Contest design and optimal endogenous entry

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Contest Design and Optimal Endogenous Entry*

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Abstract

This paper derives the effort-maximizing contest rule and the optimal endogenous entry in a context where potential participants bear fixed entry costs. The organizer is allowed to design the contest under a fixed budget with two strategic instruments: he sets the value of the prize purse, and arranges a monetary transfer (entry subsidy or fee) for each participating contestant. In other words, the budget can either be used to subsidize participation or an entry fee can be charged to fund the prize purse. The results show that the optimally designed contest attracts exactly two participating contestants in its unique subgame perfect equilibrium (when there is a positive fixed entry cost) and extracts all the surplus from participating contestants. The study also shows that the direction and amount of the monetary transfer depend on the magnitude of the entry cost: the contest organizer subsidizes entry when contestants bear substantial entry costs, but charges an entry fee to fund the prize purse whenever the entry cost is sufficiently low.

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1 Introduction

A contest is a situation in which economic agents expend costly and non-refundable resources in order to win a limited number of prizes. Numerous academic surveys and anecdotal accounts have shown that a wide variety of competitive activities can be viewed as winner-take-all contests. These include research tournaments, political lobbying, sports races and promotion tournaments in a firm's internal labor market. It has been widely recognized in the literature that the incentive to win and the resultant behavior of contestants depend largely on the competitive environment as defined by the rules of the contest. Therefore, a forward-looking organizer must set the rules of a contest strategically such that the contest structure best serves his interests.

While a contest organizer may have diverse objectives, enormous academic resources have been devoted to the design of contests that maximize the effort that has to be exerted by contestants (see Baye, Kovenock and de Vries, 1993, Gradstein and Konrad, 1999, Rosen, 1986 etc.) This paper follows in the same vein and investigates the design of the effort-maximizing contest. It is assumed that to enter the contest, potential contestants must bear a fixed sunk cost and that they must bear the cost of productive efforts that determine the probability of them winning the prize. The traditional modeling approach assumes that (i) all invested resources contribute to contestants' productive efforts and help increase their likelihood of success; and (ii) contestants choose their effort outlays rationally in order to increase the likelihood of winning. In reality, however, contestants often bear additional costs merely to participate which do not relate directly to winning. To provide an analogy of this point, while an air ticket paves the way for American tennis star Venus Williams to arrive at the courts of the Australian Open, it does not contribute to her winning the championship. Similarly, a research company may have established the necessary laboratory equipment and developed the project-specific knowledge required to participate in an innovation tournament, but its success depends largely on its subsequent efforts and the value of its creative input.

It is clear from the above examples that a potential contestant would join a contest if and only if the expected payoffs from participation are higher than the entry costs. Unlike the typical setting where there is generally a given pool of active contestants, in the context of this paper, the number of participating contestants is endogenously determined by the contest structure.[1,2]

The model that is proposed in this paper pertains to the induction of maximal total effort from a fixed pool of potential contestants, with the contest organizer being financially bound by a fixed budget. The contest organizer is allowed to design the competition with two strategic instruments: the value of the (unique) winner's purse and a direct monetary transfer to each participating contestant. Conventional wisdom suggests that (i) a larger number of contestants leads to greater total effort; and (ii) a more generous winner's purse causes each contestant to exert more effort. These insights, however, lead to a paradoxical situation where no clear implications can be provided to the contest organizer with a limited budget. The monetary transfer may be an entry subsidy aimed at mitigating contestants' entry costs. For instance, the U.S. Department of Defense (DoD) substantially subsidizes military research and development (R&D) activities conducted by contractors competing for procurement contracts.[3] An entry subsidy encourages more participation on the one hand, while absorbing funds that would otherwise be used to award the winner on the other. It is then called into question whether an entry subsidy is a desirable way to promote the effort outlay. By the way of contrast, when the monetary transfer between the contest organizer and each participating contestant moves in the opposite direction instead, the entry subsidy turns into an entry fee. This phenomenon is widely observed in many real-life tournament settings.[4] Despite the fact that entry fees discourage participation, the revenue earned nevertheless enriches the winner's purse and promotes competition among participating contestants. Hence, the direction and amount of the optimal monetary transfer have yet to be identified, and the desirable number of participants in the contest remains

[1] An imperfectly discriminatory contest with concave contest technology does not involve endogenous entry if no fixed cost is incurred upon entry. The internal equilibrium would guarantee that all participating contestants would receive positive expected payoffs.

