

MPRA

Munich Personal RePEc Archive

The beauty of "bigness" in contest design: merging or splitting?

Fu, Qiang and Lu, Jingfeng

April 2006

Online at <http://mpra.ub.uni-muenchen.de/947/>

MPRA Paper No. 947, posted 07. November 2007 / 01:25

The Beauty of “Bigness” in Contest Design: Merging or Splitting? †

Qiang Fu[‡]

Jingfeng Lu[§]

National University of Singapore

National University of Singapore

August, 2006

Abstract

This paper studies in a multiple-winner contest setting how the total efforts may vary between a grand contest and a set of subcontests. We first show that the rent-dissipation rate increases when the numbers of contestants and prizes are “scaled up”. In other words, the total efforts of a contest exhibit a striking “increasing return to scale” property: when the numbers of contestants and prizes scale up proportionally, the total efforts of the contest increase more than proportionally. Thus, the total efforts must increase when a set of identical subcontests are merged into a grand contest. Equivalently, the total efforts decrease when a grand contest is evenly divided. We further allow the grand contest to be split into uneven subcontests. We show that under a mild and plausible condition (regular contest technology), the grand contest generates more efforts as compared to any split contests.

JEL Nos: C72, D72

Keywords: Contests; Multiple-winners; Efforts; Size; Replication

1 Introduction

Contest is the situation where economic agents expend costly and non-refundable efforts in order to win a limited number of prizes. Abundant examples of such competition can be

*Very preliminary and incomplete. Please do not cite.

[†]We owe special thanks to Michael Baye for the inspiring discussion. We are grateful to Chew Soo Hong, Desheng Ouyang, Hongjun Zhong, Jimmy Chan, and all seminar participants at Shanghai University of Finance and Economics, for helpful comments and suggestions. All errors remain ours.

[‡]Qiang Fu: Assistant Professor, Department of Business Policy, National University of Singapore, 1 Business Link, SINGAPORE, 117592; Tel: (65)65163775, Fax: (65)67795059, Email: bizfq@nus.edu.sg.

[§]Jingfeng Lu: Assistant Professor, Department of Economics, National University of Singapore, 10 Kent Ridge Crescent, SINGAPORE, 119260; Tel: (65)65166026, Fax: (65)67752646, Email: ecljlf@nus.edu.sg.

observed in everyday life, such as promotion tournaments in internal labor markets inside firms, political campaigns, influence politics, college admissions, etc. Due to their ubiquity, contests have naturally attracted enormous attention from economic scholars, and a huge body of literature has been developed to explore the strategic behaviors of rent seekers in a wide variety of contexts. It has been widely recognized that the rule or the organization of a contest may pivotally influence contestants' incentives and behaviors. As argued by Gradstein and Konrad (1999), "... the contest structures are the outcome of a careful design with the view of attaining a variety of objectives, one of which is maximization of efforts by contenders". A great number of papers therefore have been devoted to the optimal design of contests that contribute to the interests of contest organizers.

Though a lion's share of these papers assume that contestants compete against all others for a single prize (winner-take-all), the assumption contrasts with many contest settings in reality. For instance, the government telecommunication regulator may issue a few operating licenses. Universities pick out thousands of freshmen from hundreds of thousands of applicants every year. A number of seats in parliament can be available to nevertheless a greater number of statesmen up for election. In all these examples, more than one prize is awarded, while each contestant may receive no more than one of them. While the design of "winner-take-all" contests has been thoroughly investigated in the literature, only a handful of papers concern themselves with the optimal structure of multi-winner contests¹.

In this paper, we focus on one particular aspect of multi-winner imperfectly-discriminatory contest design: the "size" of the contest. A contest organizer is planning to distribute a fixed number of prizes to a fixed pool of contestants. We attempt to investigate how the total efforts may vary between a grand contest and a set of subcontests. More specifically, the purpose of this paper is to study how the total efforts induced from rent-seeking contestants may vary when a set of subcontests are merged into a grand contest, or equivalently, when the grand multi-winner contest splits into "smaller" subcontests.

To pick out the prize recipients, the contest can be organized in two ways: the "grand"

¹Imperctly-discriminatory contest models that involve multiple-winner settings can be seen in the studies by Clark and Riis (1996, 1998a, 1998b), Amegashie, and Yates and Heckelman (2001). Barut and Kovenock (1998), Moldovanu and Sela (2006), and Moldovanu, Sela, and Shi (2006) investigate multiple-prize competition in perfectly-discriminatory conest setting (all-pay auction).

contest and the “split” contest. The contest organizer may administer a “grand” contest, in which every contestant competes against all others for all the prizes available. The swimming competitions in Olympic Games and World Championships tournaments exemplify such a grand contest. Swimmers are initially placed in different heats, and the eight fastest advance to the finals. The qualification to finals is not determined by a swimmer’s rank within his/her own heat, but his/her relative performance compared to all other opponents, including those who are placed in other heats. By the way of contrast, the contest organizer may split the grand contest into a set of subcontests, in each of which a subset of contestants compete against each other for a subset of the prizes. The “split” or “divisionalized” contest is not unusual either. For instance, in the recent past, the state government of Texas (as well as California and Florida) administered the “Percentage Plan” in college admissions. Under this policy, ten percent of top students in each high school are automatically admitted into The University of Texas system (or the counterparts in California and Florida), no matter how their academic performance is compared to students in other schools. As a result, the previously statewide races for admissions are more or less “downsized” to intra-school competitions.

How does the “grand” swimming competition rule differ from its “split” counterpart in providing incentives to the swimmers? Does the shift of admissions policy affect high school students’ incentive to engage in academic efforts? In general, which organizing institution demands more efforts from contestants, the “grand”, or the “split”? Does the “grand” dominate any set of “split” contests? Or does the comparison depend upon how the grand contest is divided? In particular, consider two seemingly equivalent contests with one hundred contestants and ten prizes: the first one consists of ten identical subcontests, each of which awards a single prize to ten potential recipients; while the other is a grand pooling competition that awards ten prizes to a hundred. Are they indeed identical? If they are not, which contest demands more effort from contestants?

To address these questions, we consider a grand contest C , in which $K > 1$ prizes are available to $N > K$ contestants. We allow the grand contest to be split into a set of $M > 1$ subcontests C_i . In each of them, N_i persons compete for K_i prizes, with $N_i \geq K_i \geq 1$, $\sum_{i=1}^M N_i = N$, and $\sum_{i=1}^M K_i = K$. Our main findings are summarized as follows.

