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Optimal marketing decision in a duopoly: a stochastic approach

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Summary. Let us consider two new perfect substitute durable products which are produced and sold in a market by two competing firms.
Looking at a potential buyer, we build a stochastic rule by which she purchases the good from one of the two firms (so that she becomes an *adopter*). The model is considered discrete in time and space. The probability of transition from the non adopter state to the adopter one depends on an imitation mechanism (*word-of-mouth*) as well as on the pricing and advertising policies of the producers/sellers. It is assumed that only actual information about the market determine the evolution in the subsequent time step so that a Markov process arises. Both firms maximize their expected discounted profits by choosing optimal marketing strategies. Suitable equilibria are characterized and, because of the lack of convexity in the model, the simulated annealing algorithm is proposed to compute them.

1 Introduction

The diffusion of a new product (innovation) in a market has been modelled firstly in the seminal paper by Bass (1969).
The population of potential consumers is divided into two classes: the class of the adopters - i.e. individuals which have already bought the new product and which spread information about it; the non-adopters or uninformed - i.e. individuals which are not yet informed about the innovation. The Bass equation gives the dynamic of cumulative adopters as a function of advertising and interpersonal communication. The Bass model and its early generalizations have been used, since the 1980, to address and solve optimal-control problems governed by differential equations. For a review of the decision problems related to the diffusion of new products in a market see Dockner et al. (2000) and Jørgensen and Zaccour (2004).

In this paper we firstly discuss a duopoly model of innovation diffusion. There are two competing firms which produce and sell two versions of the new product which are perfect substitutes and differ only for the brand of the producer. Advertising and interpersonal contacts also contribute to the diffusion of the new product. Moreover consumer's decisions depend on the price of the product and, since the two products are substitutes, consumers react also to the difference between the prices. The model in this paper is presented in Section 2 as a stochastic rule by which a potential buyer can become an adopter of one of the two products. The model is considered discrete in time and space. The probability of transition from the state of non-adopter to the state of adopter of one kind of products depends on an imitation mechanism (*word-of-mouth*) as well as on the pricing and advertising policies of the producer/seller. Since it is assumed that only actual information about the market determines the evolution in the next time step, then the process is a Markov one. After a brief discussion about the features of innovation diffusion dynamics, a noncooperative game between the two firms is treated.

2 The Model

We consider a population of M (potential) consumers each of which can buy at most one copy of a new durable product (innovation) choosing between two perfect substitutes P_1 and P_2. The innovation is produced and sold by two firms which can practise discrimination in prices since the two brands are differently perceived by the consumers. Let us denote by $A_{n,i}$ the number, at time n, of the individuals who have already bought the product P_i manufactured by the firm F_i, $i = 1, 2$. It is $A_{0,i} = 0$.
We assume that potential consumers are convinced to buy the new product through the advertising given by the two firms. The advertising performed by the firm F_i has a (nonnegative) influence on both the sales of the firm F_i and F_j and viceversa.
Let us denote by $\gamma_{i,n}$, $i = 1, 2$ the quantities (normalized to one) of the advertising produced by the firm F_i at time n. The effectiveness of the advertising is measured through the functions g_i and h_i. Precisely $g_i(\gamma_{i,n})$ represents the effect on the sales of the firm F_i of the advertising made by the firm F_i; whereas $h_i(\gamma_{j,n})$ measures the influence on the sales of the firm F_i given by the advertising made by the firm F_j, $i \neq j$.
Furthermore people are convinced to buy the new product through interpersonal contacts with previous adopters. This effect is modelled by parameter $k_{i,j}$ which represents the effect of word-of-mouth of P_i adopter's to convince a non adopter to buy the product P_j.
Obviously, price also influences the decision of a potential customer. Higher it is the price, lower the probability that the product is purchased. A measure of this effect is given by a so-called *price-response function* $q_i(p_{1,n}, p_{2,n})$ where $p_{i,n}$ is the price of the product P_i at time n.

