CENTRO STUDI IN ECONOMIA E FINANZA

CENTRE FOR STUDIES IN ECONOMICS AND FINANCE

WORKING PAPER NO. 117

Commitment of Monetary Policy with Uncertain Central Bank Preferences

Marcello D'Amato

April 2004

DIPARTIMENTO DI SCIENZE ECONOMICHE - UNIVERSITÀ DEGLI STUDI DI SALERNO
Via Ponte Don Melillo - 84084 FISCIANO (SA)
Tel. 089-96 3167/3168 - Fax 089-96 3169 – e-mail: csef@unisa.it

CSEF WORKING PAPER NO. 117

Commitment of Monetary Policy with Uncertain Central Bank Preferences

Marcello D'Amato *

Abstract

In this paper we analyse the equilibrium degree of commitment in monetary policy to an independent central banker whose preferences are imperfectly observed by private agents. We characterize the incentive compatible strategies by a central bank in office for two periods with no restrictions on its type space. The equilibrium level of commitment is also characterized. We show that when incentive compatibility constraints are binding for a non trivial subset of types of central banks the equilibrium level of commitment involves bunching: different types of rational governments commit monetary policy to similar institutions.

JEL Classification: E58, D82.

Keywords: Monetary policy, Delegation, Signalling games.

* CSEF, University of Salerno. E-mail: damato@.unisa.it

David Alexander Glencairn (me tendré que habituar a llamarlos alí)
era, lo sospecho, un hombre temido;
el mero anuncio de su advenimiento bastó para apaciguar la ciudad.
Ello no impidió que decretara diversas medidas enérgicas.
Jorge L. Borges, Aleph.

1 Introduction

Recent developments in the debate on the optimal degree of commitment in monetary policy has focused on the role of private information by monetary autorities both on state variables in the economy and on their own objectives. How much discretion should "society" allow in the presence of private information of the policymaker on some of the state variables in the economy is, for example, the normative issue addressed in a recent paper by Athey et al. (2003) where it is shown that the optimal social contract between society, with an agreed upon welfare function, and the delegated body in charge of monetary policy involves a simple inflation cap. Monetary policy has however also important redistributive effects and the evaluation of the trade-offs involved in monetary policy responses may be different among private agents depending on the size of wealth and income non indexed to inflation. If redistributive effects of monetary policy are important[1], the study of the issue about the optimal degree of discretion under private information has to be complemented by the analysis of the equilibrium level of discretion the political body is willing to assign to monetary targets. In a paper money system the "government" or, in a representative democracy, the Parliament have the task and the power of looking after the monetary system. In the presence of inflationary bias the government may be willing to delegate monetary policy to an independent agency (Kydland and Prescott, 1977, Barro and Gordon, 1983 and Rogoff, 1985), with chartered objectives. With central bank being a delegated power in a democracy, the institutional design of the independent body and the appointment of a specific agent in charge fo monetary policy is in the hands of a political body and may reflect political preferences. As Paul Volker once put it "Congress created us, and the Congress can uncreate us" (quoted in Stiglitz, 2003). Under no commitment to policy platforms,

[1]For a discussion of the issues related to redistributive effects of monetary policy in the US economic history see Stiglitz (2003) and references therein.

both political preferences by the government in charge and preferences by the delegated agency with chartered objectives on the trade offs associated to monetary policy actions may, however, not be accurately observed by private agents. What are the implications of imperfect observability for the equilibrium degree of discretion assigned to central bankers by the political body? That is likely to depend on the impact of partial observability of central banker's preferences on its policy actions.

The aim of this paper is to contribute to the study of how uncertainty on central banker preferences may shape incentives to pursue precommitment solutions in the presence of inflationary bias. If the central banker's objectives are not observed by the public with certainty the commiting technology is less effective and delegation may, in principle, be harder to achieve or even collapse. Under partial observability of the central banker's objectives in charge for more than one period a rational government will anticipate that monetary policy is restricted by incentive compatibility constraints and will take them into account in the delegation process. Our aim is to analyze how incentive compatibility constraints may affect the equilibrium level of commitment in a simple economy where monetary policies are affected by an inflationary bias and the commitment technology is not perfect - i.e. the objectives of the agent in charge for monetary policy are not perfectly observed by private agents.

We will consider a very simple setting where a government with specific preferences over alternative policy targets is elected, reflecting the preferences of the median voter, for two periods. In order to reduce the inflationary bias he may appoint an independent central banker serving for the same time horizon as the government. In each period the agents in the economy sign nominal contracts conditional on the information available. There are two classes of agents in the economy, one of size p have accurate information about the outcome of the delegation process and, in signing the contracts, can make full use of this information. The fraction $1-p$ of private agents, instead, is not able to observe the outcome of the delegation process in the first period, therefore, they sign first period contracts only using ex ante information, possibly taking into account equilibrium incentives to commitment by the government. In the second period they can condition their expectations on the policy outcomes observed in the first period. Being in charge for two periods as well, the Central Banker will make its choice over alternative policy targets keeping into account that its first period choices may convey information about its objectives. Subject to such incentive compatibility constraints the CB's equilibrium inflation strategies are characterized as the outcome of a Bayes- Nash equilibrium between the CB and the private sector. We restrict the analysis to equilibrium strategies that are (weakly) monotone in the central bank type. In order to fix the inflationary bias problem, after election, the government may decide to delegate monetary policy to an agent in the economy whose preferences may differ from the government's one but are not accurately observed by the general public[2].

Given the incentive compatibility constraints on the CB's strategies, the equilibrium level of commitment is characterized and the results we obtain are: 1. the equilibrium level of commitment is shown to be different from zero, i.e. the government appoints conservative CBs, even in the presence of incentive compatibility constraints on its CB's strategies provided that some observability exists; 2. the equilibrium level of commitment under incentive compatibility is larger than in the case when the monetary policy outcome resembles reputational equilibria of the Backus and Driffil (1985) type, 3. the equilibrium level of commitment under incentive compatibility constraints may be lower than in their absence given costless information transmission over time 4. there exist atoms in the equilibrium distribution of possible central bankers. The first and second results show that commitment arguments for the appointment of central bankers may not be eliminated by reputation considerations, i.e. in this simple setting reputation is not necessarily a perfect substitute of commitment (see for a different view McCallum, 1995). The third result complements some of the results in Sibert (2002) where incentive compatibility constraints on the central bank strategy do not affect incentives to commitment in the simple two type case. The fourth result shows that, for a substantial proportion of possible election outcomes, different governments will converge to the same level of commitment. This latter result seems to us of particular interest since it captures what we think is an important aspect of institutions as commitment devices in the governments hands: the insulation of the institutions' objective from the variability of po-

[2]See Alesina and Grilli (1992) for an explicit reinterpretation of the Rogoff's model in terms of a political game where a population of citizens, differing only with respect to the relative weight which they assign to inflation and stabilisation, votes upon the preferences of the "governor" to appoint. For an alternative approach on the analysis of the relationship between "society" and the agent in charge of monetary policy see Walsh (1995) and Persson and Tabellini (1993). Persson and Tabellini (1997), pp. 37 ss. offer a critical analysis of the main differences and analogies between the precommitment and the contracting approach to the normative analysis of monetary policy making.

litical objectives. We show that this is a feature of the equilibrium strategy of the government when the policies adopted by the delegated institutions are subject to credibility constraints. In the standard model of monetary policy precommitment, for example, the conservative central banker result does not necessarily rule out that the equilibrium level of commitment changes when political preferences over alternative policy targets change.

These results are, of course, limited by several caveats: the economy we model is the simplest possible economy with inflationary bias. We do not consider shocks to economic variables, the pay offs to the players simply model inflationary bias by using the usual linear quadratic preferences with no microfoundation, the incentives by private agents to acquire information about the CB type are exogenously given. However none of these limitations should affect the results obtained in a substantial way. Including shocks to real variables that are common knowledge among the public would only require that in devising the equilibrium level of commtiment, incentive compatibility constraints have to be considered in each state of the economy. This would only make the equilibrium level of commitment depend negatively on the variance of the shocks (as in Rogoff, 1985) at the cost of complicating the derivation of the results in a substantial way and without affecting the major properties of the incentive compatible strategies by the CBs. Including endogenous information acquisition may be more problematic as for the analytic solution of the model. Most of the results derived in this paper, however, are consistent with a simple fixed cost etherogeneity of information acquisition among private agents and should be confirmed in a more general model with endogenous level of observability of the central banker's type, to the extent that the (equilibrium) level observability is not perfect.

As soon recognized in the literature the behavior of policymakers in charge for monetary policy and the institutional design problem are both affected by private information on their objectives. The presence of private information may impose self discipline on the policymaker's behavior and lead to reputational equilibria that reduce or eliminate the problem of time inconsistency of monetary policy as shown, for example, in Backus and Driffil (1985), Barro (1986) and Vickers (1986). If, on the one hand, private information and reputational concerns work effectively as a discipline device, given the assigned preferences over the trade offs associated to monetary policy, it is also true, on the other hand, that incentive constraints associated to these equilibria, may induce costly distortions in policy responses that affect incentive to commtment and have to be taken into account in the design of regulatory framework of monetary policy by governments. Sibert (2002), for example, shows that, given preferences over alternative policy objectives in the society and given a conservative policymaker, the policy response by the monetary institution to a given shock can be larger under incentive compatibility constraints on the central bank's action than under complete information. She also shows, in a simple example, that, in the case with two possible types, conservative policy makers are preferred by society. She concludes that, although incentives to commitment are preserved under private information, the scope for stabilization policy turns out to be less dire than in the case of complete information as analyzed by Rogoff (1985). Commiting policy rules and objectives to agents with private information, however, is not necessarily so straightforward: the same properties of the incentive costrained policy actions that make them more responsive to shocks may affect commitment strategy at the government level. As it has been shown in the game theoretical contributions to this problem (Bagwell, 1995, Fershtmann and Kalay, 1997) incentives to commitment crucially depend on its observability when the agent is in charge for one period. More generally, when the agent is in charge for more than one period the impact of incentive compatibility constraints on policy action has to be taken into account and the equilibrium degree of commitment has to balance carefully the benefits from observability with costs associated to distortions induced by incentive compatibility constraints.

The present paper is rooted in the traditional literature (see Persson and Tabellini 2000) on the optimal degree of commitment in monetary policy but is also related to some recent developments in the field. The well known commitment approach to the inflationary bias problem has been analized (Rogoff, 1985) elaborated by the "contractual" approach to inflation targeting both in a static (Persson and Tabellini, 1993, Walsh, 1995) and dynamic framework (Svensson, 1997) relies on perfect observability of the commitment technology. The implications of private information for the institutional design of monteary policy have been discussed in recent papers both in the delegation approach, Sibert (2002) and in the contractual approach (Athey et al., 2003). As discussed above, Sibert (2002) addresses similar issues in a model with shocks and a continumm of central banker's types, but she does not characterize the equilibrium commitment strategy in the case of a continuum of types[3].

