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International Human Capital Formation, Brain Drain and Brain Gain: A Conceptual Framework

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International Human Capital Formation, Brain Drain and Brain Gain:

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by

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Abstract

A two-country, two-period model of international migration highlights microeconomic foundations for examining the interrelation between brain drain, brain gain and the location of human capital formation, at home or abroad. Ex ante choices regarding where to study depend on relative qualities of university systems, individuals' abilities, sunk educational investment costs, government grants, and expected employment prospects in both countries. The analysis underscores an inherently wide-range of conceivable positive or negative effects on domestic net welfare. These changes depend critically on the foregoing factors, as well as the optimal design of educational grant schemes, given eventual informational imperfections regarding individuals' capabilities.

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Key words: **human capital formation, brain gain, brain drain, international migration, sunk costs, educational grants, asymmetric information**

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Section I: Introduction

Spawned, notably, by the contribution of Bhagwati and Hamada (1974), there has now been considerable research concern regarding the potentially adverse impact, for a home country's growth and welfare, of the migration of skilled workers. Nonetheless, early investigations also recognized potentially advantageous effects for source countries, due to possible remittances and the eventual return of migrants with enhanced skills due to foreign job training. More recently, what Schiff (2006) has termed the "new brain drain literature", which was initiated by Mountford (1997) and Stark, Helmenstein and Prskawetz (1997), has identified another potentially important source of brain gain, which is independent from return migration. Specifically, although migration can generate a loss of domestic talent, it can also prompt an upsurge in the overall educational level of a home country, as a result of higher propensities to invest in human capital. Attractive foreign labour market conditions offer heightened incentives for domestic workers to strive to attain higher qualification levels, whether or not they ultimately find jobs abroad, thereby fostering, *ceteris paribus*, increases in average productivity levels at home.

While certain existing approaches to modelling brain drain and brain gain effects entail macroeconomic frameworks with representative agents, as in Vidal (1998), many also consider microeconomic decisions at the level of individual agents, including choices regarding optimal investment levels in education. Stark, Helmenstein and Prskawetz (1997) have proposed a framework, which demonstrates how, given the opportunity to migrate, choices regarding educational attainment will determine an individual's wage on the foreign labour market. In other modelling frameworks, as proposed by Stark, Helmenstein, and Prskawetz (1998), the potential migrant takes into account a probability of finding a job abroad, which is identical for all individuals, or, as in Stark (2004), constrained by a minimum threshold level of qualification. Mountford (1997) and Beine, Docquier, and Rapoport (2001, 2008)) propose models where an individual's decision is of a binary form – whether to undertake education or not, while the probability of finding foreign employment is exogenous. This does not allow a role for differences in individuals' characteristics, so that migrants are randomly selected. In contrast, Chiswick (1999) provides for self-selection by migrants, since, assuming two categories of individuals, the rate of return to migration is greater for those with high-ability, relative to lower-ability persons. Nonetheless, the literature has principally focused on the links between incentives to invest in human capital at home and subsequent migration flows.

The evaluation of brain drain/brain gain effects is made in the literature by assessing the impact of migration on a variety of specific

economic objectives, which, however, do not include an explicit social welfare per se. Notably, migration is shown to influence the growth rate of the home economy, as in Beine, Docquier, and Rapoport (2001), the average educational level, as highlighted by Stark et al. (1997, 1998) and Lien and Wang (2005), average productivity in Mountford (1997), as well as the wages of non-migrants in Stark (2004).

Although there is now a burgeoning number of empirical studies, assessing different dimensions of the potential impact of brain drain and gain, there remains a lack of consensus regarding the size of conjectured positive effects of migration upon levels of education, welfare and/or growth. Notably, Beine, Docquier, and Rapoport (2001, 2008) find that the proportion of migrants must be low for such effects to be apparent. According to Schiff (2006), preliminary studies by the World Bank show no positive impact, while Groizard and Llull (2006) indicate a similar finding.

A recent critique by Rosenzweig (2006), which faults existing approaches to the analysis of brain drain and gain in two crucial respects, is particularly germane for motivating the modelling framework proposed in the current research. First, he contends that the potential impact of the "'risk' of emigrating" for "domestically-educated tertiary educated person(s)" is de facto quite minimal. Second, Rosenzweig goes on to suggest that "the literature ignores the endogeneity of the emigration probability", while arguing that, in fact, "the choice of the location of tertiary education significantly affects the probability that the person can emigrate."[1] (p. 2-3) Critically, existing analytical research has paid relatively little attention to the question of whether distinctive brain drain and gain effects may arise, depending on the extent to which educational investments take place either in home and/or host countries. Nonetheless, the policy stakes of the international mobility of high-skilled workers are increasingly recognized as a source of substantial policy concern.[2]

The research in the current paper proposes a two-country model, which offers a new theoretical paradigm for understanding the nexus between locational choices regarding human capital formation, international labour market conditions, and distinctive categories of brain drain and brain gain effects. The analysis underscores an inherently wide-range of conceivable positive or negative effects on a home country's net welfare. More specifically, distinctive elements of the proposed conceptual framework include the following:

[1] While the analytical framework proposed by Rosenzweig does not allow for differences in individual abilities, his empirical findings are consistent with a number of the modeling assumptions which are subsequently invoked here. Notably, he reports evidence that students are motivated by foreign studies in order to obtain employment in a host country and that quality differences in university systems also appear to trigger the decision to study abroad.

[2] See, for example, Leipziger (2008) and Solimano (2008).

1) **Individuals, from a home country, choose whether to undertake studies abroad, which entail an incrementally higher sunk cost relative to studying at home. While foreign studies are understood to generate greater improvements in labour-market productivity, as compared with levels achievable through domestic human capital formation, the realized extent of the gains depends on an individual student's underlying abilities. If subsequently offered foreign employment, students opt to stay abroad because of higher wages, thereby generating brain drain. However, if individuals are unable to find suitable foreign employment, they still enjoy heightened productivity levels and wages, when returning home, as compared to not having studied abroad. This generates brain gain.[3]**

2) **When modelling an individual's choice of whether to study abroad or stay at home, a crucial variable is the probability of being hired in the foreign labour market. Contrarily to other models in which this probability is exogenous and identical for all graduates, it is assumed here to be a function of each individual's expected level of qualification or, alternatively, productivity level, which, in turn, depends on ability. As a consequence, migrants are "favourably self selected" to use the terminology of Chiswick (1999).**

3) **The criterion chosen to assess brain drain/brain gain effects is the net impact on the home national welfare. This is represented, in a static framework, in terms of the change in domestic value-added resulting from foreign studies and eventual migration, This welfare calculation depends, in turn, on the associated consequences for the country's level of productivity, as well as the additional costs of investment in education abroad. It is assumed there are no remittances.[4]**

4) **Since foreign studies enhance productivity and thereby potentially lead to beneficial welfare effects, public authorities in the home country may seek, under certain conditions, to encourage foreign studies by subsidizing the candidates through alternative grant schemes, subject to a given overall budgetary constraint. Welfare implications of three conceivable grant policies are compared under alternative assumptions regarding the extent of a government's knowledge of students' underlying abilities. Under a first, uniform subsidy scheme, grants are offered to an arbitrary subset of students, assuming that the government cannot observe underlying abilities. The associated net**

[3] There are certain similarities between the proposed framework and the model of Kwok and Leland (1982), but their scenario does not include a brain gain effect.

[4] It is relatively straightforward to modify the proposed modeling framework, in order to allow for remittances, which would partially offset the negative welfare effects of brain drain. While such an extension potentially impacts specific quantitative results, it does not modify the essential qualitative insights summarized in subsequent propositions.

welfare effects are then compared with those of two alternative schemes, merit and selective, which invoke the alternative assumption that the authorities can actually distinguish between students' capabilities. Whereas in the former case grants are only offered to the brightest students, in the latter scenario financing is restricted to a sub-set of students, corresponding to a particular talent-pool.

The rest of this paper is structured as follows. In Section 2 the basic modelling analysis starts with a sub-model of ex ante individual choice, regarding whether to undertake human capital formation at home or abroad. An individual's underlying ability determines known productivity gains from studying abroad, along with expected probabilities of subsequently obtaining foreign market employment at higher wages. The evaluation of the ex post net impact of brain drain and brain gain depends on the size of the sub-populations of individuals who migrate permanently, as compared with those who return home with enhanced productivity, relative to wholly domestic trained workers. Section 3 presents some comparative static results, relating to the welfare effects of changing certain model parameters, which are essential for establishing subsequent propositions. In Section 4 the relative welfare implications of alternative educational grant schemes, subsidizing studies abroad, are considered. The analysis highlights a critical role for alternative assumptions regarding the extent of a public authority's knowledge of underlying abilities, which are assumed known by the individuals themselves. A concluding section briefly summarizes certain salient findings, while identifying a number of directions for further inquiry.

Section II: Basic Modelling Framework

II. A. Sub-Model of Individual Investment in Human Capital Formation and International Migration

A two-country, two-period framework is postulated in order to focus on the implications of initial educational investment decisions, regarding where to undertake higher education, in light of anticipations regarding individuals' subsequent employment prospects, at home or abroad. In the first of two periods, individuals can choose to study in the domestic country, knowing that their job prospects will be confined to that market. Alternatively, they may elect to study abroad, albeit while incurring higher

sunk costs for their educational investments, but with known gains in productivity due to higher levels of educational attainment. Although foreign studies enhance potential work prospects in both countries, individuals face uncertainty regarding whether they will be offered employment abroad.

More specifically, out of an overall population of N individuals in the domestic country, N_0 represents the number of domestic individuals who remain at home for both their education and work, while N* is the total number of persons who choose to undertake foreign studies and, subsequently, work either at home, or abroad. Thus, there are two distinct sub-populations of N*, corresponding to the phenomena of "brain gain" and "brain drain". In particular, N_1* designates the number of domestic individuals who chose to get educated abroad and subsequently work in the foreign country, while N_1 corresponds to the number of domestic individuals who are educated abroad, but then return home to work. In sum, whereas higher values of N_1* generate greater brain drain, increases in N_1 results in more brain gain.

The overall domestic population of N individuals are understood to differ in terms of their innate intellectual and work capacities, which for the k*th* individual, can be denoted as c_k . Whereas individuals know their own abilities, alternative hypotheses will be subsequently considered regarding the extent of the public authority's information, about these capacities. The attainable productivity levels for individuals depend not only on their underlying abilities, but also on educational investments, which enhance productivity to different degrees, depending on the quality of educational systems, at home, or abroad. However, in the subsequent analysis, the quality of the domestic higher educational system, Q_1 , is hypothesized to be inferior to that offered in the foreign country, Q_2. Hence, there is an educational production function that for a fixed period of investment in human capital in a particular educational system maps individuals' capacities into their effective qualifications or productivity levels, e_k , such that $e_k=f(c_k, Q_j)$, where j=1,2.[5] This functional relation results in a range of attainable productivity levels, measured on a scale between, e_0 and e_2. For subsequent simplicity, a value of e_0 is used as a numeraire to designate an unique level of productivity for all of the N_0 domestically educated workers, regardless of their inherent capacities. However, workers trained abroad,

[5] More generally, the value of the k*th* individual's human capital investments depends on the amount of time spent on education, the quality of university educational systems and his/her ability. While the analysis here only provides for individuals undertaking higher educational studies in a single period and in only one country, it could be extended to allow for students spending different periods of time, either at home or abroad. The returns from educational investments would then, depend on the specific stage of university, or earlier, studies, as well as country-specific differences in educational quality, which could be highly variable according to educational levels.

