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Design Imitation in the Fashion Industry

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Abstract

The paper deals with the imitation of fashion products, an issue that attracts considerable interest in practice. Copying of fashion originals is a major concern of designers and, in particular, their financial backers. Fashion firms are having a hard time fighting imitations, but legal sanctions are not easily implemented in this industry. We study an alternative strategy that has been used by designers. Instead of fighting the imitators in the courtroom, designers fight them in the market. The designer markets her products in separate markets, typically a "high class" market in which the products are sold in exclusive stores at high prices. Customers in this market seek exclusivity and their utility diminishes when seeing an increasing number of copies around. Their perception of the brand tend to dilute which poses a serious threat to a fashion company. The second market is a "middle class" market in which there are many more buyers, and the fashion firm competes directly with the imitators in this market. This market can be used to practise price discrimination, to sell off left-over inventories, and to get a spin-off from the design. The paper models the decision problems of the fashion firm and the imitators as a two-period game in which firms make pricing decisions and decisions on when to introduce their products in the markets. In addition, the fashion firm decides how much efforts to spend to increase its brand image in the two markets.

1 Introduction

The paper deals with product imitation, an issue that attracts considerable interest in practice. It occurs in many industries (e.g., furniture, toys, music,

and fashion). Here we are concerned with *fashion*. A quick search of the World Wide Web suggests that a broad range of fashion designs are copied and imitations offered for sale at low prices: jewelry, handbags, leather belts and wallets, perfumes, watches, sunglasses, and mens' and womens' apparel¹. This paper considers one particular designer product.

A specific feature of fashion goods is the fact that a consumer is willing to pay a very high price for a product that she probably could buy much cheaper elsewhere. What makes this possible is the "brand integrity" or "brand image" that lies behind the fashion firm and its products. The natural enemy of brand integrity is "brand dilution" which occurs if too many people can be seen using the product, be it the original or a copy. Brand dilution leads to decreasing demand for the original product, and can be hard to reverse once the process is in motion.

The imitation of design originals is a major concern of designers and, in particular, their financial backers. Indeed, imitations of designer products can be produced and marketed very shortly after (and even before) the original appears on the market. Fashion firms wish to fight imitations, but legal actions are often infeasible. One reason is that in most countries, fashion designs cannot be patented. Various counter-strategies have been seen. Among these is the practice that a fashion firm buys up copy products and destroys them, usually in a public event witnessed by the press. (This has been done in the case of Cartier watches). Another strategy is that the fashion firm offers consumers who have bought a copy to have it replaced, at no cost, by an original. This has been seen in the case of designer T-shirts and caps of the brand Von Dutch. The strategy is useable for items produced in smaller numbers ("limited editions").

This paper studies the effects of a different strategy that has been employed by designers. Instead of fighting the imitators in the courtroom or buying up copies, designers fight imitators in the *market*. A typical instance of this practice can be described as follows. The designer firm markets its products in two separate markets. One market, *A*, is a "high-class" market in which the designer product is sold in exclusive stores at high prices and limited numbers. Typically, stores are owned or franchised by the fashion house. Customers in market *A* seek *exclusivity*, that is, their utility is highest when they know that they are the only ones who possess the product. Seeing an increasing number of "similar" products around, their perception of the brand tend to dilute.

Another factor that can decrease the strength of a fashion brand name is the use of the name on other products than the fashion items themselves. One often sees that fashion houses use their name on a wide range of products (e.g., sunglasses, perfumes). In this case the fashion firm itself

¹The fashion imitation industry seems to be particularly flourishing in India and Southeast Asia.

may contribute to the dilution of the brand.

Knowing that it cannot prevent imitations, the fashion firm enters the "middle-class" market, B . Thus, after having sold the product in market A for a limited time, the firm "moves the product" to market B . Typically this is accomplished by introducing a cheaper "diffusion line". (One example is the expensive Armani Via Borgo Nuovo line which diffuses into the less expensive Armani and Emporio Armani lines). In market B there are many more buyers and the products are sold at considerably lower prices than in market A . The stores selling the fashion product can be licensees of the fashion firm or manufacturer-owned "factory outlets".

The fashion firm can have several objectives for entering market B . One is to fight the imitations in the market, a second is to dispose of left-over inventory, and a third is to benefit from a spin-off from the original design.

This paper gives a stylized image of the marketing of original designs and imitations in the fashion industry. We consider a fashion firm, henceforth denoted by E , and confine our interest to one specific product of firm E . The firm spends considerable amounts of money in developing new designs of the product. Although the manufacturing costs may not be that high, the total costs of the firm typically include sizeable expenditures on its creative staff and on advertising.

In market B there is a competing firm (or a fringe of firms), henceforth denoted by P , producing an imitation of E 's product. The aim of P is to have a product which "looks like" that of E , with the purpose of free-riding on the brand image of the fashion firm. Firm P does not spend anything on developing new designs of its own; the firm simply adjusts the design of their product to imitate that of E . The costs of firm P are substantially lower than those of firm E and the products of firm P sell at considerably lower prices in non-exclusive stores. The imitating firm does nothing to create a demand for its product. Demand for the imitation is generated by consumers who wish to imitate the consumption of wealthy, trend-setting consumers. Thus, consumption imitation creates a demand for the imitator's product.

Studies of fashion imitation problems using game theoretic methods are very few. In fact, we only know about the one in Caulkins et al. (2003). These authors are interested in explaining the occurrence of fashion cycles and suggest an optimal control model in which state variables are the positions of the fashion firm and the imitator along a one-dimensional product space. However, the imitator is not a decision maker. She always chases the innovator and her position is determined by the position of the innovator (which, in turn, is determined by the innovator's design efforts). Thus, our work can be seen as a first exploration of the (realistic) situation in which a fashion firm faces imitators in the market place. An account of fashion imitation in practice is found in the article in *The Economist*, March 6th,

2004.

The paper proceeds as follows. Section 2 develops a two-period game model, incorporating pricing decisions of both firms and image building expenditures of the fashion firm E . The extension to a general number of periods is straightforward, but does not add to our understanding of the imitation problem, at least in the present setup. Section 3 contains the analysis of the game and Section 4 concludes.

