Tie-Breaking Rules and Divisibility in Experimental Duopoly Markets

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2007

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July 2006

Abstract

This experimental study investigates pricing behavior of sellers in duopoly markets with posted prices and market power. The two treatment variables are given by tie breaking rules and divisibility of the price space. The first treatment variable deals with the rule under which demanded units are allocated between sellers in case of a price tie. A change in divisibility is modeled by making the sellers' price space finer or coarser. The main finding is that the incidence of perfect collusion is significantly higher under the sharing tie breaking rule than under the random (coin-toss) one, especially when the price space is less divisible.

Keywords: Collusion, Tie Breaking Rules, Divisibility, Bertrand model.
JEL Classification: L1, C9.

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1 Introduction

The goal of this study is to shed some light on factors that affect pricing behavior and collusion incentives in market experiments. Previous experimental research suggests that the type of trading institution employed can prominently affect pricing behavior (e.g., Plott and Smith [28]). Furthermore, a well established experimental research program studies pricing behavior within the context of a particular market institution (see, for instance, Alger [1], Cason and Davis [5], Davis, Holt and Villamil [9], Davis and Wilson [10], Friedman and Hoggatt [18]), and seems to suggest that environmental details play an important role. This paper falls more closely in this second strand of literature, in that we keep fixed the trading institution and we focus on factors that might facilitate collusion.

On the one hand, there are several papers that study factors facilitating collusion in Cournot markets. For instance, the number of firms in a market, repeated interaction, information about rivals' actions and profits, are some of the factors affecting collusion incentives (see Huck, Normann, and Oechssler [23], Huck, Müller, and Normann [22], Feinberg and Husted [15], Huck, Normann, and Oechssler [24])[2].

On the other hand, this paper examines the effect of tie breaking rules and divisibility on tacit collusion in Bertrand duopoly markets with capacity constraints. There tends to be some tacit collusion in Bertrand duopoly anyway (see Dufwenberg and Gneezy [13])[3] but we find that, under a specific combination of the treatment variables, prices are consistently high and markets with both sellers charging the joint profit maximizing price are common.

The first treatment variable, namely the tie breaking rule, deals with the rule under which demanded units are allocated between sellers in case of equal posted prices. We explore two possible such rules. Under the random tie breaking rule (referred to as R) ties are broken randomly, i.e., the simulated buyer picks randomly which seller to approach first. On the other hand, the buyer equalizes purchases among the tied sellers under the sharing tie breaking rule (referred to as S). In the game we designed, under the assumption of risk neutrality and the Nash equilibrium concept, the tie breaking rule should not affect pricing behavior, since the ex-ante profits are the same under both

[2] Also Bertrand oligopoly markets are studied in [24].

[3] In [13], the authors investigated a game designed to "give the Bertrand model its best shot at not being rejected by the data" (p.8).

rules.[4]

Random and sharing tie breaking rules are used by government agencies in procurement auctions environments and might have an impact on collusion (see McAfee and McMillan [25] and Davis and Wilson [10]). For instance, McAfee and McMillan [25] observe that government agencies often employ a random tie breaking rule to deter collusive behavior.

Provided that tie breaking rules come into play in case of price ties, then it is natural to inquire about the divisibility of the price space since it affects the occurrence of ties. We model divisibility by making the sellers' price space coarser (Less Divisible) or finer (Continuum). The emphasis on divisibility is not minor as lower divisibility might simplify the strategy space of sellers by allowing fewer choices. There are several environments where divisibility might affect pricing behavior. An example is currency redenomination which might cause a change in money divisibility, where the term divisibility refers to the fact that money can be broken in smaller or larger units. For instance, most of the countries that adopted the Euro currency experienced a decrease in divisibility and a price increase that has involved mostly services and some small-ticket items during the changeover.[5]

The main focus of the paper is on the following question: Are lower divisibility and a sharing tie breaking rule going to facilitate tacit perfect collusion?

We answer this (and other) questions in a very simple environment.[6] In particular, we concentrate our attention on Bertrand duopoly markets (with capacity constraints) and we keep pairs fixed since this makes pricing decisions easier for subjects[7] (who have to worry only about one other player's strategy). Furthermore, since the buyer is simulated, the focus is clearly on the sellers' side of the market. We think it is reasonable to take such a duopolistic setting as a starting point.

We find that tie breaking rules have a significant effect on prices. In particular, a sharing tie breaking rule facilitates perfect collusion. As far as

[4] In contrast, in our design, note that ex-post, under R profits are unequal, while under S they are equal.

[5] Clearly, a change in divisibility is not the only factor explaining the price increase for small ticket items. Many other factors (related to the demand side as well) are involved in this phenomenon, i.e., menu costs, psychological pricing, etc.

[6] Even though the environment is simple, it has been studied in a variety of settings (see [6] or [11]) and it captures some of the features of decentralized electricity markets (see [14] or [30]).

[7] In the stage game.

divisibility is concerned, it seems to sharpen the latter result.

We think that the results concerning tacit perfect collusion are rather puzzling. In fact, a strategy supporting perfect collusion where sellers make equal profits can be chosen under both tie breaking rules. On the other hand, in order to attain equal profit, it is worth pointing out that under the sharing tie breaking rule it is sufficient that subjects post the same prices, while under the random one, subjects should additionally restrict the number of units posted for sale. That is, in our environment perfect collusion is facilitated when subjects have to worry about coordinating only on the choice of one variable (the price) rather than the choice of two variables (both price and quantity).

The layout of the paper is as follows. Section 2 provides a brief literature review. Section 3 contains a discussion of the equilibrium predictions. Then, in Section 4, we describe the experimental design and procedures. The experimental results are presented in Section 5. Finally, in Section 6, we offer some concluding remarks.

2 Literature Review

Numerous experiments illustrate that pricing behavior is strongly influenced by both market structure and the market institutional environment. A well established experimental research program studies factors affecting pricing behavior within specific market institutions. For instance, prior posted offer experiments focused on factors such as the number of sellers (see Davis, Holt and Villamil [9]), the amount of information provided to sellers (e.g., Kruse, Rassenti, Reynolds and Smith [4]), the role of subject experience (Alger [1], Benson and Faminow [2], Friedman and Hoggatt [18]), mergers (e.g., Davis and Holt [8]), and so on.[8]

Some of these studies involve duopoly markets as well (e.g., Davis, Holt and Villamil [9], Friedman and Hoggatt [18], Alger [1], Benson and Faminow [2]).

To the best of our knowledge, there are few studies focusing on the effect of tie breaking rules and divisibility. More generally, in several market environments, the sharing tie breaking rule is adopted as ties are usually broken

[8] According to Davis et al.[9], static market power leads to price increases in posted offer markets with three sellers, while the effect is not clearcut for duopolies. Both across sessions (see [2]) and within session (see [1]) subject experience seems to increase the likelihood of collusive outcomes.

by sharing the demand (e.g., Alger [1], Brown-Kruse [3], and Davis, Holt and Villamil [9]). Harrison [19] employed a random tie breaking rule in a study of contestable markets. Tie breaking rules have been introduced as treatment variables by Davis and Wilson (see [10]) in a variant of posted-offer trading rules appropriate to a procurement auction environment. In their experiment, every market lasts for forty trading periods and consists of four sellers. One of their findings is that a change in the tie-breaking rule does not affect behavior. This result, as mentioned by the authors themselves, might be driven by some features of their design which is rather elaborate. Our study seems to support their conjecture since it shows that tie breaking rules matter in a very simple environment (e.g., in our study markets are duopolistic and the induced demand and supply arrays are simpler than the ones in [10]).

On the other hand, as far as divisibility is concerned, lower divisibility generates a simplification of the price decision space. The latter has been examined in contestable markets by Brown-Kruse [3]. In this study, sellers' offers are restricted to multiples of 0.25 in one of the treatments. Under this restriction, the strategy space shrinks from 250 choices to 10 choices. In this work, a less complex specification of the choice space allows sellers to identify alternative strategies of tacit collusion, even if the impact on mean prices is not significant. They conjecture that one of the reasons why the results are not significant might rely on the specification of the design generating losses for sellers in case of price ties.[9] Note that in our environment, as opposed to theirs, sellers do not incur any loss in case of a price tie.

Thus, the contribution of our paper, relative to the existing literature, is to investigate the effect of tie breaking rules and the effect of a simplification of the choice space of sellers (on pricing and collusive behavior) in duopoly posted offer markets.

3 Equilibrium Distributions

The model consists of a finitely repeated game. The stage game is described as follows. There are two identical firms producing a homogeneous good whose

[9] "Under the decreasing average costs of a natural monopoly, if sellers match prices and share the market, this can result in substantial losses when prices are near the competitive range" ([3], p.144).

demand curve is

$$q(p) = \begin{cases} 0 & \text{if } p > 550 \\ \\ 6 & \text{if } 0 \le p \le 550. \end{cases}$$

Each firm has a capacity constraint of 4. The cost of production is 41 per unit and production is to-order as sellers incur costs only if a unit is sold. Firms simultaneously choose prices and quantities. That is, each seller has a two dimensional strategy space $B = P \times Q$, where $Q = \{0, 1, 2, 3, 4\}$, and chooses a price and a quantity $((p, q) \in B = P \times Q)$. We will consider two games whose distinction trait is the price space, namely, either $P = P_C = [0, 10000]$ or $P = P_{LD} = \{0, 50, ..., 10000\}$.[10]

Note that it is a dominant strategy for every firm to post prices exceeding 41 and offer all the units available for sale. Thus, we can treat this game as if the strategy space is in fact one dimensional and firms simultaneously choose only prices. If prices are unequal, the low-price firm sells 4 units and the high-price firm sells the remaining 2 units. If firms choose the same price, i.e., in case of a price tie, either they share the market equally, or the seller to be approached first is chosen randomly. Under the assumption that firms are risk neutral, the payoff functions are given by the following expression for $i = 1, 2$:

$$u_i(p_1, p_2) = \begin{cases} (p_i - 41) \times 4 & \text{if} \quad p_i < p_j,\, p_i \le 550 \\ (p_i - 41) \times 3 & \text{if} \quad p_i = p_j,\, p_i \le 550 \\ (p_i - 41) \times 2 & \text{if} \quad p_i > p_j,\, p_i \le 550 \\ 0 & \text{if} \quad p_i > 550 \end{cases}$$

Before characterizing the Nash Equilibrium for this game for both $P_C = [0, 10000]$ and $P_{LD} = \{0, 50, ..., 10000\}$, let us make a few remarks.

