View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Research Papers in Economics

GREQAM
Groupement de Recherche en Economie
Quantitative d'Aix-Marseille - UMR-CNRS 6579
Ecole des Hautes Etudes en Sciences Sociales
Universités d'Aix-Marseille II et III

Document de Travail
n°2010-07

MATCHING WITH PHANTOMS

Bruno DECREUSE

March 2010

DT-GREQAM

halshs-00472751, version 1 - 13 Apr 2010

halshs-00472751, version 1 - 13 Apr 2010

Matching with phantoms*

Bruno Decreuse[†]
GREQAM and University of Aix-Marseilles

This draft: February 2010

Abstract: Searching for partners involves informational persistence that reduces future traders' matching probability. In this paper, traders that are no longer available but who left tracks on the market are called phantoms. I examine a discrete-time matching market in which phantom traders are a by-product of search activity, no coordination frictions are assumed, and non-phantom traders may lose time trying to match with phantom traders. The resulting aggregate matching technology features increasing returns to scale in the short run, but has constant returns to scale in the long run. I discuss the labor market evidence and argue that there is observational equivalence between phantom unemployed and on-the-job seekers.

Keywords: Endogenous matching technology; Intertemporal and intratemporal congestion externalities; Information persistence

JEL classification: J60

*This paper benefited from the comments of participants in the 2009 SED meeting in Istanbul and participants in seminars held at GREQAM and the University of New South Wales. I wish to thank Mohamed Belhaj, Gautam Bose, Frédéric Deroïan, Jan Eeckhout, Cecilia Garcia-Penalosa, Alain Trannoy, and Alain Venditti.

[†]GREQAM - 2, rue de la charité, 13002 Marseille, France. Webpage: E-mail: decreuse@univmed.fr

halshs-00472751, version 1 - 13 Apr 2010

1 Introduction

The matching technology is a popular tool among labor market specialists and macroeconomists. The technology gives the number of jobs formed as an increasing function of the numbers of job-seekers and vacancies. This function is generally well-behaved in that it is strictly concave and has constant returns to scale. Such properties have strong empirical relevance (see Petrongolo and Pissarides, 2001) and are associated with good model outcomes, as with the independence of the unemployment rate vis-à-vis workforce size, and the saddle-path and uniqueness properties of equilibrium under rational expectations. Most of the time, the functional form of the matching technology is exogenous and can hardly be derived from elementary principles. This is unfortunate as changes in the environment like public policies or business cycles may affect the matching technology itself. The problem goes beyond the labor market case and arises whenever people must meet before trade activities take place.

Several papers provide an explicit scenario behind the aggregate matching technology. In mismatch models, workers are imperfectly mobile between sub-markets, and the distribution of traders across sub-markets governs the shape of the aggregate matching technology (see Drèze and Bean, 1990, Lagos, 2000, 2003, and Shimer, 2007). In stock-flow matching models, traders can only match with newcomers (see e.g. Taylor, 1995, Coles and Muthoo, 1998, Coles and Smith, 1998, Coles, 1999, Gregg and Petrongolo, 2005, Coles and Petrongolo, 2008, and Ebrahimy and Shimer, 2008). In urn-ball matching models, buyers independently send one buy order to each seller. As buyers do not coordinate, some sellers receive several buy orders, while others do not receive any order (see e.g. Butters, 1977, Hall, 1977, Burdett et al, 2001, and Albrecht et al, 2004, 2006, and Galenianos and Kircher, 2009, with multiple applications). Stevens (2007) makes explicit the time-consuming nature of search and endogenizes search investments. The resulting technology is CES.

As noted by Stevens (ibid), these papers rely on an implicit limited mobility assumption with an associated coordination problem. Given that workers cannot readily transfer their attention from one job (or sub-market) to another, lack of coordination generates frictions. However, another property is also involved: matching frictions result from intratemporal congestion externalities. Traders on one side of the market deteriorate search prospects for those who are currently on the same side, and improve prospects for those who are currently on the other side. In this paper, I follow the general trend in the rest of the literature as I assume that individuals have limited mobility between potential partners. However, the source of market frictions is no longer contemporaneous. I examine the complementary idea whereby matching frictions can result from informational persistence on the market about traders who have already found a match. I refer to these traders as phantom traders, or phantoms for short. Phantoms are a by-product of the search activity: when exiting the market, each trader may leave a trace that disappears over time. Phantoms result in a loss of time and resources for future traders who want to find an adequate partner. I argue that a matching technology endowed with reasonable properties can be derived from this single source of information imperfection.

There are various reasons why there may be phantom traders on the market. First, search strategies display involuntary persistence. To recruit workers, firms post ads that convey information on job offers. Ads are very useful to attract potential employees, who can thus direct their search towards the corresponding jobs. What happens to this information once a worker is recruited? The ad is likely to persist for some additional time. This may be misleading for workers who lose time and effort in prospecting

halshs-00472751, version 1 - 13 Apr 2010

a job that no longer exists. Similarly, workers send applications and register on websites. Firms may process such applications or consult websites after actual recruitment. The example of monster.fr is particularly enlightening. Ads last for one or two months, even if the position is filled in the very first minute. Firms benefit from a price reduction when they pay for two months. On February 5 2010, placing a single ad for one month cost 560€. The cost for two months was 650€. The phenomenon is probably more pervasive for free websites where the site maker has fewer incentives to clean out old ads. Second, match makers may voluntarily delay the moment they delete information about traders that have left the market. Dating websites may keep online profiles for months or even years after the person has logged on for the last time. Estate agents may showcase sold or rented houses or flats. This strategy aims at attracting customers by making the number of potential traders bigger than it really is. Third, matched traders may be incited to go on searching even though they do not want to find another trading partner. Firms that have filled in their jobs may post ads to accumulate a stock of potential applicants in case they have new vacancies. Married persons may enter a romantic online relationship without willing to go further – they certainly feel matched with their online partner, but what about this person? Finally, on-the-match seekers can be considered as phantom traders from the perspective of the unmatched. Employees for instance contact alternative employers to put pressure on their current employer to grant a pay rise. Doing so, they may create additional congestion for the unemployed.[1]

I consider a generic situation. Time is discrete and buyers and sellers try to contact each other on a unique search place. To disentangle the impacts of phantoms on the search market from more standard congestion externalities, I assume that each buyer meets one seller at most, and every trader on the short side of the market is sure to meet someone. Unfortunately, that someone may be a phantom buyer, or a phantom seller. No trade takes place in such cases. I assume that the populations of phantoms obey simple flow-stock equations, the inflow of new phantoms being proportional to the past outflow of successful traders. I examine the resulting matching pattern between the two populations of traders.