[3]The issue of commitment observability does not arise in Sibert (2002) since only two

The optimal degree of discretion under private information is studied in Athey, Atkenson and Kehoe (2003) in a normative perspective in the context of the dynamic mechanism design literature applied to monetary policy games (see references therein for other contributions in the same line). In their context, the asymmetry of information is between the delegating agent ("society" with an agreed upon social welfare function) and the monetary authority whereas, in the present paper, we study a situation where the delegating body, chooses the cb's type. They show that, once the contract is perfectly anticipated by private agents, the optimal policy is static and takes the form of an inflation cap (contingent to publicly observed states of the economy) and that agents' expectations do not vary with the monetary authority's policy choice. Below the cap the monetary autority is left complete discretion in stabilizing shocks to state variables on which it has an informational advantage with respect to public agents. Instead, in the model analyzed here, there is no asymmetry of information between the elected government and the central banker. The asymmetry of information is between the central bank and the private agents and the whole issue is about how commitment by the government and the policy choices by policymakers may affect private expectations in the future in the presence of imperfect observability. In a similar fashion we obtain that equilibrium inflation in our model is bounded from above for a subset of government types relatively more prone to temptation. The upper bound associated to bunching of commitment strategies by different types, in our case depends on the degree of commitment observability by private agents, i.e. on institutional transparency rather than on publicly observed economic variables.

The remaining of the paper is organized as follows: in section 2 we outline the model and define the separating equilibrium and the pooling equilibrium in the monetary policy sub-game; in section 3 we derive the equilibrium level of commitment in the delegation stage; in section 4 we conclude.

2 The model

As in Sibert (2002), the timing of the game is similar to the one in Rogoff (1985) except for the time horizon of the appointed policymakers: at time $t = 0$ (delegation stage) a government is elected for two periods endowed

possible CB types are considered. For the analysis of noisy commitment and finite strategy set see Bagwell (1995).

with preferences over the trade offs between inflation and surprise inflation. It may delegate monetary policy to a CB in charge for two periods whose preferences are accurately observed only by a fraction p of the population. At time $t = 1$, given agents expectations, the CB will set the inflation rate taking into account that future expectations (at time $t = 2$) by the fraction $1 - p$ of private agents will be set conditional on the observation of the CB's current choice. At time $t = 2$ the policy game is repeated.

Both in a separating and in a pooling equilibrium a given type of government has to afford costs and may obtain benefits from the presence of private information of the public about its preferences. Broadly speaking, in a separating equilibrium a tough type will afford costs in the first period in order to credibly signal its own preferences to private agents and will collect benefits in the second period when private agents' expectations are set according to its actual type. A wet type will collect benefit from private information since, in a separating equilibrium, private agents will reduce expectations about inflation in the first period and the engineering of surprise inflation is a more effective course of action. In a pooling equilibrium a tough type will be worse off in both periods depending on private agents expectations. A wet type may improve its welfare because it will benefit from surprise inflation in the second period. We will consider both separating and hybrid equilibria where some types oof monetary autorities pool their strategy and show how, in either case, the incentive to commitment is modified by the presence of private information.

The per period payoff function to the elected Government is given by

$$W_t^g = -\frac{1}{2}[\pi(\alpha)]_t^2 - \alpha_g[\pi_t(\alpha) - \pi_t^e] \tag{1}$$

with $\alpha_g \in S_g(\alpha_g) \equiv [0, A^G]$ distributed according to an arbitrary distribution function $\Gamma(\alpha_g)$. We assume $[0, A^G]$ is common knowledge among private agents. We also assume no commitment to electoral platforms by the elected government and uncertainty of the public about its preferences, i.e. about the exact identity of the median voter. This assumption implies that, at equilibrium, uninformed agents, although aware of incentives to commitment, are not able to infer the exact type of central banker by using $\digamma(\alpha_g)$ [4]. The control variable to the government is given by the central banker's type

[4] This assumption is made for simplicity. All the results are preserved to the extent some residual uncertainty about the central banker's objectives is observed. It is equivalent

α to whom monetary policy is delegated and indirectly determining the economic variables of interest, inflation (π_t), expectaions and surprise inflation ($\pi_t - \pi_t^e$) as the equilibrium outcome of the continuation monetary policy game.

The per period pay off function assigned to the central banker is given by

$$W_t = -\frac{1}{2}\pi_t^2 + \alpha(\pi_t - \pi_t^e) \tag{2}$$

where the target inflation rate has been normalised to zero and α represents the degree of temptation to surprise agents, i.e. the inflationary bias. Private agent's $\pi_t^e = pE[\pi_t \mid I_t] + (1-p)E[\pi_t \mid \Omega_t]$ where Ω_t is assumed to be coarser than I_t. In particular, for the fraction $1-p$ of uninformed agents, α is not observed and is distributed according to a beliefs distribution function $B(\alpha)$ defined over the compact support $\alpha \in S(\alpha) \equiv [\underline{\alpha}, A]$, where α is defined by a commitment function $\Gamma : \alpha_g \rightarrow \alpha$[5]. The fraction p, on the other hand collects an accurate signal about the objectives of the maginal benefits from surprise inflation. Private agents, in formulating their expectations, conditional on their information set, will minimize the forecast error, therefore the expected inflation rate at $t = 1, 2$ will be given by $\pi_t^e = pE[\pi_t \mid I_t] + (1-p)E[\pi_t \mid \Omega_t]$ where, under $\Gamma(\cdot)$, $I_t = \{\alpha = \Gamma(\cdot), \pi_{t-1},\}$ is the information set for the informed agents and $\Omega_t = \{\pi_{t-1}, \Gamma(\cdot)\}$ is the information set for the uninformed agents, where only past experience and anticipated equilibrium commitment are included.

Finally, we assume, for the sake of simplicity and without affecting the results qualitatively, that the players do not discount future so that the payoff function on the time horizon of the game are the following ones: $W = W_1 + W_2$ for the CB and $W^g = W_1^g + W_2^g$.

An equilibrium of the game will specify a commitment function and $\Gamma : S_g(\alpha_g) \rightarrow S(\alpha)$, a couple of inflation rates played by the central banker: $\overline{s} = [\pi_1(\alpha), \pi_2(\alpha)]$, a couple of equilibrium expectations $\overline{e} = [\pi_1^e, \pi_2^e]$, where:

---

to the assumption that uninformed agents in the economy only observe a noisy signal of $\Gamma(\alpha_g)$.

[5]Both the support $S(\alpha)$ and the distribution function of uninformed agents $B(\alpha)$ are endogenous and will be characterized as part of the equilibrium outcome of the game. However, since the set of incentive constraints in signalling games with a continuum of types is distribution free, most of our attention will be focused on the characterization of the equilirium support $S(\alpha)$. The only feature of interest of $B(\alpha)$ given $\Gamma(\alpha_g)$ will be the presence of atoms.

$\pi_1^e = pE[\pi_1 \mid \alpha] + (1-p)E[\pi_1]$ and $\pi_2^e = pE[\pi_2 \mid \alpha] + (1-p)E[\pi_2 \mid \pi_1]$ set according to the Bayes rule where possible.

The game is solved by backward induction, i.e. we first solve equilibrium strategies and expectations in the two periods monetary policy game and than, given the continuation game, we solve for the elected government's optimal degree of commitment at time $t = 0$. The solution concept we adopt for the monetary policy game is the Bayes-Nash concept: this is a standard signalling game with a continuity of types, whose equilibria have been characterised by Mailath (1987) and, with reference to monetary policy, by Vickers (1986) for the case of two types, D'Amato and Pistoresi (1996) and by Sibert (2002) for the case of a continuum of types. Since the specific distribution function $B(\alpha)$ is immaterial for the characterization of the incentive compatibility constraints in signalling games, we only need to define $S(\alpha)$ in order to characterize $\overline{s}$ and $\overline{e}$. Given $S_g(\alpha_g)$ we will conjecture, and then verify, an initial support $S(\alpha)$ for the equilibrium level of commitment and, given $\alpha \in S(\alpha)$, we characterize $\overline{s}$ and $\overline{e}$. After verifying that $S(\alpha)$ is indeed the equilibrium support for α under $\digamma(\cdot)$, we characterize $\digamma(\cdot)$.

Before moving to the solution of the game let us notice that inflation expectation by uninformed private agents are set after the elections and after the commitment choice by the elected government. The uninformed agents, in setting $E[\pi_1]$, can anticipate government's incentive to commit and reduce expextations accordingly. However, not being perfectly informed neither about the identity of the median voter (no commitment to electoral platforms), nor about the identity of the central banker, they cannot accurately observe the type of banker appointed in equilibrium. The equilibrium level of commitment has, therefore, to balance the cost of information transmission (incentive compatibility constraints where binding) by the monetary authority about its type and the benefits of reducing inflation expectations by both groups of agents: the informed, who accurately observe the level of commitment and the uninformed who, anticipating but not observing equilibrium commitment, also reduce expected inflation without enjoying perfect forecast.

3 Equilibria in the monetary policy game

As stated above the two period game played by the cb is a standard signalling game. As such it may have different equilibria, separating, pooling and hybrid. Since the aim of the paper is to study the effect of incentive compatibility constraints on the optimal commitment strategy by the government, we focus our attention mainly on equilibria that are separating or hybrid, i.e. equilibria such that the incentive compatibility constraints on the first period inflation choice are binding for a non trivial subset of types of bankers in the support $[0, A]$. We conjecture and verify later that if $S(\alpha_g)$ is common knowledge then $S(\alpha)$ is common knowledge too at equilibrium, for π_1 (weakly) monotone in α. We proceed by characterizing the conditions under which a pure separating strictly monotone strategy $\pi_1 = \tau(\alpha)$ exists over the support $[0, A]$. We then study the hybrid equilibrium strategy, i.e. the function $\pi_1 = h(\alpha)$, such that $h(\alpha)$ satisfies incentive compatibility over a subset $\sigma(\alpha) \subseteq S(\alpha)$. After characterizing the possible separating or hybrid equilibria in the monetary policy subgame for an arbitrary support $[0, A]$ of central bankers types, we will verify that the conjecture about $S(\alpha)$ is confirmed under the equilibrium mapping $\alpha = F(\alpha_g)$ for $\alpha_g \in [0, A_G]$. Finally the characterization of the equilibrium level of commitment will be provided.

