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PAY-TO-BID AUCTIONS

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Abstract

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1 Introduction

The relatively minor setup cost of internet websites has facilitated the creation of many varieties of auction formats. Most of these have close analogs to auctions that have existed for centuries prior, but occasionally a site develops a novel approach. Such is the case with Swoopo, a German company founded in 2005 which currently operates websites in the United Kingdom, Spain, the United States, and Austria. Swoopo's distinctive feature is that participants must pay a fee each time they place a bid.

Each auction begins at a price of zero and with a specified amount of time on a countdown clock. When a participant places a bid, the current price increments by a fixed amount (\$0.15 in the standard auction), the bidder is immediately charged a bid fee (\$0.75), and the auction

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is extended by a set amount of time (15 seconds).¹ If the time expires before another bid is placed, the last bidder pays the current price (on top of any bid fees incurred) and wins the object.

An observer's first experience with this auction format typically follows a predictable course. First, the newcomer notes that the current price on her particular item is remarkably low — indeed, the median auction closes with a final price that is 18.9% of the retail price. The clock is within seconds of expiring, yet our observer quickly realizes that the deal is elusive, as additional bids are placed every five to ten seconds. Indeed, her past bids (had she placed any) are sunk, having no bearing on her likelihood of winning once she is outbid.² Ideally, she would wait and place the last bid, but she immediately recognizes this to be the fundamental dilemma of the auction: though it is highly likely that new bids will be submitted right now, there is a small chance that none will and the auction will end.

On appreciating the low probability of winning, thoughts turn to how much revenue the auction generates: for every dollar increase in the final price, Swoopo collects five dollars in additional revenue through bid fees. To make a profit, the final price only needs to exceed one-sixth of the item's cost. This thought process has led many to question the rationality of auction participants as well as the ethics of the auctioneers. Blogs and news articles (Kato, 2009; Reklaitis, 2009; Gimein, 2009) have vented their frustrations, referring to Swoopo as gambling or an outright scam.

Even so, pay-to-bid auctions are growing in popularity. In December 2008, 11 websites conducted such auctions; by November 2009, the format had proliferated to 35 websites. Over the same time span, traffic among these sites (reported in Table 1) has increased from 1.2 million to 3.0 million unique visitors per month.³ For comparison, traffic at ebay.com fluctuated around 75 million unique visitors per month throughout that period. In other words, pay-to-bid auctions have garnered 4% of the traffic held by the undisputed leader in online auctions.

We provide a simple model of the pay-to-bid auction format that rationalizes the behavior of bidders. The key insight of this model is that the aggregate probability of being outbid is pinned down by the requirement that prior bidders had to be willing to bid ex-ante. As

¹In an English auction, this last feature is known as a *soft close*. Extending the clock ensures that participants always have an opportunity to respond after being outbid, often expressed in oral auctions as “Going once, going twice, sold!” Amazon.com used to offer English auctions with a soft close, while e-Bay uses a hard close where the ending time is fixed (See Roth and Ockenfels, 2002).

²81.3% of the unique usernames participate but never win an auction.

³Web traffic was monitored using the web analytics of compete.com. The pay-to-bid industry does not seem to be consolidating, either. Over this time period, almost all of the 35 sites experienced growth in traffic. Moreover, the concentration of traffic has fallen: in December 2008, the top two sites (including Swoopo) claimed 90% of the market. By November 2009, the top five sites (including Swoopo) jointly held 75% of the market. Bid fees have fallen, though; many (including Swoopo) are now charging \$0.60 per bid.

a consequence, we can derive a density function describing the probability that the auction ends at any given number of bids. From this, we show that the pay-to-bid auction raises an expected revenue that is nearly equal to the bidders' valuation of the item (and thus equivalent to the revenue that standard auction formats would achieve in this environment).

We test the predictions of our model using information on 34,000 auctions collected from Swoopo's website.⁴ We find that the average revenue of most items falls within one standard deviation of the expected revenue predicted by our model. Moreover, for most items, the observed distribution of ending prices fits remarkably well to the distribution predicted by the model; we fail to reject (at $p = 0.05$) that the distributions differ for 79% of the routinely auctioned items.

The one group of items which do not fit our basic model are those that involve video game systems (such as the Wii, Playstation, or Xbox), which produce more aggressive bidding and higher revenue for Swoopo. However, the model can mimic this behavior by changing bidders' utility from risk-neutral to risk-preferring. With very reasonable levels of absolute risk preference (less than half of that estimated for racetrack bettors), the theoretical expected revenue and ending price distribution line up with the observed data for 97% of the auctioned items.

We assume that the valuation of the item is known and the same across all potential bidders. Although first- or second-price auctions are seldom modeled in a full-information environment, this assumption is quite common in all-pay or war-of-attrition auctions,⁵ which are the closest relatives of the pay-to-bid auction. The common value allows us to isolate a key aspect of pay-to-bid auctions: a bidder is gambling that others who value the item *at more than the current price* may still abstain from bidding. We believe that this would still be the driving force (though less visibly) in a private valuation framework.

Furthermore, a common value is quite plausible for the types of items regularly auctioned on Swoopo. All items are new, unopened, and readily available from traditional or internet retailers. Unlike rare art or collectibles, the market prices of these items are well established. Indeed, one could interpret the valuation in our model as the lowest price for which the item may readily be obtained elsewhere (rather than as the individual's literal reservation price). With that interpretation, the payoff from placing a bid is always relative to the utility of buying the item outright.

Finally, it should be noted that our common value model matches the data very well. For instance, one might speculate that the auction of a wristwatch that sells at retail for

⁴Our empirical approach most closely resembles that of Lambson and Thurston (2006).

⁵Examples include Baye, *et al* (1996), Clark and Riis (1998), Barut and Kovenock (1998), and Konrad and Leininger (2007). Alcalde and Dahm (2009) analyze the first-price auction in a complete information environment.

\$300 is largely driven by people who only value the watch at \$50 and are hoping to win early in the auction. However, one must consider the probability of being outbid; since an identical watch is auctioned many times, we can extract that probability. The data speaks very clearly on this: whether early or late in the auction, if a person values the watch at less than \$300, his expected payoff from bidding will be less than the \$0.75 bid fee.

At first glance, one might consider the pay-to-bid auction as a mere reformulation of the all-pay auction. There, each participant pays what he bids, even though only the highest bidder wins the item. A second-price all-pay auction (or war of attrition) does the same except that the winner pays the second highest bid. These are typically modeled in a static, sealed-bid environment⁶, but their strategic properties differ significantly from the pay-to-bid auction even when set up in the same dynamic format.

The analogy arises because each time an all-pay participant raises his bid, he commits to pay that increase regardless of whether he wins, and is hence like a bid fee. However, in a pay-to-bid auction, the bid fee is distinct from the price increment, and the winning bidder must pay the final price on top of his bid fees. Moreover, in an all-pay auction, if *any* active bidder increased the bid, *every* active bidder would have to likewise increase their own bids to remain active (much like calling a bet in poker). A pay-to-bid participant only incurs a bid fee each time *he* increases the price.

Pay-to-bid auctions share more in common with work on bidding costs, defined as transaction costs that the bidder incurs but are not received as revenue for the seller. These might include legal or consultant expenses in preparing a bid or an increasing cost of financing due to capital market imperfections.⁷ In one key aspect, bid costs and bid fees are similar: a participant only places a bid if his expected payoff is sufficient to cover the certain cost. Since valuation is homogenous in our model, this imposes our indifference condition. In the bidding cost models, the cost discourages those with low valuations from participating. Indeed, when bidding costs are modeled in an ascending auction (*e.g.* Daniel and Hirshleifer, 1998), participants conserve on bid costs by making jumps in the bid rather than raising the current price by the minimum increment. In our setting, the bid increment is fixed.⁸

⁶See Amann and Leininger (1996), Baye, *et al* (1996), and Krishna and Morgan (1997). An exception is in Hörner and Sahuguet (2007), where two rounds of bidding are allowed.

⁷Che and Gale (1998) assume that only the winner of the auction incurred the bid cost, while Gaviols, Moldovanu, and Sela (2002) allow all bidders to incur the bid cost; both are set in a static, sealed-bid environment.

⁸Allowing jump bids in ascending auctions introduces a variety of strategic behaviors. One may use a jump bid to signal his private value (Daniel and Hirshleifer, 1998), to intimidate an opponent and thus induce an asymmetric equilibrium (Avery, 1998), or, for impatient participants, to finish the auction faster (Isaac, *et al*, 2007). We are confident that similar issues would arise in pay-to-bid auctions if jump bidding were allowed, though it would diminish the importance of bid fees as a source of revenue and seriously complicate the analysis.

The paper proceeds as follows: in section 2, we present the model and identify its unique symmetric equilibrium. We also discuss several variations on the model. In section 3, we discuss the data we have extracted from Swoopo’s website. Section 4 presents evidence on the model’s fit to the data and section 5 offers conclusions and some broader applications of the pay-to-bid auction.

