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## A MICROFOUNDATION FOR ADAPTABILITY RETURNS TO SCHOOLING AND TECHNOLOGICAL COMPLEXITY

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# A microfoundation for adaptability returns to schooling and technological complexity\*

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**Abstract:** In a frictional environment, technological complexity creates adaptability returns to schooling. This paper provides a microfoundation to such an argument, and revisits the impacts of worker heterogeneity and risk aversion. In our model, firms and workers are located on a knowledge space. Education widens the measure of the knowledge subset that the worker embodies, while technological complexity expands the measure of the knowledge subset that is required to operate on the job. We find uncertainty with regard to the location of the future partner motivates schooling, while it inhibits technological complexity. When workers differ in scholastic ability, the welfare of a given group increases with the proportion of workers of this group. Finally, risk aversion motivates a precautionary demand for education, which in turn creates income risk through firms' technological choices.

**Keywords:** Education; Multi-dimensional skills; Frictions; Heterogeneity; Risk aversion

**J.E.L. classification:** D24, I21, J24

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# 1 Introduction

Since Becker (1964), human capital theory views education as a production factor. More educated workers have a higher marginal productivity, and obtain higher rewards as a result of wage competition between potential employers. However, the direct contribution of education to physical production does not account for the total contribution to revenue. Education enhances allocative ability (Welch, 1970), ability to learn (Schultz, 1975, Rosenzweig 1995), and improves workers' capacity to deal with changing conditions (Nelson and Phelps, 1966). Schooling features adaptability returns, and, as suggested by Welch (1970) or Rosenzweig (1995), these returns increase with the complexity of the production process. The reason why adaptability skills and technological complexity matter relies on an assumption that is often implicit. Workers face mobility costs between differentiated jobs. Adaptability skills lower these costs, while technological complexity increases them.

This paper offers a microfoundation for adaptability returns to schooling and technological complexity. This microfoundation involves matching frictions and knowledge mismatch. We assume that the state of knowledge, viewed as a continuum of basic abilities/fields, is symbolized by a circle. Each agent, individual or firm, is characterized by a particular location on the circle, which reflects individual tastes and social background. Individuals learn at school a subset of knowledge. This subset is centered in individual location, and its measure increases with the schooling duration. Firms choose the technology, that is the subset of knowledge that is required to operate on the technology. This subset is centered in firm's location. We refer to its measure as technological complexity. More a technology is complex, more it is productive. However, a worker can operate on a job only if the subset of knowledge embodied in the technology belongs to the subset he learnt at school.

There are two sources of mismatch between jobs and workers. On the one hand, the skill requirements of technologies may outweigh the skills learnt by the workers. Firms and workers make expectations on the decisions taken by the other side of the market. If firms are too optimistic regarding the skills of the workforce, or if workers undervalue technological complexity, many employment relationships turn down. Conversely, if firms are too pessimistic, or if workers are very careful, employment relationships are not very productive, and some schooling investment may be wasted. On the other hand, the assignment process may fail to bring together workers and technologies of similar locations. Even though workers embody considerable fields of knowledge and technologies are relatively simple, employment relationships may not work well because the skills required by technologies differ a lot from workers' skills.

Such a "mismatch of talents" motivates schooling, while it inhibits technological complexity. Because workers do not know in advance which skills are required by their future

job, they have incentives to invest in education in order to become more adaptable and form matches with more firms. In a same way, since firms have to make their technological choice before they know their future employee, they can be reluctant to choose a high level of complexity. Consequently, education is increasing in technological complexity as workers get schooling in order to reduce the risk of technological failure and technological complexity is increasing in education as the risk of failure to recruit an adequate worker is lower.

We expose our results in four steps.

1. We characterize the Walrasian allocation. It is useful as a benchmark to evaluate the effects of matching imperfections. In that allocation, matching is efficient: firms of a given location only meet workers of the same location. All workers choose the same schooling duration, and all firms set the same technological complexity. The schooling duration is such that the subset of knowledge embodied in each worker coincides with the subset of knowledge required to operate the technology. Technological complexity then maximizes output.

2. The frictional allocation experiences two departures from the Walrasian allocation. Firstly, we assume Nash bargaining over match surplus. We know from Grout (1984) and Acemoglu (1996) this may lead to a hold-up problem. Each party pays the full marginal cost of investment but only obtains a share of the marginal reward. We abstract from this source of inefficiency. We assume there are no other costs to education than the time spent nonemployed, and there are no other costs to technological complexity than the risk of technological failure. The foregone earnings nature of each cost implies that both the return and cost of education are held-up and each side of the market chooses the efficient decision given the informational constraints of the economy. Secondly, we take as a starting point that efficient matching requires a degree of coordination that is unfeasible. Frictions result in the misallocation of workers to jobs. We assume random matching between jobs and workers: a given worker may meet any firm with a uniform density. Workers can no longer set education so as to match the knowledge demand of a particular technology. Equivalently, firms can no longer set technological complexity so as to match a particular bundle of worker's skills. Both strategies are too risky, as they imply a major risk of failure. Consequently, firms reduce technological complexity compared to the Walrasian allocation, while workers increase education. The presence of frictions implies less complex and less productive technologies, more time spent in education, and technological unemployment/nonemployment due to mismatch.

3. Taste heterogeneity vis-à-vis the different fields of knowledge is not the only source of heterogeneity. The presence of workers of different educational levels may alter the welfare of each person. To tackle with this issue, we introduce heterogeneity in scholastic ability. There are two types of workers: high-ability and low-ability ones. The main result

can be stated as follows: the welfare of individuals of a specific group always increases with the proportion of workers of this group. Changes in the distribution of types affect all workers through reactions of the other side of the market. Facing an increase in the share of high-ability workers, firms increase technological complexity. This benefits to the more able whose skills are under-used in equilibrium. However, it is detrimental to less able individuals who are compelled to set a longer schooling duration.

4. Education reduces the risk of technological failure. Our model, therefore, appears well-suited to the study of risk aversion. Risk aversion originates a precautionary demand for education. Facing more educated workers, firms raise technological complexity, thereby creating the risks the workers fear. As long as workers' risk aversion is higher than employers', risk aversion originates income risk. In addition, when workers differ in risk aversion, those who have higher risk aversion create a negative externality on all workers because they make firms set more complex technologies.

The interplay between adaptability and technological complexity can be viewed as a strong form of capital-skill complementary. As such, it is line with well-established empirical evidence. For instance, Bartel and Lichtenberg (1987) use the average age of capital as a proxy for technological advancement and show that more educated workers have a comparative advantage in implementing newer technologies. Dunne and Schmitz (1995) and Doms et al (1997) use plant level data and find that technologically more advanced plants employ more educated and more skilled workers. Of course, these results do not prove that more educated workers are better placed to operate on more complex technologies. However, newer or advanced technologies that embody computing equipment and/or that increase the level of automation are more complex to use and often involves multidimensional skills as minimum language skills, reading skills or basic math skills. In turn, there is evidence that the implementation of a more complex production process is associated to increased difficulties at the recruitment stage. Baron et al (1985) use data for hires collected in 1980. They show that the number of applicants interviewed per job offer increases with associated educational requirement. They also find that employers seeking to fill their positions with high educational requirements devote a larger number of hours to recruiting.

