A complementary test for ADF test with an application to the exchange rates returns

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2005

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Abstract

This study shows that augmented Dickey-Fuller (ADF) test failed to detect covariance nonstationary series. Supportive of Ahamada (2004), this study finds that the cumulative sums of squares procedure in Inclán and Tiao (1994) is useful to complement the ADF test. As illustration, the ADF test indicates that there is no unit root in the returns of Japanese yen/US dollar, British pound/ US dollar and Swiss franc/US. However, the complementary test reveals that each of these returns contains heterogeneous variance. To sum, it can be concluded that these exchange rate returns are covariance nonstationary although there is no unit root.

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1. Introduction

A basic requirement for time series modelling is that the series under study must be weakly stationary, i.e. it has constant mean and covariance. Numerous stationary tests have been developed in the past to test for stationarity and the popularly applied tests include the augmented Dickey-Fuller (ADF) test (Fuller 1976, Dickey and Fuller 1979), Phillips-Perron (PP) test (Phillips 1987, Phillips and Perron 1988) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski *et al*. 1992). Lately, Ahamada (2004) demonstrates via a simulation exercise that KPSS test fails to detect a form of nonstationarity due to a shift in the unconditional variance. They pointed out that the non-rejection of the null hypothesis of no unit root in the KPSS test does not neccesarily imply the stationarity of the data, as there is a possibility that the data may exhibit heterogeneous unconditional variance. The author further proposed a complementary test to complete the KPSS testing procedure and the complementary test was shown to be useful detecting the nonstationary covariance of the daily returns of US dollar/Euro exchange rate, in which the KPSS test has failed to do so.

Given the surprising defect in one of the most powerful stationary test, it is interesting to find out whether the most commonly utilised ADF test is robust against nonstationary covariance. As such, the this simulation study is conducted to examine whether the ADF test is able to detect nonstationary covariance. Besides, the performance of the complementary test as proposed in Ahamada (2004) in correctly identifying simulated series of nonstationary covariance is also scrutinized in this simulation study.

To preview our findings, the current study discovers that the ADF test has identified the simulated nonstationary covariance as stationary series with a unit probability. Similar finding is observed in the DF test, which is included in this simulation study for comparison purpose. On the other hand, using the complementary test as proposed in Ahamada (2004), nonstationary covariance has been correctly identified in almost all cases. Hence, this study proposes the use of this complementary test in the case of ADF test to detect nonstationary covariance if ADF test suggests no unit root in the series of interest. In this regards, the current study simulates and reports the critical values of this complementary test. In addition, this study applies the same complementary test in the case of ADF (hereafter referred as complementary ADF test) to the returns of few US dollar based exchange rate series of some developed countries to illustrate the usefulness of this complementary ADF test.

The remainder of study is structured as follows: Section 2 discusses the complementary ADF test. Section 3 explains the simulation process and presents the results of study. Section 4 illustrates the usefulness of the complementary ADF test using emperical data. Finally, Section 5 concludes this study.

2. The Complementary ADF Test

Ahamada (2004) wisely tailored the cumulative sum of square (CSS) procedure in Inclán and Tiao (1994) to formulate a complementary test for the KPSS testing procedure (hereafter, complementary KPSS test). This useful test is easily applied and interested readers may refer to Ahamada (2004)[1]. In the vein of Ahamada (2004), this study extends the application of the same CSS procedure in the case of ADF, yielding to the so-called complementary ADF test[2].

Consider the following time series $\{ y_t \}$, which is stationary around the level r_0:

$$y_t = r_0 + \varepsilon_t , \quad t = 1,...,T , \tag{1}$$

where ε_t is independent and identically distributed (i.i.d.) with a zero mean and constant variance, denoted $\varepsilon_t \sim$ i.i.d.(0, σ^2_ε).

The stationarity of $\{ y_t \}$ may be tested by the augmented Dickey-Fuller (ADF) test[3]:

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[1] Available at http://www.economicsbulletin.com/2004/volume3/EB-03C10010A.pdf.

[2] For compatibility, the current study follows closely the definitions and notations in Ahamada (2004).

