A multivariate innovations state space Beveridge Nelson decomposition

de Silva, Ashton
RMIT University

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Ashton de Silva[*]

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Abstract

The Beveridge Nelson vector innovation structural time series framework is new formulation that decomposes a set of variables into their permanent and temporary components. The framework models inter-series relationships and common features in a simple manner. In particular, it is shown that this new specification is more simple than conventional state space and cointegration approaches. The approach is illustrated using a trivariate data set comprising the GD(N)P of Australia, America and the UK.

Keywords: vector innovation structural time series, multivariate time series, Beveridge Nelson, common components.

1 Introduction

Decomposing variables into permanent and transitory components has been an important theme of economic research for more than 25 years. This paper outlines a multivariate framework that disaggregates a collection of variables into permanent and transitory components.

In this paper the vector innovation structural time series (hereafter VISTS, de Silva et al. 2007) framework is modified to extract trends and cycles in accordance with the

[*]Contact Details: Ashton.desilva@rmit.edu.au, Tel +613 9925 1313, Fax +613 9925 0000

Beveridge-Nelson (Beveridge & Nelson 1981, hereafter BN) decomposition. These models will be denoted as BN-VISTS. The specification presented is a multivariate extension to Anderson, Low & Snyder (2006). The analysis in this paper clearly shows that the proposed framework is simple and flexible.

An important advantage of this multivariate approach is that common trends or cycles can be evaluated using standard diagnostics, i.e., information criterion. This is in direct contrast to Stock & Watson (1988), Vahid & Engle (1993) who develop statistical testing procedures based on the cointegration specification.

The structure of the paper is as follows. The next section briefly summarises the background. Section 3 outlines the new framework, its models and implementation. The final section illustrates the new specification applied to the same data set considered by Anderson et al. (2006). Section 5 concludes the paper.

2 Background

Analysis of time series involves disaggregation of the observations into a set of interpretable components. A traditional decomposition is:

$$y_t = M_t + C_t + S_t + I_t \tag{2.1}$$

where M_t, C_t, S_t & I_t denote respectively the trend, cyclical, seasonal and irregular component at time t.

A more recent decomposition is that by Beveridge & Nelson (1981) who proposed that macroeconomic variables can be disaggregated into a stationary and non-stationary component. These components are denoted as the temporary and permanent components respectively.

Formally, Beveridge & Nelson (1981) tailor the permanent/transitory decomposition to the stochastic properties of the data. The permanent component always comprised a random walk with the same rate of drift as in the original data. In addition, the permanent component also comprised a disturbance term proportional to that of the original data. The transitory component represents the predictable part of the data, and was expected to dissipate as the series tends to its permanent level.

In general, two different formulations have been used to measure the trend and cyclical components, one is the ARIMA specification which is advocated by Beveridge & Nelson (1981), and the other is the structural time series approach advocated (amongst others) by Harvey & Jaeger (1993).

The results from these two procedures are typically very different. The ARIMA specification commonly yields a dominant trend whereas the structural time series approach a dominant cycle. In recent times there has been an attempt to reconcile these two approaches (see Morley, Nelson & Zivot 2003, Proietti 2002). These papers compare both the analytical and empirical aspects of the two specifications.

Analytically, the conclusion reached is that any structural time series model has an ARIMA representation, but not vice-versa. This is directly attributable to the form of the structural time series model adopted. Specifically, the random disturbances of the permanent and temporary components are constrained to be independent of each other (This issue was discussed in de Silva et al. 2007).

Using US GNP, Morley et al. (2003) find no evidence to suggest that the estimated correlation between the trend and cycle components is zero. Moreover, the estimates seem to suggest an almost perfect negative correlation between the two components. A similar outcome is observed for Italian real GDP (Proietti 2002). The implication of these outcomes is very interesting as the univariate innovation form postulated by Anderson et al. (2006) implicity assumes a perfect correlation between the trend and cycle components.

The specification presented in this paper, a multivariate version of Anderson et al. (2006), implicitly models the correlation between the innovation of each component. The degree of the correlation is not necessarily perfect unlike the univariate counterpart.

Another key feature of this new specification is its ability to consider two or more series simultaneously. There are two streams of research on the multivariate BN decompositions: the first relating to traditional state space specifications; the second focuses on modelling and testing for common trend and cycles within an cointegrating framework.

Two papers that have considered the multivariate BN decomposition in the form of traditional state space models are Morley (2002) and Sinclair (2005). Sinclair (2005) provides a detailed analysis of the American unemployment rate and GDP paying particular attention to Okun's law. Morley (2002) outlines an analytical form only.

The main contributions to the second stream are King, Plosser, Stock & Watson (1987), Stock & Watson (1988), Vahid & Engle (1993, 1997), Barillas & Schleicher (2003). These papers consider both VAR and traditional time series specifications. A particular emphasis in two of these articles (Stock & Watson 1988, Vahid & Engle 1993) is the ability to formally test for the presence of common features. In particular Stock & Watson (1988) formulate a test to determine common-trends based on autoregressive matrices. Similarly, Vahid & Engle (1993) derive a common-cycle test for series that are cointegrated.

As already indicated this new framework can also model and evaluate the presence of common components. It can be done either using a likelihood ratio test or information criterion. In the last section of this paper the AIC is used.

In addition to formulating a common-cycle test Vahid & Engle (1993) review what they define as the Beveridge-Nelson-Stock-Watson (BNSW) decomposition. This decomposition relates the cointegration specification to a structural time series framework. A brief review of the derivation is presented here.

