Consumption efficiency hypothesis and the optimality of free trade policy in a small open economy

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Abstract: The paper is designed to examine the optimality of the free trade policy in a small poor economy incorporating the consumption efficiency hypothesis in the simple two-by-two Heckscher-Ohlin-Samuelson (HOS) framework. It finds that the protectionist policy in the form of a tariff on the capital-intensive import-competing sector may improve social welfare and unambiguously raise the economy-wide effective employment.

JEL classification: J41; O15

Keywords: Consumption efficiency hypothesis; Optimality of free trade; Protectionist policy; Heckscher-Ohlin-Samuelson model; Effective employment.

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1. Introduction

It is a standard result of the theory of international trade that free trade is the optimal policy for a small open economy which has no influence over the commodity prices in the international market. Hence any impediments to free trade e.g. a tariff, a production subsidy and/or a consumption tax reduces social welfare by affecting the optimal allocation of economic resources and/or by distorting consumers' choices. The ongoing process of trade liberalization is based on this notion of optimality of free trade.

Until the mid 1980s', the less developed countries (LDCs) followed a stringent trade policy and adopted an inward-oriented strategy, making use of discriminating policies like tariffs, quotas, restricting free inflow of foreign capital and import of commodities. Only since the conclusion of the multilateral agreement and the formation of the World Trade Organization (WTO) in the Uruguay round of discussions there have been revolutionary changes in liberalizing international trade across countries whether developed or developing. Liberalization involves both inflow of foreign capital as well as reduction of protection of domestic industries and integrating the domestic market with the world market. The LDCs have been advised by the international institutions like the IMF and the World Bank to remove all impediments to free trade as removal of the protectionist policy would improve their welfare and take them into higher growth orbits. However, despite choosing free trade as their development strategy and drastic implementation of trade reforms national income as well as per capita income in most of the low income countries as per the World Development Reports (2000/2001, 2006) have hardly increased. Besides, the problem of unemployment in the developing countries including the low income countries has increased considerably over the period of economic reforms.[1]

A pertinent question is whether free trade is at all the optimal policy for a poor small LDC which is plagued by distortions of different types. The Harris-Todaro (HT, hereafter) (1970) model is usually employed in the trade and development literature in explaining the problem of unemployment in the LDCs. However, the two-sector mobile capital version of the HT model, like that of Corden and Findlay (1975), cannot explain as to why national welfare and the unemployment problem in the LDCs have not improved following trade liberalization. In that model imposition of a tariff raises the return to capital and lowers the rural sector wage following a Stolper-Samuelson effect. The aggregate wage income consequently falls[2], which lowers social welfare. On the other hand, the urban sector expands both in terms of output and employment following a Rybczynski type effect that raises the expected urban wage for a prospective rural migrant which in turn paves the way for a fresh migration from the rural to the urban sector. The level of urban unemployment rises as the number of new migrants exceeds the number of new jobs created in the urban sector.

So a theoretical framework is needed that would be able to explain the consequences of trade liberalization in the poor small open economies which are consistent with empirical findings especially when the standard two-sector mobile capital version of the HT framework fails to do so. In the circumstances, the present paper attempts to fill in this lacuna of the theoretical literature by embedding the consumption efficiency hypothesis in the simple two-sector Heckscher-Ohlin-Samuelson (HOS) model. The 'consumption

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[1] According to ILO (2007) the unemployment rates in 2006 in the Latin American and Caribbean, Sub-Saharan African and South Asian Countries were 8.0, 9.8 and 5.2, respectively. With declining employment to population ratios, official unemployment rates increased in most of the developing countries over the last two decades.

[2] In the HT framework the rural sector wage is the average wage of the workers in the economy. This is known as the 'envelope' property. So the aggregate wage income of the workers in the economy goes up if the rural wage arte rises.