N_1* or N_1 , enjoy higher final productivity levels, which are uniformly distributed on an interval from e_1 to e_2 , according to their innate abilities.

While offering the prospect of higher productivity gains, the decision to undertake foreign studies is understood to entail higher educational costs, I*, relative to the costs, I_0 , borne by students who decide to pursue further education in the domestic country. Students will be willing to incur this difference between the foreign and domestic educational costs, designated as $i = I^* - I_0$, provided such additional costs can be financed, prior to realizing expected higher returns arising from enhanced productivity gains. Accordingly, this analysis assumes perfect capital markets, since students can borrow against their expected future earnings, in order to finance the immediate sunk costs of educational investments.[6] Subsequently, scenarios are considered where there are two distinct values for I*, depending on whether a student is granted a subsidy, S, by the domestic government. The overall educational costs borne by subsidized and unsubsidized students are then designated, respectively, as I_1* and I_2*, where $I_1^* = I_2^* - S$, so that $I_1^* < I_2^*$. In addition to deciding the amount of the educational grants per student, the domestic government determines the targeted number of students to be financed, in light of an overall budget constraint, F. Note, then, that the propensity of individuals to undertake foreign studies is impacted by both quality and cost differentials. Of course, national educational pricing policies, corresponding to the variables I* and I_0 , reflect overall educational budgets and subsidies, as well as the openness of educational systems and their capacity to attract international students.[7]

Individuals' ex ante willingness to incur sunk costs of educational investments is clearly impacted by anticipations regarding the labour market conditions they face after graduating - both at home and abroad. The latter

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[6] The question of how liquidity constraints arising from the impossibility of borrowing in order to finance expected increased income resulting from human capital formation is raised by Beine, Docquier and Rapoport (2008). In their macroeconomic framework, a representative individual self-finances his/her studies in an initial period, while facing exogenously specified probabilities of subsequently earning higher pay abroad through emigration. Unlike the analysis proposed here Beine et al (2008) do not consider the implications of different government schemes for financing human capital formation, either at home, or abroad, in order to compensate for inter-temporal capital market imperfections. Clearly, the potential importance of individual and family liquidity constraints depends on social and income inequality in a given country, the standard of living, as well as educational pricing policies. It should also be noted that the possibility of personal bankruptcy linked to divergences between ex ante and ex post wage expectations is precluded from the current analysis.

[7] Game-theoretic questions, relating to the international welfare implications of fellowships being financed by, alternatively, the domestic or foreign country are not examined in this paper. Of course, university fees, while constituting only one component of the overall costs facing international students, may only partially be reflecting the overall costs and quality of foreign educational systems. Fees for international students determine the relative ease of access to national educational systems from abroad and, thereby, reflect countries' international educational and foreign policies. Yet, there is also a potential interdependency between the share of a university system's costs borne by national students, and fees required for international students, since they impact together the overall financing of a host country's university system. Furthermore, a more in-depth modeling framework could also consider how the structure of quality-adjusted, national pricing policies at different educational levels impacts individuals' overall life-time investments in human capital formation. Clearly, there are associated indirect effects on decisions to undertake further studies abroad.

are reflected not only by hiring prospects, but also by both the absolute and relative returns from working in each country. In the proposed framework, once individuals have been educated abroad, they are assumed to have the ex post option of seeking employment abroad, at a higher wage, w*, than in their home market. For the overall population of N* workers, who are educated abroad, each individual, k, faces a probability, p_k , of finding qualified employment abroad. This probability plays a crucial role in the analysis, as it delineates "brain drain" from "brain gain" effects. This is readily apparent by comparing two extreme scenarios, where, as an initial simplification, the probability values are identical for all individuals:

a) **In the first instance, all individuals from the domestic country, even if they are educated abroad, face a zero probability of being employed abroad. Consequently, all individuals undertaking studies abroad will be motivated, ex ante, by a comparison of the differential gain in wages at home, arising from foreign, instead of domestic, training, in relation to the incremental investment cost of foreign studies. This case corresponds to a pure brain gain effect. Students benefit from enhanced productivity levels procured from a higher quality foreign education, but, nonetheless, always return home to work.**

b) **A polar scenario applies when all foreign-educated individuals from the home country are sure to get a better paying job abroad, regardless of their attained productivities, so p_k=1 for all k. Provided the incremental income gain, which in this case arises from migration, fully offsets the additional cost of foreign studies, all individuals will undertake foreign studies and none will return home. Hence, this corresponds to a case of pure brain drain. In the proposed analysis, the increase in earnings is greater than in the previous case since, for the same level of qualification, foreign remuneration is assumed to be greater than in the home labour market.**

Now, a more intermediate value of p_k, comprised between 0 and 1, will be considered, under a restrictive assumption that the probability of finding a foreign job is identical for all individuals. Then again, for appropriate wage and educational cost parameters, all individuals will leave if p_k is sufficiently high. Yet, only a fraction, $1 – p_k$, will return to work in the domestic labour market. Consequently, both brain drain and brain gain will arise, respectively, in the proportions p_k and $1 – p_k$.

Nonetheless, in the proposed model, the probability of finding employment abroad varies across individuals, since it depends on their expected levels of productivity, which, in turn, are related to underlying abilities and educational choices. The values of p_k are assumed to be

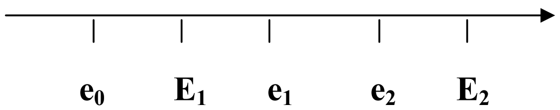
uniformly distributed across the population, so that a more complex mix of brain drain and brain gain effects needs to be examined. More specifically, each of the p_k values is taken to depend linearly on the level of the effective qualifications realized by the k*th* individual, e_k , relative to a threshold value, E_1 , reflecting a minimum standard in the foreign labour market, and negatively on the range of skill requirements, E_2 - E_1, such that:

$$\textbf{(1.)} \quad p_k = p(e_k) = \frac{(e_k - E_1)}{(E_2 - E_1)}$$

Figure 1 offers a representative illustration of the assumed distribution of effective qualification levels for domestic individuals, in relation to the skill requirements of the foreign labour market. Intermediate values for the parameters E_1 and E_2 are assumed, where these threshold values, respectively, preclude or guarantee foreign market employment. Thus, in the proposed model, each foreign-trained, domestic-origin, student faces a non-zero probability of finding employment abroad. As a simplification, it will be assumed that individuals, who chose to remain at home for their education, are unable to work abroad.[8]

Figure 1

The Assumed Structure of Skill Levels Attainable at Home or Abroad, Relative to Foreign Labour Market Requirements



The parameters, E_1 and E_2, can be understood to reflect foreign labour market conditions and policies, where employment standards abroad are influenced by the overall quality of the foreign educational system (including, for example, pre-university studies), as well as by technology-driven, labour-demand requirements. Different combinations of these parameter values can also be interpreted to represent alternative immigration policies, restricting labour market access depending on the skill intensities of available jobs in the foreign country. For instance, lower values of E_2 could, ceteris paribus, represent a situation of relative shortages for specific categories of highly skilled workers. Furthermore, lower

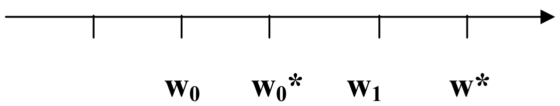
[8] Eventual rationale for this assumption include an inadequate relative quality, or high-degree of specificity, of the domestic educational system, positive social network effects on employment arising from foreign studies, and/or restrictive immigration policies, favouring students trained abroad.

(higher) values of both of these foreign market parameters can be interpreted as corresponding to alternative foreign immigration policies, facilitating (hindering) the immigration of foreign skilled workers.

Following their studies, foreign-trained domestic students have an incentive to seek employment abroad due to the higher foreign salaries, w*, for skilled jobs. In their home country returning students can only earn a lower reservation wage, w_1 .[9] For tractability, both of these salaries are assumed to be unique values, independent of the students' effective qualification levels achieved though studies abroad. Furthermore, it is assumed that this reservation wage is higher than both the remuneration offered to wholly domestically trained workers, w_0 , and the foreign wage, which they can earn in less skilled jobs abroad, w_0*. Consequently, if students are unsuccessful in finding appropriate skilled work in the foreign country, they will return home.[10] Finally, although the wage rates are taken to be exogenous, the subsequent analysis will consider comparative static changes in their values, reflecting the relative attractiveness of labour market conditions internationally. Figure 2 summarizes the overall international structure of wages, depending on job locations and educational backgrounds

Figure 2

The Structure of International Wages According to Job Location and Educational Background



The ex ante optimization problem for the representative k*th* student involves a trade-off, corresponding to an arbitrage condition. The net returns from studying and working at home, with lower overall effective qualifications, need to be compared to expected higher wage earnings, arising from enhanced productivity due to foreign studies, albeit at a greater investment cost. The expected wage remuneration involves a probability weighted average of wages for more skilled workers in the foreign and

9 An exchange rate of unity is assumed.

10 Of course, other factors, such as personal and family considerations could offset the locational incentives of these ex post wage differentials between the two countries. Such additional factors can be modeled in terms of complementary or substitutable, agent-specific assets and associated sunk costs. It can be noted that, ceteris paribus, if students have a preference to return home, there will be an increase in brain gain effects, relative to those identified in the subsequent analysis.

domestic markets. Accordingly, a representative student will decide to study in the foreign country if:

$$\textbf{(2.)}\quad p_k w^* + (1-p_k) w_1 - I^* > w_0 - I_0$$

Hence, the k*th* individual will decide to study abroad if his/her individual probability of being hired abroad, p_k is higher than a critical probability value, $\overline{p}$. This probability is assumed to depend on a student's, potentially private, information regarding his/her future productivity level, e_k. More specifically, the interrelation between this critical probability value, $\overline{p}$, and the prevailing international wage rates and educational costs are given by:

$$\textbf{(3.)}\quad \overline{p} = \frac{i-(w_1-w_0)}{w^*-w_1} \quad \textbf{if} \quad \frac{i-(w_1-w_0)}{w^*-w_1} \in [0,1]$$

$$\overline{p} = 0 \qquad \textbf{if} \quad \frac{i-(w_1-w_0)}{w^*-w_1} < 0 \textbf{, that is if } i < w_1 - w_0$$

$$\overline{p} = 1 \qquad \textbf{if} \quad \frac{i-(w_1-w_0)}{w^*-w_1} > 1 \textbf{, that is if } i > w^* - w_0$$

From (1.), it follows that the productivity level corresponding to $\overline{p}$ is: $\tilde{e} = (E_2 - E_1)\overline{p} + E_1$ However, $\tilde{e}$ does not necessarily belong to the segment of productivity levels attainable from foreign studies, [e_1 , e_2], so that the actual productivity threshold is $\overline{e}$ such that

$$\textbf{(4.)}\qquad \begin{aligned} &\overline{e} = \tilde{e} = (E_2 - E_1)\overline{p} + E_1 && \textbf{if } \tilde{e} \in [\mathbf{e_1}, \mathbf{e_2}], \\ &\overline{e} = \mathbf{e_1} && \textbf{if } \tilde{e} < \mathbf{e_1}, \\ &\overline{e} = \mathbf{e_2} && \textbf{if } \tilde{e} > \mathbf{e_2}. \end{aligned}$$

As a consequence, out of the overall population of domestic individuals, the proportion of students staying at home is given by $\frac{N_0}{N} = \frac{\overline{e}-e_1}{e_2-e_1}$, while the residual proportion pursuing studies abroad is the complementary value, equaling $\frac{N - N_0}{N} = \frac{e_2 - \overline{e}}{e_2 - e_1}$.

