Population Ageing In New Zealand: Implications for Living Standards and the Optimal Rate of Saving

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New Zealand Treasury

Working Paper 03/10

June 2003

THE TREASURY
Kaitohutohu Kaupapa Rawa

NZ TREASURY WORKING PAPER 03/10

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MONTH/YEAR

June 2003

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ACKNOWLEDGEMENTS

The paper has benefited from comments by Iris Claus, John Creedy, Richard Disney, and Brian McCulloch.

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Abstract

Over the next 50 years, New Zealand's population will age substantially. There has been wide debate about whether New Zealand should prepare for population ageing by increasing national savings. The debate had not, however, involved explicit consideration of possible time paths for savings, consumption, debt, and other relevant macroeconomic variables; nor have explicit principles been offered for determining which of these time paths are to be preferred. This paper addresses the question of choosing time paths through the use of a Ramsey-Solow model of optimal saving, adapted for investigating problems of population ageing. The results suggest that population ageing alone would not justify increases in national savings rates beyond those envisaged by current policy. The cost of ageing in terms of reduced real consumption is not large enough to justify large additional savings beyond those currently predicted, and the concomitant reduction in current consumption. The findings concerning national savings and living standards are robust to a variety of specifications of demographic conditions, interest rates, and productivity growth.

JEL CLASSIFICATION

D9 – Intertemporal choice and Growth
E21 – Consumption; Saving
J11 – Demographic Trends and Forecasts

KEYWORDS

consumption; saving; inter-temporal paths; Ramsey model; population ageing; New Zealand

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Population Ageing In New Zealand: Implications for Living Standards and Optimal Savings

1 Introduction

Over coming decades, New Zealand's population will age substantially. Statistics New Zealand (2000) suggests, for instance, that the number of people aged 65 years and over could rise from the current level of about one-eighth of the population to about one-quarter by 2051. Even if labour force participation rises, the ratio of consumers to workers is almost certain to increase. Most analysts expect that an increase in the ratio of consumers to workers will lead to an increase in the proportion of gross domestic product devoted to government social expenditures, a fall in savings, a tightening in labour markets, and perhaps slowing economic growth (Creedy and Scobie 2002, Polackova 1997, Stephenson and Scobie 2002).

Some have argued that these problems constitute an ageing 'crisis': that population ageing poses a serious threat to the consumption levels of future generations. This raises the question as to whether the level of savings expected under current and projected policies is 'adequate' to avoid placing an unreasonable 'burden' on future generations. Addressing these questions requires an economic framework or model that can trace out the likely effects of ageing on paths for consumption, savings, debt, investment, and other macroeconomic variables. Furthermore, such a framework needs to be able to weigh up the competing interests of different generations. How much weight, for instance, should be given to protecting the consumption levels of the New Zealand population in 2051? The population of 2051 may experience slower economic growth than the current population, but this must be balanced against the fact that the population of 2051 may be considerably wealthier than the current population. In short, to what extent would foregoing consumption today (through higher savings) increase welfare in the long run?

This paper addresses the question of optimal time paths through the use of a Ramsey-Solow model of optimal saving, adapted for investigating problems of population ageing (Cutler, Poterba, Sheiner and Summers 1990, Elmendorf and Sheiner 2000, Guest and McDonald 2001, 2002a). Typical of this class of models, some strong assumptions are made in order to readily identify the main macroeconomic impacts of population ageing. There is, for instance, the highest possible level of aggregation, implying no distinction between the public and private sectors. The model also implies nothing about the distribution of savings among individuals.

The model does, however, highlight the impact of demographic change on consumption possibilities, capital requirements, the return to saving, and the consequent effects on macroeconomic aggregates. Also, the model allows the analyst to evaluate the welfare implications of demographic change, by specifying an explicit social welfare function in terms of the ultimate objective of economic activity: consumption.

The results from the analysis in this paper can be summarised as follows. In our simulations of the New Zealand economy, living standards, as measured by consumption levels, approximately double by the year 2051, notwithstanding population ageing. It must be acknowledged that population ageing imposes a cost on living standards: we calculate this cost to be of the order of 12% by 2051. This magnitude varies very little under different demographic scenarios and parameter values. That is to say, the simulations suggest that if we could somehow turn back the clock, remove the post-war baby boom and maintain current rates of fertility, mortality and immigration indefinitely, then living standards in 2051 could be double their present levels, plus about 12%.

Under our benchmark simulation, it is optimal for national savings to rise by less than 0.5 percentage points of GDP for approximately the next 10 years and then to fall below current levels. Varying the demographic and macroeconomic parameters produces somewhat different results: under some parameter settings it is optimal for savings to rise somewhat higher, and under other settings it is optimal for savings to fall immediately. Given the uncertainties surrounding future demographic and economic developments, there is inevitably some uncertainty about the optimal level of national savings. In summary, however, the results suggest that population ageing *per se* would not justify increases in national savings significantly beyond those contemplated by current policies.

The remainder of the paper is organised as follows. Section Two of the paper explains the method of constructing the demographic projections under alternative assumptions about rates of fertility, mortality, and immigration. It describes the impact of the projections on the support ratio in each case. Section Three describes the Ramsey-Solow model of optimal savings. Section Four presents the results of the simulations, including an analysis of the sensitivity of the results to alternative specifications and a comparison between birth cohorts. The final section summarizes the findings and discusses the policy implications.

2 Demographic projections

2.1 Construction of population projections

The rapidity and degree of population ageing in New Zealand depend on future rates for fertility, mortality, and migration. All these rates, and hence the course of population ageing, are uncertain. Researchers have recently begun incorporating uncertainty into their models by carrying out stochastic simulations (Creedy and Scobie 2002, Lee and Edwards 2001). In this paper, however, we adopt the more conventional 'scenario' approach. The scenario approach is unable to provide probabilities for the various scenarios, and cannot cover the range of possible futures (here, for instance, we do not allow for the possibility of very low mortality combined with very low fertility.) But it is transparent and easy to implement, and it allows the contrasting effects of variation in fertility, mortality, and migration to be identified.[1]

We make assumptions about fertility, mortality, and migration, and for each we construct 'base' and 'alternative' demographic series. Combining the three base series gives a base population variant, which we use as a benchmark. Replacing each of the base series in turn with the alternative series generates three more population variants, which can be used to illustrate the effects of changes in fertility, mortality, and migration rates. The demographic series are summarized in Table 1 and their construction explained in this section. Further technical details are provided in Appendix 1.

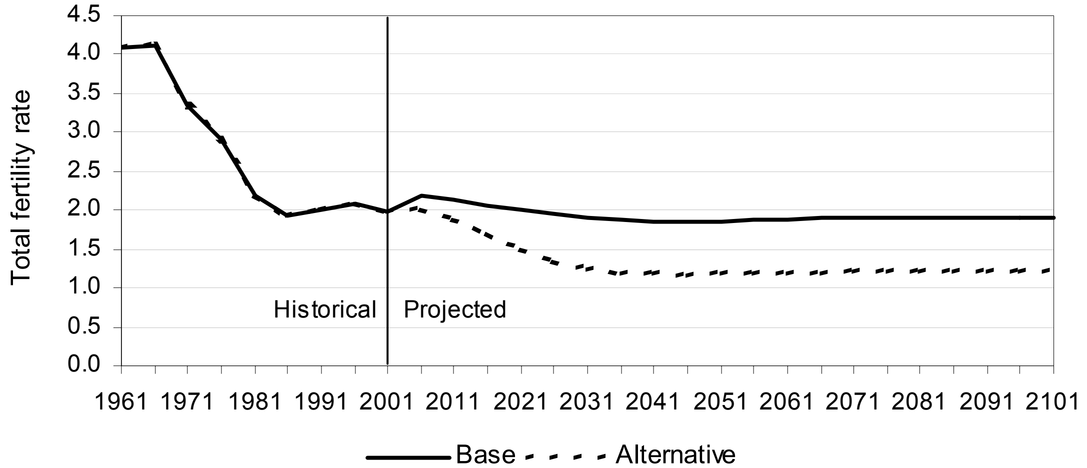
Table 1 – Summary of base and alternative demographic series

	Base	Alternative
Fertility	Total fertility rate falls from 2.0 in 2001 to 1.9 in 2025, and then remains at that level.	Total fertility rate falls from 2.0 in 2001 to 1.2 by 2025, and then remains at that level.
Mortality	Mean life expectancy rises from 78.5 years in 2001 to 86 years in 2053, and then remains at that level. (The Statistics New Zealand median assumption.)	Life expectancy increases by 2.3 years every decade for the entire projection period.
Migration	Net (inward) migration rate of approximately 1.5 per 1000.	Net (inward) migration rate of approximately 0 per 1000.

2.1.1 Fertility

The base and alternative series for fertility are shown in Figure 1, together with historical trends. Fertility is measured by the total fertility rate, the average number of children a woman would have over her lifetime if the year's age-specific fertility rates were to prevail indefinitely. Demographers usually project fertility by choosing a path for the total fertility rate and using an assumption about age-patterns to derive age-specific rates. Basing projections on the total fertility rate can, however, be problematic if the age-pattern of fertility is changing, as has been occurring in New Zealand. To avoid this problem, the fertility variants shown in Figure 1 were calculated using a cohort approach, described in Appendix 1. The base variant was designed to be similar to Statistics New Zealand's median fertility variant (Statistics New Zealand 2000), while the alternative variant brings New Zealand fertility close to the lowest-fertility countries in the OECD.

Figure 1- Total fertility rate under the base and alternative fertility series

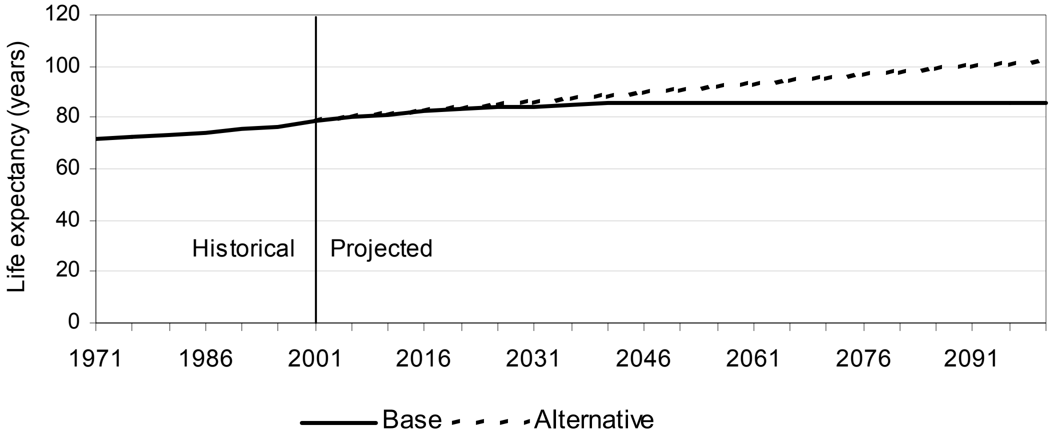


[1] The second author of this paper is currently undertaking work on the general advantages and disadvantages of using scenarios to test sensitivity to demographic variables.

2.1.2 Mortality

The base mortality series uses the same life expectancies as Statistics New Zealand's median mortality variant (Statistics New Zealand 2000) in which life expectancy is assumed to stop increasing by mid-century. The alternative variant assumes that life expectancy maintains a constant rate of increase. Although Statistics New Zealand's assumption that the increase in life expectancy will slow is still widely accepted, predictions of imminent slow-downs have been repeatedly proved wrong (Preston, Heuveline and Guillot 2001:132), and the notion that life expectancy may continue to improve past mid-century is not necessarily over-optimistic. Details of the derivation of age-specific mortality rates from the life expectancies are given in Appendix 1.