2 A Two-period Game Model

Firm E is the leading fashion firm and P is a firm that imitates the design of the product of firm E ². The product of E is sold in markets A ("high-class") and B ("middle-class"), while P sells its imitations in market B only. Thus, the two firms compete in market B , but not in market A . However, the sales volume in market B may influence the image of the designer brand in markets A (for details, see Section 2.3). For the particular product under consideration, we assume that firm E is a monopolist in market A .

2.1 Timing of Decisions

Firm E introduces a new design in market A at each of the time instants $n = 0, 1, 2$. One incentive for the continual introduction of new designs is the fact that designs will be copied by imitating firms, but this is not the main objective. Coming up continually with new designs is the *raison d'être* of fashion firms.

The game terminates at time $n = 2$. When a new design is introduced, the existing design is withdrawn from markets A and B . A design having been introduced in market A at time $n \in \{0, 1\}$ is removed from market A and introduced on market B at time t^n . Thus, at any instant of time there is only one design of firm E in the market.

Time instants t^n are decision variables of firm E and must satisfy the constraints

$$n + \kappa_E \leq t^n \leq n + 1, \quad (1)$$

where $\kappa_E \in]0, 1[$ is a constant. The interpretation of the left-hand inequality in (1) is the design original will be exclusive to the buyers on market A from time n to time t^n , and that firm E guarantees its buyers in market A that the product will be sold exclusively in that market at least until time $n + \kappa_E$. (Clearly, an imitation may appear in market B before time t^n).

²The letters designating the firms are borrowed from pursuit-evasion games: " E " for evader, " P " for pursuer. Firm P can also represent a fringe of homogeneous firms.

Note that we have assumed that a new design remain in market A at least for a time κ_E , in both periods.

Firm P observes instantaneously a new design introduced by E and starts to prepare the production and marketing of its imitation. Let τ^n denote the time of the introduction of P 's product on market B in period n and impose the constraint

$$n + \kappa_P \leq \tau^n \leq n + 1,$$

where $\kappa_P \in [0, 1[$ is a constant. Thus, the earliest time the imitation can be introduced is at $n + \kappa_P$. If $\kappa_P > 0$, the interpretation is that the imitator needs some time to prepare to market its product. If $\kappa_P = 0$, the imitator has the option of introducing its product simultaneously with the new design³. Note that we have assumed that the earliest time to introduce the imitation is the same in both periods.

For the earliest introduction times κ_E and κ_P we assume $\kappa_P < \kappa_E$. This means that the imitator could introduce its product before the time where the design original can be moved from market A to B ⁴.

2.2 Demand Functions

Let "period n " be the time interval from time n to time $n + 1$. In what follows, a superscript n indicates a period number. To specify the demand functions in the three markets we need some notation. All demands defined below are deterministic and should be interpreted as "demand rates per unit of time".

Demand	Price	Market	Product	Period
Q_A^n	p_A^n	A	original	monopoly
Q_{BE}^n	p_{BE}^n	B	original	duopoly
Q_{BMP}^n	p_{BMP}^n	B	imitation	monopoly
Q_{BP}^n	p_{BP}^n	B	imitation	duopoly

We assume that the price p_A^n of the original in market A is fixed, at some (high) level, \bar{p} . This level is the standard profit-maximizing price of a monopolist firm.

Let X_A^n and X_B^n represent the brand image of the design original, as assessed by consumers in markets A and B , respectively. Our hypothesis is that consumers in these markets have different perceptions of the brand

³Copies could actually be on the market *before* the original is for sale. Magdo (2000) writes that "as more advanced technology makes it possible to see high-quality copies appear in stores before the original has even hit the market" (Magdo (2000, p. 1)). Note that such a situation is not possible in our framework.

⁴Otherwise, firm E could have a monopoly period in market B which seems less plausible, in view of the speed at which imitations of new designs appear on the market.

and are influenced by the prices charged by the two firms. Demand rates are specified as follows:

$$Q_A^n = \eta X_A^n \quad (2)$$

$$Q_{BE}^n = \alpha_{BE} X_B^n - \theta_E p_{BE}^n - \gamma (p_{BE}^n - p_{BP}^n) \quad (3)$$

$$Q_{BMP}^n = \alpha_{BMP} X_B^n - \delta p_{BMP}^n \quad (4)$$

$$Q_{BP}^n = \alpha_{BP} X_B^n - \theta_P p_{BP}^n + \gamma (p_{BE}^n - p_{BP}^n) \quad (5)$$

in which $\eta, \alpha_{BE}, \theta_E, \gamma, \alpha_{BMP}, \alpha_{BP}, \theta_P$, and δ are positive constants.

The assumption in (2) is that demand in market A , in which firm E is a monopolist, depends on the brand image only. Buyers in this market are willing to pay, and pays, the high price \bar{p} , but their demand diminishes with their assessment of the brand image.

Equations (3) and (5) mean that the demand of both firms increase with the brand image during the duopoly period in market B . The α 's being different means that buyers of the original and the imitation may react differently to their perceptions of the brand image. The second term on the right-hand side reflects the direct influence of a firm's own price on its demand. Here, a plausible assumption could be $\theta_E < \theta_P$, that is, those customers who buy the fashion product are less sensitive to a change in product price than those who buy the imitation. The third term reflects that consumers also react on the "price differential" $p_{BE}^n - p_{BP}^n$ between the two products. Thus, *ceteris paribus*, consumers buy the product with the lowest price.

The specification in (4) means that the demand in market B , during the monopoly period of firm P , depends on the consumers' assessment of the brand and the price of the imitation.

Using the demand functions introduced above, we define the sales functions

$$S_A^n = (t^n - n) Q_A^n \quad (6)$$

$$S_{BE}^n = (n + 1 - t^n) Q_{BE}^n \quad (7)$$

$$S_{BP}^n = (t^n - \tau^n) Q_{BMP}^n + (n + 1 - t^n) Q_{BP}^n. \quad (8)$$

Equation (6) says that sales of firm E in market A are the sales rate Q_A^n over the time interval from time n to time t^n where the designer product is removed from market A . Equation (7) means that sales of firm E in market B are the sales rate Q_{BE}^n over the remaining period from t^n to $n + 1$. The specification in (8) says that the sales of firm P in market B are the sales rate Q_{BMP}^n during the monopoly period (from τ^n to t^n), plus the sales rate Q_{BP}^n over the duopoly period.

