Publicly Financed Education in an Endogenous Growth Model

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Abstract

This paper constructs an endogenous growth model, applicable largely to developing countries, based on human capital accumulation in which education is publicly provided and financed, and schooling is compulsory. Public investment in human and physical capital are financed from taxes on wage and capital income, and consumption. The equilibrium growth properties of the model are examined and the steady-state effects of education and fiscal policy are derived. The specification of the human capital production function and the strength of labour supply effects are shown to be important for the magnitude of steady-state outcomes. Simulations illustrate the model's properties.

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1 Introduction

A central tenet of education policy in developing countries is that expansion of school enrolments is desirable, for reasons of social justice, and because economic prosperity is perceived to be fostered by the accumulation of human capital *via* education.[1] This paper examines the growth effects of human capital investment achieved through publicly-provided, compulsory education, financed from income and consumption taxes. The effects on labour supply of higher tax rates, in addition to general equilibrium effects on wages and prices, are examined.

Given the aims of the paper, a number of assumptions need to be adopted which differ from those in the growth literature. Since Lucas (1988, 1990) education has been extensively examined in the context of models in which individuals allocate their time to education within an inter-temporal utility-maximising framework.[2] However, it is not obvious that this framework is the best way to capture education decisions in most developing countries. The theoretical literature on human capital and growth is generally separate from that on fiscal policy and growth. However, any analysis of the growth impact of state-provided education cannot be conducted independently of the financing implications, as dictated by the government budget constraint.[3]

Section 2 describes the basic structure of the model (with further details regarding dynamics set out in the Appendix). Section 3 examines the effects on equilibrium growth of changes in education policy. Section 4 provides numerical policy simulations to investigate the various policy trade-offs in more detail. Brief conclusions are in Section 5.

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[1] In China for example, where education expansion has been rapid, the government now claims that its nine-year compulsory education programme covers 73% of "opulated area". A problem with many developing countries is that, despite having compulsory education, participation rates are low.

[2] Aghion and Howitt (1998) and Topel (1999) provide reviews of much of this literature.

[3] Models of fiscal policy and growth recognise the government budget constraint, though relatively few examine taxes alongside productivity-enhancing or utility-enhancing public expenditures. Studies include Barro (1990), Barro and Sala-i-Martin (1992), Cashin (1995), Devarajan *et al* (1996) and Capolupo (2000). With the exception of Capolupo, human capital is either excluded or treated as a private decision in these papers. All these papers assume exogenous labour supply. Models of taxation and growth in which government expenditures have no output or utility effects include King and Rebelo, (1990), Rebelo, (1991), Mendoza *et al* (1997), Milesi-Ferretti and Roubini (1998). These allow for endogenous labour supply. For example, Milesi-Ferretti and Roubini (1998) demonstrate that the nature of leisure is important for growth predictions, depending on whether leisure requires raw labour time, quality time (time and human capital inputs) or home production (where physical capital inputs are also required).

2 The Structure of The Model

Modelling fiscal and education policy impacts on growth in an LDC context requires several assumptions that differ from those relevant to a more developed country. The majority of individuals who currently receive education in LDCs do so in public primary or secondary schools, frequently within a compulsory education regime. For those individuals not currently receiving education, this is typically because public education is unavailable rather than because families' utility-maximising calculus leads them to choose lower levels of education.[4]

Governments in LDCs also undertake substantial physical capital investment in the form of infrastructure which may be important for private sector productivity. Indeed, in many LDCs, despite recent market-orientated reforms, substantial commercial and especially investment activity is either undertaken, or controlled at the margin, by government, compared with a typical developed country. The allocation of revenues to public physical capital investment therefore needs to be included in the analysis. However, the model abstracts from private sector investment. This allows the analysis to focus on the issues of primary interest, namely the response of growth to publicly funded and provided human capital investment.[5] It results in a model analogous to Barro (1990) but where output is a function of human capital and public physical capital rather than private and public physical capital. To simplify the exposition, depreciation of human and physical capital is ignored.

Unlike Barro (1990) and similar models, labour supply is endogenous. Though choices between income and leisure may seem less relevant in an LDC context, many individuals face choices between income earned in the taxed sector and income (including subsistence activities) from the untaxed sector, or leisure. In analysing output growth, the present model focuses on labour supplied to the taxed sector and labels all other activities as leisure. In this broader sense, endogenous labour supply choices are relevant in LDCs. Since untaxed activities by an individual involve the application of the same human capital, leisure is modelled, following Mendoza *et al* (1997), as quality time, that is, time augmented by education.

It is assumed that individuals maximise their utility within each period but do not maximise inter-temporally. This reflects both the fact that poor individuals with low levels of education are unlikely to make sophisticated inter-temporal calculations and, since there is no private investment or education in the model, there is little to be gained from adopting an inter-temporal utility maximisation

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[4]However, in some LDC contexts, especially in rural agriculture, families demonstrate a preference not to send their children to school, even when available and compulsory, due to preceptions that this reduces family income.

[5]It is recognised that in some LDCs, education is publicly funded but privately provided, *via* the use of voucher schemes.

framework.[6]

2.1 Production and Investment

The model is a closed-economy general equilibrium model with endogenous labour supply, in which a single individual maximises utility from consumption of a single final good and leisure. The representative individual is initially endowed with raw labour time, N, which may be augmented by education to form human capital, H. In addition, homogeneous physical capital, K, is distributed to the representative individual. This captures the notion that individuals benefit from infrastructure and, as discussed further below, can be taxed on the imputed benefits from that consumption.

Private sector production of the final good, Q, in each period can be used for both consumption and investment. The production function takes the Cobb-Douglas form:[7]

$$Q = A_q(u_q H)^{\alpha_q} K^{1-\alpha_q} \tag{1}$$

where u_q is the proportion of human capital devoted to production of Q.