[2] Exceptions are explored in the seminal works of Baye, Kovenock and de Vries (1993) and Fullerton and McAfee (1999).

[3] The DoD's subsidies for independent military R&D projects have been empirically documented by Lichtenberg (1988).

[4] One such example is the National Scholastic Surfing Association (NSSA) National Tournament, where an entry fee applies to participating teams and individuals.

foggy.

The properties of the optimally designed contest can be illustrated by using a three-stage model. Consider a fixed pool of identical potential contestants who may choose to compete for a unique prize. In the first stage of the contest, the organizer announces the value of the winner's purse, V, as well as the amount of money to be transferred, S, to each participating contestant.[5] In the second stage, the potential contestants are informed about the rules of the contest as indicated by the contest organizer's strategy pair (V, S). They then make their entry decisions sequentially with full knowledge of the number of existing participants and incur a fixed participation cost, $C > 0$ for entering the contest. In the third stage, all participants choose their effort outlays simultaneously, and a unique winner is found through a stochastic selection procedure. The contest (V, S) needs to be **feasible** in the sense that the prize V cannot be greater than the total resources available to the contest organizer (including the revenue collected from the entry fees).

The main findings of this analysis are summarized as follows.

1. "*It takes (exactly) two to tango*": the optimally designed contest induces exactly two contestants to participate, regardless of the contest technology.

2. The contest organizer charges an entry fee when the participation cost is relatively low, and awards an entry subsidy when the participation cost is sufficiently high.

In the specific case when $C = 0$, the contest organizer does not need to restrict the number of participating contestants to exactly two in order to maximize the contestants' efforts. There are a number of equilibria in which an optimal contest can occur, involving differing entry fees, prize purse, and the number of participants required for equilibrium. All these possibilities will induce the same level of total effort, while fully utilizing the budget of the contest organizer.

This paper is inspired by and linked closely to the seminal works of Baye, Kovenock and de Vries (1993) and Fullerton and McAfee (1999). Baye, Kovenock and de Vries (1993) show that a (rent-seeking) revenue-maximizing contest organizer may strategically shortlist two finalists from a pool of candidates to participate in a contest. Fullerton and McAfee (1999)

[5] A participating contestant receives an entry subsidy if $S > 0$, but pays an entry fee if $S < 0$.

consider the optimal design of a research tournament, in which the first stage involves an all-pay auction for entry, with only the two highest bidders being allowed entry into the innovation race.

This section of the paper has introduced the topic of contest design. Section 2 sets up the model, while in Section 3, the formal analysis is presented and the results are briefly discussed. Concluding remarks are presented in Section 4.

2 Preliminaries

This section considers the design of a winner-take-all contest within a three-stage framework with endogenous entry.

The contest organizer begins with a fixed budget of Γ_0 with which to fund a contest. A fixed pool of $M(\geq 3)$ identical risk-neutral potential contestants demonstrate interest in the contest. In the first stage, the organizer announces the rules, and commits to a prize purse $V(\geq 0)$ and a direct monetary transfer $S \in \Re$ to each participating contestant. For the ease of notation, a contest will be denoted by (V, S), which also represents the contest organizer's strategy. In the second stage, contestants decide whether or not to participate. It is assumed that they enter the contest sequentially, and that they are fully aware of the number of current participants.[6] Each contestant incurs a fixed participation cost of $C > 0$ upon entry, but is either rewarded with an entry subsidy S when $S > 0$, or is charged an entry fee $|S|$ when $S < 0$. In the third stage, all contestants simultaneously submit their effort entries .

In the event that there are no contestants, the organizer simply keeps the prize. The set of contestants is denoted by Ω_N when $N(\geq 1)$ of the $M(\geq 3)$ potential contestants participate in the contest (V, S). In the event that there is only one contestant, this contestant automatically receives the prize V, regardless of the amount of effort exerted.

When there are at least two participants in a contest, the probability that a contestant

[6] Sequential entry and complete information ensure that potential contestants play pure strategies (0 or 1 probability of entry) in the entry stage of the game.