<u>II. B. Production and Welfare in the Home Country</u>

Production, or value-added at home is taken to be characterized by a linear function, reflecting a proportional relation to productivity. Thus, if individuals were not able to study abroad, national output would be $Y_0 = e_0$ N, which constitutes an essential benchmark involving only domestically trained workers. Since productivity is taken to be uniformly distributed, the

overall increase in productivity, de, resulting from an arbitrary marginal proportion, dn, of the total domestic population, N, being trained abroad, is such that $\frac{dn}{N} = \frac{de}{e_2 - e_1}$. However, only the fraction (1-p(e)) of this population, not finding better paying foreign employment, will return to work at home. Hence, the total number of foreign-educated individuals returning home is specified by $N_1 = \frac{N}{e_2 - e_1}\int_{\bar{e}}^{e_2}[1-p(e)]de$. The contribution to national production generated by these returning individuals corresponds, then, to $Y_1 = \int_{N_0}^{N} e[1-p(e)]dn = \frac{N}{e_2-e_1}\int_{\bar{e}}^{e_2} e[1-p(e)]de$, which constitutes the incremental increase in national income resulting from brain gain. Analogously, the number of foreign-educated individuals staying abroad is $N_1^* = \frac{N}{e_2-e_1}\int_{\bar{e}}^{e_2} p(e)de$. As a reminder, then, the number of individual who both study and work at home can be expressed as $N_0 = N - (N_1 + N_1^*) = N\frac{\bar{e}-e_1}{e_2-e_1}$. From the foregoing expressions, it can be readily seen that

a) N_0 is increasing with the threshold productivity level, $\bar{e}$, whereas N_1 and N_1^* are both decreasing.

b) N_1 and N_1^* increase as the lower limit of productivity attainable from a foreign education, e_1 rises, for a given e_2. Both of these populations also increase when both the limits, e_1 and e_2 , rise, for a given range of enhanced productivity levels, e_2-e_1 , arising from foreign training. The number of students staying abroad, N_1^*, is also increasing with e_2 , for given e_1 . In contrast, the number of foreign-trained returning students, N_1 has a maximum for some value of e_2. Beyond this limiting value, an improvement in the highest productivity level, attainable by the best foreign-trained domestic students, will accentuate the extent of brain drain.

A distinctive feature of the proposed analysis is the explicit consideration of how brain drain and brain gain effects, linked to international human capital formation, impact social welfare. As previously noted, an essential benchmark value is Y_0, which corresponds to a scenario where there is neither brain gain, nor brain drain. In the context of the subsequent social welfare calculations, e_0 can be viewed as the individual return in terms of the attainable level of productivity, given past and future social investment costs associated with an individual's education in the domestic country. Once international human capital formation is allowed for, the net variation of welfare, resulting from individuals studying abroad, is understood to equal the change in value-added, linked to brain gain minus the opportunity cost of losing workers abroad, or brain drain, and

subtracting the supplementary investment cost of undertaking foreign education.[11] **More formally, this change in social welfare is given by:**

$$(5.)\ \Delta W = Y_1 - e_0\ (N - N_0) - i\ (N - N_0) = Y_1 - C(N - N_0).$$

Note that the expression, $C = e_0 + i$, reflects the maximal, social opportunity cost arising from an individual studying abroad, if there is no compensatory brain gain. This expression corresponds to the associated loss of national production and the higher net educational investment cost of foreign studies, relative to the benchmark autarkic case. More explicitly, the overall change in domestic welfare equals:

$$(6.)\ \Delta W = \frac{N}{e_2 - e_1}\int_{\bar{e}}^{e_2}\langle e[1 - p(e)] - (e_0 + i)\rangle de$$

In light of expression (1.), this is equivalent to:

$$(7.)\ \Delta W = \frac{N}{e_2 - e_1}\int_{\bar{e}}^{e_2}\left[e\frac{E_2 - e}{E_2 - E_1} - (e_0 + i)\right]de = \frac{N}{e_2 - e_1}\int_{\bar{e}}^{e_2}\varphi(e)\,de = N\frac{\Phi(e_2) - \Phi(\bar{e})}{e_2 - e_1}$$

where the integrand is $\varphi(e) = e\frac{E_2 - e}{E_2 - E_1} - (e_0 + i)$, **and Φ is the primitive function of φ:**

$$\Phi(e) = -\frac{e^3}{3(E_2 - E_1)} + \frac{E_2\,e^2}{2(E_2 - E_1)} - (e_0 + i)e$$

As shown by equation (7.), the incremental change in domestic welfare is a function of all the parameters of the model. To summarize, it depends on:

- **e_0 : the productivity of less-skilled domestically-trained workers;**
- **e_1 and e_2 : the two extreme values defining the range of enhanced productivity levels for foreign-educated workers;**
- **E_1 and E_2 : parameters reflecting foreign market skill requirements and labour market access conditions, which impact the probability of finding work abroad;**
- **$\bar{e}$, the threshold value of productivity, which decides whether an individual chooses to study abroad, which, in turn, is impacted by among other factors, the wages of skilled workers employed abroad,**

[11] The educational costs for society of training students, prior to their deciding to study abroad and, subsequently, working permanently there, could also, arguably, be considered to negatively impact domestic social welfare. There would then be an additional term, negatively impacting domestic welfare, as a result of brain drain. On the other hand, the proposed specification of the social welfare function does not allow for the positive impact of remittances, which would depend on the value of w*, along with different propensities characterizing individuals' decisions to transfer funds back home.

w*, skilled workers employed at home, w_1 , and unskilled workers at home, w_0 ;[12]

_ i : the cost differential between studying abroad and at home.

The expression for the primitive function in equation (7.), Φ , which critically defines the extent of the change in domestic welfare, is of the third degree in e. The underlying reason for such a functional form is the second degree form for the integrand, $\varphi(e)$, in equation (7.), which represents the expected increase in net welfare for a representative individual. This expression involves a trade-off between the expected increase in productivity realized through brain gain, $e(1-p(e)) - e_0$, and the incremental cost of a foreign education, i. Since the former quadratic term in e assumes low values for either relatively low or high productivity values, the values of the integrand are initially negative, then positive (for sufficiently low i) and finally negative, as representative productivity levels for different individuals increase.

As illustrated in Figure 3, the general form of the primitive function Φ may first show a minimum, for e = $\hat{e}_1$, and then a maximum for e = $\hat{e}_2$. Noting, again, that the social cost of a foreign education is denoted by $C = e_0 + i$, the values of $\hat{e}_1$ and $\hat{e}_2$ are given, respectively, by $\hat{e}_1 = \frac{1}{2}\left[E_2 - \sqrt{E_2^2 - 4(E_2 - E_1)C}\right]$ and $\hat{e}_2 = \frac{1}{2}\left[E_2 + \sqrt{E_2^2 - 4(E_2 - E_1)C}\right]$. Of course, these extrema exist if and only if $E_2^2 - 4(E_2 - E_1)C > 0$, which leads to:

$$\textbf{(8.)} \quad C < \frac{E_2^2}{4(E_2 - E_1)}.$$

If the social cost of a foreign education, C, is too high, Φ is always decreasing with e, and as a consequence, the change in domestic welfare, ΔW, is always negative, so that the brain drain effect dominates that of brain gain. The value for which Φ has a minimum, $\hat{e}_1$, is relevant only if the latter is greater than E_1. Calculations show that the associated condition is simply:

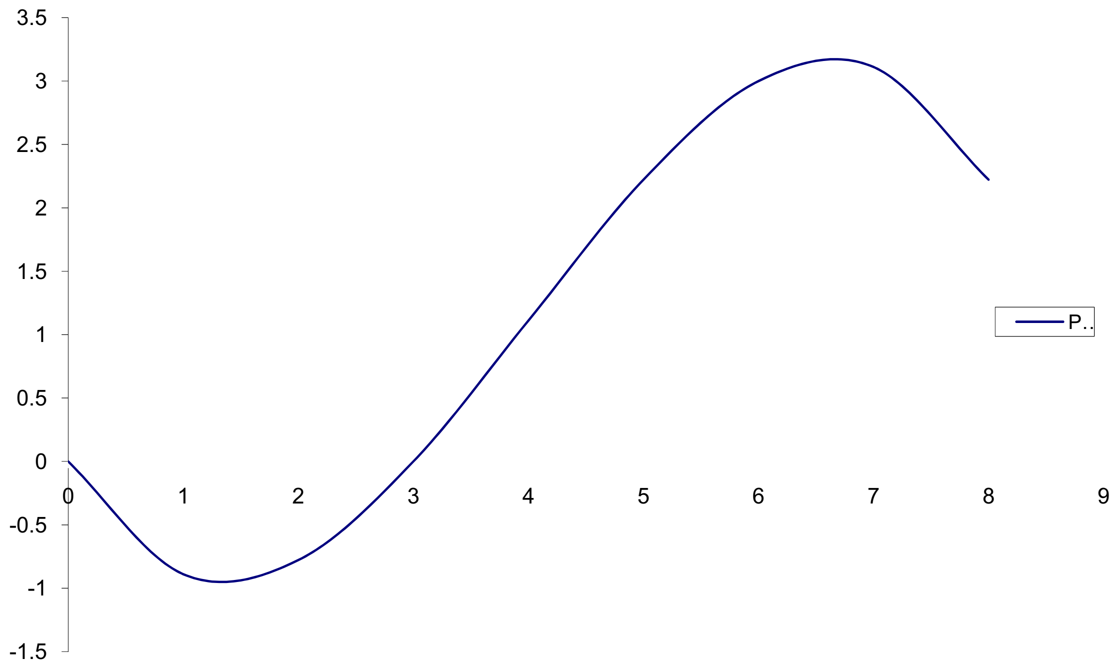
$$\textbf{(9.)} \quad E_1 < C .$$

In the rest of the paper, it is assumed that conditions (8.) and (9.) are always satisfied.