The government raises revenue from factor and consumption taxes, and spends it on three functions. First, it purchases, at market prices, an amount Q_h used as an input into human capital production together with an appropriated fraction of labour time, u_h. Adopting the Cobb-Douglas form, the production function for human capital is:

$$dH = A_h Q_h^{\alpha_h}(u_h H)^{1-\alpha_h} \tag{2}$$

The inclusion of Q_h in (2) is analogous to the use of physical capital by, for example, King and Rebelo (1990) and Milesi-Ferretti and Roubini (1998).[8]

The government undertakes physical capital investment in the form of a private (as opposed to public) capital good, whereby the government purchases, again at market prices, an amount of final output equal to Q_k.[9] To capture the

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[6]In this respect the present model is analogous to the Solow-Swan model in which savings are a fixed proportion of income. Although private savings are zero here, the private-good nature of public investment and the compulsory nature of educational time inputs, together with tax-financing, ensure that taxation and education are analogous to a compulsory savings proportion determined by government. There is the additional complication of possible tax disincentive effects on labour supply depending on the form of tax used.

[7]Throughout the following analysis, time subscripts are suppressed for convenience.

[8]As in these models, this property (in particular, the value of α_h) is important for the growth effects of fiscal policy. However, unlike these models Q is not defined here exclusively as a capital good and can represent educational inputs of a capital or recurrent nature.

[9]It can be argued that some government investment, for example on infrastructure, should be treated as a public good. However, Barro (1990) and Barro and Sala-i-Martin (1992) argue, with support from empirical evidence, that government expenditures are dominated by goods with quasi-private characteristics and are typically subject to congestion.

benefits of this physical capital to the consumer, the model adopts the device of distributing this to the individual at the start of each period.

2.2 Tax Revenue

Tax revenue not used in the production of human and physical capital is returned to the individual in the form of an untaxed transfer payment, D. This is designed to reflect the fact that much public expenditure in LDCs is more likely to affect consumption than productivity. Total revenue, R, is divided among the three expenditures in proportions θ_j, $(j = k, h, d)$ as follows:

$$
\begin{aligned}
Q_k &= dK = \theta_k R/p \\
Q_h &= \theta_h R/p \\
D &= \theta_d R = (1 - \theta_k - \theta_h)R
\end{aligned}
\tag{3}
$$

where, p is the tax-inclusive consumer price of Q.

The government balances its budget in each period, raising revenue by taxing factor incomes and consumption. As with most education systems in practice, human capital inputs into human capital production are not directly taxed in the model. The implicit income derived from the public capital, distributed to the individual, is taxed. It may be thought more appropriate to treat the private returns from the ownership of such infrastructure capital as untaxed. In practice, however, the consumption of this type of capital, such as road infrastructure, requires the use of private consumption goods (such as vehicles) which are taxed, so that capital is effectively taxed indirectly.[10]

Total tax revenue is given by:

$$
\begin{aligned}
R &= t_r rK + t_w w u_q H + t_c p'Q \\
&= t_r rK + t_w w H_q + t_c p'Q
\end{aligned}
\tag{4}
$$

where t_w, t_r are the proportional income tax rates on gross wage and rental incomes respectively; t_c is the proportional *ad valorem* tax rate on consumption of Q; and w and r are the pre-tax wage rate and rental per unit respectively. In addition, $p' = p/(1 + t_c)$ is the tax-exclusive producer price of Q, and $H_q = u_q H$ is human capital used in the production of Q. For convenience, p is normalised at unity, such that $p' = 1/(1 + t_c)$.

2.3 Consumption and Labour Supply

The representative individual maximises a Cobb-Douglas utility function in each period, expressed in terms of consumption and leisure (the latter defined to

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[10]See Brennan and Buchanan (1980) for explicit modelling of this relationship. They highlight a number of taxes on privately produced consumption goods which *de facto* tax individuals' consumption of publicly provided capital.

include all activities not giving rise to taxable incomes, as discussed above). Define $u_l = 1 - u_h - u_q$ as the proportion of human capital devoted to leisure. Thus, leisure $h = u_l H$ and:

$$U = Q_c^{\alpha_c} h^{1-\alpha_c} \tag{5}$$

where Q_c is the amount of Q consumed by the representative individual. For the reasons given above, this specification (in common with a number of other endogenous growth models) treats leisure as quality time. That is, it involves time and human capital as inputs; see Milesi-Ferretti and Roubini (1998).[11]

It follows that the individual's full income, M, is given by:

$$M = r\left(1 - t_r\right)K + w\left(1 - t_w\right)\left(1 - u_h\right)H + D \tag{6}$$

and, using standard results for Cobb-Douglas utility, the demands for goods and leisure are:[12]

$$Q_c = \alpha_c M/p \tag{7}$$
$$h = \left(1 - \alpha_c\right)M/\left\{w\left(1 - t_w\right)\right\} \tag{8}$$

Finally, the model's adding-up condition, expressed in terms of gross incomes and producer prices, requires that:

$$wu_q H + rK = p'Q \tag{9}$$

This ensures that the total value of output is equal to the total value of factor incomes, consistent with constant returns to scale. In addition, the Cobb-Douglas production function implies that the labour income share is $\alpha_q = wu_q H/pQ$ and $1 - \alpha_q = rK/pQ$ is the share of capital in total income. Physical and human capital endowments expand each period in response to public investment and taxation decisions, though human capital used in production is endogenous via the choice of u_q.

2.4   Basic Properties

Several general features of the model can usefully be stated here. First, since there are no private savings, uniform taxation of both sources of income at rate,

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[11]The human capital endowment can be thought of as raw labour time, N, augmented by education, such that $H = BN$, where in each period B represents the labour-augmenting skills acquired through education (with the initial value of B normalised to 1).