2.3 Brand Images

Brand images X_A^n and X_B^n can be seen as the combined set of beliefs (perceptions) about the brand of firm E , held by buyers in markets A and B , respectively. These perceptions influence demand in markets A and B . Brand images are supposed to evolve over time according to

$$X_A^{n+1} = X_A^n + \beta_A u_A^n - \rho_A [S_{BE}^n + S_{BP}^n]; \quad X_A^0 \text{ fixed} \quad (9)$$

$$X_B^{n+1} = X_B^n + \beta_B u_B^n; \quad X_B^0 \text{ fixed.} \quad (10)$$

In (9) and (10), $\beta_A > 0, \beta_B \geq 0, \rho_A > 0$ are constants and $u_A^n \geq 0, u_B^n \geq 0$ are efforts (e.g., image advertising) made by firm E to improve its goodwill in markets A and B .

The interpretation of (9) is as follows. The brand image in market A dilutes in proportion to the sales in market B . As customers in market A observe an increasing number of imitations being around, their image of the brand is negatively affected. This effect can be modified, however, if the fashion firm spends image building efforts ($u_A^n > 0$).

A driving force behind the demand for highly priced fashion products is the "snob" effect (Leibenstein (1950)). This effect refers to the phenomenon that some consumers' demand for a product decreases because others are consuming it. In our model, the snob effect is not modeled directly in the demand function in market A , but affects demand indirectly due to the brand image dynamics in market A .

The assumption in (10) is that the sales volume in market B does not affect (negatively) market B consumers' perception of the brand; consumers in this market do not care about the number of products (original or fake) being around.

If the fashion firm wishes to have a strong brand image, the dynamics in (9) suggest that this aim can be accomplished by (i) trying to reduce the sales of imitations and (ii) advertises to build up the brand image. Equation (9) shows that the fashion firm's *own sales* in market B are also detrimental to brand integrity (as assessed by market A customers). Firm E faces a dilemma here: It knows that its designs will be imitated, but since E cannot fight the imitator P in market A (where P 's product is not sold), firm E faces its competitor in market B . This will, however, tend to dilute the brand image in market A .

Inserting from the sales functions into (9) and (10) yields

$$X_A^{n+1} = X_A^n + \beta_A u_A^n - \rho_A (t^n - \tau^n) [\alpha_{BMP} X_B^n - \delta p_{BMP}^n] - \rho_A (n+1 - t^n) [(\alpha_{BE} + \alpha_{BP}) X_B^n - \theta_{EP} p_{BE}^n - \theta_{PP} p_{BP}^n] \quad (11)$$

$$X_B^{n+1} = X_B^n + \beta_B u_B^n. \quad (12)$$

Eqs. (11) and (12) show that having a good image in market B actually hurts the image in market A : the larger X_B^n , the smaller X_A^{n+1} . On the other hand, having a good brand image in market B stimulates demand for the product of firm E in this market - but also the demand of the imitator.

The reader should notice the implicit assumption made in (11) and (12): X_B is not affected by X_A . One may, more realistically, suppose that X_B is increasing with X_A , which means that consumers in market B follow consumers in market A in their regards for the fashion product. Then one should add some increasing function of X_A on the right-hand side of (12). We are indebted to an anonymous reviewer for making this observation. However, the suggested change in the dynamics (12) makes the problem considerably more complicated. In an analysis of the modified problem we did not succeed in obtaining any interpretable analytical results.

2.4 Cost Functions

Before the game is played, firm E is assumed to have decided to introduce new designs at the start of each of the two time periods. The implication is that E 's costs of developing new designs are sunk and can be disregarded. The variable production costs of firm E are denoted C_E^n and depend on production (sales) volume only. Assuming a linear cost function we have

$$C_E^n = c_E [S_A^n + S_{BE}^n],$$

in which c_E is a positive constant. For simplicity it is assumed that goods produced for markets A and B have the same unit production cost.

The production costs of firm P are denoted C_P^n . These costs depend on the total production (sales) volume:

$$C_P^n = c_P S_{BP}^n,$$

in which c_P is a positive constant.

Denote by K_E^n the costs of image building efforts. These costs are given by

$$K_E^n = \frac{1}{2} [k_A (u_A^n)^2 + k_B (u_B^n)^2]$$

in which k_A and k_B are positive constants. The quadratic costs reflect a hypothesis that brand image building activities are subject to diminishing marginal returns.

2.5 Profit Functions

To save a bit on notation, define the constant $\bar{\eta} \triangleq (\bar{p} - c_E)\eta$. The profit of firm E in period n is given by

$$\begin{aligned} \pi_E^n = & -\frac{1}{2} [k_A (u_A^n)^2 + k_B (u_B^n)^2] + \bar{\eta}(t^n - n)X_A^n + \\ & (p_{BE}^n - c_E)(n + 1 - t^n) [\alpha_{BE}X_B^n - \theta_{EP}p_{BE}^n - \gamma(p_{BE}^n - p_{BP}^n)], \end{aligned}$$

and that of firm P is

$$\begin{aligned} \pi_P^n = & (p_{BMP}^n - c_P)(t^n - \tau^n)[\alpha_{BMP}X_B^n - \delta p_{BMP}^n] + \\ & (p_{BP}^n - c_P)(n + 1 - t^n)[\alpha_{BP}X_B^n - \theta_P p_{BP}^n + \gamma(p_{BE}^n - p_{BP}^n)]. \end{aligned}$$

In period n , firm E has decision variables $u_A^n, u_B^n, t^n, p_{BE}^n$ and firm P has decision variables $\tau^n, p_{BMP}^n, p_{BP}^n$. Decisions of the two firms are made simultaneously and independently.

It is straightforward to demonstrate that firm P will always choose its entry time τ^n as the minimal value, $n + \kappa_P$. The reason is that the imitator gains nothing by postponing the introduction of its product. Hence, in what follows we set $\tau^n = n + \kappa_P$ for all n .