Figure 2- Life expectancy under the base and alternative mortality series



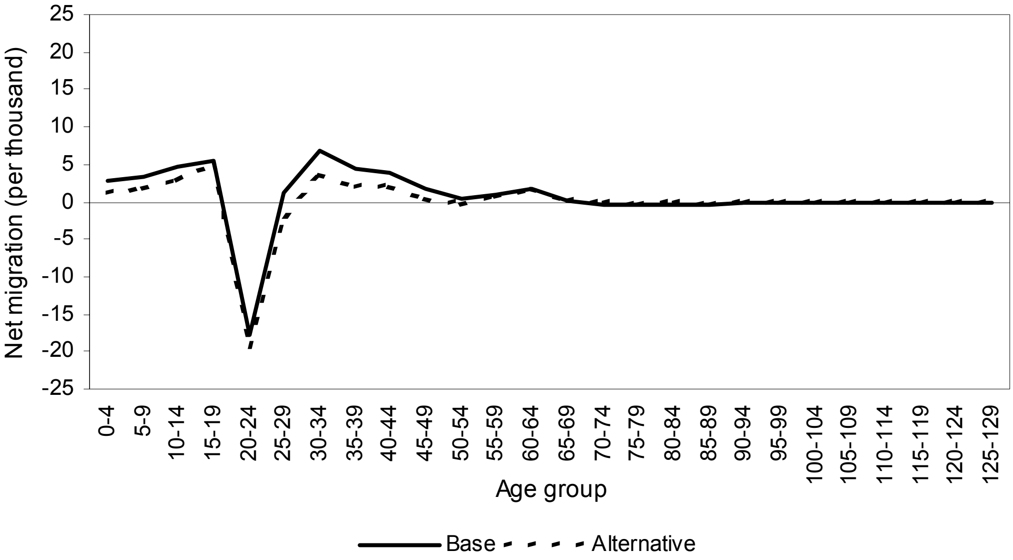
Note: 'Life expectancy' refers to the life expectancy at birth for males and females combined.

2.1.3 Migration

The migration series, which are graphed in Figure 3, are expressed as age-specific net migration rates. They show net inward migration per thousand population. When combined with New Zealand's 2001 population structure, these rates give net migration levels of slightly over 5,000 migrants per year for the base variant and 0 for the alternative variant.[2] The figure of 5,000 was chosen to match Statistics New Zealand's median variant, and the figure of 0 to reflect the fact that averages for net migration over recent decades have typically been well under 5,000. The age-specific rates are assumed to remain constant over the entire projection period, though the absolute number of net migrants changes as population size and age-structure change. The age-specific migration rates were derived by regressing individual age-sex specific rates against overall net migration levels for the years 1996-2001. Details are provided in Appendix 1. As indicated in Figure 3, rates for the 20-24 year age group are negative, even though overall net migration is by assumption positive or zero; this has in fact been the pattern in the recent years.

[2] This is somewhat simplified; for a more precise description see Appendix 1.

Figure 3 - Age-specific net migration rates in the base and alternative migration variants, 2001-2101



2.1.4 Population projections

The series for fertility, mortality, and migration were combined to give four population variants, as shown in Table 2. The projections were carried out using the standard cohort component projection methodology (Preston *et al* 2001). Data on the age-sex structure in the base year were obtained from the 2001 Census.[3]

Table 2 - Construction of population projections from fertility, mortality, and migration variants

Variant	Fertility	Mortality	Migration
Base	Base	Base	Base
Low fertility	Alternative	Base	Base
Low mortality	Base	Alternative	Base
Low migration	Base	Base	Alternative

Figure 4 shows trends in total population for the four variants. As the low-fertility variant illustrates, a sharp decline in fertility would entail rapid population decline. The low mortality variant portrays an interesting situation: constantly-improving mortality and a small migration surplus more than compensate for a fertility rate slightly below replacement level, so that population growth continues to be positive.

[3] Data obtained from the Statistics New Zealand website. Statistics New Zealand's tabulated results combined aged 85 and above. These were disaggregated using the extended life tables described in Appendix 1 and an assumption that the population was stable.

__Figure 4 - Population totals for the four population variants__

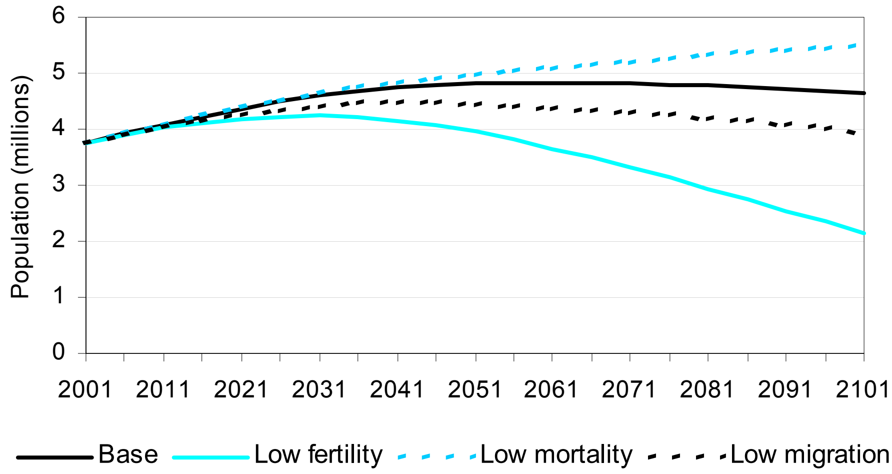
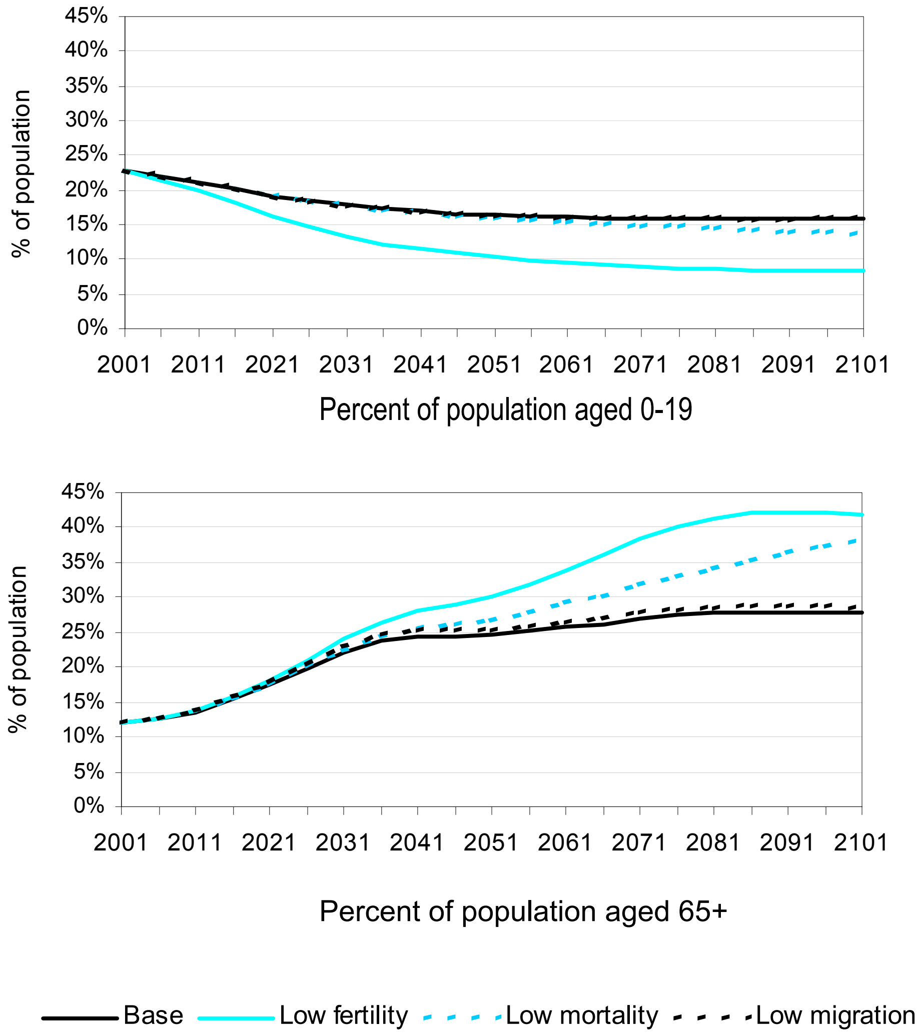


Figure 5 indicates the changes in age structure associated with each of the four variants. The population share of 0-19 year olds is virtually the same in all variants except the low-fertility variant, where it falls quickly and remains at low levels. Both the low-fertility and the low-mortality variants are associated with large increases in the population share of older people. (The idea that low fertility would raise the proportion in older age groups can appear strange; the explanation is that a fall in birth rates raises the ratio of those born many years previously to those born recently.) The population share of young people and old people is very similar under the base and low-migration variants. This is partly because the alternative migration assumption is relatively moderate, but also because small changes to migration rates do tend to affect population *size* more than they affect population *structure*.

In the base, low fertility, and low migration variants, the proportions in each age group stop changing by the end of the projection period. These populations have reached the condition known in demography as ‘stability’ whereby, in response to a sustained period of constant fertility, mortality, and migration rates, all age groups are growing or shrinking at the same speed. The low-mortality variant does not reach stability since mortality in this case is not constant.

Figure 5 - The percentage shares of selected age groups in the total population



2.2 Support ratios

2.2.1 Definition of support ratios

What is the economic significance of the differences in age structure? The index of age structure most commonly cited in discussions of population ageing is the ‘dependency ratio’, defined as the number of people outside the working ages divided by the number of people in the working ages. Cutler *et al*. (1990) introduce a more economically meaningful index called the ‘support ratio’, which incorporates information about variation by age and sex in productivity and consumption. The support ratio is defined as follows:

$$\text{Support ratio} \equiv \alpha = \frac{\begin{bmatrix}\text{Labour force in equivalent} \\ \text{worker units}\end{bmatrix}}{\begin{bmatrix}\text{Population in equivalent} \\ \text{person units}\end{bmatrix}} \equiv \frac{L}{P} = \text{Scaling factor} \times \frac{\sum_i w_i N_i}{\sum_i b_i N_i} \qquad (1)$$

where N_i is the number of people in age-sex group i, w_i is a measure of the productivity of age-sex group i, and b_i is a measure of the consumption needs of age-sex group i. Because the support ratio has no units and is a pure number, the analyst is free to choose a scaling factor. When conducting the simulations we set the scaling factor so that the support ratio equals 100 in the base year. Note that the support ratio reduces to the reciprocal of the dependency ratio if w_i is set to 1 for all those in the working ages and 0 elsewhere, and b_i is set to 1 for all those outside working ages and 0 elsewhere.

The effective labour force, effective population, and support ratio are key inputs for the model described below. To obtain series for these variables, we calculated values for the b_i's and w_i's, the consumption and productivity weights.

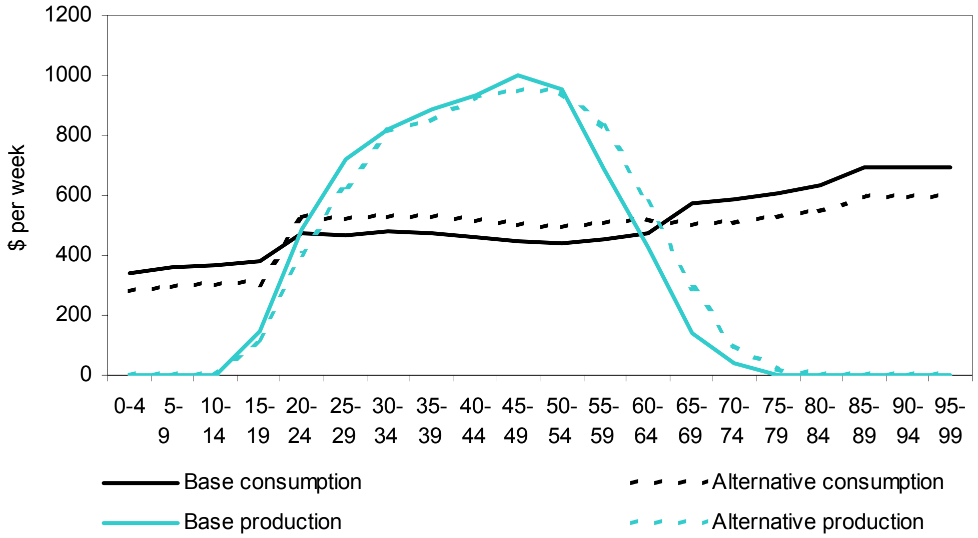
2.2.2 Consumption weights

Two sets of consumption weights—a 'base' set and an 'alternative' set—were constructed. The two sets have the same values for public consumption, and different values for private consumption. Data on public consumption by age and sex were calculated from Creedy and Scobie (2002), Ministry of Health (2002), and National Accounts data. Profiles for private consumption were constructed from adult equivalence scales, which are weights, used in analysis of household data, that are meant to capture the relative consumption needs of different age groups. The 'Revised Jensen Equivalence Scale', which has been applied in empirical work in New Zealand (Jensen 2001) was used to construct the base weights. A scale used by Guest and McDonald (2001: 123) in an application of the Ramsey model to Australia was used to construct the alternative weights. The two equivalence scales are shown in Table 3. The public and private age-sex profiles were both normalized, and a weighted mean was taken; the weights were equal to the proportion of total consumption accounted for by public and private consumption, as shown in the National Accounts. Consumption weights for males and females combined are graphed in Figure 6. The scaling factor has been chosen so that the weights equal weekly per capita consumption in the model during the initial year. Using the base weights, for instance, the average 0-4 year old has a consumption level of $340 per week in the initial year. The underlying numbers are presented in Appendix Table 1.

