T - t_1) + (1 - p^+)T]$;

- $\frac{\partial L_B}{\partial V} = pN(d_1(P,T))k[q^+(2T - t_1) + (1 - q^+)T]$;

- $\frac{\partial W_A}{\partial V} = 2pqk(T - t_1)N_2\left(d_1\left(\frac{P}{P^*_{wsA}}, t_1\right), d_1(P,T); \rho\right)$
  $+ (1 - p)qk(T - t_1)N_2\left(d_1\left(\frac{P}{P^*_{wfA}}, t_1\right), d_1(P,T); \rho\right)$;

- $\frac{\partial W_B}{\partial V} = 2qpk(T - t_1)N_2\left(d_1\left(\frac{P}{P^*_{wsB}}, t_1\right), d_1(P,T); \rho\right)$
  $+ (1 - q)pk(T - t_1)N_2\left(d_1\left(\frac{P}{P^*_{wfB}}, t_1\right), d_1(P,T); \rho\right)$;

- $\frac{\partial L_i}{\partial V} > \frac{\partial W_i}{\partial V} > 0$;

for $i = A, B$, as it is shown in the proof (1).

Proof 1 *We can observe that, when the information revalation $\rho(X,Y)$ and $\rho(Y,X)$ increase, then also the derivatives $\frac{\partial L_A}{\partial V}$ and $\frac{\partial L_B}{\partial V}$ raise because $2T - t_1 > T$. So, assuming $\rho(X,Y) = \rho(Y,X) = 0$, we can observe that $p^+ = p$ and $q^+ = q$. Moreover, as $2T - t_1 \geq 2(T - t_1)$ and $N(b) = N_2(a,b;\rho) + N_2(-a,b;-\rho)$, it's obvious that $\frac{\partial L_i}{\partial V} > \frac{\partial W_i}{\partial V} > 0$ for $i = A, B$.*

Then, the following proposition holds:

Proposition 1 *There exists, for each firm $i = A, B$, a unique critical market value V_i^W that makes $L_i(V_i^W) = W_i(V_i^W)$. Denoting by $V_W^* = \min(V_A^W, V_B^W)$ and $V_Q^* = \max(V_A^W, V_B^W)$, it results that:*

$$L_i(V) < W_i(V) \quad \text{for} \quad V < V_W^*$$
$$L_i(V) > W_i(V) \quad \text{for} \quad V > V_Q^*$$

Moreover, if A's success probability q is higher than B, for $V \in]V_W^*, V_Q^*[$ it results:

$$L_A(V) > W_A(V); \quad L_B(V) < W_B(V)$$

otherwise, if B's success probability p is higher than A, for $V \in]V_W^*, V_Q^*[$ it results:

$$L_A(V) < W_A(V); \quad L_B(V) > W_B(V)$$

Now we analyse the relation between the Follower and the Simultaneous strategies. Then, we can observe that:

- $F_i(0) = 0; \quad S_i(0) = -R$;

- $\frac{\partial F_A}{\partial V} = pq^+k(2T - 3t_1)N_2\left(d_1\left(\frac{P}{P^*_{upA}}, t_1\right), d_1(P,T); \rho\right)$
  $+ (1 - p)q^-k(T - t_1)N_2\left(d_1\left(\frac{P}{P^*_{dwA}}, t_1\right), d_1(P,T); \rho\right)$;

- $\frac{\partial F_B}{\partial V} = qp^+k(2T-3t_1)N_2\left(d_1\left(\frac{P}{P^*_{upB}}, t_1\right), d_1(P,T); \rho\right)$
$+ (1-q)p^-k(T-t_1)N_2\left(d_1\left(\frac{P}{P^*_{dwB}}, t_1\right), d_1(P,T); \rho\right);$

- $\frac{\partial S_A}{\partial V} = qN(d_1(P,T))kT[1+p];$

- $\frac{\partial S_B}{\partial V} = pN(d_1(P,T))kT[1+q]$

- $\frac{\partial F_i}{\partial V} > 0; \quad \frac{\partial S_i}{\partial V} > 0$

for $i = A, B$. In this case we have that both derivatives are positive but the intersection between Follower and Simultaneous strategies exists if $\frac{\partial S_i}{\partial V} > \frac{\partial F_i}{\partial V}$ for $i = A, B$. So the following proposition holds:

Proposition 2 *If $\frac{\partial S_A}{\partial V} > \frac{\partial F_A}{\partial V}$ then there exists a unique critical market value V^*_P that makes $S_A(V^*_P) = F_A(V^*_P)$ and it results that:*

$$S_A(V) < F_A(V) \quad \textit{for} \quad V < V^*_P$$

$$S_A(V) > F_A(V) \quad \textit{for} \quad V > V^*_P$$

otherwise, if $\frac{\partial S_A}{\partial V} \leq \frac{\partial F_A}{\partial V}$ then $S_A(V) < F_A(V)$ for every value of V.
If $\frac{\partial S_B}{\partial V} > \frac{\partial F_B}{\partial V}$ then there exists a unique critical market value V^*_S that makes $S_B(V^*_S) = F_B(V^*_S)$ and it results that:

$$S_B(V) < F_B(V) \quad \textit{for} \quad V < V^*_S$$

$$S_B(V) > F_B(V) \quad \textit{for} \quad V > V^*_S$$

otherwise, if $\frac{\partial S_B}{\partial V} \leq \frac{\partial F_B}{\partial V}$ then $S_B(V) < F_B(V)$ for every value of V.
Moreover, if A's success probability q is higher than firm B, then $V^*_P < V^*_S$ otherwise $V^*_S < V^*_P$.

