Monopoly pricing of social goods

Sääskilahti, Pekka

2007

Monopoly Pricing of Social Goods*

Pekka Sääskilahti
Intellectual Property Rights
Nokia Corporation
P.O. Box 226, FI-00045 Nokia Group
pekka.saaskilahti@nokia.com
tel: +358 50 4872487

May 5, 2007

Abstract

We analyse the roles of social network topology and size on the monopoly pricing of network goods in a market, where consumers interact with each other and are characterised by their social relations. The size effect is the well-known network externalities phenomenon, while the topological effect has not been previously studied in this context. The topological effect works against, and dominates, the size effect in monopoly pricing by reducing the monopoly's capacity to extract consumer surplus. Under asymmetric information about consumer types, the monopoly prefers symmetric network topologies, but the social optimum is an asymmetric network.

Keywords: Social relations, networks, coordination, monopoly.

JEL Classification: D42, D82, L14.

1 Introduction

Economists have synthesised network effects in positive externalities: an agent's utility increases as an additional member joins his network. However, real world networks exhibit often topological asymmetries that challenge this straightforward relation. For example, some people maintain only few close relations, whereas some people have a large number of more shallow acquaintances. In this paper, we analyse how monopoly pricing depends on the network size and topology in markets for social goods, such as personal communications equipment, which induce network effects. Where the size effects have been well covered in the previous externalities literature, we discover topological effects that have been overlooked thus far.

The conventional externalities model, building on the seminal works by Farrell and Saloner (1985), Katz and Shapiro (1985), David (1985), and Arthur (1989), assumes a

*I thank Yann Bramoullé, Juuso Välimäki, Robin Mason, Tuomas Takalo, Pekka Ilmakunnas, Olli Kauppi, Ismo Linnosmaa, Pauli Murto, Jean-Charles Rochet, Steinar Vagstad, the Editor, two anonymous Referees and seminar participants at the HECER, the FDPE, and the XXVIII Annual Meeting of the Finnish Society for Economic Research for helpful comments. I finalised the first version of the manuscript while working at the Helsinki School of Economics. I also thank the Université de Toulouse 1, where the original idea was developed, for the hospitality while I was staying there. The financial support from the European Commission and the ENTER network, the FDPE, and the Yrjö Jahnsson Foundation is gratefully acknowledged.

functional form for network effects: a network member's marginal utility is positive for an additional network member.[1] This relation constitutes the size effect. In a two-period model, Cabral et al. (1999) show how positive network externalities create an incentive for the monopolist to price so that the consumers buy in the first period rather than at the later stage. This involves an increasing price path. In general, the size effect creates an incentive for the monopolist to sell more than without network externalities in order to increase the value of the good. A similar incentive is present under negative network externalities; the monopolist limits supply in order to increase the value of the good (Kessing and Nuscheler 2006). Mason (2000) associates network externalities with economies of scale on the demand side. He shows that the monopolist prices at marginal cost, but grows the network size slower than the socially optimal rate, as it fails to internalise the full benefit of increasing the current size of the network.[2] The conventional externalities approach implicitly takes the underlying relations network as a completely connected graph, where each network member is linked with everybody else. In other words, any kind of heterogeneity in terms of social relations is absent. However, and as I show in this paper, the size effect is often exaggerated, and dominated by an effect due to network topology. Therefore, in markets where the social relations are important, the conventional approach falls short and needs to be refined.

A recent literature on social relations studies the question of how network members can benefit from their connections, when there are well-connected members and members with only a few connections. There are two classes of social relations models.[3] One class takes the network structures as exogenous to the model, and the other studies endogenous network formation. There is a very rich literature analysing models where the agents are characterised by their pre-existing, exogenous, social relations.[4] Jackson (2005) is a survey on the endogenous (undirected) network formation models. The network formation set-up is particularly fitting for e.g. firm-level interaction (see Kranton and Minehart 2001 about buyer-seller networks, and Goyal et al. 2003 about R&D collaboration networks). The problem is, however, that the economic dimension in link formation can be difficult to isolate, e.g. in personal relations. In contrast, if the network is exogenous, we can focus on the specific economic problem, such as whether to buy or not a mobile phone when the decision is affected by the network structure of personal relations. It is often the case that social relations exist prior to the decision making. For example, when we think about buying a mobile phone, we think about with whom we can use it, not how many new

[1] Consider the example by Mason and Valletti (2001): A link between two network members gives utility equal to 1. When a new member joins the network with $n-1$ existing members, the total utility increases by $2(n-1)$. The total utility in the network of n members, when n is large, is $n(n-1) \approx n^2$. This corresponds to the famous Metcalfe's Law, which states that the value of the network equals the square of the number of its members. Swann (2002) studies the functional form approach in more detail. He establishes specific conditions that the utility function and the diffusion law governing the network good's adoption rate must satisfy for Metcalfe's Law to hold.

[2] Bensaid and Lesne (1996) obtain results that also overturn the Coase conjecture of durable goods monopolist, similar to the results by Mason (2000). They illustrate how the optimal price path of the monopolist can be increasing when network externalities are delayed so that the first adopters do not benefit from network externalities while the future buyers' utility is increasing in the number of first buyers.

[3] Related to the social networks models, in local interaction games network members coordinate their actions on a fixed relations network over time (see Ellison 1993, Young 1998 ch.6, Lee and Valentinyi 2000, and Morris 2000).

[4] See Sääskilahti (2005b) for a list of references for different application fields where the social network is taken as exogenous.

friends we get by using it. We follow this line of analysis and treat the social network as exogenous in our model.

Our model differs from the previous work on (exogenous) social networks in two aspects. First, we endogenise the network members' payoffs via monopoly pricing. The question we are interested in is, how an external player (the seller) can take advantage of the (buyers') social network structure. Secondly, we introduce asymmetric information with respect to the consumer types. Sundararajan (2005) and Banerji and Dutta (2005) analyse closely related models of product adoption with local network effects. Banerji and Dutta (2005) are, to our knowledge, the only others who study endogenous pricing under local network effects. They analyse the case of a Bertrand duopoly, and show that, in certain network topologies, market segmentation with positive profits is feasible. They, however, analyse only the case of perfect information. Sundararajan (2005) analyses a model with imperfect information. He studies the existence and efficiency of equilibria in a coordination game with action complementarities where the agents are characterised by their local network connections. His model involves imperfect information with respect to both the consumer types as well as the network structure. Our paper differs from Sundararajan (2005) in that he does not analyse endogenous pricing, as the cost of adoption is fixed in his model. Tucker (2006) is the first to analyse empirically the role of social network topology in conjunction with the adoption of a network good.

We study the case where a monopoly launches a new device that constitutes an efficient medium for interaction. The product has no intrinsic value, as it is used only when two people interact with each other. As a consequence, a potential buyer needs to coordinate his actions with the other people on whether to switch to the new good or to stay using the legacy system. We think of products such as the fax machine that is a relatively drastic innovation in the sense that it is not compatible with the earlier generation products (postal and courier services). Other examples are PC and mobile phone software, and the fixed line telephony in the late 1800's. The firm decides on the device price. It understands that a low price may help in solving the buyers' coordination problem, but it erodes margins. Consumers are characterised by their exogenous personal social relations. Each person is interested only in interacting with a subset of the population, called his neighbours, who are e.g. their friends, family, and colleagues.[5] Consumers are heterogeneous with respect to the attainable interaction utility. For example, some people like to write letters (the conventional way to interact), whereas some people prefer to send e-mails (the novel product). Importantly, a consumer cannot tailor his actions vis-à-vis each neighbour. This way, he must consider the overall network structure, rather than each link separately.

We analyse two informational regimes. In the first one, all information is perfect, and in the other case, the buyers' types are private information. We give general characterisations of both cases, and apply them to three network topologies: complete graph, circle, and star. The complete graph and the circle are symmetric networks, whereas the star is asymmetric. The complete graph is the structure implicitly assumed in the conventional network externalities models.

We show how certain consumers have critical network positions through their social relations, which give them leverage against the monopoly, and are thus able to capture higher surplus than other people. Critical network members, whose connections are important from all network members' perspective, exist in asymmetric and symmetric networks

[5] See Tucker (2006) for an empirical result supporting the assumption that network benefits are predominantly a "local", rather than a "global", phenomenon.

under perfect information. In symmetric networks, the critical roles are due to consumer heterogeneity. Members who have links *with* high types are critical, as opposed to the high types themselves. When information is reduced to asymmetric, these critical roles cease to exist. On the other hand, the critical roles due to centrality in the network always capture higher utility in asymmetric networks under asymmetric information. This is not true necessarily with perfect information, as it depends on the level of consumer heterogeneity.

Our main finding is that the network topology has a dominating effect on the optimal monopoly price. The topological effect is caused by the existence of the critical roles. The firm's response to the existence of critical consumers is to set a lower price in order to guarantee a higher probability to buy for them compared to the other consumers. The more completely linked the network is, i.e. the more connections the members have, the higher the optimal price is. In other words, the topological effect dominates the size effect, and it is the stronger the less linked the network is. As a result, the implicit complete graph assumption of the conventional network externalities model risks overestimating the value of the network effects and the level of the monopoly mark-up.

Under asymmetric information, the asymmetric network topologies yield lower monopoly profits, but higher total surplus, than symmetric networks of a given link value do. Therefore, the monopolist's and the society's preferences are misaligned. The topological effect has distributional implications via monopoly pricing, as the critical and the other network members obtain opposite surplus effects when the number of network members is increased under asymmetric information. The firm can match the profits generated in symmetric networks, if price discrimination according to the network position is allowed in asymmetric topologies. Price discrimination equalises the buying probabilities of the critical and non-critical consumers, but it reduces total surplus.

In section 2, we formalise the model. We study the perfect information case in section 3. We analyse the asymmetric information case in section 4. We conclude in section 5.

2 Network structure and actions

The timing of events is that first the consumers draw their types, then the firm sets the device price, after which the consumers decide on buying. The interaction structure in the model is given by a graph $\mathcal{G} = (\mathcal{I}, \mathcal{E})$, where the set of nodes is $\mathcal{I} = (1, ..., I)$, $I \in \mathbb{N}$, and the set of undirected links, or edges, between the nodes is $\mathcal{E} \subset \mathcal{I} \times \mathcal{I}$. The nodes $i \in \mathcal{I}$ are interpreted as consumers, and the links $\{i, j\} \in \mathcal{E}$ are connections between the consumers. Two consumers connected by a link are called neighbours. The set of neighbours of the consumer i is $\mathcal{N}_i = \{j \in \mathcal{I} \setminus i : \{i, j\} \in \mathcal{E}\}$. We assume that the graph is completely connected, i.e. there exists a path between any two nodes. The total number of links $|\mathcal{E}|$ corresponds to the size, or link value, of the graph. The topology of the graph follows from the link wiring $\mathcal{E}$ between consumers.

The network inherits its structure from outside the model. The links represent personal relationships with family, friends and colleagues. We assume that these connections have been formed prior to the launch of the product, and they are not affected by the availability of a new medium for interaction. Because the network is constant over time, also the firm is able to acquire information of its structure. Hence, we assume that the structure of the graph is common knowledge.

Assumption 1 *The graph $\mathcal{G} = (\mathcal{I}, \mathcal{E})$ is common knowledge.*

The problem for the consumer i is to choose an action $a_i \in \{0,1\}$, where $1 =$ buy the new device and $0 =$ do not buy. If both end nodes of a link buy, their interaction is mediated by the new good. In this case, we call that the link becomes "active". If only one of the consumers buys or neither buy, the link remains inactive.