1. The output (total efforts) of contests exhibits a striking **“increasing-return-to-scale” property**: When the numbers of contestants and prizes scale up proportionally, the total efforts of the contest increase more than proportionally. Therefore the rent-dissipation rate increases. Consequently, our result implies that the total efforts of a contest decrease when the grand contest is evenly split into a set of identical partitions. In other words, the total efforts increase when a set of identical subcontests merge into a grand contest.
2. We further allow the grand contest to be unevenly divided. We establish a mild sufficient condition (**regular** contest technology defined in Section 3.2), under which given the total number of contestants and the total number of prizes, the grand contest dominates any split contests in terms of the total efforts induced.

The rest of the paper proceeds as follows. In Section 2, we set up the model and present the general equilibrium solutions. The research questions are analyzed in details in Section 3. Section 4 discusses the implications and applications of our results, and concludes this paper.

2 Preliminaries

2.1 Multi-Winner Contests

Denote by $C \equiv C(N, K, V)$ a contest with $N \geq 2$ identical risk-neutral contestants competing for $K \in \{1, 2, \dots, N\}$ prizes of the unit value V . The N contestants simultaneously choose their effort outlays to compete for the K prizes. In the case where $K = 1$, a single winner receives the prize. We consider a contest success function, which is axiomized by Skaperdas (1996). A contestant i wins the prize with the probability

$$P_i = \frac{f(e_i)}{\sum_{j=1}^N f(e_j)}, \quad (1)$$

given the contestants' efforts profile $\mathbf{e} = (e_1, e_2, \dots, e_N)$. Wärneryd (2001) names f the *impact function*, which indicates contestants' technology. To guarantee an interior equilibrium,

we assume the function f is strictly increasing, twice differentiable, and weakly concave. Thus, a contestant i chooses his/her effort e_i to maximize the expected payoff

$$\pi_i = P_i V - e_i. \quad (2)$$

In the case where $K > 1$, we then consider a multi-winner nested contest as studied by Clark and Riis (1996, 1998), as well as Fu and Lu (2006), but assume a general impact function $f(e)$ as defined above. A block of K prizes are awarded in a sequential lottery process. Contestants simultaneously submit their efforts, while the winners of the contest are sequentially selected in K consecutive draws. Each contestant is allowed to receive no more than one prize. Thus, once a contestant is selected to win a prize, he/she is immediately removed from the pool of candidates up for the next draw. Denote by Ω_m the set of remaining contestants for the m -th draw, with $m \leq K$. The conditional probability that a contestant $i \in \Omega_m$ wins the m -th prize is given by

$$p(e_i, e_{-i}; \Omega_M) = \frac{f(e_i)}{\sum_{j \in \Omega_M} f(e_j)}. \quad (3)$$

Denote by $P_m(e_i, e_{-i})$ the probability that contestant i is selected in the m -th draw. Note that $P_m(e_i, e_{-i}) = \sum_{\forall \Omega_l^m} [\Pr(\Omega_l^m) \Pr(i \in \Omega_l^m) p(e_i, e_{-i}; \Omega_l^m)]$, where $\Pr(\Omega_l^m)$ is the probability that the remaining contestants up for the m -th draw are Ω_l^m , and $\Pr(i \in \Omega_l^m)$ is the probability that contestant i belongs to Ω_l^m . A contestant i chooses his/her effort e_i to maximize

$$\pi_i = V \sum_{m=1}^K P_m(e_i, e_{-i}) - e_i. \quad (4)$$

2.2 Equilibrium Solutions

Denote by e the equilibrium effort each contestant exerts in a symmetric Nash equilibrium of the contest (N, K, V) , and denote by $E \equiv Ne$ the total efforts the N contestants make in the contest. With a symmetric Nash equilibrium effort e , we have

$$\frac{\partial P_m(e, \dots, e)}{\partial e_i} = \frac{(1 - \sum_{t=0}^{m-1} \frac{1}{N-t}) f'(e)}{N f(e)}. \quad (5)$$

Note that

$$\sum_{m=1}^K (1 - \sum_{t=0}^{m-1} \frac{1}{N-t}) = K - \sum_{g=0}^{K-1} \frac{K-g}{N-g}. \quad (6)$$

Define $H(e) \equiv \frac{f(e)}{f'(e)}$. Because $f(e)$ is increasing and concave, we have $H'(e) > 0$. Inserting (6) into (5), we establish the first order condition for the symmetric Nash equilibrium

$$H(e) - \frac{1}{N} \left(K - \sum_{g=0}^{K-1} \frac{K-g}{N-g} \right) V = 0. \quad (7)$$

Proposition 1 *In the symmetric Nash equilibrium of a N -person, K -prize contest (N, K, V) , each contestant makes an effort*

$$e = H^{-1} \left[\frac{1}{N} \left(K - \sum_{g=0}^{K-1} \frac{K-g}{N-g} \right) V \right]. \quad (8)$$

The total efforts the N contestants make in the contest is then given by

$$E = NH^{-1} \left[\frac{1}{N} \left(K - \sum_{g=0}^{K-1} \frac{K-g}{N-g} \right) V \right]. \quad (9)$$

3 Analysis

In this paper, we consider a situation where the contest organizer plans to give away a total of K prizes of unit value V to K recipients from a pool of a total of N contestants, with $N > K > 1$. We define $C = C(N, K, V)$ the grand contest, where the N contestants compete for the K prizes. The grand contest C can be split into $M \geq 2$ subcontests $C_i = C(N_i, K_i, V)$, where $N = \sum_{i=1}^M N_i$ and $K = \sum_{i=1}^M K_i$. In a subcontest C_i , N_i contestants compete for $K_i \in \{1, \dots, N_i\}$ prizes of the unit value V . Thus, in other words, the grand contest C is the combination of the M subcontests C_i .