Table 3 - Equivalence scales used in calculation of consumption weights

Age group	Revised Jensen Equivalence Scale (used for base weights)	Guest and McDonald scale (used for alternative weights)
0-19	0.73	0.50
20-64	1.00	1.00
65+	1.00	0.75

Figure 6 - Consumption and productivity weights, males and females combined



Note: The weights have been scaled so that they show age-specific consumption and production in the model during the initial year.

2.2.3 Productivity weights

'Base' and 'alternative' sets of productivity weights were also calculated. In both sets, real wages were assumed to equal the marginal product of labour, and the productivity weight for an age-sex group was proportional to the average wage for the group multiplied by the fraction of the group who are employed. The weights were held constant over the entire projection period. The base weights were constructed from data on wages and employment for 2001 were obtained from the Statistics New Zealand's June Quarter 2001 New Zealand Income Survey. The alternative weights were designed to reflect the fact that future cohorts are likely to be longer-lived and better-educated than their predecessors (Dowrick and McDonald 2001). For males aged 15-49, wages and employment rates were assumed to be the same as they were in the 2001 survey. Above age 49, however, the profile for wages and employment were shifted 2.5 years to the right: wages and employment rates for males aged 50-54 were assumed to equal the average of the survey figures for males aged 45-49 and 50-54; wages and employment rates for males aged 55-59 were assumed to equal the average of the survey figures for males aged 50-54 and 55-59; and so on. Wages and employment rates for females were assumed to be identical to those for males, except that females' employment rates during the prime childrearing ages of 25-39 were assumed to be only 90% as high as those of men. The base and alternative weights are shown in Figure 6. A scaling factor has been applied so that the graph shows age-specific production in the initial year of the model. The underlying data are given in Appendix Table 2.

2.2.4 Results for the support ratios

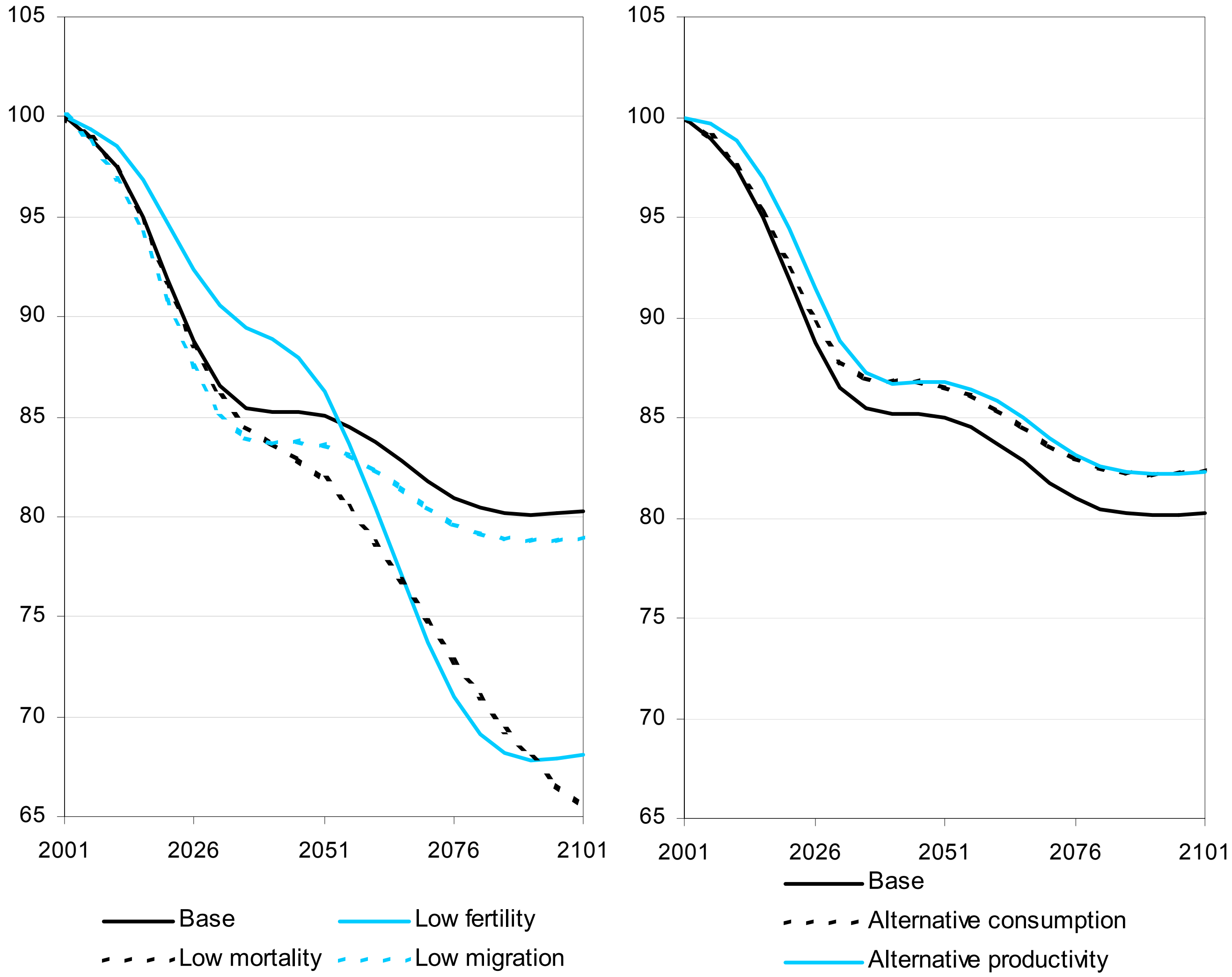
We calculated six sets of projections for the support ratio. The first four sets were obtained by combining the four population variants described above with the base consumption and productivity weights. The remaining two were obtained by combining the base population variant with the alternative variants for consumption and productivity. The various combinations are summarized in Table 4.

Table 4 - Construction of support ratio projections from demographic, consumption, and productivity variants

	Demographic rates				
Projection	Fertility	Mortality	Migration	Consumption	Productivity
Base	Base	Base	Base	Base	Base
Low fertility	Alternative	Base	Base	Base	Base
Low mortality	Base	Alternative	Base	Base	Base
Low migration	Base	Base	Alternative	Base	Base
Alternative consumption	Base	Base	Base	Alternative	Base
Alternative productivity	Base	Base	Base	Base	Alternative

Figure 7 presents the results of the calculations. In all cases apart from the low-mortality scenario, the support ratio stops changing towards the end of the projection period, as the condition of population stability (discussed above) is reached. Predictably, the support ratio falls furthest in the low-fertility and low-mortality cases. It is worth noticing, however, that the support ratio is initially *higher* in the low-fertility case than in the base case. The reason can be identified in Figure 5. In the low-fertility case, the proportion of the population aged 0-19 falls sharply from the very beginning of the projection, which pushes the support ratio upwards. This effect is eventually counteracted by the rise in the proportion aged 65 and over, but there is a delay of several decades before the rise in the population aged 65 and over occurs. Comparison of the base and alternative consumption projections shows the effects of using different consumption weightings: the support ratio falls further in the base case than in the alternative case because the Jensen equivalence scale gives a higher relative weight to the consumption of older adults than the Guest and McDonald scale. Much the same is true for the alternative productivity weights. The support ratio falls further in the base case than in the alternative case because older adults are more productive, relative to younger adults, in the alternative case.

Figure 7 - Projections of support ratios



3 Analytical framework

This section describes the Ramsey-Solow model of optimal saving, as modified to incorporate population ageing. Before giving a detailed description of the model, this section discusses two important issues for the application of the model: the existence of an endogenous interest rate premium, and labour productivity growth.

3.1 Endogenous interest rate premium

This study extends the analytical approach adopted in the seminal work by Cutler *et al*. (1990) and later by two of the same authors in Elmendorf and Sheiner (1999). They apply a Ramsey-Solow model, allowing for heterogeneous consumers and workers, to the case of both a closed economy and a small open economy. Yet neither the small open economy nor the closed economy is an entirely appropriate framework for an economy such as New Zealand. While it is both small and relatively open, New Zealand almost certainly does not face perfect capital mobility in that it does not face a perfectly elastic supply of foreign capital.

Indeed there is extensive evidence for many other countries that capital is not perfectly mobile and is in fact quite immobile.[4] The most important explanation according to Gordon and Bovenberg (1996) is asymmetric information between investors of different countries. In particular, foreign investors know less about the economic prospects of another country than do the residents of that country. Foreign investors therefore tend to earn a lower return on their investments than do domestic investors in the borrowing country. This drives a wedge between the equilibrium interest rate in borrowing and lending countries, where the size of the wedge depends on the amount of international capital flows. There is also evidence that small countries face a risk premium, the size of which depends on their level of foreign debt[5]. The result of each of these mechanisms – asymmetric information and a risk premium - is that the equilibrium interest rate in capital importing countries is higher than the interest rate in capital exporting countries.

The existence of an endogenous interest rate premium is potentially important in determining the effect of a demographic shock on optimal saving. With a constant world interest rate and a constant rate of time preference the rate of return to saving, following a demographic shock, is unaffected by the path of debt. This is not the case, however, if the interest rate is affected by the level of foreign debt for example. In that case the marginal cost of borrowing increases as debt increases and conversely falls as debt falls. Hence the rate of return to saving is a function of the path of debt and is therefore endogenous as it is in the closed economy case. The extent to which an interest rate premium matters for the path of optimal saving was investigated in Guest and McDonald (2002b) by simulations of the same model as that applied in this study. They found that the path of optimal saving in response to a demographic shock – in particular, a lower fertility rate – becomes close to that for the closed economy for quite small endogenous interest rate premia.

In representing an endogenous interest rate premium as an upward sloping supply price of foreign capital, we draw on Glenn (1997) who considers the impact of a rise in the price of an imported production input for an economy facing an interest rate that depends on its level of foreign liabilities.

3.2 Demographic change and labour productivity growth

An important question concerns the impact of demographic change on labour productivity growth. This could quite easily go either way, as discussed by Cutler *et al.* (1990: 38). On the one hand slower population growth makes innovation less profitable by reducing the gains from economies of scale through the spreading of fixed costs; and a smaller youth share of the population may reduce innovation through a loss of “dynamism”. Also, in endogenous growth models of the type in Steinman, Prskavetz, and Feichtinger (Steinman, Prskawetz and Feichtinger 1998), lower population growth results in less human capital accumulation and therefore a lower growth rate of labour productivity. On the other hand, in other endogenous growth models, slower labour force growth implies a higher relative price of labour and therefore greater incentive to innovate through capital investment or research and development (Romer 1990). Also, diseconomies of higher population growth, through congestion for example, can reduce labour productivity growth.

The empirical evidence on the effect of demographic change on labour productivity is relatively scarce. Galor *et al.* (1997), using panel data on 73 countries, find evidence that countries with smaller average family size attain higher labour productivity. They attribute

[4] For a discussion of the various explanations see Gordon and Bovenberg (1996); and for a survey of the evidence on the Feldstein-Horioka puzzle as an indicator of imperfect capital mobility see Coakley, Kulasi and Smith (1999).

[5] For Australia, see Juttner and Luedecke (1991)

this to the extra resources that parents with smaller families can provide to each of their children to finance their education. Similarly, Ahituv (2001), using panel data on 114 countries, concludes that lower population growth increases GDP per capita growth. The principal mechanism here seems to be the effect of the time required for child rearing on the labour input of their parents. Fewer children imply less time input from parents thereby freeing up labour time (although, properly measured, this would not amount to an effect on labour productivity but rather a change in labour input). On the other hand Hondroyiannis and Papapetrou (1999) find no long run relationship between the fertility rate and the output growth rate using time series data for the U.S. On the basis of this ambiguity in theory and evidence, Guest and McDonald (2002b) assume that demographic change has no net effect on labour productivity; that assumption is also adopted in this paper.