We proceed by backward induction and derive the equilibrium level of π_2 by solving the following program:

$$\begin{array}{ll} \underset{\pi_2}{Max} W_2 = & -\frac{1}{2}\pi_2^2 - \alpha(\pi_2 - \pi_2^e) \\ s.t. & \pi_2^e = pE[\pi_2 \mid \alpha] + (1-p)E[\pi_2 \mid \pi_1] \end{array}$$

which, since the expectations by private agents are taken as given by the monetary authority, yields the simple first order condition as a dominant strategy for second period inflation:

$$\pi_2 = \alpha \tag{3}$$

Private agents expectations, given their information set, can be obtained by computing the expected value of (3) over α conditional on relevant variables, α and π_1 for the informed and uninformed group respectively, which yields

$$\pi_2^e = p\alpha + (1-p)\widehat{\alpha} \tag{4}$$

where $\widehat{\alpha} = E(\alpha \mid \pi_1) = \tau^{-1}(\pi_1)$, as dictated by the Bayes rule where applicable and τ represents the strategy in the first period of the game. In a pure separating equilibrium, τ is a one to one mapping from the type space onto the strategy space satisfying an initial value condition (see Mailath, 1987). Following the literature on signalling games (Riley, 1979, Milgrom and Roberts, 1982, Mailath, 1987, Sibert 2002) we will adopt $\tau(A) = A$ i.e. the initial value condition such that the most undesirable signal is sent by the banker holding the type corresponding to the worst possible conjecture by the uninformed players. Define $\pi_1 = h$ the equilibrium strategy in a hybrid equilibrium. The hybrid equilibrium is such that there will exist a subset of types $\sigma(\alpha)$ for which incentive compatibility constraints are strictly binding with $\widehat{\alpha} = E(\alpha \mid \pi_1)$ and $h = \tau$, $h(A) = A$ for $\alpha \in \sigma(\alpha)$, where- at equilibrium- it will be shown that $\sigma(\alpha)$ is such that $\alpha \leq A$ for $\alpha \in \sigma(\alpha)$. We focus upon hybrid equilibria such that, for $\alpha \notin \sigma(\alpha)$, $h \neq \tau$ and constant.

In order to identify the conditions under which either τ or h define the equilibrium first period inflation, let us characterize τ as the function that satisfies incentive compatibility constraints, given some regularity conditions on the reduced form pay-off function of the cb. After substituting for the second period equilibrium strategies and beliefs, this reduced form is defined as:

$$\widetilde{W}(\alpha, \widehat{\alpha}, \pi_1) = -\frac{1}{2}\pi_1^2 + \alpha(\pi_1 - \pi_1^e) - \frac{1}{2}\alpha^2 + \alpha[\alpha - p\alpha - (1-p)\widehat{\alpha}]$$

the first order condition is given by:

$$-\tau + \alpha - \alpha(1-p)\frac{d\widehat{\alpha}}{d\tau} = 0 \tag{5}$$

By evaluating (5) at $\widehat{\alpha} = \alpha$, the characterization of the incentive compatible τ is given in the following:

Lemma 1 *(Incentive Compatibility).*

There exist unique a monotone function $0 < \tau(\alpha) \leq A$ *on* $\sigma(\alpha) = (\underline{\alpha}, A]$, *with* $\underline{\alpha} \geq 0$ *such that satisfies* $Arg \max_{\pi_1 = \tau} \widetilde{W}(\alpha, \tau^{-1}(\pi_1), \pi_1)$ *as the solution of the initial value problem*

$$\tau'(\alpha) = \frac{\alpha(1-p)}{\alpha - \tau}, \tau(A) = A \tag{6}$$

Proof: Sufficient conditions for existence and uniqueness of τ are given by Theorem 1,2, 3 in Mailath (1987) and we only need to check that some regularity conditions on (**??**) for the existence of τ are satisfied in our setting. These conditions are $\widetilde{W}_{\widehat{\alpha}}(.) = -(1-p)$ (belief monotonicity), $\widetilde{W}_{\alpha,\pi_1}(.) = 1$ (type monotonicity), $\frac{\partial[\widetilde{W}_{\pi_1}(.)/\widetilde{W}_{\widehat{\alpha}}(.)]}{\partial\alpha} = -\frac{\pi_1}{\alpha^2(1-p)}$ (single crossing). By type monotonicity $\tau(\alpha)$ is strictly increasing, belief monotonicity and the initial value condition imply that $\tau(A) = A$. Single crossing requires $\pi_1 > 0$. Moreover, since $\tau'(\alpha)$ is unbounded at $\alpha = A$ and type monotonicity condition is positive, by Theorem 3 in Mailath (1987), it must be: $0 < \tau(\alpha) < \alpha$, except for $\alpha = A$. The first order condition on (**??**) implies (6). Second order conditions are easily checked to require $\tau' > 1 - p$ □

A brief discussion of the monotonicity conditions will help in the interpretation of results to be provided below. The belief monotonicity condition, given by $\partial\widetilde{W}/\partial\widehat{\alpha} < 0$, suggests that the CB would prefer to be believed to be a tough type in fighting inflation because this reduces inflation expectations by uninformed agents and, coeteris paribus, makes surprise inflation a more effective course of action; the type monotonicity condition, $\partial^2\widetilde{W}/\partial\pi_1\partial\alpha = 1 > 0$, suggests that the marginal benefit of (surprise) inflation is increasing in α; the single crossing condition, $\partial[\widetilde{W}_{\pi_1}/\widetilde{W}_{\widehat{\alpha}}]/\partial\alpha = -\pi_1/[\alpha^2(1-p)] > 0$ for $\pi_1 > 0$ suggests that the marginal rate of substitution between an increase in inflation at time $t = 1$ and a reduced reputation next period (an increase in $\widehat{\alpha}$) is increasing in α. Intuitively, the weaker the banker, the more he is willing to pay in terms of future reputation for a unit increase of surprise inflation today. Under these conditions a separating equilibrium exists, is unique and is described by the solution to (6).

Notice that the conjectured initial support $S(\alpha)$ is such that there exist a type of banker $\alpha = 0$ who has a dominant strategy $\pi_1 = 0$. As noted by Sibert (2002) the presence of such a type has to be carefully taken into account in characterizing the first period inflation equilibrium schedule. Also notice that whether $\underline{\alpha}$ is strictly larger than zero or not has been left unspecified in Lemma 1[6]. There exist two possible cases: either $\underline{\alpha} = 0$ and $\tau(0) = 0$ is part of $\tau(\alpha)$, then we have a pure separating equilibrium in $[0, A]$ or $\underline{\alpha} = \alpha^s > 0$

[6]D'Amato and Pistoresi (1996) and Sibert (2002) restrict the initial support exluding so that single crossing is strictly satisfied for any type and a pure separating equilibrium always exists on the entire support. D'Amato and Salsano (2003) also study the equilibrium in the unrestricted support.

with, given strict monotonicity of $\tau(\alpha)$, α^s such that $\lim_{\alpha \searrow \alpha^s} \tau = 0$ and we may have a hybrid equilibrium, provided that a pooling strategy exists for α in $\sigma^p(\alpha) \equiv [0, \alpha^s]$. In order to characterize both cases we need preliminarily to prove that, at equilibrium, $\tau''(\alpha) > 0$.

Lemma 2 *(Convexity)* $\tau''(\alpha) > 0$ *for* $\tau(\alpha)$ *satisfying (6) and the initial value condition* $\tau(A) = A$.

Proof: Simple algebra shows that $\tau''(\alpha) \geq 0$ iff $\tau' \geq \tau/\alpha$ for $\alpha \in \sigma(\alpha)$. By evaluating the inequality at (6) it is satisfied if $x^2 - x + (1-p) \geq 0$. Where $x = \tau/\alpha$. By lemma 1 incentive compatibility requires $0 \leq x \leq 1$. By studying the last inequality we get that it is always satisfied with strict inequality if $p < 0.75$ and therefore $\tau''(\alpha) > 0$ for $0 \leq \tau/\alpha \leq 1$. Alternatively, if $p \geq 0.75$ the inequality is satisfied for $x \in [0, \frac{1-\sqrt{4p-3}}{2}] \cup [\frac{1+\sqrt{4p-3}}{2}, 1]$. Therefore, by continuity of τ' in the interior of $[0, A)$ either $x \in [0, \frac{1-\sqrt{4p-3}}{2}]$ or $x \in [\frac{1+\sqrt{4p-3}}{2}, 1]$. The initial value condition requires $\tau(A) = A$, i.e. $x = 1$ to be part of the equilibrium and hence, it must be that, for $p \geq 0.75$, $x \in [\frac{1+\sqrt{4p-3}}{2}, 1]$ which implies $\tau''(\alpha) > 0$. □

This result is an extension of Lemma 2 in Sibert (2002) in that no restriction on the support for the central banker's type is assumed. The differences in the proof are related to partial observability of cb's objectives in the present model. In plain words, incentive compatibility *and* the initial value condition is sufficient to induce a monotonically increasing and convex τ function. The lemma above is important in that it helps in characterizing α^s and the conditions on p such that either of the two possible equilibria regimes, separating and hybrid, such that incentive compatibility is binding for a non trivial subset of $[0, A]$.

A pure separating equilibrium exists provided that the degree of observability is large enough. First period inflation rate satisfies $\pi_1 = \tau(\alpha)$ and $\widehat{\alpha} = \tau^{-1}(\pi_1)$ over $[0, A]$, $\pi_1 = \pi_1^e = \alpha$.

Proposition 1 *A pure separating equilibrium strategy in the monetary policy game exists for* $p \geq \overline{p} \equiv 0.75$. *The separating strategy satisfies (6), the initial value condition* $\tau(A) = A$ *and* $\tau(0) = 0$. *Moreover,* $\tau(\alpha) \geq \frac{1+\sqrt{4p-3}}{2}\alpha$.