Throughout the paper, we are interested in the role of the exogenous structure of the social network on the activity level that results from consumers' buying decisions. The following definition characterises the activity level on the network.

Definition 2 *The activity level on the network is said to be*
(i) a complete network, when $\min a_i = 1$, $i \in \mathcal{I}$.
(ii) an empty network, when $\max a_i = 0$, $i \in \mathcal{I}$.
(iii) a partial network, when $\min a_i = 0$ *and* $\max a_i = 1$, $i \in \mathcal{I}$.

All interaction is mediated by the new product in a complete network. In a partial network some, but not all, interaction is mediated by the new product. In the empty network, no-one uses the new product.

The value of an inactive link is normalised to zero representing the utility from interaction with the help of older generation systems. Interaction generates positive utility when it is facilitated by the new device. This can be thought as an efficiency gain or additional utility obtained from the types of interaction not previously available. Importantly, the consumer i has an interest in interacting only with his neighbours $\mathcal{N}_i$. The consumer i gets utility θ_i, corresponding to his type, from each activated link he has. The value θ_i is an i.i.d. random variable drawn from the uniform distribution $F(\theta)$ with the support $\left[\theta^-, \theta^+\right]$, $\theta^- \geq 0$, for all $i \in \mathcal{I}$. We assume that θ_i is independent of the network location the consumer i occupies, because the social relations are formed prior to the launch of the new device and they are unrelated to the value the consumer puts on the device. Under perfect information, the types of all consumers are revealed to everybody, including the firm, before the firm sets the price. Under asymmetric information, the types are private information, and the firm observes nothing. This simplification is based on the assumption that the consumers know their own needs better than the firm. The distribution $F(\theta)$ is common knowledge.

Since our model is a coordination game, it has multiple equilibria.[6] The most interesting equilibrium in regard to the monopoly's pricing problem is the one that maximises the usage of the new device, because it also maximises the monopoly's profits. So the monopoly would suggest this maximal equilibrium. The maximal equilibrium is not in contradiction with efficiency in the consumers' coordination game. The coordination game is supermodular with strategic action complementarities, which guarantee that the maximal equilibrium is the Pareto-dominant one. Moreover, the consumers might be able to use the network to communicate their buying intentions in order to reach the efficient

[6] Sääskilahti (2005a) studies the existence of a unique equilibrium in network externalities models. The paper builds on the recent work on coordination games by Carlsson and van Damme (1993), Herrendorf et al. (2000), Mason and Valentinyi (2003), and Morris and Shin (2003). In the context of consumer's buying decision making, uniqueness requires that both actions, "buy" and "do not buy", are played as strictly dominant strategies simultaneously by different groups of consumers. The key to uniqueness is sufficient buyer heterogeneity with respect to non-network specific attributes. Under perfect information, heterogeneity must be real in the sense of a broad type distribution. Under imperfect information, a possibility that some consumers are of very high and very low types simultaneously is sufficient to yield a unique equilibrium.

outcome. We rationalise that all this focalises the maximal equilibrium compared to the other candidate equilibria.

In the real world, we can observe almost infinite number of different network structures. Unfortunately, large networks are analytically cumbersome typically, so we opt for analysing three primitive networks that bring out the effects missing in the conventional externalities models:

- *Complete graph*, where each consumer is connected to everybody else, $\mathcal{N}_i = \{\mathcal{I} \setminus i\}$ for all $i \in \mathcal{I}$. The complete graph is the structure used implicitly by the conventional network externalities models.
- *Circle*, where each consumer has exactly two neighbours. The links form a circle, when the nodes are indexed in ascending order, so that the consumer $i \in \{\mathcal{I} \setminus (1, I)\}$ has neighbours $\mathcal{N}_i = \{i-1, i+1\}$, and the first and the last consumer have neighbour sets $\mathcal{N}_1 = \{I, 2\}$ and $\mathcal{N}_I = \{I-1, 1\}$.
- *Star*, where one consumer is a centre with connections to everybody else, and where the peripheral consumers are linked only to the centre. The centre $C \in \mathcal{I}$ has a set of neighbours $\mathcal{N}_C = \{\mathcal{I} \setminus C\}$. A peripheral consumer's only neighbour is the centre, $\mathcal{N}_i = C$, for all $i \in \{\mathcal{I} \setminus C\}$.

The network is *symmetric* if all consumers have an equal number of links. The complete graph and the circle are symmetric, whereas the star is *asymmetric*.

The link $\{i, j\} \in \mathcal{E}$ comprises two directed links (i, j) and (j, i). With I consumers, the complete graph has $I(I-1)$, the circle $2I$, and the star $2(I-1)$ directed links. When the number of consumers is fixed, the comparison across different network topologies comprises the *size effect* (number of links) and the *topological effect* (link wiring). The way different consumer *types are configured* on the network causes a third effect. The size effect has been carefully analysed in the earlier literature, while the topological effect and the role of the type configuration are new features. The size effect relates to the understanding of network effects as demand side economies of scale, because it corresponds to the link value of the network the consumer is associated with. There are two types of critical consumers, who have connections that are important from all network members' perspective. One type are focal topology-wise, e.g. the centre in a star. The existence of these consumers induces the topological effect on monopoly pricing. The second, more subtle, critical type is focalised by high heterogeneity between the consumer's and his neighbours' types. These critical consumers induce the effect due to the type configuration. We can eliminate the size effect by controlling for the link value. This requires that the less connected networks are *compensated* by increasing the number of consumers.

3 Perfect information

The consumer types $\theta = (\theta_1, ..., \theta_I)$ are revealed to all before the firm sets the price. Let $a = (a_i, a_{-i})$ be the vector of actions, where the actions taken by the other consumers than i are denoted by a_{-i}. The net utility of the consumer $i \in \mathcal{I}$ is

$$u_i(a, \theta_i) = \sum_{j \in \mathcal{N}_i} a_i a_j \theta_i - a_i p, \tag{1}$$

where p is the unit price for the device.[7] He is indifferent between buying and not when his type is

$$\widetilde{\theta}_i = \frac{p}{\sum_{j \in \mathcal{N}_i} a_j}.$$

The coordination game Γ, parameterised by the price p, consists of consumers $\mathcal{I}$ with types $\theta = (\theta_1, ..., \theta_I)$ arranged on the graph $\mathcal{G}$, pure actions $a \in \{0, 1\}$, and payoffs (1) for all $i \in \mathcal{I}$. The consumer i's best response is $a_i^* \in \arg\max_{a_i \in \{0,1\}} u_i(a, \theta_i)$. The Nash equilibrium (NE) of Γ is the strategy profile $a^* = (a_1^*, ..., a_I^*)$ which maximises the consumer's utility $u_i\left(a_i^*, a_{-i}^*, \theta_i\right) \geq u_i\left(a_i, a_{-i}^*, \theta_i\right)$ for all $i \in \mathcal{I}$. The game Γ has multiple NE conditional on the price, the realisations of types, and the network topology. Particularly, the empty network is always a trivial NE for a positive p. In the rest of this section, we consider only the interesting cases of non-empty, or "active", NE.

Lemma 3 *The action profile $a^* = (a_1^*, ..., a_I^*)$ is an active NE of Γ if*

$$\begin{aligned} a_i^* &= 0 \text{ if } \theta_i < \widetilde{\theta}_i^* \\ a_i^* &= 1 \text{ if } \theta_i \geq \widetilde{\theta}_i^* \end{aligned}$$

where $\widetilde{\theta}_i^* = \frac{p}{\sum_{j \in \mathcal{N}_i} a_j^*}$ for all $i \in \mathcal{I}$.

We define the total consumer surplus $CS = \sum_{i \in \mathcal{I}} u_i(a^*, \theta_i)$ as the sum of utilities of all consumers in the network. There can be a number of active NE. The largest NE, which maximises the use of the new device, corresponds to efficient coordination, because Γ is supermodular with positive spillovers (action complementarity).

Lemma 4 *The coordination game Γ is supermodular with positive spillovers.*
Proof. *Steps (i)-(iii) prove the supermodularity of Γ. Positive spillovers result from (iv).*
(i) Action set $a = \{0, 1\}$ is a compact subset of $\mathbb{R}$.
(ii) If proportion $k = |a_j = 1|$, $j \in \mathcal{N}_i$ buy, the number of active links is k when i plays $a_i = 1$. The gain from $a_i = 1$ versus $a_i = 0$ is $v_i(\theta_i, k) = k\theta_i - p$, which is strictly increasing in θ_i for all $i \in \mathcal{I}$, showing increasing differences.
(iii) The payoff function $u_i : \{0, 1\} \times \theta \to \mathbb{R}$ is continuous.
(iv) The payoff gain $v_i(\theta_i, k)$ is strictly increasing in k. ■

Lemma 4 applies to both symmetric and asymmetric graphs. Now, Topkis' theorem guarantees that the supermodular game Γ has the maximal and the minimal NE elements, and due to positive spillovers, the maximal NE is Pareto-dominating (Vives 2001, p. 33-34). We have rationalised the adoption of the maximal NE by its compatibility with both the monopoly's interests and the Pareto-efficiency in the consumers' coordination game earlier. Therefore, we focus on the maximal NE when analysing the firm's problem. Denote $b(p) \in [0, I]$ as the number of consumers who buy in the maximal NE for a given p.

[7] We can write the equation (1) with links explicitly expressed $u_i(a, \theta_i) = \sum_{j \in \{\mathcal{I} \setminus i\}} g_{ij} a_i a_j \theta_i - a_i p$, where $g_{ij} = \{0, 1\}$ indicates whether i and j are neighbours ($g_{ij} = 1$) or not ($g_{ij} = 0$). If we write the equation (1) as $u_i(a, \theta_i) = \alpha + \sum_{j \in \mathcal{N}_i} a_i a_j \theta_i - a_i p$, where $\alpha = 0$ is the intrinsic utility from the good, we see that the utility function is of the de Palma and Leruth (1996) type, where the consumers have differentiated valuations of network benefits, as opposed to the original Katz and Shapiro (1985) specification, where the consumers are differentiated with respect to the intrinsic utility α.

The function $b(p)$ is decreasing in p. The firm observes the realisations of θ and maximises profits

$$V = b(p)(p - c)$$

by setting the price p. Marginal cost is constant $c \geq 0$, and there are no fixed costs. The pricing problem is interesting only if the firm sets one price for all consumers under perfect information. If price discrimination was allowed, the firm would capture all surplus from every consumer, and the resulting activity level would be a complete network. The optimal price is

$$p^* = \arg\max_p \{b(p)(p - c)\}.$$

Finally, we define the total surplus as the sum of total consumer surplus and profits $W = CS + V$. Next we apply the general framework to the complete graph, the circle, and the star. We do a comparison across networks in section 3.4.

3.1 Complete graph

The type configuration of consumers is irrelevant in a complete graph, because each consumer is connected to everybody else. Utility for the consumer i is $u_i(a, \theta_i) = \sum_{j \in \{\mathcal{I} \setminus i\}} a_i a_j \theta_i - a_i p$. The NE of Γ is expressed in lemma 5.

Lemma 5 *The action profile $a^* = (a_1^*, ..., a_I^*)$ is an active NE of* Γ, *if for all $i \in \mathcal{I}$*

$$\begin{aligned} a_i^* &= 0 \text{ if } \theta_i < \frac{p}{\sum_{j \in \{\mathcal{I} \setminus i\}} a_j^*} \\ a_i^* &= 1 \text{ if } \theta_i \geq \frac{p}{\sum_{j \in \{\mathcal{I} \setminus i\}} a_j^*} \end{aligned}$$

All activity levels are sustainable in equilibrium, conditional on p and the realisations of θ, and multiple NE are possible. The number of buyers in the maximal NE $b(p)$ is decreasing in p, with a ceiling $b\left((I-1)\theta^-\right) = I$ and a floor $b\left((I-1)\theta^+ + \varepsilon\right) = 0$, where $\varepsilon > 0$ is small. Hence, the optimal price is bounded in the range $p^* \in \left[(I-1)\theta^-, (I-1)\theta^+\right]$. Example 25 in the appendix analyses the optimal price in a four consumer complete graph.