The derivation begins by stating the Wold decomposition for a vector of series integrated of order one, that is:

$$\Delta \boldsymbol{y}_t = \boldsymbol{\mu} + C(L)\boldsymbol{\epsilon}_t \tag{2.2}$$

where $C(0) = I_N$ and $\sum_{j=1}^{\infty} j|C_j| < \infty$. Assuming $\boldsymbol{\mu} = \mathbf{0}$ for simplicity, equation (2.2) can be rewritten as:

$$\Delta \boldsymbol{y}_t = C(1)\boldsymbol{\epsilon}_t + \Delta C^*(L)\boldsymbol{\epsilon}_t \tag{2.3}$$

where $C_i^* = \sum_{j>i}^{\infty} -C_j$ for all i and $C_0^* = I_N - C_1$. Integrating both sides of (2.3) gives:

$$\boldsymbol{y}_t = C(1)\sum_{s=0}^{\infty} \boldsymbol{\epsilon}_{t-s} + C^*(L)\boldsymbol{\epsilon}_t \tag{2.4}$$

Equation (2.4) denotes the multivariate BN decomposition, where the first term depicts the permanent (trend) component and the second, the temporary (cyclical) component. If common trends exist (or in other words a cointegrating relationship between two or more variables exist) then $C(1)$ will be of reduced rank. Specifically, the rank of $C(1)$ will equal k, where $k < N$ and $C(1)$ can be expressed in the form of:

$$C(1) = \boldsymbol{\gamma}\boldsymbol{\delta}' \tag{2.5}$$

where $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ are both of rank k. The common trend BNSW decomposition is expressed as:

$$\boldsymbol{y}_t = \boldsymbol{\gamma}\boldsymbol{\tau}_t + \boldsymbol{c}_t, \tag{2.6}$$

where

$$\boldsymbol{\tau}_t = \boldsymbol{\tau}_{t-1} + \boldsymbol{\delta}'\boldsymbol{\epsilon}_t. \tag{2.7}$$

Repeated back substitution of (2.7) yields;

$$\boldsymbol{\tau}_t = \boldsymbol{\delta}' \sum_{s=0}^{\infty} \boldsymbol{\epsilon}_t. \tag{2.8}$$

Substituting (2.8) into (2.6) yields:

$$\boldsymbol{y}_t = \boldsymbol{\gamma}\boldsymbol{\delta}' \sum_{s=0}^{\infty} \boldsymbol{\epsilon}_{t-s} + C^*(L)\boldsymbol{\epsilon}_t. \tag{2.9}$$

Equation (2.9) is the common trend specification of the BN decomposition. Watson (1994) refers to this representation as the *common trends representation of the cointegrated system*. In section 3.3 it is shown how the BN-VISTS specification also relates to the BNSW form defined by (2.4) (and in the common feature case, equation (2.9)).

3 Models

In this section the general form of the VISTS framework is presented. Three special cases are also discussed which form the basis of the empirical work presented in Section 4.

3.1 VISTS general form

The VISTS general form, letting $\boldsymbol{y}_t$ denote an N-vector of observations at time t is:

$$\boldsymbol{y}_t = H\boldsymbol{x}_{t-1} + \boldsymbol{e}_t, \quad \boldsymbol{e}_t \sim^{iid} MVN(\mathbf{0}, \Sigma) \tag{3.1}$$

$$\boldsymbol{x}_t = F\boldsymbol{x}_{t-1} + A\boldsymbol{e}_t, \tag{3.2}$$

where H, F and A denote various coefficient matrices, $\boldsymbol{x}_t$ denotes a k-vector of states and $\boldsymbol{e}_t$ denotes a N-vector of innovations which are normally distributed at time t. It is assumed that the covariance matrix of $\boldsymbol{e}_t$, Σ, is diagonal.

Equations (3.1) and (3.2) denote the measurement and transition equation respectively. The measurement equation describes the vector of observations as a function of unobserved components. Each variable having its own latent permanent and temporary component is combined according to the structure matrix; H.

The transition equation, a first order recursive process describes how the components vary over time. This equation is a function of F, the transition matrix, the previous set of state values denoted by $\boldsymbol{x}_{t-1}$, and the smoothing matrix A which moderates the influence of the unexpected change.

The coefficient matrices H and F are typically partially predetermined. The smoothing matrix, A, is wholly determined by the data. The dimension of these matrices will vary according to the model estimated.

It is important here to reiterate the difference between the VISTS specification and the conventional specification (Harvey & Jaeger 1993). Namely, the VISTS framework has one source of error per series, conversely the conventional specification has multiple sources of error per series;

$$
\begin{aligned}
\boldsymbol{y}_t &= H\boldsymbol{x}_{t-1} + \boldsymbol{\eta}_t,\ \boldsymbol{\eta}_t \sim^{iid} MVN(\mathbf{0}, Q) \\
\boldsymbol{x}_t &= F\boldsymbol{x}_{t-1} + \boldsymbol{\epsilon}_t,\ \ \boldsymbol{\epsilon}_t \sim^{iid} MVN(\mathbf{0}, R).
\end{aligned} \tag{3.3}
$$

The conventional structural time series multivariate model, denoted by (3.3) assumes a different source of error for the measurement and transition equations. Furthermore, it is most commonly assumed: $E(\boldsymbol{\eta}_t, \boldsymbol{\epsilon}_s) = 0$ for $t, s = 1, \ldots, T$. This difference although appearing somewhat trivial fundamentally differentiates these specifications. The multiple source of error specification is arguably more difficult to implement. In addition it can be shown that the conventional specification is a special case of VISTS model (3.2) (see de Silva et al. 2007).