efficiency hypothesis' of Leibenstein (1957, 1958)[3] has been widely used in the development models for explaining the existence of involuntary unemployment in a poor LDC. The basic tenet of the hypothesis is that the nutritional efficiency of a worker is positively related to his consumption level at least up to a certain point. Higher consumption means higher calorie intake, an increase in body mass, a reduction in morbidity as well as greater ability to work.[4] There is now considerable evidence[5] that in a poor LDC with low levels of consumption of the workers there is a significant positive relation between workers' consumption and productivity. Therefore, an increase in the consumption level raises the nutritional efficiency i.e. productivity of the worker. Now if there is a stable relationship between the consumption level of the worker and his wage income then the worker's productivity is positively linked to the wage that he receives. If this is so, then it is in its interest the firm will not offer its profit-maximizing wage but the efficiency wage because now the wage through the nutritional efficiency function enters into the production function of the firm. The firm minimizes its unit labour cost and it is a standard result of this literature that the efficiency wage is set where the elasticity of the nutritional efficiency function of the worker is equal to unity. Hence the efficiency wage is constant. Even if there is an excess supply of labour at the efficiency wage, the firms will have no incentives to lower the wage rate. Hence the labour market does not clear and the problem of involuntary unemployment crops up.

However, this theory assumes a one commodity world. But, in a two commodity world the nutritional efficiency of a worker may depend on the consumption of one commodity only. If the two commodities are food and cloth, then the worker's nutritional efficiency depends only on his consumption of food. The present paper examines the optimality of the free trade policy in a small poor economy incorporating the consumption efficiency theory in the simple two-by-two Heckscher-Ohlin-Samuelson (HOS) framework. It also

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[3] See also Stiglitz (1976), Mazumdar (1959), Mirrlees (1975), Bliss and Stern (1978) and Dasgupta and Ray (1986).

[4] One may go through Ray (1993) for the process of efficiency build-up.

[5] See Bose (1996) in this context.

analyzes the implication of the protectionist policy on unemployment problem in the economy. It finds that free trade may not be the optimal policy for a small open economy if the consumption efficiency hypothesis is valid and that the protectionist policy in the form of an import tariff may improve social welfare. It also shows that imposition of a tariff unambiguously raises the effective employment in the economy. These results are new in the literature on trade and development.

2. The model

Let there be two commodities: X and Y. There are L number of homogeneous workers in the economy. The worker consumes both commodities but his productivity (nutritional efficiency) depends only on the consumption of commodity, X (food). The utility function is of the CES type and is given by

$$V = A\left[\delta x^{-\rho} + (1-\delta) y^{-\rho}\right]^{\frac{-1}{\rho}} \text{ with } A > 0; 0 < \delta < 1;\ -1 < \rho \neq 0 \qquad (1)$$

where V, x and y denote the utility level and the consumption levels of X and Y, respectively. A, δ and ρ are parameters. δ is the share of commodity X in the consumer's budget while ρ is the substitution parameter.

The budget constraint of the worker (consumer) is

$$P_x x + P_y^* y = (1-u)W \qquad (2)$$

where P_x, P_y^* and W are the two prices and the wage rate, respectively. Finally, u denotes the rate of unemployment of labour in the economy. So the right-hand side of (2) is the expected wage income of the worker.

Equation (1) is maximized with respect to x and y and subject to (2). Maximization exercise leads to the following demand functions for commodities X and Y, respectively.[6]

[6] Here the two commodities are gross compliments. In other words, an increase in the price of either good lowers the quantities demanded of both the commodities and vice versa.

$$x = \frac{(1-u)W}{P_x(1+Z)} \quad (3)$$

$$y = \frac{(1-u)W}{P_y^*\left[1+(1/Z)\right]} \quad (4)$$

where $Z = [(\frac{1-\delta}{\delta})^{\frac{1}{1+\rho}}(\frac{P_y^*}{P_x})^{\frac{\rho}{1+\rho}}] > 0$

The demand for X by each worker in the general form is written as

$$x = x(P_x, P_y^*, W, u) \quad (3.1)$$

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For quite straightforward reasons the demand for X is a negative function of the two prices and the unemployment rate while it is a positive function of the wage rate.

The nutritional efficiency of each worker, h, is assumed to be a positive function of his consumption level of commodity X and is given by

$$h = h[x(P_x, P_y^*, W, u)]; h^{/} > 0; h^{//} < 0 \quad (5)$$

So h increases at a decreasing rate with an increase in x.

The unit cost of labour, ω, is given by

$$\omega = \frac{W}{h} \quad (6)$$

Apart from labour, capital is used in production. Assuming capital to be perfectly mobile intersectorally and its uniform return be r economy-wide**,** each firm minimizes its unit labour cost given by (6). The first-order condition of minimization is

$$h(.) = Wh^{/}(.)x_3 \quad (7)$$

where $x_3 = (\partial x / \partial W) > 0$; with $x_{33} = 0$; and, $x_{31}, x_{32}, x_{34} < 0$.[7]

It may be checked that the second-order condition is automatically satisfied as $h^{//} < 0$.