The rest of the paper is organized as follows. Section 2 presents the model, and compares the Walrasian allocation with the frictional allocation. Section 3 considers two extensions, namely worker heterogeneity in scholastic ability and risk aversion. Section 4 relates our paper to the literature. Section 5 concludes.

## 2 The model

### 2.1 General assumptions

We describe a one-period environment where a continuum of workers normalized to one faces a continuum of firms of identical size. There is a unique consumption good. Both firms and workers are risk-neutral and make ex-ante investments: workers choose their education, while firms determine technological complexity. Each firm meets one and only one worker and reciprocally.

*Education.* Let  $\mathbb{K} \equiv [0, 1]$  denote knowledge, viewed as a collection of primary fields. Each individual has a location  $i$  on the space of knowledge, which represents her tastes vis-à-vis the different fields of knowledge. Locations are uniformly distributed on the space of knowledge. Each individual is endowed with one unit of time she divides between a schooling period  $s \in [0, 1]$  and a participation period  $1 - s$ . Schooling gives access to a subset of knowledge. The measure of this subset increases with schooling investment. Let  $\mathbb{H}_{si} \subset \mathbb{K}$  be the subset of knowledge acquired by a worker whose location is  $i$  and educational investment is  $s$ . Let also  $H(s)$  denote the Lebesgue measure of  $\mathbb{H}_{si}$ . Then,  $\mathbb{H}_{si} = [i - H(s)/2, i + H(s)/2]$ , accounting for the fact that locations are defined modulo one. The function  $H$  is endowed with the following properties.

**Assumption 1** (i)  $H'(s) > 0$  for all  $s \geq 0$ , (ii)  $H''(s) \leq 0$  for all  $s \geq 0$ , (iii)  $H(0) = 0$ , (iv)  $H(1) \leq 1$

*Technology.* A technology converts knowledge into a production process. Firms determine technological complexity. Increasing technological complexity raises job productivity. However, it reduces the probability that a given worker can operate on the job. Formally, choosing a technology consists in picking randomly a location  $j$  on the space of knowledge, and then choosing the scope of skills  $c \in (0, 1]$  required to operate on the technology. The corresponding subset of the space of knowledge is  $\mathbb{H}_{cj} = [j - c/2, j + c/2]$ . Output per unit of time spent working  $y$  depends on worker's subset of knowledge  $\mathbb{H}_{si}$  and technology-specific subset of knowledge  $\mathbb{H}_{cj}$  according to

$$y(\mathbb{H}_{cj}, \mathbb{H}_{si}) = \begin{cases} f(c) & \text{if } \mathbb{H}_{cj} \subset \mathbb{H}_{si} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Equation (1) presents an O-ring type technology (Kremer, 1993). Production involves a combination of different tasks. The number of tasks increases with technological complexity. If all tasks are completed, production takes place. If at least one task is not performed correctly, there is no output.

**Assumption 2** (i)  $f'(c) > 0$  for all  $c \geq 0$ , (ii)  $f''(c) \leq 0$ , for all  $c \geq 0$ , (iii)  $f(0) = 0$

To close the model, we need to specify how firms and workers are allocated to each other, and how output is divided between the two parties. Let  $\Omega_l(i)$  denotes the set of firms' location the worker may meet. The set  $\Omega_l(i)$  specifies a non-empty subset of  $\mathbb{K}$ , potentially restricted to a singleton, and a distribution function on that subset. Similarly, let  $\Omega_k(j)$  denotes the set of workers' locations the firm may meet.

## 2.2 Efficient matching

In this sub-section, we describe the case of efficient matching. Workers and firms are perfectly assigned to each other: workers of a given location only meet firms from the same location, and vice-versa. Formally,  $\Omega_l(i) = \{i\}$  and  $\Omega_k(j) = \{j\}$ . We first analyze the Walrasian allocation, and then study the decentralized mechanism with ex-post rent-sharing.

*Walrasian allocation.* For each pair of worker and firm, the problem is to maximize production, that is

$$(s^w(i), c^w(i)) \arg \max_{s_i, c_i} (1 - s_i) f(c_i) \quad (2)$$

subject to  $H(s_i) \geq c_i$ .

**Proposition 1** *The Walrasian allocation can be summarized by the unique pair  $(s^w, c^w) \in (0, 1) \times (0, 1)$  such that:*

$$(i) \ s^w(i) = s^w \text{ and } c^w(j) = c^w \text{ for all } (i, j) \in [0, 1] \times [0, 1]$$

$$(ii) \ H(s^w) = c^w$$

$$(iii) \ f(c^w) = (1 - s^w) f'(c^w) H'(s^w)$$

Point (i) states that all workers have the same schooling duration, and all firms set the same technological complexity. Point (ii) says that workers get schooling so as to match firms' technological complexity. Equivalently, firms determine technological complexity so as to coincide with workers' embodied knowledge. Finally, equation (iii) states that optimal schooling equates marginal cost and marginal benefit. The left-hand side is the marginal cost, which consists of foregone production. The right-hand side is the marginal benefit, equal to marginal output times the fraction of time spent working.

*Efficient matching with ex-post rent-sharing.* Output is divided between the firm and the worker according to the asymmetric Nash solution to the bargaining problem. If  $w$  denotes the wage per unit of time spent working,  $\beta \in (0, 1)$  common workers' bargaining power and  $\pi$  firm's profit, we have

$$w = \beta y \text{ and } \pi = (1 - \beta) y \quad (3)$$

Agents choose investments to maximize expected payoffs, rationally expecting the investment achieved by the other side of the market. Once the schooling period is completed, each worker is allocated to a firm. Then, output is determined according to (1).

Workers' investment in education and firms' technological complexity jointly result from:

$$\hat{s} \in \arg \max_s \{(1-s)\beta f(c), H(s) \geq c\} \quad (4)$$

$$\hat{c} \in \arg \max_c \{(1-s)(1-\beta)f(c), H(s) \geq c\} \quad (5)$$

**Proposition 2** (i) *The set of equilibria is*

$$E = \{(c, s) \in [0, H(1)] \times [0, 1], H(s) = c\} \quad (6)$$

(ii)  $(c^w, s^w) \in E$  and it strictly pareto-dominates the other allocations

Multiple equilibria suggests that there are huge market failures. For instance,  $c^* = s^* = 0$  and  $c^* = H(1)$  with  $s^* = 1$  are two highly inefficient equilibria. However, the Walrasian allocation belongs to the set of equilibria, and it pareto-dominates the other allocations.

