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An Econometric Analysis of a Production Function for New Zealand

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Abstract

Using the capital stock series recently released by Statistics New Zealand, two approaches have been employed to estimate a production function. The first approach is based on the estimation of a constant elasticity of substitution (CES) production function with a value added form. The second approach is based on a nested CES function. Using the nested structure, we allow imports as an intermediate input in the production block. The estimated results reveal that the data rejects the Cobb-Douglas specification and the use of the value added form is not justifiable.

JEL classification: C51, E23

Keywords: CES production function, Cobb-Douglas specification

Contents

AN ECONOMETRIC ANALYSIS OF A PRODUCTION FUNCTION FOR NEW ZEALAND*

1. Introduction

In this paper, we develop and estimate an appropriate analytical supply side framework, in which some important parameters of the production function such as the elasticity of substitution and productivity, can be examined for the New Zealand (NZ) economy. The concept of a production function is of some importance as it underlies the results of many recent NZ empirical studies[1] on productivity and growth theory. Furthermore, the specification of the functional form of the production function has a crucial role in determining the behaviour of various macroeconomic models in New Zealand[2].

However, many NZ models of growth and development are based on two untested hypotheses. The first one is to assume a two-factor Cobb-Douglas specification for the production function with capital and labour as inputs. Many researchers have used the Cobb-Douglas function for the linear property of the function. However, the use of the Cobb-Douglas function implies that the elasticity of substitution between capital and labour is constant and always equal to one. The elasticity of substitution is an important parameter, which measures the degree of substitutability between inputs, which in turn determines the factor demand elasticities and the trend of the relative factor shares over time. Atkinson (1969) suggested that the rate of convergence to the long-run steady state in a neo-classical growth model also depends on the elasticity of factor substitution. Duffy and Papageorgiou (2000) demonstrated that there is a possibility of long-run endogenous growth or multiple steady states when the elasticity of factor substitution is not equal to one.

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1 See, for example, Gounder (2000) and Engelbrecht and McLellan (2001)

2 For example, both the NZ Treasury's NZM model and the National Bank of NZ's DEMONZ model employ a nested CES structure while the Reserve Bank of NZ's FPS model employs a Cobb-Douglas technology.

The second untested assumption is related to taking value added as output in the production function. The use of the value added form is justifiable only under very restricted conditions such as functional separability between intermediate-good inputs and capital/labour inputs. The use of the value added form implies that the marginal product of the intermediate-good inputs is constant and equal to unity.

Therefore, the primary purposes of this paper are threefold: (1) To test the validity of the Cobb-Douglas specification with NZ data; (2) To test the validity of the use of the value added form in the production function, and (3) To provide a stronger empirical foundation for the Macro-model used by the Treasury.[3]

Given the importance of this subject, it is surprising that there have been only a few empirical studies on estimating the production function. This is possibly due to the unavailability of the official capital stock series, which was only recently released by Statistics New Zealand (SNZ). Using this provisional capital stock series, this paper aims to build on previous studies that attempt to econometrically estimate a production function for NZ.

This paper first employs the methodology proposed by Grimes (1983), which estimated a constant elasticity of substitution (CES) production function[4] using a value added approach. The CES technology has an elasticity of substitution, which is constant and takes alternative values other than unity. We should be able to test the suitability of the Cobb-Douglas specification, as the elasticity of substitution in a Cobb-Douglas is always equal to one. Allowing for gross output as the measure of output, we then estimate a nested CES structure, which has been used in the Australian Murphy Model, NZM and DEMONZ. The two approaches are compared and we test whether the use of the value added form is justified or could cause the estimates of parameters to be biased.

The paper is structured as follows: Section 2 discusses the theoretical framework used by Grimes (1983) and presents the results of the estimation. Section 3 briefly outlines the nested CES structure and presents the results of the estimation. The final Section 4

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[3] The substitution-related parameters of the NZM are imposed.

[4] Refer to Henderson and Quandt (1980) for the properties of the CES production function.

provides the conclusion.

2. The Value Added Approach

2.1 The Model

The derivation of the estimated equation starts with a CES production function as follows:

$$\mathbf{Q_t} = \gamma(\delta(\mathbf{e}^{\lambda\mathbf{T}}\mathbf{N_t})^{(\sigma-1)/\sigma} + (1-\delta)(\mathbf{e}^{\mu\mathbf{T}}\mathbf{K_t})^{(\sigma-1)/\sigma})^{\mathbf{h}\sigma/(\sigma-1)} \qquad (1)$$

where Q is the value added, N is the labour input, K is the capital input, T is a time trend, σ is the elasticity of substitution between capital and labour, h is the returns-to-scale parameter, λ and μ are the rates of labour and capital augmenting technical progress, respectively, γ is the efficiency parameter and δ is the distribution parameter.