We denote by E_i the total equilibrium efforts the N_i contestants make in each subcontest C_i , with $i \in \{1, \dots, M\}$, and denote by E the total efforts the total of N contestants make in the combined contest C . By Proposition 1, we obtain

$$E_i = N_i H^{-1} \left[\frac{1}{N_i} \left(K_i - \sum_{g_i=0}^{K_i-1} \frac{K_i - g_i}{N_i - g_i} \right) V \right], \quad (10)$$

and

$$E = NH^{-1} \left[\frac{1}{N} \left(K - \sum_{h=0}^{K-1} \frac{K-h}{N-h} \right) V \right]. \quad (11)$$

Without a merger, the N contestants make the effort $\sum_{i=1}^M E_i = \sum_{i=1}^M \left\{ N_i H^{-1} \left[\frac{1}{N_i} \left(K_i - \sum_{g_i=0}^{K_i-1} \frac{K_i - g_i}{N_i - g_i} \right) V \right] \right\}$ in the set of M smaller contests C_i .

The intent of this paper is to compare the total efforts of the grand contest to that of the set of split subcontests. Consider the equilibrium effort solution we have obtained. The comparison between $\sum_{i=1}^M E_i = \sum_{i=1}^M \{N_i H^{-1}[\frac{1}{N_i}(K_i - \sum_{g_i=0}^{K_i-1} \frac{K_i-g_i}{N_i-g_i})V]\}$ and $E = NH^{-1}[\frac{1}{N}(K - \sum_{g=0}^{K-1} \frac{K-g}{N-g})V]$ would be inconclusive, since virtually no restriction has been imposed on the form of the impact function $f(e)$.

3.1 The “Replication” of Contests

For this moment, we consider a simple but interesting case, which requires the subcontests $C_i(N_i, K_i, V)$ to be identical, with $N_i = N_j = \tilde{N}$ and $K_i = K_j = \tilde{K}, \forall i, j \in \{1, \dots, M\}$.

As a result, the grand contest $C(N, K, V)$ is therefore a “ M -fold replication”² of each single subcontest, with $N = M\tilde{N}$, and $K = M\tilde{K}$. When the set of subcontests are merged into the grand one, the ratio of the number of prizes to the number of contestants hold constant. In the symmetric equilibrium of either setting, a contestant has a chance of $\frac{K}{N}$ to receive a prize. Does the behavior of the contestants differ between a grand contest and a subcontest?

By equation (10), we have

$$E_i = \tilde{N}H^{-1}[(\frac{\tilde{K}}{\tilde{N}} - \frac{1}{\tilde{N}} \sum_{g=0}^{\tilde{K}-1} \frac{\tilde{K}-g}{\tilde{N}-g})V], \text{ and} \quad (12)$$

$$\sum_{i=1}^M E_i = M\tilde{N}H^{-1}[(\frac{\tilde{K}}{\tilde{N}} - \frac{1}{\tilde{N}} \sum_{g=0}^{\tilde{K}-1} \frac{\tilde{K}-g}{\tilde{N}-g})V]. \quad (13)$$

In the grand contest that is a “ M -fold replication” of each single subcontest, the equilibrium total effort E can be rewritten as

$$\begin{aligned} E &= M\tilde{N}H^{-1}[\frac{1}{M\tilde{N}}(M\tilde{K} - \sum_{h=0}^{M\tilde{K}-1} \frac{M\tilde{K}-h}{M\tilde{N}-h})V] \\ &= M\tilde{N}H^{-1}[(\frac{\tilde{K}}{\tilde{N}} - \frac{1}{M\tilde{N}} \sum_{h=0}^{M\tilde{K}-1} \frac{M\tilde{K}-h}{M\tilde{N}-h})V]. \end{aligned} \quad (14)$$

²Wärneryd (2001) defines a contest with rN contestants competing for a prize of the value rV as the r -fold replication of the contest with N contestants competing for a prize of the value V . In our context, which may involve more than one winner, we borrow the terminology “ r -fold replication” of the original contest, but it represents a different setting from Wärneryd (2001). We allow the number of prizes and the number of contestants vary, but keep constant the value of each single prize.

Proposition 2 *When a grand contest is split into a set of identical subcontests, with the same number of contestants competing for the same number of prizes in each of them, the total efforts induced decrease, i.e., $E > \sum_{i=1}^M E_i$.*

Proof. To show this result, we only need to compare (13) to (14) and show that $E > \sum_{i=1}^M E_i$,

for $M > 1$. We rewrite $\sum_{h=0}^{M\tilde{K}-1} \frac{M\tilde{K}-h}{M\tilde{N}-h}$ as

$$\begin{aligned} \sum_{h=0}^{M\tilde{K}-1} \frac{M\tilde{K}-h}{M\tilde{N}-h} &= \sum_{h=0}^{M-1} \frac{M\tilde{K}-h}{M\tilde{N}-h} + \sum_{h=M}^{2M-1} \frac{M\tilde{K}-h}{M\tilde{N}-h} + \cdots + \sum_{h=(\tilde{K}-1)M}^{\tilde{K}M-1} \frac{M\tilde{K}-h}{M\tilde{N}-h} \\ &= \sum_{i=1}^{\tilde{K}} \sum_{h_i=(i-1)M}^{iM-1} \frac{M\tilde{K}-h_i}{M\tilde{N}-h_i}. \end{aligned} \quad (15)$$

Because

$$\begin{aligned} \frac{M\tilde{K}-h_i}{M\tilde{N}-h_i} &= \frac{M\tilde{K}-(i-1)M}{M\tilde{N}-(i-1)M} \\ &= \frac{\tilde{K}-(i-1)}{\tilde{N}-(i-1)}, \end{aligned} \quad (16)$$

and $\frac{M\tilde{K}-h_i}{M\tilde{N}-h_i}$ is decreasing in h_i , we have

$$\sum_{h_i=(i-1)M}^{iM-1} \frac{M\tilde{K}-h_i}{M\tilde{N}-h_i} < M \frac{\tilde{K}-(i-1)}{\tilde{N}-(i-1)}. \quad (17)$$

Inequality (17) implies

$$\begin{aligned} \sum_{i=1}^{\tilde{K}} \sum_{h_i=(i-1)M}^{iM-1} \frac{M\tilde{K}-h_i}{M\tilde{N}-h_i} &< M \sum_{i=1}^{\tilde{K}} \frac{\tilde{K}-(i-1)}{\tilde{N}-(i-1)} \\ &= M \sum_{i=0}^{\tilde{K}-1} \frac{\tilde{K}-i}{\tilde{N}-i} \\ &= M \sum_{g=0}^{\tilde{K}-1} \frac{\tilde{K}-g}{\tilde{N}-g}. \end{aligned} \quad (18)$$