## 2.3 Random matching

In this sub-section, we focus on the situation where matching is random. We assume that  $\Omega(i) = \{[0, 1], U\}$  for all  $i \in [0, 1]$ , where  $U$  is the uniform law. Similarly for firms,  $\Omega(j) = \{[0, 1], U\}$  for all  $j \in [0, 1]$ . Each worker meets a firm picked from the whole set of locations, and equivalently for firms. Output is still determined by ex-post rent sharing.

*Educational investment.* Each worker maximizes her indirect utility function. As we are only interested in symmetric equilibrium, we consider a worker's  $i$  schooling investment, taking as given the common technological complexity set by firms. Optimal schooling results from

$$\hat{s}_i \in \arg \max_s \Pr [\mathbb{H}_{cj} \subset \mathbb{H}_{si} \mid c, i \text{ and } j \in \Omega(i)] (1-s)\beta f(c) \quad (7)$$

The objective is the probability of matching technology-specific skill requirements, times the output part accruing to the worker. But,

$$\Pr [\mathbb{H}_{cj} \subset \mathbb{H}_{si} \mid c, i \text{ and } j \in \Omega(i)] = \Pr [x + i + c \leq i + H(s)] \quad (8)$$

where  $x$  is the distance between worker's and firm's locations. Finally,

$$\Pr [\mathbb{H}_{cj} \subset \mathbb{H}_{si} \mid c, i \text{ and } j \in \Omega(i)] = H(s) - c \quad (9)$$



The latter probability does not depend on worker's location. Optimal schooling results from

$$\hat{s} \in \arg \max_s (H(s) - c) (1 - s) \beta f(c) \quad (10)$$

The first-order condition is necessary. It can be written as follows:

$$(H(\hat{s}) - c) \beta f(c) = (1 - \hat{s}) H'(\hat{s}) \beta f(c) \quad (11)$$

The left-hand side is the marginal cost of education, which consists in foregone expected earnings. This accounts for the matching probability  $H(\hat{s}) - c$ . The right hand-side is the marginal benefit. It is equal to the marginal effect of education on the matching probability due to increased knowledge, times the time spent working, times the wage. Simplifying, we get

$$H(\hat{s}) = c + (1 - \hat{s}) H'(\hat{s}) \quad (12)$$

Equation (12) displays several crucial features. First,  $H(\hat{s}) > c$ . Due to random matching, workers cannot afford to get schooling so as to match technological complexity because the probability of meeting a firm with the same location is nil. This result also holds when technological complexity becomes arbitrarily small (i.e.  $c = 0$ ): heterogeneity in firms' locations alone implies workers need to train. Second, workers expect some technological failure, as they bear the risk  $1 - (H(\hat{s}) - c) > 0$  of not being successful while trying to produce. Third, even though workers only maximize their share of expected output, the resulting schooling investment does neither depend on output, nor on bargaining power. This is due to the foregone earnings nature of the schooling cost, which implies both gains and costs from education are proportional to workers' share of output. In the terminology of Acemoglu (1996), both the returns and the costs from investment are held-up.

*Technological choice.* Each firm chooses independently technological complexity, taking as given the choice of other firms, as well as workers' choice. Due to the symmetry between the worker's problem and the firm's, optimal technological complexity results from

$$\hat{c} \in \arg \max_c (H(s) - c) (1 - \beta) (1 - s) f(c) \quad (13)$$

The first-order condition writes down<sup>1</sup>

$$(1 - \beta) (1 - s) f(\hat{c}) = (H(s) - \hat{c}) (1 - \beta) (1 - s) f'(\hat{c}) \quad (14)$$

The left-hand side is the marginal cost of technological complexity, which consists of a lower probability to produce successfully. The right-hand side is the marginal return to complexity, which consists of a higher productivity. Simplifying,

$$\hat{c} = H(s) - f(\hat{c}) / f'(\hat{c}) \quad (15)$$

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<sup>1</sup>In the proof of Proposition 2 exposed in the Appendix, we rigorously study firms' and workers' maximization programs.

Several points should be noted. First, technological complexity is set lower than  $H(s)$ . As in the case of education detailed previously, this property is due to random matching. Firms expect to meet workers from the whole distribution of locations. Setting technological complexity close to  $H(s)$  would expose the firm to a huge risk of technological failure. Second, technological complexity is increasing in education. An educated workforce stimulates the introduction and use of riskier and more productive technologies. Finally, technological complexity does not depend on bargaining power: the marginal cost of complexity is of the foregone-earnings type, so both the cost and return to technological complexity are proportional to earnings.

*Equilibrium.* An equilibrium is a pair  $(s^*, c^*)$  solving  $s^* = \widehat{s}(c^*)$  and  $c^* = \widehat{c}(s^*)$ .

**Proposition 3** (i) *There exists a unique non-trivial equilibrium, that is an equilibrium with  $s^* \in (0, 1)$*

(ii)  *$s^* > s^w$  and  $c^* < c^w$*

People educated longer and firms set less complex technologies in the frictional economy than in the Walrasian environment. These properties are due to the imperfection of the assignment mechanism. Nothing prevents workers and firms to meet partners very far from their location on the space of knowledge. Agents alter their behavior to reduce the extent of the mismatch. As covering the whole circle would imply to spend the whole time endowment in education, random matching means technological unemployment takes place in equilibrium. The rate of unemployment is  $u^* = 1 - (1 - s^*) H'(s^*)$ .

An important property of equilibrium is that it does not depend on workers' bargaining power. The following result proves that the decentralized allocation is constrained efficient.

**Proposition 4** *Let  $u(c, s)$  and  $v(c, s)$  denote, respectively, workers' and firms' payoffs.*

(i) *The decentralized allocation maximizes output*

(ii) *The decentralized allocation maximizes expected payoffs, i.e.  $u(c^*, s^*) > u(c, s)$  and  $v(c^*, s^*) > v(c, s)$  for all  $(c, s) \neq (c^*, s^*)$*

The decentralized allocation is efficient: not only output is maximized, but no other allocation can improve welfare. Despite agents make ex-ante investments, assignment of workers to jobs is random, and wages are determined by ex-post rent-sharing, the decentralized equilibrium is constrained efficient. This result departs from Acemoglu (1996). It can be understood by carefully examining equations (11) and (14). For each type of investment, i.e. education and technological complexity, the marginal return is typically too low as the agent only maximizes her share of output. However, and for the same reason, the marginal cost is also too low. These two distortions offset each other, and the decentralized economy achieves the efficient allocation.

### 3 Extensions

In this section, we consider two extensions: workers' heterogeneity in ability to learn, and risk aversion.

#### 3.1 Hierarchical heterogeneity

In the basic framework, workers only differ in location. We now suppose workers differ in scholastic ability  $a$ , which represents their capacity to learn. The measure of the subset of knowledge  $\mathbb{H}_{sia}$  obtained by the workers is  $H(a, s)$ .