Assuming perfect competition in the product and factor markets, the firm is a price taker. Thus each factor is utilised up to the point where its marginal product equals its real price. Hence, the labour demand function can be written as:

$$\mathbf{log}\,\hat{\mathbf{N}}_\mathbf{t} = \beta_2 + \sigma\,\mathbf{log}\,\mathbf{W_t} + ((\mathbf{h}\sigma - \sigma + 1)/\mathbf{h})\,\mathbf{log}\,\mathbf{Q_t} + \lambda(\sigma - 1)\mathbf{T} \qquad (2)$$

where a ^ indicates the desired value of a variable and W is the real wage and

$$\beta_1 = \sigma\,\mathbf{log}(\mathbf{h}\delta\gamma^{(\sigma-1)/\mathbf{h}\sigma}/\beta).$$

The role of β is to allow the economy to be short of the perfectly competitive position (see Kelejian and Black 1970).

Similarly, the demand function for capital can be expressed as:

$$\mathbf{log}\,\hat{\mathbf{K}}_\mathbf{t} = \beta_2 + \sigma\,\mathbf{log}\,\mathbf{R_t} + ((\mathbf{h}\sigma - \sigma + 1)/\mathbf{h})\,\mathbf{log}\,\mathbf{Q_t} + \mu(\sigma - 1)\mathbf{T} \qquad (3)$$

where R is the real cost of capital and

$$\beta_2 = \sigma \log(h(1-\delta)\gamma^{(\sigma-1)/h\sigma} / \phi).$$

The role of Φ is analogous to that of β

From the above equations, it is clear that the values of Q, N and K are jointly determined. Therefore, it is important to estimate all three equations as a system. In the final model specification, we assume that there is a partial adjustment mechanism in which each endogenous variable adjusts partially to its desired level.

Hence, the final model specification is shown as follows:

$$\begin{aligned}\Delta \log N_t &= -\alpha_1\sigma \log W_t + (\alpha_1(h\sigma - \sigma + 1)/h)\log Q_t + \alpha_1\lambda(\sigma-1)T \\ &\quad - \alpha_1 \log N_{t-1} + \beta_1\Delta \log N_{t-1} + C_1 + \varepsilon_{1t}\end{aligned} \tag{4}$$

$$\begin{aligned}\Delta \log K_t &= -\alpha_2\sigma \log R_t + (\alpha_2(h\sigma - \sigma + 1)/h)\log Q_t + \alpha_2\mu(\sigma-1)T \\ &\quad - \alpha_2 \log K_{t-1} + \beta_2\Delta \log K_{t-1} + C_2 + \varepsilon_{2t}\end{aligned} \tag{5}$$

$$\begin{aligned}\Delta \log Q_t &= (\alpha_3 h\sigma/(\sigma-1))\log(\delta(e^{\lambda T}N_t)^{(\sigma-1)/\sigma} + (1-\delta)(e^{\mu T}K_t)^{(\sigma-1)/\sigma}) \\ &\quad - \alpha_3 \log Q_{t-1} + C_3 + \varepsilon_{3t}\end{aligned} \tag{6}$$

where α_i represents the partial adjustment parameters, C_i is the constant and ε_{it} is the random error term (i=1,2,3).

2.2 Data and Methodology

2.2.1 Data

Except for capital stock and real cost of capital, data for all the variables used in this paper is from the NZM database[5], which in turn is derived from SNZ's 95/96 fixed weight series. As we are only interested in the private sector production function, gross

5 Excludes computers to remove the bias caused by fixed weight methodology.

output is measured as the sum of all the exports supply and all components of total demand, other than government employment and the consumption of housing services. We then derive the value added measure by subtracting the imports from the gross output.

The private business sector capital stock (PBS) series used in this paper has been constructed using SNZ's annual chain-linked productive capital stock (PCS) that begins in March 1987. The starting point in forming the private sector capital stock is derived using the following formula:

$$\mathbf{PBS} = \mathbf{PCS} - \mathbf{RCS} - \mathbf{GS} \quad (7)$$

where RCS and GS are the capital stock for residential buildings and the government sector respectively.

SNZ also provides a detailed breakdown of PCS by industry. The Government sector comprises the following industries: central government, local government, education, health and community services, cultural and recreational services and personal and other community services.

Combined with business investment and the March 1987 starting point for the capital stock, the private business capital stock is projected forward on a quarterly basis using the perpetual inventory method.[6] The depreciation rates used in the calculation are derived implicitly from the official productive capital stock series. The real cost of capital is calculated by adding real interest rates and depreciation rates.

2.2.2 Methodology

One of the important issues in dealing with macroeconomic time series is the problem of spurious regression. If the time series used in the analysis are nonstationary, the results of the estimation could be spurious as the classical t and F tests are based on

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6 The capital stock is measured using the following equation: closing stock = opening stock + investment - depreciation.

the assumption that the variables are stationary. However, non-stationary variables may be used in the regression if they prove to be cointegrated.

There are three approaches to the problem of spurious regression. The first approach is to difference the data before estimating. The second approach is to add the lags of the dependent variable. Finally, one may consider using the co-integration technique. The results of the unit root and cointegration tests are reported in Appendix 1. These suggest that all the variables contain a unit root, except the real cost of capital (R). All the series are I(I) except the capital stock series which is I(2).

There are two basic approaches to estimating the error correction model, the two-step approach developed by Engel and Granger (1987) and the multivariate approach developed by Johansen (1988). However, both approaches are incapable of estimating non-linear structural models.