It's evident that in the simmetric case in which both players have the same success probability $p = q$, it results that $V^*_W = V^*_Q$ and $V^*_S = V^*_P$.
By the Propositions (1) and (2), we are able to setting the several Nash equilibriums $v(A)$ and $v(B)$ in case of no partneship with information revelation $\rho(X,Y)$ and $\rho(Y,X)$.

4 The Cooperation between A and B

In this section we analyse the cooperation between firms A and B that allows to capture the whole information revelation and so to improve the R&D success probabilities. In particular way we assume that the value achieved by the cooperation can be trasferred from one player to the other. We show as the strategic alliance is the joint best response to the non-cooperative alternative and so the equilibriums that both firms obtain through the cooperation are Pareto-dominate all the non-cooperative ones. As we consider two players, we denote by $C(A \cup B)$ the feasible set for the coalition, namely is the set of outcome which can be obtained by the two players acting together. The cooperation value is given by the sum of two firm's payoff using the whole information revelation $\rho_{\max}$ deriving by two firms' R&D investments. Both players can agree upon several partnership contracts. For istance firms A and B can share equitably the surplus of cooperation using the Shapley values:

$$Sh_A = v(A) + \frac{C(A \cup B) - (v(A) + v(B))}{2} \tag{32a}$$

$$Sh_B = v(B) + \frac{C(A \cup B) - (v(A) + v(B))}{2} \tag{32b}$$

where $C(A \cup B) - (v(A) + v(B))$ is the surplus of cooperation. This solution looks natural in the symmetric case $p = q$ in which both firms have the same success probability otherwise, we can assume also asymmetric shares. For istance, we can split the cooperation value $C(A \cup B)$ as:

$$P_A = v(A) + \frac{q}{p+q}\left(C(A \cup B) - (v(A) + v(B))\right) \tag{33a}$$

$$P_B = v(B) + \frac{p}{p+q}\left(C(A \cup B) - (v(A) + v(B))\right) \tag{33b}$$

We can observe that, if $p = q$, then $Sh_i = P_i$ for $i = A, B$ and the efficiency property is satisfied as $Sh_A + Sh_B = P_A + P_B = C(A \cup B)$.
The cooperative information ρ_{max} influences the Leader and Follower payoffs that we denote by $L_i^C(V)$ and $F_i^C(V)$ for $i = A, B$, where C means the cooperative action. The four possible cooperation strategies are:

- Both players decide to wait to invest at time t_0. Then, their cooperation value will be:
$$C(A \cup B) = W_A(V) + W_B(V) \equiv W_C(V)$$
- The firm A invests at time t_0 while the firm B delays its decision at time t_1. The firm B obtains the overall information revelation ρ_{max}:
$$C(A \cup B) = L_A^C(V) + F_B^C(V) \equiv LF_C(V)$$
- Simmetrically, the firm B invests at time t_0 and the firm A delays its decision at time t_1. In this case it results:
$$C(A \cup B) = F_A^C(V) + L_B^C(V) \equiv FL_C(V)$$
- Both players decide to invest at time t_0. In this case, their cooperation value will be:
$$C(A \cup B) = S_A(V) + S_B(V) \equiv S_C(V)$$

The two-by-two matrix represented in the Fig.(3) summarizes the final payoffs considering both the cooperative and the non cooperative strategies. The first upper value in each cell indicates the strategic investment opportunity for A at time t_0, while the second represents the firm B's value. Moreover, in the lower part of each cell we denote the overall value that both firms can realize by cooperation according to several strategic actions.

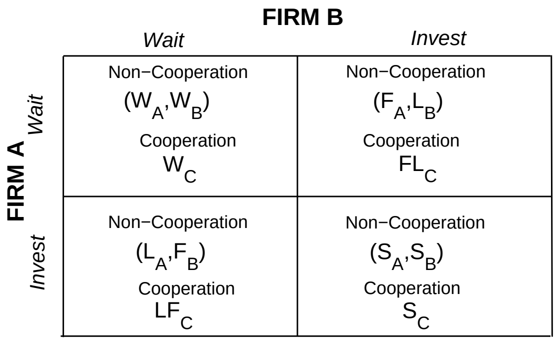


Figure 3: Final payoffs at time t_0

4.1 Cooperative Critical market values

The aim of two firm acting together is to improve their position compared with no parteship and to reach a Pareto optimal solution. To realize this objective, we have to determine the maximum value among the four cooperation strategies according to several expected market values V at time t_0. Therefore we compute the cooperative critical market values that delimite the maximum payoff $C(A \cup B)$. So it results that:

- $W_C(0) = 0; \qquad S_C(0) = -2R;$
- $LF_C(0) = -R; \quad FL_C(0) = -R;$

When the market value V is equal to zero, both firms realize a loss equivalent to the R&D investment made at time t_0. Now, we analyse the relations among the four alliance strategies. In particular way, we compute the derivatives without to consider the dividends that allow us to obtain the cooperative critical market values:

- $\frac{\partial W_C}{\partial V} = 2k(T-t_1)pq\left[N_2\left(d_1\left(\frac{P}{P^*_{wsA}}, t_1\right), d_1(P,T); \rho\right) + N_2\left(d_1\left(\frac{P}{P^*_{wsB}}, t_1\right), d_1(P,T); \rho\right)\right]$
  $+k(T-t_1)\left[(1-p)qN_2\left(d_1\left(\frac{P}{P^*_{wfA}}, t_1\right), d_1(P,T); \rho\right) + (1-q)pN_2\left(d_1\left(\frac{P}{P^*_{wfB}}, t_1\right), d_1(P,T); \rho\right)\right]$
- $\frac{\partial S_C}{\partial V} = qkTN(d_1(P,T))\left[2p + (1-p)\right] + pkTN(d_1(P,T))\left[2q + (1-q)\right];$
- $\frac{\partial S_C}{\partial V} > \frac{\partial W_C}{\partial V} > 0.$

as $N(a) > N_2(a,b;\rho)$. Now we can remark that, if $q = p$, then it results $LF_C(V) = FL_C(V)$ as $L^C_A(V) = L^C_B(V)$ and $F^C_A(V) = F^C_B(V)$. So in this case both strategies give the same value. But, if $q > p$, then we have that $LF_C(V) > FL_C(V)$ and, if $q < p$, then $LF_C(V) < FL_C(V)$. The Tables (3) and (7) illustrate some numerical examples how $LF_C(V) > FL_C(V)$ when $q > p$.
So, to determine the maximun value, we consider the cooperation strategy in which

the Leader is the firm with the highest success probability. Assuming that $q \geq p$, we take into account the cooperative strategy LF_C. It results that:

$\frac{\partial LF_C}{\partial V} = qN(d_1(P,T))k[p^+(T-t_1)+T]+qp^+k(2T-3t_1)N_2\left(d_1\left(\frac{P}{P^*_{upB}},t_1\right), d_1(P,T);\rho\right)$
$+ (1-q)p^-k(T-t_1)N_2\left(d_1\left(\frac{P}{P^*_{dwB}},t_1\right), d_1(P,T);\rho\right)$.

The proof (2) shows that $\frac{\partial LF_C}{\partial V} > \frac{\partial W_C}{\partial V} > 0$.

Proof 2 *In the case $p = q$, it results that $\rho_{\max} = 1$ and $p^+ = 1$. After some manipulation and leaving out to simplify the positive quantity of LF_C strategy:*

$$(1-q)p^-k(T-t_1)N_2\left(d_1\left(\frac{P}{P^*_{dwB}},t_1\right), d_1(P,T);\rho\right)$$

we have that $\frac{\partial LF_C}{\partial V} > \frac{\partial W_C}{\partial V}$ if:

$$Aq^2 + Bq < 0 \qquad (34)$$

where:
$A = 4(T-t_1)N_2\left(d_1\left(\frac{P}{P^*_{ws}},t_1\right), d_1(P,T);\rho\right) - 2(T-t_1)N_2\left(d_1\left(\frac{P}{P^*_{wf}},t_1\right), d_1(P,T);\rho\right)$;
$B = 2(T-t_1)N_2\left(d_1\left(\frac{P}{P^*_{wf}},t_1\right), d_1(P,T);\rho\right) - (2T-t_1)N(d_1(P,T))$
$- (2T-3t_1)N_2\left(d_1\left(\frac{P}{P^*_{up}},t_1\right), d_1(P,T);\rho\right)$
Since $P^*_{up} < P^*_{ws} < P^*_{wf}$ and therefore:

$$N_2\left(d_1\left(\frac{P}{P^*_{up}},t_1\right), d_1;\rho\right) > N_2\left(d_1\left(\frac{P}{P^*_{ws}},t_1\right), d_1;\rho\right) > N_2\left(d_1\left(\frac{P}{P^*_{wf}},t_1\right), d_1;\rho\right)$$

we have that $A > 0$ and $B < 0$ and $\frac{-B}{A} > 1$. So the disequation (34) is satisfied for every value of $0 \leq q \leq 1$. For the case $q > p$, we will give some numerical examples illustred in the Table (9).

So, the following proposition holds:

Proposition 3 *There exists a unique critical market value V^*_C such that $LF_C(V^*_C) = W_C(V^*_C)$ and:*

$$LF_C(V) < W_C(V) \quad for \quad V < V^*_C$$
$$LF_C(V) > W_C(V) \quad for \quad V > V^*_C$$

Now we analyse the several cooperative equilibriums that can be occour.

4.1.1 First case

If $\frac{\partial LF_C}{\partial V} \geq \frac{\partial S_C}{\partial V}$ then there is not intersection between the functions LF_C and S_C. Moreover, the intersection LF_C and W_C occurs before than S_C and W_C. So, in this case, we have to consider only the critical market value V^*_C given by Proposition (3) and we can state that:

- If $V < V^*_C$ the maximum payoff that both player can obtain by cooperation is

$$C(A \cup B) = W_C(V)$$

- If $V > V_C^*$ the maximum payoff attainable cooperating is

$$C(A \cup B) = LF_C(V)$$

In this case, the best strategic cooperation is the waiting policy (W_C) until the expected market value V is below the critical value V_C^* and, if $V > V_C^*$, the optimal strategy is the Leader-Follower one (LF_C) in which the firm with higher success probability realizes the R&D investment at time t_0 and the other player postpones its decision at time t_1. This is the best payoff attainable through cooperation considering both the whole information revelation $\rho_{\max}$ and the effects of network externalities.

4.1.2 Second case

If $\frac{\partial LF_C}{\partial V} < \frac{\partial S_C}{\partial V}$ then there is intersection between the functions LF_C and S_C. So the following proposition holds:

Proposition 4 *If $\frac{\partial LF_C}{\partial V} < \frac{\partial S_C}{\partial V}$ the there exists a unique critical market value V_G^* such that $LF_C(V_G^*) = S_C(V_G^*)$ and it results that:*

$$S_C(V) < LF_C(V) \quad for \quad V < V_G^*$$

$$S_C(V) > LF_C(V) \quad for \quad V > V_G^*$$

Moreover, the proof (3) shows as $V_C^* < V_G^*$ and so the intersection between LF_C and W_C happens before than LF_C and S_C.