**Assumption 3** (i)  $H_1(a, s) > 0$ , (ii)  $H_{12}(a, s) > 0$ , (iii)  $H_2(a, s) > 0$ , (iv)  $H_{22}(a, s) < 0$ , (v)  $H(a, 0) = 0$ , and (vi)  $\lim_{s \rightarrow 1} H(a, s) = 1$

(i) says that the marginal productivity of scholastic ability is positive, (ii) says that the marginal productivity is increasing in ability, (iii) to (v) correspond to properties (i) to (iii) in Assumption 1. Finally, (vi) says that agents can learn the whole space of knowledge if they spend their whole time endowment at school. This assumption ensures that all agents will have a chance of getting a job in equilibrium.

There are two types of workers:  $p_1 \equiv p \in (0, 1)$  high-ability workers whose ability is  $a_1$ , and  $p_2 \equiv 1 - p$  low-ability workers, whose ability is  $a_2 < a_1$ .

Output per unit of time spent working is now given by

$$y(\mathbb{H}_{sia}, \mathbb{H}_{cj}) = \begin{cases} f(c) & \text{if } \mathbb{H}_{cj} \subset \mathbb{H}_{sia} \\ 0 & \text{outside} \end{cases} \quad (16)$$

In the case of efficient matching, firms and workers know ex-ante which type of agents they will meet ex-post. In that case, Proposition 4 still holds: the matching place is perfectly segmented by location and schooling attainment, and the decentralized equilibrium replicates the Walrasian allocation.

The case of random matching is more interesting: firms and workers do not know ex-ante who they will meet ex-post. We now investigate this case.

Workers choose education rationally expecting firms' common technological complexity. This yields:

$$H(a_i, \hat{s}_i) = c + (1 - \hat{s}_i) H_2(a_i, \hat{s}_i) \quad (17)$$

Education is still increasing in technological complexity. The effect of scholastic ability is ambiguous: due to complementarity between ability and schooling in knowledge acquisition, the direct effect of ability is to raise schooling. However, high-ability agents need less education to get the same knowledge. Formally, it depends on  $(1 - \hat{s}_i) H_{12} - H_1$ . The

key point is that  $H(a, \hat{s})$  is strictly increasing in  $a$ . Indeed,

$$\begin{aligned} \frac{dH(a, \hat{s})}{da} &= H_1 + H_2 \frac{d\hat{s}}{da} \\ &= H_1 + H_2 \frac{(1-s)H_{12} - H_1}{2H_2 - (1-s)H_{22}} > 0 \end{aligned}$$

While setting technological complexity, firms take into account workers' heterogeneity according to

$$\hat{c} \in \arg \max_c \sum_i p_i \max \{H(a_i, s_i) - c, 0\} (1 - \beta) (1 - s_i) f(c) \quad (18)$$

Unlike the case with homogenous workers, the firm may choose technological complexity so as to exclude low-ability workers from the economic activity.<sup>2</sup> As workers' best-response prevents the occurrence of such a situation in equilibrium, we only focus on an interior maximum. The first-order condition reads as

$$\hat{c} = \bar{H}(s_1, s_2) - f(\hat{c})/f'(\hat{c}) \quad (19)$$

where

$$\bar{H}(s_1, s_2) = \frac{\sum_i p_i (1 - s_i) H(a_i, s_i)}{\sum_i p_i (1 - s_i)} \quad (20)$$

Technological complexity is strictly increasing in workers' mean subset of knowledge.

**Proposition 5** *Let  $u_i(c, s)$  denote the expected payoff of a type- $i$  worker, endowed with schooling  $s$  and facing technological complexity  $c$ .*

- (i) *There exists a unique equilibrium;*
- (ii) *Equilibrium schooling and technological complexity are increasing in the share of high-ability workers, that is  $ds_i^*/dp > 0$ , and  $dc^*/dp > 0$ ;*
- (iii) *Welfare of a given group is increasing in the share of this group, that is  $du_i(s_i^*, c^*)/dp_i > 0$ .*

As in Acemoglu (1996), agents' decisions have an impact on other agents through the reaction of the other side of the market, that is firms. High-ability workers make easier the implementation of more complex technologies. Equilibrium technological complexity, therefore, is increasing in the share of high-ability individuals. From the schooling investment equation (17), this implies education duration also rises. Those developments are similar to Acemoglu: rising the share of those who have the highest schooling return (or

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<sup>2</sup>The mechanism would be similar to Acemoglu (1999). In this contribution, there are two types of workers, skilled and unskilled, and firms set capital intensity before matching a worker. However, capital can be resold without loss on the capital market. Firms choose to do so whenever the gap between a skilled and an unskilled worker is sufficiently large.

the lowest schooling cost in Acemoglu) translates into an increase in the schooling attainment of all individuals. The crucial difference, however, is illustrated by point (iii). If the welfare of high-ability individuals is increasing in their share in the whole population, the welfare of low-ability individuals is decreasing. High-ability workers exert a negative externality on low-ability workers.

### 3.2 Risk aversion

In this sub-section, we suppose both firms and workers are risk-averse. A type  $r$  agent has the following Von Neuman-Morgenstern utility function  $u(z) = z^{1-\alpha_r}$ , where  $\alpha_r \in [0, 1]$  is the absolute risk aversion. Entrepreneurs are of type  $\alpha_k$ , while workers are of type  $\alpha_l$ . The bargained wage is given by

$$w \in \arg \max_w \{ \beta (1 - \alpha_l) \ln [(1 - s) w] + (1 - \beta) (1 - \alpha_k) \ln [(1 - s) (f(c) - w)] \} \quad (21)$$

Therefore,

$$w = \mu f(c) \quad (22)$$

where  $\mu = \frac{\beta(1-\alpha_l)}{\beta(1-\alpha_l)+(1-\beta)(1-\alpha_k)}$ . The wage is decreasing in worker's risk aversion, and increasing in entrepreneur's risk aversion (this is a standard result). Educational investment results from

$$\hat{s} \in \arg \max_s (H(s) - c) [(1 - s) \mu f(c)]^{1-\alpha_l} \quad (23)$$

This yields

$$H(\hat{s}) = c + \frac{1}{1 - \alpha_l} H'(\hat{s}) (1 - \hat{s}) \quad (24)$$

The schooling investment equation features two properties. First, and as in the case with risk-neutral individuals, the bargaining power does not affect schooling investment. Second, there is a precautionary demand for education. As the left-hand side of equation (24) is decreasing in education, optimal schooling is increasing in risk aversion.<sup>3</sup>

Similarly, technological complexity is determined by

$$\hat{c} \in \arg \max_c (H(s) - c) [(1 - \mu) (1 - s) f(c)]^{1-\alpha_k} \quad (25)$$

This yields

$$\hat{c} = H(s) - \frac{1}{1 - \alpha_k} f(\hat{c}) / f'(\hat{c}) \quad (26)$$

A crucial feature of equation (26) is that technological complexity decreases with risk aversion. The reason is similar to the one behind workers' precautionary demand for

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<sup>3</sup>Gould et al (2001) also focus on the precautionary demand for education in a two-sector model with sector-specific uncertainty and logarithmic preferences. However, education is a binary decision, firms' technology is exogenous and Gould et al do not derive the normative implications below.

education, but it runs the other way around. Risk-averse entrepreneurs have a demand for insurance. As no insurance is available, they respond to educational investments by setting a conservative technological complexity, that is lower than the one which would be chosen by a social planner aiming at maximizing output.