[12] As shown by considering equations 3 and 4.

Figure 3

Representation of the Functional Form for Φ, which Determines the Overall Change in Domestic Welfare



Section III: An Analysis of the Effects on Economic Welfare of Changes in Key Model Parameters

III.1 The Interrelation between Threshold Productivity Levels and Changes in Welfare

The initial focus here is on the welfare implications of the critical value of $\overline{e}$, which reflects the threshold productivity level for which a representative individual chooses to study abroad. The value of $\overline{e}$ in relation to $\hat{e}_2$ is potentially of key importance. Note again that $\overline{e}$ is a function of the critical threshold probability, $\overline{p}$, triggering foreign study, as well as of the foreign labour market productivity requirements, E_1 and E_2; while $\hat{e}_2$ is a function of E_1, E_2 , and the social opportunity cost of foreign

studies, C, where: $\bar{e} = (E_2 - E_1)\bar{p} + E_1$ **and** $\hat{e}_2 = \frac{1}{2}\left[E_2 + \sqrt{E_2^2 - 4(E_2 - E_1)C}\right]$**.** **Hence, it follows that** $\bar{e} > \hat{e}_2$ **for** $\bar{p} > p_{lim}$**, where** $p_{lim} = \frac{\hat{e}_2 - E_1}{E_2 - E_1} = \frac{\sqrt{E_2^2 - 4(E_2 - E_1)C} - E_1}{E_2 - E_1}$**. The value of** p_{lim} **is always inferior to one, and while it could be negative, this would mean that** $E_1 > \hat{e}_2$**. However, this corresponds to a relatively uninteresting case, where domestic welfare always declines, as a result of individuals studying abroad. For more relevant scenarios, there is an actual probability threshold beyond which** $\bar{e} > \hat{e}_2$**. It can also easily be seen that when** $\bar{e}$ **increases and** $e_2 > \hat{e}_1$**,** ΔW **has a maximum for** $\bar{e} = \hat{e}_1$**. Thus, if initially** $\bar{e} < \hat{e}_1$**, a marginal increase in** $\bar{e}$ **promotes welfare. However, if** $\bar{e} > \hat{e}_1$**, an increase in** $\bar{e}$ **reduces the number of people who study abroad, thereby reducing welfare. An examination of Figure 3 and a comparison of the values taken by the function Φ for** $\bar{e}$ **and** e_2**, leads then to the following:**

Proposition 1

For intermediate values of the threshold productivity value, $\bar{e}$***, determining whether individuals will study abroad, and of the upper limit on the associated level of enhanced productivity,*** e_2***, specifically belonging to the interval [***$\hat{e}_1, \hat{e}_2$***], the change in welfare resulting from studying abroad, ΔW, is positive. Hence, the welfare improvement from brain gain dominates the loss due to brain drain.***

In contrast, there are three cases where foreign studies generate a loss of welfare. Notably,

a) when $\bar{e}$ ***and*** e_2 ***are both very low, the return to foreign education, in terms of increased productivity, is weak and does not compensate for its social costs, even if many individuals study abroad and return home to work, e ;***
b) when $\bar{e}$ ***and*** e_2 ***are both very high, few individuals leave to study abroad, but most of these will readily find a job abroad, resulting in a dominance of the brain drain effect;***
c) when $\bar{e}$ ***is low and*** e_2 ***is high, there is an accumulation of the foregoing effects a) and b). Notably, many individuals study abroad, thereby generating high additional educational investment costs, but only those with lower-productivity gains return home.***

In sum, the welfare implications of comparative static changes in productivity levels, $\bar{e}$ **and** e_2**, are inherently ambiguous.**

III.2 The Configuration of Wages and Associated Welfare Effects

The influence of wages on domestic welfare works through changes in the critical values for $\overline{p}$ and $\overline{e}$. As can be expected, higher wages for domestically trained workers create, ceteris paribus, a disincentive to studying abroad, so when w_0 increases, both $\overline{p}$ and $\overline{e}$ increase. However, when the potential job market returns to foreign studies w* or w_1 increase, the incentives to studying abroad are increased, so that $\overline{p}$ and $\overline{e}$ are lowered. The associated consequences for domestic welfare stem from the preceding analysis of the influence of $\overline{e}$. Specifically, if $\overline{e}$ is not very low (inferior to $\hat{e}_1$), an increase (decrease) in wages for foreign-trained (domestic-trained) workers, decreases $\overline{e}$, thereby enhancing welfare.

III.3 Welfare Implications of Changes in the Relative Productivity Gains from Education at Home and Abroad

It is also straightforward to see that a heightened efficiency for domestically-trained workers, e_0, by increasing the opportunity cost of undertaking foreign studies, generates a negative influence on the net impact on welfare of brain drain and brain gain. In contrast, an increase in the lower limit of the enhanced efficiency level attained via foreign studies, e_1 , raises the returns to a foreign education, and induces a larger proportion of the population to study abroad. The effect of a variation in e_2 is more complex to assess. By widening the span of productivity values, an increase of e_2 , ceteris paribus, has a negative influence upon ΔW. If $e_2 > \hat{e}_2$, $\Phi(e_2)$ also decreases, so that the overall effect is also negative. However, if e_2 belongs to the interval $[\hat{e}_1 , \hat{e}_2]$, $\Phi(e_2)$ increases and the net effect is indeterminate. More specifically, the formula for the derivative of ΔW is:

$$\text{(10.)} \quad \frac{d\Delta W}{de_2} = \frac{N}{(e_2 - e_1)^2} \{\Phi(\overline{e}) - [\Phi(e_2) - (e_2 - e_1)\, \varphi(e_2)]\} .$$

It can be seen that, if $\overline{e} < \hat{e}_2$, so that ΔW may be positive, then the foregoing expression is negative for $e_2 = \hat{e}_2$. Consequently, the change in the domestic country's welfare has a maximum for some value of e_2 (also inferior to $\hat{e}_2$). In light of the foregoing analysis, the following holds:

Proposition 2

The change in the domestic country's welfare, ΔW, is an increasing function of the level of e_2 , the maximal level of enhanced productivity achievable by undertaking studies abroad, provided e_2 remains under a critical

level. Beyond this threshold, ΔW decreases with e_2 . Thus, too much of an improvement in human capital, or, alternatively, relative excellence in the foreign institutions, generates a dominant brain drain.

The associated critical value of e_2 is increasing with the threshold productivity level determining whether students go abroad, $\bar{e}$, and decreasing with the lower limit of the value of enhanced productivity, e_1 .

Finally, if both e_1 and e_2 increase with a constant span between the two values, ΔW has a maximum for $e_2 = \hat{e}_2$. In that case, from the perspective of domestic welfare, there is also an optimal level of relative efficiency in the foreign educative system. Any increase of efficiency above this level will diminish home national welfare.

III.4 Changes in the Sunk Cost Differential for Studying Abroad

As the additional sunk costs associated with foreign studies, i , increase, the integrand function φ decreases. There is a resulting loss of welfare (provided only values of e for which φ is positive are considered). Furthermore, the threshold probability of finding a job abroad increases, as does the corresponding threshold productivity level, $\bar{e}$. As a consequence, so long as $\bar{e} \in [\hat{e}_1 , \hat{e}_2]$, an increase in the incremental costs of studies abroad, i, reduces the home country's welfare. In contrast, for low values of $\bar{e}$ ($\bar{e} < \hat{e}_1$), an increase in i could possibly be beneficial. In such a scenario there is an excessive flight of students abroad, since, for a representative student, the productivity gains from a foreign education are high, whereas the additional costs, i, are low.

III.5 Alternative Immigrant Employment Policies in the Foreign Country

The relative ease of access to the foreign labour market is captured here by alternative values for the labour market requirement parameters, E_1 and E_2. Ceteris paribus, for higher values of either parameter it is more difficult for a domestic-origin, but foreign-trained, job-searcher with a given qualification level to be employed abroad. More specifically, when either E_1, or E_2 increase, $\bar{e}$ increases, but $p(e_k)$ decreases for any value of e_k . Crucially, there are two offsetting effects. On the one hand, fewer individuals leave to become educated abroad, but, on the other hand, a greater fraction of graduated students come back home. Thus, the total pool of foreign trained students from the domestic country is reduced. This means that the overall exposure of the domestic country to welfare changes, arising from either brain drain or brain gain, decreases. However, the relative proportion of foreign-trained students generating a brain gain

increases as a result of the more restrictive job filtering environment in the foreign country. Consequently, the net effect on domestic welfare is potentially ambiguous.

As demonstrated in Appendix 1, the following summary conclusion applies:

<u>Proposition 3</u>

Restrictions limiting entry by foreign-trained students to the host country's labour market increase home national welfare, provided the following conditions hold:

a. ***the cost differential for undertaking foreign studies is high;***
b. ***the maximum achievable productivity level, e_2, is relatively low; and***
c. ***relatively few individuals undertake studies abroad (i.e. $\overline{p}$ is near 1).***

In contrast, if the foregoing conditions are not satisfied, then less favourable foreign labour market conditions result in a negative impact on domestic welfare.

<u>Section IV: A Comparative Analysis of the Domestic Welfare Implications of Alternative Educational Grant Schemes</u>

The focus in this section is on the optimal policy design, for a home country, of educational grants, aimed at facilitating foreign study for specific categories of students. Since alternative subsidy programs change the incentives to study abroad, they potentially impact the balance between brain drain and brain gain, which, in turn, determines the net changes in domestic welfare. Three different grant schemes will be considered, which invoke alternative assumptions regarding the extent of a government's knowledge of students' underlying abilities. Under a first grant program, labelled as an uniform scheme, the public authorities have no information regarding differences in students' underlying abilities when assessing their expected future productivity gains achievable from foreign studies. Notably, such uniform grants, amounting to a value of S for each potential beneficiary, are proposed for a proportion α of individuals in the overall population, N. These awards, then, are independent of the inherent abilities of a specific grantee and, consequently, his/her expected gains in productivity. While such a program reduces the potential cost of individuals undertaking foreign studies, there are, of course, potential welfare inefficiencies, since certain grant recipients would have opted anyway to study abroad, even in the absence of such a program.

In contrast, under an alternative paradigm, where the government can discriminate ex ante between students according to their abilities, two different merit grant schemes will be considered. Under the first scheme, grants are only proposed to the most qualified individuals, who would not otherwise be able to undertake foreign studies. More specifically, such grants will be awarded to students, whose productivity is inferior to the critical level $\bar{e}$, but superior to a limit fixed by budgetary constraints. Here, again, $\bar{e}$ corresponds to the productivity threshold for which students are prepared to go abroad even in the absence of an educational award. Note that in this first case of an unrestricted merit grant scheme the targeted individuals constitute a sub-population of relatively more capable students, who have particularly good employment prospects abroad, so that there is a relatively high ex ante probability of brain drain. Hence, it is conceivably more efficient for a government to offer a second, alternative scheme of merit grants restricted to somewhat less capable individuals, since a larger proportion of those students would actually return home to work, thereby generating greater brain gain.