In this paper, all three equations in the final model specification have included the lags of the dependent variables through the partial adjustment mechanism. As in the linear model case, adding the lags of the dependent variables in the non-linear model should mitigate the problem of nonstationary variables. Therefore, we estimated the model in levels rather than in differences.

When Grimes (1983) estimated the three-equation system using non-linear three stage least squares, difficulty was found in obtaining convergence when all the parameters were unrestricted. In this paper, the estimations are implemented in TROLL, which uses sparse-matrix techniques to calculate the determinant and inverse efficiently, and all the models are estimated using full information maximum likelihood (FIML).

2.3 Results

The equations are estimated over the period 1987Q4-2000Q4. The coefficients from the estimation are presented in the Table 1. The second column (Est. 1) presents the results obtained when equations (4) to (6) are jointly estimated without any restrictions imposed on the coefficients. The results of the unrestricted model suggest that the returns-to-scale parameter is 1.094, which is significantly greater than one. We also note that the estimate for λ, the rate of labour augmenting technical progress of 0.4%

per annum appears to be smaller than expected. Furthermore, when we make the convergence criterion smaller and re-estimate, the value of h becomes larger and the value of λ becomes smaller. Therefore, it will continue to find better and better solutions with less negative log-likelihood. In other words, the value of the likelihood function can be maximized by a combination of higher values of h and lower values of λ.

As theory suggests that there are constant returns to scale in production, we will impose this restriction in the remainder of our empirical analysis. In the third column (Est. 2) of Table 1, we report the results from estimating the equations with h = 1. Except for the estimate for λ and α_3, the results obtained from the restricted model are similar to those from the unrestricted model.

In general, the estimated coefficients appear to be sensible but the Breusch-Godfrey test statistics suggest some problem of autocorrelation for the capital equation. Now, the rate of labour-augmenting technical progress (λ) is estimated to be about 1.1% per annum which is smaller than the estimated value (2.1% per annum) found by Grimes (1983). The difference in this finding could be attributed to the different estimation period.[7] The elasticity of substitution is estimated at 0.46, which is consistent with that found by Grimes. It is important to note that the estimated value of σ is significantly different from one. Therefore, the data has strongly rejected the Cobb-Douglas specification.

Another interesting finding from the estimation is that the rate of capital augmenting technical process (μ) is relatively large, but insignificantly different from zero at the 5% level. Grimes suggests that μ is proxying not only technical progress, but also other factors such as the measurement errors of the capital stock series. As μ is not significantly different from zero, we re-estimated the model by restricting μ = 0. The results of the estimation are reported in the fourth column (Est.3) of Table1. Overall, the coefficient estimates are not sensitive to the above restriction.

It is important to note that α_2, the coefficient of the partial adjustment for the capital equation, is very small but statistically different from zero for all the models. The speed

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[7] Grimes estimated the equations over the period 1962q1 to 1979q1.

of the adjustment is rather slow, which might indicate that the actual level of the capital stock can deviate from its desired level for a long time.

Finally, we have examined the robustness of our findings by reporting the results of single-equation estimation (Est. 4 – Est. 6). The results of these estimations have provided initial starting values for the system estimation. It is important to point out that the system estimates are superior to single-equation estimates for two reasons. First, when we estimate the three-equation system, we can impose cross-equation restrictions on the elasticity of substitution, the returns to scale and the rates of capital and labour augmenting technical changes. Second, single-equation estimation would yield inconsistent estimates owing to the problem of simultaneous-equation bias.

The main conclusion drawn from these results is that we can reject the hypothesis of $\sigma = 1$ in all instances. These results give us more evidence to suggest that a Cobb-Douglas specification for the aggregate production function may not be appropriate for New Zealand.

3. The Gross Output Approach

3.1 Model

The theoretical framework of this model is based on the production block developed by Murphy (1998). The production block combines three inputs (capital, labour and imports[8]) in producing two outputs (domestic goods and exports) and is composed of two CES functions and one constant elasticity of transformation (CET) function.

Those CES and CET functions[9] are expressed as follows:

$$\mathbf{Y_t} = ((\mathbf{A}_1\mathbf{e}^{\lambda \mathbf{T}}\mathbf{N_t})^{\rho} + (\mathbf{A}_2\mathbf{e}^{\mu \mathbf{T}}\mathbf{K_t})^{\rho})^{1/\rho} \quad (8)$$

$$\mathbf{T_t} = ((\mathbf{A}_3\mathbf{e}^{\pi_1 \mathbf{T}}\mathbf{M_t})^{\delta} + (\mathbf{A}_4\mathbf{Y_t})^{\delta})^{1/\delta} \quad (9)$$

$$\mathbf{T_t} = ((\mathbf{A}_5\mathbf{e}^{\pi_2 \mathbf{T}}\mathbf{E_t})^{\theta} + (\mathbf{A}_6\mathbf{YD_t})^{\theta})^{1/\theta} \quad (10)$$

where Y denotes primary factors for production, T total gross output, M imports, YD domestic sales, and E exports

Equation (8) and (9) form a nested CES function. Equation (8) represents the value of production contributed by capital and labour. Hence, Y can be interpreted as the value added. The value added (Y) is then combined with imports in another CES to yield the gross output T.