[12]In the absence of private investment, no borrowing or lending is allowed. Also the existence of the transfer payment means that it is possible to have a corner solution where the individual does not work, so (8) applies only if the wage rate exceeds a threshold level.

t_y, is equivalent to a consumption tax at rate $t_c / (1 + t_c)$. This is evident from the adding up constraint in (9) such that:

$$t_y \left(w u_q H + rK \right) = t_c p' Q = \frac{t_c}{1 + t_c} pQ \tag{10}$$

Secondly, the absence of private investment ensures that the usual distorting effects of capital taxes are absent. Physical capital is publicly purchased (out of tax revenues) and is transferred costlessly to the individual. Hence, rental income can be thought of as a transfer which may be taxed at rate t_r which has no direct incentive effects on accumulation. However, a secondary labour supply effect can arise via responses of u_q to changes in the individual's full income; see equation (6).

Thirdly, in addition to the growth-enhancing effects of compulsory savings, there is a growth-inhibiting effect of taxation on labour supply. In analysing the impact of alternative taxes, it is therefore important to separate these effects. Finally, unconditional transfers, like government consumption expenditures in the Barro (1990) and Barro and Sala-i-Martin (1992) models, are utility-enhancing but do not affect private sector production.

The dynamic properties of the model arise from the government's accumulation functions for physical and human capital. The latter acts as labour-augmenting technical progress, raising effective endowments of human capital. Since education and physical investment are funded from tax revenue, the dynamics are crucially determined by tax revenue, output and factor inputs. As with most endogenous growth models, the primary concern is whether the steady-state growth rate of per capita income is constant or diminishing in the absence of exogenous technical progress. Using the dot notation to indicate proportional changes, and letting asterisks denote equilibrium values, steady-state growth in the present model requires:

$$\dot{Q}^* = \dot{K}^* = \dot{H}^* = \dot{R}^* \tag{11}$$

This ensures that the allocation of resources between the government and the private sector is in equilibrium.[13] The Appendix examines dynamic properties in further detail, showing that the model does display equilibrium or steady-state growth with positive $\dot{Q}^*$, and that a shock to the model results in subsequent convergence towards a new equilibrium.

3 Policy Effects on Equilibrium Growth

The government chooses six fiscal policy parameters: the educational time input, u_h, two of the three expenditure proportions, θ_j $(j = k, h, d)$, and the three

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[13]The assumption of homotheticity ensures that u_q is constant in equilibrium so that if H^* is constant, H_q^* is also constant.

proportional tax rates t_r, t_w, and t_c. Changes in the public investment expenditure parameters, θ_k and θ_h, both have positive direct effects on equilibrium growth (by initially raising $\dot{K}$ and $\dot{H}$ respectively) and negative indirect effects via reductions in the relevant factor returns, r or wu_q.

3.1 Direct and Indirect Effects

To assess the role of fiscal policy, the following result can be established, as shown in the Appendix:[14]

$$\dot{Q}^* = A_h u_h \left[\frac{\theta_h w u_q}{\alpha_q} \left((1 - \alpha_q)t_r + \alpha_q t_w + \frac{t_c}{1+t_c} \right) \right]^{\alpha_h} \tag{12}$$

This result clearly shows the importance for growth of the time spent in education, u_h. It also demonstrates that the three tax rates have positive direct effects on equilibrium growth through the impact on investment in physical and human capital of higher tax revenues. However, they also have indirect labour supply effects to the extent that they affect the endogenous pre-tax wage, the proportion of human capital devoted to labour supply and the income share of capital.

The social transfer, D, does not appear in (12), but increases in θ_k and θ_h imply a reduction in the share of revenues allocated to transfers. Therefore, in common with Barro (1990), D has no direct effect on equilibrium growth. But an increase in D financed by a commensurate increase in income or consumption tax revenues is harmful for growth to the extent that there are disincentives to labour supply.[15]

It is convenient to drop asterisks from equilibrium values and work with the elasticity of equilibrium growth $(d\dot{Q}/\dot{Q})$ with respect to each policy parameter. Denoting the total elasticity of x with respect to y as $\eta_{x,y}$, differentiation of (12) for each policy parameter, say b, gives the general form:

$$\eta_{\dot{Q},b} = \frac{\partial \dot{Q}}{\dot{Q}} \frac{b}{\partial b} + \alpha_h \left[\frac{\partial W}{W} \frac{b}{\partial b} \right] \tag{13}$$

where $W = wu_q$ measures wage income per unit of total human capital. Provided labour supply is upward sloping, w and u_q are positively correlated so that an increase in either component raises W, and *vice versa*.[16]

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[14] This appears to suggest no direct growth effect from the investment proportion, θ_k. This is only because $\dot{Q}^*$ is expressed in terms of w and u_q. In view of the joint endogeneity of various terms, growth can alternatively be expressed in terms of the capital return, r, whereby $\dot{Q}^* = \theta_k r \left(t_r + \frac{\alpha_q}{1-\alpha_q}t_w + \frac{t_c}{1+t_c} \right)$. The negative indirect effect of an increase in θ_k occurs via a reduction in r.

[15] Unlike Mendoza *et al.* (1997), financing increased D by raising consumption taxes has similar effects to using income taxes because of the absense of private savings here.

[16] This condition is sufficient but not necessary since W rises as long as $\eta_{u_q,w} > -1$.

3.2 Human Capital Investment

Equation (13) demonstrates the importance of the human capital input parameter, α_h, for the magnitude of the elasticities: the more important is human capital into its own production, the greater are the potential endogenous growth effects. The lower is α_h, the less human capital growth relies on the ability to raise tax revenues. The fact that human capital is an input in its own accumulation function, unlike physical capital, means that the growth effects of θ_k and θ_h are not symmetric.