Proof: To prove that for $p \geq \overline{p} = 0.75$ there exist a unique pure separating equilibrium notice that, as shown in the previous Lemma, $\tau'' > 0$, which implies $\tau(\alpha) \geq \frac{1+\sqrt{4p-3}}{2}\alpha$. Therefore the definition of α^s as the value of α such that $\lim_{\alpha \searrow \alpha^s} \tau = 0$, must be satisfied at $\alpha^s = 0$. Therefore, the monotonic and convex function for the pure separating strategy satisfying (6) and the initial value condition $\tau(A) = A$ is defined for any $\alpha \in [0, A]\square$

The lemma states that, for the case in which the fraction of informed agents is large enough, there exist a strictly monotone and convex strategy such that the weakest and the toughest banker do not distort with respect to the complete information strategy whereas all other types in the support downward distort inflation. The banker with $\alpha = 0$ has a dominant strategy $\phi(0) = 0$ chosen independently of the parameters of the economy and $\frac{1+\sqrt{4p-3}}{2}\alpha \leq \tau(\alpha) \leq \alpha$. All types in the interior of the support $0 < \alpha < A$ will distort downward the inflation equilibrium choice and the distortion is bounded from below by $\frac{1+\sqrt{4p-3}}{2}\alpha$. The intuition is straightforward: the larger the number of informed agents in the economy the lower the cost in terms of reputation loss (increased expected inflation in the second period) given a larger surprise inflation in the first period. Therefore the lower will be the incentives for the banker to maintain reputation by distorting inflation. As a consequence, if the fraction of informed agents in the economy is large enough, the distortion due to incentive compatibility constraints will never violate single crossing properties in the support $[0, A]$ and a pure separating strategy exists over the whole support. As expected, the larger the fraction of informed agents the closer the separating strategy will be to the complete information outcome of the monetary policy game, $\pi_1 = \alpha$. This result may be compared with results in Sibert (2002) and D'Amato and Pistoresi (1996) where the support for the separating strategy was arbitrarily restricted to eliminate types such that single crossing is violated.

The separating strategy is reported in figure 1.

The equilibrium strategy is slightly harder to characterize for the case of $p < \overline{p}$. In this case it is possible to show that a semi separating strategy exists. The following lemma provides a characterization:

Proposition 2 *A hybrid equilibrium strategy $\pi_1 = h(\alpha)$ in the signalling game exists for $p < \overline{p} = 0.75$. Moreover,*

a. *There exist $\alpha^s > 0$ such that for $\alpha^s < \alpha \leq A$, the separating strategy satisfies (6) , i.e. $h(\alpha) = \tau(\alpha)$, and the initial value condition $h(A) = A$.*

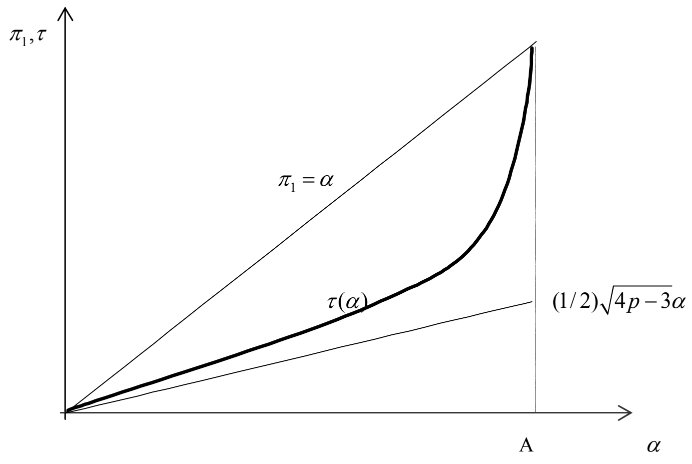

Figure 1: Pure separating equilibrium, $p \geq \overline{p}$.

b. *There exist out of equilibrium beliefs* $E[\pi_2 \mid_{\pi_1>0}] = E[\pi_2 \mid_{\{\pi_1,\alpha\}>0}] = h^{-1}(\pi_1)$ *such that, for* $0 < \alpha \leq \alpha^s \leq A/2$, $h(\alpha) = 0$.

Proof: To prove that for $p < \overline{p} = 0.75$ $h(\alpha) = \tau(\alpha)$ consider that, for $\alpha^s < \alpha \leq A$, $h(\alpha)$ solves the same problem as in proposition 1 and therefore has to satisfy the same initial value problem. Notice that for $p < \overline{p}$, $\tau(\alpha)$ is bounded from below by single crossing and therefore it must be $\tau > 0$. By computing, for $\tau(\alpha^s) = 0$, we get $\lim_{\alpha \searrow \alpha^s} \tau' = 1 - p$. By computing, for $\tau > 0$, $\lim_{\alpha \searrow 0} \tau' = 0$ and since $\lim_{\alpha \to A^-} \tau' = +\infty$, by continuity of τ and τ' it must be that $\tau' = 1 - p$ at $0 < \alpha^s < A$. This establishes that incentive compatibility is satisfied and $h(\alpha) = \tau(\alpha)$ for $\alpha^s < \alpha \leq A$. The proof that a pooling equilibrium exists for $0 < \alpha \leq \alpha^s$ at $h(\alpha) = 0$ under $E[\pi_2 \mid_{\pi_1>0}] = E[\pi_2 \mid_{\{\pi_1,\alpha\}>0}] = h^{-1}(\pi_1)$ and a discussion of these beliefs is provided in the appendix.□

The lemma characterizes the monotone hybrid equilibrium strategy represented in figure 2[7].

[7]It is easy to show that $\psi(\alpha)$ is unique in the set of monothone strategies satisfying incentive compatibility in a subset of $[0, A]$, the initial value condition $\psi(A) = A$ and $\psi(0) = 0$. However, since the game admits other pooling equilibria satisfying $\psi(A) = A$ and $\psi(0) = 0$ and no incentive compatibility and since the aim is to characterize the

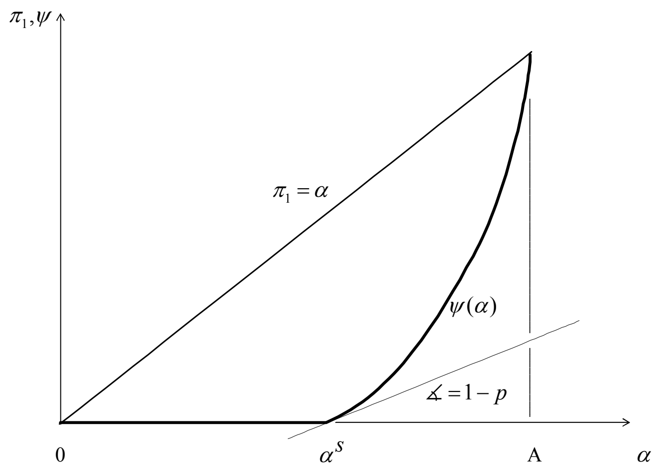

Figure 2: Semi-separating equilibrium, $p < \overline{p}$.

The economic intuition for the reason why different classes of equilibria, where incentive compatibility is strictly binding for a non negligeable subset of types, emerge depending on p is also clear. For a large enough fraction of uninformed people the trade off between the marginal benefit of increasing (unexpected) inflation in the first period and a larger marginal cost in terms of increased expected inflation in the second period is large. In order not to ruin their reputation and not to be confused with bankers tempted by inflation surprises, the level of signalling distortion that central bankers are willing to afford are larger, the lower the fraction of informed agents p. At p low enough, a hybrid equilibrium emerges since the distortion induced by incentive compatibility becomes so large that types in the interior of the support will not be able to separate themselves and will pool to zero inflation. Given that incentive compatibility forces $\alpha > \alpha^s$ to strong distortions in the signalling strategy to warrant future credibility, there exist types $\alpha \leq \alpha^s$ who limit themselves to acquire the common level of future credibility implied by a common action, $h = 0$, in the first period. This result extends similar results obtained in Sibert (2002) and D'Amato and Pistoresi (1996) for a pure separating equilibrium, where, in both cases, an arbitrary restriction of

optimal commitment strategy given incentive compatibility binds for a subset of types, for the sake of simplicity we do not focus on uniqueness here.

the type space is used to derive τ.

Having characterized the separating and the hybrid equilibrium strategy such that incentive compatibility constraints given by (6) are binding for a subset of types in the conjectured support we have to study, now, the conditions under which this conjecture is confirmed.

Before doing that however, consider that, as it is well known, a third set of possible equilibria of the game involves complete pooling by different types in the support. We are not interested in pooling per se but rather on the implications of pooling equilibria in the monetary policy game for the degree of commitment. We concentrate on a specific pooling equilibrium given by $\overline{s} = [0, \alpha]$, and $\overline{e} = [\pi_1^e, \pi_2^e]$, where, $\pi_1^e = (1-p)E[\pi_1]$ and $\pi_2^e = p\alpha + (1-p)E[\pi_1]$, that resembles (in pure strategies) the Backus and Driffil (1985) outcome in our simple economy. It is well known that monetary policy games with private information may lead to reputational equilibria in finite horizon too, when the set of possible policymakers types includes a type who will find full commitment always desirable. In our simple model we have seen that the government with type $\alpha_g = 0$ has a dominant strategy $\alpha = 0$. The presence of such a type may induce other types in the support of α to behave in the first period. Without loss of generality (same results in the equilibrium level of commitment would obtain in our simple model for the case of other pooling equilibria such that inflation rate as function of types is constant in the first period) consider the case of a pooling strategy $\pi_1^p = 0$. It is easy to show that the equilibrium can be supported by harsh punishment by the private sector as a whole (both informed and not informed) by setting second period expectations after deviation large enough. As also argued in Sibert (2002) it could be possible to rule out such an equilibrium using results in Ramey (1996); however, we are interested in the Backus and Driffil outcome of the monetary policy game to the extent it will provide a benchmark for assessing the effects of incentive compatibility on the equilibrium degree of commitment.

Finally, before studying the monetary policy equilibrium we need to establish conditions under which $S(\alpha)$ is common knowledge given $S_g(\alpha_g)$ is.

Proposition 3 *Given $\alpha_g \in [0, A_g]$ and $0 \leq p \leq 1$, for any of the two possible equilibrium strategies τ and h for the monetary policy game such that a) incentive constraints (6) are binding for a non negligeable subset of types $\sigma(\alpha)$ and b) initial value $\tau(A) = A$ is satisfied , the commitment function $\alpha = F$ (α_g) must be such that $F' \geq 0$ with F $(0) = 0$ and $F(A_g) = A_g(1-p)/2$.*

Therefore $0 \leq \Gamma(\alpha_g) \leq A_g(1-p)/2$. *If it is common knowledge that* $\alpha \in [0, A_g]$, *then* $\alpha \in [0, A]$ *is common knowledge among private agents, for any* $\Gamma(\alpha_g)$. *Proof: see appendix.*

The intuition is clear, given the strategy by any possible CB is non decreasing in its type in any of the equilibria for the monetary policy game characterized above, the trade offs faced by the government in appointing a CB are non decreasing in its type α_g. I.e. the larger the weight the government sets on unexpected inflation, the larger must be the marginal benefit in appointing a banker relatively more willing to surprise uninformed agents. This completes the characterization of the equilibrium strategies in the monetary policy game in the support $S(\alpha) \equiv [0, A]$.