$i \in \Omega_N$ wins the unique prize is

$$p_i(e_i, \mathbf{e}_{-i}; \Omega_N) = \frac{f(e_i)}{f(e_i) + \sum\limits_{j \in \Omega_N, j \neq i} f(e_j)},^{7} \quad (1)$$

where e_i is i's effort and $\mathbf{e}_{-i}$ denotes the effort vector of the other participating contestants.[8] The impact function $f(\cdot)$ represents the technology of the contestants. To guarantee the existence of a unique symmetric pure-strategy equilibrium, $f(\cdot)$ is assumed to be strictly increasing and weakly concave, with $f(0) = 0$ and $f'(0) > 0$. We define $H(\cdot) \equiv \frac{f(\cdot)}{f'(\cdot)}$. Due to the concavity of $f(\cdot)$, $H^{-1}(\cdot)$ must be strictly increasing. In addition, $\frac{dH^{-1}(x)}{dx} \in (0, 1)$. If all the participating contestants exert zero effort, it is assumed that the prize will be given away at random.

Assume that the cost of effort equals the effort itself. A potential contestant can then expect to receive a payoff of

$$\pi_i(e_i, e_{-i}; \Omega_N, V, S) = p_i(e_i, \mathbf{e}_{-i}; \Omega_N)V - e_i + S - C, \quad (2)$$

if he participates and exerts effort e_i, provided that the efforts of the other participating contestants are $\mathbf{e}_{-i}$. Every participating contestant will choose the level of effort to maximize his expected payoff.

Since all $N(\geq 1)$ participating contestants are identical, every individual contestant in symmetric equilibrium has the equilibrium probability $\frac{1}{N}$ of receiving the prize, and receives an equilibrium payoff of $\pi(N, V, S) = \frac{1}{N}V - e(N, V, S) + S - C$, where $e(N, V, S)$ denotes the equilibrium effort as a function of N, V and S. The results below indicate the participants' equilibrium individual effort and equilibrium payoff, which can be established through standard techniques.

Lemma 1 *In the unique symmetric Nash equilibrium of a contest (V, S) with N participating*

[7] This model, together with a ratio-form success function, can be applied to a wide variety of contest settings. For instance, Baye and Hoppe (2003) establish strategic equivalence between Tullock rent-seeking contests and research tournaments, as well as patent races.

[8] We assume that a nonparticipant will not be awarded the prize.

contestants, where $N \geq 1$*, each contestant exerts an effort of*

$$e(N,V,S) = \begin{cases} 0 & \text{if } N = 1, \\ H^{-1}(\frac{V}{N}(1-\frac{1}{N})) & \text{if } N \geq 2, \end{cases} \tag{3}$$

and each contestant receives an expected payoff of

$$\pi(N,V,S) = \begin{cases} V+S-C & \text{if } N = 1, \\ \frac{1}{N}V - H^{-1}(\frac{V}{N}(1-\frac{1}{N})) + S - C & \text{if } N \geq 2. \end{cases} \tag{4}$$

Based on Lemma 1, the equilibrium number of entrants in contest (V,S) is characterized in the following lemma.

Lemma 2 *A contest* (V,S) *attracts a unique number of* $N(V,S) = \underset{\{\pi(N,V,S)\geq 0, 1\leq N\leq M\}}{\arg\max}\{N\}$ *contestants to participate if* $\pi(1,V,S) \geq 0$*, since* $\pi(N,V,S)$ *strictly decreases with* $N(\geq 1)$*. If* $\pi(1,V,S) < 0$*, the contest will have no participant.*

Proof. Clearly, $\pi(1,V,S) > \pi(2,V,S)$. To show that $\pi(N,V,S)$ strictly decreases with N for any $N \geq 2$, all that is necessary is that function $g(x) = Vx - H^{-1}(x(1-x)V)$ is increasing over the interval $(0, 1/2]$. Note that $\frac{dg(x)}{dx} = V - \frac{dH^{-1}(y)}{dy}|_{y=H^{-1}(x(1-x)V)}(1-2x)V \geq 0$ as $\frac{dH^{-1}(y)}{dy} \in (0,1)$. Since contestants Ω_N enter the contest (V,S) if and only if $\pi(N,V,S) \geq 0$, $N(V,S)$ is the unique equilibrium number of entrants in contest (V,S) if $\pi(1,V,S) \geq 0$. It is then obvious that no one participates in the contest if $\pi(1,V,S) < 0$. ■

The contest organizer has a total budget of Γ_0 available from his own pocket. He has the freedom either to split the budget between the prize purse and the payment of entry subsidies up to the budget limit, or to fund the prize purse using the revenue from the entry fees. Such flexibility in resource allocation represents one of the main features of the analysis in this paper.