2 Model

An item being sold has a known, objective value of v to all potential bidders. This is initially offered at a price of 0. A customer immediately pays b dollars each time she places a bid, which also increments the price by s dollars and starts a new period. The auction concludes if no one places a bid during the current period, resulting in the last bidder paying the current price and receiving the item. Assume that potential bidders’ decisions of when (if at all) to act are distributed without atoms throughout each period; thus ties never occur. This constitutes a full information, extensive form game; we examine the symmetric subgame perfect equilibria.

When a customer places a bid, she is betting b dollars that no one else will bid after her. Consider a customer who observes the current price at $s \cdot (q - 1)$ dollars. If she were to place the q^{th} bid, her expected payoff would be $(v - s \cdot q)(1 - \mu_{q+1}) - b$, where μ_{q+1} denotes the probability that anyone else will place the $q + 1^{\text{th}}$ bid. She weighs the expected payoff of winning against the certain cost of the bid fee.⁹

Let $Q \equiv \frac{v-b}{s}$, and for simplicity, assume that Q is an integer. If $q > Q$, then this customer will strictly prefer not to bid — regardless of μ_{q+1} — because the current price plus bid fee exceeds the item’s value. Thus, $\mu_q = 0$ for all $q > Q$. If $q = Q$, this customer is indifferent about bidding (even though $\mu_{Q+1} = 0$ and hence winning is assured). Indeed, during this and *all* earlier periods of the game, the participants *must* be indifferent about bidding. If the q^{th} bidder strictly preferred bidding, then $\mu_q = 1$ and therefore the $q - 1^{\text{th}}$ bidder would strictly prefer not to bid. If the q^{th} bidder strictly preferred not bidding, then $\mu_q = 0$ and therefore the $q - 1^{\text{th}}$ bidder would strictly prefer to bid.

This indifference condition then determines unique probabilities that define equilibrium behavior. In particular, for all $1 < q \leq Q$, $\mu_q = 1 - \frac{b}{v-s \cdot (q-1)}$. Only the likelihood that anyone places the first bid, μ_1 , is non-unique, since there is no prior bidder who must be made indifferent. While any $\mu_1 \in [0, 1]$ is consistent with equilibrium, we set $\mu_1 = 1 - \frac{b}{v}$

⁹One might wonder if the expected payoff should also include an option value of placing a bid; that is, a bidder might place the q^{th} bid even if he will be outbid for certain, because it keeps the auction running and gives him a chance to place the $q + 2^{\text{th}}$ bid. However, in equilibrium, the expected payoff of placing the $q + 2^{\text{th}}$ bid (net of the bid fee) is zero, so a bidder at period q can ignore the option value.

for simplicity, assuming in effect that the first period behavior mimics that of subsequent periods. This assumption has minimal impact on our analysis, only slightly altering expected revenue.

We also recognize that other asymmetric equilibria can exist. For instance, suppose the 100th bidder threatens that she will place the 102nd bid with probability 1 if anyone outbids her. This would dissuade all other participants from placing what would be a wasted 101st bid. While such equilibria can exist, our analysis ignores them for two reasons. First, the equilibrium would be difficult to implement if bidders are anonymous, since all potential bidders would need to know the bidder’s specific threat.¹⁰ Second, if it were possible to credibly communicate such threats, the obvious solution is to employ them after the first bid. Our empirical observation does not reflect this sort of behavior, but matches the symmetric mixed strategy equilibrium very well.

2.1 Individual strategies

The probabilities μ represent the aggregate consequence of individual mixed strategies. By specifying the process by which bidders participate in the auction, we can analyze these individual decisions and interpret the aggregate probabilities better. However, the aggregate is sufficient to analyze the outcome of the auction.

First, consider an environment in which a fixed pool of n participants are watching the auction. In the q^{th} period, every individual chooses a mixed strategy β_q on the probability of placing a bid, taking the others’ choices as given. In a symmetric equilibrium, this will bear direct relation to the aggregate probabilities. Suppose that $q - 1$ bids have been placed. The $n - 1$ customers who currently are not winning must decide whether to place a bid. With probability $(1 - \beta_q)^{n-1}$, none of them do. In order to satisfy the indifference condition for the $q - 1^{\text{th}}$ bid, this must equal $1 - \mu_q$. So $\beta_q = 1 - (1 - \mu_q)^{\frac{1}{n-1}}$.

Suppose instead that bidders arrive to the auction according to a Poisson process, with rate λ per minute. Upon arrival, the bidder instantaneously decides whether to bid (employing mixed strategy β_q). If he declines to bid or someone bids after him, he departs the auction. Again, consider after the $q - 1^{\text{th}}$ bid has just been placed. If a period lasts T minutes, the probability that n customers arrive is $\frac{e^{-\lambda T}(\lambda T)^n}{n!}$, and the probability that all of them decline to bid would be:

¹⁰Andrzej Skrzypacz suggested creating a username “will_outbid_you_over_five_bucks” to announce your strategy. Perhaps to prevent such communication, Swoopo limits the length and content of usernames. Moreover, only the last ten bidders are shown, making it difficult to establish a reputation during the auction.

$$\sum_{n=0}^{\infty} \frac{e^{-\lambda T} (\lambda T)^n}{n!} \cdot (1 - \beta_q)^n = e^{-\lambda T \beta_q}$$

In order to satisfy the indifference condition for the $q - 1^{\text{th}}$ bid, this must equal $1 - \mu_q$; however, it may not be able to do so. In particular, if λ and q are sufficiently small, we could have $e^{-\lambda T \beta_q} > 1 - \mu_q$ for any β_q . Even so, equilibrium still exists, with minor adaptation. Individual mixed strategies would be $\beta_q = \min \left\{ -\frac{\ln(1 - \mu_q)}{\lambda T}, 1 \right\}$, with aggregate probability that someone bids set to $1 - e^{-\lambda T}$ when the latter case holds. In such an equilibrium, all bidders who arrive when q is low will strictly prefer to bid even knowing that those who follow will do the same, because there is a large enough chance that no one will show up at all. For high enough q , eventually the indifference condition will hold and μ_q again describes the aggregate probability.

2.2 Expected revenue

In examining the expected revenue of a pay-to-bid auction, we first consider one particular variant in which customers bid for the right to purchase an item at a predetermined price p . In practice, Swoopo still increments the price after each bid as if $s = 0.15$ (which allows us to determine the number of bids which occurred), but ignores sq in computing the final price. The special case when $p = 0$ is referred to as a 100% off auction, where nothing is paid beyond the bid fees.

The advantage of examining these fixed price auctions is that it produces a simple probability distribution. Since indifference requires that $(v - p)(1 - \mu_q) - b = 0$, we derive $\mu_q = 1 - \frac{b}{v - p}$ for all q . This conditional probability (that the q^{th} bid occurs, given that the $q - 1^{\text{th}}$ already has) allows us to construct the probability density that the auction ends at exactly q bids:

$$f(q) = \frac{b}{v - p} \left(1 - \frac{b}{v - p} \right)^q. \quad (1)$$

We then examine the expected revenue:

$$\sum_{q=1}^{\infty} (bq + p) f(q) = v - \frac{vb}{v - p} \quad (2)$$

and variance of revenue:

$$\left(v - \frac{vb}{v - p} \right)^2 f(0) + \sum_{q=1}^{\infty} \left(bq + p - v + \frac{vb}{v - p} \right)^2 f(q) = \frac{(v - p - b)((2v - p)bp + (v - p)^3)}{(v - p)^2} \quad (3)$$

We note that this auction is not any more lucrative than standard auction formats. A typical

1st or 2nd price sealed-bid auction among (nearly) identical buyers would raise a revenue close to v . Yet here, in the instance of a 100% off auction, the expected revenue is essentially the same, at $v - b$.

In the standard pay-to-bid auction, μ_q varies with q , which significantly complicates the probability density function of ending bids, $f(q)$. In particular, we obtain

$$f(q) \equiv (1 - \mu_{q+1}) \prod_{j=1}^q \mu_j = \frac{b}{v - s \cdot q} \prod_{j=1}^q \left(1 - \frac{b}{v - s \cdot (j - 1)} \right), \quad (4)$$

with $f(0) = 1 - \mu_1$. This density function is decreasing in q ; that is, the unconditional probability of ending at a given number of bids decreases as bids increase. The expected revenue is more difficult to directly derive with this density function, but this is done in the appendix, again arriving at an expected revenue of $v - b$ and variance of $\frac{b(v-b)(s+v)}{b+2s}$.

The former can also be shown via an indirect method. Assuming that the seller places no intrinsic value on the item, the total expected surplus of the auction is equal to v times the probability that someone wins the auction, or in other words, the probability that at least one bid is placed. This computation yields $v \cdot \mu_1 = v \cdot \left(1 - \frac{b}{v}\right) = v - b$. This expected surplus is split between the seller and buyers, yet by construction, the expected surplus of the buyers is zero; hence, the seller's expected revenue is $v - b$.