IV.1 Uniform Grants

Under this grant scheme a lump-sum amount, S, is proposed to a fraction α of the overall population of N individuals, without any a priori knowledge regarding their underlying abilities. While only certain of these grant recipients will actually decide to study abroad, that sub-population potentially includes individuals who would have chosen to go abroad without a grant, since unlike the public authorities, individuals know their own abilities. Thus, there is an inherent potential inefficiency in such a uniform grant program, arising from the asymmetry of information between the grantee and recipients. For grant beneficiaries, the incremental cost of foreign education will now be denoted by $i_1 = I_1^* - I_0$. However, the incremental cost for unsubsidized students remains $i = I_2^* - I_0$, where I_2^* now denotes the full-cost of foreign studies, so that $i - i_1 = I_2^* - I_1^* = S$. Clearly, then the threshold probability of studying abroad differs for the two sub-populations. Specifically, for grant recipients, that value is given by $\bar{p}_b = \frac{i_1 - (w_1 - w_0)}{w^* - w_1}$, which corresponds to a productivity threshold $\bar{e}_b$, such that $\bar{e}_b = (E_2 - E_1)\bar{p}_b + E_1$. For the unsubsidized students, this threshold remains $\bar{p} = \frac{i - (w_1 - w_0)}{w^* - w_1}$, where $\bar{e} = (E_2 - E_1)\bar{p} + E_1$, while $\bar{p}_b < \bar{p}$ and $\bar{e}_b < \bar{e}$. Furthermore, the difference in the threshold probabilities can be expressed as $\bar{e} - \bar{e}_b = (E_2 - E_1)(\bar{p} - \bar{p}_b) = \frac{E_2 - E_1}{w^* - w_1} S$. For the ex ante distribution of

productivities, the interval $\bar{e} - \bar{e}_b$ corresponds to the sub-class of grant recipients who would otherwise have stayed home without such financing, but actually decide to go abroad. Of course, individuals with expected productivity levels inferior to $\bar{e}_b$ will still stay at home, whereas those with productivity levels superior to $\bar{e}$ would have studied abroad anyway. For the latter individuals, such uniform grants are just a redistributive transfer from the state, without any net impact on welfare. [13] [14]

Essential dimensions, characterizing the overall uniform grant policy initiative, include the value, S, proposed for each identified grant recipient and the proportion, α, of the overall population, N individuals, being targeted. Together these influence the number of potential beneficiaries actually going abroad, N*, in light of the overall government budgetary constraint limiting the total expenses on this program, F_b. Clearly, a key issue here is that certain targeted grant recipients may actually decide not to study abroad. Hence, there is an interdependency between α, S, and F_b due to the government's incomplete information regarding the underlying abilities of proposed grant recipients. The expression for overall government expenditures, for such a uniform grant program, is given by:

$$\textbf{(11.)}\quad F_b = \alpha N[(e_2 - \bar{e}_b)/(e_2 - e_1)][(\bar{e} - \bar{e}_b)(w^* - w_1)/(E_2 - E_1)] = k\,\alpha(e_2 - \bar{e}_b)(\bar{e} - \bar{e}_b)\,,$$

Here, k designates a constant term equal to $[N/(e_2 - e_1)][(w^* - w_1)/(E_2 - E_1)$.

With such a uniform grant scheme, the overall variation of social welfare, resulting from individuals studying abroad, comprises two effects and is given by:

$$\textbf{(12.)}\quad \Delta W_b = \alpha \frac{N}{e_2 - e_1} \int_{e_b}^{\bar{e}} \varphi(e)\,de + \frac{N}{e_2 - e_1} \int_{\bar{e}}^{e_2} \varphi(e)\,de\,.$$

The first term of this expression is the incremental change in welfare generated by the uniform grant program per se, whereas the second term is the welfare effect resulting from foreign studies, in the absence of any such grant initiative. As a consequence, the net impact of uniform grants is

[13] This analysis abstracts from issues regarding the opportunity cost of the public expenditures used to subsidize foreign studies, relative to the private use of such funds.

[14] If the framework of the analysis were expanded to allow for a distinction between rich and poor individuals, an alternative policy option could be for a government to propose "uniform" loans for sub-populations of less financially-privileged individuals, who are not able to afford foreign studies. Clearly, if such educational loans are associated with the obligation to return home to work, they will only generate brain gain effects. At the same time, such a measure counters the potential informational asymmetry a government faces with respect to its capacity to identify students' underlying abilities, since there would be an underlying self-selection mechanism for financially constrained individuals, which implicitly reveals their abilities.

welfare increasing if and only if the values for the primitive function are such that $\Phi(\bar{e}) > \Phi(\bar{e}_b)$. The optimal grant policy maximizes ΔW_b by choosing values of α and S, for a given foreign educational budget, F_b. These, in turn, endogenously determine the new threshold efficiency level, $\bar{e}_b$, at which individuals decide to study abroad. The analysis in Appendix 2 characterizes the effects of varying the levels of $\bar{e}_b$ on the net changes in economic welfare, ΔW_b, which leads to the following:

Proposition 4

Let us consider a scenario where a government is awarding uniform grants for foreign studies, under conditions where it has no information regarding students' innate abilities and it faces a specific educational budget constraint. Then, the optimal proportion of the population, α, which should receive such awards, depends on the value of $\bar{e}$, reflecting the threshold productivity level for which individuals will chose to study abroad in the absence of foreign educational subsidies.

More specifically, for $\hat{e}_1 < \bar{e} < e_2 < \hat{e}_2$, ΔW_b is an increasing function of a new threshold value $\bar{e}_b$, determined by the uniform grant program, and, as a consequence, also a function of α. It is then optimum to choose $\alpha = 1$, that is to propose a grant to all individuals in the overall population. For $e_2 > \hat{e}_2$ and $\bar{e}$ far enough from e_2, the same result applies. However, for $e_2 > \hat{e}_2$ and $\bar{e}$ near to e_2, there is an optimal value for $\bar{e}_b$, where only a sub-population is targeted as grant recipients, such that α is inferior to 1, provided that the budget constraint, F_b, is small enough (since for given $\bar{e}_b$, α is an increasing function of F_b). Furthermore, it can be observed that in the case where $e_2 > \hat{e}_2$ and $\bar{e}$ is such that $\Phi(\bar{e}) > \Phi(e_2)$, the change of welfare, ΔW, would be negative in the absence of grants. In sum, the rationale for the introduction of uniform grants, in this final case, is that they can generate an increase in welfare, provided available funds are large enough to set $\bar{e}_b$ at a value sufficiently low to satisfy the condition $\Phi(\bar{e}_b) < \Phi(e_2)$.

IV.2 Merit Grants

In this alternative scenario, unlike in the previous case of uniform grants, the government is assumed to be omniscient, having full information regarding the underlying ability of all students. Accordingly, grants will be

allocated as a function of a candidate's ability, or, equivalently, in light of the expected gain in an individual's productivity. However, unlike the previous scenario for uniform grants, individuals whose productivity is superior to the standard threshold $\bar{e}$ will never be grant beneficiaries, since there is no need for any additional financial incentive to undertake foreign studies. Thus, an inherent informational inefficiency of the uniform grant scheme is avoided. Under a merit system, all individuals, whose productivity levels are comprised between a designated level, $\bar{e}_m$ and $\bar{e}$, will now receive a grant. The lower productivity limit for the grant recipients, $\bar{e}_m$, is, as with uniform grants, endogenously determined by the per capita value of the foreign educational subsidies and the government's overall educational budget constraint.

If $\bar{p}_m$ indicates the threshold probability for beneficiaries of a merit grant to study abroad, then, there is a standard interrelation between that value and the associated threshold productivity level, $\bar{e}_m$, such that: $\bar{e}_m=(E_2 - E_1)\ \bar{p}_m + E_1$. It follows that the interval of productivity levels characterizing merit grant recipients, $\bar{e} - \bar{e}_m$, is proportional to $\bar{p} - \bar{p}_m$, and, consequently, to the level of the grant S. Since the range of beneficiaries is proportional to $\bar{e} - \bar{e}_m$, a given grant budget can be expressed as:

$$\textbf{(13.)}\quad F_m = k\,(\bar{e} - \bar{e}_m)^2,$$

where k is the same constant term as in equation 11. Hence, for a particular value of the individual subsidy, S, this budget determines directly the threshold $\bar{e}_m$.

The overall change in welfare, induced by such a merit grant scheme, again, results from two effects and is specified by:

$$\textbf{(14.)}\quad \Delta W_m = \frac{N}{e_2 - e_1}\int_{\bar{e}_m}^{\bar{e}} \varphi(e)\, de \;+ \frac{N}{e_2 - e_1}\int_{\bar{e}}^{e_2} \varphi(e)\, de$$

As in the previous welfare analysis for uniform grants, the first term captures the incremental impact on welfare of the specific grant program, while the second relates to the overall enhancing impact of foreign human capital formation, as compared to the autarkic benchmark case. A merit grant program will have a positive effect on welfare if $\Phi(\bar{e}_m) < \Phi(\bar{e})$. However, when $\bar{e} > \hat{e}_2$, and the overall funds for foreign studies grants are relatively limited, it is possible that $\bar{e}_m > \hat{e}_2$, so that the merit grant program actually has the perverse effect of worsening the loss of welfare, relative to a

situation without any such grants. Thus, in order to be efficient, merit grants must be sufficiently large, so that the threshold level $\overline{e}_m$ becomes small enough to satisfy $\Phi(\overline{e}_m) < \Phi(e_2)$. Yet, grant schemes for which $\overline{e}_m < \hat{e}_1$ are not efficient, since there is a welfare maximum for $\overline{e}_m = \hat{e}_1$. Of course, the feasibility of implementing specific grant policies depends on there being a large enough budget, F_m, as well as the value of e_1.

The welfare effects of uniform and merit grants can now be compared on the basis of equations (11.) through (14.), which specify the expressions for the incremental welfare changes and the corresponding budgetary constraint under these alternative programs. First. it can be noted that identical welfare changes can be realized under the two schemes, such that $\Delta W_b = \Delta W_m$, by targeting the same threshold productivity, $\overline{e}_b = \overline{e}_m$. Notably, this occurs when uniform grants are offered for the entire population, $\alpha = 1$. However, the overall financial cost for such an uniform grant program is inherently higher, since $(e_2 - \overline{e}_b)\ (\overline{e} - \overline{e}_b) > (\overline{e} - \overline{e}_b)^2$. This inefficiency reflects the asymmetric information the government faces in the case of uniform grants.