For the extreme case where $\alpha_h = 0$ only human capital is used in its own production and $\dot{Q} = A_h u_h$. The human capital appropriated by the government for education is the sole determinant of equilibrium growth. Physical capital accumulation and fiscal policy variables are irrelevant, and education policy is all important. Fiscal variables affect the *level* of output via effects on equilibrium factor proportions. This outcome is similar to the Lucas (1988) result (omitting externalities to human capital) but in his case the proportion, u_h, is chosen by optimising individuals.[17]

For the other extreme case where $\alpha_h = 1$, it can be shown that $\dot{Q} = \dot{H} = A_h Q_h / H = A_h \theta_h R / H$, so that the accumulation processes for human and physical capital are now similar, and the equilibrium factor endowment ratio is determined entirely by the government's choice of expenditure proportions ($K^*/H^* = \theta_k / A_h \theta_h$).

Human capital expenditures have direct and indirect effects on growth. The indirect effect depends on how changes in θ_h affect labour supply via changes to capital and wage income in (6). For example, an increase in θ_h reduces the K/H ratio which, from (10), implies a fall in wu_q/r (and an absolute fall in wu_q) in equilibrium.[18] From (12), this latter effect impacts negatively on the equilibrium growth rate.

The direct and indirect effects are illustrated by the following results, obtained from (12):

$$
\begin{aligned}
\eta_{\dot{Q},u_h} &= 1 - \alpha_h(1 - \eta_{W,u_h}) \\
\eta_{\dot{Q},\theta_h} &= \alpha_h\left(1 + \eta_{W,\theta_h}\right)
\end{aligned}
\tag{14}
$$

These elasticities include the terms η_{W,u_h} and η_{W,θ_h} which capture the indirect effects associated with the labour supply/rental rate adjustments required for

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[17]See Lucas (1988, equations 20 and 21), who also finds steady-state growth is fully determined by the proportion of time allocated to education. This result on the ineffectiveness of fiscal policy mirrors that found by Milesi-Ferretti and Roubini (1998) where leisure is quality time, only human capital is used in human capital production and education is untaxed − the conditions that hold in this case.

[18]It can be shown that an increase in H, for given K, which reduces wu_q/r must also increase r and reduce wu_q. This can also be seen from the fact that the rise in H raises the average and marginal returns to capital.

equilibrium.[19] Hence the signs of the partial effects involving W in (14) depend on how fiscal policy adjustments initially affect the relative growth of K and H. For human capital, $\eta_{W,\theta_h} < 0$ in (14), but an upward sloping labour supply curve $(\eta_{u_q,w} > 0)$ is a sufficient condition to ensure that negative indirect effects do not outweigh positive direct effects. This follows by noting that in equilibrium $\eta_{\dot{Q},\theta_h} = \eta_{\dot{H},\theta_h}$, and, from (14):

$$
\begin{aligned}
\eta_{\dot{H},\theta_h} &= \alpha_h(1 + \eta_{w,\dot{H}}\eta_{\dot{H},\theta_h}) \\
&= \alpha_h/(1 - \alpha_h\eta_{w,\dot{H}}) \quad\quad (15)
\end{aligned}
$$

An upward sloping labour supply curve ensures that $\eta_{w,\dot{H}} < 0$; hence $\eta_{\dot{H},\theta_h} > 0$.

Consider the growth effects of increasing the proportion of time allocated to education. As mentioned above, this is negatively related to α_h, with $\dot{Q} = A_h u_h$ for $\alpha_h = 0$. However, from (14), $\eta_{\dot{Q},u_h}$ could be positive or negative depending on the size of α_h and the size and sign of η_{W,u_h}. The pure income effect associated with an increase in u_h, together with homothetic preferences, ensures that w and u_q both fall; hence $\eta_{W,u_h} < 0$. Thus larger labour-education substitutions have a *ceteris paribus* lowering effect on equilibrium growth in response to an increase in u_h, and growth is lower if $\alpha_h(1 - \eta_{W,u_h}) > 1$. Such a situation can arise where negative labour supply effects, in association with large α_h, dominate the growth-enhancing effects of increased human capital accumulation. That is, while raising u_h raises $\dot{H}$ and thereby the potential wage-income tax base, wH, it simultaneously reduces tax revenues by reducing the proportion of human capital available for production of final output. Therefore u_h is effectively the tax rate on human capital, H, with the relationship between u_h and $\dot{Q}$ representing a form of Laffer curve whereby growth rates and revenues are low at very high and low values of u_h. The model also highlights the fact that human capital is effectively taxed twice when the government appropriates educational time inputs: it is taxed once directly at rate u_h, and again at rate t_w on the income from human capital net of "education tax" and leisure, wu_qH.

Raising the tax rates, t_r, t_w, t_c, also produces growth-enhancing effects via increased revenues for investment (for given θ_k, θ_h) and negative indirect effects via labour supply incentives.

$$
\eta_{\dot{Q},t_i} = \alpha_h\left(1 + \eta_{W,t_i}\right) \quad\quad (16)
$$

where $i = r, w$; where $i = c$, the tax rate is replaced by $t_c/(1 + t_c)$. Hence the net response of growth to increases in tax rates is positive only if the elasticity of W with respect to the relevant tax exceeds -1.

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[19]For physical capital, $\eta_{\dot{Q},\theta_k} = \alpha_h\eta_{W,\theta_k} > 0$.