2.6 Behavioral Assumptions

The imitator's decisions influence the brand image X_A , but the latter does not affect the payoff of the imitator. For this reason the imitator can disregard X_A (and therefore also its dynamics). On the other hand, although the brand image X_B is payoff-relevant for the imitator, the firm knows that its decisions have no influence upon X_B . Consequently, the imitator can disregard X_B (and its dynamics). The upshot is that the imitating firm will act *myopically*, i.e., the firm makes its decisions on a period-by-period basis. The reader should notice that the imitator's myopism is a result of the structure of the game and hence we cannot foresee what would happen if the imitator did not behave myopically. To do so one would need a model in which the rational behavior of the imitator would not be myopic. Then one could suppose that the imitator was irrational and behaved myopically.

State variables X_A and X_B are payoff-relevant to firm E , and the firm's decisions influence the dynamics of both variables. Thus, if it wishes, firm E can condition its actions upon the state variables. Since the strategy of the imitator at most will depend on time, the fashion firm can without loss restrict itself to a strategy that depends on time only (Fudenberg and Tirole (1992, p. 530).

The profit function of the fashion firm E is given by

$$\Pi_E = \sum_{n=0}^{N-1} (i_E)^n \pi_E^n + (i_E)^N [\sigma_A X_A^N + \sigma_B X_B^N]$$

where $N = 2$, $i_E \in (0, 1]$ is a discount factor and σ_A, σ_B are positive constants. The term in square brackets is a salvage value function which assesses the value to the fashion firm of having brand images X_A^N and X_B^N at the end of the planning horizon.

The imitating firm P maximizes, period-by-period, its profit π_P^n for $n \in \{0, 1\}$.

3 Analysis of the Game

An equilibrium of the two-period game is defined by a pair of decision paths

$$\{\hat{u}_A^n, \hat{u}_B^n, \hat{t}^n, \hat{p}_{BE}^n\}_{n \in \{0,1\}}, \{\hat{p}_{BMP}^n, \hat{p}_{BP}^n\}_{n \in \{0,1\}}$$

such that each firm's decisions maximize its objective while taking the decision path of the other firm as given.

Firm E determines at time zero its decision path so as to maximize

$$\begin{aligned} \Pi_E = & -\frac{1}{2} [k_A(u_A^0)^2 + k_B(u_B^0)^2] + \bar{\eta}t^0 X_A^0 + \\ & (p_{BE}^0 - c_E) (1 - t^0) [\alpha_{BE}X_B^0 - \theta_E p_{BE}^0 - \gamma (p_{BE}^0 - p_{BP}^0)] + \\ & i_E \left\{ -\frac{1}{2} [k_A(u_A^1)^2 + k_B(u_B^1)^2] + \bar{\eta}(t^1 - 1)X_A^1 + \right. \\ & \left. (p_{BE}^1 - c_E) (2 - t^1) [\alpha_{BE}X_B^1 - \theta_E p_{BE}^1 - \gamma (p_{BE}^1 - p_{BP}^1)] \right\} + \\ & (i_E)^2 \{ \sigma_A X_A^2 + \sigma_B X_B^2 \}, \end{aligned}$$

subject to the brand image dynamics

$$\begin{aligned} X_A^{n+1} = & X_A^n + \beta_A u_A^n - \rho_A (t^n - k_P^n) [\alpha_{BMP} X_B^n - \delta p_{BMP}^n] - \\ & \rho_A (n + 1 - t^n) [(\alpha_{BE} + \alpha_{BP}) X_B^n - \theta_E p_{BE}^n - \theta_P p_{BP}^n] \end{aligned}$$

$$X_B^{n+1} = X_B^n + \beta_B u_B^n,$$

where $n \in \{0, 1\}$ and X_A^0, X_B^0 are given.

At time zero, firm P determines its decisions $\{\hat{p}_{BMP}^0, \hat{p}_{BP}^0\}$ so as to maximize

$$\begin{aligned} \pi_P^0 = & (p_{BMP}^0 - c_P) (t^0 - \kappa_P) [\alpha_{BMP} X_B^0 - \delta p_{BMP}^0] + \\ & (p_{BP}^0 - c_P) (1 - t^0) [\alpha_{BP} X_B^0 - \theta_P p_{BP}^0 + \gamma (p_{BE}^0 - p_{BP}^0)], \end{aligned}$$

in which X_A^0, X_B^0 are given. At time one, firm P determines its decisions $\{\hat{p}_{BMP}^1, \hat{p}_{BP}^1\}$ to maximize

$$\begin{aligned} \pi_P^1 = & (p_{BMP}^1 - c_P) (t^1 - (1 + \kappa_P)) [\alpha_{BMP} X_B^1 - \delta p_{BMP}^1] + \\ & (p_{BP}^1 - c_P) (2 - t^1) [\alpha_{BP} X_B^1 - \theta_P p_{BP}^1 + \gamma (p_{BE}^1 - p_{BP}^1)], \end{aligned}$$

in which X_A^1, X_B^1 are given.

3.1 Period 1 Equilibrium Decisions

Prices of the imitation in market B are

$$\hat{p}_{BMP}^1 = \frac{1}{2\delta} [\alpha_{BMP} X_B^1 + \delta c_P], \quad (13)$$

$$\hat{p}_{BP}^1 = \frac{1}{2(\gamma + \theta_P)} [\alpha_{BP} X_B^1 + \gamma (c_P + p_{BE}^1) + c_P \theta_P]. \quad (14)$$

Advertising efforts of the fashion firm are

$$\hat{u}_A^1 = \frac{i_E \sigma_A \beta_A}{k_A}, \quad \hat{u}_B^1 = \frac{i_E \sigma_B \beta_B}{k_B} \quad (15)$$

The results in (15) say that the marginal advertising cost in period 1 equals the (one-period) discounted value of the marginal increase in the salvage value of the brand image⁵. Efforts decrease if costs increase and increase if efficiency β and/or the salvage value σ increase. Efforts also increase if the firm becomes more far-sighted ($i_E \rightarrow 1$). These results are as expected.