I refer to the aggregate matching technology as the phantom matching technology, or PMT for short. The PMT features intratemporal and intertemporal externalities. Intratemporal externalities result from the fact that an increase in the number of agents on the long side of the market reduces the proportion of phantom traders. A larger proportion of contacts leads to matches as a result. Intratemporal externalities imply that the PMT displays increasing returns to scale in the short run. Intertemporal externalities result from the fact that current matches fuel future phantom traders. Although period-t number of traders may have an ambiguous impact on period-$(t+k)$ number of matches, intertemporal externalities combine so as to negatively affect the current number of matches. Intratemporal and intertemporal externalities balance each other, and the PMT features constant returns to scale vis-à-vis the whole set of current and past traders.

The interplay between intratemporal and intertemporal matching externalities has two implications. First, I discuss the stationary phantom matching technology (SMPT) that emerges as the steady-state PMT of an environment where the populations of traders are themselves stationary. The SMPT obeys a simple parametric form that depends on the entry rate of new phantoms and phantom death probability. The SMPT exhibits constant returns to scale. The elasticity of the matching technology vis-à-vis the number of traders on the short side of the market depends on the ratio of sellers to buyers (negatively

[1] The consideration of on-the-job search is of course not new. What is new is the assimilation of on-the-job search with the more general phenomenon of informational persistence on matching markets.

halshs-00472751, version 1 - 13 Apr 2010

if sellers are on the short side, and positively otherwise). This elasticity belongs to the interval $(1/2, 1)$. Second, I examine the effects of a temporary increase in the number of traders on the short side of the market. Owing to short-run increasing returns to scale, the temporary shock generates a matching boom in the short run. The matching boom then gives birth to phantom traders that alter the matching pattern. As the boom stops, the market is left with many more phantoms and fewer matches take place than prior to the shock. Matching probabilities gradually converge towards their steady-state values.

I further discuss the PMT through four extensions to the basic model. The first extension is devoted to the honeymoon effect that benefits new markets. New markets have no history, and feature no phantoms. Matching probabilities start very high as a result. Then, phantoms accumulate and matching probabilities deteriorate. The second extension considers another popular source of market frictions, namely coordination frictions. This allows a distinction to be made between the respective contributions of phantom traders and coordination frictions to overall matching frictions. The third extension examines the empirical implications of the PMT. The discussion is based on a general model that admits the standard Cobb-Douglas matching technology and the nonfrictional technology as particular cases. The final extension discusses the case of on-the-match search. I argue that on-the-match seekers can be considered as phantoms in the PMT framework. This implies that phantoms and on-the-match seekers are observationally equivalent. Consequently, papers studying the labor market and highlighting the role played by on-the-job search on the aggregate matching technology provide indirect evidence in favor of the phantom trader thesis.

The rest of the paper is organized as follows. Section 2 introduces the model and computes the resulting matching technology. Section 3 analyzes the interplay between intratemporal and intertemporal externalities. Section 4 discusses the honeymoon effect, studies the interplay between phantom traders and coordination frictions, looks at empirical implications, and analyzes the case of on-the-job search.

2 The model

Time is discrete and denoted by t. A population of buyers and sellers want to trade with each other. But they have to meet before trade takes place. Matching takes place every period. Every time a buyer and a seller meet and agree on match formation, they exit the market.

Let B denote the (mass) number of buyers, S the number of sellers, P^B the number of phantom buyers, and P^S the number of phantom sellers.

The matching mechanism involves two steps. In a first step, each trader on the short side of the market is assigned to a trader on the long side. This results in the following number of contacts:

$$\min\left\{B_t + P_t^B, S_t + P_t^S\right\} \tag{1}$$

In a second step, matches are derived from contacts. The rule is that only contacts between non-phantom traders lead to effective trade. The number of matches is

$$M_t = \frac{B_t}{B_t + P_t^B}\frac{S_t}{S_t + P_t^S}\min\left\{B_t + P_t^B, S_t + P_t^S\right\} \tag{2}$$

The number of contacts is multiplied by the product of the two proportions of non-phantom traders. I assume that phantoms cannot be distinguished from non-phantoms by the matching mechanism.

halshs-00472751, version 1 - 13 Apr 2010

Matching probabilities are

$$\mu_t = \frac{M_t}{B_t} = \frac{S_t}{S_t + P_t^S} \min\left\{1, \frac{S_t + P_t^S}{B_t + P_t^B}\right\} \tag{3}$$

$$\eta_t = \frac{M_t}{S_t} = \frac{B_t}{B_t + P_t^B} \min\left\{\frac{B_t + P_t^B}{S_t + P_t^S}, 1\right\} \tag{4}$$

The numbers of phantoms obey the following laws of motion:

$$P_t^B = \beta^B M_{t-1} + \left(1 - \delta^B\right) P_{t-1}^B \tag{5}$$

$$P_t^S = \beta^S M_{t-1} + \left(1 - \delta^S\right) P_{t-1}^S \tag{6}$$

with $\beta^j > 0$, and $0 < \delta^j \leq 1$, $j = B, S$. The inflow of new phantoms is proportional to former matches. The parameter β^j can be interpreted as the probability that a match gives birth to a phantom trader, or as the relative search efficiency of phantoms vis-à-vis non-phantoms. In the former case, $\beta^j \leq 1$. In the latter case, there are no additional restrictions on β^j. The outflow results from constant depreciation at rate δ^j. Phantoms face a constant probability of dying δ^j each period. Life expectancy follows a Poisson law.