Table 1: Benchmark Values For Simulations

Fiscal parameters		Other parameters	
Wage tax rate, t_w	0.25	Production function elasticity, α_q	0.70
Capital income tax rate, t_r	0.10	Production technology, A_q	1.0
Goods tax rate, t_c	0.10	Utility function elasticity, α_c	0.66
Expenditure proportions:		Human capital elasticity, α_h	0.70
Physical capital, θ_k	0.1	Human capital technology, A_h	1.0
Human capital, θ_h	0.2		
Social transfers, θ_d	0.7		
Education proportion, u_h	0.15		

4   Some Policy Simulations

This section provides numerical examples to examine the orders of magnitude of alternative fiscal policies, examining both steady-state and transitional responses.[20]
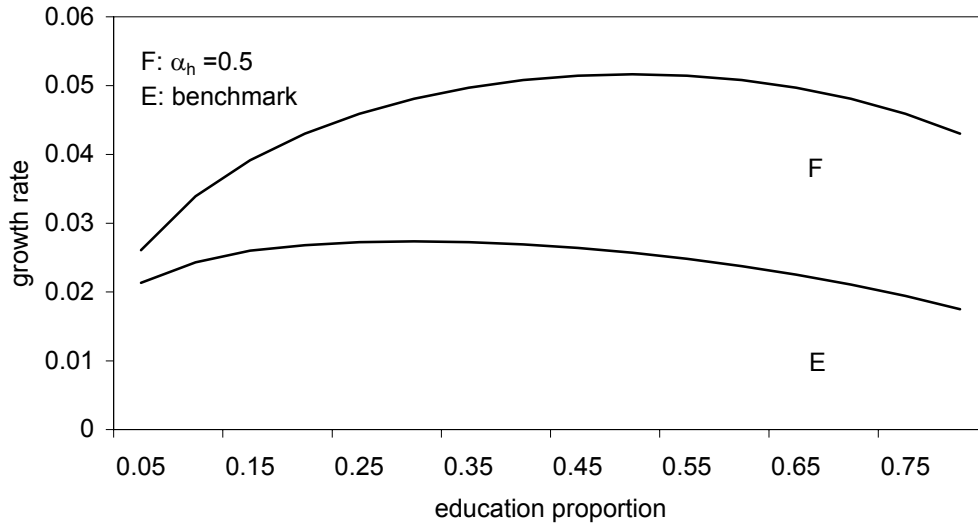
4.1   A Benchmark Case

Parameter values for a benchmark case are given in Table 1. This sets $t_w = 0.25$, and $t_c = t_r = 0.1$. Choosing a wage tax rate is complicated by the fact that marginal income tax rates in practice are typically much higher than average rates of tax (especially in LDCs), whereas in the model all taxes are proportional. Though 0.25 is higher than typical *average* wage tax rates it is more plausible as a *marginal* rate, which is more relevant to explore labour supply effects. In practice in LDCs, consumption taxes are often more important than personal income taxes, though labour supply effects of both forms of tax can be expected to be similar (and in the model are identical). Taxes on the benefits from consumption of public capital are implicit and difficult to quantify in practice; benchmark simulations set $t_r = 0.10$.

The key issue in setting values for the public expenditure proportions is distinguishing those expenditures which contribute to private sector productivity from those which affect utility. Benchmark simulations assume that 30% and 70% of expenditures are productivity and utility enhancing respectively, with the education expenditures twice those of physical capital investment. A benchmark case of $u_h = 0.15$ is selected. Other benchmark parameters reflect standard assumptions regarding factor income shares and leisure preferences. The education production function is difficult to specify, but $\alpha_h = 0.70$ was adopted for

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[20]The computer program (written in Fortran) is available from the authors.

Figure 1: Equilibrium Growth and Education Proportion

the benchmark case, giving a steady-state growth rate of just under 2.6% per period.

4.2 Time Spent in Education

The above analysis suggested that a Laffer curve effect can be expected as the education proportion, u_h, is varied; this is displayed in Figure 1. Using benchmark values for other parameters suggests a growth-maximising human capital allocation of around 30% to education (profile E), rising to around 50% if human capital inputs are more important for human capital accumulation (profile F, where $\alpha_h = 0.5$). These numbers are greater than likely to be observed in practice, though of course policy objectives other than growth maximisation are likely to play a role in determining public education provision.

4.3 Changing Expenditure Allocations

The growth trade-off between the two productive expenditures on human and physical capital is illustrated in Figure 2 which shows alternative combinations of the proportions, θ_k and θ_h, which yield constant growth. Starting with the benchmark values $(\theta_k = 0.1; \theta_h = 0.2)$, profile G represents a form of iso-growth curve, depicting combinations of θ_k and θ_h yielding a constant steady-state growth rate

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Figure 2: Iso-Growth Expenditure Proportions
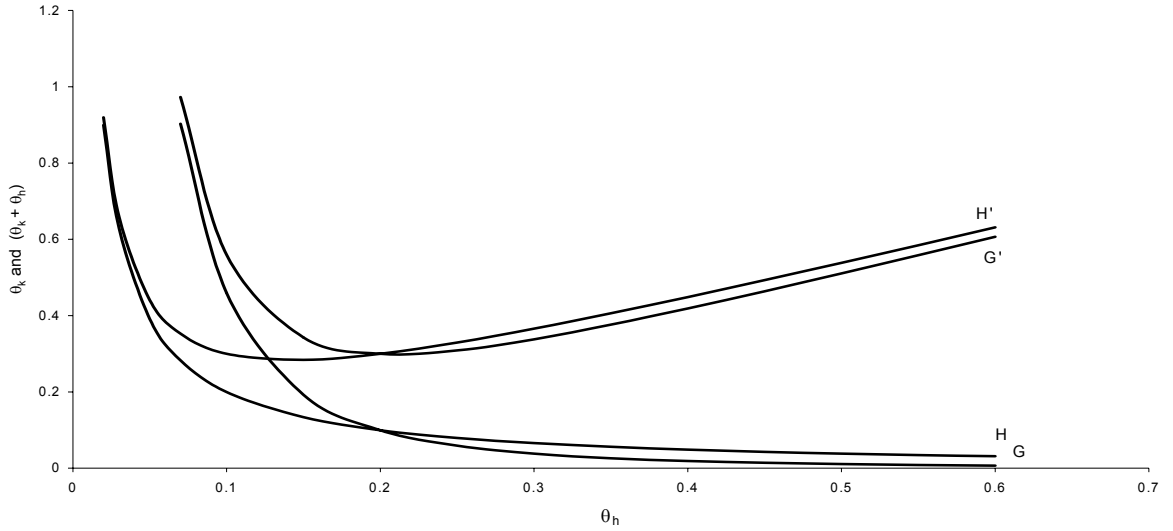
of 0.026.[21] The sum, $\theta_k + \theta_h$, identifies the remaining public resources available for social transfers, and the minimum point on profile G' indicates that the maximum resources available for social transfers without reducing long-run growth is, by coincidence, close to the benchmark values of $\theta_k + \theta_h \approx 0.3$. These profiles are not affected by changes in α_h (the elasticity of human capital production with respect to inputs of Q), though the value of the growth rate differs from the benchmark case.