3.2 Circle

In the circle, each consumer has two neighbours. The consumer i's utility is $u_i(a, \theta_i) = a_i(a_{i-1} + a_{i+1})\theta_i - a_i p$. We obtain a three-partition of consumer types. The low types never buy. The medium types buy only if both of their neighbours buy. The high types buy if at least one of their neighbours buys. The NE of Γ is expressed in lemma 6.

Lemma 6 *The action profile $a^* = (a_1^*, ..., a_I^*)$ is an active NE of* Γ, *if for all $i \in \mathcal{I}$*

$$\begin{aligned} a_i^* &= 0, \text{ if one neighbour buys and } \theta_i < p, \text{ or both neighbours buy and } \theta_i < \frac{1}{2}p \\ a_i^* &= 1, \text{ if one neighbour buys and } \theta_i \geq p, \text{ or both neighbours buy and } \theta_i \geq \frac{1}{2}p \end{aligned}$$

All activity levels are feasible as NE, conditional on p and the realisations of θ, and multiple NE can exist. The network structure matters now more than in a complete graph, as the consumer's action depends on the types of his neighbours rather than of the whole population. The optimal price is in the range $p^* \in \left[2\theta^-, 2\theta^+\right]$. See example 26 in the appendix for an example how the monopolist sets the price in a four consumer circle.

3.3 Star

The star network is asymmetric with a single central consumer connected to $I-1$ peripheral consumers, who in turn are connected only to the centre. The centre's utility is $u_C(a,\theta_C) = \sum_{i\in\mathcal{N}_C} a_C a_i \theta_C - a_C p$, $\mathcal{N}_C = \{\mathcal{I} \setminus C\}$. A peripheral consumer's utility is $u_i(a,\theta_i) = a_i a_C \theta_i - a_i p$, for all $i \in \{\mathcal{I} \setminus C\}$. An active NE requires that the centre buys.

Lemma 7 *The action profile* $a^* = \left(a_C^*, a_1^*, ..., a_{I-1}^*\right)$ *is an active NE of* Γ *if*

$$\begin{aligned} a_i^* &= 0 \text{ if } a_C = 1 \text{ and } \theta_i < p \\ a_i^* &= 1 \text{ if } a_C = 1 \text{ and } \theta_i \geq p \\ a_C^* &= 1 \text{ if } \theta_C \geq \frac{p}{\sum_{i\in\mathcal{N}_C} a_i^*} \end{aligned}$$

for the centre $C \in \mathcal{I}$ *and all peripheral consumers* $i \in \{\mathcal{I} \setminus C\}$.

Define $b(p)$ as the largest number of peripheral consumers who buy in the maximal NE for a the price p. The centre's demand $b_C(p)$ is a step-function

$$b_C(p) = \begin{cases} 0, & \text{if } p > \overline{u}_C \\ 1, & \text{if } p \leq \overline{u}_C \end{cases}$$

where $\overline{u}_C = b(p)\theta_C$ is the utility from active links. The lower and upper bounds for $b(p)$ are $b\left(\min\left\{\theta^+, (I-1)\theta_C\right\} + \varepsilon\right) = 0$, and $b\left(\theta^-\right) = I-1$, which take into account the centre's and periphery's topological differences. In order to evade the empty network, the firm must guarantee that the centre and at least one peripheral consumer buy. Hence, the firm's problem is to maximise profits, $V = [1 + b(p)](p-c)$ subject to $p \leq \overline{u}_C$. See example 27 in the appendix how the monopolist sets the price in a four consumer star.

3.4 Comparison of networks

With perfect information, the type configuration of consumers makes the comparison across network topologies impractical. For example, a complete graph with four consumers corresponds to a compensated circle with six consumers. A circle of six consumers has 720 permutations (of which half are mirror images). In order to have the topological effect stand out, we first do comparisons across networks where the type configuration is eliminated. This requires that all consumers are of the same type.

Let the consumer types be identical $\theta_i = \theta > c$ for all $i \in \mathcal{I}$. The optimal prices are simple in this case. The monopoly price is constant with respect to the number of consumers in the star and the circle, but it is increasing in I in the complete graph.

Lemma 8 *The monopolist maximises its profits by setting* $p = (I-1)\theta$ *in the complete graph,* $p = 2\theta$ *in the circle, and* $p = \theta$ *in the star, when* $\theta_i = \theta > c$ *for all* $i \in \mathcal{I}$.

Proof. *In the complete graph, a price higher than* $p = (I-1)\theta$ *yields negative net utility for any consumer* $i \in \mathcal{I}$ *even if everybody else buys. A price lower than* $p = (I-1)\theta$ *leaves positive surplus to all consumers in the maximal NE, thus the monopolist could increase the price up to* $p = (I-1)\theta$, *without changes in the consumers' NE actions. Similar argumentation applies to the price in the circle* $p = 2\theta$. *In the star, with similar argumentation, the optimal price is* $p = \theta$, *which is determined by a peripheral consumer's net utility.* ■

Next we fix the link value of the social network at $I(I-1)$, which corresponds to a complete graph of I consumers, and compensate the less connected circle and star by increasing the number of consumers in them so that the link values equal $I(I-1)$. We also assume $I > 3$ in order to differentiate between the topologies. The optimal prices are defined in lemma 8, and they produce a complete network (full activity) in the maximal NE in all topologies. The monopoly is able to extract all consumer surplus in the symmetric complete graph and circle, $CS_{CG} = CS_C = 0$. In contrast, the centre in the star is left with positive surplus, while the peripheral consumers get zero surplus, $CS_S > 0$. In the case of symmetric networks, the topological effect causes the monopoly profits to be lower in the less connected compensated circle, $V_{CG} > V_C$. This is because the monopolist incurs higher production costs for selling to a higher (compensated) number of consumers in the circle compared to the fully connected complete graph. The monopoly faces even higher costs due to the higher number of consumers in the compensated star. The monopoly profits in the compensated star are lower than in the symmetric networks also because the monopoly is unable to extract all surplus from the centre due to the pricing constraint created by the less-connected peripheral consumers. The centre benefits from his topologically focal position and his links that are important from all network members' perspective. Therefore, the profits are the lowest in the compensated star, $V_{CG} > V_C > V_S$.

Proposition 9 *For a given network link value $I(I-1)$, $I > 3$, and the consumer types $\theta_i = \theta > c$ for all $i \in \mathcal{I}$, the topological effect implies that*
(i) monopoly profits are the highest in complete graph, and the lowest in the compensated star, $V_{CG} > V_C > V_S$.
(ii) consumer surplus is zero in the symmetric networks, and positive in the compensated star, $CS_S > CS_{CG} = CS_C = 0$.
(iii) total surplus is the highest in the complete graph, and the lowest in the compensated star, $W_{CG} > W_C > W_S$.
Proof. *A complete graph of I consumers generates a link value of $I(I-1)$. A circle requires $\frac{I(I-1)}{2}$ consumers to generate $I(I-1)$ links. Similarly, a star requires $1 + \frac{I(I-1)}{2}$ consumers. The optimal monopoly prices are given in lemma 8. The reported results (i)-(iii) follow directly from the comparisons of consumer surpluses, profits, and total surpluses detailed below.*
(i) The monopoly profits in the maximal NE are $V_{CG} = I(I-1)\theta - Ic$ in the complete graph, $V_C = I(I-1)\theta - \frac{I(I-1)}{2}c$ in the compensated circle, and $V_S = \left(1 + \frac{I(I-1)}{2}\right)(\theta - c)$ in the compensated star. The difference $V_C - V_S$ is always positive for $I \geq 3$ and $\theta > c$.
(ii) The total consumer surplus in the maximal NE equals zero $CS_{CG} = CS_C = 0$ in the complete graph and in the compensated circle, and it is positive $CS_S = \left(\frac{I(I-1)}{2} - 1\right)\theta > 0$ in the compensated star.
(iii) The total surplus in the compensated star is $W_S = I(I-1)\theta - \left(1 + \frac{I(I-1)}{2}\right)c$, while the total surplus in the compensated circle is $W_C = I(I-1)\theta - \frac{I(I-1)}{2}c$ and in the complete graph $W_{CG} = I(I-1)\theta - Ic$ in the maximal NE. ■

The size effect relates to an increase in the (uncompensated) link value of the network. The effect has straightforward and predictable implications.[8] The profits and total surplus

[8] The size effect is derived by comparing the *uncompensated* networks. The optimal monopoly prices are given in lemma 8. The monopoly profits in the maximal NE are $V_{CG} = I(I-1)\theta - Ic$ in the complete

increase in the link value of the network, while consumer surplus decreases. Thus, profits and total surplus are maximised in the complete graph, whereas consumer surplus is maximised in the star. Monopoly profits and total surplus are increasing in the total number of network members in all topologies.

The more links there are, the higher is the generated value in the network. This shows how the strength of network externalities is easily overestimated. An assumption on a complete graph as the prevailing social structure, when the true social structure is something less connected, produces exaggerated estimates for the network value, thus for the total surplus and monopoly rents.

Let us next analyse a more diverse case, where we do not eliminate the type configuration. This, however, necessitates us to limit the size of the network and consider only the uncompensated networks in order to maintain the model's workability.[9] Consider a complete graph, a circle and a star of four consumers with $\theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4$, and assume $c = 0$ for expositional reasons. Figure (1) gives the firm's profits, consumer surplus, and the total surplus in the maximal NE. The social networks are given in the rows, columns correspond to the activity level.

The optimal monopoly price, which are given in the parenthesis in the profits lines, is affected by the size and the topology of the social network, and by the type configuration (e.g. circle A and B have the same topology, but are different configurations). Because the coordination game is supermodular with positive spillovers, consumer surplus is maximised in the complete network (full activity) in all social networks in the illustrated example.[10] Since profits are just transfers from consumer surplus, total surplus is maximised in the complete network.

Network topology can create topologically critical positions that constrain pricing, like the centre in the star. The critical positions due to the type configuration of consumers is illustrated by the comparison of the circles A and B. Consumer θ_2 in the circle B is an example of this. His position constrains pricing, if his type is sufficiently low relative to his neighbours' types $(2\theta_2 < \theta_3)$, despite network symmetry. Obviously, the type configuration can matter only in networks that are not completely connected.

Remark 10 *Critical consumers that constrain the optimal price have*
(i) topologically central positions (centre in star),
(ii) important connections (low types with high type neighbours),
in networks that are not completely connected graphs.

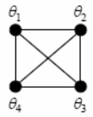
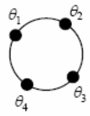
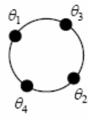
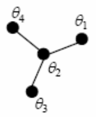
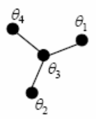
Consider the circles A and B again. Let the complete network be optimal in A, so that $8\theta_1 > \max\{3\theta_2, 2\theta_3\}$. If we also have $8\theta_1 < 6\theta_2$ and $2\theta_2 > \theta_3$ it is optimal for the firm to choose the 3-buyer network in B. Why? The firm finds it profitable to increase the price

graph, $V_C = 2I\theta - Ic$ in the circle, and $V_S = I(\theta - c)$ in the star, which all are increasing in I. Consumer surplus in the complete graph and circle equals zero, $CS_{CG} = CS_C = 0$. Consumer surplus in the star is constituted by the centre's surplus $CS_S = (I-2)\theta$, which is increasing in I. Total surplus in the complete graph and circle equals the monopoly profits, and in the star it is $W_S = 2(I-1)\theta - Ic$. The reported results in the text follow directly from the comparisons of the above values.