3.1 Equilibrium Commitment with incentive constraints on central bankers strategies

Having characterized the set of possible equilibria such that incentive compatibility is binding for a subset of types $\sigma(\alpha)$ in the arbitrary support $\alpha \in [0, A]$ we move now to studying the equilibrium level of commitment by an elected government who appoints a CB in charge for two periods whose strategies are constrained by incentive compatibility conditions in the first period of his office. This problem is interesting in that incentive compatibility may induce separating costs to the monetary authority that have to be taken into account by the government. It may seem intuitive that the equilibrium degree of commitment would be lower than that would obtain in the absence of incentive compatibility constraints. To assess if and by how much incentive compatibility constraints affect commitment we need a benchmark. In this simple model, due to the absence of shocks, it is easy to see that, in the absence of incentive compatibility constraints on the central banker's strategies, the optimal level of commitment is only limited by its observability. With perfect observability, $p = 1$, it is the optimal level of commitment is infinite, and full commitment obtains at $\alpha^{FC} = 0$. Under imperfect observability, on the other hand, the optimal level of commitment is lower (Fershtmann and Kalai, 1996).

In order to assess the effects of incentive compatibility on the equilibrium degree of commitment let us consider the following benchmark when a delayed accurate signal is acquired about the central banker's type by the fraction of uninformed agents $1 - p$ at the beginning of period 2. In this case it will be common knowledge that the cb will misregard any effect of the first period choice on the second period expected inflation, choosing $\pi_1 = \alpha$ as a dominant strategy. By anticipating this equilibrium outcome, the government, in choosing the CB, will trade off first period surprise inflation with second period inflationary equilibrium. In this case the government, upon election, will appoint a banker $\alpha = \digamma^{ds}(\cdot)$ maximizing $\widetilde{W}_g(\alpha) = -\frac{1}{2}\alpha^2 + \alpha_g[(1-p)(\alpha - E\pi_1)] - \frac{1}{2}\alpha^2$ which yields $\digamma^{ds} = \alpha_g(1-p)/2$, where the superscripts ds refers to the case where an accurate signal is received by uninformed agents with a one period delay. This value will be our benchmark to evaluate the optimal level of commitment under incentive compatibility constraints on the cb's strategy. Of course the benchmark $\digamma^{ds}$ is not normative and simply measures the effect of incentive constraints on the first period inflation rate when the second period is played under full revelation of the bankers type under a pure separating equilibrium.

Another interesting benchmark we will use is given by the equilibrium degree of commitment obtained under the Backus and Driffil (1985) type of monetary policy, with $\pi_1^{BD} = 0$ $\pi_1^{BD} = \alpha$. In the case of pooling it is easy to show that the equilibrium degree of commitment by a rational government $\alpha = \digamma^{BD}(\alpha_g)$ will be given by $\digamma^{BD}(.) = \alpha_g(1-p)$. As simple as it is, this result is not void of interest: in a simple two period model the reputation outcome (Backus and Driffill, 1985) does not rule out incentive to commit monetary policy by a government whose type is not accurately observed.

Following the characterization in the previous paragraph we have to distinguish between two cases according to the size of the fraction of informed agents in the economy. In both cases we derive the endogenous support of possible central bankers types given the exogenous support of government preferences in the economy and the distribution function of government types. To this aim, notice that the characterization of equilibrium strategies in a signalling game such as the one studied here is distribution free this will allow us to derive the optimal degree of commitment for any possible distribution function of the prior beliefs held by uninformed agents in the economy.

To derive the equilibrium degree of commitment $\digamma(\cdot)$ under private information the government solves the following problem given the continuation monetary policy game:

$$\underset{\alpha}{Max}\ W_g = \quad -\tfrac{1}{2}\pi_1^2 + \alpha_g(\pi_1 - \pi_1^e) - \tfrac{1}{2}\pi_2^2 + \alpha_g(\pi_2 - \pi_2^e)$$

$$\begin{array}{ll} s.t. & \pi_1^e = pE[\pi_1 \mid \alpha] + (1-p)E[\pi_1] \\ & \pi_2^e = pE[\pi_2 \mid \alpha] + (1-p)E[\pi_2 \mid \pi_1] \\ & \pi_2 = \alpha \\ & \pi_1 = \begin{cases} \phi(\alpha) & \text{for } p \geq 0.75 \\ h(\alpha) & \text{for } p < 0.75 \end{cases} \end{array}$$

Notice that the equilibrium degree of commitment takes into account the direct effect that the choice of the banker will have on the expectations set by informed agents, as in Rogoff (1985) and, indirectly, the incentive constraints associated to the learning effect due to equilibrium updating by the uninformed agents where relevant.

We move now to characterize the government's equilibrium strategy. We first derive the equilibrium level of commitment when the size of the fraction is large enough ($p \geq 0.75$) and incentive compatibility is binding for any possible banker in the support. In this case the equilibrium level of commitment is denoted $\digamma^s$, where the superscripts s refers to separating equilibrium obtained in the continuation monetary policy game.

We then study the equilibrium level of commitment (denoted $\digamma^h$) in the complementary case of $p < 0.75$ where the hybrid equilibrium obtains. As we will see there are interesting differences in the two cases due to the presence of pooling regions reflected in different atoms being part of the support of the equilibrium distribution of central bankers under commitment.

3.2 Equilibrium level of commitment with high observability

If the size of informed agents in the society is large enough $p \geq 0.75$ the equilibrium level of commitment, given equilibrium strategy in the continuation game, can be derived by solving the following problem

$$\underset{\alpha}{Max}\ W_g = \quad -\tfrac{1}{2}\tau(\alpha)^2 + \alpha_g(1-p)[\tau(\alpha) - E[\pi_1]] - \tfrac{1}{2}\alpha^2 + \alpha_g(1-p)(\alpha - \widehat{\alpha})$$

$$\begin{array}{ll} s.t. & \\ & \tau' = \frac{(1-p)\alpha}{\alpha - \tau} \qquad \text{for } \alpha \in [0, A] \\ & \tau(0) = 0,\ \tau(A) = A,\ \frac{1+\sqrt{4p-3}}{2}\alpha \leq \tau(\alpha) \leq A \end{array}$$

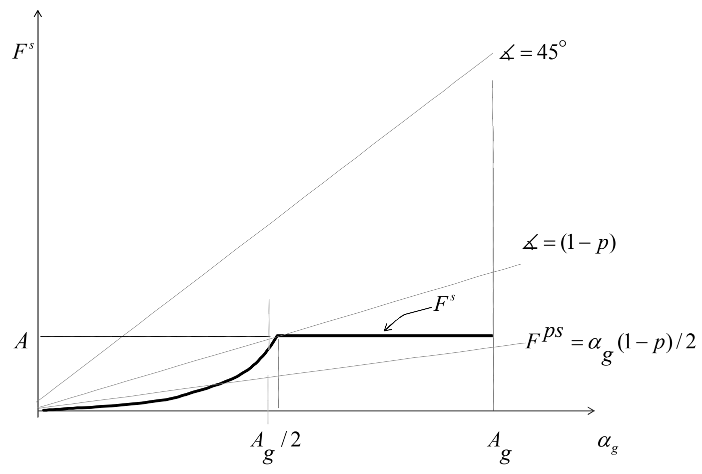

Figure 3: Equilibrium level of Commitment, $p \geq \overline{p}$.

Where the incentive compatibility constraints hold for any possible banker in the support and the pure separating strategy is characterized by Proposition 1.

We prove that F^s has to satisfy

$$-\tau\tau' + \alpha_g(1-p)\tau' - \alpha = 0 \tag{7}$$

That implictly defines $\alpha = F^s(\alpha_g)$.

Proposition 4 *For $p \geq 0.75$, there exists $\tilde{\alpha}_g = A_g/2$ such that the equilibrium degree of commitment is as follows:*

$$\begin{array}{ll} \textit{for } \alpha_g < \tilde{\alpha}_g & F^s(\alpha_g) \textit{ satisfies (7)} \\ \textit{for } \alpha_g \geq \tilde{\alpha}_g & F^s(\alpha_g) = (1-p)A_g/2 \end{array}$$

Proof: see Appendix.

The equilibrium level of commitment for $p \leq 0.75$ is plotted in figure 3.

In words, the equilibrium level of commitment is a non decreasing function of the type of government elected. It exhibits bunching at $(1-p)A_g/2$, equivalently the equilibrium distribution of possible central bankers will have to exhibit an atom in A. The intuition for the result is due to the specific behavior of the first period inflation rate as restricted by incentive compatibility. As it is well known (Mailath, 1987) the incentive compatible separating strategy is monotonic and has an unbounded first derivative at the initial value. Anticipating this effect, the temptation for the delegating government, by marginally relaxing on the degree of commitment, is increasing fast in the type of CB. There will exist a type of government in the interior of the support which may have incentive to delegate monetary policy to a CB which is less committed than the CB appointed by the worst possible government in the support. This would violate monotonicity of $\digamma^s(\alpha_g)$. Intuitively, due the inflation being fast increasing in α, for α_g close to A_g there is a strong incentive to appoint a less committed banker than $A = (1-p)A_g/2$ provided that it is interior in the support $[0, A]$[8]. But this is a self defeating strategy since by setting $\alpha > (1-p)A_g/2$ the appointed banker would define the worst possible type in the support $[0, A]$. Therefore there is no profitable deviation such that $\alpha > (1-p)A_g/2$. In other words, incentive compatibility implies monotonicity of the CB's strategy, which, in turn, implies that the equilibrium commitment is non decreasing, which requires bunching.

3.3 Equilibrium level of commitment with a low level of observability

If the size of informed agents in the society is small enough $p < 0.75$ the equilibrium level of commitment is slightly less straightforward to be characterized. After substituting for the equilibrium strategies in the continuation monetary policy game, the equilibrium commitment will solve following problem:

$$\underset{\alpha}{Max} W_g = -\tfrac{1}{2}h(\alpha)^2 + \alpha_g(1-p)[h(\alpha) - E[\pi_1]] - \tfrac{1}{2}\alpha^2 + \alpha_g(1-p)(\alpha - \widehat{\alpha})$$
$$s.t.$$
$$h' = \tfrac{(1-p)\alpha}{\alpha - h} \text{ and } h(A) = A \qquad \text{for } \alpha \in (\alpha^s, A]$$
$$h(\alpha) = 0, \qquad \text{for } \alpha \in [0, \alpha^s]$$

The hybrid strategy is characterized by Proposition 2 and the incentive compatibility constraints hold for $\alpha \in \sigma(\alpha) \equiv (\alpha^s, A]$. For $p < 0.75$, $\alpha = \digamma^h(\alpha_g)$ has to satisfy:

[8] A similar effect in Sibert (2002) implies that stabilization is larger under incentive compatible inflation rates.

$$-hh' + \alpha_g(1-p)h' - \alpha = 0 \text{ for } \alpha \in (\alpha^s, A] \quad (8)$$
$$\alpha_g(1-p) - \alpha = 0 \text{ for } \alpha \in [0, \alpha^s] \quad (9)$$

. It is easy to show, by similar arguments, that bunching occurs on the right of the support of $\Gamma(\alpha_g)$, exactly as in the case of $F^h(\cdot)$. However a bunching equilibrium also arises for a non trivial subset of government's types for $\alpha_g < A_g/2$. The next proposition characterizes $\alpha = F^h(\alpha_g)$.