The standard assumption made in macroeconomic models is that exports and domestic goods are perfectly transformable in production.[10] As NZ's exports are based significantly on primary industries, the assumption of perfect transformation seems inappropriate. Thus, these two outputs are combined in a transformation function described by equation (10).

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8 Imports are considered as intermediate materials.

9 Constant returns to scale is assumed in the production block.

10 This assumption has been employed in the previous section.

Now, the parameter ρ is related to the constant elasticity of substitution (σ_1) between capital and labour by the following formula:

$$\sigma_1 = \frac{1}{1-\rho} \qquad (11)$$

The elasticity of substitution between imports and primary factors (σ_2) is related to the parameter δ in a similar way. As δ approaches $-\infty$, σ_2 become 0, which means that imports and primary factors are imperfect substitutes. In other words, the value added is an appropriate measure of output in the production function. Another interesting case is when $\theta = 1$, firms can switch their outputs from the domestic goods to the export goods easily. In this case, the CET function becomes a linear function of E and YD. Allowing for the trend change in the import penetration and more open economy, two trend growth rates (π_1 and π_2) are introduced in the production block.

Like the value added approach, we also assume that perfect competition exists in the product and factor markets and that the firm is a price taker. Hence, firms maximize their profits subject to the production constraints. Using unit cost and revenue functions, eight first-order conditions can be derived.[11] The non-linear eight-equation system is written as follows:

$$\widetilde{\mathbf{N}} = 1/\mathbf{a}_1(\widetilde{\mathbf{Y}}^{\rho} - (\mathbf{a}_2\mathbf{K})^{\rho})^{(1/\rho)} \qquad (12)$$

$$\widetilde{\mathbf{P}}_{\mathbf{Y}} = \mathbf{WA}/\mathbf{a}_1((\mathbf{a}_1\widetilde{\mathbf{N}}/(\mathbf{a}_2\mathbf{K}))^{-\rho} + 1)^{((\rho-1)/\rho)} \qquad (13)$$

$$\widetilde{\mathbf{P}}_{\mathbf{T}} = ((\widetilde{\mathbf{P}}_{\mathbf{Y}}/\mathbf{a}_4)^{(\delta/(\delta-1))} + (\mathbf{P}_{\mathbf{M}}/\mathbf{a}_3)^{(\delta/(\delta-1))})^{(\delta-1)/\delta} \qquad (14)$$

$$\widetilde{\mathbf{P}}_{\mathbf{YD}} = \mathbf{a}_6(\widetilde{\mathbf{P}}_{\mathbf{T}}^{(\theta/(\theta-1))} - (\mathbf{P}_{\mathbf{E}}/\mathbf{a}_5)^{(\theta/(\theta-1))})^{(\theta-1)/\theta} \qquad (15)$$

$$\widetilde{\mathbf{T}} = \mathbf{a}_6\mathbf{YD}(\widetilde{\mathbf{P}}_{\mathbf{YD}}/(\mathbf{a}_6\widetilde{\mathbf{P}}_{\mathbf{T}}))^{1/(1-\theta)} \qquad (16)$$

[11] For a more detailed discussion of the first-order conditions, see Powell and Murphy (1997).

$$\tilde{\mathbf{Y}} = 1/\mathbf{a}_4(\tilde{\mathbf{T}}^{\delta} - (\mathbf{a}_3\mathbf{M})^{\delta})^{(1/\delta)} \qquad (17)$$

$$\tilde{\mathbf{E}} = \tilde{\mathbf{T}}/\mathbf{a}_5(\mathbf{P}_{\mathbf{E}}/(\mathbf{a}_5\tilde{\mathbf{P}}_{\mathbf{T}}))^{1/(\theta-1)} \qquad (18)$$

$$\tilde{\mathbf{M}} = \mathbf{a}_4\tilde{\mathbf{Y}}/\mathbf{a}_3((\mathbf{P}_{\mathbf{M}}/(\mathbf{a}_3\tilde{\mathbf{P}}_{\mathbf{T}}))^{\delta/(1-\delta)} - 1)^{-1/\delta} \qquad (19)$$

where

$$\mathbf{a}_1 = \mathbf{A}_1\mathbf{e}^{\lambda\mathbf{T}} \qquad \mathbf{a}_2 = \mathbf{A}_2\mathbf{e}^{\mu\mathbf{T}}$$

$$\mathbf{a}_3 = \mathbf{A}_3\mathbf{e}^{\pi_1\mathbf{T}} \qquad \mathbf{a}_5 = \mathbf{A}_5\mathbf{e}^{\pi_2\mathbf{T}}$$

$$\mathbf{a}_4 = \mathbf{A}_4 \qquad \mathbf{a}_6 = \mathbf{A}_6$$

WA is the nominal wage, P_i is the price variable for goods i and i = Y, T, YD, E and M. For example, P_Y is the price of the primary factors. The medium-run equilibrium variable is distinguished from the actual variable by the symbol ~.