Definition 1 *A contest design* (V,S) *is* ***feasible*** *if and only if*

$$0 \leq V \leq \Gamma_0 - N(V,S)S. \tag{5}$$

The feasibility condition (5) states that the prize purse cannot exceed the total resources available to the contest organizer. We define $E \equiv \sum_{i \in \Omega_{N(V,S)}} e_i$, where $\Omega_{N(V,S)}$ is the set of equilibrium participants who enter the contest (V, S). In this paper, it is assumed that the objective of the contest organizer is to optimally design the contest so as to induce the highest total amount of effort from the participating contestants. In other words, this is a search for the optimal contest (V^*, S^*) that maximizes the total effort exerted by the endogenously-determined number of participating contestants.

3 Analysis

The following is assumed to make the analysis more interesting.

Assumption 1 $C \leq \frac{\Gamma_0}{2}$.

The prize is assumed to be automatically awarded if there is only one contestant. This is because a contestant would exert zero effort if he turns out to be the unique participant. Therefore, a contest rule cannot create an optimal situation if less than two contestants participate. Assumption 1 guarantees that the contest organizer can induce the entry of at least two participants by providing an entry subsidy, as shown by the following Lemma.

Lemma 3 *A **feasible** contest that induces at least two potential contestants to participate exists if and only if Assumption 1 holds.*

Proof. Let S_0 denote the solution of

$$\pi(2, \Gamma_0 - 2S, S) = \frac{\Gamma_0}{2} - H^{-1}(\frac{\Gamma_0 - 2S}{4}) - C = 0. \tag{6}$$

Sufficiency First, note that $\frac{\Gamma_0}{2} - H^{-1}(\frac{\Gamma_0 - 2S}{4}) - C$ increases with S. Second, when $S = \frac{\Gamma_0}{2}$, $\pi(2, \Gamma_0 - 2S, S) = \frac{\Gamma_0}{2} - C \geq 0$ based on Assumption 1. Third, when $S = \frac{\Gamma_0}{2} - 2H(\frac{\Gamma_0}{2})$, $\pi(2, \Gamma_0 - 2S, S) = -C < 0$. Thus, there exists a unique solution $S_0 \in (\frac{\Gamma_0}{2} - 2H(\frac{\Gamma_0}{2}), \frac{\Gamma_0}{2})$ for equation (6).

Set $S = S_0$ and $V = \Gamma_0 - 2S_0 > 0$. Since $\pi(2, \Gamma_0 - 2S_0, S_0) = 0$, we have $N(\Gamma_0 - 2S_0, S_0) = 2$ by Lemma 2, which indicates that two contestants will participate in the contest $(\Gamma_0 - 2S_0, S_0)$. In addition, contest $(\Gamma_0 - 2S_0, S_0)$ is feasible according to Definition 1. It has thus been shown that Assumption 1 represents a sufficient condition for the existence of a feasible contest that induces at least two contestants to participate.

Necessity We attempt to prove that a feasible contest that induces at least two participants would not exist if Assumption 1 does not hold. We prove it by contradiction. Suppose the contrary that $C > \frac{\Gamma_0}{2}$.

Step 1: A feasible contest that attracts exactly two participants does not exist.