3.2 Beveridge-Nelson Vector Innovation Structural Time Series Models

Anderson et al. (2006) consider three univariate innovation structural times series models. These models will be referred to as the BN-univariate innovation structural time series (BN-UISTS) models.

The first model they considered was the BN-UISTS(0,1,1) model which is equivalent to an ARIMA(0,1,1) process. It is specified by equations (3.4) and (3.5):

$$
y_t = \delta + \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} m_{t-1} \\ c_{t-1} \end{bmatrix} + e_t \tag{3.4}
$$

$$
\begin{bmatrix} m_t \\ c_t \end{bmatrix} = \begin{bmatrix} \delta \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix} e_t \tag{3.5}
$$

where the trend and cycle at time t is denoted by m_t and c_t respectively. The smoothing parameter α measures the impact of unexpected changes on the trend. The response in the cycle to unexpected changes is measured by $1 - \alpha$.

The vector form of the model is labeled the BN-VISTS(0,1,1) model. It has the form:

$$\boldsymbol{y}_t = \boldsymbol{\delta} + \begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_{t-1} \\ \boldsymbol{c}_{t-1} \end{bmatrix} + \boldsymbol{e}_t \tag{3.6}$$

$$\begin{bmatrix} \boldsymbol{m}_t \\ \boldsymbol{c}_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{\delta} \\ 0 \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_{t-1} \\ \boldsymbol{c}_{t-1} \end{bmatrix} + \begin{bmatrix} A \\ I - A \end{bmatrix} \boldsymbol{e}_t \tag{3.7}$$

where I denotes an $N \times N$ identity matrix and the smoothing matrix A is of dimension $N \times N$. The 0_N matrices denote $N \times N$ null matrices. The term $\boldsymbol{\delta}$ denotes an N-vector of constants acting as a drift term. Bolded characters denote vectors of length N.

The next model Anderson et al. (2006) specified was the BN-UISTS(1,1,0):

$$y_t = \delta + \begin{bmatrix} 1 & -\phi \end{bmatrix} \begin{bmatrix} m_{t-1} \\ c_{t-1} \end{bmatrix} + e_t, \tag{3.8}$$

$$\begin{bmatrix} m_t \\ c_t \end{bmatrix} = \begin{bmatrix} \delta \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -\phi \end{bmatrix} \begin{bmatrix} m_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ 1 - \alpha \end{bmatrix} e_t, \tag{3.9}$$

$$\phi = \frac{1-\alpha}{\alpha}. \tag{3.10}$$

This model introduces a new parameter but does not increase the number of unknowns because of the condition specified in (3.10). This condition is necessary to remove the MA component (see Anderson et al. 2006). The multivariate form of the model denoted as the BN-VISTS(1,1,0) is:

$$\boldsymbol{y}_t = \delta + \begin{bmatrix} I & -\Phi \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_{t-1} \\ \boldsymbol{c}_{t-1} \end{bmatrix} + e_t \tag{3.11}$$

$$\begin{bmatrix} \boldsymbol{m}_t \\ \boldsymbol{c}_t \end{bmatrix} = \begin{bmatrix} \delta \\ 0 \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & -\Phi \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_{t-1} \\ \boldsymbol{c}_{t-1} \end{bmatrix} + \begin{bmatrix} A \\ I - A \end{bmatrix} \boldsymbol{e}_t \tag{3.12}$$

$$\Phi = (I - A)^{-1} A \tag{3.13}$$

The multivariate form of condition (3.10) is presented in (3.13).

The final model that Anderson et al. (2006) proposed was the BN-UISTS(2,1,2) specification:

$$y_t = \delta + \begin{bmatrix} 1 & -\phi_1 & -\phi_2 & \theta_1 \end{bmatrix} \begin{bmatrix} m_{t-1} \\ c_{t-1} \\ c_{t-2} \\ e_{t-1} \end{bmatrix} + e_t \tag{3.14}$$

$$\begin{bmatrix} m_t \\ c_t \\ c_{t-1} \\ e_t \end{bmatrix} = \begin{bmatrix} \delta \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\phi_1 & -\phi_2 & \theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_{t-1} \\ c_{t-1} \\ c_{t-2} \\ e_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ 1-\alpha \\ 0 \\ 1 \end{bmatrix} e_t \tag{3.15}$$

The multivariate form of this model, called the BN-VISTS(2,1,2) is:

$$\boldsymbol{y}_t = \boldsymbol{\delta} + \begin{bmatrix} I & -\Phi_1 & -\Phi_2 & \Theta_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_{t-1} \\ \boldsymbol{c}_{t-1} \\ \boldsymbol{c}_{t-2} \\ \boldsymbol{e}_{t-1} \end{bmatrix} + \boldsymbol{e}_t \tag{3.16}$$

$$\begin{bmatrix} \boldsymbol{m}_t \\ \boldsymbol{c}_t \\ \boldsymbol{c}_{t-1} \\ \boldsymbol{e}_t \end{bmatrix} = \begin{bmatrix} \delta \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & -\Phi_1 & -\Phi_2 & \Theta_1 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_{t-1} \\ \boldsymbol{c}_{t-1} \\ \boldsymbol{c}_{t-2} \\ \boldsymbol{e}_{t-1} \end{bmatrix} + \begin{bmatrix} A \\ I-A \\ 0 \\ 1 \end{bmatrix} \boldsymbol{e}_t \tag{3.17}$$

This model is relatively large requiring four parameter matrices of dimension $N \times N$ to be estimated. The matrices are denoted as Φ_1, Φ_2, Θ and A.