[9] We label the results from examples as *remarks*, in order to distinguish them from *propositions* with formal proofs.

[10] One must be careful not to compare consumer surpluses in dominated networks, so that the implied type configuration conditions on pricing have to be taken into account. Once the type conditions are factored in, the comparison is straightforward and shows that the consumer surplus is maximised in the complete network in all social structures.

Figure 1: Profits and surpluses in different social networks.

	Complete network	3-buyer network	2-buyer network
Complete graph 	$V = 4\,(3\theta_1)$ $CS = 3\,(\theta_2{+}\theta_3{+}\theta_4) - 9\theta_1$ $W = 3\,(\theta_1{+}\theta_2{+}\theta_3{+}\theta_4)$	$V = 3\,(2\theta_2)$ $CS = 2\,(\theta_3{+}\theta_4) - 4\theta_2$ $W = 2\,(\theta_2{+}\theta_3{+}\theta_4)$	$V = 2\,(\theta_3)$ $CS = \theta_4 - \theta_3$ $W = \theta_3{+}\theta_4$
Circle A 	$V = 4\,(2\theta_1)$ $CS = 2\,(\theta_2{+}\theta_3{+}\theta_4) - 6\theta_1$ $W = 2\,(\theta_1{+}\theta_2{+}\theta_3{+}\theta_4)$	$V = 3\,(\theta_2)$ $CS = (2\theta_3{+}\theta_4) - 2\theta_2$ $W = \theta_2{+}2\theta_3{+}\theta_4$	$V = 2\,(\theta_3)$ $CS = \theta_4 - \theta_3$ $W = \theta_3{+}\theta_4$
Circle B 	$V = 4\,(2\theta_1)$ $CS = 2\,(\theta_2{+}\theta_3{+}\theta_4) - 6\theta_1$ $W = 2\,(\theta_1{+}\theta_2{+}\theta_3{+}\theta_4)$	$V = 3\,(\min\{2\theta_2, \theta_3\})$ $CS = \max\{(\theta_3{+}\theta_4) - 4\theta_2,$ $(2\theta_2{+}\theta_4) - 2\theta_3\}$ $W = 2\theta_2{+}\theta_3{+}\theta_4$	Dominated
Star, 2 as centre 	$V = 4\,(\theta_1)$ $CS = (3\theta_2{+}\theta_3{+}\theta_4) - 3\theta_1$ $W = \theta_1{+}3\theta_2{+}\theta_3{+}\theta_4$	$V = 3\,(\min\{2\theta_2, \theta_3\})$ $CS = \max\{(\theta_3{+}\theta_4) - 4\theta_2,$ $(2\theta_2{+}\theta_4) - 2\theta_3\}$ $W = 2\theta_2{+}\theta_3{+}\theta_4$	Dominated
Star, 3 as centre 	$V = 4\,(\theta_1)$ $CS = (\theta_2{+}3\theta_3{+}\theta_4) - 3\theta_1$ $W = \theta_1{+}\theta_2{+}3\theta_3{+}\theta_4$	$V = 3\,(\theta_2)$ $CS = (2\theta_3{+}\theta_4) - 2\theta_2$ $W = \theta_2{+}2\theta_3{+}\theta_4$	$V = 2\,(\theta_3)$ $CS = \theta_4 - \theta_3$ $W = \theta_3{+}\theta_4$

so that θ_1 opts out. At the same time, the high types θ_3 and θ_4 induce their common neighbour θ_2 to purchase. The consumer θ_2 benefits from the links with high types θ_3 and θ_4, and the firm is able to capture some (or all) of this rent.

When we compare the 2-buyer networks, we see that in the circle B and the star with θ_2 at the centre, the 2-buyer networks are always dominated (in terms of profits) by the 3-buyer networks. The critical type θ_2 sets a pricing constraint. The firm may be forced to sell at a lower price in order to guarantee his participation. On the contrary, in the circle A and the star with θ_3 at the centre, because the high types are clustered, the 2-buyer networks are not dominated. Hence, in some graphs, the monopolist limits supply whereas in other graphs that are identical save the configuration of consumers, it covers the whole market. When consumers are relatively homogeneous, the firm prefers to have the high types scattered in the network. Scattered high types support the purchases of lower types, and therefore full coverage is more likely to occur. On the other hand, if the consumers are highly heterogeneous, so that the firm prefers to exclude the low types by setting a high price, the dispersion of high types hurts the firm as the price is constrained by the critical (low) types.

Remark 11 *The role of consumer heterogeneity:*
(i) the firm excludes the low consumer types in heterogeneous markets, whereas homogeneous markets are completely covered.

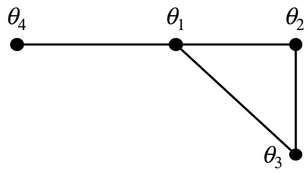


Figure 2: Modified star: "insiders - outsider"

(ii) the dispersion of the high consumer types is good for the firm in homogeneous markets, whereas in heterogeneous markets, the dispersion of the high types constrains the firm.

We close the analysis with a modification to remark 11 underlining the importance of network topology in the monopoly pricing problem. Consider the network illustrated in figure (2) with types $\theta_1 < \theta_2 < \theta_3 < \theta_4$, and $c = 0$. Let the 2-buyer network dominate the 3-buyer network, $V_2 = 2(\theta_2) > V_3 = 3(2\theta_1)$. If $\theta_4 > 3\theta_1$ and $3\theta_1 < \theta_2 < 6\theta_1$ hold, then it is true that $V_4 = 4(3\theta_1) > V_2 = 2(\theta_2)$. When this holds, the types θ_3 and θ_4 can be significantly higher than θ_1 and θ_2 (high heterogeneity) and the firm still covers the whole market. This is possible thanks to two factors. One, θ_3 and θ_4 are not neighbours, so the firm cannot sell only to them. Two, θ_1 is well connected, which compensates his low type and negates his otherwise critical position.

4 Asymmetric information

In this section, the types $\theta = (\theta_1, ..., \theta_I)$ are private information, but the structure of the social network $\mathcal{G} = (\mathcal{I}, \mathcal{E})$ and the distribution $F(\theta)$ remain common knowledge. Pure strategy for the consumer i is $a_i : [\theta^-, \theta^+] \to \{0, 1\}$. The consumer i's expected net utility is

$$\mathbb{E}[u_i(a, \theta_i)] = \sum_{j \in \mathcal{N}_i} a_i \pi_j \theta_i - a_i p, \tag{2}$$

where π_j is the probability consumer i puts on the event that his neighbour j chooses $a_j = 1$. The consumer i is indifferent between the actions when his type is

$$\widetilde{\theta}_i(\pi_{\mathcal{N}_i}) = \frac{p}{\sum_{j \in \mathcal{N}_i} \pi_j},$$

where $\sum_{j \in \mathcal{N}_i} \pi_j \neq 0$. The utility (2) is increasing in the consumer's own type θ_i and in the number of neighbours for a fixed π_j, $j \in \mathcal{N}_i$. This means that the consumer i's best response is the threshold strategy $a_i^* = 1$ if $\theta_i \geq \widetilde{\theta}_i(\pi_{\mathcal{N}_i})$, and $a_i^* = 0$ if $\theta_i < \widetilde{\theta}_i(\pi_{\mathcal{N}_i})$. The probability that i buys, given his beliefs over his neighbours' actions and price, is

$$\pi_i = 1 - F\left(\min\left\{\theta^+, \widetilde{\theta}_i(\pi_{\mathcal{N}_i})\right\}\right).$$

The coordination game Γ_{AI} with asymmetric information consists of consumers $\mathcal{I}$ arranged on the graph $\mathcal{G}$, pure actions $a = \{0, 1\}$, i.i.d. types $\theta = (\theta_1, ..., \theta_I)$ with prior distribution $F(\theta)$, and payoffs (2), for all $i \in \mathcal{I}$, and it is parameterised by the price p. As in the case of perfect information, the game Γ_{AI} has multiple equilibria conditional on the price and network topology. In particular, the empty network is always a trivial equilibrium. In the rest of this section, we consider only the interesting active equilibria. The active Bayesian Nash equilibrium (BNE) of Γ_{AI} is characterised in lemma 12.

Lemma 12 *The action profile* $a^* = (a_1^*, ..., a_I^*)$ *is an active BNE of* Γ_{AI} *if*

$$\begin{aligned} a_i^* &= 0 \text{ if } \theta_i < \widetilde{\theta}_i\left(\pi^*_{\mathcal{N}_i}\right) \\ a_i^* &= 1 \text{ if } \theta_i \geq \widetilde{\theta}_i\left(\pi^*_{\mathcal{N}_i}\right) \end{aligned}$$

where $\widetilde{\theta}_i\left(\pi^*_{\mathcal{N}_i}\right) = \frac{p}{\sum_{j\in\mathcal{N}_i}\pi_j^*}$ *and* $\pi_i^* = 1 - F\left(\min\left\{\theta^+, \widetilde{\theta}_i\left(\pi^*_{\mathcal{N}_i}\right)\right\}\right)$ *for all* θ_i *and* $i \in \mathcal{I}$.

We can further narrow down the set of interesting BNE by resorting to supermodularity that carries over to the asymmetric information regime.

Lemma 13 *The game* Γ_{AI} *is supermodular with positive spillovers.*
Proof. *Steps (i)-(iii) prove the supermodularity.*
(i) The set $\pi_i \in [0, 1]$ *is a compact subset of* $\mathbb{R}$.
(ii) The expected payoff gain from $a_i = 1$ *versus* $a_i = 0$ *is* $\mathbb{E}\left[v_i\left(\theta_i, \pi_j\right)\right] = \mathbb{E}\left[u_i\left(a, \theta_i\right)\right]$, $j \in \mathcal{N}_i$, *for all* $i \in \mathcal{I}$, *where* $\mathbb{E}\left[u_i\left(a, \theta_i\right)\right]$ *is given by equation (2). We have increasing differences in the payoffs as* $\mathbb{E}\left[v_i\left(\theta_i', \pi_j\right)\right] \geq \mathbb{E}\left[v_i\left(\theta_i, \pi_j\right)\right]$ *for all* $\theta_i' > \theta_i$.
(iii) The payoff function $\mathbb{E}\left[u_i\left(a, \theta_i\right)\right] : \{0, 1\} \times \theta \rightarrow \mathbb{R}$ *is continuous.*
Positive spillovers arise because the payoff gain is strictly increasing in the neighbours' strategies $\frac{\partial \mathbb{E}[v_i(\theta_i, \pi_j)]}{\partial \pi_j} > 0$, $j \in \mathcal{N}_i$ *for all* $i \in \mathcal{I}$. ■

Supermodularity guarantees the existence of the maximal and the minimal BNE elements. The smallest BNE is the empty network with $\pi_i^* = 0$ for all $i \in \mathcal{I}$, while any active BNE depends on the price and the network topology. Positive spillovers mean that the largest BNE, which maximises the use of the new device, is Pareto-dominating. We assume that this focalises the equilibrium, like under perfect information. We get further support for this argument by proving later that the maximal BNE is Cournot tâtonnement stable in all studied social networks.