As we noted before, μ_1 is not uniquely determined by an indifference condition, though the preceding analysis selects μ_1 to be congruent with the other μ_q . The only impact of this choice is seen in expected revenue: if for instance the first bid *always* occurred, expected revenue would be v . While such equilibria do exist, we find it less plausible that the first bid would be dramatically different from those that follow. Moreover, in our empirical findings which follow, we do in fact observe a small proportion of auctions concluding without a single bid, consistent with our selected equilibrium.

2.3 Shill bidding

One concern about this auction format is that it could be manipulated by the auctioneer to generate more revenues; indeed, a popular accusation against Swoopo is that the website employs shill bidders (who place bids for free to drive up the final price). As shown in the remainder of the paper, our model is able to explain the observed data without incorporating any malfeasance from the auctioneer, but we can offer two comments on shill bidding.

One must ask what the equilibrium response would be if shills were employed. For instance, if shills always ensure that bidding on an item continues through at least 100 bids, then real participants would (in equilibrium) have no reason to waste a bid in that initial

range, since they will certainly be outbid. This is equivalent to starting the auction with an initial price of $100s$. This increases the expected final price, but reduces the revenue from bid fees by $100b$. The net effect produces the same expected revenue of $v - b$.

More likely, a shill would operate by randomly bidding, allowing some auctions to close with very few bids. If so, in equilibrium, the added probability of bidding by shills displaces some of the probability of bidding by real participants — yet the aggregate probability of a bid occurring, μ_q , must maintain the indifference condition to keep the real participants participating. For simplicity, suppose that a shill places the q^{th} bid with probability $0 < \sigma_q < \mu_q$. After a shill places a bid, she will win the auction with probability $1 - \mu_{q+1}$; if that occurs, the auctioneer retains the item but receives no revenue from the final price, though he does collect the bid fees of all real participants.

This shill arrangement is reflected in the following expression of expected profits (*i.e.* expected revenue minus the probability of a real bidder winning times v):

$$E[\pi] = \sum_{q=1}^Q (1 - \sigma_q)(sq - v)(1 - \mu_{q+1}) \prod_{j=1}^q \mu_j + \sum_{q=1}^Q b(1 - \sigma_q) \prod_{j=1}^q \mu_j.$$

The first summation determines expected profit from the auction winner, who is a real bidder with probability $1 - \sigma_q$ and pays sq in the final price, but requires delivery of the item, causing the auctioneer to incur cost v . Neither occurs if a shill bidder wins. The remaining terms of the first summation represent the aggregate probability of ending at q bids. The second summation computes expected revenue from bid fees. The product indicates the aggregate probability that q^{th} bid occurs, which is multiplied by $b(1 - \sigma_q)$, the expected bid fee revenue from that bid since shill bidders do not actually pay.

After substituting for μ_{q+1} , this simplifies to

$$E[\pi] = - \sum_{q=1}^Q (1 - \sigma_q)b \prod_{j=1}^q \mu_j + \sum_{q=1}^Q b(1 - \sigma_q) \prod_{j=1}^q \mu_j = 0.$$

To compare this to our previous expected revenue, note that without shill bidding, the expected cost to the seller is $v \cdot \mu_1 = v - b$, which also give an expected profit of 0. Shill bidding of this form produces no additional profit.

3 Data

From the inception of their US website in September 2008 through January 2009, Swoopo.com auctioned over 34 thousand items, all via pay-to-bid auctions. Most of these auctions are

repetitions of identical objects, with only 623 unique items. Furthermore, a small handful of items are auctioned much more frequently than the others; only 65 items were auctioned more than 45 times, with the most popular (a Nintendo Wii system) being auctioned 2,728 times. In Table 2, we group these frequently-auctioned items into 6 categories (video games, computer accessories, apparel, etc.) and report the number of unique items and total number of auctions for each category.

Swoopo lists all of their ended auctions on their website. For each auction, the site provides the suggested retail price, the final auction price, the bid fees paid by the winner, and the end time. Also listed is the bid fee and the price increment that occurs with each new bid. Most auctions increase by \$0.15, but penny auctions increase by \$0.01. We divide the final price by the bid increment to get the total number of bids in each auction. We multiply the number of bids by the bid fee and add this to the final price to determine Swoopo’s total revenue for that auction.

While all Swoopo auctions involve the same bid fee, there is some variety in the rules for final prices. During this observation period, two-thirds of all Swoopo auctions followed a standard pay-to-bid auction in which the winner pays the final auction price. The others used the fixed price variation, where the winner pays a predetermined price. The most common of these (12% of all auctions) was a 100% off auction, where the winner of the auction pays nothing for the item except for the money already paid in bid fees.¹¹

In many cases the suggested retail price is significantly higher than what is readily available from other online retailers. To provide an alternative measure of retail price, we found each item on Amazon.com. By this measure, Swoopo’s suggested retail price is about 28% higher than the price on Amazon.com (weighting items by the number of auctions for that item).

Swoopo also provides the usernames of the winner and last 10 bidders of each auction.¹² Across all of the auctions in our sample, there were 15,540,820 bids placed, of which 2% occur as one of the last ten bids that we can observe. Among the last ten bidders across all auctions, there are 69,519 unique usernames with 13,001 of these users winning at least one item. Over 63% of the winners only win once and a very small number (less than 1%) have won more than 25 auctions.

A common thought upon observing a pay-to-bid auction is why anyone would be the first bidder. While it is unlikely that the first bidder will win the auction, we do have 134 auctions in which the first bidder wins the auction, 299 where the 2nd bidder does, and 8,667

¹¹Since January 10, 2009, Swoopo has stopped offering any fixed price auctions. This may have been in response to criticism that 100% off auctions bore too much resemblance to a lottery or raffle.

¹²We cannot observe the full history of bids — that is, the identity of each bidder for each period. This data would be needed to discern between the models of individual strategies developed in Section 2.1.

that are won by one of the first 9 bidders. In fact, two of the surprising predictions of the model are that a large fraction of the auctions will end during the early bids and that the (unconditional) probability of an auction ending at a given number of bids is decreasing in the number of previous bids.

Swoopo provides bidders the option to use a bid butler which will place bids for them based on preset parameters. To use the bid butler, the bidder decides at what price to start bidding on an item, at what price to stop bidding on an item, and how many bids they are willing to make. The bid butlers are designed to be as frugal as possible and will wait to bid until there are fewer than 10 seconds left on the auction clock. We find that 32% of the last ten bids are made using a bid butler but 64% of auctions are won by someone using a bid butler.

Swoopo is not the only website in the US to offer pay-to-bid auctions; ten others offered a similar service during the time period of our analysis. However, using data from a web traffic monitoring company (Compete.com), summarized in Table 1, we found that Swoopo.com accounted for almost half of the web traffic going to this group of sites. One of the primary advantages of studying Swoopo is that, unlike most competitors, it provides information on all past auctions.

4 Evidence

Our empirical objective is to test whether our model of pay-to-bid auctions appropriately describes observed bidding on Swoopo. We test this hypothesis by comparing the observed distribution of the number of bids to the theoretical predictions produced by our model.

While our data offers a little variation in bid fees and price increments, the overwhelming majority charged \$0.75 and raised the price by \$0.15; hence, no item was auctioned enough times outside of this standard setup to provide useful statistical inference. Even so, v has a much greater influence on the theoretical distribution than b or s . Note that s has no impact on expected revenue, $v - b$, and that b is typically two orders of magnitude smaller than v . Where the same item has been auctioned many times, we assume that v is held constant and that all variation in ending price is due to the equilibrium use of mixed strategies. We repeat this analysis with 70 distinct items.

We start by providing some illustrative examples in Figure 1, showing the degree to which the distribution of bidding behavior we observe in the Swoopo data matches the equilibrium predictions of our model. In each example, the numbers along the x-axis are the number of bids that occurred in an individual auction (which maps one-on-one with the revenue from the auction). The y-axis is the probability (either theoretical or actual) that the auction

ends at that number of bids.

The model predicts a distribution of ending bids based on the valuation of the item, the bid fee, and the increment by which the final price rises with each bid. The latter two are clearly specified for each auction. For the valuation, we initially use the retail price reported by Swoopo; however, it is quite possible that bidders may value an item more or less than its retail price indicates. For instance, if the same item is available from internet retailers at a discount, bidders would behave as if their true valuation is the discount price. One would expect Swoopo bidders to be sufficiently internet-savvy to have checked Amazon.com or similar sites for a price comparison. On the other hand, if the item were difficult to find through other outlets, such as the Wii video game console, the true valuation could be above retail.