In both cases the competitive equilibrium ($p = 41$ and quantity demanded $= 6$) is not a Nash equilibrium, since either of the sellers can profit from a unilateral deviation, i.e., there is static market power. Specifically, we will see that because of market power the noncooperative equilibrium for the market game leads to a cycle of prices that exceed the competitive level (e.g., see [20]). Furthermore, no pure strategy Nash Equilibrium exists in this game and noncooperative firms randomize to avoid being slightly undercut in the

[10] The last paragraph preceding Subsection 3.1 contains more details regarding the choice of the parameters of the model.

cycle. There is a unique symmetric mixed strategy equilibrium in both cases that requires mixing over the range of the Edgeworth cycle.[11] Notice that in our case, the support of the distribution coincides with the range of the Edgeworth cycle. (See [21].)

Since the stage game has a unique equilibrium, by backward induction the unique subgame perfect equilibrium of the repeated game is to play the static equilibrium in every subgame.

The parameters of the model (costs, limit price, etc.) are chosen so that the equilibria of the two games (C and LD) are not too different. For example, not only do both games admit a unique equilibrium in mixed strategies, but also the Edgeworth cycle intervals are very close ($[295.5, 550]$ in C and $[300, 550]$ in LD). We think this is a better fit to the study of divisibility per se, since it helps us isolate the effects of divisibility.

3.1 Equilibrium in the continuum case

In this section, we let $P_C = [0, 10000]$. The noncooperative equilibrium involves randomization over the range of prices called "Edgeworth cycle." The support of the noncooperative equilibrium distribution is determined as follows. The upper end of the range of the Edgeworth cycle is 550. At a price of 550, the residual demand is of two units, so that either seller can secure himself a profit of $2[550 - 41] = 1018$. If a seller chooses a price of 550, the other seller's best response is to post a price just below it and sell four units. Best responses consist in undercutting until a price of 295.5 is reached. In fact, selling four units at a price below 295.5 is not as profitable as selling two units at a price of 550 ($4[295.5 - 41] = 1018$).

Let $G(p)$ denote the probability that the price p posted by seller i is the highest price posted in the market for a period. Since any two sellers in a market have identical profit functions, we will focus on a symmetric mixed equilibrium, so that $G(p)$ can be considered as the common price distribution. Furthermore, note in this game there is a unique equilibrium in mixed strategies and that the probability of a price tie is zero (for a proof see [27] and [12]). If a price p is the highest price, the seller who posted it will sell two units and will earn $H(p) = 2[p - 41]$. On the other hand, if p is the lowest

[11] Each period each firm posts a price slightly below the prices posted in the previous period. Eventually prices decrease to a level where a firm's best response is to charge the monopoly price for the residual demand. The other firms follow and a new price cycle begins.

price, the seller will sell four units and his earnings are $L(p) = 4[p - 41]$. Hence, the expected profit for a seller is given by

$$G(p)H(p) + (1 - G(p))L(p).$$

Next, observe that sellers must be indifferent over all prices in the support of the distribution, and they can secure themselves a profit of 1018 by choosing a price of 550. By substituting for the expression $H(p)$ and $L(p)$, we obtain

$$G(p)2[p - 41] + (1 - G(p))4[p - 41] = 1018.$$

Solving for $G(p)$ we have the equilibrium cumulative distribution function of prices

$$G(p) = \begin{cases} 0 & \text{if} \quad 0 < p \leq 295.5 \\ \frac{591-2p}{41-p} & \text{if} \quad 295.5 \leq p \leq 550, \end{cases}$$

whose density function is $g(p) = \frac{509}{(41-p)^2}$, for $295.5 \leq p \leq 550$.

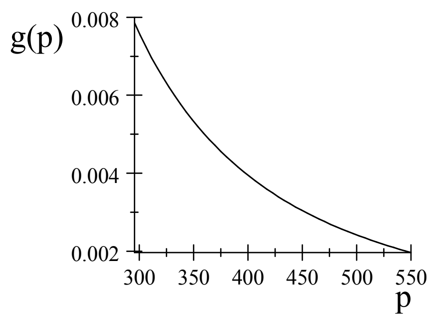

Figure 1. Density function in the continuum case.

The equilibrium density function has no probability mass point. The intuition is that, since the equilibrium is symmetric, if the density function admits probability mass points, then each seller has the incentive to undercut at the common mass point, contradicting the notion of equilibrium.

3.2 Equilibrium in the discrete case

Now, let $P_{LD} = \{0, 50, ..., 10000\}$, i.e., sellers are allowed to post only prices that are multiples of 50. We now deal with a finite game so that there exists an equilibrium. With the help of specialized software (see either [26] or [29]) we are able to find the equilibrium in mixed strategies. Furthermore, the equilibrium is unique and it is symmetric. The equilibrium probability distribution assigns zero probability to all prices below 300, and it is described next.

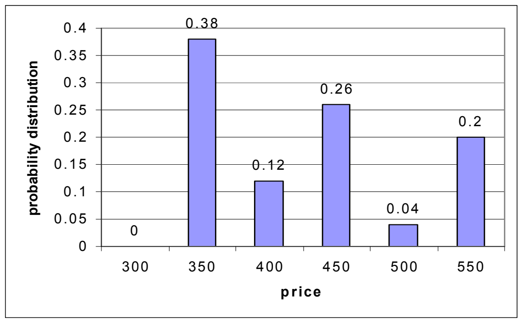

Figure 2. Equilibrium distribution in the discrete case.

A comparison of Figures 1 and 2 suggests that in the continuum case the probability distribution is slightly more skewed to the left than in the discrete case. Also, the probability that a tie occurs is zero in the continuum case (since there are no mass points), while it is positive in the discrete case (at least for prices greater than 300).

3.3 Collusive Equilibria

Theoretically, collusion is not an equilibrium of our finite horizon games but behaviorally repeated game effects are possible.[12] Thus, we comment briefly on the existence of subgame perfect equilibria of the infinitely repeated counterpart of our games. In particular, we focus on the perfectly collusive equilibrium which we consider as a benchmark. Standard Folk Theorem arguments show that, in order to sustain perfect collusion as an equilibrium in the continuum and less divisible cases, the discount rate or probability of continuation should exceed 0.499 and 0.43, respectively.[13] This suggests that is more difficult to sustain collusion under the continuum than under the less divisible

[12] The time horizon of sixty periods in the experiment may be long enough to make repeated game effects possible.

[13] This equilibrium is sustained by a trigger strategy: each player colludes until someone fails to collude, which triggers a switch to the noncooperative Nash equilibrium forever after. Here, we focused on the sustainibility of perfect collusion as a subgame perfect Nash equilibrium of the infinitely repeated game. Note that other feasible payoffs can be obtained in subgame perfect equilibria (for sufficienlty high probabilities of continuation). That is, the set of noncooperative equilibria includes cooperative outcomes that are not repetitions of Nash equilibria of the stage game.

space. This result is intuitive since the gains from undercutting (i.e., deviating from perfect collusion) are higher under the continuum than under the less divisible space.[14]

3.4 Summary of Equilibrium Predictions

Under risk neutrality, the choice of the tie breaking rule does not affect Nash equilibrium predictions. Consequently, based on the previous sections, we can summarize the equilibrium predictions in Table I. For completeness, we include also the predictions from perfectly collusive behavior.[15]

	Mean Price	Median Price	St. Dev.	Profit/Period
Perfectly Collusive	550	550	0	1527
Nash Equ. (Continuum)	393.81	380.33	71.16	1018
Nash Equ. (LD)	427.67	400	75.44	1118.69

Table I. Theoretical predictions.

From a qualitative standpoint the static Nash equilibrium predicts the following differences under the two divisibility regimes:

Hypothesis 1. A change in the tie-breaking rule does not affect the equilibrium predictions (under risk neutrality).

Hypothesis 2. Mean and median prices are higher under the Less Divisible regime than under the Continuum one.

Hypothesis 3. Price dispersion is higher under LD than under C.

Hypothesis 4. Equilibrium profit is higher under LD than under C.

Before testing for these hypotheses (in Section 5) let us describe the experimental design.

[14] This effect dominates over the one arising from the fact that the punishment from reverting to the noncooperative Nash equilibrium is harsher under the continuum than under the less divisible treatment.

[15] Perfectly collusive profits are per seller profits associated with the limit price.

4 Experimental Design and Procedures

Every market consisted of sixty trading periods during which pairs remained *fixed*. Supply and demand arrays for each market are shown in Figure 3.

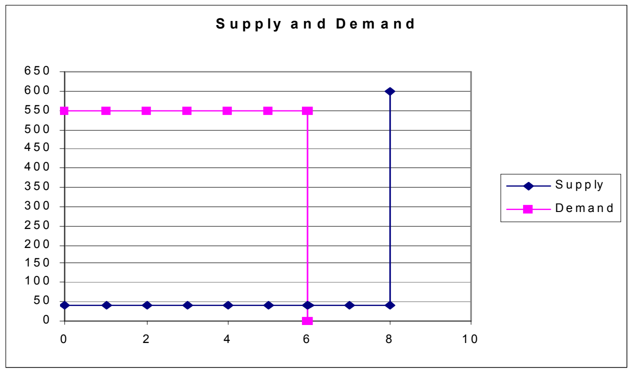

Figure 3. Induced supply and demand arrays.

Note that the two sellers in every market were symmetric. They had identical costs and identical capacity constraints, i.e., they both were endowed with four units at a cost of 41 each.[16]

All markets were conducted under posted-offer rules with a simulated, fully revealing buyer. In every period, sellers simultaneously made price/quantity decisions in each market. Every seller was allowed to post only one price at which he was willing to sell the posted units. After all prices were posted, the simulated buyer purchased up to six units in each market at prices up to 550 (and no units at a price exceeding 550). The buyer made all profitable purchases, buying first from the seller with the lowest posted price, then from the other seller.

Sellers were fully informed about other seller's cost, about the preferences and shopping behavior of the simulated buyer, and about the matching protocol. Production was to-order as sellers incurred costs only if a unit was sold. Thus, the payoff for every seller was given by

Payoff = [(selling price×number of units sold)-(production cost of units sold)].

We adopted two treatment variables. The first treatment variable changed the rules according to which the simulated buyer made purchases in the event of a price tie. In case of identical prices, in design S (S for sharing) the buyer

[16] For an explanation regarding the choice of the parameters values, see the paragraph preceding Subsection 3.1.

equalized purchases among the tied sellers, while in design R (R for random), the buyer chose randomly which seller to approach first.[17]

The second treatment variable dealt with the divisibility of sellers' strategy space (modeled by making each seller's strategy space coarser or finer).