Proof 3 *The condition to have $V_C^* < V_G^*$ is that:*

$$\frac{\partial(LF_C - W_C)}{\partial V} \geq \frac{\partial(S_C - LF_C)}{\partial V} \tag{35}$$

The first part of inequality (35) is the reduction of slope to reach the critical market value V_C^*. It's obviuos that if this reduction is faster then $\frac{\partial(S_C - LF_C)}{\partial V}$, then V_C^* will be smaller than V_G^*.
Considering to simplify the simmetrical case $p = q$ such that $p^+ = 1$, the conditions (35) holds if:

$$Uq^2 + Zq \geq 0 \tag{36}$$

where:

$$\begin{aligned}
U =& - 2kTN(d_1(P,T)) + 2k(T-t_1)N_2\left(d_1\left(\frac{P}{P_{wf}^*}, t_1\right), d_1(P,T); \rho\right)\\
&- 4k(T-t_1)N_2\left(d_1\left(\frac{P}{P_{ws}^*}, t_1\right), d_1(P,T); \rho\right);\\
Z =& 2k(T-t_1)N(d_1(P,T)) + 2k(2T-3t_1)N_2\left(d_1\left(\frac{P}{P_{up}^*}, t_1\right), d_1(P,T); \rho\right)\\
&- 2k(T-t_1)N_2\left(d_1\left(\frac{P}{P_{wf}^*}, t_1\right), d_1(P,T); \rho\right)
\end{aligned}$$

Since $t_1 \leq \frac{T}{3}$ and $N_2\left(d_1\left(\frac{P}{P_{up}^*}, t_1\right), d_1; \rho\right) > N_2\left(d_1\left(\frac{P}{P_{wf}^*}, t_1\right), d_1; \rho\right)$ it results that $U < 0$, $Z > 0$ and $-U > Z$. So the condition (35) holds for every value of $0 \leq q \leq 1$. For the case $q > p$ we give some numerical applications summarized in the Table (9).

So, using the Propositions (3) and (4) we observe that:

- If $V < V_C^*$ the maximum payoff that both firms can obtain with cooperation is

$$C(A \cup B) = W_C(V)$$

- If $V_C^* < V < V_G^*$ the maximum payoff attainable through the cooperation is

$$C(A \cup B) = LF_C(V)$$

- If $V > V_G^*$ the maximum payoff that both player can obtain cooperating is

$$C(A \cup B) = S_C(V)$$

In this case the optimal cooperation strategy is to wait to invest (W_C) when the expected market value V is below V_C^* while, if V is in the range $[V_C^*, V_G^*]$, then the maximum payoff is obtained by the cooperation strategy Leader-Follower (LF_C) and finally, if $V > V_G^*$, both players realize their R&D investment simultaneously at time t_0.

5 Real Applications

5.1 Assumptions and Inputs

To illustrate the concepts and equations presented, we develop some numerical examples for the cooperative R&D game between firms A and B with the following parameters and we focus on the several noncooperative and cooperative equilibriums according to different expected market value V deriving by R&D innovations:

- R&D Investment: R= 250 000 \$;
- Development Investment: D= 400 000 \$;
- Market and Costs Volatility: $\sigma_v = 0.93$; $\sigma_d = 0.23$;
- Proportion of D required for R: $\varphi = \frac{R}{D} = 0.625$
- Correlation between V and D: $\rho_{vd} = 0.15$;
- Dividend-Yelds of V and D: $\delta_v = 0.15$; $\delta_d = 0$;
- R&D innovation parameter $k = 0.30$
- Expiration Time of Simple Option: $T = 3$ years;
- A and B success probability: $q = 0.60$; $p = 0.55$;
- Non Cooperative Information Revelation: $\rho(X, Y) = \rho(Y, X) = 0.40$;
- Cooperative Information Revelation: $\rho_{max} = 0.9026$;

The overall investment cost D is the exercise price for the development option. We consider that the investment cost is proportional to market share, namely if the firm market share is $K_{0_S 0_S}$ then the investment cost will be $K_{0_S 0_S} D$. We assume that D follows the Brownian motion process defined in Eq. (1b).
The R&D investment R can be realized at time t_0 or t_1. If it is made in t_0, then $R = 250\,000$ \$, otherwise the investment R assumes the identical stochastic process of D, except that it occours at time t_1 and it is proportional to $\varphi = 0.625$ of D.

Appropriately, we assume that the volatility of quoted shares and traded options is an adequate proxy for the volatility of assets V and D.
According to financial options, δ denotes the opportunity cost in holding the option instead of the stock. So, in real option world, δ_v is the opportunity cost of deferring the project and δ_d is the "dividend yield"on asset D. As at the beginning the cash flows are very low, we assume that $\delta_v = 0.15$ and $\delta_d = 0$.
The time to maturity T denotes project's deferment option after that each opportunity disappears and we adopt $T = 3$ years.
We assume also that firm A has an higher and more efficient Know-How than firm B and so, firm A'success probability is $q = 0.60$ while firm B's one is $p = 0.55$ but we suppose that the intesity of noncoopertive information revelation is equal for both players and so we state $\rho(X, Y) = \rho(Y, X) = 0.40$. Moreover, using the Eq. (7), it results that the cooperative information revelation is $\rho_{max} = 0.9026$;
Finally, we assume that the R&D innovation parameter $k = 0.30$ and we analyse the two cases according to postponement time t_1. We remark that $t_1 \leq \frac{T}{3}$ to allow the development phase of R&D project and so, considering our adapted parameter values, the maximum postponemet time t_1 is 1 year.