Proposition 5 *If $p < 0.75$, there exist $\alpha_g^p > 0$ and $\alpha_g^s > 0$, with $\alpha_g^p = (1-p)\alpha_g^s$ such that the equilibrium degree of commitment is as follows:*

for $\alpha_g \in [0, \alpha_g^p]$	$\alpha = (1-p)\alpha_g$
for $\alpha_g \in (\alpha_g^p, \alpha_g^s]$	$\alpha = (1-p)^2\alpha_g^s$
for $\alpha_g \in (\alpha_g^s, A_g/2)$	$F^h(\cdot)$ satisfies (7)
for $\alpha_g \geq A_g/2$	$\alpha = (1-p)A_g/2$

Proof: see Appendix.

In words the proposition states that, if the size of the informed fraction of agents in society is not large enough, the equilibrium level of commitment will be such that the distribution of equilibrium bankers will exhibit two atoms. The intuition for the bunching region defined for $\alpha_g \geq A_g/2$ at $(1-p)A_g/2$ is the same as in the case of $p > \overline{p}$. The intuition for the second bunching region $(\alpha_g^p, \alpha_g^s]$ is also due to similar reasons: by appointing $\alpha = (1-p)\alpha_g > \alpha = (1-p)^2\alpha_g^s$ the government would appoint a banker playing positive inflation $\tau(\alpha)$ in the first period who will be recognized by private agents as such, the welfare costs associated to the inflationary equilibrium at $\alpha = (1-p)\alpha_g$ are large enough to warrant governments in the region $(\alpha_g^p, \alpha_g^s]$ to bunch at $\alpha = (1-p)^2\alpha_g^s$.

To summarize: we have derived the equilibrium level of commitment under imperfect observability of cb objectives, when the equilibria of the monetary policy subgame is restricted to satisfy incentive compatibility for at least a subset of cb's type in office for two periods. We have shown that when informational constrained strategies by cbs are taken into account by rational government the incentives to precommit monetary policy are not reduced by private information aspects of monetary policy games. In order to achieve commitment there must exist a certain degree of observability.

For any given level of observability p the effects on the optimal degree of commitment due to incentive compatibility constraints is provided in the following proposition:

Proposition 6 *The relationship between* $F^j(.)$, $j = s, h$, $F^{ds}(.)$ *and is as follows:*

1. for $p < 0.5$, $F^s(.) \geq F^{ds}(.)$ *for* $0 < \alpha_g < A_g$*, with equality holding at* $\alpha_g = 0$*, and* $\alpha_g = A$*;*

2. for $0.5 \leq p < 0.75$*, there exist* α_g', α_g'' *with* $\alpha_g^s \in [\alpha_g', \alpha_g'']$, $\alpha_g^p < \alpha_g'$ *and* $\alpha_g'' < A_g/2$

$$such\ that \begin{cases} F^s(.) \leq F^{ds}(.) \text{ for } \alpha_g \in [\alpha_g', \alpha_g''] \\ F^s(.) > F^{ds}(.) \text{ for } \alpha_g \notin [\alpha_g', \alpha_g''] \end{cases}$$

3. for $p \geq 0.75$*, there exist* $\alpha_g''' < \widetilde{\alpha}_g$

$$such\ that \begin{cases} F^s(.) \leq F^{ds}(.) \text{ for } \alpha_g \leq \alpha_g''' \\ F^s(.) > F^{ds}(.) \text{ for } \alpha_g > \alpha_g''' \end{cases}$$

Proof: see Appendix.

Compared to the case of full commitment the equilibrium degree of commitment is decreasing in the government type. However compared to the case of costless information transmission through a delayed signal acquiring to private agents in the second period the presence of incentive compatibility constraints does not necessarily decrease the equilibrium level of commitment. The distortion induced by incentive compatibility is in fact non monotonic in the government's type.

Finally we would like to compare the equilibrium level of commitment in the case of incentive compatibility with the case of commitment under pooling.

By comparing the equilibrium degree of commitment in the cases when incentive compatibility constraints are binding for a non trivial subset of types and the equilibrium level of commitment that would obtain under a Backus and Driffil type of equilibrium we obtain the following

Proposition 7 *For* $\alpha_g \in [0, A_g]$ *and* $j = s, h$, $F^j(.) \leq F^{BD}(.)$*. Moreover:*

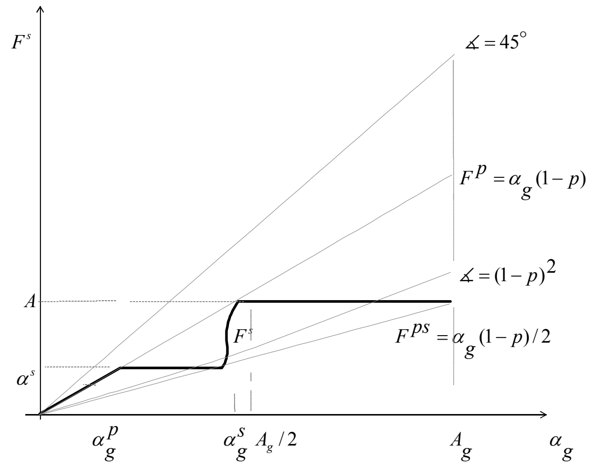

Figure 4: Equilibrium level of commitment, $p < \overline{p}$. (drawn for $p < 0.5$)

1. for $p < 0.75$, $F^s(.) = F^{BD}(.)$ at $\alpha_g = A_g/2$ and there exist α_g^p such that, for $\alpha_g \leq \alpha_g^p$, $F^s(.) = F^{BD}(.)$.

2. for $p \geq 0.75$, $F^{ps}(.) < F^{BD}(.)$ except than for $\alpha_g = A_g/2$.

Proof: the results is immediately derived from the characterization of $F^j(.)$ □

The results states that the equilibrium level of commitment in the case when credibility constraints are binding on monetary policy and some separation of types occurs is never lower than in the case of a pooling equilibrium. The result is somewhat counterintuitive in that incentive compatibility constraints induce a separation cost and one may expect these costs to allow the government to relax on commitment compared to pooling. However it turns out that the interaction between separation costs and the benefit of commitment is not that simple. Compared to a pooling equilibrium most of the separation costs evaluated using banker preferences are larger for α close to α_s. From the point of view of the government this occurs around the middle of the support (for a symmetric distribution of α_g, close to the median voter outcome). This type of government, as we have seen would actually like to relax on the equilibrium level of commitment but is disciplined by the minimum level of commitment allowed by the presence of government weaker than itself. It turns out that, for α_g close to $A_g/2$, the fear to appoint a banker ending at the upper bound of the equilibrium support of all possible bankers only allows a relaxation in the level of commitment which is exactly equal to the level of commitment that would obtain under a Bakus-Driffil type of equilibrium in monetary policy. Under this respect, the result according to the equilibrium degree of commitment is generally stronger if separation occurs than in the case when a Backus and Driffil outcome obtains may be intuitively interpreted, therefore, as a result confirming that, the stronger the anticipated reputational concerns of the monetary autority the lower the equilibrium level of commitment. In this simple model too reputation is a substitute for commitment. Not a perfect one though due to the finite horizon.

4 Conclusions and final comments

In this paper we analyzed the equilibrium level of commitment when information in the economy about government and central bankers preferences about the trade-offs of monetary policy is not perfect.

The paper provides two new results on monetary policy games with uncertain central banker preferences in a simple economy with inflationary bias. We extend some of the results in D'Amato and Pistoresi (1996) and Sibert (2002) and provide characterization for the incentive compatible strategies with no restriction on the support of preferences by a central banker serving for two periods whose type is not perfectly observed by private agents. We studied incentive compatible strategies characterizing the separating and semiseparating equilibrium. A second set of results characterizes the equilibrium level of commitment under imperfect observability and incentive compatibility constraints on the cb's strategies. We show that incentives to commitment of monetary policy by a rational government are not eliminated in all of the possible cases when equilibrium monetary policy satisfies incentive compatibility constraints. Governments strongly averse (prone) to concede to temptation increase (decrease) the equilibrium level of commitment compared to the benchmark case when costless information transmission to uninformed agents occurs in later periods.

Interestingly the equilibrium level of commitment of monetary policy is shown to be larger when incentive compatibility constraints are strictly binding for a non trivial subset of bankers in the support than in the benchmark case of a reputational equilibrium of the Backus and Driffil type with pooling occurring at early stages of incumbency.

We also show that different government may have incentive to appoint identical bankers: there exist atoms in the equilibrium distribution of possible central bankers' preferences. The closer governments are to the middle of the initial support of political preferences in the society the larger the incentive to appoint similar bankers irrespectively of the equilibrium monetary policy being a pooling, a hybrid or a separating equilibrium. Bunching in the commitment strategy and similarity in the equilibrium level of commitment across different possible monetary policy equilibrium strategies suggests that private information may play a role in explaining why small perturbations in the electoral outcome may *not* lead to drastic changes in institutional objectives under imperfectly observed delegation contracts upon election. This repeated game with short lived governents is not analyzed in this paper: the study of how incentive compatibility constraints may influence a newly appointed government in the choice between a newly appointed agent with suitable preferences but with an uncertain reputation and an established agent with inherited pre-specified preferences and a well reknown reputation is left for future work.

References

Alesina A. and V. Grilli (1992), "The European Central Bank: Reshaping Monetary Politics in Europe", in Persson T. and G. Tabellini (1995), pp. 247-277.

Alesina A. and Gatti, R. (1995) "Independent Central Banks: Low Inflation at no Costs?", *American Economic Review Papers and Proceedings,* **85**, pp.196-200.

Athey S., A. Atkeson and P Kehoe (2003), The Optimal Degree of Discretion in Monetary Policy, Nber working paper No 10109.

Backus D. and J. Driffil (1985), "Inflation and Reputation", *American Economic Review*, **75**, pp. 530-538.

Bagwell K. (1995), "Commitment and Observability in Games", *Games and Economic Behavior*, **8**, pp. 271-280.

Barro R. (1986), "Reputation in a Model of Monetary Policy with Incomplete Information", *Journal of Monetary Economics*, **17**, pp. 3-20.