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[7] See Appendix I for the derivations.

Equation (7) may be interpreted as follows. The elasticity of the nutritional efficiency function (5) is given by

$$\varepsilon_h = ((dh/dx)(x/h)) \quad (8)$$

Using (3) and (8), equation (7) can be rewritten as follows.

$$\varepsilon_h = 1. \quad (7.1)$$

So in equilibrium the efficiency wage is set where the elasticity of the nutritional efficiency function is equal to unity. This is a standard result of the efficiency wage literature. But unlike the one commodity framework, the condition here does not imply the constancy of the wage. It rather implies constancy of the consumption of commodity X on which the nutritional efficiency of a worker depends. This establishes the following proposition.

Proposition 1: In a two-commodity world where the nutritional efficiency of the worker depends positively on the consumption of one commodity only, its demand remains constant despite changes in the parameters of the system. The nutritional efficiency of the worker is also constant.

3. General equilibrium and the consumption efficiency hypothesis

Let us introduce the nutritional efficiency function into the conventional Heckscher-Ohlin-Samuelson framework. We consider a small open poor economy with two commodities, X and Y. There are two inputs of production, labour (L) and capital (K) and their endowments are given exogenously. Commodity prices, P_x and P_y, are given by the small open economy assumption. Initially there is zero tariff on the import of commodity Y. The domestic price of commodity Y is $P_y^*(=P_y(1+t))$ with $t=0$. Production functions exhibit constant returns to scale with positive but diminishing marginal productivities to each input. Markets for both commodities and capital are perfectively competitive while firms in both sectors set wages in the labour market according to equation (7). The total number of workers, L, is fixed in the economy although the effective labour force in efficiency unit is $h(.)L$. Normalizing labour in physical unit to unity the economy's effective labour force is $h(.)$.

Given the assumption of perfectly competitive commodity markets the zero-profit conditions for the two sectors are as follows.

$$\frac{W}{h}a_{Lx} + ra_{Kx} = P_x \tag{9}$$

$$\frac{W}{h}a_{Ly} + ra_{Ky} = P_y^* \tag{10}$$

where a_{ji} is the amount of the j th input required to produce one unit of output of the i th sector for $j = L, K$ and $i = X, Y$ and (W/h) and r are the wage rate per efficiency unit of labour and the return to capital, respectively. Sector X is more labour-intensive than sector Y i.e. $|\theta| = (\theta_{Lx}\theta_{Ky} - \theta_{Kx}\theta_{Ly}) > 0$ where θ_{ji} is the distributive share of the j th input in the i th sector.

Capital is fully utilized in the two sectors. The full-employment condition for capital is given by

$$a_{Kx}X + a_{Ky}Y = K \tag{11}$$

where X and Y are the levels of output in the two sectors.

There is unemployment of labour in the economy and the rate of unemployment is u. The labour endowment equation is, therefore, given by

$$a_{Lx}X + a_{Ly}Y = h(1-u) \tag{12}$$

The effective employment of labour in the economy, E, is

$$E = h(1-u) \tag{13}$$

The general equilibrium system consists of equations (5), (7) and (9) – (13). The endogenous variables are: W, h, r, u, X, Y and E. If we look at the structure we find that W, h, r and u are determined from (5), (7), (9) and (10). Once h and u are determined the effective employment of labour, E is also obtained from (13). So factor prices, nutritional efficiency of worker, the unemployment rate and the effective employment of labour in the economy depend on commodity prices but not on factor endowments. Finally, X and Y are obtained from (11) and (12).

4. Comparative statics

We are now going to analyze the consequences of tariff protection of sector Y on the endogenous variables. We consider an increase in t by $dt > 0$. Totally differentiating equations (5), (7), (9) and (10) the following results can be obtained.

$$(dh / dt) = 0 \tag{14}$$

$$(dW / dt) < 0; (dr / dt) > 0 \tag{15}$$

So imposition of an import tariff on commodity Y raises the return to capital and lowers the wage rate following the Stolper-Samuelson effect as sector Y is relatively capital-intensive.[8]

The result given by (14) has already been stated in proposition 1. As the consumption of commodity X is independent of commodity prices (and other parameters) the efficiency of the worker does not change following the imposition of a tariff.