Profiles H and H' show that the θ_k, θ_h trade-off is affected by changes in the importance of human capital in the private sector production function (α_q). In particular, profile H (lower α_q) is everywhere steeper than profile G (higher α_q). This indicates that reallocating a given proportion of expenditure from physical capital towards human capital requires a greater increase in the latter to maintain growth constant if the human capital-output elasticity is lower (profile H).

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[21]These profiles are obtained using an iterative procedure to find the required reductions in θ_k necessary to maintain constant growth for specified increases in θ_h.

Figure 3: Equilibrium Growth and Wage Taxation

A: t_w (benchmark)
B: t_w ($\alpha_c = 0.5$)
C: t_w ($\alpha_c = 0.5$; $t_r = 0$)
D: t_r (benchmark)
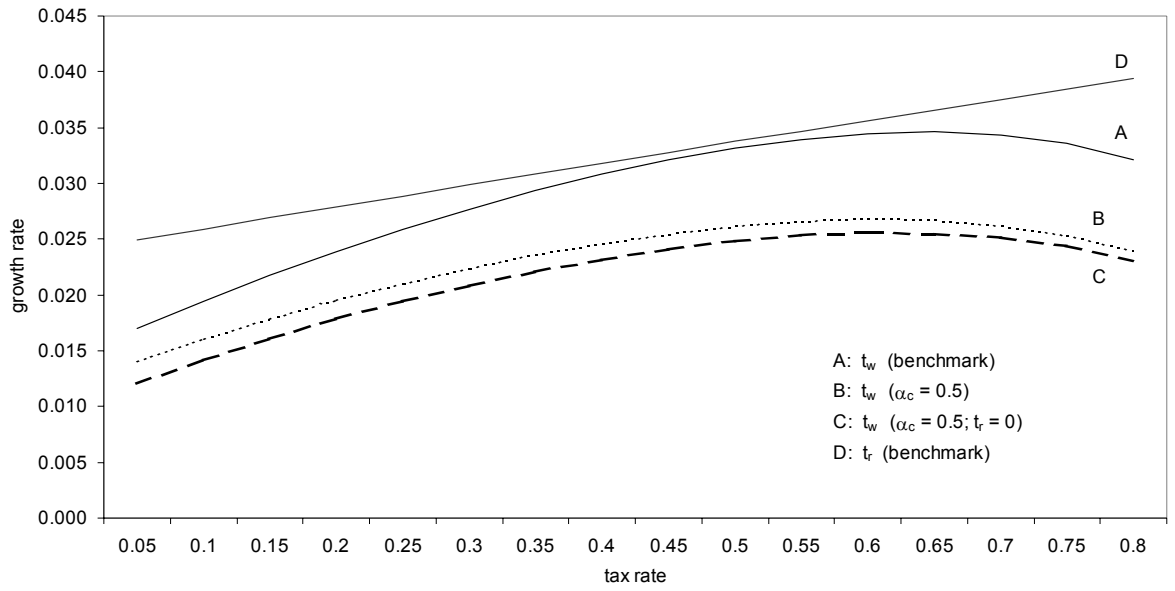
4.4 Changing Tax Rates

Figure 3 shows the relationship between the equilibrium growth rate and the tax rate on wage income.[22] These results reveal a similar pattern to those produced by the Barro (1990) model whereby the positive growth effects of productive expenditure enabled by increased tax rates initially dominate the negative growth effects of income taxes via disincentives. However, whereas in the Barro (1990) case disincentive effects operate *via* disincentives to accumulate physical capital, here it is disincentives to labour supply which eventually cause the profiles to turn down. Profile A, the benchmark case, suggests a growth maximum at around 3.5% associated with a wage tax rate of 0.65. However, lower tax rates do not involve a large growth sacrifice (for example, $\dot{Q} = 2.9\%$ at $t_w = 0.35$). With a greater leisure preference (profile B; $\alpha_c = 0.5$) maximum growth of 2.7% occurs at $t_w = 0.60$. Profile C (where $t_r = 0$) suggests that the growth effects of allowing for capital income taxation are quite small. Profile D shows how changes in t_r affect growth (for $t_w = 0.25$), and is approximately linear since there are no disincentive effects here to counteract the positive expenditure-enhancing effects of additional tax revenues.[23]

4.5 Transitional Dynamics

The transitional dynamics are, as shown in the Appendix, determined by tax revenue and labour supply effects. Shocks which generate divergences between the growth rates of tax revenue, and human and physical capital initiate convergent tendencies. Figure 4 shows the dynamic effects of reallocating public expenditures towards human capital by raising θ_h from 0.2 to 0.25. The increase in θ_h immediately raises the growth rate of human capital above that for physical capital. This in turn causes a rise in the capital growth rate but a fall in the human capital growth rate, re-establishing an equilibrium growth rate around 2.93%, with a half-life of about 36 years. This refers to the time taken for adjustment to a new equilibrium after the change induced by the initial shock. They are similar to the commonly estimated convergence rate of 2% per annum which translates into a half-life of 35 years; see Barro and Sala-i-Martin (1995, pp.26-38). Despite this length of time, the output growth rate is seen to move quickly to a value that is relatively close to its final equilibrium.