[3] ADF is the improved version of Dickey-Fuller (DF) test of the framework $\Delta y_t = \partial y_{t-1} + \omega_t$, where ω_t ~ i.i.d. (0, σ^2_ω). Here, the null hypothesis of ∂ =1 (unit root) is tested against the alternative hypothesis of $\partial < 1$ (no unit root).

$$\Delta y_t = \partial y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \eta_t , \quad (2)$$

where $\eta_t \sim$ i.i.d.(0, σ_η^2), p is the autoregressive lag length large enough to eliminate possible serial correlation in η_t and ∂ is the coefficient of interest. Conventionally, if ∂ = 0, the series contains a unit root implying nonstationary, whereas if $\partial < 0$, there is no unit root implying stationarity. In the ADF test, the null hypothesis of unit root, i.e. $H_0^{ADF} : \partial = 0$ is tested against the alternative hypothesis of no unit root, i.e. $H_A^{ADF} : \partial < 0$ using the *t* test of individual significance.

It is obvious that under the generating mechanism in (1) with $\varepsilon_t \sim$ i.i.d.(0, σ_ε^2), ∂ in (2) equals 0, thereby conventionally one may conclude that $\{ y_t \}$ is stationary. The concern of this study is whether or not the ADF test is robust against heterogeneous variance process i.e. $E(\varepsilon_t^2) = \sigma_t^2 \neq \sigma_\varepsilon^2$. In this regard, a simulation study has been conducted and we will see shortly that ADF test had identified nonstationary covariance series as stationary process[4]. A complementary test for ADF test is therefore needed to differentiate completely stationary process (mean and covariance stationary) from mean stationary but covariance nonstationary process. As in Ahamada (2004), the current study utilises the supremum $\sqrt{T/2} \mid D_K \mid$ statistic proposed in Inclán and Tiao (1994), defined as[5]:

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[4] Although striking, the results come as no surprise as Ahamada (2004) has already shown similar failure of the most powerful unit root test.
[5] With the prudent adaptation of Ahamada (2004).

$$\tau = \max_{k=1,\ldots,T} \sqrt{T/2}\, | D_K | \quad (3)$$

where $D_k = \frac{C_k}{C_T} - \frac{k}{T}$, $C_k = \sum_{t=1}^{k} e_t^2, k = 1,\ldots,T$. e_t in turn is the ordinary least squares (OLS) residuals from regressing $\{ y_t \}$ on a constant as in (1). Under the null hypothesis of e_t is independent and identically distributed with zero mean and homogeous variance, i.e. $H_0^C: e_t \sim$ i.i.d. $(0, \sigma_e^2)$, Ahamada (2004) showed that the limiting distribution of τ is given by one of the $\sup\{W_t^0\}$, where W_t^0 is a standard Brownian Bridge. It is noted here that the above assumption is also valid and therefore the distribution of $\sup\{W_t^0\}$ given by Billingsley (1968) is applicable in the current case[6]:

$$\Pr\left\{\sup | W_t^0 | \leq b\right\} = 1 + 2\sum_{k=1}^{\infty} (-1)^k e^{-2k^2 b^2}, b > 0 \quad (4)$$

where $\Pr\{A\}$ denotes the probability of event A occurs and b is the critical value. Based on simulation exercises done by Inclán and Tiao (1994), the asymptotic 10%, 5% and 1% critical values for τ are corresponding 1.224, 1.358 and 1.628[7].

[6] See proof of Proposition 1 in Ahamada (2004) and proof of Theorem 1 in Inclán and Tiao (1994).

[7] Inclán and Tiao (1994) estimated these critical values from 10000 replications of T independent N(0,1) observations. Using this specification, the simulated critical values obtained in the current study for T = 50000 are rather close to theirs, i.e. 1.225, 1.353 and 1.613, in the same order. As for different specifications of variance, these values do not vary substantially, see Appendix 1 for more simulated critical values for τ .

With the availability of this complementary ADF test, we may now conduct a complete ADF test by carrying out the following two-step procedure[8]: First, apply the ADF test. If the null hypothesis is not rejected, then we may conclude that the data is nonstationary, i.e. it contains a unit root. If the null hypothesis is rejected, there is no unit root but a shift in the variance is possible. For this case, we suggest to apply the complementary ADF test. If the τ statistic fails to reject the null hypothesis, then we have enough statistical evidence to conclude that there is a complete covariance stationarity. Otherwise, the data have variance shift and the process is not covariance stationary although there is no unit root.