The equilibrium is a fixed-point of the mutual best-response strategies.

**Proposition 6** (i) *There exists a unique non-trivial equilibrium;*

(ii)  $\partial s^*(\alpha_l, \alpha_k) / \partial \alpha_l > 0$  and  $\partial s^*(\alpha_l, \alpha_k) / \partial \alpha_k < 0$ , while  $\partial c^*(\alpha_l, \alpha_k) / \partial \alpha_l > 0$  and  $\partial c^*(\alpha_l, \alpha_k) / \partial \alpha_k < 0$ .

Workers tend to invest too much in education, while entrepreneurs tend to set too simple technologies. In both cases, inefficiency is a consequence of uninsured income risk. We may observe in equilibrium both over- and under-education, as well as too complex or too simple technologies. However, the resulting allocation never maximizes output. An interesting feature of the analysis is reported by claim (ii). Income risk originates a precautionary demand for education, yet it exacerbates risk. The reason is due to the reaction of firms that increase technological complexity when education rises.

This suggests another extension, where workers ex-ante differ with respect to risk aversion. The intuition suggests that the more risk averse workers create a negative externality on all workers, as their increased schooling duration creates incentives for firms to raise technological complexity. To show this, we consider two types of workers:  $p_1 = p \in (0, 1)$  risk haters, whose risk aversion is  $\alpha_1 \in (0, 1)$ , and  $p_2 = 1 - p$  risk tolerant, whose risk aversion is  $\alpha_2$ , with  $0 < \alpha_2 < \alpha_1$ .

**Proposition 7** *Suppose  $\alpha_k = 0$ ,  $0 \leq \alpha_2 < \alpha_1$  and let  $u_i(c, s)$  denote the expected payoff of a type- $i$  worker endowed with schooling  $s$  and facing technological complexity  $c$ .*

(i) *There exists a unique non-trivial equilibrium;*

(ii)  $ds_i^*/dp > 0$  and  $dc^*/dp > 0$ ;

(iii)  $du_i(c^*, s_i^*)/dp < 0$  for  $i = 1, 2$ .

As expected, technological complexity increases with the share of risk haters. In turn, schooling investments also increase with  $p$ . In addition, both risk haters and risk tolerant lose in terms of welfare. This result may have important implications. It shows that risk aversion may originate income risk in the society. There is a widespread belief in the society whereby economic performance strongly depends on risk-taking individuals, while growth transfers economic situations from the risk averse to the risk takers. As long as education protects from income risk and risky technologies depend on education, this may well be the exact opposite.

## 4 Related literature

Our paper relates to the literature on ex-ante investments when there are matching frictions.

Acemoglu (1996) provides the seminal analysis. He considers heterogenous agents who set their investment before they match. Frictions mean that there is uncertainty about future partner's type. The magnitude of investment cannot be contracted, and output is split ex-post between firm and worker. Acemoglu sheds light on a pecuniary externality. Due to agents' heterogeneity and random matching, increasing investment in a sub-group of individuals increases investment on the other side of the market, which improves expected payoffs for all. Our paper starts from the same market situation. However, the nature of investment differs. Rather than investments in human and physical capital, we focus on investments in adaptability skills and technological complexity. This allows us to obtain different results. Namely, education is typically longer than in the Walrasian outcome, while technological complexity is typically lower. In addition, increasing education in a sub-group of workers is generally good for firms, but bad for all the workers.

Acemoglu (1999) and Albrecht and Vroman (2002) reach conclusions that are closer to ours. They both consider a random matching model in which there are two types of workers, skilled and unskilled, and two types of jobs, complex and simple. They show that an increase in skilled proportion can make firms turn down unskilled applicants more frequently. In Albrecht and Vroman, firms increase the proportion of complex jobs, which reduces the employment prospects of the unskilled who cannot perform on such jobs by assumption. In Acemoglu, firms increase capital intensity and reject unskilled applicants because the option value of unfilled jobs is higher.<sup>4</sup> Our paper complements these earlier studies in four ways. (i) We are interested in technological choices rather than in recruitment practices or capital choices. Technological complexity is a continuous variable, and the skills required to perform on the technology are endogenous. As setting technological complexity does not involve any direct costs, having a job does not feature an option value as in Acemoglu. Applicants may be rejected because they do not have the adequate set of skills, and not because they are insufficiently productive. (ii) There is horizontal differentiation between jobs in our paper, while there is vertical differentiation in the two others. There are no good jobs and bad jobs in our model. All the jobs have the same technological complexity, but the set of required skills differs from one job to another. (iii) Horizontal differentiation implies that workers are neither always employable nor never employable. Education alters the employment probability at the margin. (iv)

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<sup>4</sup>Acemoglu first develop a static version of his model. Firms choose capital intensity before they meet a worker. However, they can resell capital once a worker is met at no cost. An increase in skilled proportion raises capital intensity, up to the point where keeping an unskilled worker is not worth paying the capital cost.

Most importantly, education and technological complexity are both endogenous in our paper. Acemoglu and Albrecht and Vroman focus on the implications of educational choices on firms' decisions. We also examine how firms' behavior alters the schooling incentives.

Nelson and Phelps' notion of adaptability has been pursued by Krueger and Kumar (2002) and Aghion et al (2002) in growth models. The former assume that general (by opposition to vocational) education reduces the probability the workers suffer a loss of task-specific productivity following the introduction of a new technology. The latter assume new technologies are sector-specific and workers must be adaptable to implement any such new technology. The longer the schooling period when young, the more likely the worker can use the new technology when old. In our paper, educated workers are more adaptable not because they can perform better on new jobs but because they can form matches with more firms. A close assumption is used by Lloyd-Ellis (1999) in a growth model with endogenous technological change. In his model, minimum skill levels are required to implement new but equiproductive technologies and workers differ in the range of technologies they can implement. However, the skill acquisition process is exogenous.

The way we model mismatch borrows from Marimon and Zilibotti (1999). This is a multi-sector matching model with ex-post Nash bargaining over the match surplus where workers have (symmetric) sector-specific skills à la Salop (1979). Firms and workers are distributed along the perimeter of a technological circle. The productivity of a pair is strictly decreasing in the distance between firm's and worker's locations. Matching is random and job offers above some critical distance are always rejected (see also Decreuse, 2008, for a different interpretation of the matching process). The relationship between the productivity of a pair and the worker-firm distance is exogenous in Marimon and Zilibotti. In a related setting in which education raises workers' adaptability, Charlot et al (2005) and Decreuse and Granier (2005) also suggest that matching frictions provide incentives to schooling. In a model without education, Acemoglu and Pischke (2000) also examine the trade-off between productivity and probability of recruiting an adequate worker. Our paper complements these earlier contributions by (i) setting microfoundations to adaptability skills and technological complexity, and (ii) examining jointly the decisions taken by the two sides of the market.