3.3 Identification and Common Component models

In this section two aspects of the BN-VISTS specification are discussed. The first is the potential problem of identification and the second, the common component specification.

The problem of identification when fitting VARMA models is well known (see Tiao & Tsay 1989). Namely, some autoregressive or moving average coefficients may be unidentifiable. For example in the case of an ARMA model the AR and MA coefficients will be unidentifiable if they have common factors. For a complete description of the identification problem with respect to vector ARMA models, and the various solutions available, refer to Athanasopoulos (2005).

The BN-VISTS(2,1,2) can exhibit the same problem, that is, the AR and MA coefficients may not be uniquely identifiable. To negate this problem the Φ and Θ matrices are constrained to be diagonal, this is a simple and effective solution to the problem of identification (Dufour & Pelletier 2005). It is important to note, that no constraint is placed on A.

The second issue addressed in this section is the common component specification. A common component occurs when the rank of the smoothing matrix is less than N and therefore;

$$A = \boldsymbol{\alpha}\boldsymbol{\beta}'. \tag{3.18}$$

For A to be uniquely identifiable either $\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$ must be fixed. In the application that follows, the values of $\boldsymbol{\beta}$ are determined a-prior according to economic rationale.

Interestingly, the reduced rank structure of A can be directly related back to the BNSW common trend specification. This is straightforward when the trend equation is back substituted (setting $\boldsymbol{\delta} = 0$) to yield :

$$\boldsymbol{m}_t = A \sum_{s=0}^{\infty} \boldsymbol{e}_{t-s} \tag{3.19}$$

Substituting equation (3.19) into any of the BN-VISTS observation equations yields:

$$\boldsymbol{y}_t = A \sum_{s=0}^{\infty} \boldsymbol{\epsilon}_{t-s} + \boldsymbol{c}_t \tag{3.20}$$

Equation (3.20) is of the same form as (2.6). Moreover if A is of reduced rank, the form is consistent with (2.9):

$$\boldsymbol{m}_t = \boldsymbol{\alpha}\boldsymbol{\beta}' \sum_{s=0}^{\infty} \boldsymbol{e}_{t-s}. \tag{3.21}$$

Therefore the same interpretation that was applied to $C(1)$ can be applied to A. The form of $\boldsymbol{c}_t$ will ultimately depend on the choice of the model. This is because the distinguishing feature between each BN-VIST model is the cycle.

3.4 Estimation

The likelihood derived in de Silva et al. (2007) can be directly employed. It is maximised using non-linear techniques. The algorithm employed in this paper was the BFGS. The likelihood function is;

$$L(\boldsymbol{\theta}, \boldsymbol{x}_0, \Sigma) = \left((2\pi)^T \prod_{i=1}^{N} \sigma_i^2 \right)^{-1/2} exp\left(-\frac{1}{2} \sum_{t=1}^{T} \sum_{i=1}^{N} e_{it}^2 \sigma_i^2 \right) \tag{3.22}$$

where $\boldsymbol{\theta}$ denotes the unknown coefficients, $\boldsymbol{x}_0$ denotes the state seed values and Σ denotes a diagonal covariance matrix of the innovations. The non-zero (diagonal) elements of Σ are denoted as σ_i, $i = 1, \dots, N$.

The initial state values, $\boldsymbol{x}_0$, are determined by running a regression on the first ten observations. The regression consists of a constant and a trend only. The constant is used for the initial trend value, m_0. The coefficient belonging to the linear time trend is reserved to be the starting value for δ. The median of the residuals is employed for c_0 (and c_{-1} when necessary).

The parameter matrices feed into (3.22) to initiate the estimation procedure are diagonal. The values are 0.3, 0.9, 0.9 and 0.25 for A, Φ_1, Φ_2 and Θ respectively.

In general, the likelihood is maximised subject to two conditions. One being invertibility:

$$mod|\lambda(D)| < 1, \quad D = F - AH, \tag{3.23}$$

and the other stationarity:

$$mod|\lambda(F_{N+1})| < 1. \tag{3.24}$$

The invertibility condition denoted by equation (3.23) refers to the modulus of the largest eigenvalue of D. Equation (3.24) denotes the stationarity condition, constraining the $N + 1$ largest eigenvalue of F to have a modulus less than one (as its first N eigenvalues are one by construction).

3.5 Interpretability of the BN-VISTS specification

A key advantage of the multivariate approach is that inter-relationships between components of different series can be gauged. The importance of this feature cannot be underestimated as it is arguably naive to consider an economic variable in isolation. This is acknowledged by most researchers including Morley et al. (2003) who close their paper with a caveat stating that additional information may affect the estimates of the trend and cycle.

An obvious measure that is often employed to measure inter-series relationships is correlation. Although the specification adopted in this paper constrains the observations to be contemporaneously independent the components are not. However, unlike the univariate case the correlations between components of a given series are not necessarily perfect.

The degree of correlation is determined indirectly by the values of smoothing parameters. It is important here to distinguish between the two types of correlations often employed. The type being discussed is the correlation between the innovations not the correlation between estimates of the permanent and cyclical components. The latter can be determined after the model is fitted whereas the former is determined implicitly during the fit of the model.