To account for this possibility, we compute a maximum likelihood estimate of the parameter v for each regularly auctioned item, chosen to maximize $\sum_i \ln f(q_i; v)$, where i represents each observed auction of that item, q_i is the ending number of bids in that auction, and f is the theoretical distribution given in Equation 4. We then repeat our statistical tests using this estimated valuation. Figure 2(b) compares the prices we obtain using our MLE method with those found for the same item on Amazon.com. Our MLE method provides estimates that are higher than the Amazon (the regression coefficient on the fitted line is 0.427) but also a very close fit along the regression line (the R^2 of this regression is 0.818). In contrast, a regression comparing Swoopo's reported retail prices with Amazon prices has a smaller slope of 0.365 and an R^2 of only 0.570 (shown in Figure 2(a)).

We should note that using the MLE estimates for v does not enable us to shape $f(q)$ at will. Increasing v will lengthen the support of f and decrease its curvature in a very particular way; moreover, it will not change that $f'(q) < 0$ and $f''(q) > 0$. Also, in what follows, we have set b and s to the objective values listed for each auction. When we simultaneously estimated those parameters, for most auctions we recovered nearly the same objective values. In those auctions where we did not, the fit was equally bad whether selecting only v or also selecting b and s .

The top row of Figure 1 displays the results from three of the most common non-video game items; the bottom row has three of the most common video game items. The upper figures provide visual evidence that the actual distribution of revenues matches very closely with that predicted by the model; and the fit is improved using the MLE valuations. A large number of auctions end with few bids (and hence low revenue), and the probability of the auction ending at q declines (at a decreasing rate) as q increases.

As a consequence, many of these auctions conclude in a net loss for Swoopo — for items similar to those depicted in the top row, as many as 69% of the auctions of a given item will

not generate enough revenue (in bid fees and final price combined) to cover the retail price. However, the long right tail generates enough compensating revenue so that, on average, these auctions raise revenue equal to the valuation. These results also indicate that (for this set of items) bidders may in fact be rational in participating in the pay-to-bid auction, even early on in the auction, because of the relatively high probability of obtaining the item at a price far below its retail price.

Items depicted in lower row of Figure 1 show that auctions for video game systems and accessories do not match the predictions of our model very well. In particular, we find there is an initial range for which the probability is *increasing* in q , which cannot be generated by this model under any parameter values. In addition, we find that the density of ending bids in the low end is much smaller than predicted, and the right tail of the distribution is much thicker.

We next turn to some statistical tests comparing the match between the prediction of our model and what we observe in the Swoopo data. We restrict our analysis to items that are sold in 45 or more standard auctions. Out of 505 distinct items (sold in the standard pay-to-bid auction), 65 meet this criterion. For each of these items, we perform a Pearson's chi-square goodness of fit test comparing the observed distribution of ending bids to the theoretical distribution. We also perform a t-test comparing the theoretical and actual mean revenue. In Table 2, we report the fraction of items for which the tests fail to reject the null hypothesis that the observed data differs from the prediction of the model (based on whether the test statistic corresponds to a p-value greater than .01, .05, and .10).

In Panel A of Table 2, we use the reported retail value of the item as our measure of bidder's common valuation for the item. We group items into five categories: computers, computer accessories, home electronics, apparel, and video games. For instance, considering computer-related items and using the .05 p -value as a threshold for our test, we find that when comparing the predicted and actual distribution, the majority of the time we fail to reject that the two distributions are statistically different. The degree to which our model matches the observed distribution of the data is much lower for video games and home electronics, where we fail to reject that the two distributions are different only about 25% of the time.

In the far right columns of Table 2 we look specifically at mean revenue. Our model does a worse job at predicting the mean revenue for an item, with us failing to reject that the two means are different 42% of time for computers and even less often for the other items. However, in this panel the valuation of the item is determined by the reported retail price, which may be a poor proxy for the true valuation. Consider graph (c) of Figure 1, which shows the ending bid distribution for an Invicta Diver Watch with a reported retail price of

\$370. Our MLE estimate suggests that the true valuation is actually more like \$256, which is also in line with other online retailers. When we use the MLE-based valuation, the plot of our predicted distribution matches the data much more closely.

In Panel B of Table 2, we use our MLE-based valuations and report the fraction of the items in each category for which we fail to reject at each of the three common statistical cut-offs. The goodness of fit of our model goes up for all categories; for example, with all apparel auctions, we fail to reject a difference between the theory and data. On the other hand, with video game items, we still only fail to reject that our predictions differ from the data 46% of the time. In aggregate, auctions of 57 of the 72 items (or 79%) can be explained by our theory (at the .05 p -value threshold).¹³

As a final piece of evidence, in Figure 3 we compare the observed average revenue with the theoretical expected revenue, reporting the difference in standard deviation units. We find that most of the non-video game items clump very close to zero, with 90% of these items having average revenue within one standard deviation of the prediction of our model. For the video-game items, we also see the mode occurring near zero but there is a large right tail to this distribution, with 40% of the items having average revenue greater than one standard deviation above the model's prediction.

5 Risk preferences

To this point, we have assumed bidders to be risk neutral. We now make a minor adaptation of our model by incorporating preferences towards risk. Since wealth is unobserved, we sidestep this by assuming CARA utility: $u(w) = \frac{1 - e^{-\alpha w}}{\alpha}$. From here, we follow the same process as before, imposing indifference between placing a risky bid versus not participating, then solving for μ :

$$(1 - \mu_{q+1}) \frac{1 - e^{-\alpha(w+v-sq-b)}}{\alpha} + \mu_{q+1} \frac{1 - e^{-\alpha(w-b)}}{\alpha} = \frac{1 - e^{-\alpha w}}{\alpha}, \quad (5)$$

which yields:

$$\mu_q = \frac{1 - e^{\alpha(b+s(q-1)-v)}}{e^{\alpha b} - e^{\alpha(b+s(q-1)-v)}}. \quad (6)$$

¹³We also used a Kolmogorov-Smirnov test and found very similar results, with a slightly higher likelihood of rejecting the null for some categories.

5.1 100% off auctions

As before we can construct a probability density function from these μ . In the special case of 100% off auctions, this becomes:

$$f(q) = \left(\frac{1 - e^{\alpha(v-b)}}{1 - e^{\alpha v}} \right)^q \left(\frac{e^{\alpha v} - e^{\alpha(v-b)}}{1 - e^{\alpha v}} \right) \quad (7)$$

which generates an expected revenue of $b \left(\frac{e^{\alpha(b-v)} - 1}{1 - e^{\alpha b}} \right)$. Note that expected revenue is decreasing in α ; the auction generates more revenue as agents become less risk averse.

We can perform a maximum likelihood estimation on $f(q)$ as before; however, α is not separately identifiable from v or b . This is because the distribution has the form $f(q) = (1 - \mu)\mu^q$. Thus, the MLE procedure can only identify μ .

Fortunately, all of the items where Swoopo has routinely used 100% off auctions have an obvious objective value for v . These items include cash (\$1,000 or \$80) and vouchers for free Swoopo bids (either 300 or 50, worth \$0.75 per bid). The bid fee b is also exogenously set, so we can use the MLE procedure to identify α .

For these items, we find α to be slightly negative, between -0.001 and -0.01. This implies that Swoopo participants are risk loving. The resulting theoretical distribution is depicted in Figure 4; with the MLE risk preferences, the fit is remarkable. Using risk neutral bidders (and the objective value of v), the theoretical distribution would predict too few bids at every point and hence too high a probability of ending early. With the addition of risk-loving preferences, bidders become a bit more aggressive at every point of the auction.

5.2 Standard auctions

After incorporating risk preferences in a standard pay-to-bid auction, the resulting probability density is far less tractable:

$$f(q) \equiv (1 - \mu_{q+1}) \prod_{j=1}^q \mu_j = \frac{1 - e^{\alpha b}}{e^{\alpha b} - e^{\alpha(b+s(q-v))}} \prod_{j=1}^q \left(\frac{1 - e^{\alpha(b+s(j-1)-v)}}{e^{\alpha b} - e^{\alpha(b+s(j-1)-v)}} \right). \quad (8)$$

Direct analytic solutions for expected revenue are no longer possible. Indeed, even the indirect method used in the risk neutral case is no longer applicable, since the auction not only creates value by transferring an item to someone who values it, but also creates risk, whether for good ($\alpha < 0$) or ill ($\alpha > 0$). However, numerical computation is relatively simple, revealing a few key features of f .

First, the support of f is the same as before, placing positive probability everywhere

from $0 \leq q \leq Q$. Increases in v have essentially the same effect that they did in the risk-neutral case: it will increase the support and flatten the distribution. When $\alpha > 0$ (*i.e.* risk aversion), f has a similar convex shape to the risk neutral density function, only with greater curvature as α rises.

The distribution behaves quite differently for $\alpha < 0$ (*i.e.* risk-loving). When α is very close to zero, an inflection point \hat{q} is introduced near zero such $f'' < 0$ below \hat{q} ; thus f is no longer strictly convex. As α decreases, this inflection point takes on higher values, and eventually, creates a hump-shaped distribution. As α becomes more negative, the q which maximizes f increases.

To perform maximum likelihood estimation, we use numerical approximation of the density function. For each auctioned item, we estimate both the valuation and risk parameter, which are identified by distinct first-order conditions of this generalized pay-to-bid distribution. Note that even with these two degrees of freedom, we are not capable of replicating every unimodal distribution, but are constrained to the particular shape described above.