Suppose it is possible for exactly two contestants to participate. This means that there must exist a feasible contest (V, S) with $0 \leq V \leq \Gamma_0 - 2S$, such that $\pi(2, V, S) \geq 0$ holds. It implies the following: firstly, $S \leq \frac{\Gamma_0}{2}$ since $0 \leq \Gamma_0 - 2S$; secondly, $\pi(2, V, S) \leq \pi(2, \Gamma_0 - 2S, S)$ since $V \leq \Gamma_0 - 2S$. Because $\pi(2, \Gamma_0 - 2S, S)$ increases with S, $S = \frac{\Gamma_0}{2}$ would maximizes $\pi(2, \Gamma_0 - 2S, S)$ with $\pi(2, 0, \frac{\Gamma_0}{2}) = \frac{\Gamma_0}{2} - C$, which is strictly negative if $C > \frac{\Gamma_0}{2}$. This result conflicts with the existence of a feasible contest (V, S) that induces the entry of exactly two participants.

Step 2: A feasible contest that attracts more than two participants does not exist.

Since $\pi(N, V, S)$ decreases with $N(\geq 2)$, it follows that $\pi(N, V, S) \leq \pi(2, V, S)$ for any feasible contest (V, S) for $N > 2$. Thus, in any feasible contest (V, S), a participant would expect to receive a payoff $\pi(N, V, S) \leq \frac{\Gamma_0}{2} - C$, which is strictly negative as long as $C > \frac{\Gamma_0}{2}$. As a result, N would not be the equilibrium number of participants. This means that there exists no feasible contest that induces $N(> 2)$ contestants to participate.

In conclusion, we show that Assumption 1 also represents a necessary condition for the existence of a feasible contest that induces the participation of at least two contestants. ■

Clearly, when C is small, more than two potential contestants can be induced to participate. The contest organizer therefore has more freedom in term of the (desirable) number of participants he can attract. Lemma 4 and Lemma 5 characterize two intuitive necessary conditions for the optimal feasible contest. The formal proofs are laid out in the Appendix.

Lemma 4 *In the optimal feasible contest (V^*, S^*), every participating contestant breaks even, i.e.* $\pi(N(V^*, S^*), V^*, S^*) = 0$.

Lemma 5 *In the optimal feasible contest (V^*, S^*), the contest organizer must put all the resources available in the prize purse, i.e.,* $V^* = \Gamma_0 - N(V^*, S^*)S^*$.

From Lemmas 4 and 5, it follows that the effort-maximizing contest exhausts the resources available to the contest organizer, while causing all participating contestants to break even. As a result, the effort-maximizing contest (V^*, S^*) must satisfy $V^* = \Gamma_0 - N(V^*, S^*)S^*$ and $\pi(N(V^*, S^*), V^*, S^*) = 0$.

3.1 Main Results

Next, the optimal number of entrants $N(V^*, S^*)$ needs to be revealed. In the optimal contest (V^*, S^*), each contestant receives an equilibrium payoff of

$$\pi(N(V^*, S^*), V^*, S^*) = \frac{\Gamma_0 - N(V^*, S^*)S^*}{N(V^*, S^*)} - e(N(V^*, S^*), V^*, S) + S^* - C.$$

Thus,

$$\begin{aligned} & N(V^*, S^*)\pi(N(V^*, S^*), V^*, S^*) \\ = \; & (\Gamma_0 - N(V^*, S^*)S^*) - N(V^*, S^*)e(N(V^*, S^*), V^*, S^*) + N(V^*, S^*)S^* - N(V^*, S^*)C, \end{aligned}$$

which leads to

$$\begin{aligned} E &= N(V^*, S^*)e(N(V^*, S^*), V^*, S^*) \\ &= \Gamma_0 - N(V^*, S^*)\pi(N(V^*, S^*), V^*, S^*) - N(V^*, S^*)C. \end{aligned}$$

Combining Lemma 4, the following important fact can thus be established:

$$E = \Gamma_0 - N(V^*, S^*)C. \tag{7}$$

Note the importance of Equation (7). It states that in the optimally designed contest, the equilibrium total effort is given by the difference between the total budget of the contest organizer and the total entry costs incurred by participating contestants, regardless of the contest technology. In addition, the right-hand side of Equation (7) strictly decreases with $N(V^*, S^*)$, the equilibrium number of participating contestants, for any $N(V^*, S^*) \geq 2$. Hence, it can be deduced that the equilibrium efforts are bound from above by $\overline{E} = \Gamma_0 - 2C$. The following result is now ready to be established.