In the Continuum treatment, sellers were allowed to post prices that were numbers up to three decimal places, while in the Less Divisible treatment, sellers were allowed to post only prices that were multiples of 50. That is, in the Continuum treatment (C thereafter) $p \in \widetilde{P}_C = [0, 0.001, ..., 10000]$ and $q \in Q = \{0, 1, 2, 3, 4\}$. On the other hand, in the Less Divisible treatment (LD), $p \in P_{LD} = [0, 50, ..., 10000]$ and $q \in Q = \{0, 1, 2, 3, 4\}$.[18]

We conducted fifty-six homogeneous-product duopoly markets, run in ten sessions with 8, 10, 12, 14 or 16 participants (see Table II).[19]

The experiments are divided into four cells based on the two treatment conditions.

	Continuum (C)	Less Divisible (LD)
Sharing (S)	7 markets (1 session)	16 markets (3 sessions)
Random (R)	16 markets (3 sessions)	17 markets (3 sessions)

Table II. Matrix of treatments and sessions summary.

Note that the matrix in Table II also displays the number of sessions run under each treatment. All the sessions took place at the Vernon Smith Experimental Economics Laboratory at Purdue University. The experiment was programmed and conducted with the software z-Tree (See [16]). Subjects for the experiments were recruited from undergraduate students at Purdue University. Subjects were inexperienced, where experience refers to previous participation in one or more of the posted offer experiments.[20] Instructions

[17] In case of risk neutrality of the sellers this treatment variable does not affect the outcome. Under risk aversion and discrete strategy space, on the other hand, this might affect behavior. Indeed, note that if two sellers price tie between 50 and 550 and post four units each, then under S they will sell three units each, while under R, one of them (randomly selected) will sell four, and the other only two.

[18] In both treatments the price space is large, even though participants should not price above 550. The intention is not to impose any ex ante restriction on the participants' pricing behavior (reflected also in the instructions that do not mention any upper bound for the price space). It might be worth noticing that none of the posted prices exceeded 10000.

[19] Seven markets (1 session) have been conducted under the Sharing Continuum treatment. The purpose of this session is to check whether, given the sharing tie breaking rule, tacit perfect collusion survives under the continuum (price) strategy space.

[20] On the other hand, if by experience we mean previous participation to any type of

were read aloud to participants as they followed along in their own copies.[21] Subjects were given explicit information regarding the purchasing decision of the simulated buyer as well as the fact that sellers were identical. Furthermore, they were told that they were paired with the same person throughout the whole experiment,[22] that the experiment would consist of 60 periods[23] and would run for up to one hour and a half. A typical experiment lasted about an hour. Earnings ranged between \$11 and \$22 per subject. Average earnings were \$15.95.

5 Experimental Results

In this section, we will adopt the following notation:

$rc =$ random tie breaking rule and continuum price space
$rld =$ random tie breaking rule and less divisible price space
$sc =$ sharing tie breaking rule and continuum price space
$sld =$ sharing tie breaking rule and less divisible price space.

Before providing a detailed discussion of our results, let us point out that in our experiment markets consisted of fixed pairs, so that we can treat markets as statistically independent observations. This is important for nonparametric tests, where we use exactly one summary statistic value for each market. That is, the number of observations in each treatment is given by $N_{rc} = 16$, $N_{rld} = 17$, $N_{sc} = 7$, and $N_{sld} = 16$.

For the sake of completeness, we tested our hypotheses by using the price medians as well as the mean posted prices.[24] The most important conclusions of our analysis are derived from mean and median posted prices, from a regression analysis and from a probit model.

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economics experiment, our subjects were experienced. As a matter of fact, two sessions were run with subjects who had never participated before to any type of experiment. The results from these sessions appear significantly different from the ones obtained from the others, suggesting the introduction of a third (experience) treatment variable, that goes beyond the scope of this study.

[21] The instructions for the treatments Sharing Less Divisible, Random Less Divisible and Random Continuum are contained in Appendix B. The instructions for the treatment Sharing Continuum is obtained as an obvious modification of the ones included.

[22] Note that participants were seated at visually isolated booths.

[23] That is, the stopping rule of 60 periods was publicly announced.

[24] Note that the noncooperative equilibrium predictions for the one-shot game regard posted prices.

When prices are pooled within each treatment, the median prices for periods 11-60[25] are plotted in Figure 2A (in Appendix A). Figure 3A (also in Appendix A) plots mean prices in 10 period intervals pooled by treatments.[26]

The data are characterized by the following features:

(i) In all treatments, prices do not follow a specific monotonic trend, but they exhibit an unstable pattern.

(ii) The quantitative theoretical predictions are rejected by the data. Mean and median market prices are higher than predicted by the noncooperative Nash Equilibrium and lower than monopolistic ones, in all four treatments.[27]

(iii) The treatment *sld* is characterized by the highest incidence of median prices equal to 550.

The next sections contain a more detailed analysis of the data by focusing first on the results regarding tie breaking rules, and then on the results related to divisibility.

5.1 Tie Breaking Rules

Does a change in the tie breaking rule affect pricing behavior? Recall that under the assumption of risk-neutrality, tie breaking rules (since we are dealing with duopolies) should not affect prices. On the other hand, our data analysis seems to support the opposite. In what follows, we focus on the effect of tie breaking rules on perfect tacit collusion (Section 5.1.1), as well as on posted prices (Section 5.1.2).

In order to study the effect of tie breaking rules on perfect tacit collusion, we analyze the data collected under the less divisible (LD) and continuum (C) treatments.[28]

5.1.1 Perfect tacit collusion

In this section we test the following Hypothesis.

[25] We chose to focus on this range to account for some initial noise and end-game effects.

[26] We pooled mean prices by 10 period intervals since it makes easier comparisons across treatments.

[27] According to the Wilcoxon signed-rank test. That is, we tested whether a particular random sample came from a population with a specified mean or median (see [7], Chapter 5). Here, the random sample is given by individual markets mean or median of prices posted after period 10, i.e., we have one summary statistic value per market. The specified mean or median is given by the theoretically predicted values of Table I.

[28] Recall that fewer observations were collected under the continuum with sharing tie breaking rule treatment, relative to the other treatments.

Hypothesis 1. A change in the tie breaking rule does not affect the equilibrium predictions under risk neutrality.

This hypothesis seems not to be supported by the data. Indeed, tie breaking rules affect the rates of perfect tacit collusion. In particular, perfect tacit collusion, captured by ties at 550 (i.e., both sellers post a price equal to 550), occurs more frequently under the sharing tie breaking rule (see Figure 4).

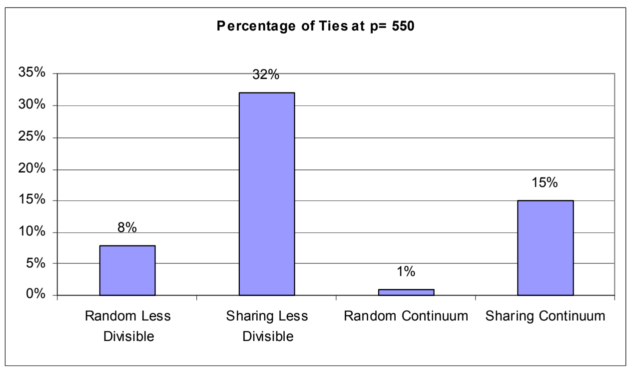

Figure 4. Percentages of Ties at 550 pooled by treatments.

For instance, a look at Figure 5 shows that when the sharing rule is employed and prices are less divisible, 7 markets (out of 16 markets) converged to perfect collusion for 21, 21, 27, 40, 41, 49, and 54 consecutive periods respectively.[29] In five of them perfect collusion was broken in the last period, and in two of them in the last two periods, providing some evidence of endgame behavior. On the other hand, when the random rule is employed, perfect collusion was sustained by only one of the 17 markets (with 16 consecutive ties), and in some others it was attempted without success.

[29] This provides us with the highest number of consecutive ties. The *total* number of ties for these markets were 28, 41, 43, 43, 44, 49, 54, respectively.

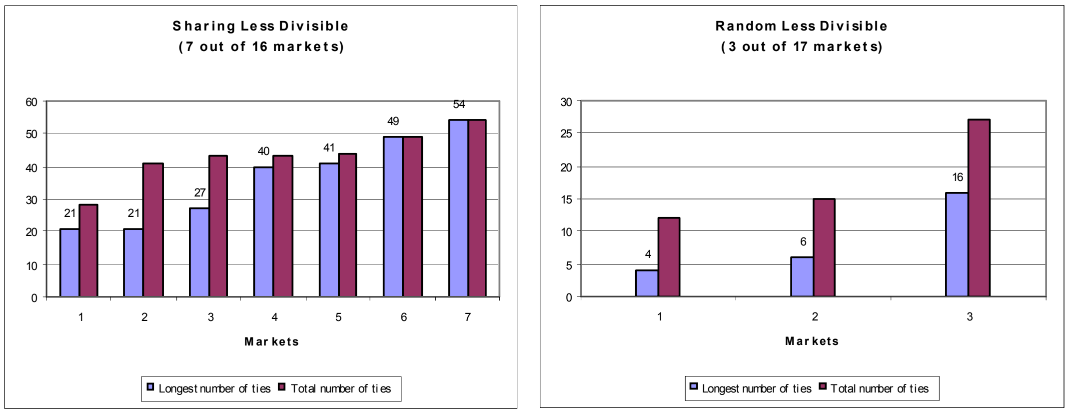

Figure 5. Markets exhibiting tacit perfect collusion under the sharing and random (less divisible) treatments.

When prices are more divisible, 3 out of 7 markets converged to perfect collusion[30] under the sharing tie breaking rule, while none of the 16 markets did under the random one. It seems that the sharing tie breaking rule facilitates perfect collusion, regardless of the degree of divisibility of the price space. Perfect collusion might be slightly more fragile under a continuum price space but still survives (see Figure 4).

Furthermore, markets under the sharing rule exhibit a more stable pattern of perfect collusion since the latter is sustained except for the very last periods. For instance, one of the markets converged to perfect collusion after the first five periods and remained there until the last period. On the other hand, when the random rule is employed, perfect collusion is more unstable, i.e., it is broken and resumed several times. Also, it seems to be sustained in only one market, where the longest sequence of 16 ties occurs in the last twenty periods.[31] That is, the random rule makes perfect collusion more difficult to be attained and, in case of success, more fragile.

[30] To be precise, one of the three markets displayed "almost perfect" collusion, in the sense that one seller posted 3 units for sale at a price of 549, and the other seller posted 4 units at 550. It involved 26 periods (periods 34-59).

[31] Under the random less divisible treatment three markets tied for 12, 15 and 27 periods, but not consecutively. In particular, the longest sequences consisted of 4, 6 and 16 ties respectively. In the former two markets, perfect collusion was broken in the second half of the experiment. In the latter market, the longest sequence of ties (16) occurred in the last twenty periods.