After MLE, the resulting risk parameter α indicates that bidders on video game items are mildly risk-loving, primarily in the range of -0.001 to -0.006, with a few as low as -0.025. These values are significantly lower than the estimated risk preferences of bettors at horse race tracks: using the same functional form and assuming a bet (*i.e.* our bid fee) of £1, Jullien and Salanié (2000) estimate $\alpha = -0.055$.¹⁴ The resulting MLE valuations v are also plausible for each item, generally below the retail price with the exception of items that were in short supply during this data period. Figure 2(c) compares these to Amazon.com prices; the associated regression has a steeper slope (0.571) than the risk-neutral MLE valuation in Figure 2(b), and a slightly lower R^2 of 0.756.

The improvement in fit is again remarkable. Figure 5 illustrates the new theoretical distribution, demonstrated on the same three items from the lower panel of Figure 1. The risk neutral model was unable to match the hump shape of the distribution because for any choice of the parameter v , $f(q)$ was decreasing in q . In the altered model, $f(q)$ will first rise then fall if α is sufficiently large; thus, early bidders are more likely to bid than risk neutrality would imply.

Intuitively, this occurs because when q is low, there is greater variance in the outcome, because the payoff if no one else bids, $v - sq$, is large. A risk-loving participant will be willing to bid in spite of unfair odds as long as the gamble has a highly skewed payoff. Thus μ_q can be larger than in the risk neutral model (more aggressive bidding) while still satisfying our

¹⁴Similar studies uses a functional form $u(w) = aw^b$, which has constant relative risk aversion. Ali (1977) assumes an initial wealth $w = 1$ and estimates $b = 1.178$, or an absolute risk aversion (comparable to our α) of -0.178. Kanto, *et al* (1992) and Golec and Tamarkin (1998) arrive at similar estimates.

indifference condition, and as a result, it is less likely that the auction ends with low q . As q increases, though, the variance in outcome diminishes, and the same risk-loving participant will require closer to fair odds in order to bid. Indeed, as the final price sq approaches v , the resulting μ_q approaches the risk neutral result.

Panel C of Table 2 reports the degree of fit for all auctions after MLE allowing for risk-loving preferences. Note that nearly all of the auctions can be explained by the model once risk preferences are included. In particular, nearly 50% more video game items fit the adapted model.¹⁵

5.3 Risk-loving preferences

It is not particularly surprising that Swoopo participants would have a preference for risk; after all, this auction is essentially a form of gambling. Like a slot machine, the bidder deposits a small fee to play, aspiring to a big payoff (of obtaining the item well below its value). The only difference is that the probabilities of winning are endogenously determined.

The idea that a gambler would voluntarily take on risk, paying more than the expected payout to play, is puzzling. Economists have tried to rationalize such behavior in one of two routes: by assuming some intrinsic utility from placing the gamble or by assuming convex preferences with respect to wealth.¹⁶ Regarding the former, it is noteworthy that Swoopo prominently advertises itself as “entertainment shopping.” While many economists have referred to intrinsic utility as motivation for gambling, very few have provided formal modeling of the concept. Diecidue, Schmidt, and Wakker (2004) provide a brief but useful survey of these works and offer their own formal model. They conclude that utility from gambling would necessarily contradict stochastic dominance.

We employ the alternative: assuming convex preferences (that is, increasing marginal utility of wealth within some range). Friedman and Savage (1948) provided an early formal model of this phenomenon. Their justification for increasing marginal utility was that major increases in wealth would promote a person into a new social and economic status, while minor increases in wealth would maintain a person within the same class and hence exhibit diminishing marginal utility. This view was supported by Gregory (1980) and Brunk (1981).

More recent work by Golec and Tamarkin (1998) and Garrett and Sobel (1999) proposes that what appears as *risk-loving* in the Friedman-Savage utility should be more aptly named *skewness-loving*. In an empirical study of horse-track betting and US lottery games, respec-

¹⁵Again, Kolmogorov-Smirnov tests provided nearly identical results.

¹⁶In addition, much of behavioral economics is inspired by these and similar puzzles. These lines of work offer alternatives to expected utility, such as prospect theory, ambiguity aversion, or bounded rationality, to name a few. We omit discussion of these theories because their application in a game theoretic setting is not a settled matter.

tively, both papers conclude that gamblers experience disutility from variance in payoffs (*i.e.* are still risk averse), but gain utility from skewness in payoffs. Golec and Tamarkin show that this will appear as risk-loving preferences when estimating parameters for CRRA utility.

This view of risk-loving behavior seems appropriate for our model, particularly for items such as video games. The bidders may not be able to justify (to their spouse, their parent, or themselves) spending \$300 on a Wii at a retail store; however, the potential to win the Wii early in the auction for only a fraction of that makes it worth the \$0.75 gamble, even at unfair odds. As the number of bids (and hence the final price) increase, the skewness in outcome decreases and these gamblers are less tolerant of unfair odds. Indeed, as the final price approaches the retail price, μ_q eventually adjusts so that bidding becomes a fair bet.

Figure 6 provides evidence of this behavior, plotting the expected payoff for bidding on a Wii game system conditional on the current number of bids. The conditional expected payoff is the probability that the auction will end before the next bid (conditional on the auction reaching the current bid) times the net value derived by the winner (the valuation minus the final price). The theoretical plot derives conditional probabilities from the indifference condition, while the observed plot extracts this information from the data.

The plot of the expected payoff using the observed data is based on a two-step procedure. First, we use a kernel regression to smooth the cumulative density function of the number of bids at which the Wii auction ends, $F(q)$. This is used to construct the expected payoff, $(v - sq) \frac{f(q)}{1-F(q)}$, where the latter term is the probability of the auction ending, conditional on the auction reaching that number of bids. We then smooth this expected payoff function using a kernel regression and plot this function up to 1600 bids (the 95th percentile, or a final price that is 70% of the valuation).¹⁷ The expected payoff from bidding is very low at the start of the auction (around 22 cents) but rises because the probability of winning increases faster than the net value of winning falls.

For more practical items such as computers or apparel, bidders behave as if they are risk neutral.¹⁸ This probably reflects that they intend to purchase the item from a traditional retailer if unsuccessful at Swoopo.

¹⁷For both kernel regressions we use a Epanechnikov kernel and a bandwidth of 75. The pattern in figure 6 is the same when we use alternative bandwidths or use a local polynomial regression.

¹⁸The distinct behavior in video game auctions naturally raises the question of whether a different set of bidders participate in video game versus other auctions, or if instead the same people participate in both but behave more aggressively in video game auctions. We were not able to follow every bid on every auction, but for a set of 1,084 auctions, we were able to record the usernames of most of the auction participants (as opposed to only the last ten). Unfortunately, even for those auctions, we lack sufficient observations for each bidder to provide any meaningful insight on this question; only 2% of the bidders place more than 10 bids.

6 Conclusion

This paper presents a simple model of bidder choices in a pay-to-bid auction. In the symmetric subgame perfect equilibrium, potential bidders are indifferent about participating, and the exact mixed strategy is determined by this indifference condition. Using these mixed strategies we can establish that expected revenue will be near the bidders' valuation of the auctioned item.

Even when restricted to risk neutral bidders, the model is able to explain a strong majority of the observed data. However, items associated with video game systems routinely produce more aggressive bidding and higher revenue than can be explained by this model. These exceptions are the most profitable of Swoopo's auctions, and are among the most frequently run. Once we allow bidders to have preferences towards risk, the model can explain almost all of the observed auctions. Buyers of video game systems have small preferences for risk (which may be interpreted as a love of skewness), which makes them aggressive in the early phase of the bidding.

On a broader level, the pay-to-bid auction describes an incremental king-of-the-hill contest. By incurring a sunk cost (*e.g.* bid fee), anyone can become the current king-of-the-hill; yet that title only becomes permanent if all challengers give up. The contest is incremental because each replacement of a king reduces the hill's value to the eventual winner.

For instance, this could describe a particular form of competition among rent seekers. Suppose several firms were seeking the same exclusive license from a bribe-accepting regulator. The regulator could require an up-front bribe each time a firm wishes to make an overture; moreover, to displace the previous overture, the current firm promises to return a greater portion of the future rents to the regulator. The license is awarded once no additional overtures are attempted, and is given to the firm with the last (and hence best) overture. A similar story could be told for competition among suitors, showering gifts or attention on a potential mate.

Applied to these situations, our risk-neutral model would predict that the regulator captures essentially the full value of the license (in expectation). The firms would be indifferent about participation *ex-ante*; yet *ex-post*, the winning firm will typically enjoy large rents, with a final price well below the full value. If the firms are risk loving, the regulator can expect to extract even more than the full value, making this a far more profitable means of allocating the license than other auction formats. Indeed, risk-loving (or skewness-loving) preferences might well be applicable if the residual rents would significantly alter the winner's social class or if, in the case of a suitor, the contest gives him a shot at a woman normally far outside his league.