An interesting feature of the BN-VISTS formulation is that the effect of shocks can also be considered by way of impulse response function analysis. Moreover, the impulse response function of the temporary component also indicates the half-life of shocks, another important economic artifact.

The magnitudes of the relative components is gauged by comparing the variance of each component. Different economic theories exist as to which component is more dominant. As already discussed different specifications will yield different conclusions. Innovation specifications (i.e., ARIMA and innovation structural time series models Morley et al. 2003) typically produce dominant permanent components. Arguably, most economists believe that the permanent component should be dominant in times of economic and political stability (Sinclair 2005).

In addition to the inter-series insights discussed so far, the dates of troughs estimated by the model can also be compared to official recession dates (ie. NBER recession dates). This type of comparison however can be problematic, and care should be taken when performing this type of inference. In general there are two type of cycles, the *classical business* and the *growth* cycle.

The classical business cycle is defined as the presence of hills and valleys in the levels of the series. The growth cycle is defined similarly, however applied to the detrended levels. For a detailed discussion of this issue refer to Pagan (1997).

In the following section classical business cycle dates are compared to estimated permanent components. Although the extracted component does not precisely match the classical business cycle definition it does provide useful and interesting diagnostic tool.

4 Application of the Beveridge Nelson Vector Innovation Structural Time Series Models

The data employed in this example corresponds to the data used by Anderson et al. (2006); quarterly real GD(N)P for UK, Australia and USA. The natural logarithm was taken before the models were fitted, these are displayed in Figure 1. The data spans from the first quarter of 1980 to the last quarter of 2002. This time span is slightly shorter than the smallest time span considered by Anderson et al. (2006).

The plot of the series indicate that real GDP is growing steadily over the 22 years. All three series are close in scale and therefore there is no need to standardise these series before the multivariate model is fitted.

The analysis is conducted in three stages. The first stage fits a series of univariate models and compares them to Anderson et al. (2006). In the second stage, an unrestricted multivariate version of the BN-VISTS framework is fitted. In the final stage the smoothing matrix is constrained to be of reduced rank and therefore models two independent common features.

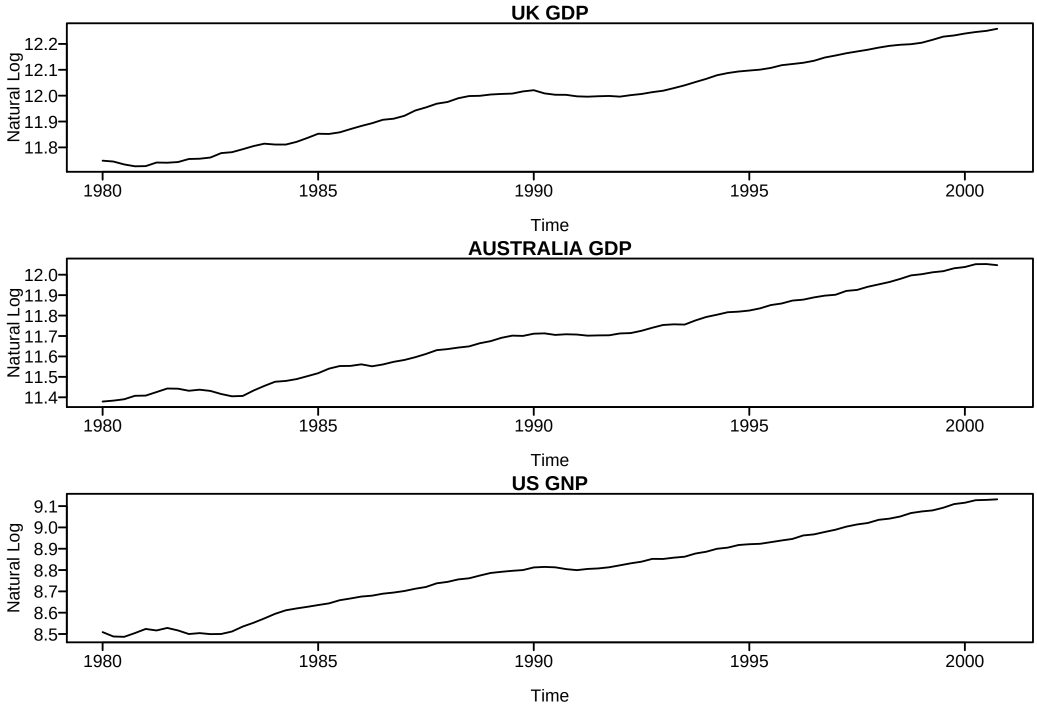

Figure 1: The Natural Logarithm of Real GDP of United Kingdom, Australia & America.

ARIMA order	Economy	ϕ_1	ϕ_2	θ	α	δ	$\sum_{i=1}^{T} e_t^2$	R^2	AIC
(0,1,1)	UK				1.088	0.006	3.525	0.986	-10.205
(1,1,0)	UK	-0.516			2.066	0.005	3.111	0.724	-10.331
(2,1,2)	UK	-0.551	-0.327	-1.044	4.510	0.003	2.897	0.636	-10.330
(0,1,1)	AUST				1.112	0.008	6.684	0.980	-9.566
(1,1,0)	AUST	-0.349			1.535	0.007	6.170	0.860	-9.646
(2,1,2)	AUST	-0.810	0.403	0.267	1.419	0.008	5.974	0.859	-9.606
(0,1,1)	USA				1.427	0.008	4.182	0.843	-10.035
(1,1,0)	USA	-0.160			1.191	0.007	6.501	0.953	-9.593
(2,1,2)	USA	-1.213	0.878	0.008	1.148	0.007	5.550	0.950	-9.680

Table 1: Univariate estimates

4.1 Univariate model

4.1.1 First Stage: Univariate Model

As a first step the BN-UISTS models were fitted to the data. The results are slightly different from Anderson et al. (2006) as the sample sizes have been reduced.