Finally, the idea whereby income risk creates a social demand for education is not new. Gould et al (2001) propose a two-sector growth model that features such incentives to schooling. However, the growth process is exogenous. Our paper goes a step further by considering the implications of a more educated workforce on the incentives to set more complex technologies.



## 5 Conclusion

Why does education provide adaptability skills? There is a large body of evidence suggesting that education is more than a production factor. In particular, education offers a bundle of skills that reduces the extent of mismatch and allow firms to use complex technologies. However, the relationship between education, adaptability skills, and technological complexity is most of the time a black box. The purpose of this paper is to provide an explicit microeconomic scenario in which the technological environment is endogenous and there are adaptability returns to schooling. Our scenario is based on two components: labor market frictions and knowledge mismatch. Firms and workers are located on a knowledge space. Education widens the measure of the knowledge subset that the worker embodies, while technological complexity expands the measure of the knowledge subset that is required to operate on the job. Matching frictions result in location mismatch.

We find uncertainty with regard to the location of the future partner motivates schooling, while it inhibits technological complexity. Matching frictions, therefore, raise the need for adaptability skills and lower incentives to adopt complex technologies. When workers differ in scholastic ability, an increase in the proportion of high-ability individuals increases technological complexity. The welfare of a given group (low-ability or high-ability individuals) increases with the proportion of workers of this group. Finally, risk aversion motivates a precautionary demand for education, which in turn creates income risk through firms' technological choices.

Our model is sufficiently simple to be used in a dynamic setting. The knowledge space could expand over time and implications for growth could be derived. Innovations' location in the knowledge space could be biased, worsening the mismatch problem. The description of matching frictions could also be enriched. These extensions are on our research agenda.

## APPENDIX: PROOFS

**Proof of proposition 1** Immediate. ■

**Proof of proposition 2** (i) From the maximization program (4), workers' best response to firms' technological complexity  $c \in (0, H(1))$  is  $\hat{s} \equiv \hat{s}(c) = H^{-1}(c)$ . Similarly, from the maximization program (5), firms' best response to workers' education  $s \in (0, 1)$  is  $\hat{c} \equiv \hat{c}(s) = H(s)$ . An equilibrium is a fixed-point of mutual best responses. Therefore, all  $(s, c) \in (0, 1) \times (0, 1)$  such that  $H(s) = c$  are equilibria.

(ii) From Proposition 1, the Walrasian allocation satisfies  $H(s^w) = c^w$ . Therefore  $(s^w, c^w) \in E$ . As the Walrasian allocation maximizes output and expected payoffs are proportional to aggregate output, the Walrasian allocation is the unique allocation that maximizes firms' and workers' expected payoffs. ■

**Proof of proposition 3** We first establish the properties of workers' and firms' investments discussed in the text. Then, we show (i) and (ii).

Preliminary. The expected payoffs of workers and firms are, respectively,  $u(c, s) = (1 - s)(H(s) - c)\beta f(c)$  and  $v(c, s) = (1 - s)(H(s) - c)(1 - \beta)f(c)$ .

*Workers' choice.* Consider the worker's maximization problem (10). If  $c = 0$ ,  $u(c, s) = 0$  for all  $s \in [0, 1]$ . If  $c > H(1)$ ,  $u(c, s) < 0$  for all  $s \in [0, 1]$  and the worker rationally chooses not to get schooling, so that  $\hat{s} = 0$ . If  $c = H(1)$ , the worker is indifferent between not participating and setting  $s = 1$ . Finally, if  $c \in (0, H(1))$ , then worker's optimal choice results from the first-order condition (12). To show this, note that  $u(c, 0) = u(c, 1) = 0$ , while  $u(c, s) > 0$  for all  $s \in (H^{-1}(c), H(1))$ . By continuity, the first-order condition is necessary. Now, consider the function  $\phi_1$  such that

$$\phi_1(s, c) = H(s) - c + (1 - s)H'(s) \quad (27)$$

We have  $\phi_1(0, c) < 0$  and  $\phi_1(1, c) > 0$  since  $c < H(1)$  by assumption and

$$\frac{\partial \phi_1(s, c)}{\partial s} = 2H'(s) - (1 - s)H''(s) > 0 \quad (28)$$

from Assumption 1. By continuity, there exists a unique  $\hat{s}(c)$  such that  $\phi_1(\hat{s}(c), c) = 0$ . Therefore,

$$\hat{s} = \begin{cases} [0, 1] & \text{if } c = 0 \\ \hat{s}(c) & \text{if } c \in (0, H(1)) \\ \{0, 1\} & \text{if } c = H(1) \\ 0 & \text{if } c > H(1) \end{cases} \quad (29)$$

When  $c \in (0, H(1))$ , the implicit function theorem implies that

$$\begin{aligned}\widehat{s}'(c) &= -\frac{\partial \phi_1(\widehat{s}(c), c) / \partial c}{\partial \phi_1(\widehat{s}(c), c) / \partial s} \\ &= \frac{1}{2H'(\widehat{s}(c)) - (1 - \widehat{s}(c))H''(\widehat{s}(c))} > 0\end{aligned}\quad (30)$$

Moreover,  $\lim_{c \rightarrow H(1)} \widehat{s}(c) = 1$ , while  $\lim_{c \rightarrow 0} \widehat{s}(c) = s_0 > 0$ , where  $s_0$  is the unique solution to

$$H(s_0) = (1 - s_0)H'(s_0) \quad (31)$$

*Firms' choice.* Consider the firm's maximization problem (13). A similar analysis to the worker's problem yields that

$$\widehat{c} = \begin{cases} [0, 1] & \text{if } s = 0 \\ \widehat{c}(s) & \text{if } s \in (0, 1) \\ [0, 1] & \text{if } s = 1 \end{cases} \quad (32)$$

where  $\widehat{c}(s)$  is the unique solution to (15). To show this latter point, consider the function  $\phi_2$  such that

$$\phi_2(s, c) = c - H(s) + f(c)/f'(c) \quad (33)$$

We have  $\phi_2(s, 0) < 0$  and  $\phi_2(s, H(1)) > 0$  since  $s < 1$  by assumption and

$$\frac{\partial \phi_2(s, c)}{\partial c} = 2 - f(c)f''(c)/f'(c)^2 > 0 \quad (34)$$

from Assumption 2. By continuity, there exists a unique  $\widehat{c}(s)$  such that  $\phi_1(\widehat{s}(c), c) = 0$ . The implicit function theorem implies that

$$\widehat{c}'(s) = \frac{H'(s)}{2 - f(c)f''(c)/f'(c)^2} > 0 \quad (35)$$

Finally,  $\lim_{s \rightarrow 0} \widehat{s}(c) = 0$ , while  $\lim_{s \rightarrow 1} \widehat{c}(s) = c_1 < H(1)$ , where  $c_1$  is the unique solution to

$$c_1 = H(1) - f(c_1)/f'(c_1) \quad (36)$$

(i) Remark first that there are two trivial equilibria:  $(s^*, c^*) = (0, 0)$  and  $(s^*, c^*) = (1, H(1))$ . Non-trivial equilibria are fixed points of the functions  $\widehat{s}(c)$  and  $\widehat{c}(s)$ . Consider the function  $\psi$  such that

$$\psi(c) = c - H(\widehat{s}(c)) + f(c)/f'(c) \quad (37)$$

A non-trivial equilibrium solves  $\psi(c^*) = 0$ ,  $c^* \in (0, H(1))$ , and  $s^* = \widehat{s}(c^*)$ . But  $\psi(0) < 0$ ,  $\psi(H(1)) > 0$  and

$$\begin{aligned}\psi'(c) &= 1 - H'(\widehat{s}(c))\widehat{s}'(c) \\ &= 1 - \frac{H'(\widehat{s}(c))}{2H'(\widehat{s}(c)) - (1 - \widehat{s}(c))H''(\widehat{s}(c))} > 0\end{aligned}$$