Hence,

$$\begin{aligned} \frac{\tilde{K}}{\tilde{N}} - \frac{1}{M\tilde{N}} \sum_{h=0}^{M\tilde{K}-1} \frac{M\tilde{K}-h}{M\tilde{N}-h} &= \frac{\tilde{K}}{\tilde{N}} - \frac{1}{\tilde{N}} \left[\frac{1}{M} \sum_{h=0}^{M\tilde{K}-1} \frac{M\tilde{K}-h}{M\tilde{N}-h} \right] \\ &> \frac{\tilde{K}}{\tilde{N}} - \frac{1}{\tilde{N}} \sum_{g=0}^{\tilde{K}-1} \frac{\tilde{K}-g}{\tilde{N}-g}. \end{aligned} \quad (19)$$

As $H^{-1}(\cdot)$ is an increasing function, we have

$$E > \sum_{i=1}^M E_i. \quad (20)$$

Q.E.D. ■

Proposition 2 contends that the grand contest induces more efforts as compared to the set of identical subcontests. This result requires only the weak concavity on f , therefore it holds under fairly general settings. Because all contestants are identical, it reveals that in the symmetric equilibrium, a contestant behaves more competitively in the grand contest, although in the equilibrium he/she has the same chance to receive a prize in either contest setting. Our analysis therefore sheds light on a more fundamental question: How does the structure of a multiple-winner contest affect contestants' incentives to make efforts? We argue that the equilibrium effort of a contest exhibits the following **“increasing-return-to-scale”** property.

Theorem 1 *In a multiple-winner contest, holding constant the unit prize value, when the number of contestants and the number of prizes increase by a common integer factor t ,*

- (i) *each contestant increases his/her equilibrium effort;*
- (ii) *the total efforts end up with increasing by more than t times.*

The equilibrium rent-dissipation rate for a contest is defined as the ratio of total effort and total prizes, i.e. $\frac{Ne}{KV}$. The following implication naturally arises.

Corollary 1 *When the number of contestants and the number of prizes increase proportionally, the rent-dissipation rate strictly increases.*

We contend that a “bigger” contest would demand more efforts than contestants even if the number of prizes increase in proportion to the number of contestants. Theorem 1 yields important insights for economic studies on contests. It implies that contestants behave differently when the “scale” of the contest varies. Thus, insights obtained from relatively small contest settings may not naturally extend to contests of large scale.

To understand the intuition behind the result, consider two identical smaller contests, C_1 and C_2 . Suppose in each of them, N identical risk-neutral contestants compete for $K < N$

prizes of the unit value V , while in the merged contest $2N$ identical risk-neutral contestants compete for $2K < 2N$ prizes of the unit value V . In contests C_1 and C_2 , the efforts exerted by the contestants increase their winning probabilities for the first K draws. By the way of contrast, in the merged contest, the efforts exerted by the contestants increase their winning probabilities for the first $2K$ draws, which yields higher marginal return as compared to the smaller contest. The increased marginal return to effort therefore leads to higher equilibrium efforts as the marginal costs of effort remain unchanged. It should be clarified that the higher individual effort in the “scaled-up” contest does not stem from the escalated competition among a larger number of contestants. This is clear from the equilibrium effort solution as given by equation (8): When $f(e) = e$ and $K = 1$ in (8), we have $e = \frac{1}{N}(1 - \frac{1}{N})$, which decreases with N for $N \geq 2$.

3.2 Uneven Subcontests

Our previous results show that the total efforts decrease when a grand contest is evenly split into a set of identical subcontests. In this part, we extend our analysis to the setting where the grand contest can be unevenly divided. Consider a simple example with a grand contest $C = C(10, 4, 1)$ and a linear impact function $f(e) = e$. The grand contest C can be split into two subcontests $C_1 = C(7, 3, 1)$ and $C_2 = C(3, 1, 1)$. From Proposition 1, we have $e = 0.287$, $e_1 = 0.291$ and $e_2 = 0.222$. Once the grand contest is split into C_1 and C_2 in the above way, contestants allocated to C_1 increase effort, while contestant in C_2 reduce efforts. But nevertheless, when it comes to the total efforts induced, we see that $\sum_{i=1}^2 E_i = 2.704 < E = 2.87$, and the grand contest C dominates the set of two split contests. However, a contest can be structured and split in numerous ways. Thus, an interesting question arises: is the dominance of the grand contest we observed from this example merely an artifact of the particular setting, or does it stem from any regularity that applies in broader contexts? Alternatively, if such regularity exists, then to what extent does it hold?

To this purpose, we consider the class of contests that satisfy the following **regularity** condition.

Definition 1 *A contest is **regular** if and only if*

(i) contestants' impact function $f(e)$ is strictly increasing, concave and third-order differentiable;

(ii) $H^{-1}(\cdot)$ is concave, where $H(\cdot) = \frac{f(\cdot)}{f'(\cdot)}$.³

To compare $\sum_{i=1}^M E_i$ with E , we first present the following key Lemma, which summarizes an interesting property of the series $\{\frac{K-g}{N-g}\}_{g=0}^{K-1}$.

Lemma 1 $\sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1} + \sum_{g_2=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2} > \sum_{g=0}^{K_1+K_2-1} \frac{K_1+K_2-g}{N_1+N_2-g}$, $\forall N_1 \geq K_1 > 0, N_2 \geq K_2 > 0$, with $N_1 + N_2 > K_1 + K_2$.

Please refer to the appendix for the proof of Lemma 1. The main idea of the proof is quite straightforward. Define S to be the set composed of all the $K_1 + K_2$ elements in series $\{\frac{K_1+K_2-g}{N_1+N_2-g}\}_{g=0}^{K_1+K_2-1}$, and \tilde{S} to be the set composed of all the $K_1 + K_2$ elements in the combined series of $\{\frac{K_1-g_1}{N_1-g_1}\}_{g_1=0}^{K_1-1} \cup \{\frac{K_2-g_2}{N_2-g_2}\}_{g_2=0}^{K_2-1}$. In the proof, we constructively set up a one-to-one correspondence between S and \tilde{S} , such that any element in S is smaller than or equal to its counterpart in \tilde{S} .

With the property revealed by Lemma 1, we obtain the following result.

Theorem 2 *The grand contest induces more total efforts than the set of subcontests, i.e.*
 $E > \sum_{i=1}^M E_i, \forall M \geq 2$.