Proposition 1 *In each period t, the number of matches is given by*

$$\ln M_t = \ln S_t + \ln B_t - \ln\left[X_t + \beta_t \sum_{k=0}^{\infty} (1 - \delta_t)^k M_{t-k-1}\right] \tag{PMT}$$

with $\begin{cases} \beta_t = \beta^B,\ \delta_t = \delta^B \text{ and } X_t = B_t \text{ if } \min\left\{B_t + P_t^B, S_t + P_t^S\right\} = S_t + P_t^S \\ \beta_t = \beta^S,\ \delta_t = \delta^S \text{ and } X_t = S_t \text{ if } \min\left\{B_t + P_t^B, S_t + P_t^S\right\} = B_t + P_t^B \end{cases}$.

Proof. Suppose that $\min\left\{B_t + P_t^B, S_t + P_t^S\right\} = S_t + P_t^S$. This yields the following matching technology

$$M_t = \frac{S_t B_t}{B_t + P_t^B} \tag{7}$$

We have

$$P_t^B = \beta^B \sum_{k=0}^{\infty} \left(1 - \delta^B\right)^k M_{t-k-1} \tag{8}$$

This gives

$$\ln M_t = \ln S_t + \ln B_t - \ln\left[B_t + \beta^B \sum_{k=0}^{\infty} \left(1 - \delta^B\right)^k M_{t-k-1}\right] \tag{9}$$

Now suppose that $\min\left\{B_t + P_t^B, S_t + P_t^S\right\} = B_t + P_t^B$. This yields the following matching technology

$$M_t = \frac{B_t S_t}{S_t + P_t^S} \tag{10}$$

We have

$$P_t^S = \beta^S \sum_{k=0}^{\infty} \left(1 - \delta^S\right)^k M_{t-k-1} \tag{11}$$

This gives

$$\ln M_t = \ln S_t + \ln B_t - \ln\left[S_t + \beta^S \sum_{k=0}^{\infty} \left(1 - \delta^S\right)^k M_{t-k-1}\right] \tag{12}$$

This closes the proof.

The phantom matching technology (PMT) collapses into the usual non-frictional technology whenever $\beta_t = 0$. The novelty comes from the inclusion of the weighted sum of former matches in the last term. The weights depend on survival probabilities $(1-\delta_t)^k$ and entry rate of new phantoms β_t.

Market history may start at a finite date, say $t = 0$, without loss of generality. Equation (PMT) must be modified accordingly:

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[X_t + \beta_t \sum_{k=0}^{t-2} (1-\delta_t)^k M_{t-k-1} + \beta_t (1-\delta_t)^{t-1} P_0^t \right] \tag{13}$$

where the relevant initial number of phantoms is $P_0^t = \begin{cases} P_0^S \text{ if } \min\left\{B_t + P_t^B, S_t + P_t^S\right\} = S_t + P_t^S \\ P_0^B \text{ if } \min\left\{B_t + P_t^B, S_t + P_t^S\right\} = B_t + P_t^B \end{cases}$.

The role played by market history is parameterized by δ_t. As δ_t tends to 0, phantoms are almost infinite-lived and old phantoms have a large impact on current matches. Conversely, with full depreciation $\delta^B = \delta^S = 1$, phantoms live for one period and the PMT reduces to

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[X_t + \beta_t M_{t-1}\right] \tag{14}$$

3 Intertemporal vs intratemporal externalities

In this section, I examine the matching externalities featured by the phantom matching technology. The combination of intratemporal and intertemporal externalities implies that the technology has increasing returns to scale in the short run and constant returns in the long run. I study these properties in three steps.

3.1 Intratemporal externalities

Proposition 2 *Without loss of generality, assume that $S_t + P_t^S < B_t + P_t^B$ for all t. In each period t,*

(i) $d \ln M_t / d \ln S_t = 1$

(ii) $d \ln M_t / d \ln B_t = \dfrac{\beta^B \sum_{k=0}^{\infty} (1-\delta^B)^k M_{t-k-1}}{B_t + \beta^B \sum_{k=0}^{\infty} (1-\delta^B)^k M_{t-k-1}}$

Proof. This results from direct computation.

The phantom matching technology has constant returns with respect to the number of traders on the short side of the market. This property is typical of non-frictional matching models. Meanwhile, the PMT has positive returns with respect to the number of traders on the long side. The reason is that additional traders reduce the proportion of phantom traders. The more phantoms there are, the greater the effect.

Intratemporal externalities imply that the matching technology exhibits increasing returns to scale in the short run. Indeed, $d \ln M_t / d \ln S_t + d \ln M_t / d \ln B_t > 1$. The magnitude of increasing returns to scale is parameterized by β^B (driving phantom births), δ^B (governing phantom deaths), and by the history of matching flows $\{M_{t-k-1}\}_{k=0}^{\infty}$ (fueling potential phantoms).

halshs-00472751, version 1 - 13 Apr 2010

halshs-00472751, version 1 - 13 Apr 2010

3.2 Intertemporal externalities

Proposition 3 *Without loss of generality, assume that* $S_t + P_t^S < B_t + P_t^B$ *for all* t. *In each period* t,

(i) $\partial \ln M_t / \partial \ln M_{t-k-1} = -\frac{\beta^B \left(1-\delta^B\right)^k M_{t-k-1}}{B_t + \beta^B \sum\limits_{k=0}^{\infty} (1-\delta_t)^k M_{t-k-1}}$

(ii) $d \ln M_{t+k} / d \ln B_t = \sum\limits_{j=0}^{k-1} \left(\partial \ln M_{t+k} / \partial \ln M_{t+j}\right) \left(d \ln M_{t+j} / dB_t\right)$

(iii) $d \ln M_{t+k} / d \ln S_t = \sum\limits_{j=0}^{k-1} \left(\partial \ln M_{t+k} / \partial \ln M_{t+j}\right) \left(d \ln M_{t+j} / dS_t\right)$

(iv) $\sum_{k=1}^{\infty} \left(d \ln M_t / d \ln B_{t-k} + d \ln M_t / d \ln S_{t-k}\right) = -\frac{\beta^B \sum\limits_{k=0}^{\infty} \left(1-\delta^B\right)^k M_{t-k-1}}{B_t + \beta^B \sum\limits_{k=0}^{\infty} \left(1-\delta^B\right)^k M_{t-k-1}}$

Proof. Points (i) to (iii) result from direct computation. Point (iv) results from the fact that $d \ln M_t / d \ln M_{t-1} = \sum_{k=1}^{\infty} \left(d \ln M_t / d \ln B_{t-k} + d \ln M_t / d \ln S_{t-k}\right)$.