The following properties are assumed to hold about the functions g_i, h_i:

1. $g_i(0) = 0; \qquad i = 1, 2;$
2. $h_i(0) = 0; \qquad i = 1, 2;$
3. $0 \leq g_i(\gamma_{i,n}) + h_i(\gamma_{j,n}) \leq 1 \qquad i, j = 1, 2$

Moreover the functions g_i and h_i are assumed to be increasing and concave in their arguments to incorporate decreasing advertising returns.

We assume that the *price-response function* $q_1(p_1, p_2)$ is increasing with respect to p_2 and that it is decreasing with respect to p_1. Furthermore $0 \leq q_1(p_1, p_2) \leq 1$. The analogous properties hold for q_2. In the rest of the paper we choose price-response function as follows

$$\begin{cases} q_1(p_1, p_2) = \exp(-\alpha_1 p_1)\varphi_1(p_2 - p_1) \\ q_2(p_1, p_2) = \exp(-\alpha_2 p_2)\varphi_2(p_1 - p_2) \end{cases}$$

where α_i are positive constants and φ_i are increasing functions. So, potential consumers react to the price of the product they are going to buy but they also react to the difference between the prices of the two products.

We provide a stochastic rule for a potential buyer to become an adopter of one of the two products P_1, P_2. The process is considered discrete in time and we assume that decisions are taken at time $n \in \{0, 1, ..., T-1\}$, where $T \in \mathbb{N}$.

Let us define, for $j = 1, \ldots, M$ and $n = 0, \ldots, T$, $i = 1, 2$, the random variables $X^j_{n,i}$ as follows:

$$X^j_{n,i} = \begin{cases} 1 & \text{if the } j\text{-th individual is an adopter} \\ & \text{of product } P_i \text{ at time } n \\ & \\ 0 & \text{if the } j\text{-th individual is not yet an adopter} \\ & \text{of product } P_i \text{ at the time } n. \end{cases}$$

Let

$$X^j_n = (X^j_{n,1}, X^j_{n,2}).$$

Thus the number of adopters at time n is given by

$$A_n = \sum_{j=1}^{M} X^j_n.$$

where

$$A_n = (A_{n,1}, A_{n,2}).$$

If the j-th individual is not yet an adopter at time n, then she becomes an adopter at time $n+1$ with probabilities

$$r_{n,A_n,1} := P\left(X_{n+1}^j = (1,0)|X_n^j = (0,0), X_n^1, \ldots, X_n^M\right) =$$

$$q_1(p_{1,n}, p_{2,n})\left(1 - (1 - g_1(\gamma_{1,n}) - h_1(\gamma_{2,n}))\left(1 - \frac{k_{1,1}}{M}\right)^{A_{n,1}}\left(1 - \frac{k_{1,2}}{M}\right)^{A_{n,2}}\right)$$

$$r_{n,A_n,2} := P\left(X_{n+1}^j = (0,1)|X_n^j = (0,0), X_n^1, \ldots, X_n^M\right) =$$

$$q_2(p_{1,n}, p_{2,n})\left(1 - (1 - g_2(\gamma_{2,n}) - h_2(\gamma_{1,n}))\left(1 - \frac{k_{2,1}}{M}\right)^{A_{n,1}}\left(1 - \frac{k_{2,2}}{M}\right)^{A_{n,2}}\right)$$

Moreover we have

$$\begin{cases} P\left(X_{n+1}^j = (0,0)|X_n^j = (0,0), X_n^1, \ldots, X_n^M\right) = 1 - r_{n,A_n,1} - r_{n,A_n,2} \\ P\left(X_{n+1}^j = (0,0)|X_n^j = (1,0), X_n^1, \ldots, X_n^M\right) = 0 \\ P\left(X_{n+1}^j = (0,1)|X_n^j = (1,0), X_n^1, \ldots, X_n^M\right) = 0 \\ P\left(X_{n+1}^j = (0,0)|X_n^j = (0,1), X_n^1, \ldots, X_n^M\right) = 0 \\ P\left(X_{n+1}^j = (1,0)|X_n^j = (0,1), X_n^1, \ldots, X_n^M\right) = 0 \\ P\left(X_{n+1}^j = (1,0)|X_n^j = (1,0), X_n^1, \ldots, X_n^M\right) = 1 \\ P\left(X_{n+1}^j = (0,1)|X_n^j = (0,1), X_n^1, \ldots, X_n^M\right) = 1 \end{cases}$$

The first equality means that the probability to remain non adopter for an individual who has not already adopted the innovation is $1 - r_{n,A_n,1} - r_{n,A_n,2}$. The last two equalities mean that if the j-th individual is an adopter at time n, then she remains an adopter for all the future time. The meaning of the other equalities is obvious.