Proof. We first consider two subcontests C_1 and C_2 , which belong to the set of M contests. Without loss of generality, we assume $K_1 + K_2 < N_1 + N_2$. We denote by $C_{1 \cup 2} = (N_1 + N_2, K_1 + K_2, V)$ the contest that combines C_1 and C_2 . By Proposition 1, without combination, the total efforts induced by the two subcontests are given by

$$E_1 + E_2 = N_1 H^{-1}\left[\frac{1}{N_1}\left(K_1 - \sum_{g_1=0}^{K_1-1} \frac{K_1 - g_1}{N_1 - g_1}\right)V\right] + N_2 H^{-1}\left[\frac{1}{N_2}\left(K_2 - \sum_{g_2=0}^{K_2-1} \frac{K_2 - g_2}{N_2 - g_2}\right)V\right]. \quad (21)$$

In contrast, in the combined contest, the total efforts contestants make amount to

$$E_{1 \cup 2} = (N_1 + N_2) H^{-1}\left[\frac{1}{N_1 + N_2}\left(K_1 + K_2 - \sum_{h=0}^{K_1+K_2-1} \frac{K_1 + K_2 - h}{N_1 + N_2 - h}\right)V\right]. \quad (22)$$

³This property is not necessary for the results in section 3.1.

Step 1: $E_1 + E_2 \leq (N_1 + N_2)H^{-1}\left\{\left[(K_1 + K_2) - \left(\sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1} + \sum_{g_2=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2}\right)\right]V\right\}$.

Because $H(e) = \frac{f(e)}{f'(e)}$ is concave, it follows that $H^{-1}(\cdot)$ is convex in its argument. By Jensen's inequality, we establish

$$\begin{aligned}
& E_1 + E_2 \\
&= N_1 H^{-1}\left[\frac{1}{N_1}\left(K_1 - \sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1}\right)V\right] + N_2 H^{-1}\left[\frac{1}{N_2}\left(K_2 - \sum_{g_2=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2}\right)V\right] \\
&\leq (N_1 + N_2)H^{-1}\left\{\frac{N_1}{N_1 + N_2} \cdot \left[\frac{1}{N_1}\left(K_1 - \sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1}\right)V\right] + \frac{N_2}{N_1 + N_2} \cdot \left[\frac{1}{N_2}\left(K_2 - \sum_{g_2=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2}\right)V\right]\right\} \\
&= (N_1 + N_2)H^{-1}\left\{\frac{1}{N_1 + N_2}\left[(K_1 + K_2) - \left(\sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1} + \sum_{g_2=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2}\right)\right]V\right\}. \tag{23}
\end{aligned}$$

Step 2: $(K_1 + K_2) - \left(\sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1} + \sum_{g_2=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2}\right) < (K_1 + K_2) - \sum_{g=0}^{K_1+K_2-1} \frac{K_1+K_2-g}{N_1+N_2-g}$.

This inequality directly follows from Lemma 1.

Step 3: $E_1 + E_2 < E_{1\cup 2}$.

Because $H^{-1}(\cdot)$ is strictly increasing, we have

$$\begin{aligned}
& H^{-1}\left\{\left[(K_1 + K_2) - \left(\sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1} + \sum_{g_2=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2}\right)\right]V\right\} \\
&< H^{-1}\left\{\left[(K_1 + K_2) - \sum_{g=0}^{K_1+K_2-1} \frac{K_1+K_2-g}{N_1+N_2-g}\right]V\right\} = E_{1\cup 2}, \tag{24}
\end{aligned}$$

which implies $E_{1\cup 2} > (N_1 + N_2)H^{-1}\left\{\left[(K_1 + K_2) - \left(\sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1} + \sum_{g_2=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2}\right)\right]V\right\} > E_1 + E_2$.

Step 4: $E > \sum_{i=1}^M E_i$.

Consider another contest $C_3 = C(N_3, K_3, V)$. Combine $C_{1\cup 2}$ with C_3 . Then by the result of **Step 3**, we must have $E_{1\cup 2\cup 3} > E_{1\cup 2} + E_3$. In other words, the combined contest $C_{1\cup 2\cup 3}$ generates higher efforts than the two separate contests $C_{1\cup 2}$ and C_3 . Following this argument, we have $E > \sum_{i=1}^M E_i, \forall M \geq 2$.

Q.E.D. ■

Theorem 2 establishes a mild sufficient condition (**regular** contest technology) for the dominance of the grand contest: A grand contest always generates strictly more efforts than the split “smaller” subcontests, no matter how the pool of contestants are divided or how the prizes are allocated across subcontests. Alternatively, merging “smaller” contests always

creates more competition and induces contestants to exert more efforts, no matter how these “smaller” contests are constructed. But nevertheless, the **regularity** condition, i.e. the weak concavity of $H(e)$, is by no means a strong restriction. It is satisfied by the class of power functions $f(e) = e^\alpha$, which have been commonly assumed as the contest technology in the literature. Hence, the result of Theorem 2 holds for a wide class of contest settings.

Contestants in different subcontests may respond differently after the grand contest is split. Some of them may have to exert more efforts, while others exert less, depending on the particular structures of the subcontests. However, the “gain” of efforts in some subcontests always comes at the cost of “loss” in the others, and the “gain” must be more than offset by the “loss”. As a consequence, the total efforts unambiguously decreases as the grand contest is split.

Theorem 2 directly implies the following corollary.

Corollary 2 *The grand contest generates a higher rent-dissipation rate than the set of split contests.*

4 Discussion and Conclusion

In this paper, we show that, compared to any split contests, the grand contest maximizes the total efforts induced. Thus, if the efforts exerted by contestants accrue to the benefits of the contest organizer, then a grand contest of a greater size better serves the interest of the contest organizer. Besides the theoretical contribution, our results shed light on many real-life situations that resemble the competition as we modeled in this paper.