A more general comparative analysis of the welfare effects of the two programs needs to consider the conditions under which uniform grants can generate a greater increase in welfare, than with merit grants, $\Delta W_b > \Delta W_m$, subject to identical budgetary requirements, $F_b = F_m = F$. The analysis of the comparison between ΔW_b and ΔW_m is developed in Appendix 3 and leads to the following conclusion:

<u>Proposition 5</u>

In comparison to uniform grants, where a government lacks any information regarding students' abilities, a system of merit grants, which presupposes full knowledge of abilities, is inherently superior, provided that $\overline{e} < \hat{e}_2$. This sufficient inequality condition stipulates that the productivity level, reflecting the threshold for which individuals will chose to study abroad in the absence of foreign educational subsidies, must be less than a specific critical value.

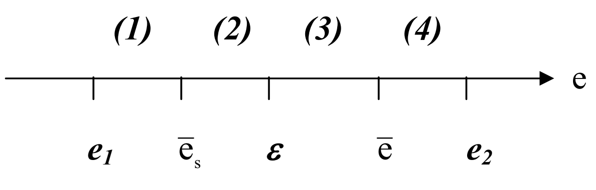
Finally, if $\overline{e} > \hat{e}_2$, there are some situations for which uniform grants are actually more efficient than merit grants. These are typically associated with a combination of low (but not excessively so) values for the overall budget and high values of the productivity threshold.

<u>IV.3 Selective Grant Policies Targeting a Specific Subset of Students</u>

A problem with the foregoing merit grant program is that there is an apparent risk that educational subsidies offered to only the brightest students will foster excessive brain drain, relative to brain gain, given their relatively stronger employment prospects in higher-wage foreign labour markets. Hence, it is conceivable that grants which target somewhat less-qualified students may promote greater brain gain effects, since such students are more likely to return home to work. The analysis here considers a selective grant policy, targeting specific categories of students, assuming that the government knows, ex ante, students' abilities, as in the case of merit grants. However, unlike the latter scenario the student beneficiaries are now not necessarily among the most capable students, who are prevented from going abroad by a lack of funding. The welfare effects of such an optimally designed program will be compared with those arising from a merit grant scheme. Under this restricted merit scheme it is postulated that the grants are aimed at a sub-population of individuals having a hypothetical maximum productivity level equal to ε; where, of course, $\varepsilon < \bar{e}$, in order to ensure that there is a sufficiently high level of brain gain. For a specific value of the foreign educational subsidy, there is potentially a subset of students, relative to the targeted group, with relatively lower abilities for whom the privately anticipated probability of finding employment abroad may not be large enough to warrant incurring the additional costs of foreign studies. Accordingly, such individuals will not accept a grant. As a result, for any given proposed values for the individual foreign study grants and an overall budget for the government grant program, the subset of students actually going abroad can be designated as having productivity levels comprised between a lower threshold value, $\bar{e}_s$, and ε, as shown in Figure 4.

Figure 4

Structure of the population for selective merit grants, targeting somewhat less capable students



zone 1 corresponds to individuals for whom the proposed grant is not sufficient to convince them to go abroad
zone 2 corresponds to grant recipients who undertake foreign studies due to the grant
zone 3 corresponds to individuals who do not receive a grant and study at home
zone 4 corresponds to individuals who while not receiving any grant, still undertake foreign studies

Under a selective grant scheme, the variation of welfare generated by enabling additional individuals to study abroad, again, results from two effects, such that:

$$(15.)\quad \Delta W_s = \frac{N}{e_2 - e_1}\int_{\bar{e}_s}^{\varepsilon} \varphi(e)\, de + \frac{N}{e_2 - e_1}\int_{\bar{e}}^{e_2} \varphi(e)\, de\ .$$

Note that while $\bar{e} - \bar{e}_s$ is proportional to the amount of the grant, the number of beneficiaries is reflected by the productivity interval $\varepsilon - \bar{e}_s$. Consequently, the educational budgetary constraint can be expressed as:

$$(16.)\quad k(\bar{e} - \bar{e}_s)(\varepsilon - \bar{e}_s) \leq F_s\ .$$

The maximum value of ΔW_s is reached for values of $\bar{e}_s$ and ε such that $\bar{e}_s \geq \hat{e}_1$ and $\varepsilon \leq \hat{e}_2$. Otherwise, were $\bar{e}_s$ to be inferior to $\hat{e}_1$, it would be possible to increase ΔW_s by increasing $\bar{e}_s$ with ε constant, while reducing the educational budgetary expenditures. Analogously, if ε were to be superior to $\hat{e}_2$, it would be possible to increase ΔW_s by decreasing ε with $\bar{e}_s$ constant, while reducing again budgetary expenditures. Nonetheless, a maximum value for ΔW_s with $\varepsilon = \hat{e}_2$ may not be feasible since, by construction, it must be the case that $\varepsilon \leq \bar{e}$.

If it is assumed that $\bar{e} \geq \hat{e}_2$, then ε can reach $\hat{e}_2$, which is its unconstrained optimal value. Provided the level of available funds permits such a value for ε and $\bar{e}_s$, the optimum will then be $\varepsilon = \hat{e}_2$ and $\bar{e}_s = \hat{e}_1$, that is if $F_s \geq k(\bar{e} - \hat{e}_1)(\hat{e}_2 - \hat{e}_1)$. If the available public funds are too low, the budgetary constraint will be binding and the optimum value of ε will be inferior to $\hat{e}_2$. Yet, in any case the optimal value of ε will be strictly inferior to $\bar{e}$.

Instead, if it is assumed that $\bar{e} < \hat{e}_2$, the constraint $\varepsilon \leq \bar{e}$ becomes binding. The optimum then corresponds to $\varepsilon = \bar{e}$ and $\bar{e}_s = \hat{e}_1$, conditional on the level of funds being large enough, that is if $F_s \geq k(\bar{e} - \hat{e}_1)^2$. However,

when the budgetary constraint is binding, the maximum value of ΔW_s can be determined by taking the derivative of the foregoing expression for the change in welfare with respect to $\bar{e}_s$, such that:

$$\textbf{(17.)}\quad \frac{d\Delta W_s}{d\bar{e}_s} = \frac{N}{e_2 - e_1}[\varphi(\varepsilon)\frac{d\varepsilon}{d\bar{e}_s} - \varphi(\bar{e}_s)]\,.$$

Along the budget constraint, it follows that:

$$(18.)\quad \varepsilon = \frac{F_s/k}{\bar{e}-\bar{e}_s} + \bar{e}_s \ \textbf{and}\ \frac{d\varepsilon}{d\bar{e}_s} = \frac{F_s/k}{(\bar{e}-\bar{e}_s)^2} + 1\,.$$

It is shown in Appendix 4 that there exists a threshold productivity value, $\bar{e}_{min}$, and associated intervals of values for $\bar{e}$ and F_s, such that ΔW_s has a maximum for some value of $\bar{e}_s$, where the corresponding ε is inferior to $\bar{e}$. When these conditions are not satisfied, ΔW_s is always increasing in $\bar{e}_s$. Since ε is always inferior or equal to $\bar{e}$, the optimum is associated with the maximum value for $\bar{e}_s$, which corresponds to $\varepsilon = \bar{e}$, and is given by $\bar{e}_s = \bar{e} - \sqrt{F_s/k}$. This means that the segment of productivity values $[\bar{e}_s, \bar{e}]$ is fully covered by the allocation of the grants, while the value of $\bar{e}_s$ depends on the level of the available funds.

Such a system of restrictive merit grants can now be compared with the program of unrestricted merit grants analyzed earlier. It can first be remarked that when $\varepsilon = \bar{e}$, the two systems are equivalent, since $\bar{e}_s$ is simply being substituted for $\bar{e}_m$. As a consequence, an optimal program of restricted grants cannot be worse than the unrestricted merit grants in terms of overall welfare. Indeed, the former grant scheme will actually be superior when, at the optimum, $\varepsilon < \bar{e}$. In the alternative situation where there is no interior optimum, the restricted merit grants are inferior, or at best identical, to unrestricted merit grants.

The foregoing analysis, comparing the two grant programss, can be summarized, as follows:

<u>Proposition 6</u>

When a government has full knowledge regarding students' abilities, a scheme of restrictive merit grants, targeting individuals who are not among the most qualified, can dominate a merit scheme, under specific conditions. In particular, the welfare effect of such restricted grants are preferable when

either of the following combination of productivity and budgetary conditions apply: either $\overline{e} > \hat{e}_2$, or $\overline{e} \in [\overline{e}_{\min}, \hat{e}_2]$ and $F \in [F_s^1, F_s^2]$.

Section V: Conclusion

Certain of the principal insights from this research can now be summarized. First, in general, it is very difficult to assert whether the net welfare impact of foreign studies, in the absence of educational grants, will be positive or negative. This is due to the non-linearity of welfare effects, reflecting associated brain drain and brain gain effects, in relation to the distribution of workers' productivity levels. Nevertheless, when the threshold minimum productivity value, determining whether individuals leave, and the maximum attainable for the population of foreign-educated students are both relatively average, in comparison with the productivity requirements for foreign employment, the net welfare effect resulting from foreign human capital formation is positive, i.e. brain gain dominates brain drain. In the foregoing case, welfare is a decreasing function of the threshold probability of finding a job abroad, and, thereby, of the investment cost differential between foreign and domestic studies. Welfare is also an increasing function of wages paid to foreign-educated skilled workers, working in either the home, or foreign labour markets, and a decreasing function of wages paid to less-skilled domestic-trained workers at home. In contrast, either very low, or relatively large values for the fore-mentioned productivity parameters may generate detrimental welfare effects from undertaking foreign studies. Furthermore, the welfare consequences of most parameter values are the inverse of what has been observed in the central zone, so that, now, brain drain dominates brain gain.

The analysis has subsequently examined the efficiency and domestic welfare effects of alternative public initiatives, undertaken by a home country, which are aimed at assisting students to finance their studies abroad. A consideration of three different grant schemes, with alternative assumptions about the extent of a government's information regarding candidates' underlying abilities, suggests that different foreign-study grant schemes are generally efficient and may provide incentives which generate an overall positive effect on welfare, provided that the available funds are sufficient. Nonetheless, a number of subtleties, concerning the specific conditions under which a specific grant scheme can dominate the other schemes, are identified. More specifically, with uniform grants, given the asymmetric information between the government and grant recipients, it is optimum in most situations to propose relatively smaller amounts of

financing to all individuals in the population. Furthermore, with an identical public budget constraint and an average value of the productivity threshold without grants ($\bar{e}$), a merit grant scheme, wherein the government can identify the most capable candidates, is superior to a program of uniform grants. Yet, for high enough critical values for the productivity threshold and for the availability of public funding, uniform grants can generate relatively larger increases in welfare, despite the informational asymmetries. Indeed, when the productivity threshold is rather high, a restrictive merit grant scheme, which targets students who are not the best candidates, but for whom there is a lower propensity for brain drain, may be the most welfare enhancing, provided the level of available funds belongs to some critical interval.