These first order conditions cannot be used directly for estimation because four of the variables (Y, $P_{Y,}$ T and P_T) are not observable. By substitution, we can have four equations in the four variables regarded as endogenous in the medium term: N ,P_{YD} E and M. In other words, the firm has to make decisions on those four variables in the medium-term and expects other (exogenous) variables to remain at a given level. Those exogenous variables are P_E, P_M, YD, WA and K.

3.2 Methodology

In the previous section, a partial adjustment mechanism has been employed to model the dynamic structures. However, it is not possible to express each endogenous variable as a function of other variables in this model because of highly non-linear structures. Therefore, we need to use four first-order conditions directly in the estimation. However, applying FIML directly to this non-linear four-equation system is not appropriate because residuals from each equation will exhibit autocorrelation.

The problem can be overcome by a 2-step method. Let us express the four-equation system in a matrix form:

$$\mathbf{D_t} = \mathbf{f}(\mathbf{D_t}, \mathbf{S_t}) + \boldsymbol{\varepsilon}_\mathbf{t} \qquad (20)$$

where D_t is a vector of the endogenous variables (N ,P_{YD}, E and M) at time t, S_t the vector of all the exogenous variables and ε_t is the vector for the residuals. Assuming the residuals obey a first-order autoregressive scheme such as

$$\boldsymbol{\varepsilon}_\mathbf{t} = \mathbf{R}\boldsymbol{\varepsilon}_{\mathbf{t-1}} + \mathbf{u_t} \qquad (21)$$

where R is a 4x4 matrix of coefficients of the autoregressive scheme, and u_t is a vector random process which is assumed to have a jointly normal distribution for each t, with zero mean and variance-covariance given by Σ and no correlation across time periods. We further assume that the off-diagonal elements of the matrix R are zero. It means that there is no cross-autocorrelation in the scheme.

Substituting (20) into (21) yields the following equation:

$$\mathbf{D_t} - \mathbf{f}(\mathbf{D_t}, \mathbf{S_t}) - \mathbf{R}(\mathbf{D_{t-1}} - \mathbf{f}(\mathbf{D_{t-1}}, \mathbf{S_{t-1}}) = \mathbf{u_t} \qquad (22)$$

As u_t is distributed independently across time periods[12], we can estimate equation (22) with FIML.

Hence, the first step of the method is to apply FIML to equation (20). The estimation will yield a time series of residuals for each equation. Regressing each time series on itself lagged one period, yields an estimate of R, which is then substituted in equation (22). The second step of the procedure is to apply FIML to equation (22) with the value of R as given.

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12 Rae (1994) has estimated a similar system with FIML without adjusting the problem of autocorrelation. However, the assumption of sequentially independent errors is crucial to the method of FIML.

3.3 Results

As in the previous section, the estimation of the model is performed over the period 1988Q4–2000Q4. All data is from the NZM database except for the capital stock series. The results of the unit root tests for all variables are presented in the Appendix 1.

When we first attempted to estimate the whole model with FIML, we encountered difficulty in obtaining convergence because of the complexity of the model. As the parameters of interest in this paper are ρ, δ and θ, we decided to allocate the parameters into two vectors: A and B. The first vector A is composed of A_1, A_2, A_3, A_4, A_5, and A_6. The second vector B consists of ρ, θ, δ, λ, μ, π_1, and π_2.

The maximum likelihood estimates of A and B can be obtained as follows: Start with an initial value for B and maximize the likelihood function with respect to A. Using the estimate of A, we then estimate the likelihood function with respect to B. These procedures can be repeated until convergence is reached.

Preliminary estimations suggest that convergence[13] comes after a few iterations. We also find that the results are sensitive to the starting values of ρ, δ, and θ, and the convergence is to a local optimum rather than a global one. From the results of the previous section, we have estimated the value of ρ. Therefore, we decide to conduct a two-dimensional grid search on both δ and θ for the starting values. For δ, the grid search is carried out at an interval of 0.2 between –1 and –2.6. The range of θ is covered from 2 to 3 at an interval of 0.5.

The results of the grid search[14] are summarized in Table 2a, Table 2b and Table 2c. The values of δ and θ that maximize the log-likelihood function, are -2.51 and 2.66 respectively and the resulting residuals are used to estimate the autoregressive coefficients. The diagonal elements of R are estimated to be 0.71, 0.77, 0.72 and 0.52. Taking those estimates, we are able to estimate equation (22). The final results of the estimation are presented in Table 3.

[13] Convergence criterion is set at 0.001 in this paper.

Looking at Table 3, the estimated value of ρ is -0.883, which is significantly different from zero, implying that the elasticity of capital and labour is 0.53 and is similar to the estimate from the previous section.

The most important finding is that the estimated value of δ is found to be –2.502. This result suggests that the elasticity of substitution between primary factors and import is 0.29, which is consistent with the estimate of 0.36 found by Rae (1994), but is smaller than 0.75 employed by NZM. This result provides evidence that there is some substitutability between primary factors (value added) and imports, and that the use of the value added approach is not justifiable in NZ.

