Trading Mechanism Selection with Directed Search when Buyers are Risk Averse

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ABSTRACT: We endogenize the trading mechanism selection in a model of directed search with risk averse buyers and show that the unique symmetric equilibrium entails all sellers using fixed price trading. Mechanisms that prescribe the sale price as a function of the realized demand (auctions, bargaining, discount pricing, etc.) expose buyers to the "price risk", the uncertainty of not knowing how much to pay in advance. Fixed price trading eliminates the price risk, which is why risk averse customers accept paying more to shop at such stores.
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JEL: D4, D81, D83

1 Introduction

The objective of this note is to endogenize the pricing mechanism selection in a model of directed search where buyers are risk averse. The adoption of a particular mechanism signals how the seller intends to share the surplus ex-post, which in turn influences the attractiveness of the store and pins down the expected demand. So, it is natural to ask which trading protocol sellers would choose when given the option.

Existing models endogenizing the trading mechanism selection using competitive search seem to agree that a large number of mechanism are payoff equivalent. Kultti [6], for instance, presents a model where sellers compete for risk neutral buyers via fixed pricing or auctions and demonstrates that in equilibrium both mechanisms yield the same expected payoff. Eeckhout and Kircher [3] show that payoff equivalence is not specific to price posting and auctions, rather it holds for a continuum of trading protocols all of which may be offered in equilibrium.[1] These models, however, exclusively focus on risk neutral

[1] The literature on competing mechanisms may be divided into two categories. The first category focuses on a monopolist seller who selects a trading mechanism in order to maximize his expected profit. The set of alternatives typically includes auctions, bargaining and price posting. The approach is 'partial equilibrium' in that the demand is taken as given and mechanisms feature exogenous costs. Among others see Wang [11, 12] and the references therein. The second category, to which this paper belongs, has competitive environments where demand at a store endogenously depends on the trading mechanism in place; for instance see Eeckhout and Kircher [3], Kultti [6], McAfee [7], Peters [8].

buyers. To the best of our knowledge this is the first paper that considers risk averse buyers.

Risk aversion matters for the following reason. With directed search the equilibrium matching function has the urn-ball form, as such some sellers receive multiple customers while others receive no customer at all. If a trading protocol prescribes the sale price as a function of the realized demand then it exposes customers to what we call the *price risk,* the risk of not knowing how much to pay in advance. Take for instance second price auctions, a frequently used mechanism in the literature (e.g. Julien et al. [5]). It stipulates that if a single customer is present then a reserve price is charged and in case of excess demand bidding ensues; the winning customer pays a higher price than the reserve. Customers who contemplate visiting such a store face an uncertainty regarding how much to pay as they do not know in advance how many other buyers will turn up at that store. Bargaining, add-on pricing, discount pricing etc. are likewise; they all carry some degree of price risk.

Risk averse buyers dislike such uncertainty; therefore, all else equal, they are more likely to shop at fixed price stores. The marginally higher demand allows such stores earn more than their competitors which is why in the unique symmetric equilibrium all sellers compete via fixed pricing.

2 Model

The setup is a standard directed search model where buyers are risk averse. The economy is large and it consists of B identical buyers and S identical sellers. Each seller is endowed with one unit of a good and wants to sell at a price above his reservation price of zero. Similarly each buyer wants to purchase one unit of an indivisible good and is willing to pay up to his reservation price, which is normalized to one. The game proceeds as follows. First, sellers simultaneously and independently announce a price schedule $\mathbf{p} = \{p_n\}_{n=1}^{B}$ that specifies a sale price for each demand realization, i.e. p_n is to be charged if n customers demand the good. Then, buyers observe sellers' selections and choose one store to visit; however once they reach a store they cannot move elsewhere. If n customers show up at the same location then each customers has a chance $1/n$ of obtaining the good. If trade takes place at price p then the seller realizes payoff p and the buyer $u(1-p)$, where u satisfies $u' > 0$ and $u'' < 0$. For simplicity let $u(0) = 0$. Agents who do not trade earn zero. Once players realize their gains the game ends.[2]

Pricing Mechanisms: The vector $\mathbf{p}$ assigns a unique price p_n for each $n = 1, 2, .., B$. We do not impose any restrictions on $\mathbf{p}$ except for requiring $p_n \in [0, 1]$. As such, our specification captures a wide range of pricing mechanisms and provides sellers with substantial freedom in determining the terms of trade. Some intuitive mechanisms are:

[2] The model is static; however, the result goes through in steady-states of a dynamic model.