Former matches generate phantom traders. In turn, phantoms deteriorate the current matching process. These intertemporal externalities imply that the whole market history affects current matches. Intertemporal externalities are characterized by points (ii) and (iii).

Current matches may positively or negatively alter future matches. To understand this property, I consider the case where phantoms only last one period, i.e. $\delta^B = \delta^S = 1$. Then,

$$d \ln M_{t+k} / d \ln B_t = (-1)^k \prod_{j=0}^{k-1} \frac{\beta^B M_{t+j}}{B_{t+j} + \beta^B M_{t+j}} \tag{15}$$

The magnitude of this elasticity decreases with horizon period k. Its sign depends on $(-1)^k$, which is negative for even k and positive for odd k. An increase in the number of period-t traders increases the number of period-$t+1$ phantoms, thereby reducing the flow of matches in period $t+1$. For a similar reason, this increases the flow of matches in period $t+2$.

Point (iv) shows that the sum of intertemporal externalities is negative. This compensates for the positive intratemporal externality that is discussed in Proposition 3. Intratemporal and intertemporal externalities combine so that the matching technology has constant returns to scale with respect to the whole set of current and former traders.

3.3 Stationary phantom matching technology

I assume that whenever a buyer and a seller get matched they are replaced by a similar pair of agents. I show that the phantom matching technology (PMT) converges towards a stationary technology, the stationary phantom matching technology (SPMT).

The number of traders follows $B_t = B$ and $S_t = S$ for all t. The number of matches follows the PMT. Without loss of restriction, sellers are on the short side of the market and

$$\ln M_t = \ln S + \ln B - \ln \left[B + \beta^B \sum_{k=0}^{\infty} \left(1 - \delta^B\right)^k M_{t-k-1} \right] \tag{16}$$

halshs-00472751, version 1 - 13 Apr 2010

Proposition 4 *Let $B + P_t^B > S + P_t^S$ for all t. The sequence M_t converges towards the stationary number of matches*

$$M = m(B, S) = B \frac{-1 + \left[1 + 4\beta^B/\delta^B (S/B)\right]^{1/2}}{2\beta^B/\delta^B} \tag{SPMT}$$

Proof. In steady state, $M_t = M$ and solves

$$\beta^B M^2/\delta^B + BM - BS = 0$$

Resolution gives (SPMT). To establish convergence, note that $M_t = BS/(B + P_t)$. This implies that the sequence $\{M_t\}$ converges towards M if and only if the sequence $\{P_t\}$ converges towards $P = \beta^B M/\delta^B$. But,

$$P_{t+1} = \phi(P_t) \tag{17}$$

with $\phi(x) = \beta^B BS/(B + x) + (1 - \delta)x$. As $\phi(0) > 0$ and $0 < \phi'(x) < 1$ for all $x \geq 0$, $\{P_t\}$ converges towards P for all $\beta \geq 0$ and all $\delta \in (0, 1]$.

The SPMT features standard properties. First, it is strictly increasing in the numbers of traders on each market side. Second, it has constant returns to scale. This property results from the constant intertemporal returns to scale discussed previously. Third, the elasticity of the matching technology with respect to the ratio of sellers to buyers is $\varepsilon(S/B) = \frac{2(\beta^B/\delta^B)S/B}{-(1+4(\beta^B/\delta^B)S/B)^{1/2}+(1+4(\beta^B/\delta^B)S/B)} \in (1/2, 1)$. This elasticity decreases with $\left(\beta^B/\delta^B\right) S/B$. If buyers were on the short side of the market, the elasticity would be decreasing in $\left(\beta^B/\delta^B\right) S/B$.

3.4 Dynamic implications

I consider a temporary increase in the number of sellers. This allows the results shown by Propositions 2 to 4 to be illustrated. From time t_0 to time $t_1 > t_0$, the number of sellers goes from S to $S(1 + \varepsilon)$. It then returns to S. Initial numbers of phantoms are set at their stationary numbers. I distinguish three different matching technologies. In all cases, I consider deviations vis-à-vis the log of the stationary number of matches $\ln M$:

$$\ln M_t/M = \ln S_t/S + \ln[B + M] - \ln\left[B + .5\sum_{k=0}^{\infty}(.5)^k M_{t-k-1}\right] \tag{PMT1}$$

$$\ln M_t/M = \ln S_t/S + \ln[B + M] - \ln[B + M_{t-1}] \tag{PMT2}$$

$$\ln M_t/M = \ln\frac{-1 + [1 + 4(S_t/B)]^{1/2}}{2} - \ln S - \ln[B + M] \tag{SPMT}$$

In technology PMT1, half of the matches give birth to phantom traders and the depreciation rate is 50%, i.e. $\beta^B = .5$ and $\delta^B = .5$. This technology has unlimited memory. In technology PMT2, all matches originate phantom traders, but phantoms only last one period, i.e. $\beta^B = 1.0$ and $\delta^B = 0$. This technology has limited memory. The technology SPMT is the stationary phantom matching technology corresponding to PMT1 and PMT2. This technology does not depend on former matches.