Appendix

Mean and Variance of Revenue

To facilitate the analysis of the first and second moments of the f distribution, we make use of the *ascending factorial* or Pochhammer symbol: $(a)_k \equiv a(a+1)\cdots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)}$, where $\Gamma(\cdot)$ denotes the Gamma function.

Expected revenue is:

$$\begin{aligned}
 ERev &= \sum_{q=1}^Q (b+s)q(1-\mu_{q+1}) \prod_{j=1}^q \mu_j = \sum_{q=1}^Q (b+s)q \prod_{j=1}^q \mu_j - \sum_{q=1}^Q (b+s)q \prod_{j=1}^{q+1} \mu_j \\
 &= \sum_{q=0}^{Q-1} (b+s)(q+1) \prod_{j=1}^{q+1} \mu_j - \sum_{q=1}^Q (b+s)q \prod_{j=1}^{q+1} \mu_j \\
 &= \sum_{q=0}^{Q-1} (b+s) \prod_{j=1}^{q+1} \mu_j + \sum_{q=0}^{Q-1} (b+s)q \prod_{j=1}^{q+1} \mu_j - \sum_{q=1}^Q (b+s)q \prod_{j=1}^{q+1} \mu_j \\
 &= \sum_{q=0}^{Q-1} \left((b+s) \prod_{j=1}^{q+1} \mu_j \right) - (b+s)Q \prod_{j=1}^{Q+1} \mu_j
 \end{aligned}$$

The last term drops out entirely since $\mu_{Q+1} = 0$. Since $\mu_j = 1 - \frac{b}{v-s \cdot (j-1)} = \frac{\frac{b-v}{s} + j - 1}{-\frac{v}{s} + j - 1}$, this can be rewritten as:

$$ERev = (b+s) \sum_{q=1}^Q \prod_{j=1}^q \mu_j = (b+s) \sum_{q=1}^Q \frac{\left(\frac{b-v}{s}\right)_q}{\left(-\frac{v}{s}\right)_q}.$$

An identity of ascending factorials provides that $\frac{(a)_j}{(a)_k} = (a+k)_{j-k}$. Hence,

$$ERev = \frac{b+s}{\left(-\frac{v}{s}\right)_Q} \sum_{q=1}^Q \frac{\left(\frac{b-v}{s}\right)_q}{\left(-\frac{v}{s}\right)_q} \left(-\frac{v}{s}\right)_Q = \frac{b+s}{\left(-\frac{v}{s}\right)_Q} \sum_{q=1}^Q \left(\frac{b-v}{s}\right)_q \left(-\frac{v}{s} + q\right)_{Q-q}.$$

Another transformation of ascending factorials establishes that $(a)_{n+1} - (x)_{n+1} = (a-x) \sum_{k=0}^n (a)_k (x+k+1)_{n-k}$. Applying it here, let $a = \frac{b-v}{s}$, $x = -\frac{s+v}{s}$, and $n = Q$, which yields:

$$\left(\frac{b-v}{s}\right)_{Q+1} - \left(-\frac{s+v}{s}\right)_{Q+1} = \frac{b+s}{s} \sum_{q=0}^Q \left(\frac{b-v}{s}\right)_q \left(-\frac{s+v}{s} + q + 1\right)_{Q-q},$$

or, since $(a)_0 \equiv 1$, this becomes:

$$\left(\frac{b-v}{s}\right)_{Q+1} - \left(-\frac{s+v}{s}\right)_{Q+1} = \frac{b+s}{s} \left(\left(-\frac{v}{s}\right)_Q + \sum_{q=1}^Q \left(\frac{b-v}{s}\right)_q \left(-\frac{v}{s} + q\right)_{Q-q} \right).$$

This allows us to substitute for the summation in expected revenue to obtain:

$$ERev = \frac{b+s}{\left(-\frac{v}{s}\right)_Q} \left(\frac{s}{b+s} \left(\left(\frac{b-v}{s}\right)_{Q+1} - \left(-\frac{s+v}{s}\right)_{Q+1} \right) - \left(-\frac{v}{s}\right)_Q \right).$$

Since $(a)_{n+1} = (a+n) \cdot (a)_n$ and $(a-1)_n = \frac{a-1}{a+n-1} \cdot (a)_n$, this simplifies to:

$$\begin{aligned} ERev &= \frac{1}{\left(-\frac{v}{s}\right)_Q} \left(s \left(\frac{b-v}{s}\right)_{Q+1} + (s+v) \left(-\frac{v}{s}\right)_Q - (b+s) \left(-\frac{v}{s}\right)_Q \right) \\ &= \frac{s \left(\frac{b-v}{s}\right)_{Q+1}}{\left(-\frac{v}{s}\right)_Q} + (v-b) = \frac{s \left(\frac{b-v}{s} + Q\right) \cdot \left(\frac{b-v}{s}\right)_Q}{\left(-\frac{v}{s}\right)_Q} + (v-b) \end{aligned}$$

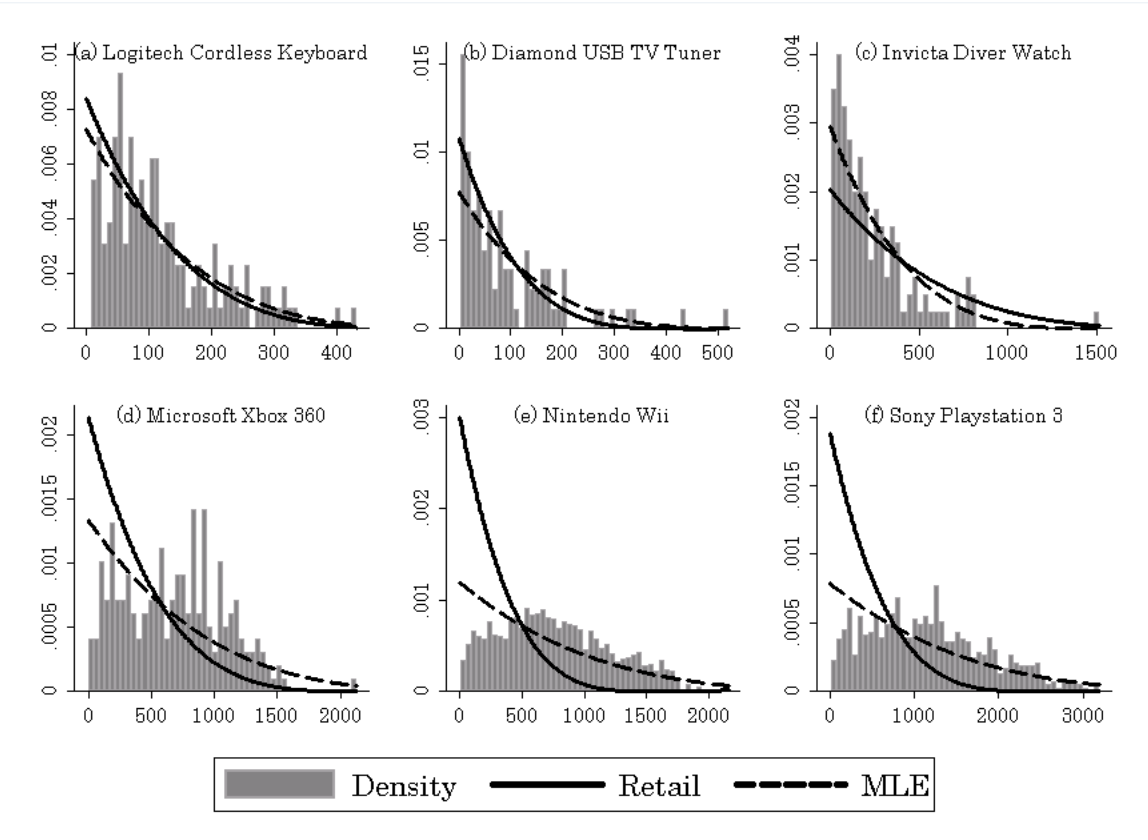
But since $\frac{b-v}{s} + Q = 0$, this yields $ERev = v - b$.

Variance is is similarly derived:

$$\begin{aligned} Var &= \sum_{q=0}^Q ((b+s)q - v + b)^2 f(q) \\ &= (v-b)^2 - 2(v-b) \sum_{q=1}^Q (b+s)q f(q) + \sum_{q=1}^Q (b+s)^2 q^2 f(q) \\ &= -(v-b)^2 + (b+s)^2 \sum_{q=1}^Q q^2 (1 - \mu_{q+1}) \prod_{j=1}^q \mu_j \end{aligned}$$

With similar manipulation as above, this last sum is equal to $\frac{(b-2v)(b-v)}{(b+s)(b+2s)}$. Hence, $Var = \frac{b(v-b)(s+v)}{b+2s}$.