- *Fixed Price Trading.* The same price is charged for all demand realizations i.e. $p_n = p$ for all n.

- *Second price auctions.* A "reserve price" p is charged if a single customer shows up; in case of excess demand bidding ensues, i.e. $p_1 = p$ and $p_n = 1$ if $n \geq 2$.[3]

- *Bargaining.* Focus on the last stage of the game where customers have already arrived at stores and consider a complete information bargaining game G_n between a seller (central player) and $n \geq 1$ buyers (peripheral players) who wish to share a pie of size 1. Let q_n be the resulting equilibrium price. If G_n has multiple equilibria and therefore generates multiple sale prices then we assume that there is an equilibrium selection device that uniquely pins down q_n. The game, the selection device and the resulting price schedule $\{q_n\}_{n=1}^{B}$ are all common knowledge. So if a seller communicates to the market that he competes via bargaining, then he can be thought as posting $\mathbf{p} = \{q_n\}_{n=1}^{B}$.[4]

- *Add-on pricing.* The sale price consists of a "base price" p plus an extra bit that rises in excess demand. Consider for instance $p_n = p + (1-p)\frac{n-1}{n}$.

- *Discount pricing.* The seller offers discounts by promising to charge only a fraction of a "sticker price" p; the fewer the buyers the larger the discount. Consider for example $p_n = p\left(1 - \frac{0.6}{n+1}\right)$, which implies that a 30% discount off the sticker price applies if a single customer turns up, a 20% discount applies if two customers turn up, etc.

We would like to remind that set of available mechanisms is not restricted to above; indeed $\mathbf{p}$ can be some esoteric sequence with no particular pattern at all.

[3] In the second price auction buyers bid their true valuation even if they are risk averse, so the seller's expected revenue is the same as with risk neutral bidders; see Fudenberg and Tirole [4], Ch 6.

[4] We remain agnostic about the nature of G_n; it can be strategic or axiomatic. An example for such a game can be found in Camera and Selcuk [2] who consider a discrete-time alternating offers game (à la Rubinstein [10]) between a seller and $n \in \mathbb{N}$ customers. The game starts with the seller making the initial offer q to a buyer. If the offer is accepted the seller obtains payoff q, the buyer obtains $1-q$ and the game ends. Otherwise, in subsequent bargaining periods a random device selects either the seller (with probability γ) or a buyer to propose a new price. One can interpret the parameter $\gamma \in (0,1)$ as the seller's bargaining power. Agents discount the future by $\beta \in (0,1)$ and the game continues until an agreement is reached. This game has a unique subgame perfect equilibrium where agreement is reached immediately at

$$q_n(\gamma,\beta) = \frac{(n-\beta)\left[1-\beta(1-\gamma)\right]}{n(1-\beta)+\beta\gamma(n-1)}.$$

Observe that q_n rises in n: multiple buyers "compete" for a single item and the price rises as the competition stiffens.

3 Analysis

We focus on strongly symmetric outcomes, where, on and off the equilibrium path, buyers are indifferent to where they shop, and direct their search independently across all sellers. Consider a seller who posts $\mathbf{p}$ and let v be the probability that a representative buyer visits him. He meets exactly n buyers with probability

$$z_n(B,v) = \binom{B}{n} v^n (1-v)^{B-n} \text{ for } n = 0,1,..B. \tag{1}$$

Observe that in a small market the demand co-varies across sellers—if a store has many customers then the next store is likely to have few customers. In a large market, instead, the covariance dies out; hence the demand is i.i.d. across sellers.