5.2 Numerical application of First case

Assuming that the R&D investment decision can be delay at time $t_1 = 0.5$ year, we obtain, using the Eqs. (9a)-(9f), the following growth market coefficients:

$$K_{0_S 0_S} = 1.8;\ K_{0_S 1_S} = 1.65;\ K_{1_S 1_S} = 1.50;\ K_{1_S 0_S} = 1.35;\ K_{0_S} = 0.90;\ K_{1_S} = 0.75$$

As we can show in the Fig.(4), the $\frac{\partial S_C}{\partial V} < \frac{\partial LF_C}{\partial V}$ and so, using the Proposition (3), we compute the critical market value V_C^* to determine the best cooperation strategy. For our adapted number, it results that $V_C^* = 700\,037$. So, if $V < 700\,037$ both players decide to wait to invest and $C(A \cup B) = W_C(V)$ otherwise, if $V > 700\,037$ the best cooperation strategy is the Leader-Follower one in which firm A invests at time t_0 and firms B delays its decision at time t_1, so $C(A \cup B) = LF_C(V)$.

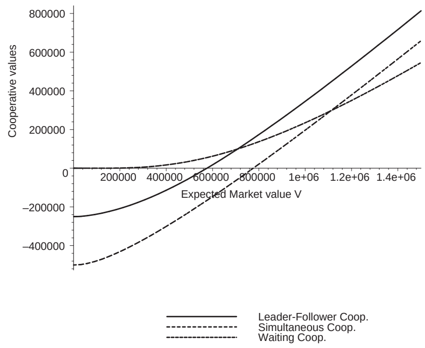


Figure 4: First case

Now, to determine the partnership shares (Sh_A, Sh_B) and (P_A, P_B), we need to compute the non cooperative critical market values V_W^*, V_Q^*, V_P^* and V_S^* that allow to determine the Nash equilibriums. So, using the Propositions (1) and (2), it results that:

$$V_W^* = 1\,028\,380; \quad V_Q^* = 1\,066\,240; \quad V_P^* = 1\,200\,470; \quad V_S^* = 1\,268\,650;$$

The Fig.(5) summarizes the relations among the four non cooperative strategies that allow to determine the Nash equilibriums. We can observe that, if $V < 1\,028\,380$ the waiting policy (W_A, W_B) is optimal in Nash meaning for both player at time t_0, if $1\,028\,380 < V < 1\,066\,240$ and $1\,200\,470 < V < 1\,268\,650$ we have one Nash non cooperative equilibrium (L_A, F_B) in which the firm A, that has an higher success probability, decides to invest in R&D earlier than player B, if $1\,066\,240 < V < 1\,200\,470$ then we obtain two Nash equilibriums (L_A, F_B) and (F_A, L_B) and at last, if $V > 1\,268\,650$ it results one Nash equilibrium (S_A, S_B) in which both player decide to invest simultaneously in R&D at time t_0.

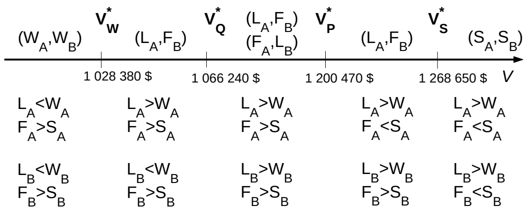


Figure 5: Relations among the non cooperative strategies with $t_1 = 0.5$

Let us examine the partnership between firms A and B combining the cooperative and non cooperative critical market values. The Tables (1) and (2) summarize the non-cooperative payoffs of both firms considering the most notable several expected market values. With these values we are able to compute the Nash-equilibriums $v(A)$ and $v(B)$ that are listed in the second and third column of Table (4). Moreover the Table (3) summarizes the cooperative values $C(A \cup B)$ according to four strategic cooperations and, in particulare way, the bold type values are the maximum ones deriving by the optimal strategic alliance. Using the Eqs. (32) and (33), firms A and B can split the cooperative value $C(A \cup B)$ by the Shapley (Sh_A, Sh_B) or the Asimmetric (P_A, P_B) values that are shown in the Table (4). Comparing the cooperative and the non cooperative values, we can observe that the partnership is favorable for both players since each firm improve its payoff deriving from non cooperative Nash equilibrium. So we can state that the couples (Sh_A, Sh_B) and (P_A, P_B) are Pareto optimals with respect to $(v(A), v(B))$. Only if $V < 700\,037$, and so $V = 600\,000$, then the partnership does not add value to each player because the surplus of cooperation $W_C(V) - (W_A(V) + W_B(V))$ is equal to zero. So we can state that the waiting policy is optimal for both players at time t_0 also in cooperative alternative and firms A and B prefer to wait better market conditions.
Finally, the Fig.(6) represents the overall situation assuming $V = 1\,400\,000$. In particular way, the black line denotes the the feasible set of partnership, namely it represents all the combinations to split $C(A \cup B)$. But only the segment T-H is interesting, because otherwise firms have the incentive to deviate from cooperation. In fact we can observe that Shapley (Sh_A, Sh_B) and Asimmetric (P_A, P_B) values belong to the segment T-H. Moreover, the Fig.(6) shows the four non cooperative strategies and in particular way the Nash-equilibriums (S_A, S_B). We can notice that the segment joins the couples

(S_A, S_B) and (Sh_A, Sh_B) has a 45° slope since, by the Shapley value, A and B share equitably (simmetrically) the surplus of cooperation $C(A \cup B) - (v(A) + v(B))$. So, if firms agree to split the surplus differently, then other solutions will be chosen on the segment T-H.