To test for the effect of tie breaking rules on perfect collusion, we calculated for each market both the total number of price ties at 550 and the highest number of *consecutive* ties at 550.[32] We think that the latter number is a more accurate measure of perfect collusion, since it is better suited to capture the willingness to collude (it is less sensitive to scattered random ties). Table IA in Appendix A contains information both on the total number of ties and the highest number of consecutive ties, for every market in every treatment. A Mann-Whitney two-tailed test carried on individual markets' highest number of consecutive ties confirms that perfect collusion is significantly higher under the sharing tie breaking rule than under the random one when the price space is less divisible (p-value= 0.06). Under the continuum treatment this effect is not significant.[33]

This suggests that perfect collusion is more likely when the sharing tie breaking rule is employed. To estimate the likelihood of perfect collusion (i.e., price ties at 550) in the last thirty periods we employ a probit model. Probit analysis is conducted using the following random effects model:

$$Ties30_{i,t} = \beta_0 + \beta_1 Ties30_{i,t-1} + \beta_2 s30_{i,t} + \varepsilon_{i,t},$$

where i refers to markets and t to periods. We use a random effects specification to account for unobserved markets heterogeneity.[34]

We focus on the effect of tie breaking rules on the probability of perfect collusion in the last thirty periods. We run two separate regressions to account for the two different levels of divisibility, i.e., less divisible and continuum. So, given a divisibility level, the dependent variable $Ties30_{i,t}$ is a binary variable capturing perfect collusion, and it is equal to 1 in case of a tie at 550 and 0 otherwise. The explanatory variables are $Ties30_{i,t-1}$ (i.e., the lagged variable of $Ties30_{i,t}$), that accounts for whether there was perfect collusion or not in

[32] In a given period, an observation qualifies as a price tie at 550 if both sellers posted a price of 550. Thus, for a market, the highest number of consecutive ties at 550 is the longest sequence of consecutive periods where both sellers in that market posted the joint profit maximizing price of 550.

[33] Recall that we are using one summary statistic value for each market. This result might be due to the low number of observations collected under the sharing continuum treatment. Furthermore, note that if we include in our analysis the market that colluded almost perfectly (see footnote 30), this effect is significant also for the continuum treatment (Mann-Whitney, p-value=0.08).

[34] The individual effect in our case can be assumed to be uncorrelated with the regressors.

the previous period, and $s30_{i,t}$ which is a dummy variable whose value is 1 if the market is under the sharing treatment.

Dependent variable: tacit perfect collusion

Random effects probit

	Less Divisible	Continuum
Constant (β_0)	-2.33^{***} (0.42)	-3.13^{***} (0.34)
Perfect collusion last period (β_1)	1.18^{***} (0.21)	1.29^{***} (0.31)
Sharing dummy (β_2)	1.57^{***} (0.60)	1.32^{**} (0.61)
Wald $\chi^2(2)$	41.50^{***}	22.50^{**}
Marginal effects of sharing dummy	0.30^{**}	0.042
Obs.	957	667

Table III. Random effects probit results on perfect collusion.

Notes: Standard errors in parentheses.

***Statistically significant at 1% level. **Statistically significant at 5% level.

The results reported in Table III provide evidence that inertia (β_1) and the sharing tie breaking rule (β_2) affect significantly the probability of perfect collusion in the last thirty periods, under both divisibility treatments.[35] Nonetheless, the effect of the sharing tie breaking rule is stronger under the less divisible price space. In fact, the marginal effects on the probit regression indicate that the sharing tie breaking rule significantly increases the probability of perfect collusion by 30% under the less divisible price space. On the other hand, this increase is 4.2% under the continuum one, and it is not significant.

Using our results from the nonparametric and parametric analysis of our data we draw the following conclusion.

Conclusion: Tacit perfect collusion is significantly higher in the Sharing treatment than in the Random one.

[35] The null that the appropriate model contains only a constant is rejected (see the Wald $\chi^2(2)$ in the table). Alternative specifications do not change our conclusion. For example, this specification is robust to omitting the lagged dependent variable, and probit results are robust to assumptions about within-market correlation (that takes care of the fact that in our data consecutive observations within the same market are not independent).

This result seems to suggest that it is easier for subjects to coordinate their actions when coordination involves the choice of only one variable (e.g., price) rather than two variables (e.g., price and quantity). In fact, sellers can achieve equal profits and perfect collusion under both tie breaking rules. The only difference is that under the sharing rule it is sufficient that subjects post the same price (equal to 550), while under the random one, subjects should additionally restrict the number of units posted for sale. For example, if two subjects tie at a price of 550, then they sell three units each under the sharing tie breaking rule (getting a profit of 1527 each). To obtain the same outcome under the random tie breaking rule, not only each subject should post a price of 550, but also he should post a quantity equal to 3.

This observation motivates us to have a closer look at quantity restrictions below capacity, i.e., below 4 units.[36] In order to investigate whether subjects attempted to coordinate and share through output restrictions, we calculate the longest sequence of consecutive periods where output was restricted below 4 units. We use the longest sequence of periods rather than the total number of periods since we think it is a more accurate measure of the willingness to coordinate and share. We calculate this number for every seller so that, at the market level, this produces two numbers. We pick the largest of the two, which gives the longest sequence of consecutive periods with quantity restrictions for every market, i.e., we have one summary statistic value for every market.[37] For every treatment, according to the Wilcoxon test, the median of the distribution of the longest sequence of consecutive periods with output rationing is significantly greater than zero, where zero corresponds to no output restrictions. Thus, there is evidence that subjects tried to coordinate by limiting their output, in all treatments.[38]

Furthermore, if we further restrict our attention to sequences that exceed 3 consecutive periods,[39] 13 out of 33 markets attempted to coordinate and

[36] We thank a referee for this suggestion.

[37] Note that we do not use the number of periods where both sellers were simultaneously rationing their output since the occurrence of such an event was very rare. That is, in most cases, output rationing was unilateral.

[38] Recall that $N_{sc} = 7$, $N_{sld} = 16$, $N_{rc} = 16$, and $N_{rld} = 17$. This result is significant at the 1% level in all treatments but *sc*, where it is significant at the 5% level. It is also confirmed by the Kolmogorov-Smirnov test comparing the empirical distribution of the sample with a theoretical distribution which is degenerate at 0 (corresponding to no output restrictions).

[39] Since output rationing is not necessarily observable by sellers, the longer is the sequence

share under the random rule treatment (8 under the less divisible price space and 5 under the more divisible one). Under the sharing treatment, 8 out of 23 markets attempted to collude through quantity restrictions (7 under the less divisible space and 1 under the more divisible one). This analysis indicates that subjects tried to coordinate and share by rationing their output. Notice that regardless of whether the output rationing was visible[40] or not to the competitor, it did not facilitate perfect collusion in the random rule treatment,[41] suggesting that coordination requiring output rationing is more difficult to achieve.

5.1.2 Prices

The empirical distributions of posted prices are different under the two tie breaking rules (see also Figure 1A in Appendix A). For instance, the frequency of prices equal to 550 is 43.7 under the sharing tie breaking rule, and 26.08 under the random one (given a less divisible price space).

The effects of tie breaking rules on prices is not as clear-cut as those on collusion. Tie braking rules seem to have opposite effects depending on the divisibility of the price space (see Figure 3b and Figure 3d in Appendix A).

To test for these effects we employ both nonparametric and parametric analysis. To carry nonparametric tests, we use as observations the mean of prices posted after period ten in each individual market. That is, we use prices posted in each individual market and we average them over periods 11-60, so that we have exactly one summary statistic value for each market.[42] We find that, when the price space is less divisible, market means are marginally significantly higher under the sharing rule than under the random one (Kolmogorov-Smirnov, p-value=0.09). When the price space is continuum, we have the opposite (Kolmogorov-Smirnov, p-value=0.08).[43]

We investigate the effects of tie breaking rules on posted prices also by using a subject-specific regression model. Since observations from the two

of consecutive periods where it occurs, the stronger may be the willingness to signal a desire for cooperation.

[40] The number of units posted for sale was not displayed on the competitor's screen. So, unilateral output restrictions could have been inferred only indirectly. For example, if the seller restricting the output also posted a lower price.

[41] Only one market converged to perfect collusion under the random rule treatment, and it did not exhibit output rationing.

[42] Recall that $N_{rld} = 17$, $N_{sld} = 16$, $N_{rc} = 16$, and $N_{sc} = 7$.

[43] The tests on market medians have the same direction but are not significant.

sellers in one market are highly dependent and consecutive observations over time are not independent, we accounted for cross-sectional correlation and heteroskedasticity across panels, as well as autocorrelation within panels.

The statistics are calculated by running a regression with posted price as the dependent variable. Dummy variables and the inverse of the time period are the explanatory variables. The regression makes use of all the price data except for period 1 and the last two periods.[44] We estimate the following model

$$p_{it} = \alpha_0 + \alpha_1(1/t) + \alpha_2 rc_{it} + \alpha_3 sld_{it} + \alpha_4 sc_{it} + \varepsilon_{it}.$$

where i refers to individual sellers and t to time periods.

The variable $(1/t)$ is an explanatory variable equal to the inverse of the trading period. We employ four dummy variables, one, rc_{it}, is equal to 1 if the price is observed under the treatment with random tie breaking rule and continuum choice space, while sld_{it} is 1 if the price is observed under the treatment with sharing tie breaking rule and less divisible choice space. Similarly, the dummy variable sc_{it} equals 1 if the posted price is observed in the treatment with sharing tie breaking rule and continuum choice space.[45]

The next table contains the results of our estimation. Standard errors are printed below the coefficient estimates.

Dependent variable: posted price

α_0	$\alpha_1(1/t)$	$\alpha_2(rc)$	$\alpha_3(sld)$	$\alpha_4(sc)$
440.30^{***}	-152.40^{***}	17.93^{***}	27.64^{***}	-16.26^{***}
(1.29)	(7.02)	(1.69)	(2.31)	(3.14)

***Statistically significant at 1% level. Obs.=6384. $\chi^2(4) = 681.21$.

Table IV. Regression results on posted prices.

The negative and significant coefficient on t implies an upward trend in prices for the early periods. The positive and significant coefficient on sld indicates that sellers under the sharing tie breaking rule posted higher prices than the ones posted by sellers under the random one (given the less divisible choice space). On the other hand, a look at the difference between the coefficient estimates of rc and sc, suggests the opposite under the continuum

[44] We excluded the first and last periods to account for noise and endgame effects, respectively. This procedure is used in the analysis of panel data in experimental economics (e.g., [4]). Excluding the first ten periods does not change the results qualitatively.

[45] The dummy variable rld_{it} is the omitted category.

treatment.[46] This suggests that pricing behavior tends to be affected by the tie breaking rule.