3. Simulation Procedures and Results

Consider the following data-generating processes (*DGP*) specified in Ahamada (2004):

$$DGP_{H_0}: \; x_t = 0.01 + \varepsilon_t \,, \tag{5}$$

where ε_t ~N(0,1) for t = 1, …, 200; and

$$DGP_{H_A}: y_t = 0.01 + \varepsilon_t^{'} \,, \tag{6}$$

[8] The null and alternative hypothesis of KPSS test is the reverse of ADF test, see Ahamada (2004) for complementary KPSS test.

where $\varepsilon_t^{'}$~N(0,1) for t = 1, …, 100 and $\varepsilon_t^{'}$~N(0,1.5) for t = 101, …, 200.

Note that the series $\{x_t\}$ is stationary around the level 0.01 but $\{y_t\}$ is nonstationary as the variance varies. The estimated rejection rate of the null hypothesis of nonstationary at 1%, 5% and 10% level for both series for 1000 replications of each *DGP* is given in Table 1.

TABLE 1. Rejection Rate of the Null Hypothesis of Nonstationary

Series	DF Test			ADF Test[a]			Complementary Test		
	10	5%	1%	10%	5%	1%	10	5%	1%
$\{x_t\}$	1.000	1.000	1.000	1.000	1.000	1.000	0.116	0.051	0.012
$\{y_t\}$	1.000	1.000	1.000	1.000	1.000	1.000	0.883	0.957	0.989

Note: [a] Results reported are for p= 4. Similar results (not shown) are obtained with other specifications of p.

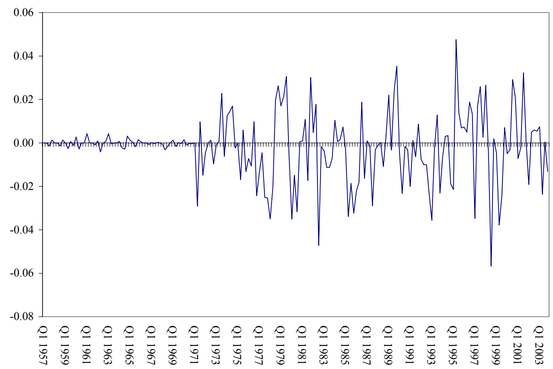
Table 1 shows that both the DF and ADF test correctly reject the null hypothesis of unit root (implying stationarity) in the $\{x_t\}$ series, whereas the performance of the complementary test is well close to the nominal levels. On the other hand, both the DF and ADF test errorneously reject the null hypothesis of unit root in the nonstationary $\{y_t\}$ series. Nonetheless, the complementary test is able to correctly identify the nonstationary variance and the performance is again as good as nominal levels. Thus, the complementary test has good size and power of test, but the DF and ADF have only satisfactory size of test.

4. Illustrations of Complementary ADF Test

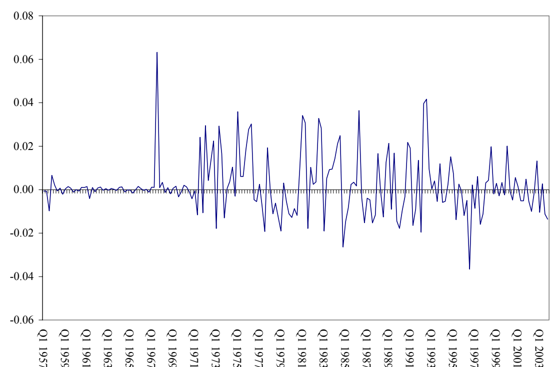
To demonstrate the potential usefulness of the complementary ADF test, this study applies it to the the returns of three US dollar based nominal exchange rate series of developed countries, namely the Japanese yen, British pound and Swiss franc. Quarterly data of these nominal bilateral exchange rates covering 1957Q1 to 2004Q1 (amounting to 188 usable observations) are obtained from the International Financial Statistics. The returns of these series computed from $X_t = \log(S_t / S_{t-1})$ where S_t is Japanese yen/US dollar, British pound/ US dollar or Swiss franc/ US dollar are plotted in Figure 1. It is seen from Figure 1 that these returns series are rather stationary around the level 0 but there is obviously a shift in variance in all cases. Based on the formal DF and ADF tests, in which the results are summarised in Table 2, the null of unit root has been rejected at 1% significance level in all cases. However, as argued earlier, this finding does not automatically implies stationarity since the homogeneity condition of variance is yet to be determined. In this respect, further application of the complementary test is obligatory to complete the ADF testing procedure and the results are also given in Table 2. In line with our earlier observation (eye-inspection), strong evidence of heteroscastic variance in all returns series are given by the complementary test. Thus, we may conclude that while there is no unit root in all the returns series under study, they are actually covariance nonstationary. Our results are supportive of Ahamada (2004), which reports similar finding on the daily returns of US dollar/Euro exchange rate by the complementary KPSS test.