Totally differentiating (5), (7), (9) and (10) and then (13) the following results can be obtained.[9]

$$(du / dt) < 0; \text{and}, (dE / dt) > 0. \tag{16}$$

These lead to the following proposition.

Proposition 2: A tariff lowers the unemployment rate and raises the economy-wide effective employment of labour unambiguously.

Proposition 2 can be intuitively explained as follows. An increase in the tariff, t, by $dt > 0$ on sector Y raises the domestic price of commodity Y i.e. $P_y(1+t)$. It lowers the demand for X (i.e. x) as commodities are gross compliments. The wage rate per

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[8] Actually, the efficiency wage, $\omega(= W / h)$ decreases following the Stolper-Samuelson effect. But as the nutritional efficiency of the workers, h, does not change a decrease in ω implies a fall in the wage rate per worker, W.

[9] See Appendix II for detailed derivations.

worker, W, falls as sector Y is more capital-intensive vis-à-vis sector X which in turn lowers the demand for X. There are two negative effects on x when t rises. However, from proposition 1 we find that the net effect of any parameter changes on the demand for commodity X must be zero. Therefore, so as to neutralize the negative effects of an increase in t on x, the unemployment rate, u, must fall which raises the demand for good X by increasing the expected wage income of the worker.

As the nutritional efficiency, h, of each worker remains unchanged despite an increase in the tariff rate, t, (see proposition 1), the effect of a change in t on the economy's effective employment level depends solely on how the increase in the tariff rate affects the unemployment rate, u. As u falls following an increase in t, the economy-wide effective employment, E, rises as t rises.

We are now going to analyze the consequences of tariff protection of the import-competing sector on the welfare of the economy. The demand side of the model is represented by a quasi-concave social utility function. Let I denote the social utility that depends on the consumption demands for the two commodities denoted by, D_X and D_Y. Thus, it is shown as

$$I = I(D_x, D_y) \quad . \tag{17}$$

The balance of trade equilibrium requires that

$$P_x D_x + P_y D_y = P_x X + P_y Y \tag{18}$$

or equivalently,

$$P_x D_x + P_y^* D_y = P_x X + P_y^* Y + tP_y M \tag{18.1}$$

The demand for the importables (commodity Y) and the volume of import are given by the following two equations, respectively.

$$D_y = D_y(P_x, P_y^*, N) \tag{19}$$

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$$M = D_y(P_x, P_y^*, N) - Y \tag{20}$$

where the national income of the economy at domestic prices, N , is given by

$$N = P_x X + P_y^* Y + tP_y M \tag{21}$$

or equivalently,

$$N = W(1-u) + rK + tP_Y M \tag{21.1}$$

where $Wh(1-u)$ and rK are the aggregate wage income and rental income on capital, respectively. Finally, $tP_Y M$ measures the tariff revenue collected by the government and transferred to the consumers as lump sum payments. With $t = 0$, $tP_Y M$ is equal to zero.

Differentiating equations (17), (18.1), (20), (21.1) and using the expression for (dE / dt) the following proposition can be established.[10]

Proposition 3: A tariff protection of the import-competing sector improves social welfare unequivocally if $t = 0$ initially. Social welfare may improve if initially $t > 0$.

Proposition 3 is interesting as it states that free trade may not be the optimal policy for a small poor open economy. This result can be explained in the following fashion. If $t = 0$ initially there will be only one effect. As the unemployment rate falls and the economy-wide effective employment of labour rises there will be an increase in the aggregate wage income. Social welfare, therefore, improves unambiguously. On the other hand, when $t > 0$ initially, the aggregate factor income rises. This also raises the demand for the importables and hence the tariff revenue given the production of commodity Y . These two effects work favourably on national welfare. On the other hand, as the domestic price of Y rises, the import-competing sector expands and also the demand for the importables falls. These two factors tend to lower the import demand and hence the tariff revenue. So an increase in the domestic price of Y exert downward pressure on welfare as it leads to misallocation of economic resources from the export sector to the import-competing sector and also affects the consumers' choices. So both the supply side and demand side

[10] This has been proved in Appendix III.

distortionary costs of tariff increase. These two effects work negatively on welfare. So there are four effects on welfare in total. If the sum of the first two positive effects dominates over the combined magnitude of the other two, social welfare improves.