Conclusion: Sellers tend to post higher prices under the sharing tie breaking rule than under the random one under the less divisible regime, and the reverse under the continuum.

5.2 Divisibility

What about the effect of divisibility on tacit perfect collusion? It seems to strengthen the incidence of tacit perfect collusion. When the sharing tie-breaking rule is employed, the percentage of price ties at 550 is 32% under the less divisible space and 15% under the continuum (see Figure 4).

Also, the fraction of markets that exhibited tacit perfect collusion is 7/16 under the less divisible space and 3/7 under the continuum (under the sharing tie breaking rule). Under the continuum treatment tacit perfect collusion still survives but is more fragile.[47]

This is confirmed also by the probit analysis provided in Table III. In particular, the sharing tie breaking rule significantly increases the probability of perfect collusion by 30% under the less divisible price space. This increase is 4.2% under the continuum one, and it is not significant.

The effect of divisibility on posted prices is less clear-cut. To see this, following the Nash Equilibrium prediction (See Table I), we test three hypotheses.

Hypothesis 2: Mean and median prices are higher under the Less Divisible regime than under the Continuum one.

The results are not conclusive. For instance, a look at Figure 6 suggests that this hypothesis is not supported by the data, under the random rule.

[46] The qualitative results of our estimation are robust to the addition of the lagged dependent variable as explanatory variable.

[47] For example, in one of the markets in the continuum treatment tacit perfect collusion exhibited an unstable pattern: sellers would break collusion with a cyclicality of few periods, would punish each other, and would resume collusion again.

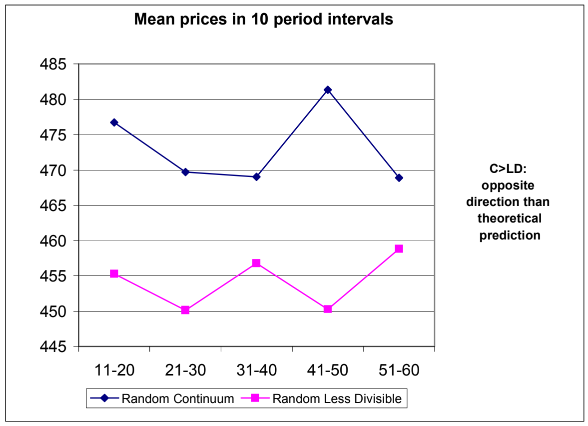

Figure 6. Mean Prices in ten period intervals.

To test for this hypothesis, we calculated the mean and median of prices posted after period ten,[48] so that we have exactly one observation per market.

Even though markets' mean prices are not significantly different, median market prices are significantly higher under *rc* than under *rld* (according to Kolmogorov-Smirnov test, p-value=0.03).[49] However, under the sharing rule, median and mean market prices tend to be-not significantly- higher when the space is less divisible (see also Figures 2c and 3c in Appendix A).[50]

According to parametric analysis, the positive coefficient on *rc* in Table IV confirms that sellers under the continuum choice space tend to post higher prices than under the less divisible choice space, given the random tie breaking rule. Under the sharing tie breaking rule, the opposite is true.

The intuition behind these results might be that, under the random rule, the continuum price space allows sellers to gently shade each other while the less divisible price space leads to abrupt undercutting behavior. On the other hand, given the sharing tie breaking rule, collusion occurs more frequently when the space is less divisible.

Hypothesis 3: Price dispersion is higher under LD than under C.

This hypothesis does not receive strong support. The units of observation are the standard deviations of prices posted in every market with the

[48] We did not include the first ten periods to account for noisy behavior in the initial periods.

[49] The Kolmogorov-Smirnov test is a two-sample test of equality of distributions. The p-value of 0.03 refers to the combined test. According to the Mann-Whitney tests there is no significant difference.

[50] This effect is not significant (Kolmogorov-Smirnov test, p-value=0.69)

exclusion of the first 10 periods.[51] The Mann-Whitney test fails to reject the null hypothesis that there is no difference between market prices standard deviations in the less divisible treatment versus the continuum one, both under the random tie breaking rule (p-value=0.80) and under the sharing one (p-value=0.14).[52]

Hypothesis 4: Equilibrium profit is higher under LD than under C.

This hypothesis is not supported by the data as well. Individual markets average profits are not significantly different across divisibility levels, given a tie breaking rule.[53]

Conclusion: None of the theoretical predictions regarding divisibility is strongly supported by the data. For instance, under the random tie breaking rule, prices tend to be higher under the continuum regime than under the less divisible one.

6 Conclusions

We examine the effect of a change in tie breaking rules and divisibility on prices in simple experimental duopoly markets with posted prices and simulated buyer behavior. We explore two possible tie breaking rules, sharing and random, and we model a change in divisibility by making the sellers' price space finer or coarser.

Theoretically, in our duopoly model, prices should be affected by divisibility but not by tie breaking rules. Our results show that a change in the tie breaking rule significantly affects pricing behavior. In particular, the sharing tie breaking rule facilitates tacit perfect collusion. Moreover, we find some evidence that in our design divisibility has an opposite effect than predicted by the Nash equilibrium, under the random tie breaking rule. This might be driven by the fact that, under a random tie breaking rule, the continuum

[51] We excluded the prices posted in the first 10 periods to account for noisy behavior.

[52] If we use the Kolmogorov-Smirnov test, this is the case under the random rule. On the other hand, under the sharing rule, standard deviations are significantly lower when the price space is less divisible (p-value= 0.07).

[53] In particular, contrary to the hypothesis, when the random rule is employed, they tend to be lower under the less divisible than under the continuum space.

This difference is not significant under both Mann-Whitney and Kolmogorov-Smirnov two-sample tests. Here, the two samples are given by individual markets average profits under the two divisibility treatments, given the tie breaking rule. That is, $N_{rld} = 17$ and $N_{rc} = 16$ in one comparison, while $N_{sld} = 16$ and $N_{sc} = 7$, in the other one.

choice space allows sellers to smoothly undercut each other and consequently keep higher prices by slowing down the Edgeworth cycle.

The result regarding tie breaking rules is striking since, in our game, a strategy supporting perfect collusion where sellers make equal profits can be chosen under both tie breaking rules. Under the sharing tie breaking rule it is sufficient that subjects post the same prices, while under the random one, subjects should additionally restrict the number of units posted for sale. For example, if two sellers tie at a price of 550, then they would sell three units each under the sharing tie breaking rule (getting a profit of 1527 each). To obtain the same outcome under the random tie breaking rule, not only each seller should post a price of 550, but also a quantity equal to 3.

That is, in our environment, perfect collusion is facilitated when subjects have to worry about coordinating only on the choice of one variable (the price) rather than the choice of two variables (both price and quantity). Thus, from the behavioral viewpoint, tie breaking rules affect coordination incentives, and, consequently, also collusion incentives.

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Appendix A

rld **(17 Markets)**		***sld*** **(16 Markets)**		***rc*** **(16 Markets)**		***sc*** **(7 Markets)**	
Ties	*Longest Sequence Of Ties*	*Ties*	*Longest Sequence Of Ties*	*Ties*	*Longest Sequence Of Ties*	*Ties*	*Longest Sequence Of Ties*
0	0	28	27	0	0	12	4
15	6	4	1	0	0	51	49
6	4	44	21	0	0	0	0
3	1	2	1	1	1	0	0
27	16	49	49	2	1	0[1]	0
8	2	54	54	0	0	0	0
0	0	0	0	0	0	2	1
12	4	2	1	9	5		
4	1	1	1	0	0		
0	0	1	1	3	3		
3	1	43	40	0	0		
0	0	2	1	0	0		
0	0	43	21	1	0		
2	2	41	41	0	0		
1	1	1	1	0	0		
1	1	1	1	0	0		
3	1						

Table IA. Total number of ties and highest number of consecutive ties.

[1] This market exhibited almost perfect collusion, since one seller consistently posted a price of 549 and a quantity of 3, while the other posted a price of 550 and a quantity of 4. If we count the periods when almost perfect collusion took place, both the number of Ties and the Longest Sequence of Ties would be 26.