Another interesting finding is that the estimate of labour productivity is now about 2% per annum, which is much higher than the estimate of 1.2% per annum from the previous section. When we look at the results of the grid search, we find that there is a positive relationship of λ and δ. This result implies that imposing a restriction of no substitutability between value added and imports could lead to a downward bias on the estimate of labour productivity.

Finally, the value of θ is estimated to be 2.666, suggesting that the elasticity of transformation is about 0.6. The elasticity of transformation between the domestic good and exports is set at 0.75 and 1 in NZM and DEMONZ respectively. Therefore, our estimate is slightly less than those imposed on the models.

4 Conclusions

In this paper, two approaches have been used in estimating the production function. The first approach is based on the estimation of a CES function with the use of value added form. The second approach estimated a nested CES function, allowing for the substitution between value added and imports. The main implications are summarised below:

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[14] As the estimates of ρ, δ and θ do not alter significantly with the iteration limit, all the estimations are based on one iteration.

- The results of both approaches reject the Cobb-Douglas specification.
- There is strong evidence that there is some substitutability between value added and imports. This casts some doubt over the use of the value added approach.
- Imposing a restriction of no substitutability between value added and imports *could* bias labour productivity estimates downwards.

This paper raises an important issue that failure to include imports as inputs in the production function could lead to a downward bias on the estimate of labour productivity. In particular, the post-1990 period was marked by higher imports penetration ratio and changing composition of imports. That is why it is essential to use the gross output approach in constructing models of growth over the 1990s period.

References

Atkinson, A.B. (1969), 'The timescale of economic models: How long is the long run?', *Review of Economic Studies*, 36,137-152.

Duff, J. and Papageorgiou, C. (2000), 'A cross-country empirical investigation of the aggregate production function specification', *Journal of Economic Growth*, 5, 87-120.

Engel, R. F. and Granger, C.W. J. (1987), 'Cointegration and error correction: representation, estimation, and testing', *Econometrica*, 55(2), 251-276.

Engelbrecht, H and McLellan, N. (2001), 'Economic growth revisited: identifying a class of growth models for the New Zealand Economy', forthcoming.

Gounder, R. (2000), 'Determinants of New Zealand's total factor productivity, 1960 to 1998', Conference paper presented at the New Zealand Association of Economists Conference, Wellington, July 2000.

Grimes, A. (1983), 'The Reserve Bank real wage-employment relationship: A reply and further results', *New Zealand Economic Paper*, 17, 67-78.

Hamilton, J. (1994), *Time Series Analysis*, Princeton University Press.

Henderson, J. M. and Quandt, R. E. (1980), *Microeconomic Theory: A Mathematical Approach*,

Kelejian, H.H. and Black, S.W. (1970). 'A Macro Model of the U.S. Labour Market', *Econometrica*, 38(50), September.

Johansen, S. (1988). 'Statistical analysis of cointegration vectors', *Journal of Economic Dynamics and Control* 12, 231-254.

Murphy, C. (1998), 'New Zealand Model 1998', *A Report to the New Zealand Treasury* Wellington, September 1998.

Powell, A. and Murphy, C. (1997), *Inside a Modern Macroeconomic Model*, Berlin: Springer-Verlag.

Rae, D. (1994), 'NBNZ-DEMONZ dynamic equilibrium model of New Zealand', Working Paper, The National Bank of New Zealand.

APPENDIX 1: UNIT ROOT TESTS

There are three forms of the Augmented Dickey-Fuller regression equation. They are shown as follows:

$$\Delta \mathbf{Y_t} = \alpha_0 + \alpha_1 \mathbf{Y_{t-1}} + \alpha_2 \mathbf{t} + \sum_{\mathbf{j}=1}^{\mathbf{P}} \gamma_{\mathbf{j}} \Delta \mathbf{Y_{t-j}} \qquad \text{(a)}$$

$$\Delta \mathbf{Y_t} = \alpha_0 + \alpha_1 \mathbf{Y_{t-1}} + \sum_{\mathbf{j}=1}^{\mathbf{P}} \gamma_{\mathbf{j}} \Delta \mathbf{Y_{t-j}} \qquad \text{(b)}$$

$$\Delta \mathbf{Y_t} = \alpha_1 \mathbf{Y_{t-1}} + \sum_{\mathbf{j}=1}^{\mathbf{P}} \gamma_{\mathbf{j}} \Delta \mathbf{Y_{t-j}} \qquad \text{(c)}$$

Equation (b) and (c) are just the special cases of equation (a), which includes both a constant and a linear time trend. The null of a nonstationary series Y is rejected if the T-Test statistic for α_1 is smaller than the critical value. The general principle is to choose a specification that is a plausible description of the data under both the null and alternative hypotheses (Hamilton 1994). As all the variables seem to contain a trend, we test the stationarity of the series in levels using equation (a) and use the Schwartz (SC) information criteria to select the lag length. If we do not reject the unit root hypothesis, we take the first difference of the series and rerun the test with equation (b). If the unit root tests are still rejected, we have to run the test with the second differences of the series. The following tables report the results of the unit root tests.