Table 1 displays the parameter estimates for each BN-UISTS model. The columns of particular interest are α and R^2. These columns depict the direction and degree of the initial reaction in the long-run (permanent) component to a 1% change in GDP.

Specifically, the R^2 values are calculated by regressing the first difference in the extracted permanent component against the first difference of the observations (this is in accordance with Anderson et al. 2006). Therefore R^2 depicts the amount of variation captured by the trend component.

As previously stated this model implicitly depicts the correlation between the latent components as being perfect. The direction of this correlation is ultimately determined by α which can be thought of as the long-run coefficient. If $\alpha > 1$ then the correlation is negative, conversely if $\alpha < 1$ this implies that the correlation is positive. In all instances it appears that the trend and the cycle of a given series are perfectly negatively correlated.

The choice of which BN-UISTS model is overall most appropriate is not clear. The UK data seems to be modeled appropriately either by an BN-UISTS of order (1,1,0) or (2,1,2). For the US data the AIC chooses the BN-UISTS(2,1,2) model whereas the BN-UISTS(1,1,0) seems most appropriate for the Australian data. Note that the BN-UISTS(2,1,2) nests the BN-UISTS(1,1,0) model, and furthermore it is the most appropriate for two out of three cases. Hence the BN-UISTS(2,1,2) is considered to be (in general) the most appropriate model.

4.1.2 Second Stage: Multivariate Model

In this section the BN-VISTS(2,1,2) is fitted. The form is slightly different from (3.17) as $A^* = I - A$. The reason for this becomes apparent in the third stage where it is shown that there is no evidence for A being of reduced rank, but there is for A^*. The resulting estimates are:

$$\begin{aligned}
\hat{\Phi}_1 &= diag\begin{bmatrix} 0.393 & -0.507 & 0.757 \end{bmatrix}, \ \hat{\Phi}_2 = diag\begin{bmatrix} 0.111 & 0.132 & 0.943 \end{bmatrix}, \\
\hat{\Theta} &= diag\begin{bmatrix} -0.192 & 0.098 & -0.010 \end{bmatrix}, \ \hat{\Sigma} = diag\begin{bmatrix} 0.031 & 0.052 & 0.055 \end{bmatrix}, \\
\hat{A}^* &= \begin{bmatrix} -0.528 & 0.086 & 0.003 \\ -0.748 & -0.258 & -0.321 \\ -0.05 & -0.003 & -0.047 \end{bmatrix}, \ I - \hat{A}^* = \begin{bmatrix} 1.528 & -0.086 & -0.003 \\ 0.748 & 1.258 & 0.321 \\ 0.05 & 0.003 & 1.047 \end{bmatrix} \qquad (4.1)
\end{aligned}$$

4.1.3 Third Stage: Multivariate Common Feature Model

A well known mathematical decomposition is the singular value decomposition. This decomposition expresses any real matrix into three component matrices, one of which is diagonal (see Schott 1997, for further details). The values in the diagonal matrix (which are sometimes referred to as singular values) are typically arranged in descending order and can be interpreted in a similar way as eigenvalues. Namely, a value of zero indicates the matrix is of reduced rank.

The singular values of A^* are 0.971, 0.273 and 0.029. Although none of these values are zero the smallest value, 0.029, is very close to zero. (Conversely the three singular values relating to A are 1.83,1.83 and 0.96.) Therefore it seems appropriate to constrain A^* to have a rank of two and hence,

$$A^* = \boldsymbol{\alpha}\boldsymbol{\beta}'. \tag{4.2}$$

Restricting the rank of A^* means it has the structure depicted by equation (4.2). As a result the likelihood must be maximised conditional on $\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$ as they cannot be uniquely determined. We choose to specify $\boldsymbol{\beta}$ according to economic theory prior to fitting the new model.

Specifically, the three economic variables considered in this example all measure the level of economic activity. In other words GD(N)P can be loosely considered as an economic barometer. As these economies are all wealthy and have strong trade and political links, it is reasonable to expect them to move together over time. This is confirmed visually by Figure 1. In other words, $\boldsymbol{\beta}$ should specify error correction mechanisms between the series, thereby constraining the real GD(N)P values to move together over time:

$$\boldsymbol{\beta} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}. \tag{4.3}$$

The form of the restriction in equation (4.3) depicts two common trends are present. Each trend is error correcting constraining two of the three series to move together over time. Enforcing this restriction yields the following parameter estimates:

$$\begin{aligned}
\hat{\Phi}_1 &= diag\begin{bmatrix} 0.026 & -0.699 & 0.803 \end{bmatrix}, \ \hat{\Phi}_2 = diag\begin{bmatrix} 0.097 & -0.021 & 0.897 \end{bmatrix}, \\
\hat{\Theta} &= diag\begin{bmatrix} -0.082 & 0.359 & -0.006 \end{bmatrix}, \ \hat{\Sigma} = diag\begin{bmatrix} 0.033 & 0.050 & 0.054 \end{bmatrix}, \\
\hat{\gamma} &= \begin{bmatrix} -0.175 & -0.080 \\ -0.134 & -0.900 \\ 0.021 & 0.022 \end{bmatrix}
\end{aligned} \tag{4.4}$$