The stationary numbers of traders are $B = 2.0$ and $S = 1.0$. The shock consists of a 10% increase in the number of sellers, i.e. $\varepsilon = .1$. Initial numbers of phantoms are set at their stationary values.

halshs-00472751, version 1 - 13 Apr 2010

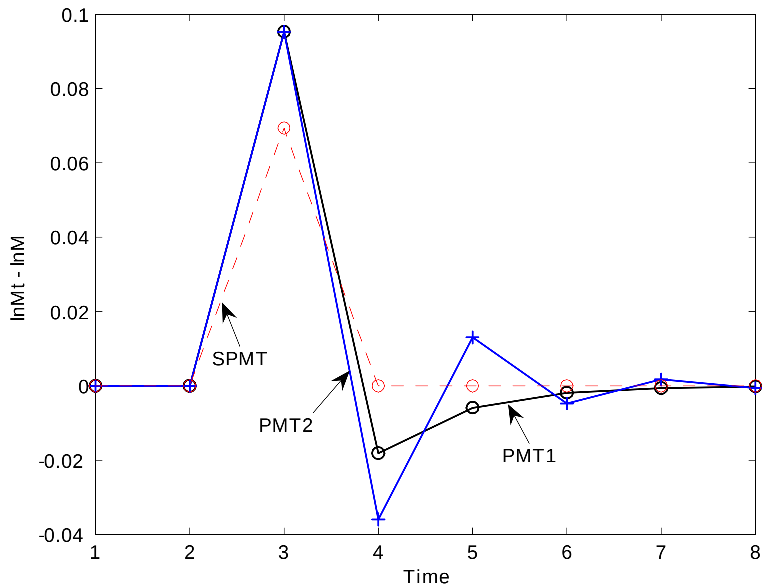

Figure 1: Changes in buyers' matching probability following a one-period shock. The shock takes place at time $t_0 = 3$. Initial conditions: $S = 1.0$, $B = 1.0$, $M_t = M$ for all $t < t_0$. Shock $\varepsilon = .1$. Parameters are $\beta^B = .5$ and $\delta^B = .5$ in the case PMT1, and $\beta^B = 1.0$ and $\delta^B = 0$ in the case PMT2.

I first consider a one-period shock. The shock takes place at period $t_0 = 3$. Figure 1 depicts the resulting trajectories of buyers' matching probabilities. With the SPMT, the matching probability increases at the time of the shock, and subsequently goes down to its stationary value. The elasticity of the matching probability with respect to the ratio S/B is about .7. With the other technologies, Proposition 2 shows that the short-run elasticity of the matching probability with respect to S/B is one. This result explains why the spike at the time of the shock is higher with PMT1 and PMT2 than with the SPMT. Changes in the phantom proportion then alter the matching probabilities, which converge towards the SPMT. The matching probability undershoots its long-run value at period $t_0 + 1 = 4$. With PMT1, phantoms die at a constant rate, and there is monotonic convergence towards the steady-state value. With PMT2, phantoms only last one period. This implies oscillations of decreasing magnitude around the steady-state value, as discussed after Proposition 3.

I then consider a five-period shock. The shock occurs from $t_0 = 3$ to $t_1 = 7$. Figure 2 shows that the phantom matching technologies rapidly converge towards the SPMT. This implies oscillations with PMT2, and monotonic convergence with PMT1. Both technologies originate the same negative effect in period $t = 8$, that is once the negative shock has elapsed. Technology PMT1 compensates a low phantom birth rate by a large survival probability. Overall, the stock of phantoms is the same for PMT1 and PMT2 in $t = 8$.

These examples illustrate two general phenomena. First, the matching technology has increasing returns to scale in the short run. The PMT magnifies temporary shocks with respect to matching technologies that have constant returns to scale in the short run. Second, the accumulation of phantoms and the resulting negative intertemporal externality imply that matching probabilities fall below their

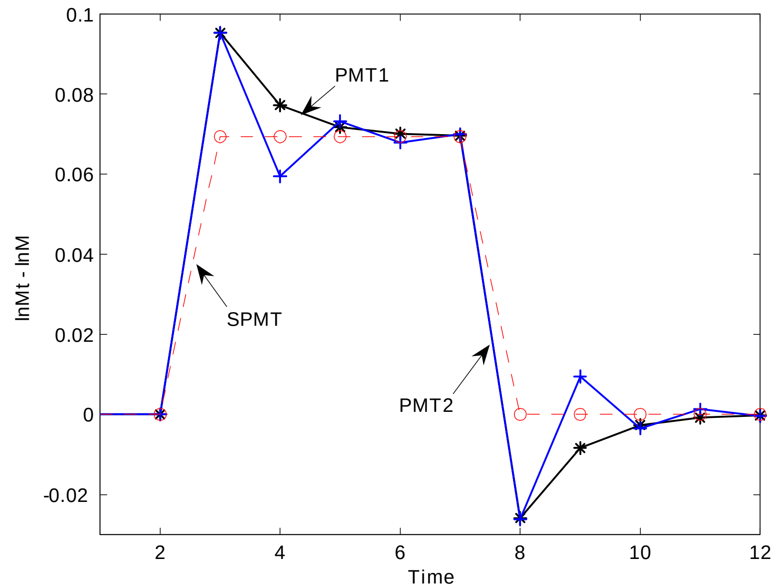

Figure 2: Changes in buyers' matching probability following a five-period shock – The shock takes place at time $t_0 = 3$ and lasts until $t_1 = 7$. Initial conditions: $S = 1.0$, $B = 1.0$, $M_t = M$ for all $t < t_0$. Shock $\varepsilon = .1$. Parameters are $\beta^B = .5$ and $\delta^B = .5$ in the case PMT1, and $\beta^B = 1.0$ and $\delta^B = 0$ in the case PMT2.

stationary level after the shock.

4 Discussions

I discuss four aspects of the phantom matching technology (PMT). First, I argue that a new matching place benefits from a honeymoon effect because there are no phantoms haunting the place. Second, I augment the model with another source of matching frictions, namely coordination frictions. Third, I turn to empirical implications. Finally, I compare the PMT framework to matching technologies that account for on-the-match search.

4.1 Market birth and the honeymoon effect

Given increasing returns to scale in the short run, a new market benefits from a honeymoon effect. Without phantoms in the very beginning of market history, traders easily get matched. However, the phantom stock grows and the matching technology deteriorates.