Table 4a. Unit root tests (Levels)

	Lag length p	ADF test statistic
Q	0	-2.316
K	1	-2.639
N	0	-3.375
W	2	-1.826
R	3	-5.889*
YD	0	-2.145
E	0	-3.601**
M	0	-2.675
P_E	1	-1.100
P_M	1	-0.215
P_{YD}	2	-2.373
WA	3	-2.667

Table 4b. Unit root tests (First Differences)

	Lag length p	ADF test statistic
Q	0	-6.645*
K	3	-2.185
N	1	-3.019**
W	1	-5.393*
R		
YD	0	-7.395*
E		
M	0	-7.547*
P_E	0	-3.840**
P_M	0	-3.321**
P_{YD}	1	-3.799**
WA	2	-3.316**

Table 4c. Unit root tests (Second Differences)

	Lag length p	ADF test statistic
K	0	-7.480*

* significantly different from 0 at 1% level.

** significantly different from 0 at 5% level.

*** significantly different from 0 at 10% level.

The critical values are based on MacKinnon critical values for unit root tests.

Table 1. FIML estimates for the 3-equation system

	Est. 1	Est. 2	Est. 3	Est.4	Est. 5	Est. 6
Coefficients	Unrestricted Equation	H =1	H =1 μ = 0	Eq (4)	Eq (5)	Eq (6)
h	1.094* (0.0059)					
λ	0.004 (0.0024)	0.011* (0.0017)	0.011* (0.0016)	0.004 (0.0083)		0.012* (0.0015)
μ	0.016** (0.0063)	0.015*** (0.0077)			0.018** (0.0077)	0.836** (0.3346)
σ	0.497* (0.0061)	0.460* (0.0335)	0.425* (0.0841)	0.594 (0.3632)	0.501* (0.0299)	0.324*** (0.1794)
α_1	0.233* (0.0147)	0.226* (0.0237)	0.227* (0.0671)	0.234* (0.0511)		
α_2	0.009* (0.0021)	0.010* (0.0022)	0.007* (0.0007)		0.008* (0.0020)	
α_3	0.040* (0.0116)	0.272* (0.0255)	0.269* (0.0411)			0.396* (0.1023)
δ	0.001 (0.0096)	0.325* (0.0883)	0.221** (0.0964)			0.585* (0.1604)
C_1	-0.359* (0.0200)	-0.322* (0.0353)	-0.341* (0.0478)	-0.270 (0.2077)		
C_2	0.0030* (0.0006)	0.006* (0.0021)	0.006** (0.0023)		0.004** (0.0016)	
C_3	-0.678* (0.0162)	0.396* (0.0552)	0.348* (0.0822)			0.848* (0.2539)
β_1	0.178*** (0.0945)	0.264** (0.1094)	0.270** (0.1070)	0.138 (0.1212)		
β_2	0.758* (0.0554)	0.739* (0.0592)	0.785* (0.0491)		0.777* (0.0586)	
BG eq(4)	4.497	4.996	5.412	4.478		
BG eq(5)	8.547***	9.567**	14.913*		8.454	
BG eq(6)	4.556	3.068	2.864			4.373

Notes: Standard errors are given in parentheses.

* significantly different from 0 at 1% level.

** significantly different from 0 at 5% level.

*** significantly different from 0 at 10% level.

BG is the test statistics of the Breusch-Godfrey test which is used to test for 4^{th} order of autocorrelation. The critical value $\chi^2_{0.05}(4) = 9.488$.

Table 2a. Results of grid search with θ =2.0

	Initial values θ =2.0								
δ	-1	-1.2	-1.4	-1.6	-1.8	-2	-2.2	-2.4	-2.6
ρ	-0.87995	-0.88255	-0.88478	-0.88666	-0.88821	-0.88951	-0.89061	-0.89154	-0.89235
δ	-1.02864	-1.23378	-1.43808	-1.6415	-1.84397	-2.04552	-2.24615	-2.44588	-2.64471
θ	1.847419	1.856233	1.861833	1.866179	1.869739	1.872716	1.875229	1.877386	1.879204
λ	0.022725	0.021719	0.021051	0.020492	0.020045	0.019681	0.019377	0.019119	0.018896
μ	0.010264	0.009676	0.009188	0.008839	0.008568	0.008347	0.008167	0.008018	0.007886
π_1	-0.00839	-0.00822	-0.00804	-0.00793	-0.00785	-0.00778	-0.00773	-0.00769	-0.00766
π_2	-0.02837	-0.02646	-0.02507	-0.02396	-0.02308	-0.02237	-0.02178	-0.02128	-0.02086
A_1	13.43927	13.36234	12.56548	12.34219	12.17318	12.14195	12.18288	12.21648	12.33892
A_2	0.453944	0.451026	0.424404	0.416581	0.4106	0.409273	0.410383	0.411247	0.415103
A_3	9.356734	7.729223	5.440164	4.442561	4.008598	3.788134	3.678481	3.613171	3.611034
A_4	1.636376	1.569861	1.300695	1.167933	1.134439	1.127598	1.134971	1.148751	1.168627
A_5	1.31097	1.326678	1.078399	0.981509	0.963574	0.974146	0.99964	1.02811	1.068266
A_6	0.870789	0.880581	0.715864	0.651439	0.639432	0.646346	0.663162	0.681954	0.708498
Log-likelihood	-841.08	-839.72	-838.89	-838.46	-838.27	-838.24	-838.31	-838.45	-838.65