Market Value V	Leader's Value L_A	Follower's Value F_A	Simultaneous Value S_A	Waiting Value W_A
600 000	-63 344	41 217	-68 466	33 244
900 000	71 204	110 957	62 390	96 736
1 050 000	141 889	154 226	131 135	137 826
1 100 000	165 819	169 582	154 408	152 591
1 250 000	238 525	217 964	225 119	199 561
1 400 000	312 391	269 253	296 958	249 879

Table 1: Firm A's final payoffs assuming $k = 0.30$ and $t_1 = 0.5$

Market Value V	Leader's Value L_B	Follower's Value F_B	Simultaneous Value S_B	Waiting Value W_B
600 000	-73 104	37 018	-78 226	29 024
900 000	54 409	101 802	45 595	86 609
1 050 000	121 398	142 306	110 644	124 373
1 100 000	144 077	156 705	132 666	138 001
1 250 000	212 981	202 120	199 575	181 499
1 400 000	282 984	250 310	267 552	228 291

Table 2: Firm B's final payoffs assuming $k = 0.30$ and $t_1 = 0.5$

5.3 Numerical application of Second case

If we assume now that $t_1 = 0.8$ year, so the postponement time increases, using Eqs. (9a)-(9e) we have that the growth market coefficients are:

$K_{0_S0_S} = 1.8;\ K_{0_S1_S} = 1.56;\ K_{1_S1_S} = 1.32;\ K_{1_S0_S} = 1.08;\ K_{0_S} = 0.90;\ K_{1_S} = 0.66$

As is shown in the Fig.(7), the $\frac{\partial S_C}{\partial V} > \frac{\partial LF_C}{\partial V}$ and so we have two critical market values V_C^* and V_G^*.

Numerically we compute that $V_C^* = 815\,710$ and $V_G^* = 1\,796\,130$ and, using the Propositions (3) and (4), we are able to state the optimal cooperation strategy. So, if $V < 815\,710$ the best partnership strategy is to wait to invest and $C(A \cup B) = W_C(V)$, if $815\,710 < V < 1\,796\,130$ then both player choise the cooperation form LF_C beneficing of information revelation $\rho_{\max}$ and network externalities and hence $C(A \cup B) = LF_C(V)$ and finally, if $V > 1\,796\,130$ then both player prefer to invest simultanoeusly at time t_0 and so $C(A \cup B) = S_C(V)$.

Now, to determine the partnership shares (Sh_A, Sh_B) and (P_A, P_B), we need to com-

Market Value V	Leader-Follower Value LF_C	Follower-Leader Value FL_C	Simultaneous Value S_C	Waiting Value W_C
600 000	17 412	12 486	-146 693	**62 269**
900 000	**257 854**	248 968	107 985	183 345
1 050 000	**390 083**	378 605	241 780	262 199
1 100 000	**435 386**	422 974	287 075	290 593
1 250 000	**574 220**	558 837	424 694	381 060
1 400 000	**716 600**	698 053	564 510	478 170

Table 3: Firms A and B cooperative payoff assuming $k = 0.30$ and $t_1 = 0.5$

Market Value V	Non-Coop. $v(A)$	Non-Coop. $v(B)$	Shapley Value Sh_A	Shapley Value Sh_B	Asim. Value P_A	Asim. value P_B
600 000	33 244	29 024	33 244	29 024	33 244	29 024
900 000	96 736	86 609	133 990	123 863	135 610	122 244
1 050 000	141 889	142 306	194 833	195 250	197 135	192 948
1 100 000	165 819	156 705	222 250	213 136	224 704	210 682
1 100 000	169 582	144 077	230 445	204 940	233 092	202 294
1 250 000	238 525	202 120	305 312	268 907	308 216	266 004
1 400 000	296 958	267 552	373 003	343 597	376 309	340 291

Table 4: Shapley and Asimmetric values assuming $k = 0.30$ and $t_1 = 0.5$

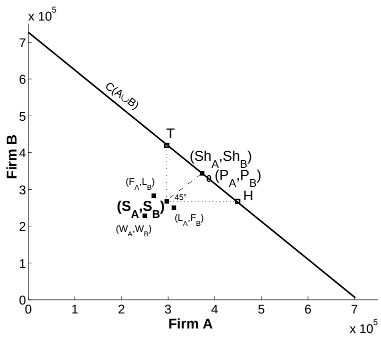


Figure 6: A and B equilibriums when $V = 1\,400\,000$

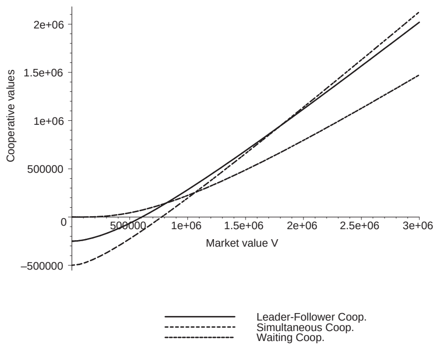


Figure 7: Second case

pute the four non cooperative critical market values V_W^*, V_Q^*, V_P^* and V_S^* by Propositions (1) and (2). So, it results that:

$$V_P^* = 1\,019\,230; \quad V_S^* = 1\,064\,060; \quad V_W^* = 1\,075\,210; \quad V_Q^* = 1\,120\,840;$$

The Fig.(8) shows the relations among the non cooperative strategic values and the several Nash equilibriums. We can observe that, if $V < 1\,064\,060$, both players prefers to wait (W_A, W_B) and to defer their R&D decision at time t_1, if $1\,064\,060 < V < 1\,075\,210$ we obtain two Nash equilibriums (W_A, W_B) and (S_A, S_B) and finally, if $V > 1\,075\,210$, then the simultenous R&D investment (S_A, S_B) at time t_0 is optimal in Nash meaning.