I consider the case where the total population of matched and unmatched agents is fixed. This case corresponds to the marriage market, with an equal number of men and women. Suppose that a new marketplace opens. Then, N men and N women enter the market, with S_0 individuals unmatched (singles) and $N - S_0$ matched (in couples) on each side of the market. Matched men and matched women originate phantoms with equal probability β, and phantoms of both gender die with equal probability δ.

halshs-00472751, version 1 - 13 Apr 2010

halshs-00472751, version 1 - 13 Apr 2010

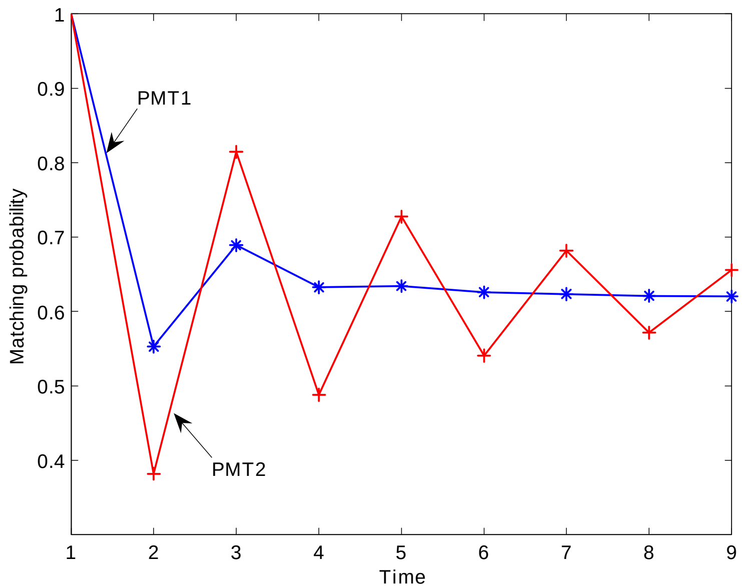

Figure 3: Parameters are $\beta^B = .5$ and $\delta^B = .5$ in the case PMT1, and $\beta^B = 1.0$ and $\delta^B = 0$ in the case PMT2.

The initial number of phantoms is 0. Once matched, men and women enjoy the benefits from being in couple until they separate. The separation probability is q.

Populations of traders obey the following motions:

$$S_t = S_{t-1} - M_{t-1} + q\left(N - S_{t-1}\right) \tag{18}$$

$$\ln M_t = 2\ln S_t - \ln\left[S_t + \beta \sum_{k=0}^{t-2} (1-\delta)^k M_{t-k-1}\right] \tag{19}$$

In steady-state, $S_t = S$ and $M_t = M$. This gives $M = m(S,S)$ and $S = (qN - M)/q$. It follows that

$$\frac{M}{S} = \frac{-1 + (1 + 4\beta/\delta)^{1/2}}{2\beta/\delta} \tag{20}$$

$$S = qN/\left[\frac{-1 + (1 + 4\beta/\delta)^{1/2}}{2\beta/\delta} + q\right] \tag{21}$$

I assume that the initial population of singles is the steady-state population, i.e. $S_0 = S$. The total population N of each gender is normalized to 1. I consider the two matching technologies PMT1 and PMT2 used in subsection 3.4. PMT1 corresponds to $\beta = 1$ and $\delta = 1$. PMT2 corresponds to $\beta = .5$ and $\delta = .5$. Figure 3 depicts the resulting patterns of the matching probabilities. These patterns feature the honeymoon effect. Without phantoms, the first-period matching probability is one. The matching probability subsequently falls and converges towards its stationary value.The honeymoon effect may apply to various match-making industries, as with online dating, real estate, or even temporary work agencies. This effect predicts that newcomers in those markets may build on their initial advantage and easily conquer market shares in a first step. However, they should suffer from negative intertemporal externalities in a second step, leading to high mortality rates. The honeymoon effect may also contribute to explaining why old and established match makers are not necessarily very efficient despite their experience and visibility for the unmatched traders.

4.2 Phantoms and coordination frictions

I examine how phantom traders interact with an alternative source of market frictions. In the urn-ball matching (UBM) model, agents on one side of the market try to contact agents on the other side. However, they do not coordinate, resulting in coordination frictions. The PMT framework and the UBM model complete each other so as to offer a rich description of market frictions.

Assume that each buyer, including phantoms, sends a buy order to one of the sellers, including phantoms. The probability that a particular seller receives a buy order from a particular buyer is $1/\left(S+P^S\right)$. Where a seller receives multiple offers, two cases must be analyzed. Either the seller can detect phantom buyers or he cannot.

If the seller can detect phantom buyers, the number of matches is

$$M = S\left[1-\left(1-\frac{1}{S+P^S}\right)^B\right] \tag{22}$$

As $B, S, P^S \to \infty$, this gives

$$M = S\left[1-\exp\left(-\frac{B}{S+P^S}\right)\right] \tag{23}$$

This technology still features increasing returns to scale vis-à-vis B and S, as an increase in S allows the phantom proportion on the sellers' side to be reduced. In the long run, the SPMT is

$$M = S\left[1-\exp\left(-\frac{B}{S+\beta^S M/\delta^S}\right)\right] \tag{24}$$

This equation implicitly defines $M = m(B,S)$. The SPMT has constant returns to scale.

If the seller cannot detect phantom buyers, the number of matches is

$$M = \frac{B}{B+P^B} S\left[1-\exp\left(-\frac{B+P^B}{S+P^S}\right)\right] \tag{25}$$

The corresponding SPMT is

$$M = \frac{B}{B+\beta^B M/\delta^B} S\left[1-\exp\left(-\frac{B+\beta^B M/\delta^B}{S+\beta^S M/\delta^S}\right)\right] \tag{26}$$

The implicit function $M = m(B,S)$ also features constant returns to scale. The latter technology highlights the contributions of phantom traders and coordination frictions to overall market frictions. The term $BS/\left(B+\beta^B M/\delta^B\right)$ captures the direct role played by phantom traders, while the term $1-\exp\left(-\frac{B+\beta^B M/\delta^B}{S+\beta^S M/\delta^S}\right)$ relies on coordination frictions. Coordination frictions themselves are parameterized by the stocks of phantoms on each market side.