The firm's expected profits are

$$\mathbb{E}(V) = \sum_{i\in\mathcal{I}} \pi_i^* \left[p\left(\pi^*\right) - c\right].$$

The firm cannot choose the activity level directly, as it could under perfect information. Instead, it maximises profits by choosing π_i^*. The inverse demand $p(\pi^*)$ is derived from the BNE condition of Γ_{AI}.

Next we apply the general framework to the complete graph, the circle, and the star.

4.1 Symmetric networks

Symmetric networks are analytically identical under asymmetric information, because the role of type configuration on the social network is eliminated. We work through a generalised version of a symmetric graph where all consumers have n neighbours. For the complete graph $n = I - 1$ and for the circle $n = 2$. Note that some constructions are infeasible. For example, it is impossible to construct a symmetric graph of five consumers each having three neighbours. The generalised version applies to complete graphs and circles of any number of consumers, though.

Lemma 14 *An active BNE in a symmetric network is characterised by a unique probability to buy π, for each consumer $i \in \mathcal{I}$, and it satisfies*

$$\pi = 1 - F\left(\min\left\{\theta^+, \frac{p}{n\pi}\right\}\right) \tag{3}$$

Proof. *Let $n \in [1, I-1]$ be the number of neighbours for the consumer i in a population arranged on a symmetric graph $\mathcal{G}^{sym}$. By symmetry, n is the number of neighbours for all consumers. Lemma 12 gives the BNE probability that the consumer i buys π_i^*. Assume that the probabilities are different so that for all other consumers except i, the probability to buy is π and for i it is $\pi_i < \pi$. The consumer i's expected utility is*

$$\begin{aligned}\mathbb{E}\left[u_i(a, \theta_i)\right] &= \sum_{k=0}^{n} a_i \binom{n}{k} \pi^k (1-\pi)^{n-k} k\theta_i - a_i p \\ &= a_i \pi n \theta_i - a_i p.\end{aligned}$$

Similarly, the expected payoff for the consumer $j \in \mathcal{N}_i$ is

$$\begin{aligned}\mathbb{E}\left[u_j(a, \theta_j)\right] &= \sum_{k=0}^{n-1} a_j \binom{n-1}{k} \pi^k (1-\pi)^{(n-1)-k} k\theta_j + a_j \pi_i \theta_j - a_j p \\ &= a_j \left[(n-1)\pi + \pi_i\right] \theta_j - a_j p.\end{aligned}$$

The equilibrium condition that the consumer i buys is $z_i(\pi) = 1 - F\left(\min\left\{\theta^+, \frac{p}{n\pi}\right\}\right)$, and for all $j \in \mathcal{N}_i$ it is $z_j(\pi_i, \pi) = 1 - F\left(\min\left\{\theta^+, \frac{p}{(n-1)\pi + \pi_i}\right\}\right)$. The functions $z_i(\pi)$ and $z_j(\pi_i, \pi)$ are increasing in π and in (π, π_i) respectively. If the initial assumption $\pi_i < \pi$ holds, then it must be that $z_i(\pi) > z_j(\pi_i, \pi)$ which leads to a contradiction. The case $\pi_i > \pi$ leads to a corresponding contradiction. Hence, in the BNE it must be that $\pi_i = \pi$ for all $i \in \mathcal{I}$. ■

Asymmetric information eliminates the effects caused by the type configuration. Unlike with perfect information, under asymmetric information the consumer cannot condition his behaviour with respect to the realisations of his neighbours' types, and he can base his action on the number of neighbours only. When the network is symmetric, every consumer has the same number of neighbours. In this situation, a consumer holds that all his neighbours face an identical situation to his own, and therefore a consumer of type θ has the same BNE strategy independent of his network location. As a result, asymmetric equilibria, where identical consumer types would buy with different probabilities, are ruled out. This is in contrast with the case with perfect information, which allows asymmetric NE thanks to the effects by the type configuration.

The introduction of asymmetric information has reduced the number of equilibria to three at most. In addition to the empty network, there can be at most two active BNE in the interval $\pi \in \left]\frac{p}{n\theta^+}, 1\right]$. To check the existence of active BNE, we solve the equation (3) for the positive π. Real roots exist when $\left(\theta^+ n\right)^2 - 4\left(\theta^+ - \theta^-\right) np \geq 0$, with equality yielding a unique active BNE. The firm operates in the region where the price is determinate, so that the equation (3) gives the inverse demand

$$p = n\pi\left[\theta^+ - \left(\theta^+ - \theta^-\right)\pi\right]. \tag{4}$$

The firm maximises expected profits $\mathbb{E}(V) = I\pi[p(\pi) - c]$ by choosing the optimal level for π. The first order condition gives the standard monopoly mark-up rule

$$\frac{p(\pi^*) - c}{p(\pi^*)} = \frac{1}{\eta}, \tag{5}$$

where π^* is the optimum and $\eta = -\frac{\partial \pi^*}{\partial p}\frac{p(\pi^*)}{\pi^*}$ is the price elasticity of demand.

Consider the special case of $c = 0$. The equation (5) gives

$$\pi^* = \frac{2\theta^+}{3(\theta^+ - \theta^-)},$$

and the equation (4) gives

$$p(\pi^*) = \frac{2(\theta^+)^2}{9(\theta^+ - \theta^-)}n, \tag{6}$$

which satisfy second order conditions.[11] The derived values represent the maximal BNE. When the price (6) is plugged back into equation (3), we can solve again for the corresponding equilibrium probabilities. As suggested, there exist two active BNE

$$\pi = \frac{\theta^+ \pm \frac{1}{3}\theta^+}{2(\theta^+ - \theta^-)}.$$

The maximal BNE and the empty network are Nash tâtonnement stable, whereas the smaller active BNE is an unstable one.[12] Hence, the convergence occurs towards zero or the maximal BNE, unless the tâtonnement process begins exactly at the lower active BNE. Denote the maximal, focal, BNE as π_+^*. The expected profits are in that case[13]

$$\mathbb{E}(V_+^*) = \frac{4}{27}\left(\frac{\theta^+}{\theta^+ - \theta^-}\right)^2 \theta^+ In. \tag{7}$$

Total expected consumer surplus in the maximal BNE is given by

$$\begin{aligned} \mathbb{E}(CS) &= I\int_{\widetilde{\theta}(\pi_+^*)}^{\theta^+} f(\theta)\left[n\pi_+^*\theta - p^*\right]d\theta \\ &= \frac{4}{27}\left(\frac{\theta^+}{\theta^+ - \theta^-}\right)^2 \theta^+ In \end{aligned} \tag{8}$$

which shows that it equals expected profits $\mathbb{E}(CS) = \mathbb{E}(V_+^*)$.

We are ready to compare the asymmetric information model (with $c = 0$) with the results from the perfect information case. We see from equations (7) and (8) that the profits and consumer surplus are increasing in the number of neighbours (i.e. in the link value) and in the overall number of consumers. Hence, the size effect guarantees that the complete graph generates the highest total surplus, and therefore the size effect agrees with the results under perfect information.

---

[11] $\left.\frac{\partial^2 \mathbb{E}(V)}{\partial \pi^2}\right|_{\pi = \frac{2\theta^+}{3(\theta^+ - \theta^-)}} = -2\theta^+ In < 0.$

[12] The checks for stability are provided in the appendix 7.2.1.

[13] The difference in realised profits between the maximal BNE and the smaller active BNE is $(\pi_+^* - \pi_-^*)\,p(\pi^*) = \frac{1}{2}\mathbb{E}(V_+^*)$. The empty network yields zero profits of course.

Proposition 15 *The size effect implies that*
(i) monopoly price increases as the number of neighbours increases.
(ii) expected consumer surplus and profits increase in the number of neighbours, with the complete graph supporting the highest expected consumer surplus and profits in the maximal BNE.
(iii) profits and consumer surplus increase in the total number of consumers in symmetric graphs.
Proof. *Follows directly from equations* (6) , (7) *and* (8). ■

The type configuration of consumers is irrelevant since the consumers are ex ante symmetric. Hence, the critical roles that existed in symmetric networks under perfect information are removed. It is obvious now that the complete graph corresponds to the conventional network externalities model, where the underlying social structure is abstracted away. When we take the probability π as the fraction of the total population who buy, we arrive at a basic membership externality model, where the consumer's utility increases with the number of people joining the (global) network at any location.

Proposition 16 *Type configuration does not affect the optimal monopoly price in symmetric graphs under asymmetric information.*
Proof. *Follows from lemma 14.* ■

We apply a mean-preserving spread $\left[\theta^- - x, \theta^+ + x\right]$, $x > 0$, on $F(\theta)$ to see what the impact of consumer heterogeneity is. Increased heterogeneity reduces the probability to buy

$$\frac{\partial \pi_+^*}{\partial x} = -\frac{2\left(\theta^+ + \theta^-\right)}{3\left(\theta^+ - \theta^- + 2x\right)^2} < 0.$$

This results in a lower monopoly price in general.

$$\frac{\partial p^*}{\partial x} = -\frac{4\left(\theta^+ + x\right)\left(\theta^- - x\right)}{9\left(\theta^+ - \theta^- + 2x\right)^2},$$

which is negative when $\theta^- > x > 0$, but positive with $\theta^- = 0$. An increase in heterogeneity causes two effects. First, higher heterogeneity induces higher monopoly price as in the standard case of monopoly pricing with unit demand. Second, higher heterogeneity increases the uncertainty about the neighbours' buying decisions. The second effect induces the monopoly to reduce its price to counter the reduction in the neighbours' probability to buy. Because the second effect dominates in general, the total effect is negative.

We can write the expected consumer surplus and profits in the maximal BNE as

$$\mathbb{E}(CS) = \mathbb{E}\left(V_+^*\right) = \frac{4}{27}\left(\frac{\theta^+ + x}{\left(\theta^+ + x\right) - \left(\theta^- - x\right)}\right)^2 \left(\theta^+ + x\right) In. \tag{9}$$

Since the spread increases uncertainty about neighbours' purchasing decisions, expected consumer surplus decreases despite the reduction in price. For the firm, higher uncertainty leads to lower demand and lower price, thus lower profits. The firm cannot distinguish between networks where the high consumer types are clustered and where they are dispersed. Hence, it is incapable of taking advantage of clusters of high types, unlike it was under perfect information.

Proposition 17 *Increased consumer heterogeneity, i.e. uncertainty, decreases expected consumer surplus and profits in the maximal BNE.*

Proof. *From equation* (9) *we get* $\frac{\partial \mathbb{E}(CS)}{\partial x} = \frac{\partial \mathbb{E}\left(V_+^*\right)}{\partial x} = \frac{4}{27} In \frac{\left(\theta^+ + x\right)^2}{\left(\theta^+ - \theta^- + 2x\right)^3} \left[-\left(\theta^+ + x\right) - 3\left(\theta^- - x\right)\right]$, *which is negative when* $\theta^- \geq 0$ *and* $x > 0$ *is small.* ■

4.2 Star

For the star, we obtain an equilibrium system that comprises two distinct probabilities for buying. One is for the centre and the other for the peripheral consumers. The firm has to choose a price that applies to all consumers, but we allow price discrimination in the section 4.3.

The consumers' utilities are $\mathbb{E}\left[u_C\left(a, \theta_C\right)\right] = \sum_{j \in \mathcal{N}_C} a_C \pi_{Cj} \theta_C - a_C p$ for the centre $C \in \mathcal{I}$, and $\mathbb{E}\left[u_i\left(a, \theta_i\right)\right] = a_i \pi_{iC} \theta_i - a_i p$ for the peripheral consumer $i \in \{\mathcal{I} \diagdown C\}$. Since the peripheral consumers are ex ante symmetric, their behaviour is characterised by a common probability.