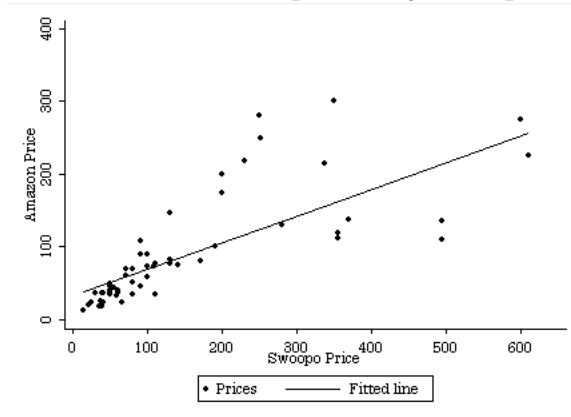
Figure 1: Representative examples: comparison of the theoretical and actual distribution of ending bids for each item.



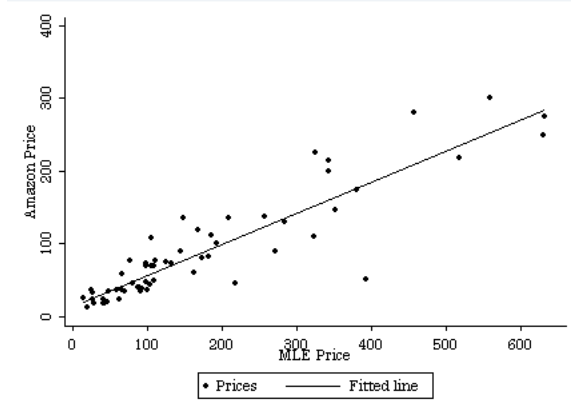
Notes: The unit of observation is the individual auctions for each item. In each figure, the x-axis denotes the final number of bids placed for the item. The y-axis is the probability that the auction concludes at that number of bids. Bars denote the observed frequencies; the solid line gives the theoretical frequencies using the retail value and the dashed line does the same using the MLE estimated value.

Figure 2: Comparison of item valuations to Amazon.com prices

A. Retail value reported by Swoopo



B. MLE-estimated valuation



C. Risk MLE-estimated valuation

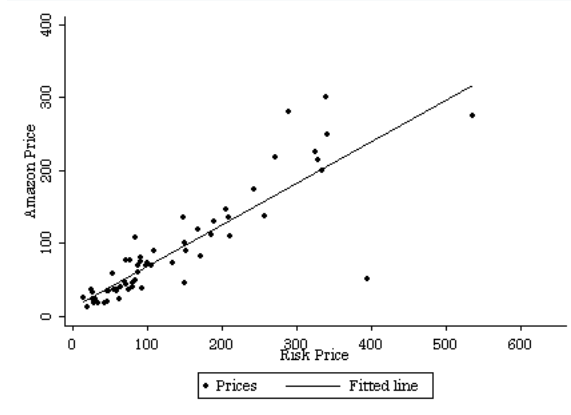
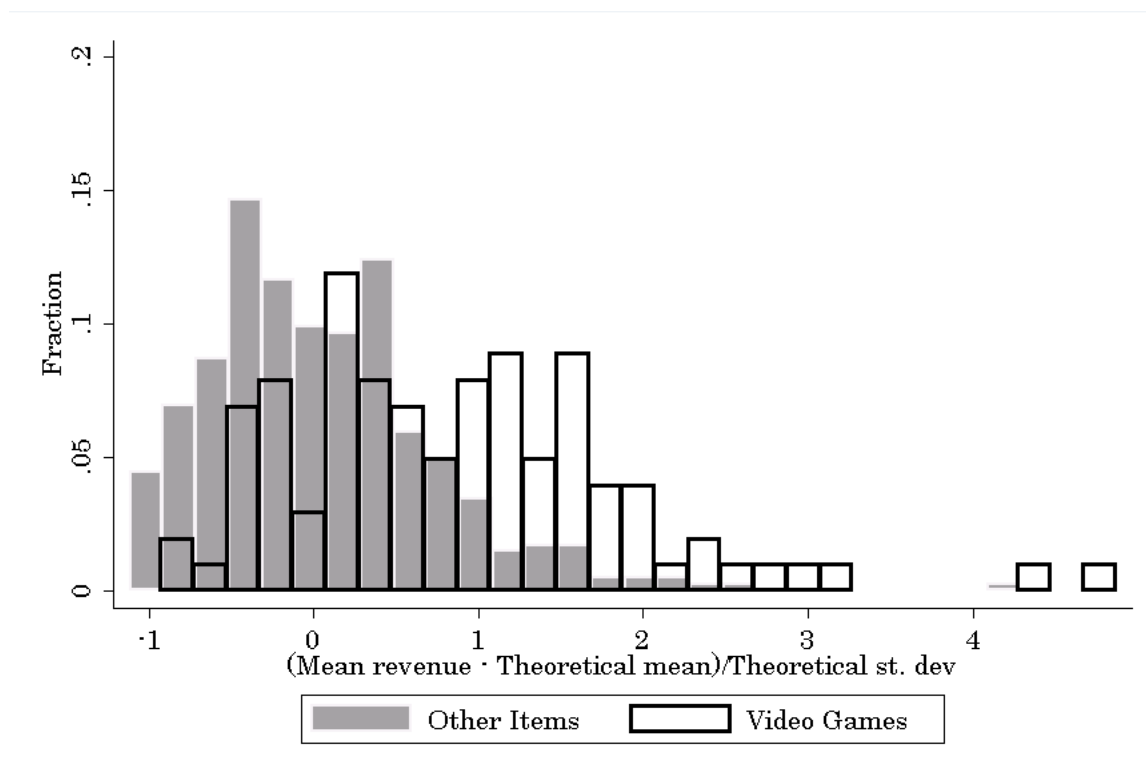


Table 1: Monthly traffic on pay-to-bid websites

Website	# of unique visitors	
	Dec. 2008	Nov. 2009
Swoopo.com	508,237	560,871
Ubid.com	550,802	276,649
Bidcactus.com	0	788,497
Beezid.com	0	325,906
Quibids.com	0	320,998
All other sites	164,792	742,431
Total # of sites	11	35
All Pay-to-Bid Sites	1,223,842	3,015,352
Ebay.com	79,806,830	74,067,835
Pay-to-Bid Site Traffic/Ebay Traffic	1.5 %	4.1 %

Notes: Data from compete.com, a web traffic monitoring company.

Figure 3: Deviation from Expected Revenue in risk neutral model (separately for video games and other items)



Notes: The unit of observation is each unique item. The x-axis indicates by how many standard deviations the observed average revenue differs from the theoretical expected revenue. The y-axis indicates the fraction of items auctioned with that observed outcome.

Table 2: Statistical tests comparing theoretical distribution with observed data.

A. Using retail price as item valuation.

			Pearsons χ^2 Test (compares distributions)			<i>t</i> -Test (compares means)		
Item Type	<i>N</i>	<i>obs</i>	$p \geq .10$	$p \geq .05$	$p \geq .01$	$p \geq .10$	$p \geq .05$	$p \geq .01$
Computers	7	521	0.571	0.571	0.714	0.286	0.429	0.429
Computer Accessories	24	3434	0.375	0.417	0.583	0.292	0.333	0.417
Home Electronics	7	1651	0.286	0.286	0.429	0.143	0.143	0.286
Apparel	14	1045	0.286	0.357	0.357	0.071	0.071	0.143
Movies	2	97	1.000	1.000	1.000	1.000	1.000	1.000
Video Games	18	9831	0.111	0.111	0.222	0.000	0.056	0.167