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[22]Given the correspondence noted earlier between t_w and $t_c/(1 + t_c)$, growth effects from changes in t_w and t_c are similar.

[23]Some secondary disincentive effects occur with t_r to the extent that capital income taxes change relative factor prices and hence labour supply decisions. Simulations suggest these are small.

Figure 4: A Shift in Public Expenditure Towards Human Capital
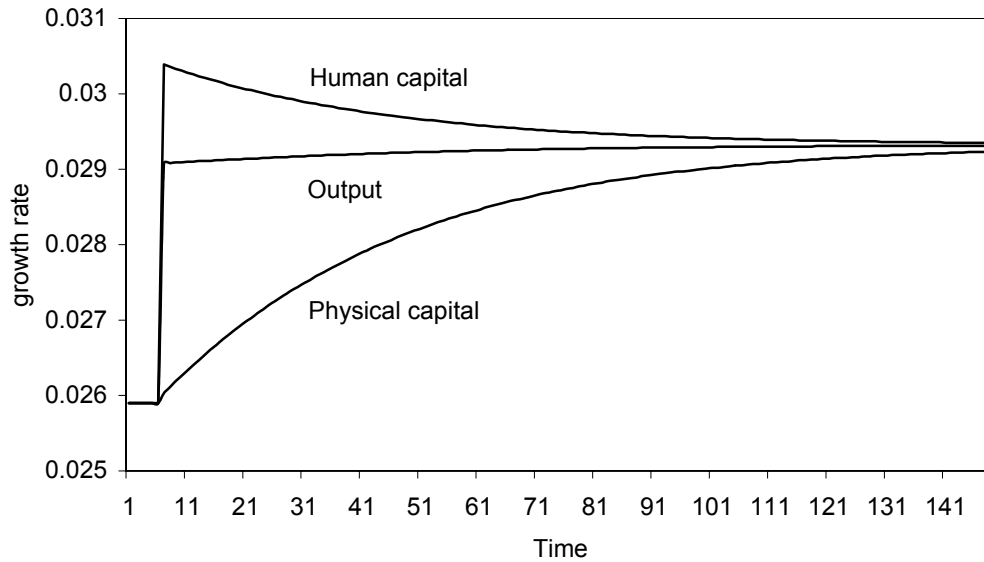
5 Conclusions

This paper has examined the role of publicly provided and tax-financed human capital accumulation in the context of a general equilibrium endogenous growth model. A key difference from previous models of endogenous human capital accumulation in the Lucas (1988, 1990) tradition is that schooling is compulsory and therefore exogenous to the representative individual.[24] Consumption and labour supply choices are based on maximisation of a static (single period) utility function. Arguably these assumptions are more relevant for a developing country seeking to extend compulsory schooling. Education and public physical capital investment are financed from taxes on wage and capital income, and consumption.

Direct and indirect effects of increasing both the proportion of time devoted to education, and the proportion of tax revenue used in human capital production, were identified. It was found that, as in the models of Lucas (1988, 1990), Stokey and Rebelo (1995) and Milesi-Feretti and Roubini (1998), the specification of the human capital production function and the strength of labour supply effects are important for the magnitude of steady-state outcomes. In addition, with an endogenous supply of labour, the proportion of time compulsorily devoted to

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[24]The analysis assumed that the compulsion is effective.

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education acts as a form of distortionary tax on human capital.

Numerical analyses found that, for benchmark parameter values, the growth maximising wage tax rate appear to be somewhat higher than those observed in practice, but the growth sacrifice associated with lower tax rates is not large. In the absence of incentives to private accumulation in the model, growth-maximising tax rate predictions should be treated with caution. Nevertheless, the model points to the possibility that, where taxes are used partly to fund growth-enhancing expenditures, growth-maximising tax rates can be quite high, even allowing for strong labour supply responses.

Finally it was shown that the model's transitional dynamics have strong convergence properties in response to fiscal policy changes. These arise essentially because the government budget constraint ensures that tax revenues, and public physical and human capital accumulation, are jointly endogenously determined. As a result, any divergences between their rates of growth are temporary. Simulations using plausible labour supply assumptions suggest typical half-lives of around 30-40 years; that is, similar to empirical regression-based estimates.

Appendix: The Dynamics of The Model

This appendix examines the dynamic properties of the model. Given the nature of the interdependencies in this general equilibrium framework, it is not possible to solve analytically for the equilibrium output and prices. This also applies to the dynamics. However, output growth can be expressed in terms of what Mendoza *et al* (1997) refer to as semi-reduced forms. Such equations for output growth yield insights into the influence of fiscal policies on equilibrium growth.