FIGURE 1. The exchange rate returns

Japanse yen/US dollar



British pound/US dollar



Swiss franc/US dollar

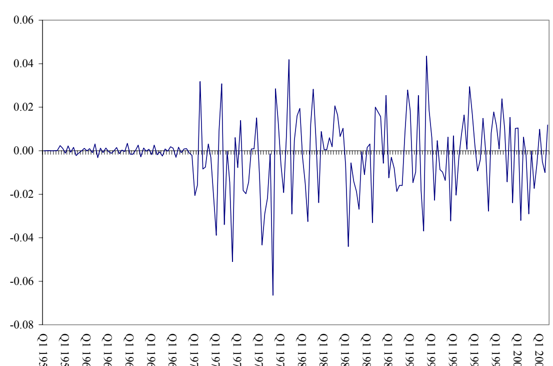


TABLE 2. DF, ADF and complementary tests results with simulated critical values

Exchange Rate	DF	ADF	Complementary Test
Yen/US dollar	-11.300*	-7.037*	3.298*
Pound/US dollar	-12.269*	-5.940*	2.012*
Swiss Franc/US dollar	-13.500*	-5.563*	2.821*
Simulated Critical Values[a]			
1%	-4.293	-3.691	1.556
5%	-3.488	-3.225	1.311
10%	-3.025	-2.875	1.174

Note: [a] Estimated from 1000 replications of 188 independent N(0,1) observations. Asterisk (*) denotes significant at 1% level.

5. Conclusion

This study demonstrates through a simulation study that the most commonly applied ADF test failed to detect covariance nonstationary series. This finding is not surprising as Ahamada (2004) has already shown that the KPSS test, one of the most powerful stationary test has similar deficiency. Following Ahamada (2004), this study utilises the cumulative sums of squares in Inclán and Tiao (1994) to form a complementary test for the ADF test. Simulation results show that this complementary test has the desired good size and power of test, but not the ADF test. Hence, a two-step testing procedure starting from the ADF test and ending with the complementary test is essential for a complete stationary test. This study considers the returns of Japanese yen/US dollar, British pound/ US dollar and Swiss franc/US dollar for illustration of this two-step procedure. The ADF test indicates that there is no unit root in these returns. However, the complementary test identifies that each of these returns contains a shift in variance. Summing up both test results, it is concluded that these exchange rate returns are covariance nonstationary although there is no unit root.

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APPENDIX 1

TABLE 3. Critical values of τ statistic for various sample size, *T*.

Sample size, *T*	Critical values		
	10%	5%	1%
20	1.042	1.166	1.414
40	1.111	1.236	1.478
60	1.126	1.262	1.505
80	1.141	1.266	1.527
100	1.145	1.281	1.537
120	1.156	1.293	1.548
140	1.162	1.298	1.550
160	1.169	1.304	1.552
180	1.173	1.307	1.554
200	1.175	1.315	1.558
400	1.180	1.324	1.570
800	1.191	1.325	1.573
1600	1.199	1.330	1.597
10000	1.209	1.352	1.611
50000	1.225	1.353	1.613

Note: Estimated from 10000 series that are replicated from independent random errors with N(0,1) distribution. Each series contains *T* usable observations.

TABLE 4. Critical values of τ statistic for various residuals variance, σ_ε^2.

σ_ε^2	Critical values		
	10%	5%	1%
0.1	1.197	1.336	1.597
1	1.205	1.352	1.612
10	1.211	1.355	1.616
100	1.212	1.356	1.622
1000	1.214	1.357	1.625

Note: Estimated from 10000 series that are replicated from independent random errors with $N(0, \sigma_\varepsilon^2)$ distribution. Each series contains 10000 usable observations.