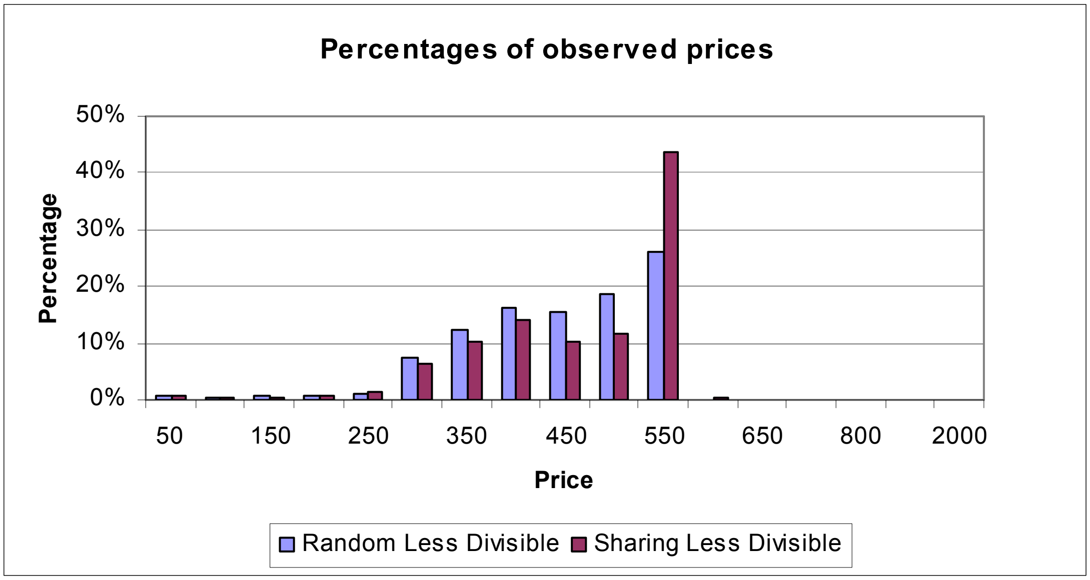
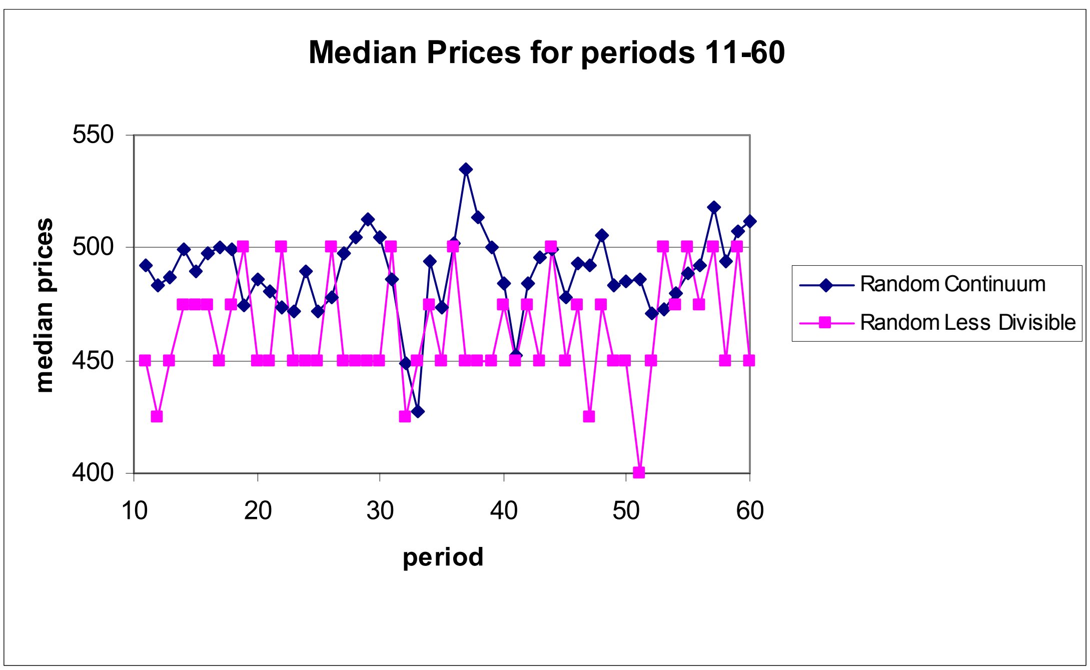
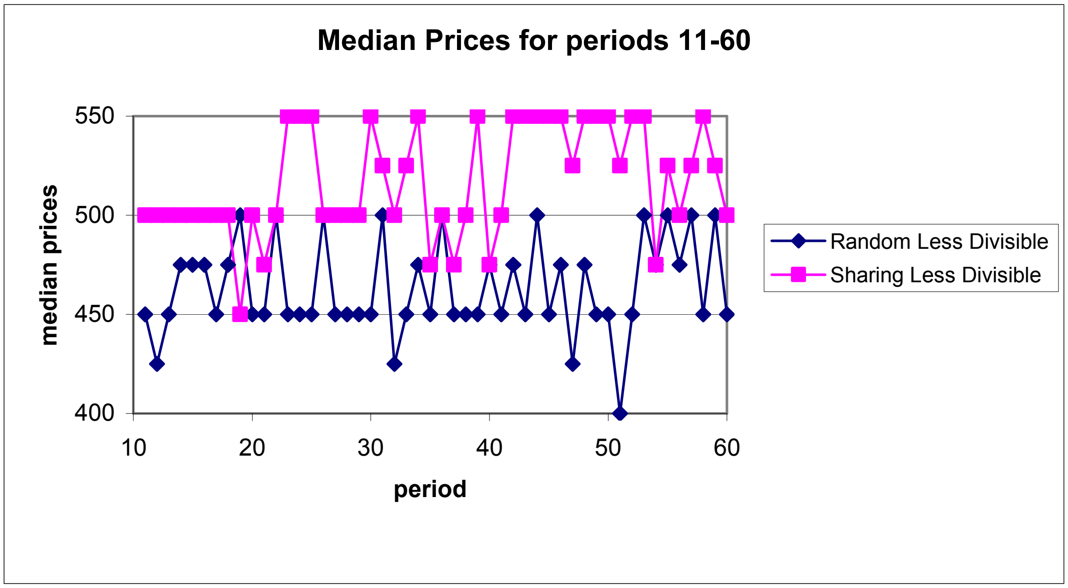


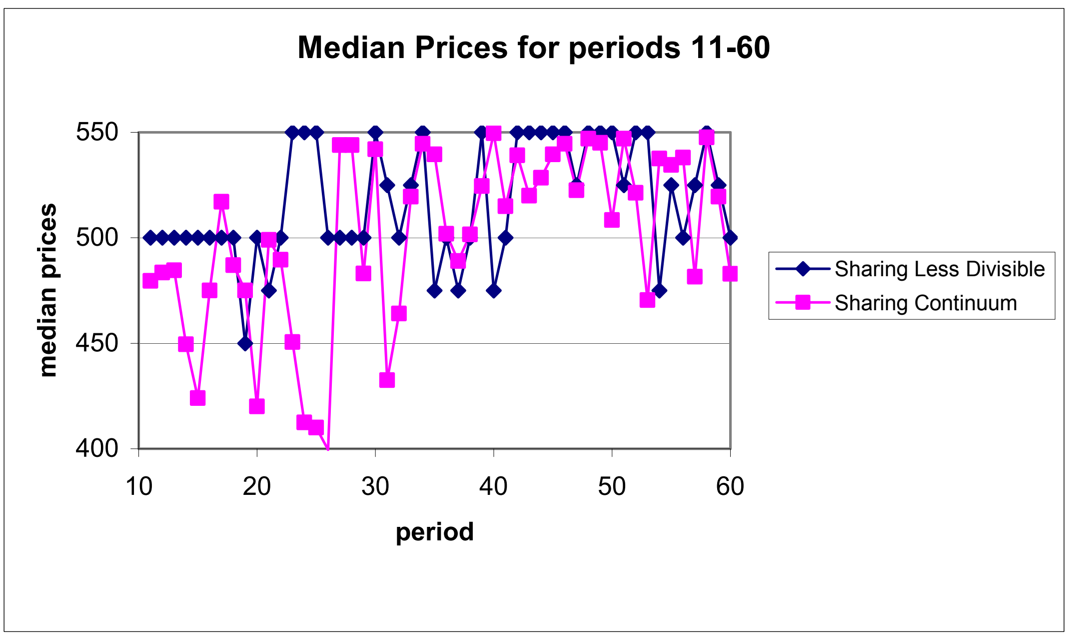
Figure 1A. Percentages of observed prices under the less divisible treatment.



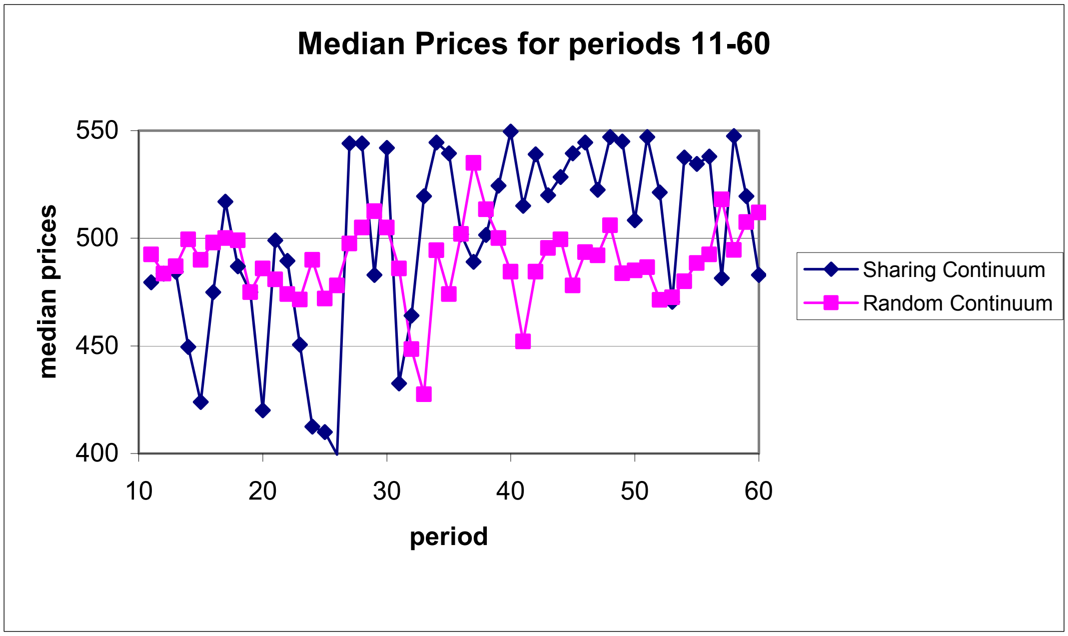
2a) Random Continuum vs. Random Less Divisible