Lemma 18 *An active BNE in a star is characterised by* (π_C, π), *where* π_C *is the probability that the centre* $C \in \mathcal{I}$ *buys and* π *is the probability that a peripheral consumer* $i \in \{\mathcal{I} \diagdown C\}$ *buys, and they satisfy*

$$\begin{aligned} \pi_C &= 1 - F\left(\min\left\{\theta^+, \tfrac{p}{(I-1)\pi}\right\}\right) \\ \pi &= 1 - F\left(\min\left\{\theta^+, \tfrac{p}{\pi_C}\right\}\right) \end{aligned} \tag{10}$$

Proof. *Proof follows directly from lemma 14 and uses the symmetry property.* ■

We get the market clearing price and the centre's probability to buy as a function of π from the system (10).

$$\begin{aligned} p\left(\pi\right) &= \pi_C\left(\pi\right)\left[\theta^+ - \left(\theta^+ - \theta^-\right)\pi\right] \\ \pi_C\left(\pi\right) &= \frac{\theta^+\left(I-1\right)\pi}{\theta^+ + \left(I-2\right)\left(\theta^+ - \theta^-\right)\pi} \end{aligned}$$

The difference between the probabilities $\pi_C\left(\pi\right) - \pi$ is always non-negative, which indicates that the centre's probability to buy is higher.

The firm maximises expected profits $\mathbb{E}\left(V\right) = \left[\pi_C\left(\pi\right) + \left(I-1\right)\pi\right]\left[p\left(\pi\right) - c\right]$ by choosing the probability π. The FOC gives a modified inverse elasticity rule

$$\frac{p\left(\pi^*\right) - c}{p\left(\pi^*\right)} = \frac{1}{\eta}\left\{\frac{\left[2\theta^+ + \left(I-2\right)\left(\theta^+ - \theta^-\right)\pi^*\right]\left[\theta^+ + \left(I-2\right)\left(\theta^+ - \theta^-\right)\pi^*\right]}{\left(\theta^+\right)^2 + \left[\theta^+ + \left(I-2\right)\left(\theta^+ - \theta^-\right)\pi^*\right]^2}\right\}, \tag{11}$$

where $\eta = -\frac{\partial \pi^*}{\partial p}\frac{p(\pi^*)}{\pi^*}$ is the price elasticity of demand of a peripheral consumer.

Because the rule (11) is difficult to use analytically, let us consider the specific case with $c = 0$, $\theta \sim Unif\left[0,1\right]$, and a non-degenerate star $I \geq 3$. In this case, the equation (11) has only one real root in the range $\pi \in (0,1)$, which yields positive profits, and the corners $\pi = \{0,1\}$ yield zero profits. Hence, the root in the range $\pi \in (0,1)$ is the global maximum.[14] Because the derivative $\frac{\partial \mathbb{E}(V)}{\partial \pi}$ at point $\pi = \frac{2}{3}$ $\left(\pi = \frac{1}{3}\right)$ is positive (negative), the optimal π must be in the range $\frac{1}{3} < \pi^* < \frac{2}{3}$. So, the probability to buy for a peripheral consumer is less than the probability to buy in symmetric graphs. Respectively, the monopoly achieves a higher mark-up associated with the periphery than the mark-up in the symmetric graphs. This means that the topological effect on the monopoly price is never latent under asymmetric information. On the other hand, we see from lemma 18 that the type configuration is irrelevant in pricing due to the same reasons it was in the case of a symmetric network under asymmetric information.

The firm always takes into account the topologically focal centre by guaranteeing him a higher probability to buy. At the same time, the firm balances the lower centre-specific revenues with a higher mark-up for the periphery. With perfect information, similar bias depends on the type configuration of consumers and occurs only if the centre's type is sufficiently low inducing a pricing constraint.

Proposition 19 *(i) A consumer in the periphery has a lower probability to buy, and the centre has a higher probability to buy, compared with a consumer in a symmetric network. (ii) The type configuration does not influence the optimal monopoly price in the star under asymmetric information.*

We have verified numerically that the optimal π^* is decreasing in the number of peripheral consumers I, whereas the optimal $\pi_C^* \equiv \pi_C(\pi^*)$ is growing in I. Therefore, the centre benefits the more people join his neighbourhood, but a peripheral consumer is negatively affected by an additional peripheral consumer, even though the additional consumer is not his neighbour. Why? The centre's probability to buy increases when a peripheral consumer is added. The firm can compensate this addition by increasing the price. The price increase, however, does not capture the whole increase in the centre's utility, because the firm takes into account the topologically critical position. The centre's market power and relative position against the periphery increases in importance as I grows. By leaving more surplus to the centre, thus increasing the centre's probability to buy, the firm indirectly increases the periphery's expected utility. The price increase, however, is high enough that an individual peripheral consumer gets a negative surplus effect in total, as a result of his reduced significance to the whole network. When I grows very large, the optimal π^* approaches $\frac{1}{2}$, and the optimal π_C^* approaches $\frac{I-1}{I} \approx 1$. In the minimal case where $I = 3$, the optimal values are $\pi^* \approx 0.5971$ and $\pi_C^* \approx 0.7478$. The monopoly price is the lowest at $I = 3$, where it equals $p(\pi^*) \approx 0.3012$. As the periphery becomes very large, the optimal price approaches $\frac{1}{2}$.

The size effect in the star agrees with the size effects in the perfect information regime, and with the symmetric network case under asymmetric information. Profits and the total expected consumer surplus (centre's plus periphery's surpluses) increase in I. At the same time, the difference between the centre's and the periphery's expected surpluses becomes larger. Hence, the network topology has distributional effects on consumer surplus via the monopoly's pricing strategy. This is an important difference compared to the symmetric network case under asymmetric information. We summarise the size effect in remark 20.

Remark 20 *The effects of changes in the size of the periphery are:*

[14] Second order conditions for the maximal profits are satisfied for the active BNE. This can be checked numerically for the particular case $c = 0$, $\theta^+ = 1$, $\theta^- = 0$. We have $\frac{\partial^2 \mathbb{E}(V)}{\partial \pi^2} < 0$ for $I \geq 3$. A stability check for the BNE is in the appendix 7.2.2.

(i) the centre benefits the larger the periphery is.
(ii) a peripheral consumer is adversely affected by an additional peripheral consumer.
(iii) total consumer surplus increases as the periphery grows, driven by the increase in the centre's surplus.
(iv) the price and the monopoly's expected profits increase as the number of peripheral consumers increases.

To measure the effects of higher consumer heterogeneity, we consider the spread $\theta \sim Unif\,[-x, 1+x]$, $x > 0$ on the type distribution. A numerical run shows that the firm chooses a higher optimal price for a small x. This is in contrast with the result from the symmetric network case. In the star, an increase in heterogeneity induces a price increase in the standard way, as it did in the symmetric networks case. The negative effect of higher uncertainty about neighbours' buying decisions is now weaker thanks to the asymmetric network topology. The firm is able to limit the negative effect by contrasting the centre and the periphery, which results to a positive price change in total. However, the increase in uncertainty has a negative effect on profits and consumer surplus in total, as in a symmetric graph.

Remark 21 *Small increase in consumer heterogeneity, i.e. uncertainty, decreases equilibrium profits, and total consumer surplus associated with the periphery and the centre.*

4.3 Comparison and price discrimination

We compare the symmetric networks and the star in the case of a uniform distribution of θ over $[0, 1]$, $c = 0$, and $I > 3$. Figure (3) illustrates the size effect: the complete graph (dotted line) generates far higher total surplus (profits plus consumer surplus) than the *uncompensated* circle (dashed line) or star (solid line). This is because each additional consumer creates $2\,(I-1)$ new links in the complete graph whereas only two links in the circle and the star. In other words, the same network value exaggeration problem presents itself with asymmetric information as with perfect information. If we adopt the complete graph as a postulate, when the true network is something less connected, we end up exaggerating the network's value.

Remark 22 *The complete graph generates the highest total surplus.*

For small numbers of consumers, the circle produces higher total surplus compared to the star, but for large networks, the star generates higher total surplus. The solid line crosses the dashed line just before the number of consumers reaches $I = 30$. The circle has always two links (one two-directional link) more than the star, which returns a higher consumer surplus for small networks. The star, however, supports a lower optimal monopoly price than the symmetric networks, even if the monopoly increases its price as I grows. Therefore, as the size difference becomes less determining, consumer surplus becomes higher in the star than in the circle in large networks. Because the firm maintains a lower price in the star than in a symmetric network, and because there are less links in the star, the firm's profits are the lowest in the star for a given number of consumers.

We can isolate the topological effect by comparing the *compensated* networks. Let us fix the link value of the complete graph with I consumers. A compensated circle has $\frac{I(I-1)}{2}$ consumers and a compensated star $1 + \frac{I(I-1)}{2}$ consumers.

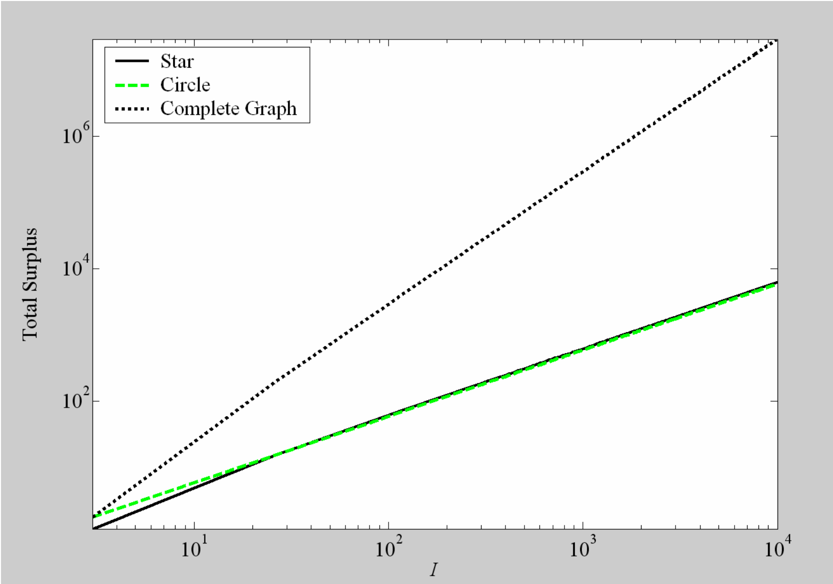


Figure 3: Uncompensated total surplus (log scale), $\theta \sim Unif\,[0,1]\,,\, c = 0$.

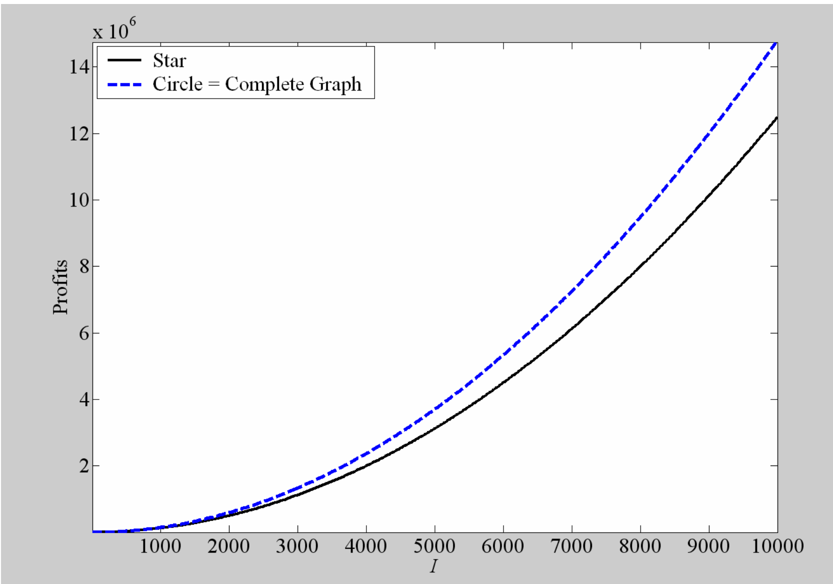


Figure 4: Compensated profits, $\theta \sim Unif\,[0,1]\,,\, c = 0$.