3.3 The model

The initial year for the model is 2001. We follow the approach in Cutler et al. (1990) in assuming that all variables, including demographic variables, are initially on steady state paths. As Cutler et al. (1990) acknowledge, this does not represent reality because demographic variables are not stable at the present time; but as they say "it is not obvious how best to model [demographic change] as a single shock" (p.23). The approach they and others (e.g. Elmendorf and Sheiner, 2000) adopt is to assume that the population has been stable—that all age groups have been growing at the same rate—until 2001, at which point an unanticipated demographic shock occurs so that employment and population follow the projections described above. Table 5 sets out the definitions of the variables used in the model

It is assumed that a central planner maximises an inter-temporal, time-additive social welfare function of general form:

$$V = \int_{t=0}^{\infty} N_t \frac{c_t^{1-\beta}}{1-\beta} e^{-\theta t} dt \qquad (2)$$

Output is produced according to a Cobb-Douglas production function with constant returns to scale:[6]

$$Y = AK^{\gamma} L^{1-\gamma} \qquad (3)$$

which can be expressed in terms of the labour force in efficiency workers as:

$$y = f(k) = k^{\gamma} \qquad (3a)$$

Variables are defined in Table 5. Labour, *L*, is assumed to be supplied exogenously. This ensures that the labour market clears: shifts in the marginal product of labour lead to equal shifts in the real wage. Capital and debt accumulate according to the following accumulation equations:[7]

[6] A Cobb-Douglas production function implies an elasticity of substitution in production of 1. An alternative elasticity of 1.33 was considered as a sensitivity exercise and was found to have a negligible effect on the consumption path (see Table 7).
[7] The non-Ponzi game condition in which the level of debt does not grow as fast as the interest rate is also assumed to hold. The symbol '·' denotes differentiation with respect to time.

Table 5 - Symbols for variables used in the model

N	Population in natural units
L	Labour force in equivalent worker units (refer to Section 2.2.1)
P	Population in equivalent person units (refer to Section 2.2.1)
α	Support ratio = L/P*
n	Growth rate of N
l	Growth rate of L
p	Growth rate of P
A	Total factor productivity
g	Rate of Harrod-neutral technical progress or labour productivity growth
L^E	Labour force in efficiency units $L_t^E = A_0 e^{gt} L_t$
P^E	Population in efficiency units $P_t^E = A_0 e^{gt} P_t$
Y	Output
K	Aggregate capital stock
D	Aggregate foreign liabilities, denominated here as debt
C	Aggregate consumption
y	Output per worker measured in efficiency units (Y/L^E)
k	Capital stock in efficiency units per worker ($k = K/L^E$)
d	Debt in efficiency units per worker ($d = D/L^E$)
c	Consumption in efficiency units per worker ($c = C/P^E$)
V	Measure of welfare maximized by social planner
r	Rate of interest
θ	The social planner's rate of time preference
δ	Rate of depreciation
γ	Capital elasticity of output
β	Degree of aversion to variability in average living standards over time[8]
q	The shadow price of capital
i	Investment in efficiency units
$J(i)$	The units of output required to increase the capital stock by i units, measured in efficiency units
μ	Parameter in the adjustment cost function
λ	Interest rate premium

[8]Because consumption levels grow steadily over time due to the assumed constant rate of productivity growth, a dollar of future consumption is discounted because its marginal social welfare is lower than a dollar of consumption today. For example, an extra dollar to us is not valued as much as an extra dollar to one of our grandparents, simply because our material standard of living is much higher than theirs. The parameter, β, measures this rate of discount.

$$\dot{d} = (r(d) - l - g)d + \frac{c}{\alpha} + J(i) - f(k) \quad (4)$$

and

$$\dot{k} = i - (l + g + \delta)k \quad (5)$$

Equation (4) is the national income accounting identity in efficiency units per worker. The left hand side is the change in net foreign assets, or current account deficit. The first term on the right hand side, *r(d)*, is the interest charge on the outstanding stock of foreign liabilities; the term *(l+g)d* is subtracted because the growth of effective workers reduces debt per effective worker; the term (c/α) is consumption converted into per worker units by dividing by the support ratio; and *J(i)* represents investment and is the units of output required to increase the capital stock by *i* units. Hence *J(i) – i* represents the adjustment costs in terms of output required to transform goods into output. Equation (5) describes the additions to the capital stock from new investment after deducting depreciation and the growth rate of effective workers. In the simulations we adopt an adjustment cost function of the form:

$$J(i) = i(1 + 0.5\mu\frac{i}{k}) \quad (6)$$

and a simple linear function for *r(d):*[9]

$$r = \bar{r} + \lambda d \quad (7)$$

Investment, consumption and debt are determined simultaneously. Debt is determined by (4), and consumption by

$$\frac{\dot{c}}{c} = \frac{1}{\beta}\left(\frac{\partial r}{\partial d}d + r - \theta + n - p\right) - g \quad (8)$$

Equation (8) is the standard Euler equation which is derived by equating the marginal rate of substitution in consumption with the return to saving. Investment is determined by equating the social value of an additional unit of investment with the social cost of capital, which gives[10]

$$\dot{q} = (r + \delta)q - \left(f'(k) + \frac{(q-1)^2}{2\mu}\right) \quad (9)$$

where

$$\frac{i}{k} = \frac{q-1}{\mu} \quad (10)$$

and *q* is the shadow price of capital.

The steady state implies that $\dot{d} = 0$, $\dot{q} = 0$, $\dot{c} = 0$ and $\dot{k} = 0$ which yields the following steady state equations:

[9] The linear form can be seen as an approximation to a non-linear upward sloping function for the interest rate. Simulations suggested that the optimal paths of living standards and saving are very insensitive to alternative, non-linear, forms.
[10] See Barro and Sala-i-Martin (1995: 123).

$$c = \alpha\big(f(k) - J(i) - (r(d) - l - g)d\big) \quad (11)$$

$$f'(k) + \frac{(q-1)^2}{2\mu} = (r+\delta)q \quad (12)$$

$$\theta = r - \beta g \quad (13)$$

By substituting (5) into (10),

$$q = 1 + \mu(n + g + \delta) \quad (14)$$

The value of μ is found by substituting (14) into (12); this gives $\mu = 3.0$.

Equation (11) illustrates the three effects of ageing on steady state consumption. The most significant effect turns out to be the dependency effect reflected in the change in α (see below for a more complete discussion of the decomposition of the effects of ageing on living standards). There are two other less significant effects. The capital widening effect is given by the change in employment growth, *l*. The third and, it turns out, the smallest effect is given by changes in values of *k* and *d* in the new steady state. Consumption is boosted to the extent that *k* is higher and *d* is lower in the initial steady state level. (This in turn depends on the assumption about the interest rate premium. See Appendix 2 for a discussion.)

Other parameter values are set equal to those in the Reserve Bank of New Zealand (RBNZ) core model, described in Black, Cassino, Drew, Hansen, Hunt, Rose, and Scott (1997), unless specified otherwise. The RBNZ model is similar in structure to the model presented here in that firms produce, and consumers consume, a single composite good that is produced according to a Cobb-Douglas production function with constant returns to scale; and consumption is based on inter-temporal optimisation in both models. One important difference is that in the RBNZ model 30% of consumers are not forward-looking but, instead, base consumption decisions on a "rule of thumb" which is defined consumption equal to 100% of household disposable income.[11]

Both models are calibrated to an initial steady state with the following common parameter values: capital to output ratio = 1.7, δ=0.085, β=1.52,[12] α=0.35, *r*=0.05, *g*=0.015. The steady state ratio of foreign liabilities to GDP in the RBNZ model is 1.05 whereas 0.8 is chosen in this model because this is approximately the current value and in our judgement a value of 1.05 seemed a little too high as a sustainable long run value. Another significant departure from the RBNZ model is the assumption that the interest rate premium is endogenous implied in (6). Our base case value is λ=0.02 but, given uncertaintly about a true value for this parameter, we conduct sensitivity to several alternative values.

Once the initial steady state values of endogenous variables are determined the optimal path following the demographic shock is found by a grid-search algorithm that determines the initial values of consumption and the capital stock that lead all endogenous variables to their new steady state values. This implies that, immediately following the shock, the values of all endogenous variables jump on to their optimal paths.

[11] The RBNZ model incorporates other complexities that are ignored in the model in this paper, such as an exchange rate and a government sector.

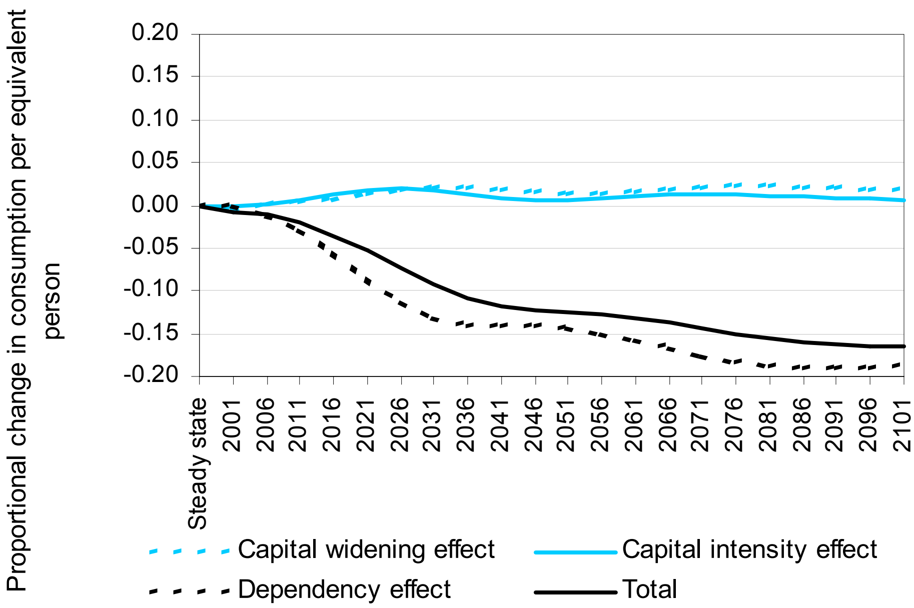
[12] The effect of the value of β on the optimal consumption path is largely offset by changes in the rate of time preference via (13). The discount rate, r, is the sum of two components: the planner's rate of time preference and a term βg which is the rate at which the marginal social value of income declines.

4 Simulation results

4.1 The impact of ageing on living standards

We define living standards as consumption per equivalent person, using the age-specific consumption weights b_i described above to calculate the population in equivalent persons. Ageing affects living standards through three mechanisms: the dependency effect, the capital-widening effect, and the capital intensity effect (Elmendorf and Sheiner 1999). The dependency effect describes the lower consumption possibilities that result from fewer workers relative to consumers. This is evident from (11) in which a lower support ratio, α, implies lower steady state consumption for any given capital-output ratio. The capital widening effect refers to the lower capital requirements of a more slowly growing labour force. In terms of (11), *J(i)* is lower allowing a higher value of *c*. Hence the capital-widening effect increases consumption possibilities. The capital intensity effect is a transitional effect as the smaller labour force temporarily increases the capital-labour ratio (or capital intensity). The higher capital-labour ratio lowers the marginal product of capital which reduces investment relative to saving and hence lowers the current account deficit. The lower current account deficit implies lower foreign liabilities, other things constant, implying a lower interest rate via (7). This in turn lowers the return to saving and raises current consumption. This effect unwinds as the economy works off the excess capital. Hence both the capital intensity effect and the capital widening effect provide temporary boosts to consumption which partially offset the dependency effect.

Figure 8 - Decomposition of the effects of ageing on living standards for base population variant



The paths of the three effects on living standards are illustrated in Figure 8. The size of each effect is calculated by decomposing the following identity:

$$c \equiv \left(\frac{c}{y}\right)\left(\frac{y}{\alpha}\right)\alpha \qquad (15)$$

where y=*f(k)*. The dependency effect is approximated[13] by $(c/\alpha)^0\left[\alpha^1-\alpha^0\right]$, where 0 refers to the initial levels and 1 refers to the post-shock levels. This gives the net effect of a reduction in youth dependency and an increase in old age dependency. The capital widening and capital intensity effects are given by, respectively, $(y\alpha)^0\left[(c/(y\alpha))^1-(c/(y\alpha))^0\right]$ and $(c/y)^0\left[y^1-y^0\right]$. As Figure 8 illustrates, the dependency effect is negative throughout the planning horizon and far outweighs the other two effects.