There are a number of potentially fruitful directions for extending the analysis proposed here by incorporating additional modelling features. These include admitting the possibility that domestically educated students, distinguished by individual abilities and associated educational attainment, can seek employment on the foreign labour market.[15] A critical consideration would then be the differential probability of finding a foreign job, which depends on the gap between the productivity distributions for home and foreign-educated domestic workers, as well as the specificity of training to employment in different countries. The latter could be captured by iceberg effects impacting the degree of convertibility of qualifications across labour markets. Clearly, a further crucial consideration may be the extent to which the educational system in the home country enables particularly capable students to enhance substantially their productivity levels, or, in other words, the extent of educational elitism. A more detailed analysis of the interrelation between alternative educational policies in the home country and the extent of brain drain and gain could examine the interrelation between the quality of education offered at different educational levels, the pricing of such studies and the extent of their subsidization – both at home and abroad. A basic presumption would be that there are potential welfare trade-offs between the budgetary expenses of improving national educational offerings and allocating funds for educating students abroad, which could depend on the associated net balances between brain drain and gain. An extended framework could also permit an analysis of the strategic interactions arising from alternative educational budgetary and policy initiatives in both the home and foreign countries. Alternative scenarios relate to the extent of government subsidies, the pricing of tuition

[15] Although brain drain and brain gain effects for domestically trained individuals are not explicitly modeled in this paper, such an extension is relatively straightforward for the special case where there is a fixed probability of being hired abroad and given wage differentials, which do not depend on either individuals' abilities, or productivity levels. Notably, such an extension would entail incorporating additional constant terms, which do not significantly impact the principal qualitative propositions that have been reported.

in relation to overall costs for both domestic and foreign students, and the overall quality of educational offerings for different educational levels in each country.

In light of well-known market failures for financing investments in human capital, initial income distributions could play a critical role in determining whether individuals are prepared to study abroad without government funding. Consequently, an additional policy option could be analyzed either in the existing modelling framework or a more general extension by incorporating alternative hypotheses regarding income and asset distributions and introducing unconditional and/or conditional loans for less wealthy students. If educational loans specify that recipients must return home to work, they generate only brain gain, thereby enabling governments to counter issues of asymmetric information regarding their knowledge of individuals' underlying abilities, since more talented students would, ceteris paribus, tend to accept such loans. Finally, a dynamic modelling perspective could highlight how alternative growth paths for the home economy depend on the extent of both domestic and foreign human capital formation, eventual migration, and endogenous adjustments in wages.

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APPENDIX 1
Consequences of Alternative Employment Policies in the Foreign Country

The analysis here examines the effects of changing the foreign labour market requirement parameters, E_1 and E_2. The specific demonstration of Proposition 3 starts by considering a comparative static change in E_2, for a given value of E_1 : $\frac{d\Delta W}{dE_2} = \frac{N}{e_2 - e_1}\left[\int_{\bar{e}}^{e_2} \frac{\partial \varphi}{\partial E_2}(e)\ de - \varphi(\bar{e})\frac{\partial \bar{e}}{\partial E_2}\right]$ **. The foregoing expression contains two terms, which can be simplified as follows:** $\frac{\partial \varphi}{\partial E_2} = \frac{e(e-E_1)}{(E_2-E_1)^2} = \bar{p}^2 \frac{e(e-E_1)}{(\bar{e}-E_1)^2}$ **and** $\frac{\partial \bar{e}}{\partial E_2} = \bar{p}$ **. As a result, one obtains:** $\frac{e_2-e_1}{N}\frac{d\Delta W}{dE_2} = \bar{p}^2 \int_{\bar{e}}^{e_2} \frac{e(e-E_1)}{(\bar{e}-E_1)^2} de - \bar{p}\ [\bar{e}(1-\bar{p}) - C]$ **. By defining** $G(\bar{e}) = \bar{p}\int_{\bar{e}}^{e_2} \frac{e(e-E_1)}{(\bar{e}-E_1)^2} de$ **, and reexpressing the term algebraically it follows that** $\frac{1}{\bar{p}}\frac{e_2-e_1}{N}\frac{d\Delta W}{dE_2} = G(\bar{e}) - [\bar{e}(1-\bar{p}) - C]$**. Note that G is a positive decreasing function of $\bar{e}$ such that $G(e_2) = 0$. Consequently, if $e_2 < \frac{C}{1-\bar{p}}$, $\frac{d\Delta W}{dE_2}$ is always positive $\forall \bar{e} < e_2$. Accordingly, the change in domestic welfare, ΔW, is always increasing with $\bar{e}$, and so also with E_2. However, if $e_2 > \frac{C}{1-\bar{p}}$, there is a threshold value for $\bar{e}$ such that beyond this value, ΔW is decreasing when E_2 and $\bar{e}$ increase. Yet, this threshold value may be inferior to e_1 , in which case ΔW is always decreasing with E_2 . Furthermore, qualitatively similar results hold for an increase in E_1 , or for an increase in both E_1 and E_2 , when, in the latter case, a constant span $E_2 - E_1$ is assumed.**

APPENDIX 2
Uniform Grants

The derivative of the welfare changes, ΔW_b , with respect to the threshold productivity level for recipients of an uniform grant, $\bar{e}_b$, can be determined, as follows:

$$\frac{e_2-e_1}{N}\frac{\partial \Delta W_b}{\partial \bar{e}_b}=\frac{d\alpha}{d\bar{e}_b}\int_{e_b}^{\bar{e}}\varphi(e)\,de-\alpha\,\varphi(\bar{e}_b)$$

$$=\frac{d\alpha}{d\bar{e}_b}\,[\,\Phi(\bar{e})-\Phi(\bar{e}_b)]-\alpha\,\varphi(\bar{e}_b)$$

$$=\frac{d\alpha}{d\bar{e}_b}\,(\bar{e}-\bar{e}_b)\;\psi(\bar{e},\bar{e}_b)-\alpha\,\varphi(\bar{e}_b)$$

with $\psi(\bar{e},\bar{e}_b)=\dfrac{\Phi(\bar{e})-\Phi(\bar{e}_b)}{\bar{e}-\bar{e}_b}$ **. Differentiation of the constraint, which yields** $\dfrac{1}{\alpha}\dfrac{d\alpha}{d\bar{e}_b}=\dfrac{1}{e_2-\bar{e}_b}+\dfrac{1}{\bar{e}-\bar{e}_b}$**, shows that along the constraint, α is an increasing function of $\bar{e}_b$. It can then be seen that** $\dfrac{\partial \Delta W_b}{\partial \bar{e}_b}=0$ **is equivalent to:**

(i) $$\left[\frac{\bar{e}-\bar{e}_b}{e_2-\bar{e}_b}+1\right]\psi(\bar{e},\bar{e}_b)=\varphi(\bar{e}_b)\,.$$

If there is a solution to the foregoing equation, it determines the optimum value of $\bar{e}_b$. As is subsequently shown, this equation is equivalent to:

(ii) $$(e_2-\bar{e}_b)^2=-3(E_2-E_1)\frac{\Phi(e_2)-\Phi(\bar{e})}{e_2-\bar{e}}\quad.$$

From the latter formulation, it is apparent that such a solution exists only if $\Phi(\bar{e}) > \Phi(e_2)$. However, this is not a sufficient condition, since $\bar{e}_b$ must also be inferior to $\bar{e}$, which yields $\bar{e}>e_2-\sqrt{-3(E_2-E_1)\dfrac{\Phi(e_2)-\Phi(\bar{e})}{e_2-\bar{e}}}$**. Essentially, this latter inequality means that the value of $\bar{e}$ must not be too far away from e_2 . If either of these conditions is not met,** $\dfrac{\partial \Delta W_b}{\partial \bar{e}_b}$ **is always positive. The equivalence of equations (i) and (ii) now needs to be demonstrated. The point of departure is equation (i), which may also be written as:**

(iii) $$\psi(\bar{e},\bar{e}_b)=(e_2-\bar{e}_b)\frac{\varphi(\bar{e}_b)-\psi(\bar{e},\bar{e}_b)}{\bar{e}-\bar{e}_b}$$

Since Φ is a third-degree polynomial in e, it can be further elaborated by developing it between $\bar{e}_b$ and $\bar{e}$:

$$\Phi(\bar{e}_b) = \Phi(\bar{e}) - (\bar{e} - \bar{e}_b)\Phi'(\bar{e}) + 1/2(\bar{e} - \bar{e}_b)^2\, \Phi''(\bar{e}) - 1/6(\bar{e} - \bar{e}_b)^3 \Phi'''(\bar{e})$$

$$= \Phi(\bar{e}) - (\bar{e} - \bar{e}_b)\, \varphi(\bar{e}) + 1/2(\bar{e} - \bar{e}_b)^2\, \varphi'(\bar{e}) - 1/6(\bar{e} - \bar{e}_b)^3\, \varphi''(\bar{e})$$

It follows that:

$$\text{(iv)} \quad \psi(\bar{e},\bar{e}_b) = \frac{\Phi(\bar{e}) - \Phi(\bar{e}_b)}{\bar{e} - \bar{e}_b} = \varphi(\bar{e}) - 1/2(\bar{e} - \bar{e}_b)\, \varphi'(\bar{e}) + 1/6(\bar{e} - \bar{e}_b)^2\, \varphi''(\bar{e}),$$

which yields:

$$\psi(\bar{e},\bar{e}_b) - \varphi(\bar{e}_b) = \varphi(\bar{e}) - \varphi(\bar{e}_b) - 1/2(\bar{e} - \bar{e}_b)\, \varphi'(\bar{e}) + 1/6(\bar{e} - \bar{e}_b)^2\, \varphi''(\bar{e})$$

However, by developing also φ between between $\bar{e}_b$ and $\bar{e}$, it follows that:

$$\varphi(\bar{e}_b) = \varphi(\bar{e}) - (\bar{e} - \bar{e}_b)\, \varphi'(\bar{e}) + 1/2(\bar{e} - \bar{e}_b)^2\, \varphi''(\bar{e}) .$$

The latter expression is equivalent to:

$$\psi(\bar{e},\bar{e}_b) - \varphi(\bar{e}_b) = 1/2(\bar{e} - \bar{e}_b)\, \varphi'(\bar{e}) - 1/3\, (\bar{e} - \bar{e}_b)^2\, \varphi''(\bar{e})$$

It follows that:

$$\frac{\varphi(\bar{e}_b) - \psi(\bar{e},\bar{e}_b)}{\bar{e} - \bar{e}_b} = -1/2\, \varphi'(\bar{e}) + 1/3\, (\bar{e} - \bar{e}_b)\, \varphi''(\bar{e})$$

By substituting the latter expression into equation (iii), it follows that:

$$\text{(v)} \quad \psi(\bar{e},\bar{e}_b) = (e_2 - \bar{e}_b)\, [\, -1/2\, \varphi'(\bar{e}) + 1/3\, (\bar{e} - \bar{e}_b)\, \varphi''(\bar{e})]$$