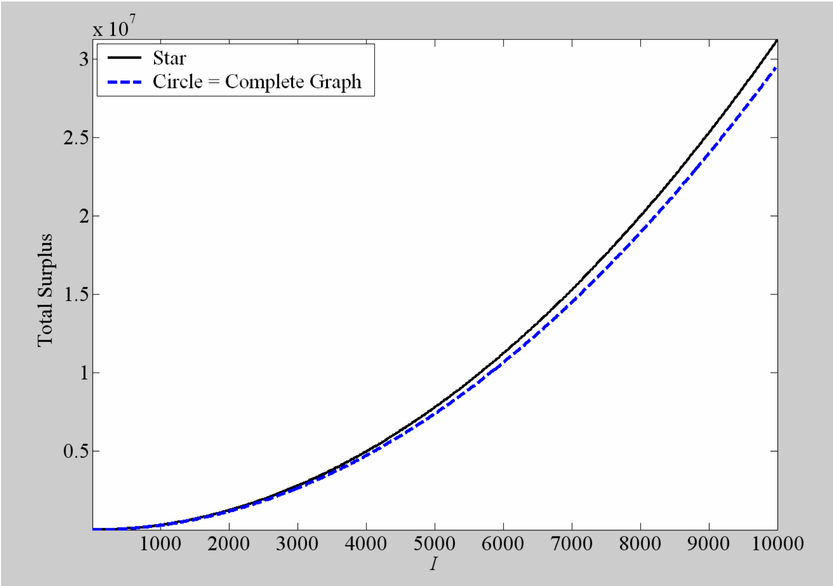


Figure 5: Compensated total surplus, $\theta \sim Unif\,[0,1]\,,\, c = 0.$

We can read from figure (4) that the firm is worse off in the compensated star. The asymmetric network topology constrains the firm as it has to leave more surplus to the centre by setting a relatively lower price. As a result, the consumer surplus is higher in the compensated star than in the circle or the complete graph. Total surplus is higher in the compensated star, since higher consumer surplus dominates lower profits. Yet, the centre amasses the surplus at the expense of the peripheral consumers and the firm. Hence, there is a misalignment between the socially optimal and the monopoly-preferred network topology. This is seen by comparing figures (4) and (5) .

Remark 23 *Misalignment of private and social preferences for network topology:*
(i) the firm prefers the symmetric compensated network topology.
(ii) total surplus is maximised in the asymmetric compensated star.

Remark 23 does not agree with the results under perfect information. The total surplus and monopoly profits were both maximised in the complete graph, and minimised in the star, under perfect information. So, the social and private preferences over the network topology were aligned.

The bias in favour of the centre in the star raises the question whether the firm could benefit by price discriminating with respect to the network location. With price discrimination, the active BNE probability system is

$$\begin{aligned} \pi_C &= 1 - F\left(\min\left\{\theta^+, \frac{p_C}{(I-1)\,\pi}\right\}\right) \\ \pi &= 1 - F\left(\min\left\{\theta^+, \frac{p}{\pi_C}\right\}\right) \end{aligned}$$

where p_C is the price for the centre and p for the periphery. The firm maximises expected profits $\mathbb{E}\,(V) = \pi_C\,[p_C\,(\pi_C, \pi) - c] + (I-1)\,\pi\,[p\,(\pi_C, \pi) - c]$ by choosing $(\pi, \pi_C)\,.$

For zero unit costs $c = 0$, the optimal probabilities are

$$\pi_C^* = \pi^* = \frac{2\theta^+}{3\left(\theta^+ - \theta^-\right)}.$$

Proposition 24 *Price discrimination with respect to network location removes the bias in favour of the centre.*

The firm of course captures a larger share of the value generated in the network by price discriminating. In the case with $\theta \sim Unif\,[0,1]$, $c = 0$, and compensated networks, it is straightforward to compute that price discrimination increases the firm's profits to the same level as in the compensated symmetric networks. Respectively, total consumer surplus falls to the level in symmetric networks. There is an efficiency loss due to price discrimination as the total surplus is reduced.

5 Conclusions

We have analysed the monopoly pricing of social goods when the market is characterised by buyers' social relations. Our model is a stylised version of coordination goods, such as mobile phones, for which the buyers' social relations determine the patterns of usage and consequently the demand for the good. We have shown that in markets where social relations are important, the parametric approach, used in the conventional network externalities models, falls short and needs to be refined. In particular, the implicit assumption of a completely connected graph that does away all topological asymmetries can result in a serious overestimation of the strength of the network effects. Consequently, both the achievable monopoly rents and the total surplus generated in the market are exaggerated. In addition, an asymmetric network topology induces distributional effects as certain consumers benefit at the expense of others from the monopoly pricing compared to a case with a symmetric network topology.

Critical consumers who have important connections capture a higher surplus compared to the more peripheral agents. A critical position is either due to a central network location (network topology) or due to important neighbours (type configuration). Under perfect information, critical positions exist in symmetric and asymmetric networks, but they always depend on the type configuration and the level of heterogeneity between the consumers. The critical positions in symmetric networks are eliminated once the consumers' types are private information, because the type configuration cannot affect the consumers' decision making. In contrast, the topologically central consumers in asymmetric networks always capture higher surplus than the peripheral consumers.

Under perfect information, the monopoly's pricing strategy depends on the network size and topology, on the heterogeneity of the consumers and their type configuration on the network. When the heterogeneity is high and the high types are clustered, the firm can charge a high price from them and exclude the low types. Profits increase as heterogeneity increases, but only if the high types are clustered. If the high types are scattered in the network, the low types' participation is needed, and therefore, the firm does not benefit from increased heterogeneity.

Asymmetric information removes the role of type configuration, and the network size and topology remain the only network-specific parameters affecting the optimal price. The monopoly price is increasing in the number of links. Higher heterogeneity equals higher uncertainty, which reduces consumer surplus in all network topologies. The firm cannot identify those networks with clusters of high types, where it would benefit from higher consumer heterogeneity in the way it could under perfect information. Therefore, higher heterogeneity reduces also profits.

In asymmetric networks, the firm sets a price that guarantees a higher probability to buy for the topologically central consumers under asymmetric information. As the size of the periphery increases, the centre becomes relatively more important, while an individual peripheral consumer becomes relatively less important from the whole network's perspective. An additional peripheral consumer increases the expected utility of the centre. A peripheral consumer is not directly affected by the additional consumer, however, increased price decreases the peripheral consumer's expected utility.

When we compare the compensated networks under asymmetric information, we see that the star is the social optimum, but the firm prefers a symmetric network. If the firm is allowed to price discriminate with respect to network location, its profits equal the level it obtains in the symmetric networks. This has a social cost, because price discrimination reduces total surplus as consumer surplus drops more than the profits increase.

We made a relatively strong assumption that the size and topology of the underlying social network are common knowledge. If we assume that the social relations are private information, as in Sundararajan (2005), the consumers have to take expectations on the sizes of their neighbours' neighbourhoods, neighbours' neighbours' neighbourhoods, and so on. On the other hand, if the firm observes nothing, it applies the same expectations on all neighbourhoods. The complexity level may not be too much increased for networks that present some regularity, suggesting an interesting area for future research.

Our model enables a number of interesting extensions.[15] We assumed that each link generates equal value independent of the interaction partner. Utility could, however, be dependent on the interaction partner as well, so that the value of a link is the random variable instead of the consumer type. We have focused on the static properties of social networks, and an obvious extension would be to expand the model in time. A multiperiod model would shed light on the optimal price paths and how the firm uses the network to diffuse information about the new device. This could be done in conjunction with an extension to richer forms of social networks. Finally, it would also be interesting to understand how the firm could use two-part tariffs for screening.

6 References

Arthur WB, 1989. Competing Technologies, Increasing Returns, and Lock-In by Historical Events. Economic Journal, 99(394), 116-131.

Banerji A, Dutta B, 2005. Local Network Externalities and Market Segmentation. Warwick Economic Research Papers 725.

Bensaid B, Lesne J-P, 1996. Dynamic Monopoly Pricing with Network Externalities. International Journal of Industrial Organization, 14(6), 837-855.

Cabral LMB, Salant DJ, Woroch GA, 1999. Monopoly Pricing with Network Externalities. International Journal of Industrial Organization, 17(2), 199-214.

Carlsson H, van Damme E, 1993. Global Games and Equilibrium Selection. Econometrica, 61(5), 989-1018.

[15] See Sääskilahti (2005b pp.115-120) for a more elaborate discussion on possible extensions.

David PA, 1985. Clio and the Economics of QWERTY. American Economic Review, 75(2), 332-337.

De Palma A, Leruth L, 1996. Variable Willingness to Pay for Network Externalities with Strategic Standardization Decisions. European Journal of Political Economy, 12(2), 235-251.

Ellison G, 1993. Learning, Local Interaction, and Coordination. Econometrica, 61(5), 1047-1071.

Farrell J, Saloner G, 1985. Standardization, Compatibility, and Innovation. RAND Journal of Economics, 16(1), 70-83.

Fudenberg D, Tirole J, 1991. Game Theory. MIT Press: Cambridge Massachusetts.

Goyal S, Konovalov A, Moraga-González JL, 2003. Hybrid R&D. Tinbergen Institute Discussion Paper TI2003-041/1.

Herrendorf B, Valentinyi Á, Waldman R, 2000. Ruling Out Multiplicity and Indeterminancy: The Role of Heterogeneity. Review of Economic Studies, 67(2), 295-307.

Jackson MO, 2005. A Survey of Models of Network Formation: Stability and Efficiency. In: Demange G, Wooders M (Eds), Group Formation in Economics: Networks, Clubs and Coalitions, Cambridge University Press: Cambridge United Kingdom.

Katz ML, Shapiro C, 1985. Network Externalities, Competition, and Compatibility. American Economic Review, 75(3), 424-440.

Kessing SG, Nuscheler R, 2006. Monopoly Pricing with Negative Network Effects: The Case of Vaccines. European Economic Review, 50(4), 1061-1069.

Kranton RE, Minehart DF, 2001. A Theory of Buyer-Seller Networks. American Economic Review, 91(3), 485-508.

Lee IH, Valentinyi A, 2000. Noisy Contagion without Mutation. Review of Economic Studies, 67(1), 47-56.

Mason R, 2000. Network Externalities and the Coase Conjecture. European Economic Review, 44(10), 1981-1992.

Mason R, Valentinyi Á, 2003. Independence and Heterogeneity in Games of Incomplete Information. University of Southampton Discussion Papers in Economics and Econometrics 0307.

Mason R, Valletti TM, 2001. Competition in Communication Networks: Pricing and Regulation. Oxford Review of Economic Policy, 17(3), 389-415.

Morris S, 2000. Contagion. Review of Economic Studies, 67(1), 57-78.