The stochastic process $A = (A_n)_{n=0,\ldots,T}$ is a Markov chain. The state space is given by the set

$$S_M := \{(l,j) \in \mathbb{N}^2 | l + j \leq M\}.$$

S_M has $\sigma_M \equiv \frac{(M+1)(M+2)}{2}$ elements.

Note that the Markov chain is nonhomogeneous: that is the transition probabilities are non-stationary because at any time n they depend on the advertising rates $\gamma_{i,n}$ and on the selling prices $p_{i,n}$, $i = 1, 2$.

Since there are no adopters at time $n = 0$, we have that the distribution of the initial state is the vector $\pi \in \mathbb{R}^{\sigma_M}$, defined by

$$\pi_{s_l} = P(A_0 = s_l) = \begin{cases} 1 & \text{if } s_l = (0,0) \\ 0 & \text{if } s_l \neq (0,0). \end{cases}$$

Setting $\mathbf{i} = (i_1, i_2) \in S_M$ and $\mathbf{j} = (j_1, j_2) \in S_M$, the transition probabilities are

$$P(A_{n+1} = \mathbf{j} \,|A_n = \mathbf{i}) = \begin{cases} \pi^n_{i,j} & \text{if } i_1 \le j_1 \text{ and } i_2 \le j_2 \\ 0 & \text{otherwise} \end{cases}$$

where $\pi^n_{i,j}$ are given by

$$\frac{(M - i_1 - i_2)!}{(j_1 - i_1)!(j_2 - i_2)!(M - j_1 - j_2)!}\, r^{j_1 - i_1}_{n,i,1}\, r^{j_2 - i_2}_{n,i,2}\, (1 - r_{n,i,1} - r_{n,i,2})^{M - j_1 - j_2}$$

If the advertising levels and the selling prices are constant in time, then the Markov chain is homogeneous. In this case let $\gamma_{i,n} \equiv \gamma_i$. If $\gamma_1 + \gamma_2 > 0$, then, by standard Markov chain asymptotic properties, it follows that

$$\lim_{n \to +\infty} P(A_{n,1} + A_{n,2} = M) = 1.$$

This means that, as the time horizon tends to infinity, the whole population adopts one of the two new products with probability one. If $\gamma_1 + \gamma_2 = 0$, then nobody becomes an adopter.

3 The dynamic game

Let $c_{p,i}$ be the per unit cost of the product P_i. Moreover let us indicate by $c_{\gamma,i}$ the unitary costs, paid by the firm F_i for the advertising made during the period $[0, T]$. Here $c_{p,i}$ and $c_{\gamma,i}$ are given positive constants. Let $\delta_i > 0$ be the (constant in time) one period instantaneous discount rate. The stochastic discounted returns to the firm F_i in the planning period, given a price-advertising policy, are then

$$\sum_{k=0}^{T-1} e^{-\delta_i k}(p_{i,k} - c_{p,i})(A_{k+1,i} - A_{k,i}) - c_{\gamma,i}\gamma_{i,k}.$$

The firms perform control of a common stochastic discrete time dynamic system which is a non-homogeneous Markov chain with one-step transition matrix depending on control parameters. At every time step, each player makes decision in order to maximize her total discounted payoff for the planning period assuming that the other player does the same. We suppose that each player knows the current state of the system (symmetric complete information). We describe the game by the dynamic Nash equilibrium.