Theorem 1 *The unique optimal contest induces exactly two potential contestants to participate, and induces the total effort of* $\overline{E} = \Gamma_0 - 2C$.

Proof. Equation (7) shows that only a contest that attracts two contestants to participate may induce the total effort of $\overline{E}$.

Thus, it is only necessary to show that a feasible contest (V^*, S^*) exists that induces exactly two participants and satisfies the conditions given by Lemmas 4 and 5. To this end, it is necessary only to show that there exists an S^* that satisfies the following condition

$$\begin{aligned}\pi(2, \Gamma_0 - 2S^*, S^*) &= \frac{\Gamma_0}{2} - H^{-1}(\frac{\Gamma_0 - 2S^*}{4}) - C \\ &= (\frac{\Gamma_0}{2} - C) - H^{-1}(\frac{\Gamma_0 - 2S^*}{4}) = 0. \end{aligned} \tag{8}$$

The existence and uniqueness of such an S^* have been established in the proof of Lemma 3. ■

Theorem 1 shows that a unique optimal contest exists that maximizes the total amount of effort exerted in the contest. The optimal contest attracts exactly two contestants and induces the equilibrium total effort $\Gamma_0 - 2C$ regardless of the contest technology. The following theorem further characterizes the properties of (V^*, S^*).

Theorem 2 *The optimally designed contest awards a unique equilibrium prize purse of* $V^* = 4H(\frac{\Gamma_0}{2} - C)(> 0)$. *When* $C \leq \frac{\Gamma_0}{2} - H^{-1}(\frac{\Gamma_0}{4})$, *the contest organizer charges an entry fee of* $S^* = [\frac{\Gamma_0}{2} - 2H(\frac{\Gamma_0}{2} - C)](\leq 0)$ *to each contestant. When* $\frac{\Gamma_0}{2} - H^{-1}(\frac{\Gamma_0}{4}) < C < \frac{\Gamma_0}{2}$, *the contest organizer awards an entry subsidy of* $S^* = [\frac{\Gamma_0}{2} - 2H(\frac{\Gamma_0}{2} - C)](> 0)$ *to each contestant.*

Proof. Equation (8) implies $S^* = \frac{\Gamma_0}{2} - 2H(\frac{\Gamma_0}{2} - C)$. This leads to $V^* = \Gamma_0 - 2S^* = 4H(\frac{\Gamma_0}{2} - C)$. Thus $S^* \geq 0$ if and only if $\frac{\Gamma_0}{2} - 2H(\frac{\Gamma_0}{2} - C) \geq 0$. ■

It is worth pointing out that the critical value $\frac{\Gamma_0}{2} - H^{-1}(\frac{\Gamma_0}{4})$ represents an individual contestant's equilibrium surplus $\pi(2, \Gamma_0, 0)$ in a feasible contest $(V, S) = (\Gamma_0, 0)$ with two participating contestants. At least two contestants are willing to participate in the contest $(\Gamma_0, 0)$ when $C \leq \pi(2, \Gamma_0, 0)$. An entry fee can then be imposed to enhance the prize purse while maintaining sufficient participation (two contestants). On the other hand, when $C > \pi(2, \Gamma_0, 0)$, only one contestant is willing to participate in $(\Gamma_0, 0)$. Thus, an entry subsidy is required in this situation in order to induce sufficient participation.

It has been assumed thus far that the contest organizer attempts solely to maximize the amount of total effort exerted. However, a contest organizer may seek other objectives as well, such as maximizing the effort exerted by each individual (symmetric) contestant. For example, the organizer of a design competition would be more concerned about the quality of the potential supplier who secures the contract, rather than the overall amount of effort exerted by the entire pool of competitors. It turns out that the optimal contest that has been derived above serves this objective as well. Equation (7) implies that the individual effort of a participating contestant is bound from above by $\bar{e} = \frac{\Gamma_0}{2} - C$, which can be achieved if and only if the contest is organized as defined by Theorem 2.