Morris S, Shin HS, 2003. Global Games: Theory and Applications. In: Dewatripont M, Hansen LP, Turnovsky SJ (Eds), Advances in Economics and Econometrics, Theory and Applications, Eight World Congress, 1, 56-114, Cambridge University Press: Cambridge United Kingdom.

Sundararajan A, 2005. Local Network Effects and Network Structure. Center for Digital Economy Research Working Paper CeDER-05-02.

Swann GMP, 2002. The Functional Form of Network Effects. Information Economics and Policy, 14(3), 417-429.

Sääskilahti P, 2005a. Buying Decision Coordination and Monopoly Pricing of Network Goods. HECER Discussion Papers, 64.

Sääskilahti P, 2005b. Essays on the Economics of Networks and Social Relations, A-263. Helsinki School of Economics: Helsinki Finland.

Tucker C, 2006. Interactive, Option-Value and Domino Network Effects in Technology Adoption. Mimeo.

Vives X, 2001. Oligopoly Pricing. MIT Press: Cambridge Massachusetts.
Young HP, 1998. Individual Strategy and Social Structure, an Evolutionary Theory of Institutions. Princeton University Press: Princeton New Jersey.

7 Appendix

7.1 Perfect information examples

Example 25 (Complete graph) *Consider a four consumer complete graph with types $\theta_1 < \theta_2 < \theta_3 < \theta_4$, and $c < 3\theta_1$ so that costs do not constrain the firm's decisions, and focus on the maximal NE. Complete network is feasible only if $p \leq 3\theta_1$. Partial network with three buyers is feasible if $\max\{3\theta_1, \theta_3\} < p \leq 2\theta_2$, and with two buyers if $\max\{3\theta_1, 2\theta_2\} < p \leq \theta_3$. The firm's profits are $V_4 = 4(3\theta_1 - c)$, $V_3 = 3(2\theta_2 - c)$, and $V_2 = 2(\theta_3 - c)$ respectively. Depending on the relative values of θ_1, θ_2 and θ_3 (the highest type does not matter), the firm chooses between a complete network and a partial network of either 2 or 3 buyers. The optimal activity level is*
(i) complete network if $V_4 > V_3$ and $V_4 > V_2$, i.e. $\theta_1 > \max\left\{\frac{1}{2}\theta_2 + \frac{1}{12}c, \frac{1}{6}\theta_3 + \frac{1}{6}c\right\}$.
(ii) 3-buyer network if $V_3 > V_4$ and $V_3 > V_2$, i.e. $\theta_2 > \max\left\{2\theta_1 - \frac{1}{6}c, \frac{1}{3}\theta_3 + \frac{1}{6}c\right\}$.
(iii) 2-buyer network if $V_2 > V_4$ and $V_2 > V_3$, i.e. $\theta_3 > \max\left\{6\theta_1 - c, 3\theta_2 - \frac{1}{2}c\right\}$.

A comparison of profits suggests that the complete network is the optimal activity level, if the consumers are homogenous in regard to their types, as the firm benefits from high sales volumes. Partial 3-buyer network is chosen when the middle types θ_2 and θ_3 are close together, but significantly higher than θ_1. 2-buyer network is chosen when there is a large difference between the two lowest and the two highest types. If the consumers are heterogeneous, it pays off to exclude the low types by charging a high price.

Example 26 (Circle) *Consider a circle with four consumers with types $\theta_1 < \theta_2 < \theta_3 < \theta_4$, and $c < 2\theta_1$ so that costs do not interfere pricing, and focus on the maximal NE. There are two cases that yield different results. In the case A, the high types θ_3 and θ_4 are neighbours (a circle where θ_1 has neighbours θ_2 and θ_3, and where his neighbours are θ_2 and θ_4 yield identical results). In the case B, they are not. The network structure sets limits to the firm's choices in the circle B, because the consumer θ_2 located between θ_3 and θ_4, holds a potentially critical position. Any non-empty NE must include him. In both cases, complete network occurs if $2\theta_1 \geq p$, and the firm's profits are $V_4 = 4(2\theta_1 - c)$. Partial network with three buyers is feasible in the circle A if $2\theta_1 < p \leq \theta_2$, and in the circle B if $2\theta_1 < p \leq \min\{2\theta_2, \theta_3\}$. Partial network with two buyers is feasible in the circle A if $\max\{2\theta_1, \theta_2\} < p \leq \theta_3$. Two buyer network is always dominated by the other structures in the circle B.*

The firm chooses the complete network only when the consumers' types are sufficiently close together.
(ia) Complete network in A if $V_4 > V_3^A$ and $V_4 > V_2^A$, i.e. $\theta_1 > \max\left\{\frac{3}{8}\theta_2 + \frac{1}{8}c, \frac{1}{4}\theta_3 + \frac{1}{4}c\right\}$.
(ib) Complete network in B if $V_4 > V_3^B$, i.e. $\theta_1 > \frac{3}{8}\min\{2\theta_2, \theta_3\} + \frac{1}{8}c$.

The firm chooses a three buyer network in both cases, if the lowest type is significantly lower, and the other consumers' types are not too different from each other. Type configuration is important, as the firm benefits if the high types (θ_3 and θ_4) are dispersed in the network. We have $V_3^A < V_3^B$ always. High types support the purchases of their common neighbour θ_2, so that the type configuration relaxes firm's pricing constraint.
(iia) 3-buyer network in A if $V_3^A > V_4$ and $V_3^A > V_2^A$, i.e. $\theta_2 > \max\left\{\frac{8}{3}\theta_1 - \frac{1}{3}c, \frac{2}{3}\theta_3 + \frac{1}{3}c\right\}$.

(iib) 3-buyer network in B if $V_3^B > V_4$, i.e. $\min\{2\theta_2, \theta_3\} > \frac{8}{3}\theta_1 - \frac{1}{3}c$.

The firm chooses a 2-buyer network in the circle A when the two highest types are significantly higher compared with the two lowest types. In the circle B, a 2-buyer network is always dominated either by the complete network or the 3-buyer network.

(iii) 2-buyer network in A if $V_2^A > V_4$ and $V_2^A > V_3^A$, i.e. $\theta_3 > \max\left\{4\theta_1 - c, \frac{3}{2}\theta_2 - \frac{1}{2}c\right\}$.

When the difference in types of the two highest and the two lowest types grows large, so that $\theta_3 > \max\left\{4\theta_1 - c, \frac{3}{2}\theta_2 - \frac{1}{2}c\right\}$, the firm strictly prefers the 2-buyer network. In the circle A, this causes no problems to the firm as it can exclude θ_1 and θ_2. However, in the circle B, segregation between the two highest and the two lowest types is blocked by the type configuration. It is forced to sell to three consumers, which yields lower profits when $\theta_2 < \frac{1}{3}\theta_3 + \frac{1}{6}c$. In this case, the firm prefers the case where θ_3 and θ_4 are neighbours (circle A). Respectively, if $\theta_3 < \max\left\{4\theta_1 - c, \frac{3}{2}\theta_2 - \frac{1}{2}c\right\}$ holds, then the firm is better off if the high types are dispersed in the network (circle B), as they support the purchases of their common neighbour θ_2.

In general, when the consumers are homogenous, the firm prefers the complete network, and the higher the heterogeneity is, the lower is the activity level on the network in the NE. Due to the incompletely connected circular network structure, this relation is conditional on type configuration, since the segregation between the low and the high types may be blocked.

Example 27 (Star) *Consider a four consumer star with a centre θ_C and three peripheral agents. Let the peripheral consumers' types be $c < \theta_1 < \theta_2 < \theta_3$, and focus on the maximal NE.*

(i) Complete network is optimal if $\begin{cases} \min\{\theta_1, 3\theta_C\} > \frac{3}{4}\left(\min\{\theta_2, 2\theta_C\}\right) + \frac{1}{4}c \\ \min\{\theta_1, 3\theta_C\} > \frac{1}{2}\left(\min\{\theta_3, \theta_C\}\right) + \frac{1}{2}c \end{cases}$

(ii) 3-buyer network is optimal if $\begin{cases} \min\{\theta_2, 2\theta_C\} > \frac{4}{3}\left(\min\{\theta_1, 3\theta_C\}\right) - \frac{1}{3}c \\ \min\{\theta_2, 2\theta_C\} > \frac{2}{3}\left(\min\{\theta_3, \theta_C\}\right) + \frac{1}{3}c \end{cases}$

(iii) 2-buyer network is optimal if $\begin{cases} \min\{\theta_3, \theta_C\} > 2\left(\min\{\theta_1, 3\theta_C\}\right) - c \\ \min\{\theta_3, \theta_C\} > \frac{3}{2}\left(\min\{\theta_2, 2\theta_C\}\right) - \frac{1}{2}c \end{cases}$

From (i)-(iii) we see that higher heterogeneity in θ supports partial networks, whereas if the consumers are sufficiently homogeneous in terms of θ, the firm chooses a complete network. The centre's topologically critical position is emphasised, as the firm must guarantee his participation. If the centre's type is low, the firm may be forced to price low although segregation between the high and the low types might be otherwise desired.

7.2 Stability of equilibria under asymmetric information

We provide checks for BNE stability against small perturbations, based on a Nash tâtonnement process (see e.g. Fudenberg and Tirole 1991).

7.2.1 Symmetric networks

The BNE condition (3) can be deconstructed into two equations $\pi = z$ (the 45-degree line) and $z = 1 - F\left(\min\left\{\theta^+, \frac{p}{n\pi}\right\}\right)$, which must be equal in the equilibrium. The deconstructed

BNE condition is

$$\begin{cases} \pi = z \\ z = 1 - F\left(\frac{p}{n\pi}\right) \end{cases} \quad \text{for active equilibria}$$

$$\begin{cases} \pi = z \\ z = 0 \end{cases} \quad \text{for the empty network.}$$

The condition for asymptotic stability is $\left|\frac{\partial \pi}{\partial z}\right| \left|\frac{\partial z}{\partial \pi}\right| < 1$. We have for the maximal BNE $\left|\frac{\partial \pi}{\partial z}\right| \left|\frac{\partial z}{\partial \pi}\right|_{\pi=\pi_+^*} = \frac{1}{2}$, and the equilibrium is stable. For the lower positive BNE we have $\left|\frac{\partial \pi}{\partial z}\right| \left|\frac{\partial z}{\partial \pi}\right|_{\pi=\pi_-^*} = 2$, which indicates that the equilibrium is unstable. The empty network is also stable since $\left|\frac{\partial \pi}{\partial z}\right| \left|\frac{\partial z}{\partial \pi}\right|_{\pi=0} = 0$.

7.2.2 Star

We study only the case $c = 0$, $\theta^+ = 1$, and $\theta^- = 0$, discussed in the main text. In the region where an active BNE can exist, the BNE conditions (10) can be written as

$$\begin{aligned} \pi_C &= 1 - F\left(\frac{p}{(I-1)\,\pi}\right) \\ \pi &= 1 - F\left(\frac{p}{\pi_C}\right) \end{aligned}$$

Since the model does not give out explicit equilibrium values that would be easily applied to the stability check $\left|\frac{\partial \pi}{\partial \pi_C}\right| \left|\frac{\partial \pi_C}{\partial \pi}\right| < 1$, we resort to a numerical test. When the equilibrium values π_C^*, π^* and $p\left(\pi_C^*, \pi^*\right)$ are substituted into the stability equation, we can plot the curve $s = \left|\frac{\partial \pi}{\partial \pi_C}\right| \left|\frac{\partial \pi_C}{\partial \pi}\right|$ for different values of I. It turns out that s remains below one for $I \geq 3$, and it approaches zero as I grows very large. Hence, the BNE is stable. The other BNE, namely the empty network, is obviously a stable one as well.