2b) Random Less Divisible vs. Sharing Less Divisible

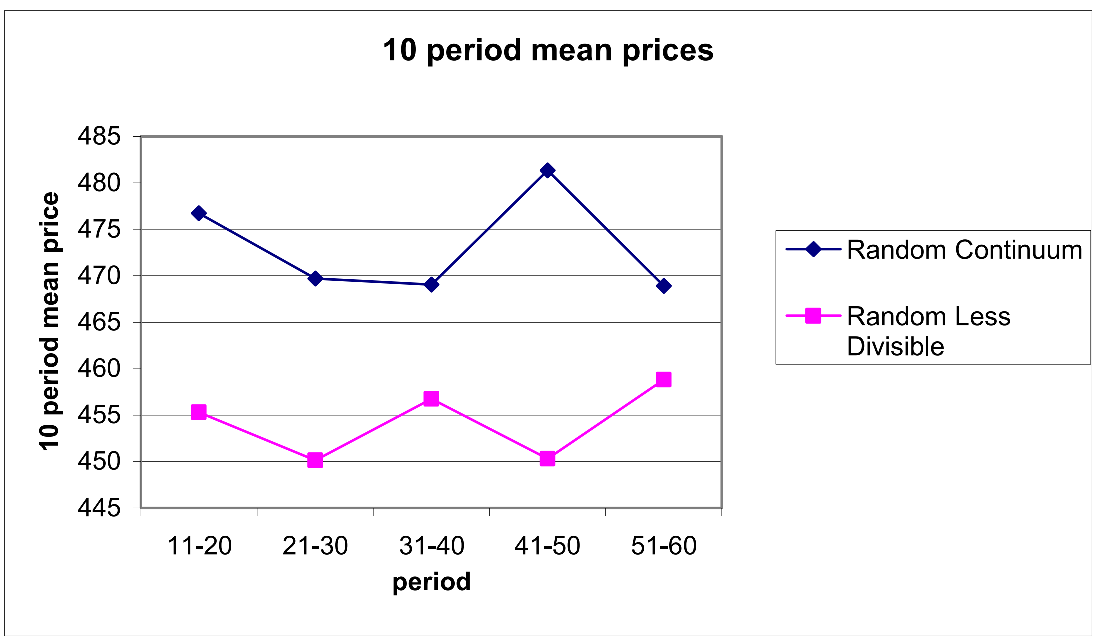


2c) Sharing Less Divisible vs. Sharing Continuum

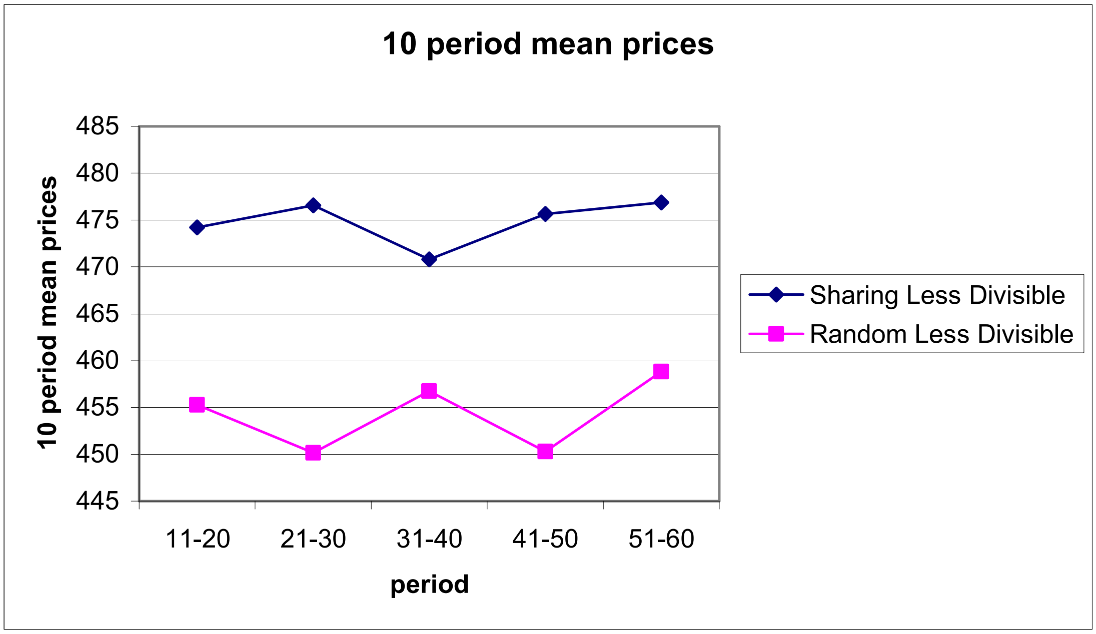


2d) Sharing Continuum vs. Random Continuum

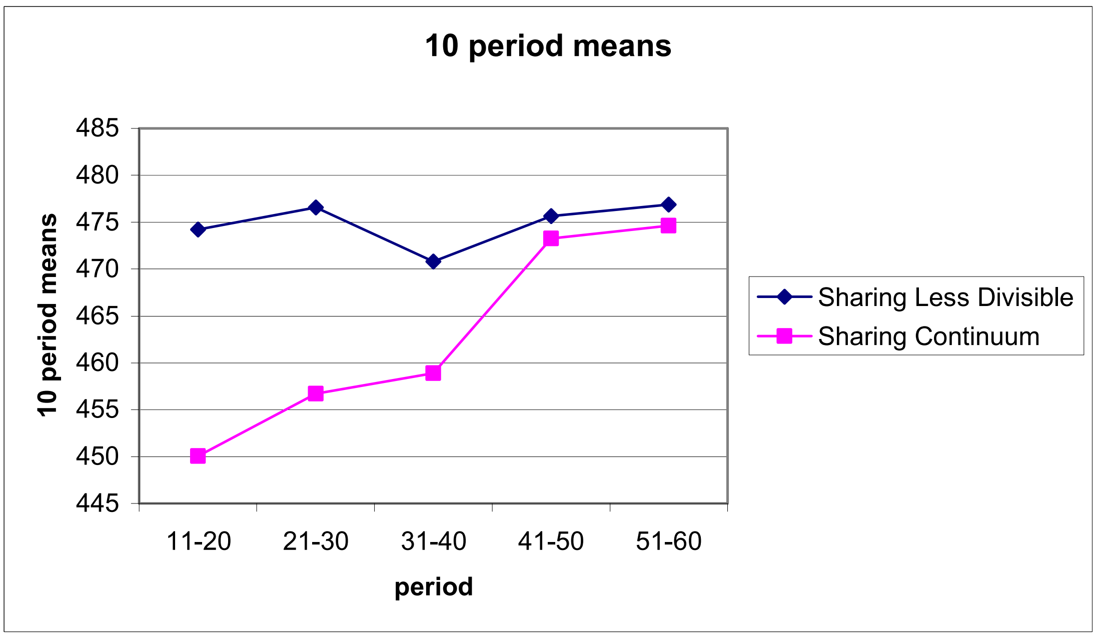
Figure 2A. Median Prices for periods 11-60 pooled by treatment.



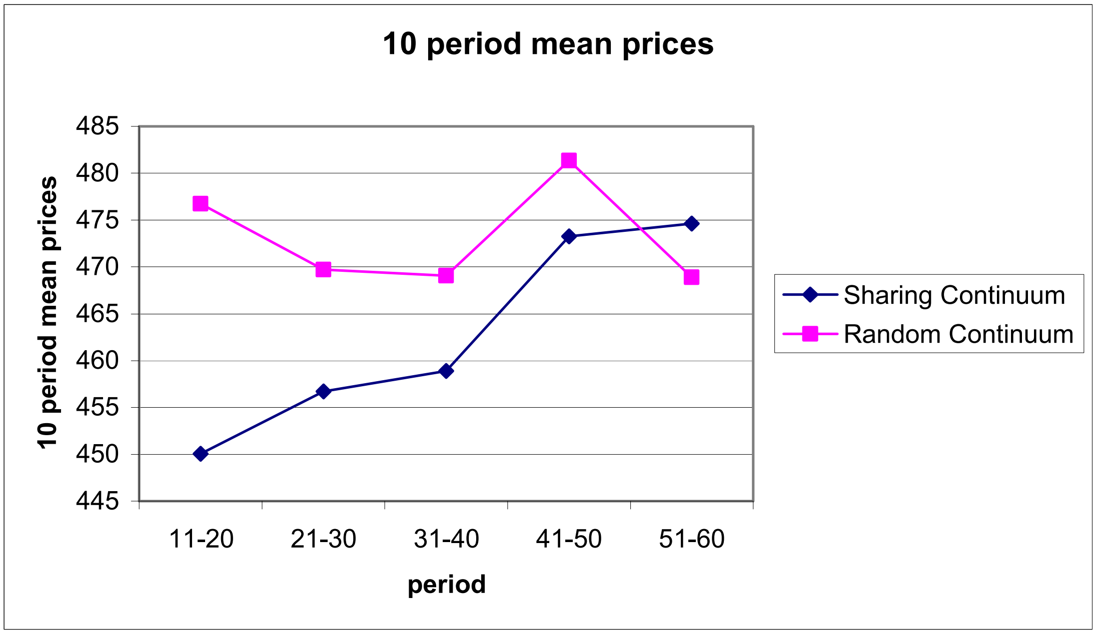
3a) Random Continuum vs. Random Less Divisible



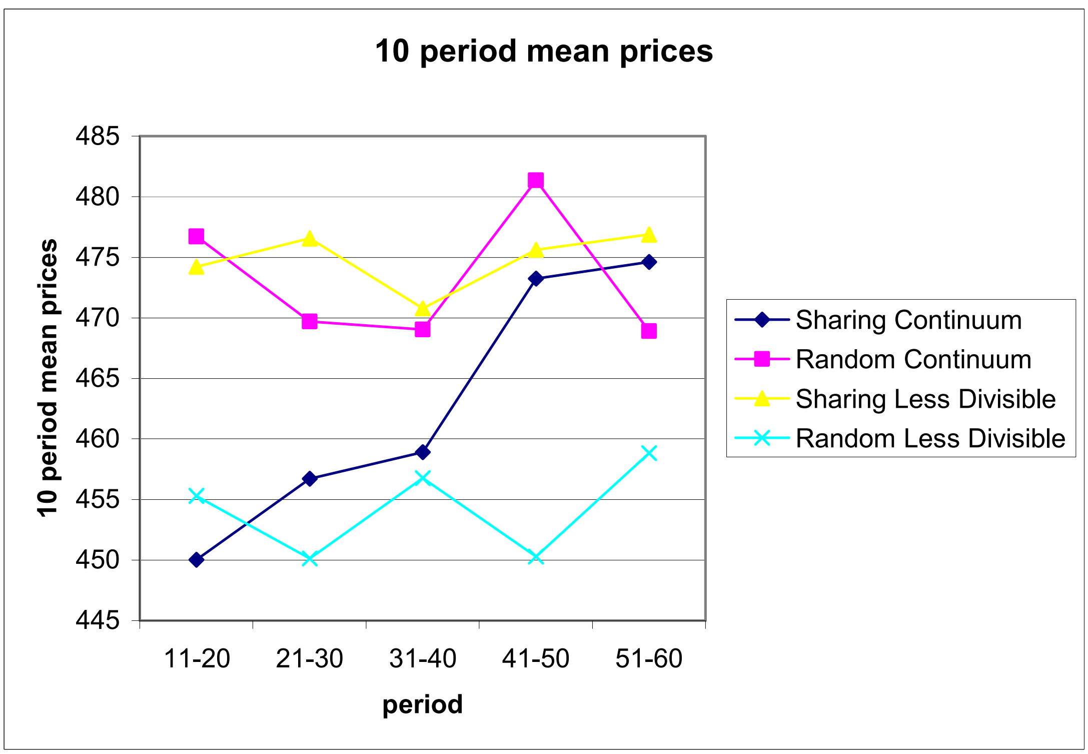
3b) Sharing Less Divisible vs. Random Less Divisible



3c) Sharing Less Divisible vs. Sharing Continuum



3d) Sharing Continuum vs. Random Continuum



3e) All treatments

Figure 3A. Ten period mean prices pooled by treatments.

Appendix B

Instructions

General

This is an experiment in the economics of market decision making. The instructions are simple and if you follow them carefully and make good decisions you will earn money that will be paid to you privately in cash. All earnings in your computer screens are in Experimental Dollars. These Experimental Dollars will be converted to real Dollars at the end of the experiment, at a rate of ________ Experimental Dollars = 1 real Dollar.

We are going to conduct a set of markets in which you will be a participant in a sequence of 60 trading periods where you can sell units of a good X. There are *two* sellers in every market. You will be a seller in today's experiment, and you will remain in this role throughout the experiment. During each trading period you will be free to sell units of the good X as you choose. Sellers earn money from selling units that cost a known amount.

Attached to these instructions you will find a sheet labeled Personal Record Sheet, which will also help you keep track of your earnings based on the decisions you might make. You are not to reveal this information to anyone. It is your own private information.