The price of the fashion product in market B is

$$\hat{p}_{BE}^1 = \frac{1}{2(\gamma + \theta_E)} [\alpha_{BE} X_B^1 + \gamma (c_E + p_{BP}^1) + c_E \theta_E + i_E \theta_E \rho_A \sigma_A]. \quad (16)$$

Eqs. (14) and (16) show that the price of one product increases if the price of the other increases. This is the standard result in a pricing game.

Solving (14) and (16) yields the duopoly prices, expressed as functions of X_B^1 :

$$\begin{aligned} \hat{p}_{BE}^1 &= \frac{1}{3\gamma^2 + 4\gamma(\theta_E + \theta_P) + 4\theta_E \theta_P} \{2c_E(\gamma + \theta_E)(\gamma + \theta_P) + \\ &\quad c_P \gamma(\gamma + \theta_P) + 2i_E \theta_E \rho_A \sigma_A (\gamma + \theta_P) + [2\alpha_{BE}(\gamma + \theta_P) + \gamma \alpha_{BP}] X_B^1\} \\ \hat{p}_{BP}^1 &= \frac{1}{3\gamma^2 + 4\gamma(\theta_E + \theta_P) + 4\theta_E \theta_P} \{2c_P(\gamma + \theta_E)(\gamma + \theta_P) + \\ &\quad c_E \gamma(\gamma + \theta_E) + i_E \theta_E \rho_A \sigma_A \gamma + [2\alpha_{BP}(\gamma + \theta_E) + \gamma \alpha_{BE}] X_B^1\}. \end{aligned} \quad (17)$$

The higher the brand image X_B^1 at the start of period 1, the higher the prices. The idea is that the the negative impact on demand of a high price can be offset by a strong brand image. We note that although the imitating firm does not care about the brand image, it enjoys the benefits of a strong image.

The time to move the fashion original from market A to B is determined by the sign of the derivative

$$\begin{aligned} \frac{\partial \Pi_E}{\partial t^1} &= i_E \{ \bar{\eta} X_A^1 - (\hat{p}_{BE}^1 - c_E) [\alpha_{BE} X_B^1 - \theta_E \hat{p}_{BE}^1 - \gamma (\hat{p}_{BE}^1 - \hat{p}_{BP}^1)] \} + \\ &\quad (i_E)^2 \sigma_A \rho_A [-(\alpha_{BMP} X_B^1 - \delta p_{BMP}^1) + (\alpha_{BE} + \alpha_{BP}) X_B^1 - \\ &\quad (\theta_E \hat{p}_{BE}^1 + \theta_P \hat{p}_{BP}^1)] \end{aligned} \quad (18)$$

⁵If there were no salvage values, firm E would have no incentive to increase the brand images in period 1, and no advertising should be done in that period.

and we have

$$\frac{\partial \Pi_E}{\partial t^1} \left\{ \begin{array}{l} > \\ = \\ < \end{array} \right\} 0 \iff t^1 \left\{ \begin{array}{l} = 2 \\ \in [1 + \kappa_E, 2] \\ 1 + \kappa_E \end{array} \right\}. \quad (19)$$

The result in (19) means that the fashion original should be moved from market A to market B either as early or as late as possible. Clearly, this result is an extreme one and excludes a fine-tuning of the decision t^1 . It is due to the linearity of the profit function with respect to t^1 .

The derivative in (18) exhibits three effects of an increase in t^1 :

- (1) $\bar{\eta} X_A^1$ is the increased profit earned on market A in period 1
- (2) $-(\hat{p}_{BE}^1 - c_E) [\alpha_{BE} X_B^1 - \theta_E \hat{p}_{BE}^1 - \gamma (\hat{p}_{BE}^1 - \hat{p}_{BP}^1)]$ is the loss of profit in market B in period 1
- (3) $i_E^2 \sigma_A \rho_A [-(\alpha_{BMP} X_B^1 - \delta p_{BMP}^1) + (\alpha_{BE} + \alpha_{BP}) X_B^1 - (\theta_E \hat{p}_{BE}^1 + \theta_P \hat{p}_{BP}^1)]$ is the present value of the gain (or loss) of salvage value of X_A^2 at the end of period 1.

For a sufficiently large value of the brand image X_A^1 , the derivative in (18) is positive and the product is never introduced on market B . This is intuitive since when the brand has a strong image in market A it is more profitable to keep the product in this market as long as possible. On the other hand, if the lost profit in market B is sufficiently large, the product should be moved to market B as early as possible.

Note that

$$\begin{aligned} & \rho_A [-(\alpha_{BMP} X_B^1 - \delta p_{BMP}^1) + (\alpha_{BE} + \alpha_{BP}) X_B^1 - (\theta_E \hat{p}_{BE}^1 + \theta_P \hat{p}_{BP}^1)] \\ &= \frac{\partial X_A^2}{\partial t^1} \end{aligned}$$

is the effect on the final brand image X_A^2 of keeping the product on market A for a marginally longer time. This effect can be positive or negative: if there is a sufficiently large gain of salvage value, it pays to keep the product in market A throughout period 1. On the other hand, if there is a large loss of salvage value, it pays to move the product to market B as soon as possible. If this effect is positive and sufficiently large, the product should be on market A throughout period 1.

Remark 3.1. For another interpretation, suppose that $\theta_E = \theta_P = 0^6$. Then, if $\alpha_{BE} + \alpha_{BP} > \alpha_{BMP}$, we have

$$-(\alpha_{BMP} X_B^1 - \delta p_{BMP}^1) + (\alpha_{BE} + \alpha_{BP}) X_B^1 > 0,$$

that is, there is a gain of salvage value.

⁶This means that the demand functions in the duopoly period in market B depend on the price differential $\gamma (p_{BE}^1 - p_{BP}^1)$ only.