A buyer's expected payoff at a store posting $\mathbf{p}$ equals to

$$\begin{aligned} U(\mathbf{p},v) &= \sum_{n=0}^{B-1} z_n(B-1,v)\frac{u(1-p_{n+1})}{n+1} \\ &= \frac{1}{Bv}\sum_{n=1}^{B} z_n(B,v)\,u(1-p_n). \end{aligned} \tag{2}$$

With probability z_n the buyer encounters n other buyers, so that the sale price is p_{n+1} and the probability of being served is $1/(n+1)$. The second line is obtained using the fact that $z_n(B-1,v) = z_n(B,v)/Bv$.

The problem of a seller is given by

$$\max_{\mathbf{p}\in[0,1]^B,\ v\in[0,1]} \sum_{n=1}^{B} z_n(B,v)\,p_n \text{ subject to } U(\mathbf{p},v) = \overline{U}. \tag{3}$$

The seller's objective is to maximize his profit, denoted by $\Pi(p,v)$, subject to the indifference constraint $U(\mathbf{p},v) = \overline{U}$, where $\overline{U}$ is the "market utility" of buyers. Given some $\mathbf{p}$ the probability of visit v adjusts to satisfy the indifference condition.[5]

Lemma 1 *Fix v and $\overline{U}$. A seller achieves the highest expected profits if he commits to charge the same price for all n i.e. if he uses fixed price selling.*

Proof. Once v is fixed the seller's problem reduces to

$$\max_{\mathbf{p}\in[0,1]^B} \mathfrak{L} = \max_{\mathbf{p}\in[0,1]^B} \sum_{n=1}^{B} z_n(B,v)\,p_n + \mu\left[\sum_{n=1}^{B} z_n(B,v)\,u(1-p_n) - Bv\overline{U}\right].$$

[5] In a large economy the market utility $\overline{U}$ is not affected by a deviation. The reason is that the covariance of demand across stores vanishes; hence a change in the probability of visiting a particular store does not affect the distribution of demand at other stores (see Burdett et al. [1], Peters [9]).

The FOC with respect to p_n is given by

$$\frac{\partial \mathfrak{L}}{\partial p_n} = z_n(B,v) - \mu z_n(B,v)\, u'(1-p_n) = 0.$$

Observe that

$$\frac{\partial \mathfrak{L}}{\partial p_n} = 0 \Leftrightarrow u'(1-p_n) = 1/\mu \text{ for all } n = 1, 2, .., B.$$

Hence, $1 - p_n$ must be constant for all n. This means that $p_n = p$, i.e. the seller must charge the same price no matter what the demand (fixed pricing).

Note that the objective function $\Pi(\mathbf{p}, v)$ is linear in $\mathbf{p}$ (recall that v is fixed) whereas the constraint $U(\mathbf{p}, v) = \overline{U}$ is concave in $\mathbf{p}$. Indeed the Hessian of the constraint is given by

$$H = \begin{bmatrix} z_1 u''(1-p_1) & 0 & ... & 0 \\ 0 & z_2 u''(1-p_2) & ... & 0 \\ . & . & & . \\ . & . & ... & . \\ . & . & & . \\ 0 & 0 & ... & z_B u''(1-p_B) \end{bmatrix}.$$

Observe that H is negative definite (recall that $u'' < 0$); therefore the solution to the FOC yields the unique maximum. ■