As stated above, steady-state growth requires:

$$\dot{Q}^* = \dot{K}^* = \dot{H}^* = \dot{R}^* \tag{17}$$

Endogenous steady-state growth also requires $\dot{Q}^*$ to be positive and constant. Given the constant returns to reproducible factors, K and H, in (1), this is achieved if accumulation functions are linear (non-decreasing), which depends on the relationship between inputs and tax revenue. From the first line of (3):

$$\dot{K} = \frac{\theta_k R}{pK} \tag{18}$$

and combining (2) with the second line of (3):

$$\dot{H} = A_h u_h \left(\frac{\theta_h R}{pH} \right)^{\alpha_h} \tag{19}$$

From the steady-state definition in (17), if an equilibrium growth rate exists, $(R/H)^*$ and $(R/K)^*$ are both positive constants yielding self-sustaining growth. Equating (18) and (19), it can be shown that:

$$\dot{Q}^* = \dot{K}^* = u_h \left[A_h \left(\frac{\theta_h K^*}{\theta_k H^*} \right)^{\alpha_h} \right]^{1/(1-\alpha_h)} \tag{20}$$

In equilibrium K^*/H^* is constant and hence, for given technological and fiscal parameters, $\dot{Q}^*$ is a positive constant.[25]

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[25]It was argued above that tax revenue in this model can be expected to display some of the properties of private savings in conventional endogenous growth models. This is shown by substituting in (18), and using (1) to substitute for Q/K, to give $\dot{Q}^* = A_q \theta_k \left(\frac{R}{pQ} \right) \left(\frac{u_q H}{K} \right)^{\alpha_q}$. Since R/pQ captures the compulsory saving rate, this shows that equilibrium growth is a positive function of the saving rate and the H/K ratio, as in Rebelo (1991). Not all of these compulsory savings are devoted to investment, as the government spends on social transfers. A broader concept of compulsory saving could be employed here, as in Rebelo (1991), to include the compulsory fraction of time, u_h, used for educational investment, such that: $S/pQ = (R - D + u_h w H)/pQ$.

Substitute for K^*/H^* in (20) to obtain an alternative semi-reduced form in terms of exogenous tax rates and the endogenous labour supply variables, w and u_q. Using (10) and (4), equation (12) can be obtained, after some re-arranging.

It is also necessary to examine whether the income growth resulting from arbitrarily chosen fiscal parameters converges towards the steady-state rate or whether initial fiscal policy choices display knife-edge properties. First, equating factor marginal products from (1) gives:

$$\frac{w}{r} = \left(\frac{\alpha_q}{1 - \alpha_q}\right) \frac{K}{H_q} \tag{21}$$

in equilibrium, so that for given α_q:

$$\dot{w} + \dot{H}_q = \dot{r} + \dot{K} \tag{22}$$

Differentiating the expression for total tax revenue in (4) gives:

$$\dot{R} = \beta \left(\dot{r} + \dot{K}\right) + \gamma \left(\dot{w} + \dot{H}_q\right) + (1 - \beta - \gamma) \dot{Q} \tag{23}$$

where β and γ are the shares of rental and wage income tax revenues in total tax revenue respectively. Hence, in the steady-state, where (22) holds, $\dot{R} = \dot{Q}$. Out of equilibrium, revenue growth may exceed or fall short of output growth unless uniform income tax rates apply.[26]

To examine transition properties, consider an initial equilibrium in which $\dot{Q}^* = \dot{K}^* = \dot{H}^*$. A shock to a fiscal parameter, such as an increase in the proportion of revenues allocated to capital investment, θ_k, causes: $\dot{K} > \dot{Q} > \dot{H}$. For the case of uniform tax rates, $\dot{R} = \dot{Q}$ and therefore $\dot{K} > \dot{R} > \dot{H}$, implying that R/K must fall and R/H must rise. Equations (18) and (19) show that this induces a reduction in $\dot{K}$ and an increase in $\dot{H}$, that is, a convergence towards equilibrium. If fiscal policy were to cause both factor inputs to grow more slowly than output (for example $\dot{K} < \dot{H} < \dot{Q} = \dot{R}$), R/K and R/H would both rise, restoring equilibrium.

In the case of non-uniform income tax rates, a shock away from the steady-state may cause $\dot{R}$ to exceed or fall short of $\dot{Q}$. However, what matters for the transitional dynamics is the relation of $\dot{R}$ to $\dot{K}$ and $\dot{H}$. As with the uniform tax case, if $\dot{R} < \dot{K}, \dot{H}$ then R/K and R/H both rise until $\dot{R}^* = \dot{H}^* = \dot{K}^*$, restoring equality with $\dot{Q}^*$. A converse equilibrating process occurs if $\dot{R} > \dot{K}, \dot{H}$. However, if initially, $\dot{K} < \dot{R} < \dot{H}$, $\dot{K}$ rises and $\dot{H}$ falls to restore equilibrium.[27]

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[26]This can be seen by differentiating (4) for the uniform income tax rate case to give:

$$\dot{R} = (\beta + \gamma) \left(\dot{r} + \dot{K} + \dot{w} + \dot{H}_q\right) + (1 - (\beta + \gamma)) \dot{Q} = \dot{Q}$$

[27]Revenue growth does not in general remain constant during the transitional process since $\dot{R}$ is a positive function of $\dot{K}$ and $\dot{H}$.

Consider, for example, a case where a fall in θ_k leads to a reduction in $\dot{K}$ from an initial equilibrium in which $\dot{R}^* = \dot{Q}^* = \dot{K}^* = \dot{H}^*$. The reduction in $\dot{K}$ reduces revenue growth but by less than the fall in $\dot{K}$. This is because capital income is only one source of tax revenue and because relative factor price adjustments ensure that the growth of capital income, rK, falls by less than the growth of the capital stock.[28] As a result, the higher revenue-to-capital ratio, R/K, generates a temporary increase in capital growth so long as the new investment proportion, θ_k, remains unchanged. Thus a new steady-state is established in which all variables grow at a lower rate. The transitional dynamics of the model are therefore essentially determined first by the government's budget which determines the pace of factor accumulation; and secondly labour supply responses to the associated relative factor price changes.

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[28]The shock to capital growth via the reduction in θ_k has no effect on human capital growth but affects the human capital used in production of Q, as relative factor price changes (due to reduced $\dot{K}$) induce labour supply changes.

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