5. Concluding remarks

This paper has examined the validity of the proposition as to whether free trade is the optimal policy for a poor small open economy by incorporating the consumption efficiency hypothesis in the simple two-sector general equilibrium model. The nutritional efficiency of the workers depends on the consumption of one commodity only although they consume both the commodities. In this setup the implications of the protectionist policy on the economy-wide effective employment of labour and on welfare have been studied. It has been found that the imposition of a tariff on the capital-intensive import-competing sector not only raises the effective employment of the economy but also may produce favourable effect on social welfare. If the initial tariff rate is zero, then a tariff unambiguously improves social welfare by raising the effective employment. So this theoretical analysis justifies the desirability of retaining some degree of protection in the poor open economies from the view points of both welfare and unemployment problem. These results are interesting also as these are completely opposite to what are obtained in the standard two-sector mobile capital HT model like that of Corden and Findlay (1975).

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Appendix I:

The problem of the consumer is as follows:

$$\underset{x,y}{Max}\ V = A\left[\delta x^{-\rho} + (1-\delta)y^{-\rho}\right]^{\frac{-1}{\rho}}; \quad (A>0;\ 0<\delta<1;\ -1<\rho \neq 0\ ) \tag{1}$$

subject to $P_x x + P_y^* y = (1-u)W$ (2)

The first-order condition is

$$(\frac{y}{x})^{1+\rho} = (\frac{P_x}{P_y^*})(\frac{1-\delta}{\delta}) \tag{A.1}$$

Solving (A.2) and (A.3) the optimum demand for good X is obtained as follows.

$$x = \frac{(1-u)W}{P_x(1+Z)} \tag{3}$$

where: $Z = [(\frac{1-\delta}{\delta})^{\frac{1}{1+\rho}}(\frac{P_y^*}{P_x})^{\frac{\rho}{1+\rho}}] > 0$ (A.2)

In general form the demand for X by each worker is written as

$$x = x(P_x, P_y^*, W, u) \tag{3.1}$$

Differentiating (3) the following results are obtained.

$$\left.\begin{aligned} x_1 &= (\frac{\partial x}{\partial P_x}) = -x\frac{[1+\frac{Z}{1+\rho}]}{P_x(1+Z)} < 0;\ x_2 = (\frac{\partial x}{\partial P_y^*}) = \frac{-xZ}{P_y^*(1+Z)}(\frac{\rho}{1+\rho}) < 0; \\ x_3 &= (\frac{\partial x}{\partial W}) = \frac{(1-u)}{P_x(1+Z)} > 0;\ x_4 = (\frac{\partial x}{\partial u}) = \frac{-W}{P_x(1+Z)} < 0; \end{aligned}\right\}$$

$$x_{31} = (\frac{\partial^2 x}{\partial W \partial P_x}) = -x\frac{[1+\frac{Z}{1+\rho}]}{P_x(1+Z)W} < 0;\ x_{33} = (\frac{\partial^2 x}{\partial W^2}) = 0; \tag{A.3}$$

$$x_{32} = (\frac{\partial^2 x}{\partial W \partial P_y^*}) = \frac{-x}{WP_y^*}(\frac{Z}{1+Z})(\frac{\rho}{1+\rho}) < 0;\ x_{34} = (\frac{\partial^2 x}{\partial W \partial u}) = -\frac{1}{P_x(1+Z)} < 0;$$

$$(x_2 - Wx_{32}) = 0;\ \ (x_1 - Wx_{31}) = 0;\ (x_4 - Wx_{34}) = 0$$

Appendix II:

Totally differentiating equations (9), (10), (5) and (7) and writing in a matrix notation one gets the following.

$$\begin{bmatrix} -\theta_{Lx} & \theta_{Lx} & \theta_{Kx} & 0 \\ -\theta_{Ly} & \theta_{Ly} & \theta_{Ky} & 0 \\ Wx_3 & -Wx_3 & 0 & -x_4 u \\ 0 & W^2 h^{//} x_3^2 & 0 & uWh^{//} x_3 x_4 \end{bmatrix} \begin{bmatrix} \hat{h} \\ \hat{W} \\ \hat{r} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} 0 \\ dt \\ (P_y x_2)dt \\ (-P_y Wh^{//} x_3 x_2)dt \end{bmatrix} \quad (A.4)$$

Solving (A.4) and simplifying one gets the following expressions.