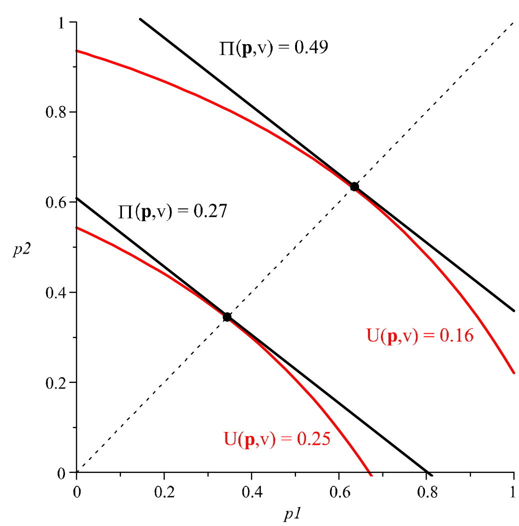

Figure 1

Figure 1 illustrates the Lemma under the following restriction (which is needed for a 2D illustration): $\mathbf{p}$ consists of two prices; p_1 is to be charged if a single customer is present and p_2 is to be charged if there are multiple customers. The concave curves consist of combinations of p_1 and p_2 satisfying the indifference constraint $U(\mathbf{p}, v) = \overline{U}$, whereas the linear lines are the isoprofit curves.[6] Observe that the optimal set of prices lie on the 45^0 line which means that setting $p_1 = p_2$ yields the maximum profits.

The intuition behind the Lemma is this. The sale price p_n generally changes with the realized demand n. As such, customers face some uncertainty regarding what price to pay, a notion which we label as the *price risk*. The price risk critically depends on the trading mechanism in place; the more p_n "fluctuates" the higher the risk. The only mechanism that does not exhibit such risk is fixed pricing; therefore, all else equal, risk averse customers are more likely to show up at such stores. Fixed price sellers trade off this additional demand with marginally higher prices and earn more than their competitors.

We can now state the main result of the paper.

Proposition 2 *The unique symmetric equilibrium entails all sellers using fixed pricing.*

The proposition is based on the previous Lemma, so it has the same intuition. Unlike the Lemma, though, the probability of visit is variable (as it should be), which potentially complicates the comparison of expected profits across different mechanisms. The proof tackles this issue.

Proof. We first show that no seller uses a trading mechanism other than fixed price selling, then we argue uniqueness. By contradiction, suppose that there is an equilibrium where a seller competes with some pricing mechanism $\mathbf{p}$ where $p_n \neq p_{\widetilde{n}}$ for some $n, \widetilde{n}$. Let $\Pi(\mathbf{p}, v)$ denote his expected profits. The seller must provide buyers with the market utility $\overline{U}$, so v must satisfy $U(\mathbf{p}, v) = \overline{U}$. Below we argue that if the seller switches to fixed price trading then he can attain a higher level of profits than $\Pi(\mathbf{p}, v)$ rendering this outcome unsustainable as an equilibrium. The main obstacle in reaching this conclusion is that if the seller switches mechanisms then v changes, which renders the comparison of expected profits non-trivial. To get around this issue we show that there exists a particular fixed price $r \in (0, 1)$ that satisfies $U(r, v) = \overline{U}$. This means that if the seller posts r, instead of $\mathbf{p}$, then he can provide buyers with the same $\overline{U}$ *and* he would still be visited with probability v. Once v is controlled for, the result follows from Lemma 1. Below we make these arguments precise.

To start, observe that

$$U(r, v) = \frac{1 - (1 - v)^B}{Bv} u(1 - r),$$

which is obtained by substituting $p_n = r$, $\forall n$ into (2). In addition we have

$$0 < U(\mathbf{p}, v) < \frac{1 - (1 - v)^B}{Bv} u(1). \qquad (4)$$

[6] The Figure is drawn for $B = 100$, $v = 1.5\%$, $\overline{U} = 0.25$ as well as $\overline{U} = 0.16$. Buyers are assumed possess exponential utility $u(x) = 1 - e^{-x}$.

Indeed, given some v, the expected utility $U(\mathbf{p}, v)$ is maximized when $p_n = 0$ for all n. Substituting $\mathbf{p} = \mathbf{0}$ into (2) yields the inequality on the right hand side. The inequality is strict because $\mathbf{p}$ is not a fixed pricing scheme; hence at least some p_n must be greater than 0. The second inequality is obtained by substituting $\mathbf{p} = \mathbf{1}$ into (2) (recall that $u(0) = 0$). Now define $\Delta(r) := U(r, v) - \overline{U}$ and observe that Δ decreases in r. Furthermore notice that

$$\Delta(0) = \frac{1 - (1 - v)^B}{Bv} u(1) - \overline{U} > 0 \quad \text{and} \quad \Delta(1) = -\overline{U} < 0.$$

These inequalities follow from (4) and the fact that $U(\mathbf{p}, v) = \overline{U}$. The Intermediate Value Theorem implies that there exists some $r \in (0, 1)$ satisfying $U(r, v) = \overline{U}$.