Therefore, there exists a unique non-trivial equilibrium.

(ii) Let us rewrite the equilibrium conditions as follows:

$$f(c) = (1-s)f'(c)H'(s) \quad (38)$$

$$H(s) = c + (1-s)H'(s) \quad (39)$$

Then, the only difference between the decentralized allocation with random matching and the Walrasian allocation comes from the second term of the right-hand side of (39). As this term only depends on  $s$ , we have  $s^w \neq s^*$  and  $c^w \neq c^*$ . Suppose  $c^* > c^w$ . From (38), it implies  $s^* < s^w$ . But, this requires from (39) that  $c^* < c^w$ , a contradiction. Therefore  $c^* < c^w$ , which implies  $s^* > s^w$ . ■

**Proof of proposition 4** Let  $y(s, c) = (1-s)[H(s) - c]f(c)$  denote aggregate output. As expected payoffs of firms and workers are given by  $\beta y(s, c)$  and  $(1-\beta)y(s, c)$ , any allocation that maximizes aggregate output also maximizes workers' and firms' expected payoffs. It comes that the decentralized allocation is the unique allocation that maximizes output. ■

**Proof of proposition 5** (i) Let  $\hat{s}_i(c)$  denote the best response of a type- $i$  worker to technological complexity  $c$ . Details concerning the properties of function  $\hat{s}_i(c)$  are similar to those exposed in the preliminary of the proof of Proposition 3. Therefore, we do not expose them twice. A non-trivial equilibrium is a fixed-point of mutual best responses, with  $s_i^* \neq 0$ ,  $i = 1, 2$ . Consider the function  $\psi$  such that

$$\psi(c) = c - \bar{H}(\hat{s}_1(c), \hat{s}_2(c)) + f(c)/f'(c) \quad (40)$$

An equilibrium is a vector  $(s_1^*, s_2^*, c^*)$  such that  $\psi(c^*) = 0$ , and  $s_i^* = \hat{s}_i(c^*)$ ,  $i = 1, 2$ . As  $\psi$  is continuous,  $\psi(0) < 0$  and  $\psi(1) > 0$ , there exists an equilibrium. We now prove  $\psi$  is strictly increasing. Hence,

$$\psi'(c) = 2 - \frac{\bar{H}_1}{2H_2^1 - (1-s_1)H_{22}^1} - \frac{\bar{H}_2}{2H_2^2 - (1-s_2)H_{22}^2} - f(c)f''(c)/f'(c)^2 \quad (41)$$

where  $s_i \equiv \hat{s}_i(c)$ ,  $\bar{H}_i \equiv \bar{H}_i(\hat{s}_1(c), \hat{s}_2(c))$ ,  $H_2^i \equiv H_2(a_i, \hat{s}_i(c))$ , and  $H_{22}^i \equiv H_{22}(a_i, \hat{s}_i(c))$ . Assumptions 2 and 3 imply

$$\psi'(c) > 2 - \frac{\bar{H}_1}{2H_2^1} - \frac{\bar{H}_2}{2H_2^2} \quad (42)$$

Taking the derivative of  $\bar{H}$  given by (20) with respect to  $s_i$ , we get

$$\bar{H}_i = \frac{p_i(1-s_i)}{p(1-s_1) + (1-p)(1-s_2)} H_2^i < H_2^i \quad (43)$$

Therefore,

$$\psi'(c) > 2 - \frac{H_2^1}{2H_2^1} - \frac{H_2^2}{2H_2^2} > 0 \quad (44)$$

(ii) Let  $\bar{H} \equiv \bar{H}(p, s_1, s_2)$  to make the dependence vis-à-vis  $p$  explicit. From the implicit function theorem,  $dc^*/dp$  has the sign of  $\partial \bar{H}(p, s_1^*, s_2^*)/\partial p$ . But,

$$\frac{\partial \bar{H}(p, s_1^*, s_2^*)}{\partial p} = \frac{(1-s_1)(1-s_2)(H_1-H_2)}{[p(1-s_1) + (1-p)(1-s_2)]^2} > 0 \quad (45)$$

As  $\widehat{s}'_i(c) > 0$ , we also have  $ds_i^*/dp > 0$ .

(iii) We have

$$\frac{du_i(c^*, s_i^*)}{dp} = \frac{\partial u_i(c^*, s_i^*)}{\partial c} \frac{dc^*}{dp} \quad (46)$$

But,

$$\frac{\partial u_i(c^*, s_i^*)}{\partial c} = (1-\beta)(1-s_i^*)f'(c^*)\{H(a_i, s_i^*) - c^* - f(c^*)/f'(c^*)\} \quad (47)$$

Using (19),

$$\frac{\partial u_i(c^*, s_i^*)}{\partial c} = (1-\beta)(1-s_i^*)f'(c^*)\{H(a_i, s_i^*) - \bar{H}(s_1^*, s_2^*)\} \quad (48)$$

The result follows from the fact  $H(a_2, s_2^*) < \bar{H}(s_1^*, s_2^*) < H(a_1, s_1^*)$ . ■

**Proof of proposition 6** (i) We proceed similarly to previous proofs. From (24), we can define the implicit function  $\widehat{s}(c)$ . It is strictly increasing and such that  $\widehat{s}(0) = s_{\alpha_l}$  and  $\widehat{s}(H(1)) = 1$ , where  $s_{\alpha_l} \in (0, 1)$  is implicitly defined by

$$H(s_{\alpha_l}) = \frac{1}{1-\alpha_l} H'(s_{\alpha_l})(1-s_{\alpha_l}) \quad (49)$$

Let

$$\psi(c) = c - H(\widehat{s}(c)) + \frac{1}{1-\alpha_k} \frac{f(c)}{f'(c)} \quad (50)$$

We have  $\psi(0) < 0$  while  $\psi(H(1)) > 0$ . In addition,

$$\psi'(c) = 1 - H'(\widehat{s}(c)) \frac{d\widehat{s}(c)}{dc} + \frac{1}{1-\alpha_k} \frac{f'(c)^2 - f(c)f''(c)}{f'(c)^2} \quad (51)$$

But,

$$\frac{d\widehat{s}(c)}{dc} = \frac{1}{H'(\widehat{s}(c)) + \frac{1}{1-\alpha_l} [H'(\widehat{s}(c)) - (1-\widehat{s}(c))H''(\widehat{s}(c))]} < \frac{1}{H'(\widehat{s}(c))} \quad (52)$$

Therefore  $\psi'(c) > 0$ .