Combing $\boldsymbol{\beta}$ and $\hat{\alpha}$ yields;

$$\hat{A}^* = \begin{bmatrix} -0.175 & 0.255 & -0.080 \\ -0.134 & 1.034 & -0.900 \\ 0.021 & 0.002 & -0.022 \end{bmatrix}, \ I - \hat{A}^* = \begin{bmatrix} 1.175 & -0.255 & 0.080 \\ 0.134 & -0.034 & 0.900 \\ -0.021 & -0.002 & 1.022 \end{bmatrix} \tag{4.5}$$

The estimates contained presented in (4.5) measure how the trend/cycle responds to an unanticipated shock in each of the three variables considered. Before the model can be interpreted the appropriateness of the restriction (4.3) is evaluated.

This can be done in two ways, either conducting a likelihood test or using an information criterion. The AIC for the unrestricted and restricted model is -29.577 and -29.633 respectively. Therefore we can conclude that there is evidence that the restriction is valid.

The parameter estimates belonging to USA GNP are all quite small with the exception of the value 1.022 relating to the permanent component and how it reacts to unexpected changes in USA GNP. This suggests the USA GNP (economy) is relatively insensitive to unexpected changes in the UK and Australian economies. This observation is consistent with the impulse response function analysis which will be discussed shortly.

The amount of variation captured by the trend component is 0.79, 0.18 and 0.96 for the UK, Australian and USA economies respectively. Interestingly, regressing the change in Australian GDP against the change in USA GNP yields an R^2 of 0.21. This suggests that the changes in the level of activity in the Australian economy is slightly more influenced by changes in the USA economy. This observation seems reasonable, as although Australia being a wealthy economy, it is a small economy by world standards and sensitive to larger economies, especially the USA.

Table 2 presents the implied correlations between the latent innovations. The lower triangle of the table indicates the direction of the linear association. The correlations are calculated from the conditional variance of the state components, that is:

$$var(\boldsymbol{x}_t|I_{t-1}) = \begin{bmatrix} I - A^* \\ \cdots \\ A^* \end{bmatrix} \Sigma \begin{bmatrix} I - A^* & \vdots & A^* \end{bmatrix} \tag{4.6}$$

In general, the correlations in Table 2 are consistent with the findings of Morley et al.

	m_{UK}	m_{AUS}	m_{USA}	c_{UK}	c_{AUS}	c_{USA}
m_{UK}		0.20	0.07	-0.69	0.32	0.49
m_{AUS}	+		0.99	-0.33	-0.70	0.73
m_{USA}	+	+		-0.27	-0.67	-0.81
c_{UK}	-	-	-		0.84	0
c_{AUS}	+	-	-	+		0.54
c_{USA}	+	+	-	0	+	

Table 2: Correlation between latent innovations

(2003) and Proietti (2002), that is the correlation between the latent components of a given series is negative. Specifically, the correlation between the latent components of UK, Australian and USA output are -0.69, -0.70 and -0.81 respectively. According to Morley et al. (2003), economically this is consistent with a shock in output which can shift the trend such that output is behind the trend until it catches up.

Two other interesting observations are: the trend in Australian GDP is almost perfectly correlated with the trend in USA GDP, and the UK trend appears to be only weakly correlated with the trends of USA and Australian GDP. The direction and strength of correlations between components of different series vary considerably.

Figure 2 displays the estimated latent permanent components. The grey shading indicates periods of recession (peak to trough episodes) as identified by the *Economic Cycle Research Institute* (2007). Although the recession periods differ for each economy the recessions dates correspond to the character of the permanent component. More specifically the recessions coincide with a flattening out or a slight decrease in the permanent