4.3 Empirical implications

The PMT can be confronted to labor market data. Without loss of generality, let buyers be the unemployed and sellers be the vacancies, and consider the case where phantom traders are the only source of

halshs-00472751, version 1 - 13 Apr 2010

halshs-00472751, version 1 - 13 Apr 2010

frictions. For simplicity, I consider the case where the phantom and non-phantom unemployed always outnumber the sum of phantom and non-phantom vacancies.[2]

Assuming that (i) the number of matches that take place in t can only be observed in $t+1$, (ii) S_t the number of registered vacancies is proportional to the actual stock of vacancies (that includes nonregistered vacancies), and (iii) there is unbiased measurement error on the number of sellers/vacancies, the statistical model can be expressed as follows:

$$\ln M_{t+1} = \alpha_0 + \alpha_1 \ln S_t + \alpha_2 \ln B_t - \alpha_3 \ln \left[B_t + \beta \sum_{k=1}^{K} (1-\delta)^k M_{t-k} \right] + \omega_t \tag{27}$$

where ω_t is the error term. The number of lags has been arbitrarily limited to some constant $K \geq 1$ so that the model can be estimated. When $K = 1$, parameters β and δ cannot be identified and phantoms last one period. The constant α_0 is due to the fact that S_t does not measure the total number of vacancies.

The model (27) allows the PMT to be tested against popular alternatives. When $\alpha_1 = 1 - \alpha_2 > 0$ and $\alpha_3 = 0$, the matching technology is Cobb-Douglas with constant returns to scale. When $\alpha_1 = 1$, $\alpha_2 = \alpha_3$ and $\beta = 0$, matching is non-frictional and the number of matches equals the number of vacancies. Finally, the PMT results when $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and $\beta > 0$.

Of course, I do not expect the restrictions $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and $\beta > 0$ to hold. The PMT abstracts from many other sources of matching frictions that have been emphasized in the literature, like geographic and skill mismatch or coordination frictions. Adding those various complementary matching frictions would modify the theoretical matching technology, and the resulting technology could be compatible, for instance, with α_1, α_2, and α_3 differing from one. However, the key particularity of the PMT is the presence of the lagged numbers of matches among the explicative variables. These variables must negatively affect the current number of matches, creating the type of dynamic externalities emphasized in this paper.

In addition to alternative sources of market frictions, I could consider alternative technologies of phantom formation / dissolution. The phantom stock obeys a linear dynamic equation, i.e. current phantom stock equals former stock minus depreciation plus new phantoms. A more general formulation would be

$$P_t = f\left(M_{t-1}, P_{t-1}\right) \tag{28}$$

Additional restrictions need to be imposed on function f. First, f must be increasing in the number of matches M_{t-1} and in the former phantom stock P_{t-1}. Second, to avoid the phantom stock growing to infinity, the partial derivative with respect to P_{t-1} must be less than one. Moreover, f must have constant returns to scale. Indeed, the stationary number of phantoms solves $P = f(M, P)$. Owing to the fact that $0 < f_P < 1$, P can be expressed as an increasing function of the stationary number of matches M. That is $P = g(M)$. The SPMT can be written

$$M = \frac{BS}{B + g(M)} \tag{29}$$

The PMT has constant returns to scale if and only if $g(M) = Mg(1)$. Put otherwise, g must be linear in M. In turn, this restriction implies that f must have constant returns to scale.

[2] If this condition were not satisfied the estimation technique would have to take into account the regime change that may occur when the short side of the market becomes the long side.

halshs-00472751, version 1 - 13 Apr 2010

Alternative technologies of phantom formation would affect model (27). The sequence $\{M_{t-k}\}_{k=1}^{K}$ would enter in a non-additive way. However, the key property of the PMT would be preserved: former matches diminish the number of current matches.

4.4 Phantoms and on-the-match search

On-the-match seekers form a particular type of phantom traders. In this subsection, I make two points. On the one hand, the PMT is a natural framework to analyze on-the-match search. On the other hand, usual empirical strategies to account for on-the-job search fail to distinguish on-the-job seekers from other types of phantom traders.

On-the-match search occurs when matched traders go on searching for alternative partners. They may do so for various reasons largely discussed in the literature, as with expanding their information set, changing partner, or bargaining a larger share of match surplus. On-the-match seekers may alter the search of unmatched agents through congestion or crowding-out effects.

On-the-match search is usually captured as follows. Let E denote the number of matched traders. The number of matches between unmatched traders is $\widetilde{m}(B, S, E)$. The dependence vis-à-vis E is typically nonpositive.

Adopting the terminology in use in this paper, on-the-match seekers can be seen as phantoms. Hereafter, the total population of matched and unmatched agents are $N_t^B = B_t + E_t$ and $N_t^S = S_t + E_t$, where E_t denotes the total number of matched agents. Matched agents separate with probability q. There are no other phantoms than matched agents.

I assume that matched agents always go on searching for alternative partners. This may be so as to improve their information on the distribution of potential partners, or to increase their share of match surplus through alternative offers and counteroffers. I also assume that matched agents provide β efficient units of search. In the PMT framework, this corresponds to $\beta^B = \beta^S = \beta$ and $\delta^B = q$. The PMT is

$$\ln M_{t+1} = \ln S_t + \ln B_t - \ln \left[B_t + \beta \sum_{k=0}^{\infty} (1-q)^k M_{t-k-1} \right] \tag{30}$$

By definition, the total number of matches is the sum of all former matches weighted by the probability that they have not separated. Therefore, $E_t = \sum_{k=0}^{\infty} (1-q)^k M_{t-k-1}$ and

$$\ln M_{t+1} = \ln S_t + \ln B_t - \ln [B_t + \beta E_t] \tag{31}$$

The matching technology directly derives from the PMT. As such, it features intratemporal increasing returns to scale vis-à-vis B and S. It also has intertemporal constant returns to scale once the negative dependence vis-à-vis E_t is taken into account.