(ii) Let  $\widehat{s}(c) \equiv \widehat{s}(\alpha_l, c)$  to highlight the dependence vis-à-vis  $\alpha_l$ . From the implicit function theorem,  $dc^*/d\alpha_l$  has the sign of  $\partial\widehat{s}(\alpha_l, c^*)/\partial\alpha_l > 0$ . It follows that

$$\frac{ds^*}{d\alpha_l} = \frac{\partial\widehat{s}(\alpha_l, c^*)}{\partial\alpha_l} + \frac{\partial\widehat{s}(\alpha_l, c^*)}{\partial c} \frac{dc^*}{d\alpha_l} > 0 \quad (53)$$

Similarly, it is easy to show that  $dc^*/d\alpha_k > 0$  and  $ds^*/d\alpha_k > 0$ . ■

**Proposition 7** Let

$$\lambda \equiv \frac{p(1 - \mu_1)}{p(1 - \mu_1) + (1 - p)(1 - \mu_2)} \quad (54)$$

A non-trivial equilibrium  $(c^*, s_1^*, s_2^*)$  solves

$$H(s_i) = c + \frac{1}{1 - \alpha_i} H'(s_i)(1 - s_i), \quad i = 1, 2 \quad (55)$$

$$c = \overline{H}(s_1, s_2) - f(c)/f'(c) \quad (56)$$

where

$$\overline{H}(s_1, s_2) \equiv \frac{\lambda(1 - s_1)H(s_1) + (1 - \lambda)(1 - s_2)H(s_2)}{\lambda(1 - s_1) + (1 - \lambda)(1 - s_2)} \quad (57)$$

(i) From (24), we can define two implicit functions  $\widehat{s}_1(c)$  and  $\widehat{s}_2(c)$ , such as in the proof of proposition 6, with  $\widehat{s}_1(c) > \widehat{s}_2(c)$  for all  $c \in [0, 1)$ , and  $\widehat{s}_1(1) = \widehat{s}_2(1) = 1$ . Let

$$\psi(c) = c - \overline{H}(\widehat{s}_1(c), \widehat{s}_2(c)) + f(c)/f'(c) \quad (58)$$

We have to show there exists a unique  $c^* \in (0, 1)$  such that  $\psi(c^*) = 0$ . But,  $\psi(0) < 0$  and  $\psi(1) > 0$ . Moreover,  $\psi$  is derivable and

$$\begin{aligned} \psi'(c) &= 2 - \overline{H}_1 \frac{d\widehat{s}_1(c)}{dc} - \overline{H}_2 \frac{d\widehat{s}_2(c)}{dc} - f(c)f''(c)/f'(c)^2 \\ &> 2 - \overline{H}_1 \frac{d\widehat{s}_1(c)}{dc} - \overline{H}_2 \frac{d\widehat{s}_2(c)}{dc} \end{aligned}$$

But

$$\frac{d\widehat{s}_i(c)}{dc} = \frac{1}{H'(\widehat{s}_i(c)) - \frac{1}{1 - \alpha_i} [H''(\widehat{s}_i(c))(1 - \widehat{s}_i(c)) - H'(\widehat{s}_i(c))]} < \frac{1}{H'(\widehat{s}_i(c))} \quad (59)$$

Hence,

$$\psi'(c) > 2 - \frac{\overline{H}_1}{H_1} - \frac{\overline{H}_2}{H_2} \quad (60)$$

The result follows from the fact  $\overline{H}_i < H_i$  for  $i = 1, 2$ .

(ii) It is sufficient to show that  $dc^*/dp > 0$ , which in turn implies that  $ds_i^*/dp > 0$ . In this goal, let  $\overline{H}(s_1^*, s_2^*) \equiv \overline{H}(\lambda, s_1^*, s_2^*)$  to make the dependence vis-à-vis  $\lambda$  explicit. The implicit function theorem implies that  $dc^*/dp$  has the sign of

$$\frac{\partial\overline{H}(\lambda, s_1^*, s_2^*)}{\partial p} = \frac{\partial\overline{H}(\lambda, s_1^*, s_2^*)}{\partial\lambda} \frac{d\lambda}{dp} > 0 \quad (61)$$

since  $s_1^* > s_2^*$ .

(iii) We have

$$\frac{du_i(c^*, s_i^*)}{dp} = \frac{\partial u_i(c^*, s_i^*)}{\partial c} \frac{dc^*}{dp} \quad (62)$$

As  $dc^*/dp > 0$ , it is sufficient to show  $\partial u_i(c^*, s_i^*)/\partial c > 0$ . But,

$$\frac{\partial u_i(c^*, s_i^*)}{\partial c} = \mu (1 - s_i^*)^{1-\alpha_i} \frac{f'(c^*)}{f(c^*)} f(c^*)^{1-\alpha_i} \left\{ -\frac{f(c^*)}{f'(c^*)} + (1 - \alpha_i) [H(s_i^*) - c^*] \right\} \quad (63)$$

As  $c^* = \bar{H}(s_1^*, s_2^*) - f(c^*)/f'(c^*)$ ,

$$\text{sign} \left\{ \frac{\partial u_i(c^*, s_i^*)}{\partial c} \right\} = \text{sign} \left\{ -\alpha_i \frac{f(c^*)}{f'(c^*)} + (1 - \alpha_i) [H(s_i^*) - \bar{H}(s_1^*, s_2^*)] \right\} \quad (64)$$

Since  $f' > 0$  and  $H(s_2) < \bar{H}(s_1, s_2)$ ,  $\partial u_2(c^*, s_2^*)/\partial c < 0$ .

Now, remark,  $H(s_1^*) - \bar{H}(s_1^*, s_2^*) = \frac{1}{1-\alpha_1} H'(s_1^*) (1 - s_1^*) - f(c^*)/f'(c^*)$ . Therefore,  $\partial u_1(c^*, s_1^*)/\partial c$  has the sign of

$$-\frac{f(c^*)}{f'(c^*)} + (1 - s_1^*) H'(s_1^*) \quad (65)$$

But the strict concavity of  $H$  implies

$$(1 - s_1^*) H'(s_1^*) < (1 - s_2^*) H'(s_2^*) = (1 - \alpha_1) \left[ H(s_2^*) - \bar{H}(s_1^*, s_2^*) + \frac{f'(c^*)}{f(c^*)} \right] \quad (66)$$

It follows that  $\partial u_1(c^*, s_1^*)/\partial c < 0$ . ■

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