Firstly, our results directly apply to the “Percentage Plan” admission policy undertaken by the states of Texas, California and Florida. The “Percentage Plan” guarantees admissions to a fixed portion of top students from each high school in the state. This policy has been generally regarded as the natural alternative to affirmative action to maintain ethnic diversity in the state universities’ student body. However, this policy has been controversial ever since its very inception. Despite of the high profile of the debate, claims have long been centered on the effectiveness of this policy as a means to achieve diversity, but nevertheless, its ramifications on the efficiency of the education system has yet to be investigated. Our model

brings forth the possibility to assess the “Percentage Plan” on the ground of academic quality. How does the admission scheme affect high school students’ incentive to exert academic efforts? Our results predict that “downsizing” the admission competitions among students from the state level to school level weakens students’ incentives to engage in academic efforts. High school students tend to be less willing to invest in academic efforts. As a result, the prediction may raise additional concerns to the policy makers, because the potential benefits of the policy may come at the cost of the schooling systems’ educational output. A comprehensive and fair assessment of the policy would be difficult. Yet our results may provide a novel view towards the “Percentage Plan”: its incentive effect!

Secondly, our finding is also relevant to the organization of internal labor market inside a firm. Team production has become an increasingly popular mechanism in organizing working force. However, promotion-based compensation schemes are universally adopted to motivate workers, while the evaluation of workers relies on their comparative performance, which resembles a contest or a tournament. Thus, when workers are organized into a number of teams, how should the firm pick out and reward top-performing workers? In particular, the firm may distribute a number of “prizes” (higher-level positions, bonus, or other compensation packages) among these teams, thus top-performing workers within each team receive the awards; by contrast, the firm may ignore workers’ ranks within their own teams, but allocate the prizes based on the performance comparison across the entire working force. Our results suggest that workers’ intra-team rankings should be assigned lesser weights. Of course, teams may perform different tasks, and therefore the management may lack a universally acceptable criterion to evaluate workers from different teams. However, if the outputs of different teams can be compared on a common ground, the comparative performance should be evaluated beyond each individual team, in order to provide a stronger incentive for workers to exert productive efforts.

Thirdly, our papers are closely related to the studies by Amegashie (2000), and Moldovanu and Sela (2006). Amegashie (2000) compares two ways of “shortlisting” in two-stage imperfectly-discriminatory contests. To select K finalists from a pool of N contestants in the preliminary stage, one shorting-listing procedure is to run a grand contest, in which each of the N contestants compete against all others. The other procedure is to evenly divide the pool of contestants into K groups. In each group one winner survives to the final. Amegashie

(2000) shows the former dominates the latter and induces more efforts if contestants have linear contest technology. We confirm his important insight and extend this thread of thinking in a context that allows for general contest technology and flexible division of the grand contest. In a perfectly-discriminatory contest setting, Moldovanu and Sela (2006) show that evenly splitting a grand contest into parallel subcontests do not benefit the contest organizer if his/her payoff is given by the total expected effort outlays.

By the way of contrast, when a grand contest is split into a set of subcontests, in each of which a subset of contestants compete for a subset of prizes, the total efforts decrease. Hence, to the extent that the rent-seeking activity is considered to be wasteful and undesirable, our results suggest that the contest organizer can successfully reduce the waste of loud lobbying by dividing the grand contest into a set of subcontests of lower scales. Hence, our results provide a rationale for “quota” systems, which are widely practiced when government agencies allocate public resources. When public resources are distributed among different regions or groups in fixed quota, the rent-seeking activities are therefore downgraded to a set of “intra-region” or “intra-group” competition. Thus, our paper is also related to Wärneryd (1998), and Inderst, Müller, and Wärneryd (2005). Both of these papers suggest that distributional conflicts can be reduced, if the jurisdictional organizations are more hierarchal.

In the practice of “quota” system, the prizes are often distributed among regions or groups with the number of prizes in proportion to the populations of the regions or the sizes of the groups. For instance, University of Texas system guarantees admissions to students among the top four percent of each high school in the State of Texas. Such rules of “proportional representations” mainly address the equity issue in allocation. However, our results raise additional concerns regarding the implementation of a “quota” system: Does the rule of “proportional representation” (assigning prizes proportional to the size of the groups) guarantee fair re-distribution? We show by Theorem 1 that the larger the size of the group, the more demanding the contest, and the more efforts contestants in that group have to expend in order to win the prizes. Consequently, those who are in a larger group receive less surplus than those who are in a smaller group, although in the symmetric equilibria, their expected gross payoffs are not different from the contestants in smaller groups.

As a result, our results confirm the conventional wisdom that “the first in village is better than the second in Rome” from an alternative angle. Theorem 1 implies that being among

the top ten of a hundred requires a contestant of more efforts than being the unique winner of ten. The success among the mass usually rewards more than the success among a few, and yet it as well demands more sacrifices. Thus, one may prefer to stay in “village”, instead of going to “Rome” to struggle for a rise, in spite of the abundant opportunities for success in “Rome”.

Finally, our paper leaves tremendous room for future extensions. A major caveat of our paper is that we assume all contestants are identical. One challenging extension would be to allow contestants to differ in ability.

Appendix: The proof of Lemma 1

Without loss of generality, we assume $\frac{K_1}{N_1} \geq \frac{K_2}{N_2}$. The case $\frac{K_1}{N_1} = 1$ is trivial. If $\frac{K_1}{N_1} = 1$,

then $\sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1} = K_1 > \sum_{g_1=0}^{K_1-1} \frac{K_1+K_2-g}{N_1+N_2-g}$. The rest of the sequence $\left\{ \frac{K_1+K_2-g}{N_1+N_2-g} \right\}_{g=0}^{K_1+K_2-1}$ starts

from the term $\frac{K_2}{N_2}$. This leads to $\sum_{g_1=K_1}^{K_1+K_2-1} \frac{K_1+K_2-g}{N_1+N_2-g} = \sum_{g_1=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2}$. Therefore, we obtain

$\sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1} + \sum_{g_2=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2} > \sum_{g=0}^{K_1+K_2-1} \frac{K_1+K_2-g}{N_1+N_2-g}$. Next we focus on the case $\frac{K_2}{N_2} \leq \frac{K_1}{N_1} < 1$, which leads to that $1 \leq K_2 < N_2$ and $1 < K_1 < N_1$.

For any $1 \leq L_1 < M_1, 1 \leq L_2 < M_2$ and integer $t \geq 0$, we have the following properties.

Property 1: $\frac{L_1}{M_1} \leq \frac{L_1+L_2}{M_1+M_2} \leq \frac{L_2}{M_2}$ if and only if $\frac{L_1}{M_1} \leq \frac{L_2}{M_2}$; equivalently, $\frac{L_1-t}{M_1-t} \leq \frac{L_1+L_2-t}{M_1+M_2-t} \leq \frac{L_2}{M_2}$ if and only if $\frac{L_1-t}{M_1-t} \leq \frac{L_2}{M_2}$, where $L_1 - t \geq 1$.