There is observational equivalence between phantoms and on-the-match seekers. This statement casts doubt on the interpretation of estimated matching technologies that explicitly account for on-the-job search. The general problem is that the number of phantoms is correlated with recent hires. The fact that traders of the past affect current recruitments does not prove that employees create congestion effects for the unemployed. This may also result from the type of informational persistence that is advocated in this paper.

Suppose for instance that there are two types of phantoms: on-the-match seekers and regular phantoms. The birth rate of regular phantoms is β^R, while the dying rate is δ^R. Similarly, a matched person seeks with search intensity β^O, while the match destruction probability is q. The PMT is

$$\ln M_{t+1} = \ln S_t + \ln B_t - \ln \left[B_t + \beta^O \sum_{k=0}^{\infty} (1-q)^k M_{t-k-1} + \beta^R \sum_{k=0}^{\infty} \left(1-\delta^R\right)^k M_{t-k-1} \right] \tag{32}$$

Focusing on the labor market case, the parameter q can be identified using data on separation rates. However, I cannot identify parameters β^O and β^R. I may try to use data on job-to-job movements to control for the effects of on-the-job seekers. However, this strategy is misleading. On the one hand, many employed job-seekers do not want to change jobs but are seeking an alternative offer so as to pressure their current employer into making a counteroffer. On the other hand, employed job-seekers do not necessarily compete with the unemployed. Many of them seek jobs that are only available to already employed people - that is, they search for jobs on a different search place.

A key difference between phantom traders and on-the-match seekers is the fact that phantoms consist of a backward variable, while part of on-the-match seekers consist of a forward variable. Burgess (1993) and Anderson and Burgess (2000) argue that a large proportion of employees do not seek jobs. The proportion that seeks jobs is actually procyclical.[3] However, phantoms may also adapt to changing market conditions. Parameters β and δ may be endogenous, reflecting the behavior of match-makers as well as the behavior of job-seekers. I leave the corresponding extensions for future work.

5 Conclusion

This paper shows that information persistence on search markets can generate market frictions, and that such market frictions give birth to a matching technology that has convenient properties from an applied perspective. The key idea is that each new match gives birth to a pair of phantom traders. In turn, phantom traders haunt the search place for some random period, inducing wasted resources spent by unmatched traders that desperately try to contact them. The resulting aggregate matching technology features increasing returns to scale in the short run, and constant returns in the long run.

The research can be extended in two directions. First, one may endogenize the parameters that govern phantom death. Match makers may spend time and money to clean their websites or to advertise for available trade partners. Non-phantom traders may send signals to be distinguished from phantoms. Second, the interplay between intra and intertemporal externalities should have implications for turnover externalities. Match formation is unlikely to account for phantom birth. Similarly, match destruction should not be affected by phantom proportion reduction.

[3] See Sunde (2007) for a discussion on the implications of endogenous search behavior.

halshs-00472751, version 1 - 13 Apr 2010

halshs-00472751, version 1 - 13 Apr 2010

Albrecht, J., Gautier, P., Vroman, S., 2006. Equilibrium directed search with multiple applications. Review of Economic Studies 73, 869-891

Albrecht, J., Tan, S., Gautier, P., Vroman, S., 2004. Matching with multiple applications revisited. Economics Letters 84, 311-314

Anderson, P.M., Burgess, S.M., 2000. Empirical matching functions: Estimation and interpretation using state-level data. Review of Economics and Statistics 82, 90-102

Burdett, K., Shi, S., Wright, R., 2001. Pricing and matching with frictions. Journal of Political Economy 109, 1060-1085

Burgess, S.M., 1993. A model of competition between unemployed and employed job searchers: An application to the unemployment outflow in Britain. Economic Journal 103, 1190-1204

Butters, G., 1977. Equilibrium distribution of sales and advertising prices. Review of Economic Studies 44, 465-491

Coles, M., 1999. Turnover externalities with marketplace trading. International Economic Review 40, 851-868

Coles, M., Muthoo, A., 1998. Strategic bargaining and competitive bidding in a dynamic market equilibrium. Review of Economic Studies 65, 235-260

Coles, M., Smith, E., 1998. Marketplaces and matching. International Economic Review 39, 239-254

Coles, M., Petrongolo, B., 2008. A test between unemployment theories using matching data. International Economic Review 49, 1113-1141

Drèze, J., Bean, C., 1990. Europe's unemployment problem: introduction and synthesis. In Drèze, J., Bean, C., (eds), European unemployment: lessons from a multi-country econometric study, MIT Press, Cambridge

Ebrahimy, E., Shimer, R., 2009. Stock-flow matching. Mimeo

Galenianos, M., Kircher, P., 2009. Directed search with multiple job applications. Journal of Economic Theory 144, 445-471

Gregg, P., Petrongolo, B., 2005. Stock-flow matching and the performance of the labor market. European Economic Review 49, 1987-2011

Hall, R., 1977. An aspect of the economic role of unemployment. In G.C. Harcourt (eds), Microeconomic foundations of Macroeconomics, MacMillan, London

Lagos, R., 2000. An alternative approach to search frictions. Journal of Political Economy 108, 851-873

Lagos, R., 2003. An analysis of the market for taxicab rides in New York city. International Economic Review 44, 423-434

Petrongolo, B., Pissarides, C., 2001. Looking into the black box: a survey of the matching function. Journal of Economic Literature 39, 390-431

Shimer, R., 2007. Mismatch. American Economic Review 97, 1074-1101

Stevens, M., 2007. New microfoundations for the aggregate matching function. International Economic Review 48, 847-868

Sunde, U., 2007. Unobserved bilateral search on the labor market: A theory-based correction for a common flaw in empirical matching studies. Economica 74, 537-560

Taylor, C., 1995. The long side of the market and the short end of the stick: bargaining power and price formation in buyers', sellers', and balanced markets. Quarterly Journal of Economics 110, 837-855