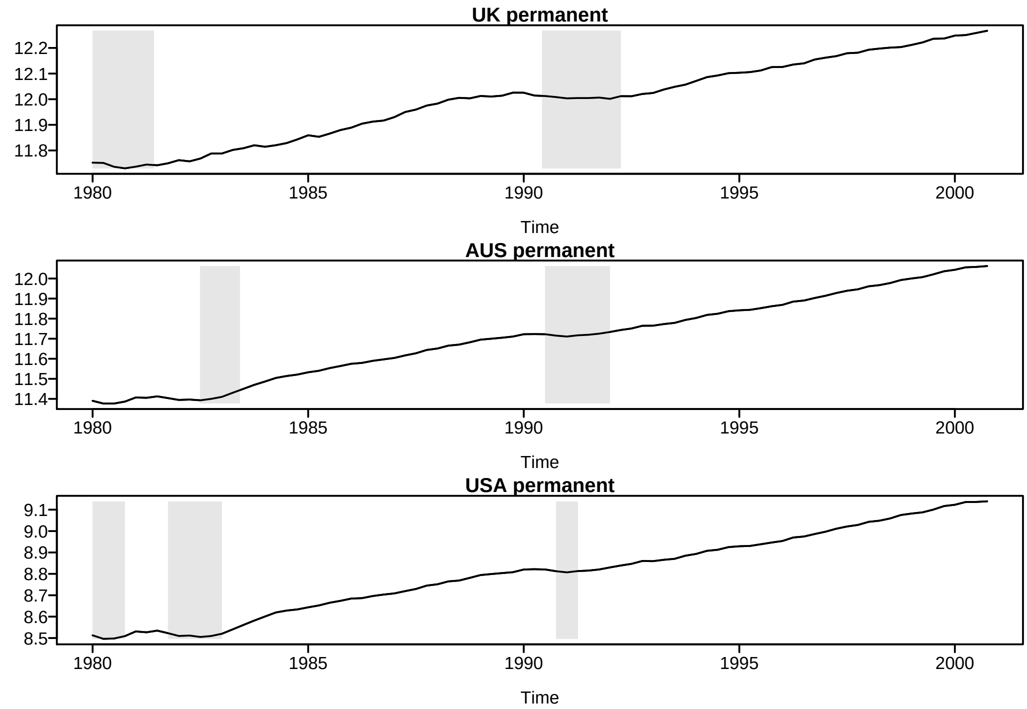

Figure 2: Comparision of estimated permanent component with ECRI classical business cycle dates

component. Therefore it is apparent that the extracted latent components are consistent with the character of the classical business cycle. This is an important observation as it helps verify this decomposition.

PANEL	SERIES	SHOCK
A	UK	UK
B	UK	AUST
C	UK	USA
D	AUST	UK
E	AUST	AUST
F	AUST	USA
G	USA	UK
H	USA	AUST
I	USA	USA

Table 3: Index to impulse response functions, Figure 3

The final aspect of model interpretation is the impulse response functions of the cyclical component, presented in Figure 3. These plots map the response in the trend component with respect to a 1% shock. An index of the nine plots is presented in Table 3. It is important to note that only three impulse response functions are possible if a univariate

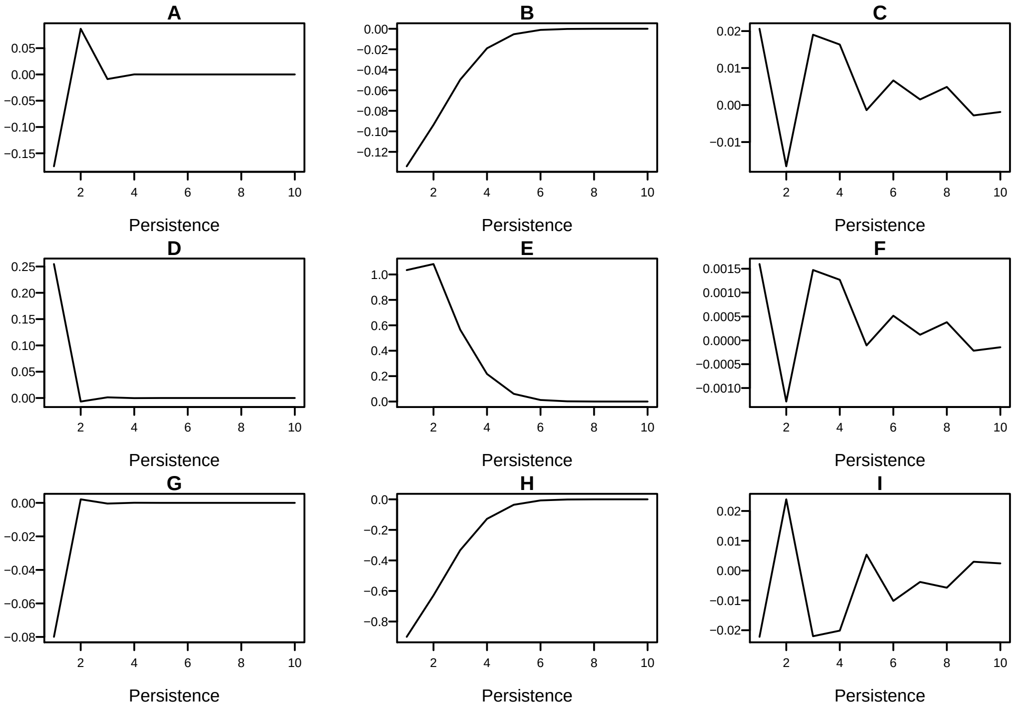

Figure 3: Impulse Response Function of temporary components.

model is fitted. These three correspond to the diagonal of Figure 3.

The first column of Figure 3 indicates that the persistence of a shock in UK GDP is relatively short lived, lasting at most four periods. The reaction to the UK shock of the Australian temporary component is a mirror image of the USA temporary component. The initial impact of UK GDP with respect to a 1% shock in UK GDP is a decrease which is followed by a rapid increase which over compensates the initial reaction.

On comparing the three graphs in the second column it is clear that the reaction to a 1% shock in Australian GDP persists for up to six periods. The half life (where life is defined by the time taken for the shock to fully dissipate) is approximately three periods for all three components. The US and UK reaction to a 1% increase in GDP is almost identical only differing slightly in scale. In these cases the immediate reaction is negative before a gradual return to the base line.

The last set of three panels gauge the response to a 1% increase in USA GNP. The return to the base line in all three cases is slightly more erratic than compared to the previous cases. Importantly, the scale of the reactions are considerably smaller in these three panels. This confirms the observation made earlier; the US is relatively insensitive.

5 Conclusion

This paper has illustrated a multivariate BN decomposition that is simple and flexible. This new approach is an extension of the newly introduced vector innovation structural time series framework (de Silva et al. 2007). It has been shown that the inference from this new approach is consistent with economic rationale.

Specifically, this analysis has shown how to exploit inter-series relationships. Two key features illustrated are the capability to model inter-series dependencies and common features.

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