Corollary 1 *The optimally designed contest that maximizes the total amount of effort also maximizes the amount of individual effort exerted by a representative participating contestant.*

4 Discussion and Extensions

4.1 Why "Two": When Fixed Entry Cost Is Present

The fixed entry cost C plays an essential role in determining the optimal contest structure. As equation (7) implies, the main results discussed in the earlier sections of this paper stem from the existence of a positive entry cost, while the optimal contest rule depends largely on the size of the fixed cost. Theorem 2 states that an entry subsidy is desirable in order to invite participation and to maintain a sufficient level of competition, if and only if the entry cost is prohibitively high. Thus, the result applies directly to the design competition for military procurement: R&D projects with a military purpose would arguably require substantial initial set-up investment, which could play a large part in deterring the entry of independent contractors, notwithstanding the generous potential rewards. Thus, a subsidy would be an effective way to maintain the optimal amount of competition.

An entry subsidy would not be justifiable if the level of the fixed entry cost falls below the threshold value $\frac{\Gamma_0}{2} - H^{-1}(\frac{\Gamma_0}{4})$. The contest organizer would instead charge an optimal entry fee to restrict the level of participation to the unique optimum of two participants, and attach the in-flow of cash to the winner's purse. For example, although a civilian R&D project may involve a fixed set-up costs, the investment may not be completely sunk because it could most likely be used for alternative purposes. Consequently, an entry fee that restricts entry may successfully enhance the quality of competition for the design of a civilian product through enhanced prize value. Hence, the results of this paper are consistent with the optimal research tournament design suggested by Fullerton and McAfee (1999), where the setting involves differential types and incomplete information.

4.2 Why Not "Two": When No Entry Cost Is Present

Although it has explicitly been assumed that $C > 0$, the analysis up to Equation (7) applies to the specific case where $C = 0$, in which participation involves no sunk costs. As Equation (7) implies, the entry of additional participants does not reduce the maximum amount of effort that can be possibly induced in an optimally designed contest. Hence, the optimal contest structure would not be unique, and the optimal number of participating contestants would not necessarily be two. The contest organizer can allow any number of contestants (but no less than 2) to participate, and simply charge each of them an appropriate entry fee to extract all the expected surplus they would enjoy from the contest. Thus, the optimal monetary transfer (entry fee) S^* satisfies

$$\frac{1}{N}V^* - H^{-1}(\frac{V^*}{N}(1-\frac{1}{N})) + S^* = 0, \forall\ N \in \{2, ..., M\}. \tag{9}$$

Since the contest organizer at the optimum directs all revenue towards the prize purse, Equation (9) is equivalent to

$$\frac{\Gamma_0}{N} = H^{-1}(\frac{(\Gamma_0 - NS^*)}{N}(1-\frac{1}{N})),\ N \in \{2, ..., M\}. \tag{10}$$

Theorem 3 *When a pool of $M \geq 3$ potential contestants are up for a contest, and each of them bears zero entry costs, the optimal contest can take a variety of forms $(V^*(N), S^*(N))$, $\forall N \in \{2, ..., M\}$. In an optimal contest $(V^*(N), S^*(N))$, the contest organizer charges an entry fee of*

$$S^*(N) = \frac{\Gamma_0}{N} - \frac{N}{N-1}H(\frac{\Gamma_0}{N}) < 0, \tag{11}$$

and awards a prize of

$$V^*(N) = \frac{N^2}{N-1}H(\frac{\Gamma_0}{N}) > 0. \tag{12}$$

In the contest $(V^*(N), S^*(N))$, exactly N contestants participate and each of them enjoys zero surplus. All these contests induce the same total amount of effort, Γ_0.

Theorem 3 defines a wide variety of optimal contest structures that differ in terms of their entry fees, prize purse and the equilibrium level of participation. When contestants bear negligible entry costs, the contest organizer has complete flexibility to design the contest. Optimally designed contests may attract any feasible level of participation, although they all yield an equivalent outcome where the entire budget is dissipated, Γ_0. Thus, our analysis does not lose its bite in those settings where a "more-than-two" participation rate could be considered optimal as well.

5 Concluding Remarks

This paper has investigated the design of an effort-maximizing contest where contestants bear a fixed entry cost and have the freedom to decide whether or not to participate. The findings indicate that contest organizers will subsidize entry when contestants bear substantial entry costs, while charging an entry fee to fund the prize purse when the entry cost is sufficiently low. Interestingly, the optimally designed contest invites exactly two participants as long as the entry cost is positive. Thus, this paper provides a clear rationale for the contest structure involving only two contestants that is widely assumed in contest literature. This optimal participation is assumed to be attributed to the presence of a fixed entry cost. In addition, it has been proven that in the absence of a fixed entry cost, the organizer of an optimally designed contest can invite any feasible number of contestants (from 2 to M) to participate, with all of equilibria yielding an equivalent outcome in terms of the total efforts induced.