Table 6 - The effects of ageing on living standards

Year	Demographic assumptions				Sensitivity analysis with base variant demographics					
	Base	Low fertility	Low mortality	Low migration	λ=0.0	λ=0.01	λ=0.03	Elasticity of substitution in production = 1.33	Elasticity of substitution in consumption = 2.00	Elasticity of substitution in consumption = 0.33
Percentage chance in consumption per effective person relative to the case without population ageing										
2011	-2.1	0.1	-2.4	-1.9	-6.0	-2.5	-2.0	-1.5	-1.4	-2.9
2021	-5.6	-2.5	-6.0	-5.9	-7.0	-5.5	-5.6	-4.5	-6.4	-5.3
2031	-9.8	-5.3	-10.3	-10.1	-8.3	-9.4	-10.0	-8.5	-11.5	-8.7
2041	-12.0	-7.5	-13.8	-13.0	-9.0	-11.7	-12.2	-10.9	-13.0	-10.9
2051	-12.7	-9.3	-16.0	-13.7	-9.3	-12.6	-12.8	-11.7	-13.0	-12.1
2101	-17.0	-25.6	-31.0	-17.9	-10.0	-16.9	-17.0	-15.9	-17.1	-16.6
Net change in living standards given population ageing and 1.5% per annum productivity growth										
2011	13.9	16.2	13.7	14.2	10.1	13.6	14.1	14.5	14.6	13.2
2021	29.1	32.2	28.7	28.8	27.7	29.2	29.1	30.2	28.3	29.4
2031	46.5	51.0	46.0	46.2	48.1	46.9	46.3	47.8	44.8	47.6
2041	69.4	73.9	67.6	68.4	72.4	69.7	69.2	70.5	68.4	70.5
2051	97.8	101.2	94.5	96.8	101.2	97.9	97.8	98.9	97.6	98.5
2101	326.2	317.6	312.2	325.3	333.2	326.3	326.2	327.3	326.1	326.6
Effect of ageing on living standards in equivalent average annual productivity growth*										
2011	-0.21	0.01	-0.23	-0.19	-0.58	-0.25	-0.20	-0.15	-0.14	-0.29
2021	-0.27	-0.12	-0.29	-0.29	-0.34	-0.27	-0.27	-0.22	-0.31	-0.26
2031	-0.31	-0.17	-0.33	-0.32	-0.26	-0.30	-0.32	-0.27	-0.36	-0.28
2041	-0.28	-0.18	-0.32	-0.30	-0.22	-0.28	-0.29	-0.26	-0.31	-0.26
2051	-0.24	-0.18	-0.30	-0.26	-0.18	-0.24	-0.24	-0.22	-0.24	-0.23
2101	-0.16	-0.23	-0.27	-0.16	-0.10	-0.16	-0.16	-0.15	-0.16	-0.15

Figure 8 shows total optimal consumption reaching a new steady state some 16% below the initial steady state. This is the long run impact of population ageing on living standards. The speed at which consumption adjusts to the new steady state level depends on the change in the rate of return on saving (i.e. the interest rate) in response to ageing and on the elasticity of substitution in consumption (equal to $1/\beta$). The change in the return on saving is determined by the parameter λ in (7). In the case where the return to saving is exogenous, as it is in the case of a zero interest rate premium (where λ=0),

[13] Ignoring second-order terms.

the return to saving does not respond at all, allowing complete consumption smoothing; hence in that case consumption adjusts instantly to its new steady state level.[14] The elasticity of substitution in consumption affects the path of consumption through the degree of consumption smoothing: the higher the elasticity, the more that future consumption is discounted, implying more consumption smoothing and hence a faster speed of adjustment of consumption to its new steady state level.

Table 6 gives the magnitude of the effect of ageing on living standards at different stages over the next 100 years. It gives the results for the demographic assumptions described above: the base case, a low fertility scenario, a low mortality scenario and a high immigration scenario. It also gives results for base case demographics under alternative values of the elasticities of substitution in production and consumption, and for alternative values of λ, including the small open economy case where λ=0.

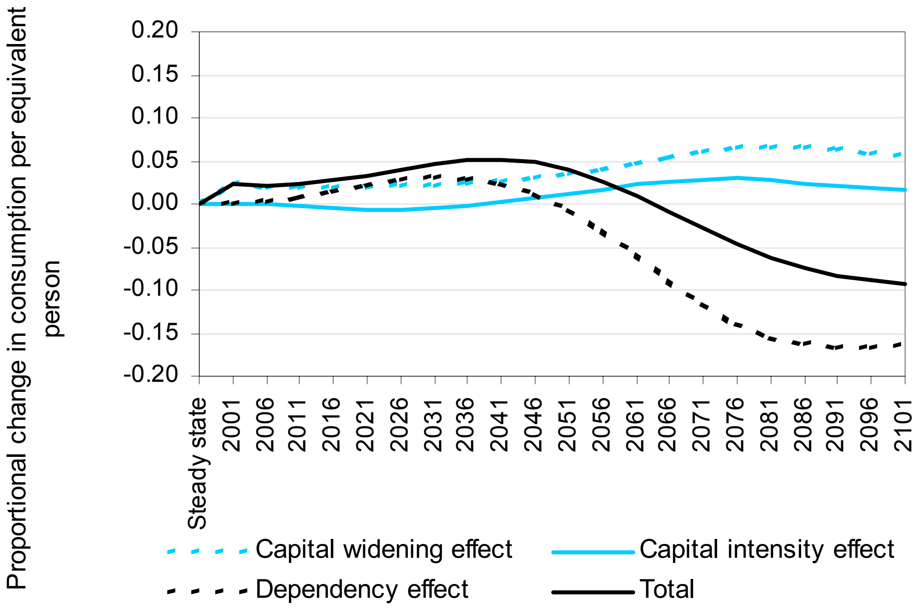
As indicated in the top left hand column of Table 6, by the year 2051, assuming base case demographics, population ageing will have reduced living standards by 12% from the level that they would have reached in the absence of population ageing. Nevertheless, living standards can be expected to approximately double by 2051 under base case demographics (see middle section of Table 6, left hand column). As the table shows, this result is quite insensitive to alternative demographic assumptions and alternative parameter values. Another way of describing the ageing effect is in terms of the equivalent loss of labour productivity growth. As the bottom section of Table 6 indicates, by 2051 population ageing will have reduced living standards by the equivalent of an annual reduction in productivity growth[15] of 0.24% with base case demographics. This would amount to a reduction from, for example, 1.5% p.a. to 1.26% p.a. Another way of saying this is that productivity growth would have to fall from 1.5% p.a. to 0.24% p.a. – that is, almost zero productivity growth year after year for 50 years – for living standards in 2051 to be below their level in 2001 on account of population ageing.

Such an apparently sanguine assessment of the effect of ageing on living standards is only as robust as the assumptions allow. However, the sensitivity analysis reported in Table 6, for alternative demographic scenarios and alternative parameter values, shows little variation in the effect of ageing on living standards. The largest variation in living standards compared with the base case occurs in the small open economy scenario (λ=0). This is because the adjustment to a lower rate of consumption occurs immediately resulting in a relatively big initial loss of living standards but a smaller loss of living standards later on. For example, the loss of living standards by 2011 is nearly three times as big as in the base case but by 2051 living standards are 25% higher than in the base case.

[14] The adjustment of consumption in not quite instant, because of the difference between the growth of the population in natural units, n, and in equivalent consumption units, p, (see (11)).

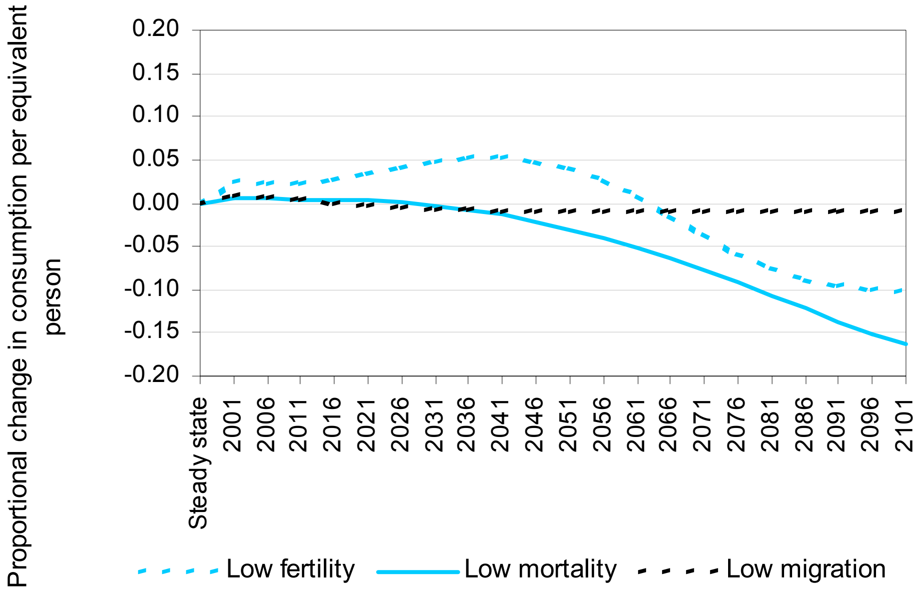
[15] Productivity growth is defined as labour augmenting.

Figure 9 - Decomposition of the difference in living standards between the base population variant and the low-fertility population variant



The other significant departure from the base case occurs for the low fertility scenario. In this case living standards are higher than in the base case until at least 2050. By 2100, however, living standards are lower. This is an interesting case in view of concerns about falling fertility rates in New Zealand as in many industrialised countries. The analysis here suggests that such concerns are ill-founded, which supports similar results for other countries (see, for example, Weil (1999) and Guest and McDonald (2002b)). In general, a lower (higher) fertility rate results in a higher (lower) standard of living for an initial period, which may last for decades, before the reverse occurs. Raising the fertility rate will always result in a transitory cost to living standards. As discussed in Section 2.2.4 reducing fertility initially raises the support ratio because the initial increase in youth dependency occurs without a commensurate reduction in old age dependency. Figure 9 illustrates the effect of lower fertility on living standards for the next 100 years. The net effect is decomposed into its three components: the dependency effect, the capital widening effect and the capital intensity effect. The dependency effect dominates the capital intensity and widening effects. When added to the dependency effect the latter two effects extend the period over which low fertility outperforms base fertility in terms of living standards.

Figure 10 - The effect of population variants on living standards, relative to the base variant



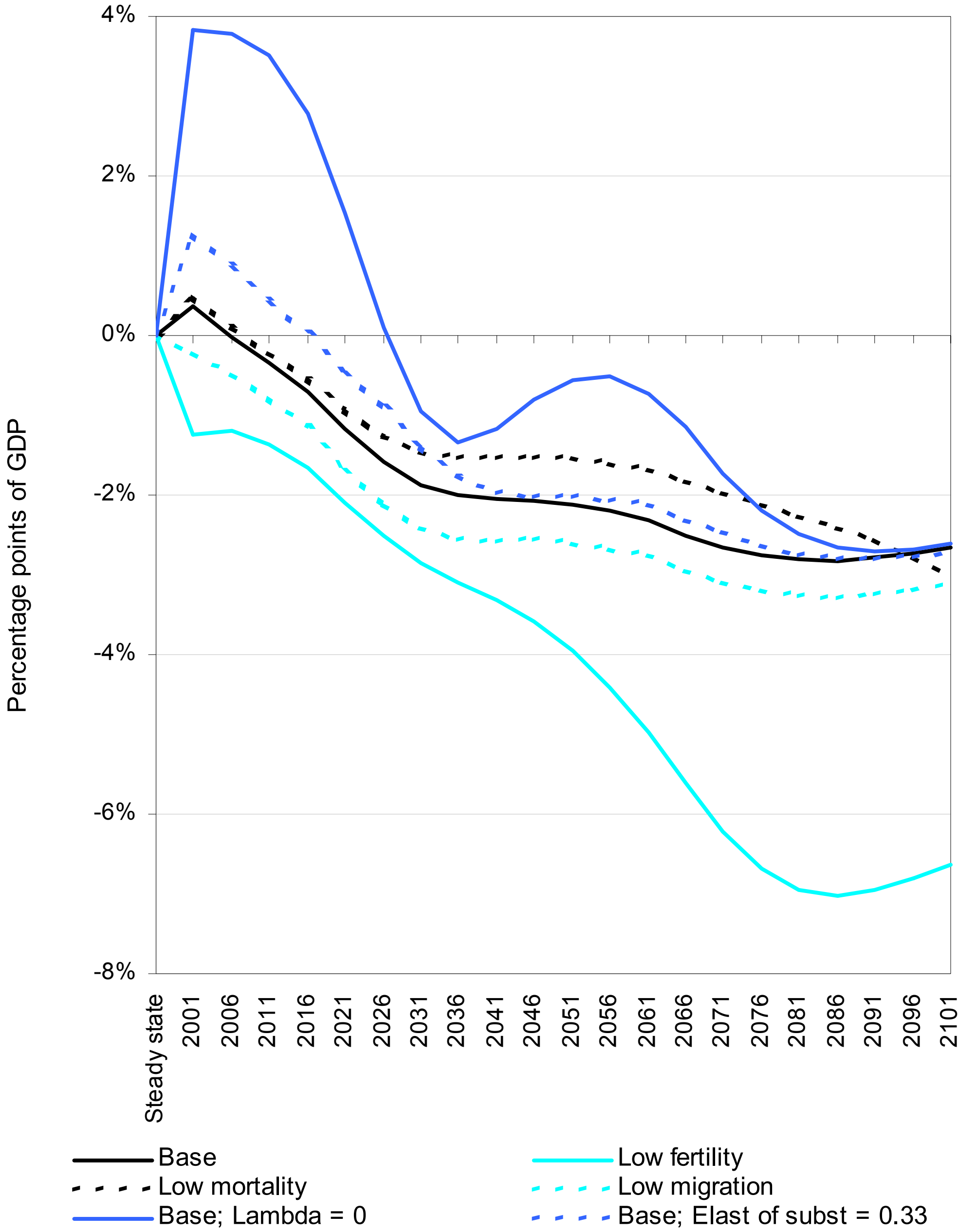
The effect of lower fertility and the other two alternative demographic scenarios on living standards over the next 100 years is illustrated in Figure 10. In both the low-fertility and low-mortality variants, living standards towards the end of the projection period are significantly lower than in the base variant. However, the low-mortality variant does not have the compensating advantage of an initial period of high consumption. Figure 10 shows low migration having much less effect on living standards than low fertility or mortality. This is partly because the alternative migration series is less extreme than the alternative fertility and mortality series. But it also reflects the fact that changes in migration levels generally have less effect on age-structure, and hence support ratios, than comparable changes in fertility and mortality.