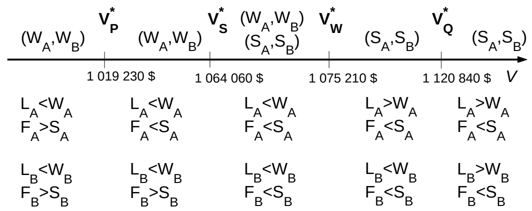


Figure 8: Relations among the non cooperative strategies with $t_1 = 0.8$

As we have seen for the first case, the Tables (5) and (6) include the non cooperative payoffs considering the most notable expected market that allow us to compute the Nash equilibriums $v(A)$ and $v(B)$ summarized in the Table (8). Moreover, in the Table (7) are listed the partnership values $C(A \cup B)$ according to four cooperative strategies and, in particular way, the bold type values are the maximum payoffs deriving by best alliance. Both players can split the cooperative value $C(A \cup B)$ by the Shapley (Sh_A, Sh_B) or the Asimmetric (P_A, P_B) values (see Eqs. (32) and (33)) that are shown in the Table (8). We can observe that, if $V = 600\,000$ and more generally $V < 815\,710$, then the cooperation does not add any value because the cooperation surplus $W_C(V) - (W_A(V) + W_B(V))$ is equal to zero. So wait and see policy is optimal also considering the cooperation way between A and B. Even if $V = 1\,900\,000$ and more

generally $V > 1\,796\,130$, then the cooperative gain $S_C(V) - (S_A(V) + S_B(V))$ is equal to zero. So the simultaneous R&D investment at time t_0 is preferable both in the cooperative strategy and in the non-cooperative one.
Moreover, the Fig.(9) shows the overall situation when $V = 1\,200\,000$. Also in this case we can remark as only the segment T-H is interesting for the splitting of cooperation value $C(A \cup B)$ otherwise firms A and B have the incentive to deviate and to leave the alliance. Also in this case we can observe that the segment joins the couples (S_A, S_B) and (Sh_A, Sh_B) has a 45° slope, since by the Shapley value A and B share equitably (simmetrically) the surplus of cooperation $C(A \cup B) - (v(A) + v(B))$.

Market Value V	Leader's Value L_A	Follower's Value F_A	Simultaneous Value S_A	Waiting Value W_A
600 000	-71 689	36 723	-68 466	37 169
900 000	56 844	90 807	62 390	94 187
1 040 000	119 817	121 657	126 501	127 095
1 070 000	133 495	128 638	140 425	134 565
1 100 000	147 230	135 738	154 408	142 169
1 200 000	193 398	160 209	201 411	168 420
1 900 000	528 189	356 418	542 253	380 026

Table 5: Firm A's final payoffs assuming $k = 0.30$ and $t_1 = 0.8$

Market Value V	Leader's Value L_B	Follower's Value F_B	Simultaneous Value S_B	Waiting Value W_B
600 000	-81 449	33 170	-78 226	33 138
900 000	40 049	83 037	45 595	85 282
1 040 000	99 575	111 633	106 259	115 633
1 070 000	112 504	118 113	119 434	122 539
1 100 000	125 487	124 707	132 666	129 576
1 200 000	169 129	147 452	177 142	153 905
1 900 000	485 595	330 439	499 659	351 367

Table 6: Firm B's final payoffs assuming $k = 0.30$ and $t_1 = 0.8$

5.4 Sensitivity analysis

In this section we study the effects that the parameters k, t_1 and p have on the equilibriums and, in particular way, on the interval in which the optimal cooperation strategy is LF_C. We recall that only the partnership LF_C allows to benefit of a cooperation gain deriving by the whole information revelation $\rho_{\max}$ unlike the waiting W_C and the simultaneous S_C policies.
As it is shown in the Table (9), we assume several combinations of k and t_1 that give the respectives growth market coefficients K. Supposing that $q = 0.60$, we propose

Market Value V	Leader-Follower Value LF_C	Follower-Leader Value FL_C	Simultaneous Value S_C	Waiting Value W_C
600 000	-2 942	-9 300	-146 693	**70 307**
900 000	**206 691**	195 683	107 985	179 469
1 040 000	**313 413**	299 940	232 760	242 729
1 070 000	**336 830**	322 809	259 860	257 105
1 100 000	**360 419**	345 844	287 075	271 745
1 200 000	**440 190**	423 726	378 553	322 325
1 900 000	1 032 079	1 001 380	**1 041 912**	731 393

Table 7: Firms A and B cooperative payoffs assuming $k = 0.30$ and $t_1 = 0.8$

Market Value V	Non-Coop. $v(A)$	Non-Coop. $v(B)$	Shapley Value Sh_A	Shapley Value Sh_B	Asim. Value P_A	Asim. value P_B
600 000	37 169	33 138	37 169	33 138	37 169	33 138
900 000	94 187	85 282	107 798	98 893	108 390	98 301
1 040 000	127 095	115 633	162 437	150 975	163 974	149 439
1 070 000	134 565	122 539	174 428	162 402	176 161	160 669
1 070 000	140 425	119 434	178 910	157 919	180 584	156 246
1 100 000	154 408	132 666	191 080	169 338	192 675	167 744
1 200 000	201 411	177 142	232 229	207 960	233 569	206 621
1 900 000	542 253	499 659	542 253	499 659	542 253	499 659