The random transition from the current state A_n to the next one A_{n+1} depends only on the actions of players:

$$\Phi_{i,n} := (\gamma_{i,n}(A_n), p_{i,n}(A_n))$$

and the current state A_n. We denote by

$$u_{i,n}(A_{n+1} - A_n, \Phi_{1,n}, \Phi_{2,n}) := (p_{i,n} - c_{p,i})(A_{n+1,i} - A_{n,i}) - c_{\gamma,i}\gamma_{i,n} \quad (1)$$

the firm current payoff. We observe that the strategies of the firms at every time step depend only on the current state of the system.

At time step $n = 0, 1, \ldots, T-1$, each firm chooses an optimal strategy:

$$\Psi_{i,n}(s) = (\Phi_{i,n}(s), \Phi_{i,n+1}(s), \ldots, \Phi_{i,T-1}(s)) \quad \forall s \in S_M$$

where

$$\Phi_{i,n}(s) = (\gamma_{i,n}(s), p_{i,n}(s)), \quad \forall s \in S_M,$$

maximizing the expected discounted sum of future one-period payoffs (1), given the policy of the other firm:

$$U_{i,n}(s, \Psi_{1,n}, \Psi_{2,n}) = E_{n,s} \sum_{k=n}^{T-1} e^{-\delta_i(k-n)} u_{i,k}(A_{k+1} - A_k, \Phi_{1,k}, \Phi_{2,k}). \quad (2)$$

Here $E_{n,s}$ is the expected value conditioned on $[A_n = s]$, that is the firms make decisions observing the state of the system at time n.

The game solution consists in a dynamic Nash equilibrium. At time n, a pair $(\hat{\Psi}_{1,n}, \hat{\Psi}_{2,n})$ is a Nash equilibrium if, for all $s \in S_M$,

$$\begin{cases} U_{1,n}(s, \hat{\Psi}_{1,n}, \hat{\Psi}_{2,n}) = \max\limits_{\Psi_{1,n}} \; U_{1,n}(s, \Psi_{1,n}, \hat{\Psi}_{2,n}) \\ U_{2,n}(s, \hat{\Psi}_{1,n}, \hat{\Psi}_{2,n}) = \max\limits_{\Psi_{2,n}} \; U_{2,n}(s, \hat{\Psi}_{1,n}, \Psi_{2,n}) \end{cases} \quad (3)$$

From the elementary properties of Markov chains, we have

$$E_{n,s} f(s, A_{n+1}, \ldots, A_T) =$$

$$\sum_{i_{n+1},\ldots,i_T} f(s, A_{n+1}, \ldots, A_T) \pi^n_{s,i_{n+1}} \pi^{n+1}_{i_{n+1},i_{n+2}} \cdots \pi^{T-1}_{i_{T-1},i_T} =$$

$$\sum_{i_{n+1}} \pi^n_{s,i_{n+1}} \sum_{i_{n+2}} \pi^{n+1}_{i_{n+1},i_{n+2}} \cdots \sum_{i_T} \pi^{T-1}_{i_{T-1},i_T} f(s, A_{n+1}, \ldots, A_T)$$

where $\pi^n_{i,j} = P(A_{n+1} = \mathbf{j} | A_n = \mathbf{i})$. Let's

$$g_{T-n-1}(s, \mathbf{i_n}, \ldots, \mathbf{i_{T-1}}) := E_{T-1,i_{T-1}} \; f(s, \mathbf{i_n}, \ldots, \mathbf{i_{T-1}}, A_T)$$

$$g_{T-n-2}(s, \mathbf{i_n}, \ldots, \mathbf{i_{T-2}}) := E_{T-2,i_{T-2}} \; g_{T-n-1}(s, \mathbf{i_n}, \ldots, \mathbf{i_{T-2}}, A_{T-1})$$

$$\vdots$$

$$g_1(s, \mathbf{i_{n+1}}) := E_{n+1,\mathbf{i_{n+1}}} \; g_2(s, \mathbf{i_{n+1}}, A_{n+2}).$$

Then we have

$$E_{n,s} f(s, A_{n+1}, \ldots, A_T) = E_{n,s} g_1(s, \mathbf{i_{n+1}})$$

and hence

$$E_{n,s}\ f(s, A_{n+1}, \dots, A_T) = E_{n,s}\ E_{n+1,A_{n+1}} \dots E_{T-1,A_{T-1}}\ f(s, A_{n+1}, \dots, A_T).$$