Barro R. and D. Gordon (1983), "A Positive Theory of Monetary Policy in a Natural Rate Model", *Journal of Political Economy,* **91**, pp. 589-610.

Canzonery M. (1985), "Monetary Policy Games and the Role of Incomplete Information", *American Economic Review*, **75**, pp. 1056-1070.

Cukierman A. (1993), *Central Bank Strategy, Credibility, and Independence: Theory and Evidence*, Cambridge, Mass., The MIT Press.

D' Amato M. and B. Pistoresi (1996), "La Riservatezza del Banchiere Centrale è un Bene o un Male?", *Politica Economica,*

Driffill J. (1987b) "Macroeconomic Policy Games with Incomplete Information - A Survey", *Warwick Economic Research Papers*, **288**.

Fershtman C. and E. Kalai (1997), Unobserved delegation, *International Economic Review*, **38**, pp. 763-774.

Kydland F. and E. Prescott (1977), "Rules Rather than Discretion: the Inconsistency of Optimal Plans", *Journal of Political Economy*, **85**, N.3, pp. 473-491.

Milgrom P. and J. Roberts, (1982), Limit Pricing and Entry under incomplete information: an equilibrium analysis, *Econometrica*, **50**, pp. 443-459.

Mailath G. (1987), "Incentive Compatibility in Signaling Games with a Continuum of Types", *Econometrica*, **55**, N. 6, pp. 1349-1365.

_ (1992), "*Signaling Games*", in Creedy et al. (ed.), *Recent Developments in Game Theory*, Edward Elgar Publishing Limited, pp. 65-93.

McCallum B.T. (1995), "Two Fallacies Concerning Central Bank Independence", *American Economic Review Papers and Proceedings,* ***85***, May, pp. 207-211.

Persson T. and G. Tabellini (1990), *Macroeconomic Policy, Credibility and Politics*, London, Harwood Academic Publishers.

_ – _ (1993), *Designing Insitutions for Monetary Stability, Carnegie- Rochester Conference Series on Public Policy*, in Persson T. and G. Tabellini (1995).

_ – _ (2000), Political Economics, Mit Press.

Riley J. (1979), Informational Equilibria, *Econometrica*, **47**, pp. 331-359.

Rogoff K. (1985), "The Optimal Degree of Commitment to an Intermediate Monetary Target", *Quarterly Journal of Economics*, **100**, N.4, pp. 1169-1189.

_ (1987), "Reputational Constraints on Monetary Policy" *Carnegie-Rochester Conference Series on Public Policy*, **26**, pp. 141-182.

Sibert A. (2002), "Monetary Policy with uncertain central bank preferences", *European Economic Review*, **46**, pp. 1093-1109.

Stiglitz J. (2003), "The Roaring Nineties", Penguin, 2003.

Sundaram R. K. (1996), "A first course in Optimization Theory", Cambridge University Press.

Vickers J. (1985), "Signalling in a Model of Monetary Policy with Incomplete Information", *Oxford Economic Papers*, **38**, pp. 443-455.

Walsh C. (1995), "Optimal Contracts for Central Bankers", *American Economic Review*, **85**, n.1, pp. 150-167.

Appendix

Proposition 2.Existence of a pooling region in the semiseparating equilibrium such that $h(\alpha) = 0$ for $0 \leq \alpha \leq \alpha^s$.

To prove that a pooling equilibrium exists we need to prove the following lemma.

Lemma 3 *In a semiseparating equilibrium the pooling region is restricted in* $\alpha^s \leq A/2$

The exact value of α^s depends on the initial value condition and the parameter p. Instead of integrating (by separating the variables) and forcing the initial value on (6) we show that an upper bound for α^s exists and is given by $A/2$ using a simple argument holding under equilibrium properties of $h(\alpha)$ given (6) is binding. Given monotonicity and convexity of $h(\alpha)$ for $\alpha^s < \alpha \leq A$ and for the initial value $h(A) = A$, it must be

$$(1-p)(\alpha - \alpha^s) \leq h(\alpha) \leq \frac{A}{A-\alpha^s}(\alpha - \alpha^s)$$

and therefore

$$\int_0^A t dt - \int_{\alpha^s}^A [t - \frac{A}{A-\alpha^s}(t-\alpha^s)]dt \geq \int_0^A h(t)dt \geq \int_{\alpha^s}^A (1-p)(t-\alpha^s)dt$$

which holds for $\alpha^s \in [-\frac{p}{1-p}\frac{A}{2}, \frac{A}{2}]$. Therefore, given the definition of α^s at (6), it must be true that $\alpha^s \leq A/2$.□

We are now able to prove that there exist beliefs such that for $0 < \alpha \leq \alpha^s$, $h(\alpha) = 0$ is part of $h(\alpha)$ defined for $0 < \alpha \leq A$, in the semiseparating equilibrium in Proposition 2**.** Consider the following pooling equilibrium strategy $\pi_1^p = h(\alpha) = 0$, with expectations given by $\pi_1^{e,p} = p\pi_1^p + (1-p)E[h(\alpha)]$, where $E[h(\alpha)] = \int_0^A h(\alpha)dB(\alpha)$.

To prove that a pooling equilibrium exist for $0 < \alpha \leq \alpha^s$ and $h(\alpha) = 0$ consider the optimal deviation given the out of equilibrium beliefs, where after observing positive inflation the both the uninformed and the informed agents set $h^{-1}(\pi_1) = \widehat{\alpha}$.

This specification of out of equilibrium beliefs is compelling for the fraction of agents who do not observe α. Any deviation to positive inflation rate by types $0 < \alpha \leq \alpha^s$ is interpreted as an equilibrium strategy $h(\alpha) = \tau(\alpha) > 0$ played by $\alpha' > \alpha^s$.

The same specification of out of equilibrium beliefs by informed agents is less natural and deserves a comment: we are assuming here that after deviation by a type they know it should have pooled at $h(\alpha) = 0$ they misregard the information about α and choose to rely on the observed inflation rate to forecast future inflation. Therefore: $E[\pi_2 \mid_{\pi_1 > 0}] = E[\pi_2 \mid_{\{\pi_1, \alpha\} > 0}] = h^{-1}(\pi_1) = \widehat{\alpha}$. For such out of equilibrium beliefs the optimal deviation is:

$$\begin{aligned} Arg \max_{\pi_1^d} W^d &= -\frac{(\pi_1^d)^2}{2} + \alpha[\pi_1^d - p\pi_1^p + (1-p)E[h(\alpha)] \\ &\quad -\frac{\alpha^2}{2} + \alpha[(1-p)\left\{\alpha - E[\pi_2 \mid_{\{\pi_1 > 0\}}]\right\} \end{aligned}$$

the first order condition is given by $\pi_1^D = \alpha$, whereas the global second order condition is given by $\partial^2 W^d / (\partial \pi_1^d)^2 = \alpha p > 0$. Therefore if a deviation from $\pi_1^p = 0$ exists, it must be $\pi_1^d = A$.

Given second period equilikbrium strategy and beliefs after deviation, a deviation pays-off:

$$W^d = -\frac{A^2}{2} + \alpha[A - (1-p)E[\pi_1] - \frac{\alpha^2}{2} + \alpha(1-p)\left\{\alpha - E[\pi_2 \mid_{\{\pi_1 > 0\}}]\right\} \tag{10}$$

Given second period equilibrium strategy and beliefs, pooling at $\pi_1 = 0$ pays-off

$$W^p = -(1-p)E[\pi_1] - \frac{\alpha^2}{2} + \alpha[(1-p)\left\{\alpha - E[\pi_2 \mid_{\{\pi_1 = 0\}}]\right\} \tag{11}$$

whereas benefits from deviation, under the specified out of equilibrium conjecture are given by

$$W^d = -\frac{\alpha^2}{2} + \alpha(\alpha - (1-p)E[\pi_1] - \frac{\alpha^2}{2} + \alpha(1-p)(\alpha - A) \tag{12}$$

For $0 < \alpha \leq \alpha^s$, $h(\alpha) = 0$ is an equilibrium if no profitable deviation exists given the specified out of equilibrium conjecture, i.e. if $W^p \geq W^d$. This inequility, after some simple algebra, can be shown to hold for $0 < \alpha \leq \alpha^s$ if $(\alpha^s)^2 \leq A^2/2$ which is true at equilibrium, as shown by the previous lemma.□

Proof of Proposition 3

To show that $F(\alpha_g)$ is monotonic in α_g we have to show that $W^G(\alpha, \alpha_g)$ satisfies increasing differences and $W^G_{\alpha,\alpha_g} \geq 0$ (see Sundaram, p.257) for any α and for any π_1, π_2 and $E(\pi_2 \mid \pi_1)$ on the equilibrium path of the continuation monetary policy game . To see that this is indeed the case write

$$W^g(\alpha) = -\frac{1}{2}\pi_1^2 + \alpha_g[\pi_1 - pE(\pi_1 \mid \alpha) - (1-p)E(\pi_1)] + \\ -\frac{1}{2}\pi_2^2 + \alpha_g[\pi_2 - pE(\pi_2 \mid \alpha) - (1-p)E(\pi_2 \mid \pi_1)]$$

The sufficient condition for $F(\alpha_g)$ to be weakly monotone in α_g is

$$W^G_{\alpha,\alpha_g} = (1-p)[\frac{d\pi_1}{d\alpha} + \frac{d\pi_2}{d\alpha} - \frac{dE(\pi_2 \mid \pi_1)}{d\pi_1}\frac{d\pi_1}{d\alpha} \geq 0 \qquad (13)$$

for any π_1, π_2 and $E(\pi_2 \mid \pi_1)$ on the equilibrium path. In any equilibrium of the monetary policy game $\pi_2 = \alpha$.

In the pure separating equilibrium defined in proposition 1 $\pi_1 = \tau(\alpha)$ and $E(\pi_2 \mid \pi_1) = \widehat{\alpha} = \tau^{-1}(\alpha)$. Therefore (13) is satisfied since $\frac{dE(\pi_2|\pi_1)}{d\pi_1}\frac{d\pi_1}{d\alpha} = 1$ and $W^G_{\alpha,\alpha_g} = (1-p)\tau'(\alpha) > 0$ for $\tau' \geq \frac{1+\sqrt{4p-3}}{2}$ at equilibrium.