Table 8: Shapley and Asimmetric values assuming $k = 0.30$ and $t_1 = 0.8$

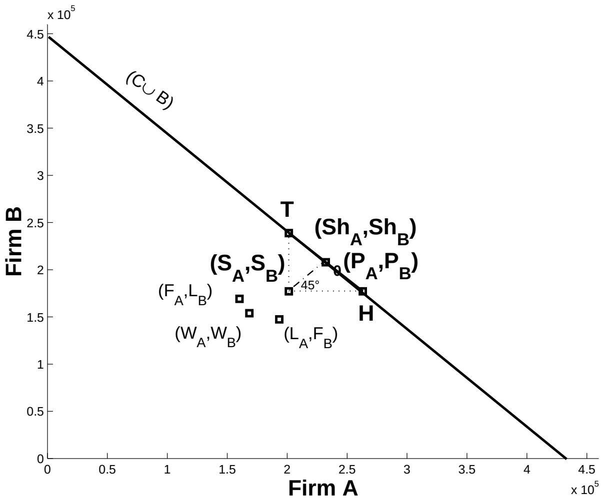


Figure 9: A and B equilibriums when $V = 1\,200\,000$

three level of firm's B success probability: $p = 0.60$ that give a cooperative information revelation $\rho_{\max} = 1$, $p = 0.55$ with $\rho_{\max} = 0.9026$ and finally $p = 0.50$ with $\rho_{\max} = 0.8165$.
When $t_1 = 0.25,\ 0.50$, we obtain the first case in which the cooperation strategy LF_C is optimal for every $V > V_C^*$. So we can observe that, when the R&D innovation parameter k increases, then the threshold value V_C^* decreases and hence the region $]V_C^*, +\infty[$ enlarges. But, if the postponement time t_1 increases, then the compound european exchange option (CEEO) and the growth market coefficients related to t_1 decrease. In other words, the follower values decreases and so the critical market value V_C^* raises. Moreover we can observe that, if the probability p decreases and so also the cooperative information $\rho_{\max}$, then the critical market V_C^* increases.
Finally, when $t_1 = 0.75,\ 1$, we have the second case and so the LF_C strategy is bounded by the critical market values $]V_C^*, V_G^*[$. In this situation, the follower value decreases and so the simultaneous R&D investment is profitable for $V > V_G^*$. We can note that, when the postponement time t_1 increases, then V_C^* enlarges and V_G^* decreases and so the range $]V_C^*, V_G^*[$, in which the optimal strategy is LF_C, goes down. While, if k raises, then both V_C^* and V_G^* go down and we can observe that the length of interval $]V_C^*, V_G^*[$ decreases. We can observe also that, if probability p decreases, then both the thresholds V_C^* and V_G^* go up. This means that, both the critical market value V_C^* until is better to wait and the threshold V^G from which is profitable the simultaneous R&D investment increase their values.
Moreover, we can note that V_C^* is always smaller than V_G^*.

k; t_1	$p = 0.60$	$p = 0.55$	$p = 0.50$
0.25; 0.25	]686 846, +∞[	]719 123, +∞[	]755 413, +∞[
0.25; 0.50	]774 857, +∞[	]808 293, +∞[	]845 441, +∞[
0.25; 0.75	]876 617, 2 345 271[	]909 670, 2 520 902[	]945 839, 2 731 506[
0.25; 1.00	]994 768, 1 317 283[	]1 024 608, 1 408 848[	]1 056 646, 1 517 632[
0.50; 0.25	]421 901, +∞[	]440341, +∞[	]460 997, +∞[
0.50; 0.50	]476 010, +∞[	]495 107, +∞[	]516 256, +∞[
0.50; 0.75	]537 974, 1 344 397[	]556 746, 1 438 431[	]577 229, 1 550 730[
0.50; 1.00	]609 405, 786 341[	]626 119, 837 313[	]644 013, 897 603[
0.75; 0.25	]322 244, +∞[	]335 757, +∞[	]350 864, +∞[
0.75; 0.50	]363 666, +∞[	]377 661, +∞[	]393 135, +∞[
0.75; 0.75	]410 794, 987 646[	]424 509, 1 053 888[	]439 450, 1 132 786[
0.75; 1.00	]464 869, 590 763[	]476 979, 627 524[	]489 922, 670 892[
1.00; 0.25	]267 910, +∞[	]278 827, +∞[	]291 016, +∞[
1.00; 0.50	]302 420, +∞[	]313 729, +∞[	]326 218, +∞[
1.00; 0.75	]341 490, 799 562[	]352 549, 851 599[	]364 582, 913 457[
1.00; 1.00	]386 159, 485 529[	]395 866, 514 887[	]406 229, 549 459[

Table 9: Interval $]V_C^*, V_G^*[$ in which is optimal LF^C cooperation strategy

6 Concluding Remarks

In this paper we have proposed an R&D cooperation between two firms using the real option approach to value their payoffs. By the alliance, the information revelation is wholly revealed and captured by two players. Moreover, we have shown that the unique cooperation strategy that allows to increase the information revelation with respect to the non cooperative situation is the Leader-Follower strategy, in which one firm realizes the R&D invesment at time t_0 and other one postpones its decision at time t_1. In particular way, as the mutual information gain implies positive network externalities, we have shown that the Leader role is assumed by the firm with the highest success probability.
Finally, computing the non cooperative and the cooperative critical market values, we are able to determine the range game in which is optimal every partnership strategy and also the combinations to split the surplus of cooperation. Using the Shapley value both firms split equitably the surplus but they can agree upon several partnership contracts, such as the asymmetric shares P_A and P_B based on different success probability.

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