4.2 The impact of ageing on optimal national saving

The size of the response of optimal saving is determined by the degree of consumption smoothing. The higher the degree of consumption smoothing, the more that any temporary fluctuations in income from a demographic shock are absorbed into saving. The degree of consumption smoothing is, as described above, determined by the elasticities of substitution in consumption and production and, in this model, by the responsiveness of the interest rate (the return to saving) to changes in foreign liabilities.

The case of a zero interest rate premium, with the highest possible degree of consumption smoothing, implies the biggest response of optimal saving. This is illustrated in Figure 11 by the prominent hump shape for the λ=0 case. In this case, optimal saving rises by nearly 4 percentage points of GDP. However, this is probably only a hypothetical case because, as discussed in the Section 3.1, the assumption of a zero interest rate premium is very unlikely to be met in practice.

Figure 11 - Percentage point change in national savings from the initial steady state



In the base case optimal saving rises by about one half of one percentage point for the first six years and then falls steadily over the next thirty years by a total of eight percentage points. The pattern is more extreme for the low fertility scenario. In that case saving falls by about 20% over the next thirty years and then rises by at least 40% over the following fifty years to a point that is about 25% above its current level by 2080. As mentioned above the low fertility scenario implies fertility levels equal to the lowest-fertility countries in the OECD. The simulation of a smaller elasticity of substitution in consumption (from 0.66 to 0.33) shows a slightly larger hump response in optimal saving than in the base case (see Table 7).

These results have implications for the change in savings needed to achieve any given target growth in living standards. For the base case parameter values, substantial growth of living standards can be achieved with no increase in current saving. In the unlikely case of a zero endogenous interest rate premium, or where the elasticity of substitution in consumption is very low, the conclusion is less clear. In those cases the results show that an increase in the current national saving rate of several percentage points for perhaps the twenty years would be required to optimise the growth of living standards. The growth of living standards in those cases is nevertheless substantial.

Table 7 – Percentage point change in national saving from the initial steady state

	Demographic assumptions				Base variant demographics		
Year	Base	Low fertility	Low mortality	Low migration	Lambda = 0.0	Elasticity of substitution in production = 1.33	Elasticity of substitution in consumption = 0.33
2004	0.30	-1.05	0.42	-0.23	3.92	0.04	1.19
2006	-0.02	-1.19	0.11	-0.50	3.78	-0.30	0.87
2011	-0.34	-1.38	-0.22	-0.79	3.52	-0.22	0.44
2021	-1.17	-2.09	-0.92	-1.66	1.53	-1.16	-0.44
2031	-1.87	-2.84	-1.47	-2.43	-0.96	-1.74	-1.39
2041	-2.05	-3.31	-1.54	-2.58	-1.17	-2.27	-1.97
2051	-2.12	-3.95	-1.54	-2.60	-0.56	-2.46	-2.03
2101	-2.66	-6.64	-3.03	-3.13	-2.61	-2.97	-2.72

Calculations not shown here suggest that if saving and investment rates remained at their current levels indefinitely, living standards would be at least maintained over the next 50 years provided labour productivity growth is greater than 0.28%, given base case demographics. That is, the cost of ageing would be 0.28% in terms of lost productivity growth. Recall from Table 6 that the corresponding figure for the optimal consumption path is 0.24% (under base case demographics). This implies that the cost of not optimising (that is, not smoothing consumption), but instead maintaining current saving and investment rates, is only 0.04% per annum in productivity growth – for example, a reduction from 1.5% p.a. to 1.46% p.a. An alternative way of describing the cost of maintaining current saving and investment rates, instead of following the optimal path, is to say that living standards would be, on average, 1.8% lower over the next 50 years, compared with living standards along the optimal path. It is emphasised that this 1.8% figure is not a cumulative figure like the 0.04% productivity growth figure – rather it is the average difference in levels of consumption per person at any time. Both of these numbers are small; they indicate that savings and investment would not have to change dramatically to attain optimal levels.

4.3 Comparison of the consumption profiles of successive cohorts

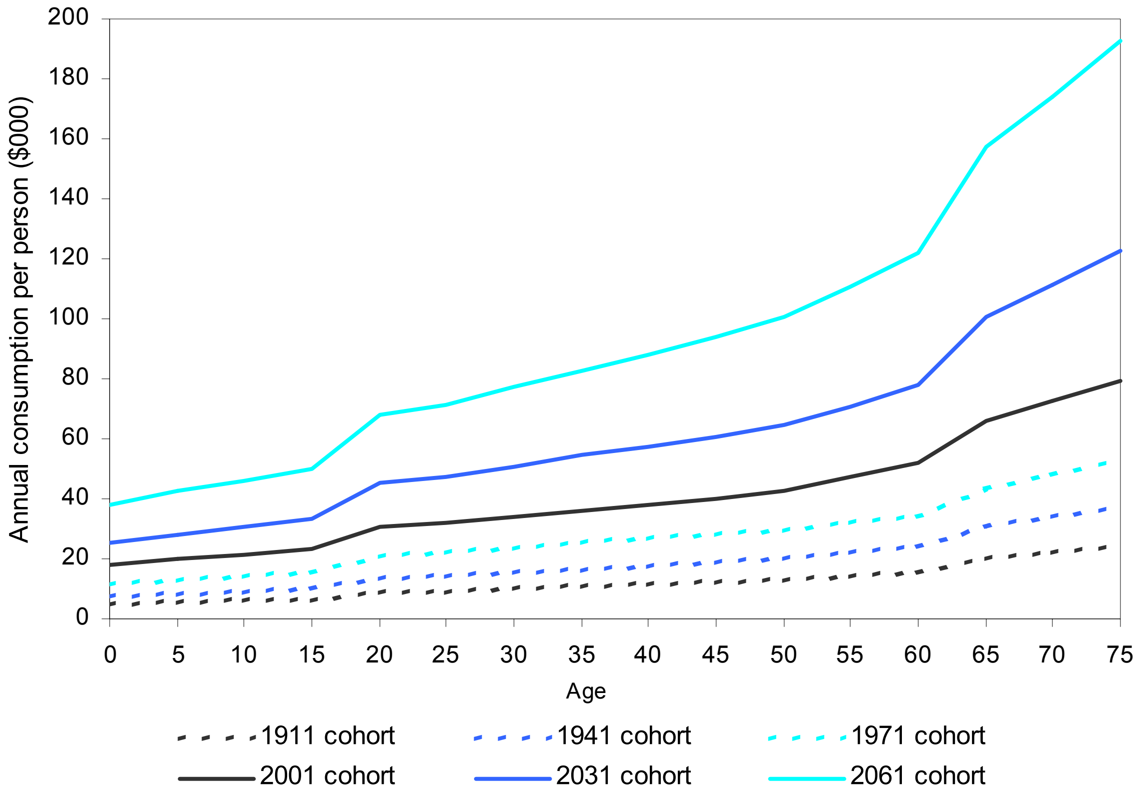
So far this paper has concentrated on aggregate consumption. Much of the policy debate over population ageing is, however, concerned with how the costs of population ageing are shared among generations (Auerbach, Baker, Kotlikoff and Walliser 1995). Questions about sharing among generations require a more disaggregated approach.

This section compares the welfare of different generations by examining the lifetime consumption paths of successive birth cohorts. A birth cohort is a group of people born in a given year. A cohort consumption path shows a cohort's consumption as it moves through its life cycle. The consumption path of the year 2000 birth cohort, for instance, is constructed from the consumption levels of 0-year-olds in the year 2000, the consumption levels of 1-year-olds in 2001, the consumption levels of 2-year-olds in 2002, and so on. The requisite data on age-specific consumption is not produced by the Ramsey-Solow model, which provides only average consumption across all age-sex-groups, in equivalent person terms, *C*/*P*. However, age-specific consumption levels can be calculated from the *C*/*P*'s by using the age-sex specific consumption weights shown in Equation 1 (the b_i's).

Note that these consumption paths do not reflect age-specific income profiles. It is not, in fact, possible to calculate age-specific income profiles from the model, since the model contains no measures of age-specific income comparable to the age-specific consumption weights.

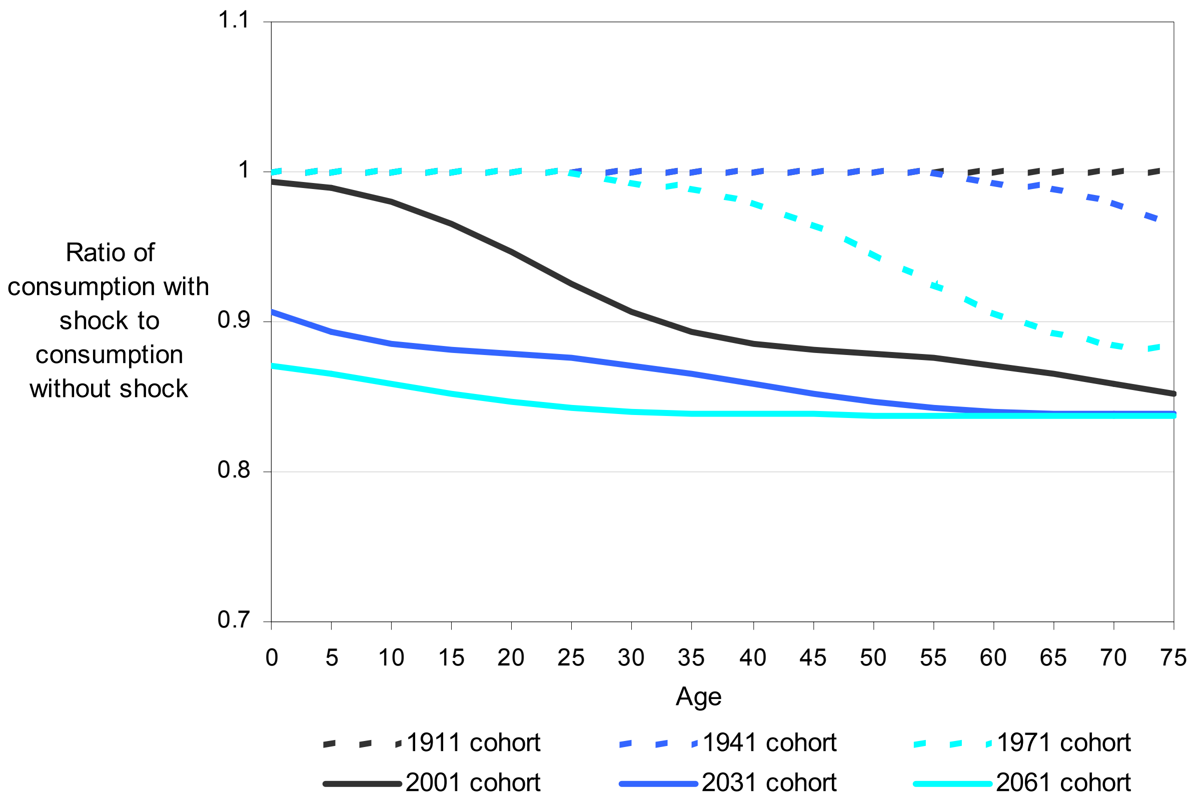
Consumption paths for males under the base case scenario are presented in Figure 12. The cohorts are 30 years—approximately one generation—apart. The horizontal axis shows age and the vertical axis shows annual per capita consumption. Upward kinks appears when each cohort moves from youth to the working ages, and from the working ages to old age. The figure clearly shows how, despite the ageing shock, each cohort achieves consumption levels substantially higher than the cohort born 30 years earlier. From the time it is born, for instance, the 2061 cohort has an annual consumption at least twice as high as the 2001 cohort.