An analogous elaboration between e_2 and $\bar{e}$, yields:

$$\Phi(e_2) = \Phi(\bar{e}) + (e_2 - \bar{e})\, \varphi(\bar{e}) + 1/2\, (e_2 - \bar{e})^2\, \varphi'(\bar{e}) + 1/6\, (e_2 - \bar{e})^3\, \varphi''(\bar{e})$$

and

$$\text{(vi)} \quad \psi(e_2,\bar{e}) = \frac{\Phi(e_2) - \Phi(\bar{e})}{e_2 - \bar{e}} = \varphi(\bar{e}) + 1/2\, (e_2 - \bar{e})\, \varphi'(\bar{e})$$

$$+ 1/6\, (e_2 - \bar{e})^2\, \varphi''(\bar{e})$$

A combination of equations (iv) and (vi), then results in:

$$\psi(e_2,\bar{e}) - \psi(\bar{e},\bar{e}_b) = 1/2\, (e_2 - \bar{e}_b)\, \varphi'(\bar{e}) + 1/6\, [(e_2 - \bar{e})^2 - (\bar{e} - \bar{e}_b)^2]\, \varphi''(\bar{e})$$

Using the expression of $\psi(\bar{e},\bar{e}_b)$ given by (v), it then follows that:

$$\psi(e_2,\bar{e}) = 1/3\, (e_2 - \bar{e}_b)\, (\bar{e} - \bar{e}_b)\, \varphi''(\bar{e}) + + 1/6\, [(e_2 - \bar{e})^2 - (\bar{e} - \bar{e}_b)^2]\, \varphi''(\bar{e})$$

and

$$\text{(vii)} \quad \psi(e_2,\bar{e}) = 1/6\, (e_2 - \bar{e}_b)^2\, \varphi''(\bar{e}) \quad .$$

Finally, since $\varphi''(\bar{e}) = -\dfrac{2}{E_2 - E_1}$**, it is easy to see that equation (vii) is equivalent to equation (ii).**

APPENDIX 3
Merit Grants

For given $\bar{e}$, in the case of the merit program, the value of F determines $\bar{e}_m$, whereas for given $\bar{e}_b$, the value of α is given by $\alpha = \frac{F/k}{(e_2 - \bar{e}_b)(\bar{e} - \bar{e}_b)} = \frac{(\bar{e} - \bar{e}_m)^2}{(e_2 - \bar{e}_b)(\bar{e} - \bar{e}_b)}$. Consequently, the condition $\Delta W_b >$ ΔW_m may be written as, $\frac{(\bar{e} - \bar{e}_m)^2}{(e_2 - \bar{e}_b)(\bar{e} - \bar{e}_b)}[\Phi(\bar{e}) - \Phi(\bar{e}_b)] > \Phi(\bar{e}) - \Phi(\bar{e}_m)$.

Some mathematical manipulations lead to:

$$e_2 < e_{2lim}(\bar{e}_b) = \psi(\bar{e},\bar{e}_b)\frac{\bar{e} - \bar{e}_m}{\psi(\bar{e},\bar{e}_m)} + \bar{e}_b \ .$$

Assuming first that $\bar{e} < \hat{e}_2$, the condition $\Delta W_b > \Delta W_m$ then implies that $\bar{e}_b <$ $\bar{e}_m$. It will now be shown that e_{2lim} is an increasing function of $\bar{e}_b$ for $\bar{e}_b \leq \bar{e}_m \leq \bar{e}$. First, after defining the expression $\xi(e) = \frac{\psi(\bar{e},e)}{\bar{e} - e}$, one has

$$e_{2lim}(\bar{e}_b) = \xi(\bar{e}_b)\frac{\bar{e} - \bar{e}_b}{\xi(\bar{e}_m)} + \bar{e}_b \ .$$

Now, as seen in Appendix 2, on the basis of equation 4, it follows that:

$$\psi(\bar{e}, e) = \frac{\Phi(\bar{e}) - \Phi(e)}{\bar{e} - e} = \varphi(\bar{e}) - 1/2(\bar{e} - e)\,\varphi'(\bar{e}) + 1/6(\bar{e} - e)^2\,\varphi''(\bar{e}) \ ,$$

so that

$$\xi(e) = \frac{\varphi(\bar{e})}{\bar{e} - e} - 1/2\,\varphi'(\bar{e}) + 1/6(\bar{e} - e)\,\varphi''(\bar{e}) \ .$$

Since φ'' is always negative, ξ is an increasing function of e, for $e \leq \bar{e}$.

The derivation of e_{2lim} with regard to $\bar{e}_b$ leads to:

$$\xi(\bar{e}_m)\frac{de_{2lim}}{d\bar{e}_b} = \xi(\bar{e}_m) - \xi(\bar{e}_b) + \xi'(\bar{e}_b)(\bar{e} - \bar{e}_b)$$

Since $\bar{e}_b \leq \bar{e}_m$, $\xi(\bar{e}_m) \geq \xi(\bar{e}_b)$ and $\xi'(\bar{e}_b) \geq 0$, the derivative is positive, which means that e_{2lim} is increasing in $\bar{e}_b$.

It follows that $e_{2lim}(\bar{e}_b) \leq e_{2lim}(\bar{e}_m)$ for $\forall \bar{e}_b \leq \bar{e}_m$. However, in this instance $e_{2lim}(\bar{e}_m) = \bar{e}$ and $\bar{e} \leq e_2$. Together, the latter two conditions are incompatible with $e_2 < e_{2lim}(\bar{e}_b)$, which precludes a relatively larger increase in welfare with uniform grants, as compared to a merit scheme, so $\Delta W_b >$ ΔW_m.

Now, if $\bar{e} > \hat{e}_2$, there are some values of F for which the maximum value of ΔW_b is larger than ΔW_m . This can be seen by observing that, for given $\bar{e}_b$ and $\bar{e}$, e_{2lim} becomes infinite for a limiting value of $\bar{e}_m$, $\bar{e}_{ml}$ (associated with a corresponding limit value F_l of F), such that $\Phi(\bar{e}_{ml})=\Phi(\bar{e})$ and $\bar{e}_{ml}=\bar{e}-\sqrt{F/k}$. Thus, there is a range of values of $\bar{e}_m$, inferior to $\bar{e}_{ml}$, and a range of values of F, superior to F_l, for which e_{2lim} will be superior to e_2. Nonetheless, if the level of funds F_l is very low, the values of $\bar{e}_b$ and $\bar{e}_m$ will be so high that $\Phi(\bar{e}_b)$ and $\Phi(\bar{e}_m)$ will be larger than $\Phi(e_2)$. Accordingly, ΔW_b and ΔW_m will both be negative and both grant schemes are inefficient.

APPENDIX 4
Restrictive Merit Grants

A starting point for the analysis is the expression of the derivative of ΔW_s with respect to $\bar{e}_s$. Since the value of the parameter $\frac{N}{e_2 - e_1}$ does not matter here, it can be arbitrarily set equal to 1, in order to simplify the notation. Accordingly, the following expression applies:

$$\frac{d\Delta W_s}{d\bar{e}_s} = \varphi(\varepsilon)\frac{d\varepsilon}{d\bar{e}_s} - \varphi(\bar{e}_s).$$

Along the budget constraint, $\varepsilon = \frac{F_s/k}{\bar{e} - \bar{e}_s} + \bar{e}_s$ and $\frac{d\varepsilon}{d\bar{e}_s} = \frac{F_s/k}{(\bar{e} - \bar{e}_s)^2} + 1$.

It can be seen that $\frac{d\Delta W_s}{d\bar{e}_s}$ is positive for $\bar{e}_s = \hat{e}_1$ and negative for $\bar{e}_s = \hat{e}_2$. Of particular interest here is the case where $\bar{e} < \hat{e}_2$. The upper limit value for ε is, then, $\bar{e}$, which corresponds to $\bar{e}_s = \bar{e} - \sqrt{F_s/k}$. If $\frac{d\Delta W_s}{d\bar{e}_s}(\bar{e})$ is negative, ΔW_s has a maximum for a value of ε strictly inferior to $\bar{e}$. On the contrary, if $\frac{d\Delta W_s}{d\bar{e}_s}(\bar{e})$ is still positive, it means that the optimum corresponds to the limit value $\varepsilon = \bar{e}$. For $\varepsilon = \bar{e}$, one has: $\frac{d\Delta W_s}{d\bar{e}_s}(\bar{e}) = \varphi(\bar{e})[\frac{F_s/k}{(\bar{e} - \bar{e}_s)^2} + 1] - \varphi(\bar{e}_s)$

with $\bar{e}_s = \bar{e} - \sqrt{F_s/k}$ and, thus, $\frac{d\Delta W_s}{d\bar{e}_s}(\bar{e}) = 2\varphi(\bar{e}) - \varphi(\bar{e} - \sqrt{F_s/k})$. Given then that $\varphi(e)$ has a maximum for $e = E_2/2$, it follows that if $\bar{e} \leq E_2/2$, $\frac{d\Delta W_s}{d\bar{e}_s}(\bar{e})$ is certainly positive. Actually this is still true, provided $\bar{e} \leq \bar{e}_{min}$, where $\bar{e}_{min} \in [E_2/2, \hat{e}_2]$ is such that $\varphi(\bar{e}_{min}) = 1/2\, \varphi(E_2/2)$.

When $\bar{e} \in [\bar{e}_{min}, \hat{e}_2]$, the sign of $\frac{d\Delta W_s}{d\bar{e}_s}(\bar{e})$ depends on the value of F_s. More precisely, this derivative is negative when F_s belongs to an interval $[F_s^1, F_s^2]$, for which the limits are functions of $\bar{e}$, and solutions of the equation.2 $\varphi(\bar{e}) - \varphi(\bar{e} - \sqrt{F_s/k}) = 0$. Furthermore, the higher the value of $\bar{e}$, the wider

is the interval, which has a maximum for $\bar{e} = \hat{e}_2$, corresponding to $F_s^1 = 0, F_s^2 = k(\hat{e}_2 - \hat{e}_1)^2$. This shows also that for $\bar{e} \in [\bar{e}_{min}, \hat{e}_2]$, all F_s belonging to $[F_s^1, F_s^2]$ meet the condition $F_s \leq k(\bar{e} - \hat{e}_1)^2$, while this condition is satisfied as an equality only for $\bar{e} = \hat{e}_2$, and $F_s = F_s^2$. As a result, when $\frac{d\Delta W_s}{d\bar{e}_s}(\bar{e}) <$ 0, the change in welfare, ΔW_s, has again an interior maximum for values of $\bar{e}_s$ and ε satisfying the equations $\frac{d\Delta W_s}{d\bar{e}_s} = \varphi(\varepsilon)[\frac{F_s/k}{(\bar{e}-\bar{e}_s)^2} + 1] - \varphi(\bar{e}_1) = 0$ and $k(\bar{e} - \bar{e}_s)(\varepsilon - \bar{e}_s) = F_s$. When F_s does not belong to the forementioned interval, $\frac{d\Delta W_s}{d\bar{e}_s}(\bar{e}) \geq 0$, so that the maximum value of ΔW_s corresponds to $\bar{e}_s = \bar{e} - \sqrt{F_s/k}$ and $\varepsilon = \bar{e}$.