In the semiseparating equilibrium defined in proposition 2 $\pi_1 = h(\alpha)$,

for $0 \leq \alpha \leq \alpha^s$, $h(\alpha) = 0$ and $\frac{dE(\pi_2|\pi_1)}{d\pi_1} = 0$ since $E(\pi_2 \mid \pi_1) = E(\alpha \mid \alpha \leq \alpha^s)$ and therefore $W^G_{\alpha,\alpha_g} = 1 > 0$;

for $\alpha^s \leq \alpha \leq A$, $h(\alpha)$ satisfies (6) and $h(A) = A$, $E(\pi_2 \mid \pi_1) = \widehat{\alpha} = h^{-1}(\alpha)$. Therefore $\frac{dE(\pi_2|\pi_1)}{d\pi_1}\frac{d\pi_1}{d\alpha} = 1$ and $W^G_{\alpha,\alpha_g} = (1-p)h'(\alpha) > 0$.

We have established that, on the conjectured support $S(\alpha) \equiv [0, A]$, both in the separating equlibrium and in the semiseparating equilibrium, sufficient conditions for $F(\alpha_g)$ being non monotonic are satisfied. Therefore, at $\alpha_g = A_g$ it must be $A = F(A_g)$, $\pi_1 = A$, $\pi_2 = A$, $E(\pi_2 \mid \pi_1) = A$. By maximising $W^g(\alpha) = -A^2 + A_g(1-p)(A - E(\pi_1)$ with respect to A , we get $A = (1-p)A_g/2$. For $\alpha_g = 0$ the dominant strategy is $\alpha = 0$. Therefore if $\alpha_g \in [0, A_g]$ then $\alpha \in [0, A]$. Which confirms the conjecture. □

Proof of Proposition 4 ($p \geq 0.75$)

To prove proposition 4 notice that $F^s(0) = 0$ and $F^s(A_g) = A_g(1-p)/2$ and $F^{s'} \geq 0$ for $0 \leq \alpha_g \leq A_g$. Let us start by noticing that, for any $0 < \alpha_g < A_g$ and for any $\pi_1 = \tau$, $\pi_2 = \alpha$ and $E(\pi_2 \mid \pi_1) = \tau^{-1}$ holding on the equilibrium path of the continuation monetary policy game the pay-offs to the government can be written as

$$W^g(\alpha, \alpha_g) = -\frac{1}{2}\tau^2 + \alpha_g(1-p)[\tau - E(\pi_1)] - \frac{1}{2}\alpha^2 + \alpha_g(1-p)(\alpha - \widehat{\alpha})$$

$W^g(\alpha, \alpha_g)$ is increasing for:

$$-\tau\tau' + \alpha_g(1-p)\tau' - \alpha > 0$$

and decreasing otherwise, equivalently $W^g(\alpha, \alpha_g)$ is increasing for

$$\alpha < [\alpha_g(1-p) - \tau]\tau'$$

that, by using (6), can also be rewritten as

$$\frac{\alpha}{\alpha - \tau}[\tau p + \alpha_g(1-p)^2 - \alpha] > 0$$

Since, at equilibrium $\alpha > \tau$, for $F^s \leq A_g(1-p)/2$, F^s must satisfy

$$\alpha < \tau p + \alpha_g(1-p)^2$$

that is (7) evaluated at (6). Notice that $W^g(\alpha, \alpha_g)$ exhibits local non concavity at $\alpha = \alpha_g(1-p)^2/(1+p^2)$ and this explains why increasing differences were used proving in proposition 3. Notice also that for $\alpha_g \nearrow A_g$, (7) evaluated at (6) yields $A = A_g(1-p)$. To prove the proposition, therefore we have to establish that, since there exist a type of government $\widetilde{\alpha}_g$ such that for $\alpha_g \geq \widetilde{\alpha}_g$ commitment occurs at $A = A_g(1-p)/2$.

Define the candidate equilibrium pay-offs for $\widetilde{\alpha}_g$ from $F^s(\widetilde{\alpha}_g) = A$

$$W^g(A, \widetilde{\alpha}_g) = -A^2 + \alpha_g(1-p)[A - E(\pi_1)]$$

and the pay-offs to $\widetilde{\alpha}_g$ to appointing $\tilde{A} > A$.

$$W^g(\tilde{A}, \widetilde{\alpha}_g) = -\tilde{A}^2 + \alpha_g(1-p)[\tilde{A} - E(\pi_1)]$$

where $E(\pi_1)$ by uninformed are set given the candidate equilibrium. Simple algebra shows that $W^g(A, \widetilde{\alpha}_g) \geq W^g(\tilde{A}, \widetilde{\alpha}_g)$ for $\frac{A_g}{2} \leq \widetilde{\alpha}_g < A_g$. The definition of $\widetilde{\alpha}_g$ implies that for $0 \leq \alpha_g < \widetilde{\alpha}_g$, $\digamma^s\ (\alpha_g)$ satisfies (7) evaluated at (6).□

Proof of Proposition 5 ($p < 0.75$).

Prosposition 5 can be proved by using the same arguments used to prove proposition 4. As before notice that $\digamma^h(0) = 0$ and $\digamma^h(A_g) = A_g(1-p)/2$ and $\digamma^h \geq 0$ for $0 \leq \alpha_g \leq A_g$. Let us start by noticing that, for any $0 < \alpha_g < A_g$ and for $\alpha \leq \alpha^s$, $\pi_1 = 0$, $\pi_2 = \alpha$ and $E(\pi_2 \mid \pi_1 = 0) = E(\alpha \mid \alpha \leq \alpha^s)$, whereas for $\alpha > \alpha^s$, $\pi_1 = h$, $\pi_2 = \alpha$, and $E(\pi_2 \mid \pi_1 > 0) = h^{-1}$ holding on the equilibrium path of the continuation monetary policy game, the pay-offs to the government can be written as

$$\begin{aligned}\overline{W}^g(\alpha, \alpha_g) = &-\tfrac{1}{2}h^2 + \alpha_g(1-p)[h - E(\pi_1)] \\ &-\tfrac{1}{2}\alpha^2 + \alpha_g(1-p)(\alpha - \widehat{\alpha}) \qquad \text{for } \alpha > \alpha^s\end{aligned}$$

and

$$\begin{aligned}\underline{W}^g(\alpha, \alpha_g) = &-\alpha_g(1-p)E(\pi_1) \\ &-\tfrac{1}{2}\alpha^2 + \alpha_g(1-p)[\alpha - E(\alpha \mid \alpha \leq \alpha^s)] \quad \text{for } \alpha \leq \alpha^s\end{aligned}$$

Remember that for $p < 0.75$, it must be that $\alpha^s > 0$.

For $\alpha > \alpha^s$ similar results as in the previous proposition apply and therefore, since $\digamma^{h'} \geq 0$, there will exist $\tilde{\alpha}_g$ such that $\digamma^h = A_g(1-p)/2$. Same arguments as in the proof of proposition 4 show that $\tilde{\alpha}_g = A_g/2$. Due to the existence of a pooling region, however, the characterization of $\digamma^h(\cdot)$ for $\alpha_g < \tilde{\alpha}_g$ requires more careful analysis.

In particular we prove that bunching has to occurr for $\alpha_g \in [\alpha_g^p, \alpha_g^s]$, $\alpha_g^p = (1-p)\alpha_g^s$ and α_g^s defined as $\digamma^h(\alpha_g^s) = \alpha^s$. To prove the result we proceed in four steps by cheking a few inequalities at equilibrium.

Step 1. We prove that $\alpha_g \in [\alpha_g^s, \tilde{\alpha}_g)$ it must be that $\digamma^h$ satisfies (7) evaluated at (6) for $(1-p)^2\alpha_g \leq \digamma^h < A_g(1-p)/2$. Since $\lim\limits_{\alpha \searrow \alpha^s} \digamma^h(\cdot) =$

$\alpha_g(1-p)^2$ and $\digamma^h$ is weakly monotone with $\digamma^h(A) = A_g(1-p)/2$, there exist α_g^s such that $\alpha^s = \alpha_g^s(1-p)^2$.

Step 2 For $\alpha \leq \alpha^s$ it is immediate to show that the first order condition on $\underline{W}^g$ require $\digamma^h(\cdot) = \alpha_g(1-p)$. By weak monotonicity of $\digamma^h(\cdot)$, there exist α_g^p such that for $\alpha_g \leq \alpha_g^p$, $\digamma^{ss}(\cdot) = \alpha_g(1-p) \leq \alpha_g^s(1-p)^2$. Therefore α_g^p solves the following equation $\alpha_g(1-p) \leq \alpha_g^s(1-p)^2$ and hence $\alpha_g^p = \alpha_g^s(1-p)$.

Step 3 For $\alpha_g \in [\alpha_g^p, \alpha_g^s]$, $\underline{W}^g(\alpha_g^s(1-p)^2, \alpha_g) \geq \underline{W}^g(\alpha_g(1-p), \alpha_g)$

Since $\alpha_g^s(1-p)^2 = \alpha^s$ and taking into account equilibrium strategies, $h(\alpha^s) = 0$, after some simple algebra it is possible to write $\underline{W}^g(\alpha_g^s(1-p)^2, \alpha_g) = -\alpha_g(1-p)E(h) - (\frac{\alpha^s}{2})^2 + \alpha_g(1-p)(\alpha^s - E(\pi_2 \mid \pi_1 = 0)$ which,

since $E(\pi_2 \mid \pi_1 = 0) \leq \alpha^s$, is at least equal to $\underline{W}^{\underline{g}}(\alpha^s, \alpha_g) = -\alpha_g(1-p)E(h) - (\frac{\alpha^s}{2})^2$ whereas, given first period expectations $E(h)$ are evaluated at equilibrium candidate $\digamma^h(\cdot)$, $\underline{W}^g(\alpha_g(1-p), \alpha_g) = -(\frac{h}{2})^2 - \alpha_g(1-p)[h - E(h)] - (\frac{\alpha}{2})^2 + \alpha_g(1-p)(\alpha - E(\pi_2 \mid \pi_1 > 0)$.

Since, for $\alpha > \alpha_g^s(1-p)^2$, $E(\pi_2 \mid \pi_1 > 0) = h^{-1} = \alpha$, this latter expression can be written as $\underline{W}(\alpha_g(1-p), \alpha_g) = -(\frac{h}{2})^2 - \alpha_g(1-p)E(h) - (\frac{\alpha}{2})^2$.

Therefore $\underline{W}^{\underline{g}}(\alpha^s, \alpha_g) \geq \underline{W}^g(\alpha_g(1-p), \alpha_g)$ can be written as $\alpha - h \geq \alpha^s$. Evaluated at $\alpha^s = \alpha_g^s(1-p)^2$, $h(\alpha)$ at $\alpha = \alpha_g(1-p)$ is indeed satisfied for $\alpha_g \in [\alpha_g^p, \alpha_g^s]$. For $0 \leq \alpha_g < \alpha_g^p$ is not satisfied and therefore $\digamma^h(\cdot) = \alpha_g(1-p)$ in this interval.□