$$(\frac{dh}{dt}) = 0 \quad (A.5)$$

$$\hat{u} = (\frac{dt}{|\theta| u x_4})[Wx_3\theta_{Kx} - P_y |\theta| x_2] < 0 \quad (A.6)$$

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$$\hat{W} = -\frac{\theta_{Kx}}{|\theta|} dt < 0 \quad (A.7)$$

$$\hat{r} = \frac{\theta_{Lx}}{|\theta|} dt > 0 \quad (A.8)$$

where: $|\theta| = (\theta_{Lx}\theta_{Ky} - \theta_{Kx}\theta_{Ly}) > 0$. (A.9)

(as sector X is more labour-intensive relative to sector Y)

Now we recall equation (13)

$$E = h(1-u) \quad (13)$$

Differentiating (13), using (A.5) and (A.6) and simplifying the following expression is obtained.

$$\hat{E} = -(\frac{u}{1-u})(\frac{dt}{|\theta| u x_4})[Wx_3\theta_{Kx} - P_y |\theta| x_2] > 0 \quad (A.11)$$

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Appendix III:

Differentiating (17) and (18.1) we get the following two expressions, respectively.

$$\frac{dI}{I_1} = dD_x + (\frac{I_2}{I_1})dD_y = \frac{1}{P_x}[P_x dD_x + P_y^* dD_y] \quad \text{(A.12)}$$

$$P_x dD_x + P_y^* dD_y = P_x dX + P_y^* dY + tP_y dM \quad \text{(A.13)}$$

Differentiating the expression for national income at domestic prices, given by (21) one obtains

$$dN = P_x dX + P_y^* dY + YdP_y^* + tP_y dM + P_y Mdt \quad \text{(A.14)}$$

By differentiating production functions and considering (11) and (12) we have

$$[P_x dX + P_y^* dY] = [P_x(F_L^x dL_x + F_K^x dK_x) + P_y^*(F_L^y dL_y + F_K^y dK_y)]$$

$$= [((W/h)dL_x + rdK_x) + ((W/h)dL_y + rdK_y)] = (W/h)(dL_x + dL_y) = (W/h)dE \quad \text{(A.15)}$$

[Note that from (11) and (12) $(dK_x + dK_y) = 0$; and, $[(dL_x + dL_y) = (W/h)dE]$. $P_i F_j^i$ is the value of marginal product of the j th factor in the i th sector, which is equal to the factor price.]

Using (A.15), equation (A.14) is rewritten as follows.

$$dN = [\frac{W}{h}dE + P_y(D_y dt + tdM)] \quad \text{(A.16)}$$

Differentiating (20) one gets

$$dM = \frac{\partial D_y}{\partial P_y^*}dP_y^* + \frac{\partial D_y}{\partial N}dN - dY \quad \text{(A.17)}$$

Using (A.16), equation (A.17) can be rewritten as follows.

$$dM = \frac{\partial D_y}{\partial P_y^*}dP_y^* + \frac{\partial D_y}{\partial N}[\frac{W}{h}dE + P_y(D_y dt + tdM)] - dY$$

$$= V[HP_y dt - dY + \frac{W}{h}\frac{\partial D_y}{\partial N}dE] \quad \text{(A.18)}$$

where: $V = \frac{(1+t)}{1+t(1-m)} > 0$;

$m = (P_y^* \frac{\partial D_y}{\partial N})$ is the marginal propensity to consume commodity Y.

H= $(\frac{\partial Dy}{\partial P_y^*} + \frac{\partial D_y}{\partial N} D_y)$ is the Stutsky's pure substitution term; and, $H < 0$.

Using (A.12), (A.13) and (A.18) and simplifying we get

$$\frac{1}{I_1}(\frac{dI}{dt}) = [\frac{W}{hP_x}(1+TmV)\frac{dE}{dt} + t(\frac{P_y}{P_x})V\{HP_y - \frac{dY}{dt}\}] \qquad \text{(A.19)}$$

Now if $t = 0$ initially, (A.19) reduces to

$$\frac{1}{I_1}(\frac{dI}{dt}) = [(\frac{W}{hP_x})(\frac{dE}{dt})] \qquad \text{(A.19.1)}$$

Using (A.11), (A.19.1) can be rewritten as follows.

$$\frac{1}{I_1}(\frac{dI}{dt}) = (\frac{-W}{|\theta| x_4 P_x})(Wx_3\theta_{Kx} - P_y|\theta| x_2) > 0.$$

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