3.2 Period 0 Equilibrium Decisions

Prices of the imitation are structurally equivalent to that in period 1:

$$\hat{p}_{BMP}^0 = \frac{1}{2\delta} [\alpha_{BP}X_B^0 + \delta c_P] \quad (20)$$

$$\hat{p}_{BP}^0 = \frac{1}{2(\gamma + \theta_P)} [\alpha_{BP}X_B^0 + \gamma (p_{BE}^0 + c_P) + c_P\theta_P] \quad (21)$$

Advertising effort in market A is

$$\hat{u}_A^0 = \frac{i_E\beta_A [(\hat{t}^1 - 1)\bar{\eta} + i_E\sigma_A]}{k_A}. \quad (22)$$

The result says that the marginal advertising cost $k_A\hat{u}_A^0$ is the sum of the present value of the marginal revenue obtained in market A in period 1, $i_E\beta_A(\hat{t}^1 - 1)\bar{\eta}$, and the discounted marginal salvage value, $(i_E)^2\beta_A\sigma_A$, of the terminal brand image X_A^2 .

Sensitivity results are, by and large, the same as for \hat{u}_A^1 , but we note the additional term $(\hat{t}^1 - 1)\bar{\eta}$. This means that there is a positive relationship between advertising effort in market A in period 0 and the length $\hat{t}^1 - 1$ of the period during which the product is sold in market A in period 1. If the product is sold for a longer time in market A in period 1, more advertising effort should be used in period 0. The intuition is that then it pays to build up the brand image X_A^1 on which demand in period 1 will depend.

Advertising effort in market B is

$$\hat{u}_B^0 = \frac{i_E\beta_B(2 - \hat{t}^1)(\hat{p}_{BE}^1 - c_E)\alpha_{BE}}{k_B} + \frac{(i_E)^2\beta_B}{k_B} \{ \sigma_B - \sigma_A\rho_A [(\hat{t}^1 - (1 + \kappa_P))\alpha_{BMP} + (2 - \hat{t}^1)(\alpha_{BE} + \alpha_{BP})] \}. \quad (23)$$

Using (12) and (17) shows that \hat{u}_B^0 is linearly increasing in X_B^0 . What drives this result is the fact that brand image has a positive impact on demand. A similar result as that in (23) occurs in, e.g., Jørgensen et al. (2001) where advertising also increases with the brand image (although at a decreasing rate).

Remark 3.2. Note that brand image X_B^0 has two impacts: a high level of X_B^0 provides, by the brand image dynamics in (12), a high level of X_B^1 , but it also implies a high level of effort u_B^0 (cf. (23)) which increases X_B^1 .

The price of the fashion original is

$$\hat{p}_{BE}^0 = \frac{1}{2(\gamma + \theta_E)} [\alpha_{BE}X_B^0 + \gamma (c_E + p_{BP}^0) + \theta_E c_E + i_E\theta_E\rho_A (\bar{\eta}(t^1 - 1) + i_E\sigma_A)]. \quad (24)$$

Solving (21) and (24) provides the duopoly prices, expressed as functions of X_B^0 :

$$\begin{aligned}\hat{p}_{BE}^0 &= \frac{1}{3\gamma^2 + 4\gamma(\theta_E + \theta_P) + 4\theta_E\theta_P} \{2c_E(\gamma + \theta_E)(\gamma + \theta_P) + c_P\gamma(\gamma + \theta_P) + \\ &\quad 2(\gamma + \theta_P)i_E\theta_E\rho_A [\bar{\eta}(\hat{t}^1 - 1) + i_E\sigma_A] + \\ &\quad [2\alpha_{BE}(\gamma + \theta_P) + \gamma\alpha_{BP}] X_B^0\} \\ \hat{p}_{BP}^0 &= \frac{1}{3\gamma^2 + 4\gamma(\theta_E + \theta_P) + 4\theta_E\theta_P} \{2c_P(\gamma + \theta_E)(\gamma + \theta_P) + c_E\gamma(\gamma + \theta_E) + \\ &\quad \gamma i_E\theta_E\rho_A [\bar{\eta}(\hat{t}^1 - 1) + i_E\sigma_A] + \\ &\quad [2\alpha_{BP}(\gamma + \theta_E) + \gamma\alpha_{BE}] X_B^0\}.\end{aligned}\quad (25)$$

The time to move the fashion original to market B is determined by the sign of the derivative

$$\begin{aligned}\frac{\partial \Pi_E}{\partial t^0} &= \bar{\eta}X_A^0 - (\hat{p}_{BE}^0 - c_E) [\alpha_{BE}X_B^0 - \theta_E\hat{p}_{BE}^0 - \gamma(\hat{p}_{BE}^0 - \hat{p}_{BP}^0)] + \\ &\quad i_E\rho_A [\bar{\eta}(\hat{t}^1 - 1) + i_E\sigma_A] [-(\alpha_{BMP}X_B^0 - \delta p_{BMP}^0) + \\ &\quad (\alpha_{BE} + \alpha_{BP})X_B^0 - (\theta_E\hat{p}_{BE}^0 + \theta_P\hat{p}_{BP}^0)]\end{aligned}\quad (26)$$

and we have

$$\frac{\partial \Pi_E}{\partial t^0} \begin{cases} > \\ = \\ < \end{cases} 0 \iff \hat{t}^0 \begin{cases} = 1 \\ \in [\kappa_E, 1] \\ \kappa_E \end{cases}.$$

As in period 1, the fashion product should be moved from market A to market B as early or as late as possible. The derivative in (26) exhibits four effects of an increase in t^0 (that is, by extending marginally the period during which the product is kept on market A in period 0):

- (1) $\bar{\eta}X_A^0$ is the increased profit earned in market A in period 0
- (2) $-(\hat{p}_{BE}^0 - c_E) [\alpha_{BE}X_B^0 - \theta_E\hat{p}_{BE}^0 - \gamma(\hat{p}_{BE}^0 - \hat{p}_{BP}^0)]$ is the loss of profit in market B in period 0
- (3) $i_E\rho_A\bar{\eta}(\hat{t}^1 - 1) [-(\alpha_{BMP}X_B^0 - \delta p_{BMP}^0) + (\alpha_{BE} + \alpha_{BP})X_B^0 - (\theta_E\hat{p}_{BE}^0 + \theta_P\hat{p}_{BP}^0)]$ is the (present value of the) effect on profit in market A in period 1
- (4) $i_E^2\sigma_A\rho_A [-(\alpha_{BMP}X_B^0 - \delta p_{BMP}^0) + (\alpha_{BE} + \alpha_{BP})X_B^0 - (\theta_E\hat{p}_{BE}^0 + \theta_P\hat{p}_{BP}^0)]$ is the present value of the gain (or loss) of salvage value of X_A^2 .