PERIODS 1-60

Buyers

The buyers' side of the market in today's experiment is simulated by a computer.

There is a single buyer. The buyer will make purchases according to the following rules.

1) The buyer will purchase a total of 6 units, if 6 units are available at prices of $550 or below. The buyer will purchase no units priced above $550.

2) The buyer will purchase first from the seller posting the lowest price, then from the seller posting the second lowest price. Once a seller has been selected, the buyer will purchase all units that can be afforded from that seller. If the buyer finishes making purchases from one seller and still wishes to buy more units, then the buyer will switch to the other seller.

3) If two sellers post identical prices and the total number of units offered exceeds 6 units, then the buyer will buy the same number of units from both sellers. For example, if each seller offers 4 units at the same price, then the buyer will purchase an equal amount from each seller.

Sellers

In this experiment there are *two* sellers in every market who are paired throughout the 60 periods. If you are a seller, your computer screen displays your costs —one cost value for each unit you might sell. The sellers are *identical*. That is, sellers incur the *same* per-unit cost when they sell a unit. Sellers may sell at most four units. See Figure 1 (the costs of the units you might sell on this example screen are completely different from the actual costs used in the experiment).

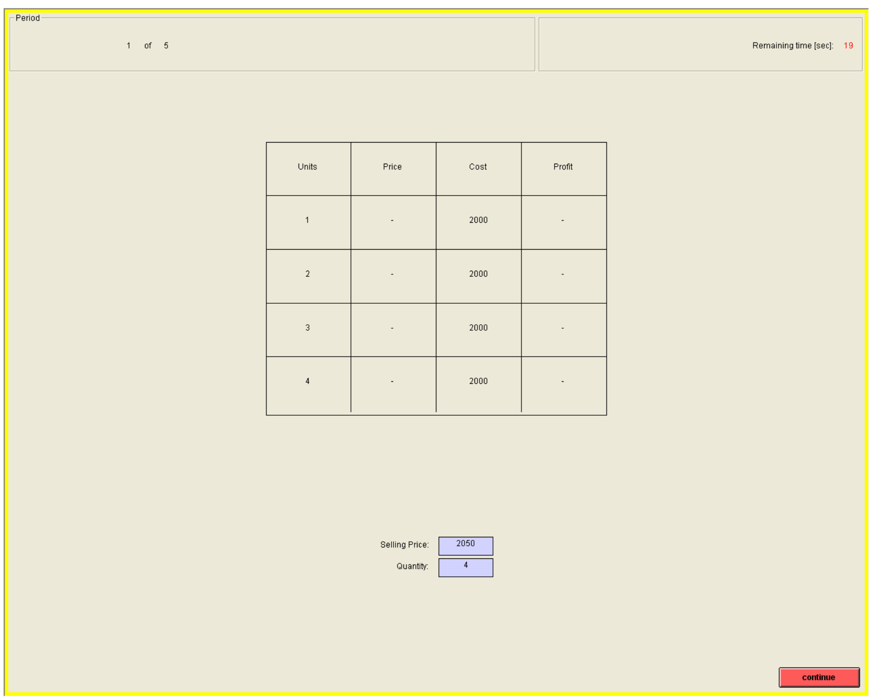


Figure 1: Example Market Trading Screen for Sellers

The profits from sales in each period (which are yours to keep) are computed by taking the difference between the amount of revenue you receive from the buyer minus the necessary production costs.

The revenue you receive from the buyer equals, of course, the price you charge times the quantity you actually sell. The buyer chooses how much to buy from you, up to the maximum quantity you have chosen to offer to the market. Your production cost is based on the units purchased from you.

That is,

[your earnings = (selling price × number of units sold) – (production cost of units sold)].

Suppose, for example, that the cost for your first unit is 2000, and the cost of your second unit is 2000. If you sell one unit at a price of 2050 your earnings are:

Earnings = (2050 × 1) – (2000) = 50.

If you sell two units at a price of 2050 your earnings are:

Earnings = (2050 × 2) – (2000+2000) = 100.

Your earnings per period and from all periods will be updated at the end of every period at the bottom of your computer screen, and are labeled *Profit this period* and *Total profit from all periods*, respectively. At the end of every period your computer screen will also display *Your price*, the *Other seller's price*, the *Number of units you sold*, and the *Total quantity sold in the market.* (See Figure 2.)

Figure 2: Example Market Trading Screen for Sellers

Notice that if a unit costs more than the amount for which you sell it then you suffer a loss in earnings on that unit. If you do not sell any units in a period then your earnings are zero for that period. Importantly, you do not incur the cost of a unit unless you sell that unit.

How to Sell

In each period you post a SELLING PRICE. Note that both you *and* the seller you are paired with must post a selling price which is a multiple of 50. You also select a QUANTITY. This limit represents the maximum number of units that you are willing to sell AT THE SELLING PRICE. You may offer as many units as you have available. However, if the posted price does not exceed the cost of all offered units, you lose earnings. You will enter selling prices and quantity using your computer. Figure 1 shows the market trading screen as seen by sellers. You submit selling prices and quantity limits using the "Selling Price" and "Quantity" box in the lower center of the screen, and then clicking on the "Continue" button. Once the selling price and the quantity are submitted, they are binding in the sense that the buyer can buy some or all of the units offered. This results in an immediate trade at the posted price.

The selling prices and the quantities of all the sellers are then given to the buyer, and the buyer may then purchase as much as he wishes from those goods that have been made available to him. A period ends when the buyer finishes making purchases, or when all sellers are out of units.

Recording Rules for Sellers

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Summary

- Sellers post selling prices and quantities. In making a price/quantity posting, the seller indicates a willingness to sell the posted number of units at the selling price
- There is one buyer who is played by the computer
- The buyer will purchase a total of 6 units, if 6 units are available at prices of $550 or below. The buyer will purchase no units priced above $550
- In the event that the two sellers post the same price and the total number of units offered exceeds 6 units, the buyer will buy the same number of units from both sellers
- Seller earnings = (selling price × number of units sold) – (production cost of units sold)
- A period ends when the buyer finishes to make his purchases, or when all sellers are out of units
- At the end of the period your computer screen displays:
  (1) Your price
  (2) The price posted by the other seller
  (3) Number of units you sold
  (4) Total quantity sold in the market
  (5) Your profit this period
  (6) Your profit from all periods
- Sellers should record these on Record Sheets at the end of each period

Are there any questions now before we begin the experiment?

Personal Record Sheet for Subject ______

Period (1)	Your Price (2)	Other Seller's Price (3)	Units you sold (4)	Units sold in the market (5)	Period Profit (6)	Total Profit from all periods (7)
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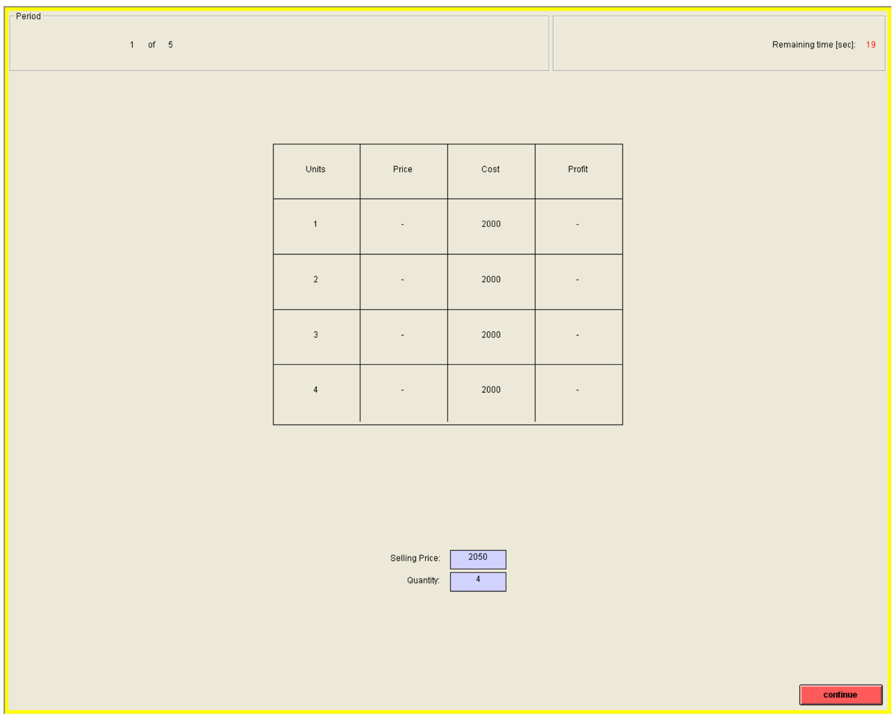

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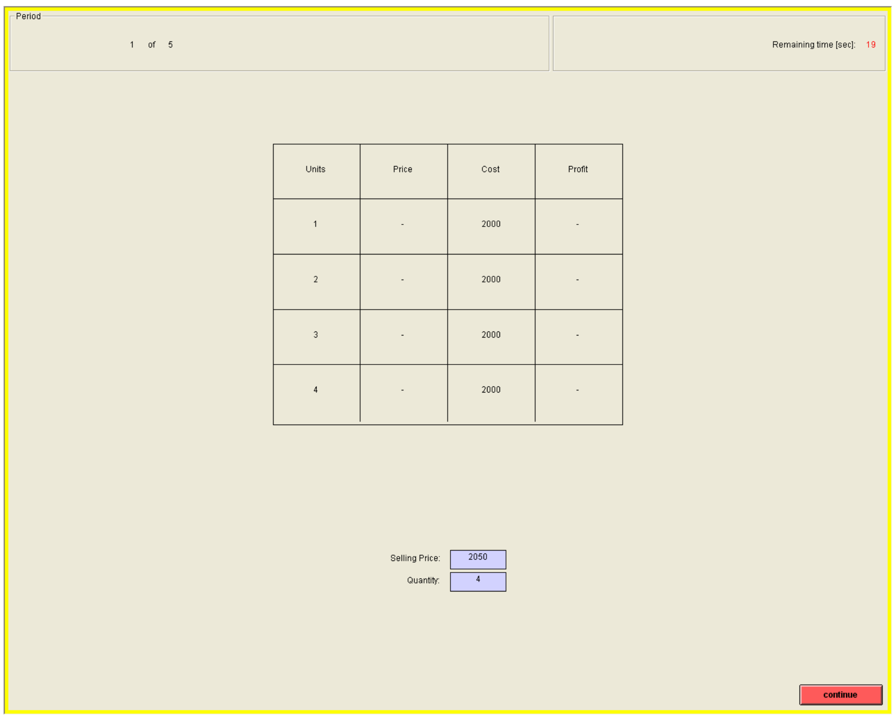

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