Given that $U(\mathbf{p}, v) = U(r, v)$, Lemma 1 implies that $\Pi(r, v) > \Pi(\mathbf{p}, v)$, which means that the seller can profitably deviate to fixed pricing. Hence there is no equilibrium where another mechanism is offered.

The uniqueness of the symmetric equilibrium immediately follows from the facts that (i) agents are homogenous (ii) all sellers must use fixed price trading, and (iii) buyers use symmetric visiting strategies. ■

The proposition implies that the only possible equilibrium of this game is where all sellers compete via fixed pricing. To characterize this equilibrium suppose that u satisfies $u(x) = x^\alpha$, where $\alpha \in (0, 1)$ and observe that a lower value of α means that buyers are more risk averse. Using rather standard techniques (e.g. see Burdett et al. [1]) one can show in the unique symmetric equilibrium all sellers post

$$r^* = \frac{1 - z_0 - z_1}{1 - z_0 - (1 - \alpha) z_1}$$

and all buyers visit each seller with probability $v^* = 1/S$. Observe that $dr^*/d\alpha < 0$ which means that sellers ask for more as buyers become more risk averse; indeed $\lim_{\alpha \to 0} r^* = 1$.

Risk Neutral Buyers. One can show that if buyers are risk neutral then any price schedule $\mathbf{p}^*$ that satisfies

$$\Pi(\mathbf{p}^*, 1/S) = 1 - z_0 - z_1 \tag{5}$$

may be posted in equilibrium. Indeed any such $\mathbf{p}^*$ provides buyers with the same market utility $\overline{U} = (1 - 1/S)^{B-1}$; therefore buyers visit each store with the same probability $v^* = 1/S$.

The key to this result is that risk neutral buyers are not sensitive to the aforementioned price risk. Any mechanism that delivers the expected utility $\overline{U}$ receives the same attention from the customers, which is why in equilibrium a continuum of mechanism may be offered.

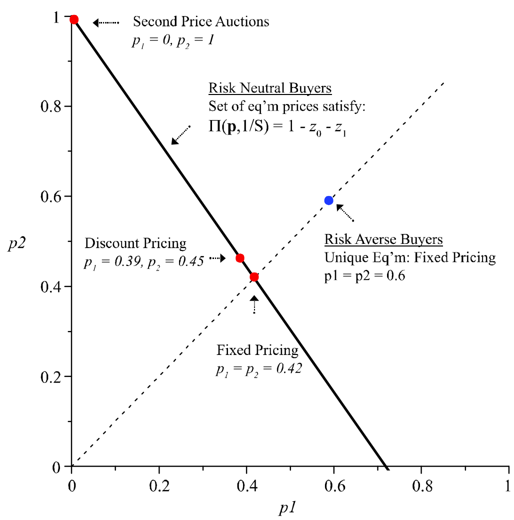

Figure 2

Figure 2 illustrates this result for $B = S = 100$. Similar to the previous picture, the figure is drawn under the restriction that $\mathbf{p} = \{p_1,p_2\}$, where p_1 is the price for a single customer and p_2 is the price for multiple customers. The downward sloping solid line consists of combinations of p_1 and p_2 satisfying (5) which means that, when buyers are risk neutral then any point on that line maybe offered in equilibrium. For comparison we have highlighted the equilibrium prices for some of the mechanisms outlined in the introduction (fixed price trading, discount pricing and auctions).

4 Conclusion

The message of this note is simple. Fixed price trading is the best performing protocol when sellers compete for risk averse buyers, because it eliminates the price risk, the uncertainty of not knowing how much to pay in advance.

The result however comes with a caveat. The proofs depend on the "market utility" assumption: each seller takes as given that he must provide buyers with a certain level of utility and understands that a deviation by a particular store simply cannot not affect buyers' outside options. This is indeed true in an large market. However in a small market with few buyers and sellers the market utility assumption does not hold.

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