Property 2: $\frac{L_1-t}{M_1-t}$ strictly decreases with t ; while $\frac{L_2-t}{M_2-t}$ and $\frac{L_1+L_2-t}{M_1+M_2-t}$ strictly decrease with t .

Property 3: Assume $L_1 - t \geq 1$. If $\frac{L_1-t}{M_1-t} \leq \frac{L_1+L_2-t}{M_1+M_2-t}$, then $\frac{L_1-(t+1)}{M_1-(t+1)} < \frac{L_1+L_2-(t+1)}{M_1+M_2-(t+1)}$; equivalently, if $\frac{L_1-(t+1)}{M_1-(t+1)} \geq \frac{L_1+L_2-(t+1)}{M_1+M_2-(t+1)}$, then $\frac{L_1-t}{M_1-t} > \frac{L_1+L_2-t}{M_1+M_2-t}$.

Our proof proceeds as follows.

Step 1: Because $\frac{K_1}{N_1} \geq \frac{K_2}{N_2}$, by Property 1, $\frac{K_1}{N_1} \geq \frac{K_1+K_2}{N_1+N_2}$. We define $t_1 = \max_{t \in \{0, 1, \dots, K_1\}} \left\{ t \mid \frac{K_1-t}{N_1-t} \geq \frac{K_2}{N_2} \right\}$, where $\frac{K_1-t}{N_1-t}$ decreases with t from Property 2. Note that t_1 is well defined as when $t = 0$, $\frac{K_1-t}{N_1-t} \geq \frac{K_2}{N_2}$, and when $t = K_1$, $\frac{K_1-t}{N_1-t} < \frac{K_2}{N_2}$. Thus we have $t_1 \in \{0, 1, \dots, K_1 - 1\}$. from Property 1, we have $\frac{K_1-t}{N_1-t} \geq \frac{K_1+K_2-t}{N_1+N_2-t}$ for $t \in [0, t_1]$. Thus

$$\sum_{h=0}^{t_1} \frac{K_1 + K_2 - h}{N_1 + N_2 - h} \leq \sum_{h=0}^{t_1} \frac{K_1 - h}{N_1 - h}. \quad (25)$$

We separate two cases: $t_1 = K_1 - 1$ and $0 \leq t_1 < K_1 - 1$.

Case 1: $t_1 = K_1 - 1$. In this case, the sequence $\left\{ \frac{K_1-g_1}{N_1-g_1} \right\}_{g_1=0}^{K_1-1}$ is exhausted. $\sum_{h=0}^{t_1} \frac{K_1+K_2-h}{N_1+N_2-h} \leq$

$$\sum_{h=0}^{t_1} \frac{K_1-h}{N_1-h} \text{ is equivalent to } \sum_{g=0}^{K_1-1} \frac{K_1+K_2-g}{N_1+N_2-g} \leq \sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1}.$$

The rest of the sequence $\left\{ \frac{K_1+K_2-g}{N_1+N_2-g} \right\}_{g=0}^{K_1+K_2-1}$ starts from the term $\frac{K_1+K_2-t_1-1}{N_1+N_2-t_1-1} = \frac{K_2}{N_1+N_2-K_1}$.

Clearly $\frac{K_2-h}{N_2-h} > \frac{K_2-h}{N_1+N_2-K_1-h}, \forall h \in [0, K_2 - 1]$ as $N_1 > K_1$.

Hence, we obtain $\sum_{g_1=0}^{K_1-1} \frac{K_1-g_1}{N_1-g_1} + \sum_{g_2=0}^{K_2-1} \frac{K_2-g_2}{N_2-g_2} > \sum_{g=0}^{K_1+K_2-1} \frac{K_1+K_2-g}{N_1+N_2-g}$, which completes the proof.

Case 2: $0 \leq t_1 < K_1 - 1$. In this case, by the definition of t_1 , we have $\frac{K_1 - t_1}{N_1 - t_1} \geq \frac{K_2}{N_2}$ and $\frac{K_1 - t_1 - 1}{N_1 - t_1 - 1} < \frac{K_2}{N_2}$. Thus, by Property 1, we have $\frac{K_2}{N_2} > \frac{K_1 + K_2 - (t_1 + 1)}{N_1 + N_2 - (t_1 + 1)} > \frac{K_1 - t_1 - 1}{N_1 - t_1 - 1}$.

Then we define $t_2 = \max_{t \in \{0, 1, \dots, K_2\}} \{t \mid \frac{K_2 - t}{N_2 - t} \geq \frac{K_1 - (t_1 + 1)}{N_1 - (t_1 + 1)}\}$. Note that t_2 is well defined and $t_2 \in [0, K_2 - 1]$. Thus, we obtain $\sum_{h=0}^{t_2} \frac{K_1 + K_2 - (t_1 + 1) - h}{N_1 + N_2 - (t_1 + 1) - h} < \sum_{g=0}^{t_2} \frac{K_2 - g}{N_2 - g}$, which implies

$$\sum_{g=0}^{t_1 + t_2 + 1} \frac{K_1 + K_2 - g}{N_1 + N_2 - g} < \sum_{g_1=0}^{t_1} \frac{K_1 - g_1}{N_1 - g_1} + \sum_{g_2=0}^{t_2} \frac{K_2 - g_2}{N_2 - g_2}. \quad (26)$$

The rest of sequence $\{\frac{K_1 + K_2 - g}{N_1 + N_2 - g}\}_{g=0}^{K_1 + K_2 - 1}$ starts from the term $\frac{K_1 + K_2 - (t_1 + 1) - (t_2 + 1)}{N_1 + N_2 - (t_1 + 1) - (t_2 + 1)}$. Again, we consider two possibilities.