This framework leaves tremendous room for the extension of research. One possible avenue for further research is to allow for different types of contestants. Indeed, this is a future research concern for the authors of this paper.

Appendix

Proof of Lemma 4

Proof. The lemma is proven by contradiction. Suppose the contrary that $\pi(N(V^*, S^*), V^*, S^*) > 0$. Two possible cases are considered.

Case I: $N(V^*, S^*) = M$.

In this case, there exists a transfer $S < S^*$ such that $\pi(M, V^*, S) > 0$ holds, since $\pi(M, V^*, S)$ is continuous with respect to S. This leads to $\pi(M, V^* + M(S^* - S), S) > \pi(M, V^*, S) > 0$. Thus, $N(V^* + M(S^* - S), S) = M$. It is clear that $(V^* + M(S^* - S), S)$ is feasible since (V^*, S^*) is feasible. However, contest $(V^* + M(S^* - S), S)$ induces a larger total amount of effort since the prize is higher and the number of potential participants who enter the contest does not change.

Case II: $2 \leq N(V^*, S^*) < M$.

By Lemma 2, we must have $\pi(N(V^*, S^*), V^*, S^*) > 0$ and $\pi(N(V^*, S^*) + 1, V^*, S^*) < 0$. There must exist a $\varepsilon > 0$ which is small enough such that $\pi(N(V^*, S^*), V^* + N(V^*, S^*)\varepsilon, S^* - \varepsilon) > 0$ and $\pi(N(V^*, S^*) + 1, V^* + N(V^*, S^*)\varepsilon, S^* - \varepsilon) < 0$ since function $\pi(N, V, S)$ is continuous with respect to all its arguments. We thus have $N(V^* + N(V^*, S^*)\varepsilon, S^* - \varepsilon) = N(V^*, S^*)$. It is clear that $(V^* + N(V^*, S^*)\varepsilon, S^* - \varepsilon)$ is feasible since (V^*, S^*) is feasible. However, $(V^* + N(V^*, S^*)\varepsilon, S^* - \varepsilon)$ induces a larger total amount of effort since the prize is higher and the number of potential participants who enter the contest does not change.

Based on the above arguments, $\pi(N(V^*, S^*), V^*, S^*) = 0$ for the optimal feasible contest (V^*, S^*). ■

Proof of Lemma 5

Proof. The lemma is proven by contradiction. Suppose $V^* < \Gamma_0 - N(V^*, S^*)S^*$. We consider two possible cases.

Case I: $N(V^*, S^*) = M$

The contest organizer has the option to allocate the balance of $(\Gamma_0 - N(V^*, S^*)S^*) - V^*$ to the prize without inducing the entry of additional participants, while increasing the total amount of effort induced.

Case II: $2 \leq N(V^*, S^*) < M$.

In this case, we have $\pi(N(V^*, S^*), V^*, S^*) = 0$ by Lemma 4, and $\pi(N(V^*, S^*)+1, V^*, S^*) < 0$ by the definition of $N(V^*, S^*)$. By the continuity of $\pi(N, V, S)$, there exists a small $\varepsilon > 0$ such that $V^*+\varepsilon \leq \Gamma_0 - N(V^*, S^*)S^*$, $\pi(N(V^*, S^*), V^*+\varepsilon, S^*) > 0$ and $\pi(N(V^*, S^*)+1, V^*+\varepsilon, S^*) < 0$. Thus $(V^* + \varepsilon, S^*)$ is feasible and $N(V^* + \varepsilon, S^*) = N(V^*, S^*)$ holds. However, $(V^* + \varepsilon, S^*)$ induces a larger amount of total effort since the prize is higher and the number of participants does not change. Based on the above arguments, $V^* = \Gamma_0 - N(V^*, S^*)S^*$ for the optimal feasible contest (V^*, S^*). ■

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