From (2) we have (we omit some functional dependencies)

$$U_{i,n}(s) = E_{n,s}\left(u_{i,n}(A_{n+1}-s) + e^{-\delta_i}\sum_{k=n+1}^{T-1} e^{-\delta_i(k-(n+1))}u_{i,k}\right) =$$

$$E_{n,s}\ u_{i,n}(A_{n+1}-s)+$$

$$e^{-\delta_i}E_{n,s}\left(E_{n+1,A_{n+1}} \dots E_{T-1,A_{T-1}}\sum_{k=n+1}^{T-1} e^{-\delta_i(k-(n+1)}u_{i,k}\right) =$$

$$E_{n,s}\ u_{i,n}(A_{n+1}-s) + e^{-\delta_i}E_{n,s}\ U_{i,n+1}(A_{n+1})$$

where $n = 0, 1, \dots, T-1$, $s \in S_M$ and $U_{i,T} \equiv 0$.

Hence we can try to solve the problem by a dynamic programming algorithm. Let's

$$\hat{U}_{i,n}(A_n, \Phi_{1,n}, \Phi_{2,n}) := U_{i,n}(A_n, (\Phi_{1,n}, \hat{\Psi}_{1,n+1}), (\Phi_{2,n}, \hat{\Psi}_{2,n+1}))$$

the payoff of a firm corresponding to a given policy at time t and optimal strategies for time $t+1, \dots, T-1$.

If $(\hat{\Psi}_{1,n+1}, \hat{\Psi}_{2,n+1})$ is a Nash equilibrium for (3) at time $n+1$ and $(\hat{\Phi}_{1,n}, \hat{\Phi}_{2,n})$ is a (unique) Nash equilibrium, for all $s \in S_M$, of the following problem:

$$\begin{cases} \hat{U}_{1,n}(s, \hat{\Phi}_{1,n}, \hat{\Phi}_{2,n}) = \max\limits_{\Phi_{1,n}}\ E_{n,s}\ u_{1,n}(A_{n+1}-s, \Phi_{1,n}, \hat{\Phi}_{2,n}) + \\ \qquad\qquad e^{-\delta_1}E_{n,s}\hat{U}_{1,n+1}(A_{n+1}, \hat{\Phi}_{1,n+1}, \hat{\Phi}_{2,n+1}) \\ \hat{U}_{2,n}(s, \hat{\Phi}_{1,n}, \hat{\Phi}_{2,n}) = \max\limits_{\Phi_{2,n}}\ E_{n,s}u_{2,n}(A_{n+1}-s, \hat{\Phi}_{1,n}, \Phi_{2,n}) + \\ \qquad\qquad e^{-\delta_2}E_{n,s}\hat{U}_{2,n+1}(A_{n+1}, \hat{\Phi}_{1,n+1}, \hat{\Phi}_{2,n+1}) \end{cases} \tag{4}$$

then $(\hat{\Psi}_{1,n}, \hat{\Psi}_{2,n}) = ((\hat{\Phi}_{1,n}, \hat{\Psi}_{1,n+1}), (\hat{\Phi}_{2,n}\hat{\Psi}_{2,n+1}))$ is a Nash equilibrium for (3) at time n.

The previous expected values can be rewritten (again we omit some functional dependencies) as:

$$\eta_{i,n}(\Phi_{1,n}, \Phi_{2,n}) := E_{n,s}\ \left(u_{i,n}(A_{n+1}-s) + e^{-\delta_i}\hat{U}_{i,n+1}(A_{n+1})\right) =$$

$$\sum_{h_1=0}^{M_s}\sum_{h_2=0}^{M_s-h_1}\left(u_{i,n}((h_1,h_2)) + e^{-\delta_i}\hat{U}_{i,n+1}(s+(h_1,h_2))\right)\pi^n_{s,s+h}$$

where

$$s = (s_1, s_2,), \quad h = (h_1, h_2), \quad M_s = M - (s_1 + s_2)$$

$$\pi^n_{s,s+h} = P(A_{n+1} = s + (h_1, h_2) | A_n = s) =$$

$$\frac{M_s!}{h_1! h_2! (M_s - h_1 - h_2)!} \, r^{h_1}_{n,s,1} r^{h_2}_{n,s,2} (1 - r_{n,s,1} - r_{n,s,2})^{M_s - h_1 - h_2}$$