Case 2.1: $t_2 = K_2 - 1$. In this case, the sequence $\{\frac{K_2 - g_2}{N_2 - g_2}\}_{g_2=0}^{K_2 - 1}$ is exhausted. The rest of sequence $\{\frac{K_1 + K_2 - g}{N_1 + N_2 - g}\}_{g=0}^{K_1 + K_2 - 1}$ starts from the term $\frac{K_1 - (t_1 + 1)}{N_1 + N_2 - K_2 - (t_1 + 1)}$, while the rest of sequence $\{\frac{K_1 - g}{N_1 - g}\}_{g=0}^{K_1 - 1}$ starts from the term $\frac{K_1 - (t_1 + 1)}{N_1 - (t_1 + 1)}$. Clearly, $\frac{K_1 - (t_1 + 1) - h}{N_1 - (t_1 + 1) - h} > \frac{K_1 - (t_1 + 1) - h}{N_1 + N_2 - K_2 - (t_1 + 1) - h}$ for all $h \in [0, K_1 - t_1 - 2]$ as $N_2 > K_2$. Hence we have

$$\sum_{h=0}^{K_1 - t_1 - 2} \frac{K_1 - (t_1 + 1) - h}{N_1 - (t_1 + 1) - h} > \sum_{h=0}^{K_1 - t_1 - 2} \frac{K_1 - (t_1 + 1) - h}{N_1 + N_2 - K_2 - (t_1 + 1) - h}, \quad (27)$$

which implies

$$\sum_{g_1=t_1+1}^{K_1-1} \frac{K_1 - g_1}{N_1 - g_1} > \sum_{g=t_1+K_2+1}^{K_1+K_2-1} \frac{K_1 + K_2 - g}{N_1 + N_2 - g}. \quad (28)$$

Combine inequalities (25), (26) and (28), we have

$$\sum_{g_1=0}^{K_1-1} \frac{K_1 - g_1}{N_1 - g_1} + \sum_{g_2=0}^{K_2-1} \frac{K_2 - g_2}{N_2 - g_2} > \sum_{g=0}^{K_1+K_2-1} \frac{K_1 + K_2 - g}{N_1 + N_2 - g}, \quad (29)$$

which completes our proof.

Case 2.2: $t_2 < K_2 - 1$. We go to Step 2.

Step 2: In the **Case 2.2**, we have $0 \leq t_2 < K_2 - 1$ and $0 \leq t_1 < K_1 - 1$. From (26), In order to show Lemma 1, we only need to show

$$\sum_{g=t_1+t_2+2}^{K_1+K_2-1} \frac{K_1 + K_2 - g}{N_1 + N_2 - g} < \sum_{g_1=t_1+1}^{K_1-1} \frac{K_1 - g_1}{N_1 - g_1} + \sum_{g_2=t_2+1}^{K_2-1} \frac{K_2 - g_2}{N_2 - g_2}. \quad (30)$$

Equivalently,

$$\begin{aligned}
& \sum_{g=0}^{[K_1-(t_1+1)]+[K_2-(t_2+1)]-1} \frac{\{[K_1-(t_1+1)]+[K_2-(t_2+1)]\}-g}{\{[N_1-(t_1+1)]+[N_2-(t_2+1)]\}-g} \\
< & \sum_{g_1=0}^{[K_1-(t_1+1)]-1} \frac{[K_1-(t_1+1)]-g_1}{[N_1-(t_1+1)]-g_1} + \sum_{g_2=0}^{[K_2-(t_2+1)]-1} \frac{[K_2-(t_2+1)]-g_2}{[N_2-(t_2+1)]-g_2}. \quad (31)
\end{aligned}$$

Note that there are at most $K_1 - 2$ and $K_2 - 2$ terms in the two sequences on the right hand side, since $t_1 > 0$ and $t_2 > 0$. By definition of t_2 , we have $\frac{K_2-t_2}{N_2-t_2} \geq \frac{K_1-(t_1+1)}{N_1-(t_1+1)}$ and $\frac{K_2-(t_2+1)}{N_2-(t_2+1)} < \frac{K_1-(t_1+1)}{N_1-(t_1+1)}$. In other words, the first term in the first sequence on the right hand side of (31) is greater than the counterpart of the second sequence. The procedure in Step 1 can thus be applied again to the two sequences on the right hand side of (31).

We repeat the procedures in Steps 1 and 2 until we reach the end of sequence $\{\frac{K_1-g_1}{N_1-g_1}\}_{g_1=0}^{K_1-1}$ or $\{\frac{K_2-g_2}{N_2-g_2}\}_{g_2=0}^{K_2-1}$. Then we can apply the reasoning in Case 2.1 of Step 1 to complete the proof.

Q.E.D.

References

- [1] Amegashie, J.A., Some Results on Rent-Seeking Contests with Shortlisting, *Public Choice*, 2000, 105, 245-253.
- [2] Barut, Y., and Kovenock, D., The Symmetric Multiple-Prize All-Pay Auction with Complete Information, *European Journal of Political Economy*, 1998, 14, 627-644.
- [3] Clark, D.J., and Riis, C., A multi-winner Nested Rent-Seeking Contest, *Public Choice*, 1996, 87, 177-184.
- [4] Clark, D.J., and Riis, C., Influence and the Discretionary Allocation of Several Prizes, *European Journal of Political Economy*, 1998a, 14, 605-615.
- [5] Clark, D.J., and Riis, C., Competition Over More Than One Prize, *American Economic Review*, 1998b, 276-289.
- [6] Fu, Q., and Lu, J., The Optimal Multiple-Stage Contest, working paper, 2005, National University of Singapore.
- [7] Gradstein, M., Optimal Contest Design: Volume and Timing of Rent-Seeking in Contests, *European Journal of Political Economy*, 1998, 14, 575-585.
- [8] Inderst, R., Müller, H.M., and Wärneryd, K., Influence Costs and Hierarchy, *Economics of Governance*, 2005, 6, 177-197.
- [9] Moldovanu, B., and Sela, A., Contest Architecture, *Journal of Economic Theory*, 2006, 70-96.
- [10] Moldovanu, B., Sela, A., and Shi, X., Contests for Status, 2006, working paper, University of Bonn.
- [11] Skaperdas, S., Contest Success Function, *Economic Theory*, 1996, 7, 283-290.
- [12] Wärneryd, K., Distributional Conflict and Jurisdictional Organization, *Journal of Public Economics*, 1998, 69, 435-450.
- [13] Wärneryd, K., Replicating Contests, *Economics Letters*, 2001, 71, 323-27.

- [14] Yates, A.J., and Heckelman, J.C., Rent-Setting in Multiple Winner Rent-Seeking Contests." *European Journal of Political Economy*, 2001, 17, 835-852.