Figure 12 - Consumption paths of selected birth cohorts



Consumption levels for later cohorts are nevertheless lower than they would have been in the absence of an ageing shock. This is illustrated in Figure 13. The figure shows the extent to which the ageing shock reduces cohorts' consumption below the level they would have attained without the shock. It shows, for instance, that by the time the 2000 cohort is aged 75, its consumption levels are only about 86% as high as they would have been without the shock. For the cohorts chosen, the later a cohort is born the greater the proportional reduction in its consumption. Although this is not shown on the graph, when the system finally reaches the steady state, consumption is 84% lower at every age with the ageing shock than it would have been without the shock.

Figure 13 - The ratio between consumption with the ageing shock and consumption without the ageing shock, for selected birth cohorts



Figures 12 and 13 combined indicate that the ageing shock causes later generations to forgo more consumption than earlier generations, but that, even with the shock, later generations still enjoy consumption levels substantially higher than earlier ones.

5 Discussion and Conclusion

The New Zealand population is ageing, implying an increase in the ratio of consumers to workers. This paper considers how saving rates should adjust in order to maximise long-run welfare. The paper also examines the extent to which population ageing reduces long-run welfare, compared to a hypothetical situation in which ageing does not occur.

We approach these issues using a Ramsey-Solow model of optimal savings, a model that has been adopted extensively in the literature (Cutler, Poterba, Sheiner and Summers 1990, Elmendorf and Sheiner 2000, Guest and McDonald 2001, 2002a). As is standard in the literature, we make a number of important simplifications. There is only one sector, with no distinction between private and public savings. We concentrate exclusively on the distribution of consumption and savings across periods and ages, and ignore the distribution of consumption and savings across individuals of the same age at the same point in time. We ignore uncertainty, except as a possible explanation for the interest rate premium. We treat the economy as if it is in a steady state in the initial year of projections. There is no distinction between tradable and non-tradable goods, and no labour-supply response to price changes. The model is evidently simple and highly aggregative.

A major advantage of simple, highly aggregative models is that the mechanisms at work are readily identifiable. More complex models rely on a greater number of parameters that are not only unknown in magnitude, but as in the case of the effect of ageing on labour productivity growth, unknown in sign. In such models the underlying economic logic can easily become obscured or invalidated (Black *et al* 1997: 11).

The simplifications do, however, need to be taken into account when interpreting the results from the model. One implication is that a high degree of precision ought not be ascribed to these results – they are indicative only. Another implication is that the present version of model is unsuited to examining some important questions concerning savings and public policy. The model does not, for instance, distinguish between public and private savings, so it cannot be used to assess the optimality of the current mix between the two. Also, the model considers only the impact of ageing on the path of optimal saving; it does not consider whether the current level of saving is optimal or whether future saving ought to be higher or lower on account of factors other than population ageing.

We are currently extending the basic model described here, to investigate the consequences of distinguishing between tradable and non-tradable goods, and allowing labour supply to respond to price signals. Other aspects of savings and policy public not addressed with the present model require different approaches altogether. Treasury is, for instance, using data from the Household Net Worth survey to examine differences in savings between individuals of the same age.

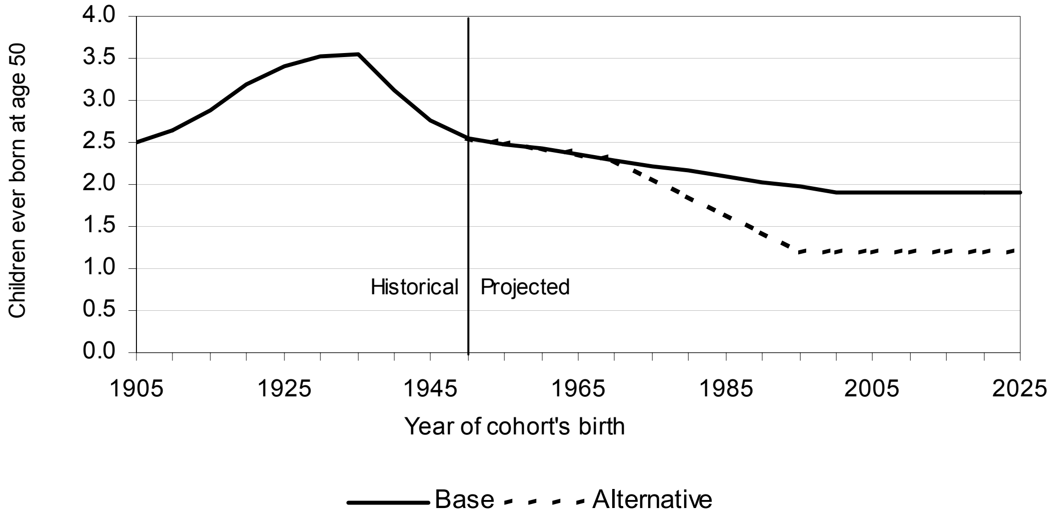
The basic model can, however, be used to establish some important points about population ageing, savings, and future living standards in New Zealand. Based on our results, it appears that future productivity increases will easily outweigh the reduction in consumption possibilities attributable to population ageing. Indeed, under the benchmark scenario, consumption levels in 2051 are almost twice as high as current levels. Under some parameter settings, including our benchmark scenario, it is optimal for savings to increase. Under the benchmark scenario, however, the increase constitutes less than 0.5 percentage points of GDP for the next 10 years. In contrast, under other parameter settings, it is optimal for savings to fall immediately. The main reason why the increases in savings are relatively muted, and why savings eventually decline, is the slower labour force growth brought about by lower birth rates. Slower labour force growth means that fewer savings are required to equip new workers with any given quantity of capital per worker. The model results suggest, moreover, that fears about population ageing having large negative effects on future consumption levels are probably exaggerated. Our results cast doubt on any assumption that population ageing *per* se necessitates large increases in national savings beyond those contemplated by current policies.

Appendix 1 Further details on the demographic projections

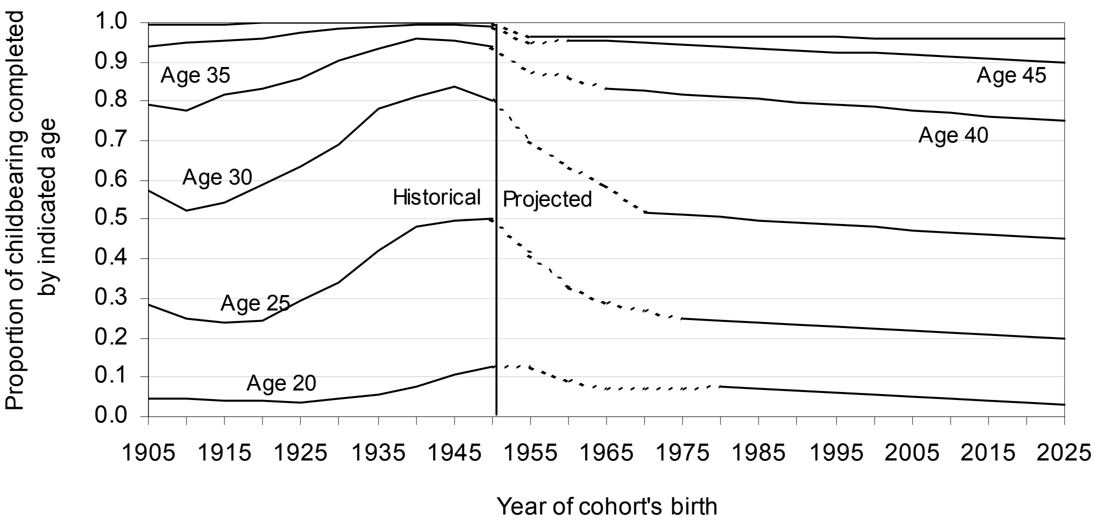
5.1 Fertility

The most common fertility indicator is the total fertility rate. An alternative, however, is the number of children successive cohorts of women had borne by the time they reached the end of their childbearing at age 50, which is known as 'completed fertility'. Completed fertility rates for New Zealand are graphed in Appendix Figure 1. The figure shows, for instance, that women born in 1905 had given birth to an average of 2.5 children by the time they turned 50.

Appendix Figure 1 – Children ever born



Appendix Figure 2 - Age-distribution of childbearing, for cohorts of women



Note: Hatched lines denote 'partly pre-determined values.' See text for details.

Appendix Figure 2 shows how successive cohorts distributed their childbearing over their reproductive lives. It shows, for instance, that the cohort born in 1905 had achieved only

30% of their completed fertility by age 25, whereas the cohort born in 1950 had completed 50%.

Demographers have traditionally projected fertility by constructing a future path for the total fertility rate, making an assumption about the age-distribution of fertility, and then deriving the age-specific rates. This approach is simple and transparent. It may not, however, perform well for populations like that of New Zealand where women seem to be systematically shifting childbearing later into their reproductive years. Demographers have traditionally incorporated these effects in an informal way: the UN Population Division, for instance, projects an increase in European fertility rates partly on the grounds that women are delaying childbearing.

Many demographers have become dissatisfied with the informal approach. Some have attempted to develop systematic procedures for stripping out the 'tempo' effects (Bongaarts and Feeney 1998), but these are still controversial, and require data not available for New Zealand. In this paper, a simple type of the 'cohort completion' (Morgan and Chen 1992) approach was used.

The first step was to construct paths for completed fertility, shown in Appendix Figure 1. Two series were constructed. In the 'base' series, completed fertility was assumed to decline linearly from 2.5 for the cohort born in 1950 to 1.9 for the cohort born in 2000, and to remain constant for subsequent cohorts. In the 'alternative' variant, completed fertility was assumed to decline to 1.2 for the cohort born in 1995.

The second step was to choose values for the age-pattern of fertility, shown in Appendix Figure 2. Cohorts born between 1950 and 1980 had had at least some of their children by 2000. This means that, once values for completed fertility had been chosen, some or all of the age-pattern was already determined. These partially pre-determined values are shown as hatched lines in Appendix Figure 2. Trends in age patterns were extended beyond the hatched lines by choosing a pattern for the 2025 and linearly interpolating. The pattern chosen for the 2025 cohort was similar to that of the early 20th Century cohorts, though with somewhat higher fertility in the 40s, on the assumption that medical progress will make delayed childbearing increasingly feasible.

Once completed fertility and its age-pattern had been chosen, rates for age-specific fertility and the total fertility rate could be read off. The results for the total fertility were shown in Figure 1 in the main text. In the base series, total fertility rate actually rises somewhat over the period 2000-2010. This is a result of the assumption, depicted in Appendix Figure 2, that the backward shifting of fertility will start to taper off by this time.

5.2 Mortality

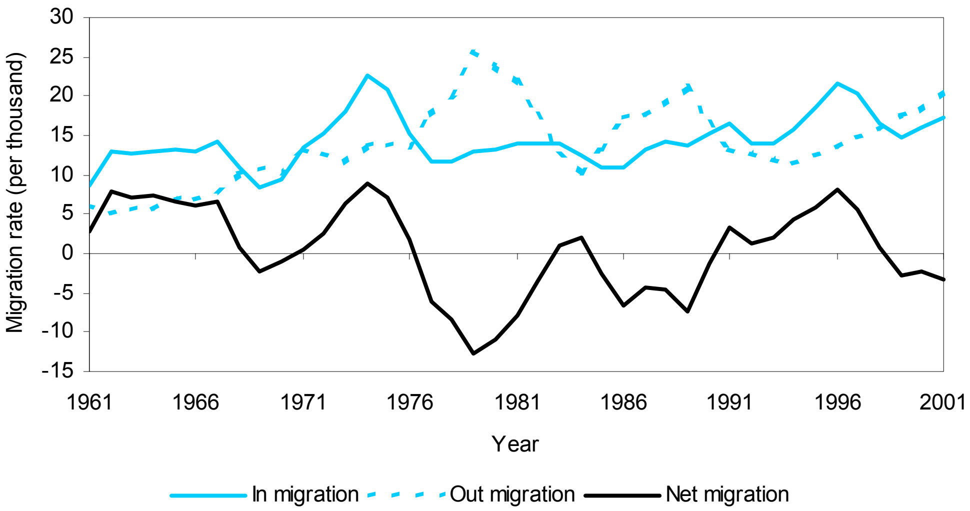
The main text discussed the projection of life expectancies, but not age-specific mortality rates. The relationship between life expectancies and age-specific mortality rates is complex, and there are variety of methods for deriving age-specific rates consistent with a given level of life expectancy and with a population's historical age-pattern of mortality. Statistics New Zealand scales all single-year mortality rates by a single percentage until the desired life expectancy is reached. We used a 'logit life table system' (Preston *et al* 2001: 119-201), with the 1998-2000 New Zealand life table as the standard, and the governing the relationship between child mortality and adult mortality held constant. Because the alternative scenario had life expectancies rising to very high levels by the end of the projection period, it was necessary to extend the standard life table into higher age-groups. This was done using the 'Kannisto model' as described in Thatcher, Kannisto, and Vaupel (1998).