Using the dynamic programming techniques, we solve the problem (3) by backward recursion starting from the last stage. At each stage n we solve the problem (4), given that the stage $n+1$ has already been solved. In other words at stage n, the values $\hat{U}_{i,n+1}(s)$ are known for all $s \in S_M$ and for the last stage we have $\hat{U}_{i,T} \equiv 0$.

In order to obtain a numerical solution of the problem (4), we use the following iterative algorithm (see Golubtsov et al. 2003). We choose an initial policy $\Phi^{2k}_{1,n}$ (where $k = 0$) for the firm F_1 and we determine the optimal response of the firm F_2 finding

$$\Phi^{2k+1}_{2,n} := \arg\max_{\Phi_{2,n}} \, \eta_{2,n}(\Phi^{2k}_{1,n}, \Phi_{2,n})$$

Furthermore we compute the optimal response of the firm F_1 for this policy:

$$\Phi^{2(k+1)}_{1,n} := \arg\max_{\Phi_{1,n}} \, \eta_{1,n}(\Phi_{1,n}, \Phi^{2k+1}_{2,n})$$

Under suitable conditions the sequence $(\Phi^{2k}_{1,n}, \Phi^{2k+1}_{2,n})$, obtained iterating the previous steps, converges to the solution of the problem (4). Because of the complexity of the functions η_i it is hard to obtain analytical information i.e. monotonicity, convexity etc. Also the existence and uniqueness of the optimal response remains an open problem. We solve numerically the previous global maximum problems using at each step k the Simulated Annealing algorithm. We iterate on k until the differences $||\Phi^{2(k-1)}_{1,n} - \Phi^{2k}_{1,n}||$ and $||\Phi^{2(k-1)+1}_{2,n} - \Phi^{2k+1}_{2,n}||$ are small according to a given precision.

In our simulation we consider the following functions for the advertising effects:

$$g_i(\gamma_i) := \rho_i \log(1 + \gamma_i)/\log(2) \qquad h_i(\gamma_j) := \xi_i \log(1 + \gamma_j)/\log(2)$$

where $\rho_i \geq 0$, $\xi_i \geq 0$ and $\phi_i := \rho_i + \xi_i < 1$. We choose $\delta_i = 0$, $M = 20$ and $T = 10$. The other parameters are listed in table (1).

In the following pictures the expected returns of the firms, given the information at the initial time and when optimal price/advertising strategies are performed, are plotted against the time. Precisely we can see the returns on the left figure and the price/advertising profiles on the right figure. The expected values

Table 1. Simulation parameters

Figure	$c_{p,1}$	$c_{p,2}$	$c_{\gamma,1}$	$c_{\gamma,2}$	ρ_1	ρ_2	ξ_1	ξ_2	α_1	α_2
1	0.40	0.50	1.20	0.80	0.40	0.25	0.02	0.03	0.45	0.50
2	0.40	0.50	1.50	0.80	0.40	0.25	0.02	0.03	0.45	0.50
3	0.40	0.40	1.00	0.80	0.30	0.25	0.02	0.03	0.50	0.50
4	0.40	0.40	1.20	0.80	0.40	0.25	0.02	0.03	0.50	0.50

$$G_i(n) := E_0\left(\sum_{k=0}^{n} e^{-\delta_i k}(\hat{p}_{i,k} - c_{p,i})(A_{k+1,i} - A_{k,i}) - c_{\gamma,i}\hat{\gamma}_{i,k}\right)$$

are computed by a Monte Carlo simulation generating a sample path of the Markov chain $(A_n)_{n=0,\dots,T}$. Note that, according to the standard literature, the advertising profiles are decreasing in time while the price profiles are decreasing at the beginning and then are definitively increasing.