Effects 1, 2 and 4 have the same interpretations as in period 1. As to the third effect, note that

$$\begin{aligned}\frac{\partial X_A^1}{\partial t^0} &= i_E\rho_A [-(\alpha_{BMP}X_B^0 - \delta p_{BMP}^0) + (\alpha_{BE} + \alpha_{BP})X_B^0 - \\ &\quad (\theta_E\hat{p}_{BE}^0 + \theta_P\hat{p}_{BP}^0)]\end{aligned}$$

is the effect on brand image X_A^1 of keeping the product on market A for a (marginally) longer time during period 0. If this effect is positive and sufficiently large, the product should be on market A throughout period 0.

3.3 Comparing Decisions Over Time

We shall need the following

ASSUMPTION 1: $\theta_E = 0$.

The assumption means that buyers of the fashion product in market B react on price differences only (and not on the absolute price of the fashion product).

Using (17) and (25), invoking Assumption 1, shows that

$$\hat{p}_{BE}^1 \geq \hat{p}_{BE}^0, \quad \hat{p}_{BP}^1 \geq \hat{p}_{BP}^0 \quad (27)$$

which means that duopoly prices are nondecreasing over time. Actually, for $\hat{u}_B^0 > 0$ they increase and for $\hat{u}_B^0 = 0$ they are constant over time. Thus, by its advertising expenditure in period 0, the fashion firm can influence the price development in the duopoly period in market B . The result is driven by the fact that $X_B^1 = X_B^0 + \beta_B \hat{u}_B^0$ and that duopoly prices in period 1 depend on X_B^1 .

Since the brand image X_B is nonincreasing, one obtains from (13) and (20)

$$\hat{p}_{BMP}^1 \geq \hat{p}_{BMP}^0.$$

The imitator increases its price [keeps it constant] in the monopoly period in market B if $\hat{u}_B^0 > 0$ [$\hat{u}_B^0 = 0$].

To compare the fashion firm's advertising efforts over time we need

ASSUMPTION 2: $i_E = 1$.

The assumption says that the fashion firm does not discount the future. It is easy to see from (15) and (22) that

$$\hat{u}_A^0 > \hat{u}_A^1$$

Advertising efforts are decreasing over time in market A . The reason is that advertising in period 0 has two impacts: to increase the image X_A^1 and to increase the salvage value. Advertising in period 1 only affects the salvage value.

Using (15) and (23) provides

$$\hat{u}_B^0 \left\{ \begin{array}{l} > \\ = \\ < \end{array} \right\} \hat{u}_B^1 \iff (2 - \hat{t}^1) (\hat{p}_{BE}^1 - c_E) \alpha_{BE} \left\{ \begin{array}{l} > \\ = \\ < \end{array} \right\} \sigma_A \rho_A [(\hat{t}^1 - (1 + \kappa_P)) \alpha_{BMP} + (2 - \hat{t}^1) (\alpha_{BE} + \alpha_{BP})] \quad (28)$$

Suppose that $\hat{t}^1 = 2$, i.e., the product is kept in market A throughout period 1. Then (28) yields $\hat{u}_B^0 < \hat{u}_B^1$. Thus, advertising in market B is *increased* in period 1, despite the fact that the product is kept in market A throughout period 1. This may seem counterintuitive. What provides the result is the positive term $\sigma_A \rho_A (2 - (1 + \kappa_P)) \alpha_{BMP}$ which is the loss of salvage value of X_A^2 caused by the imitator is having a monopoly in market B throughout period 1.

The time to move the fashion product from market A to B are given by the signs of the derivatives

$$\begin{aligned} \frac{\partial \Pi_E}{\partial t^0} = & \bar{\eta} X_A^0 - (\hat{p}_{BE}^0 - c_E) [\alpha_{BE} X_B^0 - \theta_E \hat{p}_{BE}^0 - \gamma (\hat{p}_{BE}^0 - p_{BP}^0)] + \\ & i_E \rho_A [\bar{\eta} (\hat{t}^1 - 1) + i_E \sigma_A] [-(\alpha_{BMP} X_B^0 - \delta p_{BMP}^0) + \\ & (\alpha_{BE} + \alpha_{BP}) X_B^0 - (\theta_E \hat{p}_{BE}^0 + \theta_P p_{BP}^0)] \end{aligned} \quad (29)$$

$$\begin{aligned} \frac{\partial \Pi_E}{\partial t^1} = & i_E \{ \bar{\eta} X_A^1 - (\hat{p}_{BE}^1 - c_E) [\alpha_{BE} X_B^1 - \theta_E \hat{p}_{BE}^1 - \gamma (\hat{p}_{BE}^1 - \hat{p}_{BP}^1)] \} + \\ & (i_E)^2 \sigma_A \rho_A [-(\alpha_{BMP} X_B^1 - \delta p_{BMP}^1) + (\alpha_{BE} + \alpha_{BP}) X_B^1 - \\ & (\theta_E \hat{p}_{BE}^1 + \theta_P p_{BP}^1)]. \end{aligned} \quad (30)$$

The right-hand side of (29) shows that the derivative $\partial \Pi_E / \partial t^0$ depends on the time t^1 at which the product is moved from A to B in period 1. The larger the t^1 , the larger the value of $\partial \Pi_E / \partial t^0$. This means that if the product is moved late (or never) to market B in period 1, the likelihood increases that the product is moved late (or never) to market B in period 0.

The signs of the derivatives in (29) and (30) provide four cases:

	$\frac{\partial \Pi_E}{\partial t^1} > 0$	$\frac{\partial \Pi_E}{\partial t^1} < 0$
$\frac{\partial \Pi_E}{\partial t^0} > 0$	(a): $t^0 = 1, t^1 = 2$	(b): $t^0 = 1, t^1 = 1 + \kappa_E$
$\frac{\partial \Pi_E}{\partial t^0} < 0$	(c): $t^0 = \kappa_E, t^1 = 2$	(d): $t^0 = \kappa_E, t^1 = 1 + \kappa_E$

In Case (a), the fashion product is never introduced on market B . In Case (d), the product is always introduced on market B as early as possible. In Case (b), the product is not introduced on market B in period 0, but is put on market B as early as possible in period 1. In Case (c), the situation is the opposite: the product is put on market B as early as possible in period 0, but is not introduced on market B in period 1.