5.3 Migration

In principle, it would be preferable to make separate in-migration and out-migration projections, rather than a single net-migration variable. There would, however, be little material gain in exchange for the added complexity, as migration has far less influence on age-structure, and hence support ratios, than either fertility or mortality. (The reason is that migration occurs at a wide range of ages, whereas everyone is born at exactly age 0, and most people die at ages 60 and over).

This paper is somewhat unusual in that the migration projections have been carried out in terms of rates rather than numbers. For short-term projections, the choice of numbers or rates in fact makes little difference. For long term projections, it can be argued that the use of rates is more sensible. This is most obvious for out-migration, which is clearly related to the size of the population. However, for New Zealand, it is likely to be true for in-migration as well. A significant proportion of in-migration to New Zealand consists of New Zealanders returning from overseas, and the number of New Zealanders overseas is correlated with the size of the resident population. Moreover, in-migration by non-New Zealanders is at least partly determined by government immigration policies, and future governments will presumably look at the size of the population when they make immigration policy.

Appendix Figure 3 - Migration rates 1961-2001



Appendix Figure 3 shows historical net migration rates. As is apparent in the figure, net immigration has fluctuated widely, with outflows roughly balancing inflows (the mean for the period shown is about 0.5 per thousand.) In contrast to fertility and mortality rates, there is no generally accepted summary index for a schedule of age-sex-specific immigration rates. (This may be one reason why demographers have traditionally used numbers rather than rates.) In the projections we therefore resorted to a somewhat awkward index: the overall net migration rates which would result if the schedule of age-sex-specific rates were applied to the New Zealand 2001 population. In the base variant, this measure is assumed to be constant at the historical average of 0.5 per thousand from 2001 to 2171; in the alternative variant it is assumed to be 0.

Age-sex-specific net migration rates were related to the summary index using the model

$$m_i = a_i + b_i m \qquad (16)$$

where m_i is the net migration rate for age-sex group i, m is the net migration rate index described above, and a_i and b_i are coefficients for age-sex group i. These coefficients were estimated from age-sex-specific migration data for the period 1996-2001.

5.4 Consumption weights

Appendix Table 1 - Consumption weights

	Base		Alternative	
	Male	Female	Male	Female
0-4	342	339	279	277
5- 9	362	361	296	295
10-14	366	366	300	299
15-19	374	381	306	311
20-24	463	489	514	535
25-29	450	487	503	533
30-34	454	497	506	542
35-39	452	496	505	541
40-44	444	473	499	522
45-49	440	455	495	507
50-54	436	443	492	497
55-59	448	458	501	510
60-64	462	485	513	532
65-69	576	573	499	496
70-74	593	585	513	506
75-79	612	604	528	522
80-84	630	633	543	546
85-89	671	700	577	600
90-94	671	700	577	600
95-99	671	700	577	600
100-104	671	700	577	600
105-109	671	700	577	600
110-114	671	700	577	600
115-119	671	700	577	600
120-124	671	700	577	600
125-129	671	700	577	600

5.5 Productivity weights

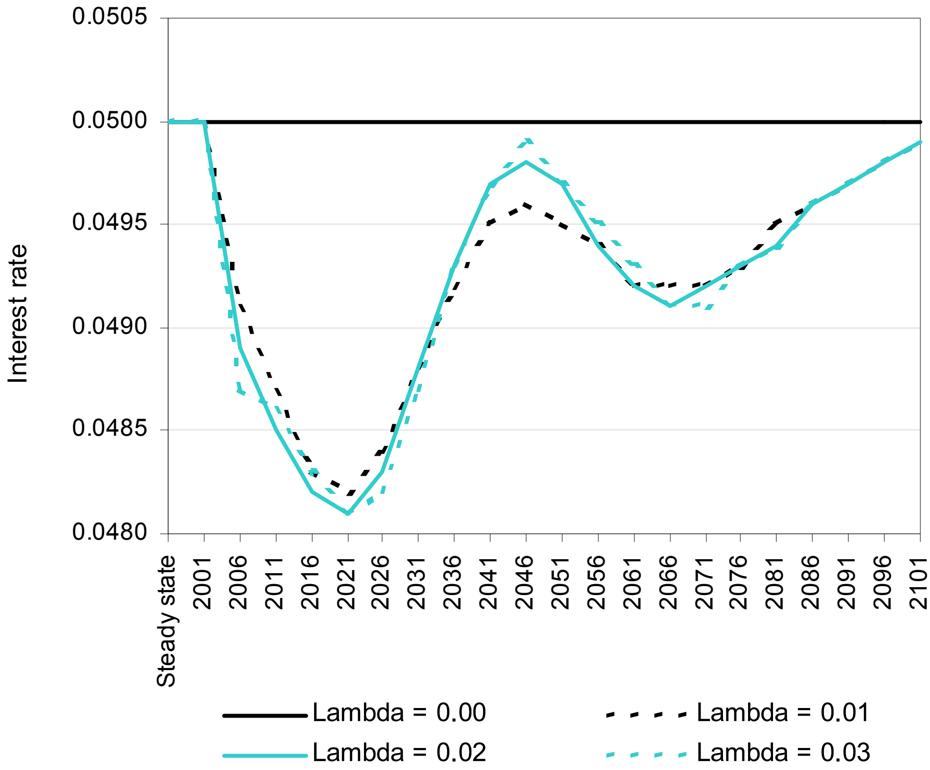
Appendix Table 2 - Productivity weights

	Base		Alternative	
	Male	Female	Male	Female
0-4	0	0	0	0
5- 9	0	0	0	0
10-14	0	0	0	0
15-19	159	133	119	119
20-24	542	428	403	403
25-29	900	560	670	603
30-34	1149	522	856	770
35-39	1201	602	894	805
40-44	1249	638	930	930
45-49	1274	732	949	949
50-54	1272	641	948	948
55-59	955	421	826	826
60-64	614	241	577	577
65-69	209	69	287	287
70-74	58	18	99	99
75-79	0	0	22	22
80-84	0	0	0	0
85-89	0	0	0	0
90-94	0	0	0	0
95-99	0	0	0	0
100-104	0	0	0	0
105-109	0	0	0	0
110-114	0	0	0	0
115-119	0	0	0	0
120-124	0	0	0	0
125-129	0	0	0	0

Appendix 2 Further details on the simulations

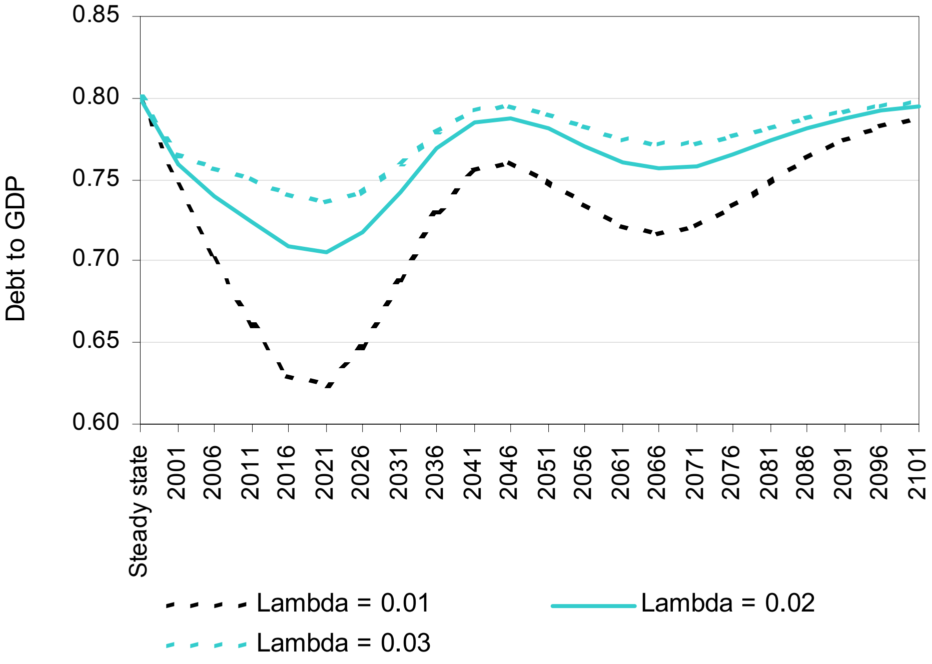
In this appendix we provide some supplementary results from the simulations. We show the paths for consumption and wealth on a phase diagram, and paths for debt to GDP and the interest rate under alternative values of λ. The parameter λ measures the responsiveness of the interest rate to the level of debt to GDP, which in this model indicates the degree of capital immobility.

Appendix Figure 4 - Time paths for interest rates for selected values of λ, the interest rate premium parameter



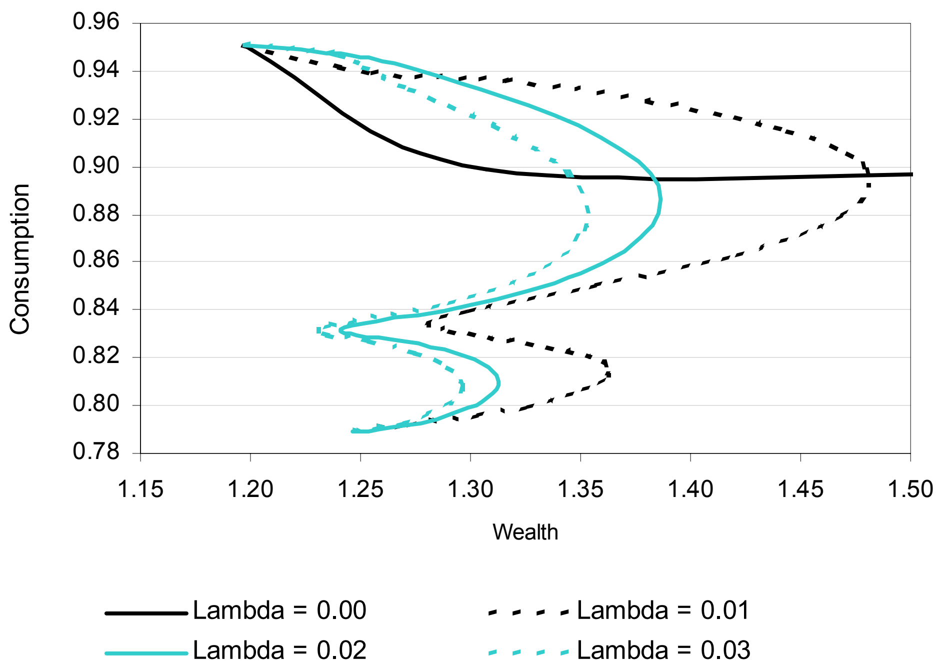
Appendix Figures 4 and 5 show time paths for debt and the interest rate under alternative values of the capital immobility parameter λ. This parameter links the pattern of population ageing with the return to saving, via Equation (7), which models the interest rate as a function of debt. In the case of a zero endogenous interest rate premium, λ=0, there is no link between ageing and the return to saving. The country borrows and lends at an unchanging interest rate. As a result there is perfect consumption smoothing and consumption drops immediately to its new steady state level. If, on the other hand, λ>0 and capital is not perfectly mobile, a fall in consumption (a rise in saving) lowers the return to saving which moderates the initial fall in consumption. The greater the value of λ, the greater the response of the interest rate to a given change in debt, which in turn moderates the change in debt. This illustrated in Appendix Figures 4 and 5. The larger the change in debt the larger the change in wealth (for a given capital stock). Hence greater values of λ imply smaller changes in wealth. This is apparent in the phase diagram in Appendix Figure 5.

Appendix Figure 5 - Time paths for the debt-to-GDP ratio for selected values of λ, the interest rate premium parameter



The most striking feature of the simulations illustrated in Appendix Figures 4, 5 and 6 is the sensitivity of optimal paths in debt and in consumption-wealth to small variations in the path of the interest rate. The path of the interest rate is itself fairly insensitive to changes in the parameter for interest rate premium, λ, for values greater than 0.01, but is highly sensitive to movement away from the λ=0 case. This illustrates the knife-edge characteristic of the λ=0 case, in that minor variations from this hypothetical case can materially affect the path of the endogenous variables.

Appendix Figure 6 – Phase diagram showing consumption-wealth paths for selected values of λ, the interest rate premium parameter



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