B. Using MLE valuation of item

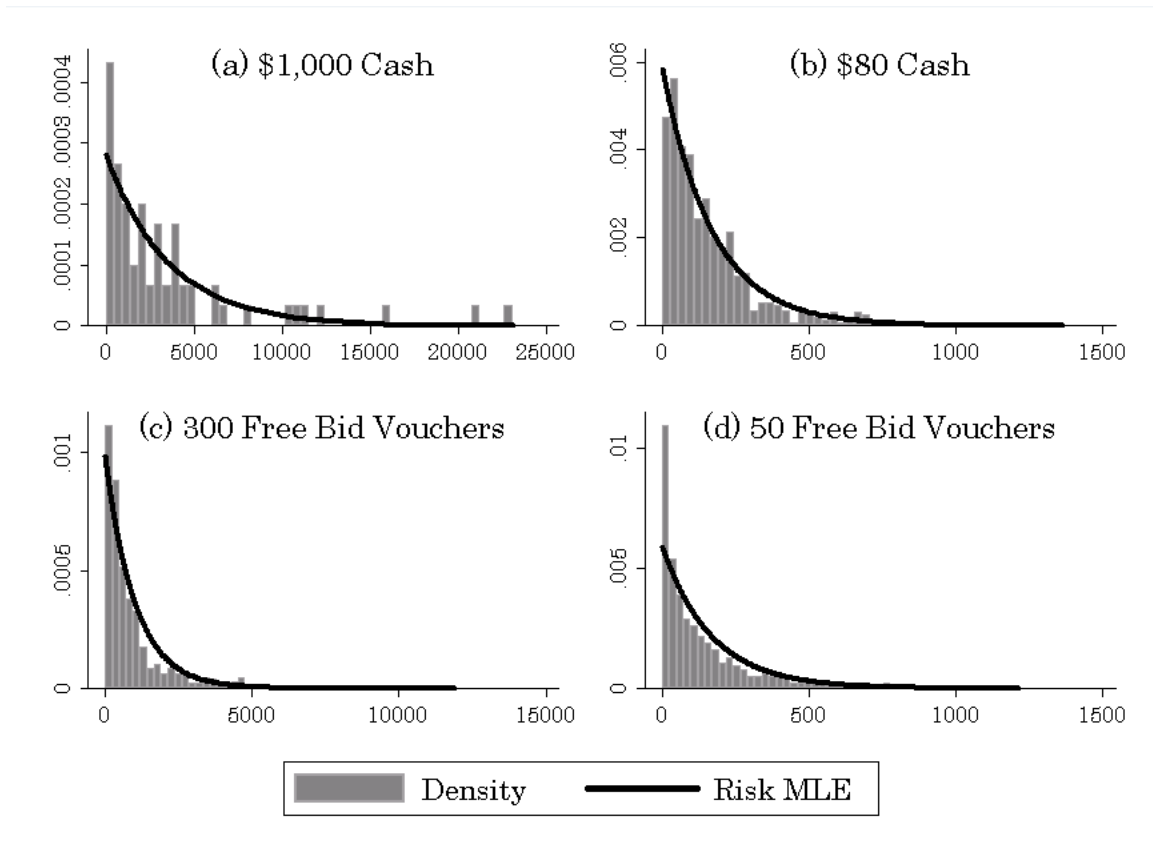
			Pearsons χ^2 Test			<i>t</i> -Test		
Item Type	<i>N</i>	<i>obs</i>	$p \geq .10$	$p \geq .05$	$p \geq .01$	$p \geq .10$	$p \geq .05$	$p \geq .01$
Computers	7	521	1.000	1.000	1.000	1.000	1.000	1.000
Computer Accessories	24	3434	0.708	0.833	0.875	0.833	0.833	0.917
Home Electronics	7	1651	0.714	0.714	0.857	0.857	0.857	0.857
Apparel	14	1045	1.000	1.000	1.000	0.857	1.000	1.000
Movies	2	97	1.000	1.000	1.000	1.000	1.000	1.000
Video Games	18	9831	0.500	0.500	0.611	0.500	0.611	0.667

C. Using MLE risk preferences and valuation of item

			Pearsons χ^2 Test			<i>t</i> -Test		
Item Type	<i>N</i>	<i>obs</i>	$p \geq .10$	$p \geq .05$	$p \geq .01$	$p \geq .10$	$p \geq .05$	$p \geq .01$
Computers	7	521	1.000	1.000	1.000	1.000	1.000	1.000
Computer Accessories	24	3434	0.875	0.917	0.917	0.833	0.917	0.958
Home Electronics	7	1651	1.000	1.000	1.000	1.000	1.000	1.000
Apparel	14	1045	1.000	1.000	1.000	0.769	0.769	0.923
Movies	2	97	1.000	1.000	1.000	1.000	1.000	1.000
Video Games	18	9831	1.000	1.000	1.000	1.000	1.000	1.000

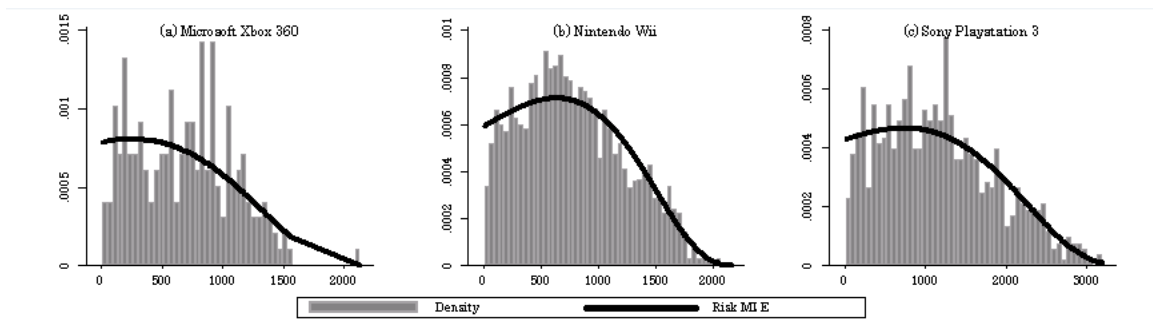
Notes: The number reported in each cell is the fraction of items for which the particular test statistic has a p-value larger than the threshold indicated in each column. *N* refers to the number of unique items in each category, while *obs* refers to the number of observed auctions within that category.

Figure 4: 100% Off Auctions with risk-loving bidders: comparison of the theoretical and actual distribution of ending bids for each item.



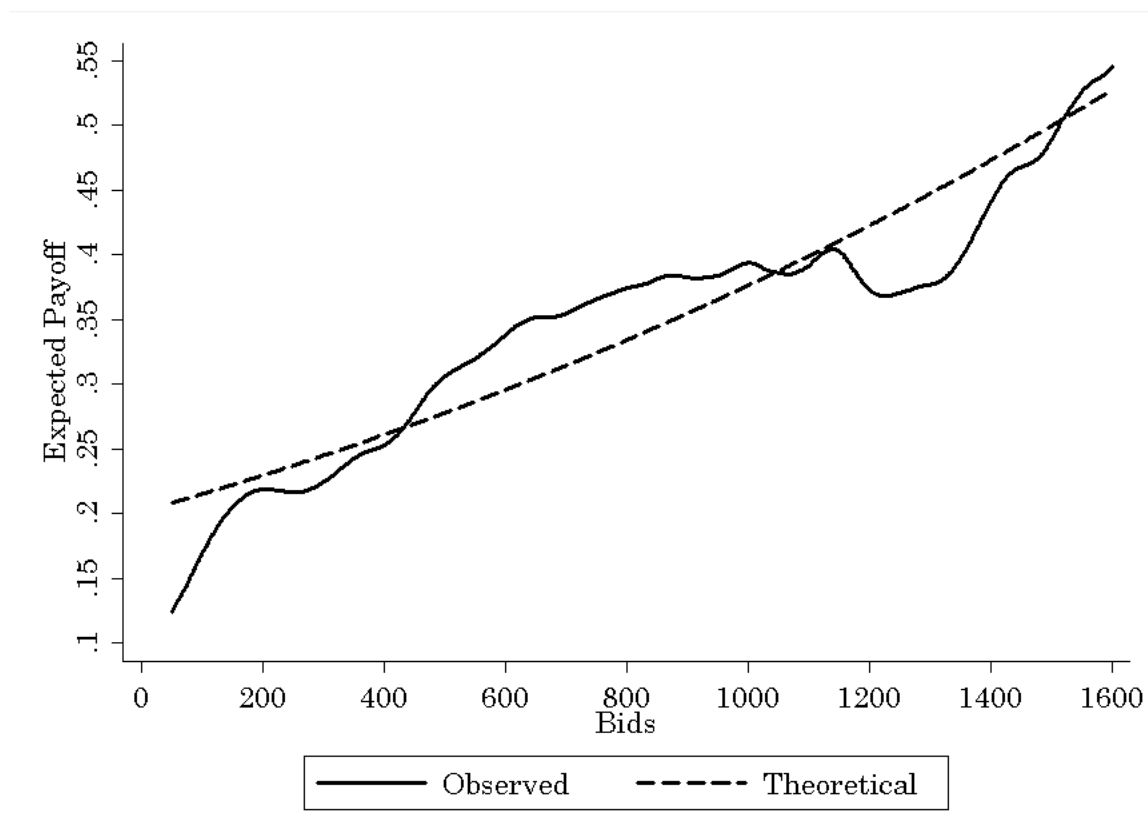
Notes: Risk MLE denotes the theoretical distribution of ending bids when MLE-estimated bidder risk preferences is used; item value is assumed to be retail value.

Figure 5: Standard auctions with risk-loving bidders: comparison of the theoretical and actual distribution of ending bids for each item.



Notes: Risk MLE denotes the theoretical distribution of ending bids when MLE-estimated item value and bidder risk preferences are used.

Figure 6: Expected payoff at each point during the auction for a Wii video game unit.



Notes: The expected payoff is the probability of the auction ending at each bid conditional on the auction arriving at that point times the value of winning the item which is the valuation minus the final price. The expected payoff must equal the bid fee of \$0.75 to be actuarially fair, which the theoretical curve reaches at 2260 bids (*i.e.* when the final price nearly equals the valuation). We smoothed the observed expected payoff function using a kernel regression. Bids above 1600 are excluded from the smoothing due to the sparseness of data in that range (fewer than 5% of the observations).

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