Next, we study the derivative in (30) as a function of X_B^1 . For this purpose we need

ASSUMPTION 3: $\theta_E = \theta_P = 0$

Define

$$\frac{\partial \Pi_E}{\partial t^1} = f(X_B^1)$$

and recall that equilibrium prices p_{BE}^1, p_{BP}^1 , and p_{BMP}^1 are functions of X_B^1 . Differentiation in (30) with respect to X_B^1 yields

$$f'(X_B^1) \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } X_B^1 \begin{cases} < \\ = \\ > \end{cases} \Gamma \quad (31)$$

where

$$\Gamma = \frac{1}{2\alpha_{BE} + \alpha_{BP}} \left[2\gamma(c_E - c_P) + \frac{9\gamma i_E \sigma_A \rho_A (\alpha_{BE} + \alpha_{BP} - 0.5\alpha_{BMP})}{2(2\alpha_{BE} + \alpha_{BP})} \right]$$

is a constant which is positive under

ASSUMPTION 4: (i) $c_E > c_P$, (ii) $\alpha_{BE} + \alpha_{BP} \geq 0.5\alpha_{BMP}$.

Item (i) of the assumption means that the fashion firm has the larger unit production cost. This is very likely. Item (ii) states that the joint impact of brand image X_B on the duopoly demands is not smaller than half the impact of the image on the imitator's monopoly demand in market B . This is also very likely.

Function $f(X_B^1)$ is strictly concave and it holds that

$$f(0) = i_E \left[\bar{\eta} X_A^1 - \frac{\gamma(c_P - c_E)^2}{9} \right] + (i_E)^2 \sigma_A \rho_A \frac{\delta}{2} c_P. \quad (32)$$

The sign of $f(0)$ is not unique and there are three possibilities:

- (1) $f(0) > 0, f(\Gamma) > 0 \implies \exists \alpha_1 > \Gamma$ such that $f(\alpha_1) = 0$. We have $f(X_B^1) > 0$ for $X_B^1 \in [0, \alpha_1)$, implying $\hat{t}^1 = 2$, and $f(X_B^1) < 0$ for $X_B^1 > \alpha_1$, implying $\hat{t}^1 = 1 + \kappa_E$. Qualitatively speaking, for X_B^1 sufficiently large, it pays to move the fashion product as early as possible to market B . The intuition is clear: The brand image X_B^1 has a large value which the firm exploits by moving the product to market B as soon as possible.
- (2) $f(0) < 0, f(\Gamma) > 0 \implies \exists \alpha_{21} < \Gamma$ and $\alpha_{22} > \Gamma$ such that $f(\alpha_{21}) = f(\alpha_{22}) = 0$. We have $f(X_B^1) < 0$ for $X_B^1 \in [0, \alpha_{21})$, implying $\hat{t}^1 = 1 + \kappa_E$, $f(X_B^1) > 0$ for $X_B^1 \in (\alpha_{21}, \alpha_{22})$, implying $\hat{t}^1 = 2$, and $f(X_B^1) < 0$ for $X_B^1 > \alpha_{22}$, implying $\hat{t}^1 = 1 + \kappa_E$. Here it pays to move the product to market B if the brand image X_B^1 has an intermediate value; if the value is small or large, the product should remain on market A .
- (3) $f(0) < 0, f(\Gamma) < 0 \implies f(X_B^1) < 0$ for all $X_B^1 \geq 0$, implying $\hat{t}^1 = 1 + \kappa_E$ always. It pays to move the product to market B as early as possible, irrespective of the value of the brand image.

Consider (32). A key determinant of the value of $f(0)$ is the brand image X_A^1 . For a sufficiently large value of X_A^1 we are in Case (1). Then, if the brand image in market B is "small" (i.e., below the threshold α_1), the product will not be moved to this market because it is more profitable to exploit the brand image in market A . Note that Case (1) also emerges if $c_P \approx c_E$.

3.4 Comparing Pricing Decisions Across Firms

The only comparison to be made is between the duopoly prices in market B . We invoke Assumptions 1 and 4. Using (17) and (25) then yields

$$\hat{p}_{BE}^i > \hat{p}_{BP}^i \text{ for } i \in \{0, 1\},$$

that is, the fashion original should be higher priced than the imitation. The result seems to confirm what can be observed in real life. What drives the result are Assumption 3 and item (ii) of Assumption 6: When the brand image has a "high" impact on the demand for the fashion product, and the latter is not affected by the product's absolute price, the product can be higher priced.

4 Conclusions

The paper has identified equilibrium strategies for price, advertising, and entry decisions in a two-period game. A fashion company introduces new designs regularly and competes in one market, B , with a myopic imitator who copies the design of the fashion company. The fashion firm has a monopoly market, A , and the imitator may have a monopoly in market B throughout a period if the fashion firm chooses not to move its product from market A to B . Demand in markets A and B depend on the respective brand images. Among our findings are the following

- Equilibrium prices the duopoly period in market B are proportional to the brand image in that market: the stronger the image, the higher the prices
- The time of entry of the fashion firm into market B depends on the strength of its image in markets A and B
- Image advertising efforts of the fashion firm in the first period depends on the time it moves its product from market A to B in period 1. Efforts depends positively on the brand image in the first period.
- Equilibrium prices in the duopoly market decrease over time
- Image advertising efforts in market A are decreasing over time
- The fashion product is higher priced in market B than the imitation.

A possible avenue for future research is to include product quality. Here one could assume that the quality of the fashion product is fixed, but the imitator can decide which quality to manufacture. The better the quality, the longer it will take before copy can be introduced on market B . On the other hand, a better quality stimulates the demand for the imitation. This problem is currently investigated by the authors. Due to the increased complexity of the model, results are derived by numerical simulations.

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