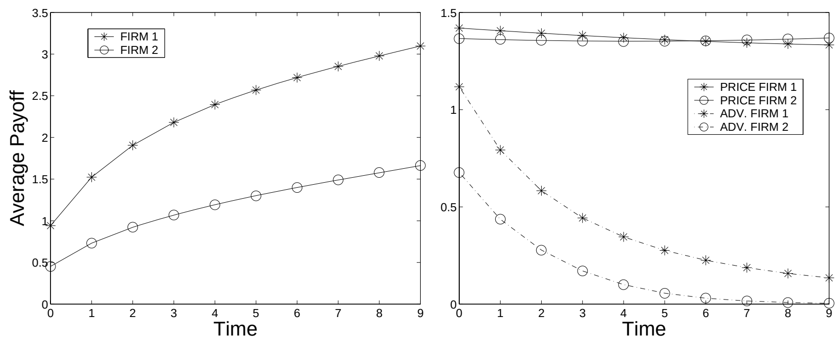

Fig. 1. Average payoffs and average price/advertising strategies

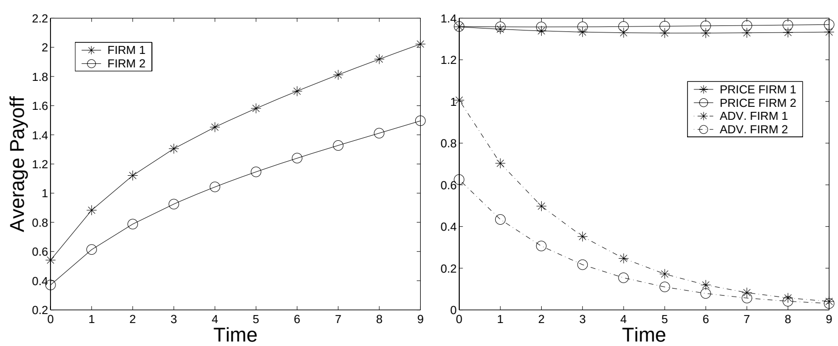

Fig. 2. Average payoffs and average price/advertising strategies

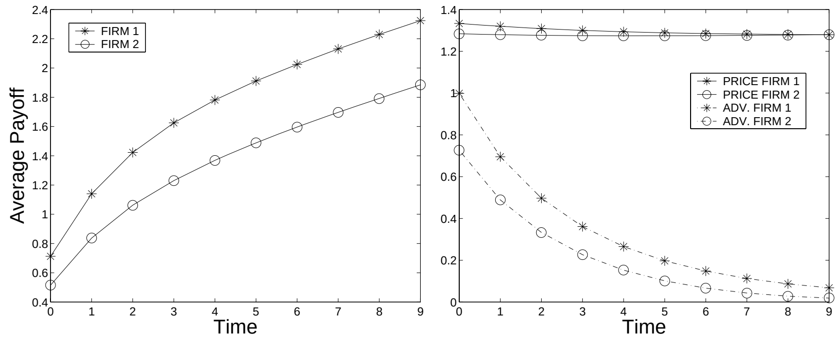

Fig. 3. Average payoffs and average price/advertising strategies

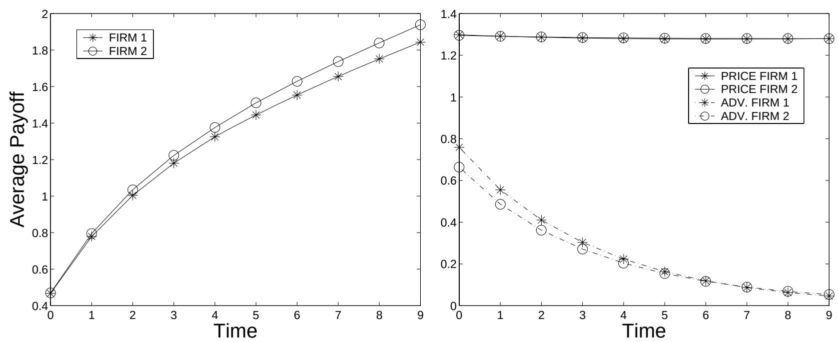

Fig. 4. Average payoffs and average price/advertising strategies

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