Table 2b. Results of grid search with θ =2.5

Initial values θ =2.5									
δ	-1	-1.2	-1.4	-1.6	-1.8	-2	-2.2	-2.4	-2.6
ρ	-0.87919	-0.88226	-0.88464	-0.88659	-0.88817	-0.88948	-0.89058	-0.89151	-0.8923
δ	-1.03608	-1.24397	-1.45221	-1.6595	-1.86628	-2.07256	-2.27834	-2.48365	-2.68849
θ	2.237806	2.250208	2.254759	2.261009	2.266247	2.2707	2.274533	2.277863	2.280779
λ	0.022217	0.021196	0.020579	0.020047	0.019626	0.019284	0.019001	0.018761	0.018556
μ	0.009877	0.009297	0.008786	0.00843	0.00815	0.007924	0.00774	0.007587	0.007459
π_1	-0.00982	-0.00967	-0.00945	-0.00933	-0.00923	-0.00915	-0.00908	-0.00902	-0.00898
π_2	-0.02901	-0.02707	-0.02568	-0.02457	-0.02368	-0.02296	-0.02237	-0.02187	-0.02145
A_1	13.165	13.21816	12.51993	12.27046	12.15395	12.08856	12.14214	12.16358	12.18783
A_2	0.449875	0.450913	0.427603	0.418925	0.414787	0.412392	0.414056	0.414623	0.415285
A_3	9.6084	7.798098	5.47271	4.538213	4.06582	3.861521	3.751892	3.702221	3.585795
A_4	1.715303	1.60161	1.313347	1.19992	1.152126	1.154012	1.160832	1.181346	1.173879
A_5	1.514838	1.506387	1.22004	1.127428	1.098838	1.116376	1.146168	1.184172	1.192398
A_6	0.932402	0.926384	0.750891	0.69383	0.676172	0.686897	0.705158	0.728469	0.733461
Log-likelihood	-835.27	-833.66	-832.63	-832.01	-831.65	-831.48	-831.44	-831.5	-831.62

Table 2c. Results of grid search with θ =3.0

	Initial values θ =3.0								
δ	-1	-1.2	-1.4	-1.6	-1.8	-2	-2.2	-2.4	-2.6
ρ	-0.87957	-0.88264	-0.88528	-0.88726	-0.88886	-0.89017	-0.89127	-0.89219	-0.89297
δ	-1.04064	-1.24939	-1.46059	-1.6702	-1.87953	-2.08862	-2.29746	-2.50608	-2.71448
θ	2.600997	2.618746	2.623376	2.631826	2.639	2.645157	2.650512	2.655202	2.659349
λ	0.021862	0.020971	0.020223	0.019703	0.019294	0.018964	0.018691	0.018461	0.018265
μ	0.009606	0.008939	0.008508	0.008148	0.007865	0.007639	0.007455	0.007302	0.007174
π_1	-0.01078	-0.01056	-0.01041	-0.01028	-0.01017	-0.01008	-0.01	-0.00994	-0.00988
π_2	-0.02943	-0.02756	-0.02606	-0.02495	-0.02406	-0.02333	-0.02274	-0.02223	-0.0218
A_1	13.06751	13.07411	12.52839	12.24971	12.16109	12.04882	12.10784	12.14375	12.15222
A_2	0.448722	0.449356	0.429856	0.420189	0.417036	0.41307	0.414974	0.416083	0.416251
A_3	9.385727	7.763136	5.553555	4.619654	4.072455	3.917231	3.820824	3.770594	3.675075
A_4	1.687907	1.607863	1.332012	1.223591	1.153324	1.174454	1.185381	1.204943	1.206396
A_5	1.599296	1.618052	1.3382	1.240417	1.18953	1.223904	1.261409	1.303319	1.320628
A_6	0.935667	0.94622	0.782819	0.725579	0.69577	0.715827	0.73771	0.762166	0.772234
Log-likelihood	-833.23	-831.45	-830.3	-829.54	-829.07	-828.81	-828.69	**-828.67**	-828.72

Table 3. FIML estimates for the system equation

Coefficients	Value	SE
ρ	-0.883*	0.0193
δ	-2.502*	0.0585
θ	2.666*	0.1431
λ	0.020*	0.0029
μ	0.006	0.0038
π_1	-0.008*	0.0027
π_2	-0.021*	0.0020
Log-likelihood	-756.22	

DW♦ for N equation = 2.03

DW for P_{YD} equation = 2.04

DW for E equation = 2.06

DW for M equation = 1.67

Notes: Standard errors are given in parentheses.

* significantly different from 0 at 1% level.

** significantly different from 0 at 5% level.

*** significantly different from 0 at 10% level.

♦ DW is the Durban-Watson test statistics.