NBER WORKING PAPER SERIES

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Working Paper 17847
http://www.nber.org/papers/w17847

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2012

We thank participants to the ESA 2011 meeting in Tucson and to seminars at Caltech, NYU, the Paris School of Economics, and Ecole Polytechnique for helpful comments. We gratefully acknowledge financial support from the National Science Foundation (SES-0617820, SES-0617934, and SES-0962802), the Center for Experimental Social Science at NYU, and the Social Science Experimental Laboratory at Caltech. Palfrey thanks the Betty and Gordon Moore Foundation for financial support; Casella thanks the Paris School of Economics for its hospitality, and the Alliance Program. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. C72,C78,C92,D70,P16

ABSTRACT

Two groups of voters of known sizes disagree over a single binary decision to be taken by simple majority. Individuals have different, privately observed intensities of preferences and before voting can buy or sell votes among themselves for money. We study the implication of such trading for outcomes and welfare when trades are coordinated by the two group leaders and when they take place anonymously in a competitive market. The theory has strong predictions. In both cases, trading falls short of full efficiency, but for opposite reasons: with group leaders, the minority wins too rarely; with market trades, the minority wins too often. As a result, with group leaders, vote trading improves over no-trade; with market trades, vote trading can be welfare reducing. All predictions are strongly supported by experimental results.

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1 Introduction

Imagine a group collectively choosing between two alternatives through majority voting. Suppose that before voting all votes can be freely traded for money: individuals feeling strongly about the decision can buy votes from those who are less concerned about the outcome. To concentrate on vote trading per se, suppose also that none of the voters is budget constrained so that they all can express the intensity of their preferences through the price they are willing to pay. In this scenario, where inequality and credit constraints do not play a role, is vote trading a good idea?

There are at least three reasons why the question is interesting. First, as is well-known, majority voting fails to account for the intensity of preferences: a lukewarm majority will win over an intense minority. How can the minority be protected not only in its rights, which can be safeguarded by the courts, but also in the expression of its strongest preferences? Second, economic theory teaches that markets typically work well in allocating goods to those who most value them. It is natural to ask whether this insight extends to votes. More than a natural curiosity, it is a fundamental question in political economy. Third, markets for votes exist beyond the hazy interiors of smoke-filled rooms. Corporate shares *are* traded in markets and come not only with rights to dividends and future profits, but also to votes. Different classes of shares exist, with different voting rights. To what extent does the inherent trading of votes affect share prices and trades? It is difficult to answer this question without understanding the fundamental forces operating in a market for votes.[1]

It is not surprising, then, that questions about vote markets, whether mediated by money or by promises of future support (log-rolling), intrigued the early scholars in modern political economy: Buchanan and Tullock (1962), Coleman (1966, 1967), Park (1967), Wilson (1969), Tullock (1970), Haefele (1971), Kadane (1972), Riker and Brams (1973), Mueller (1973), Bernholtz (1973, 1974).[2] Writing in 1974, however, Ferejohn summarized the sad state of knowledge on the subject succinctly: "[W]e really know very little theoretically about vote trading. We cannot be sure about when it will occur, or how often, or what sort of bargains will be made. We don't know if it has any desirable normative or efficiency properties" (p. 25).

The crux of the problem is that votes have characteristics that make them very

[1] See for example, Demichelis and Ritzberger (2007) and Dhillon and Rossetto (2011) and the references they cite.

[2] The papers had different methodological approaches (for example, cooperative versus non-cooperative games; or log-rolling versus markets for votes) and often focused on specific examples. McKelvey and Ordeshook (1980) report a laboratory experiment that studies the Riker and Brams (1973) logrolling example.

different from standard goods. Votes are indivisible and intrinsically worthless; their value depends on the influence they provide on the group decision-making, and therefore on the holdings of votes by all other individuals. Thus, demands are interdependent, and payoffs discontinuous at the point at which a voter becomes pivotal. These unique features pose a major theoretical obstacle to understanding vote trading. Both in a market for votes and in log-rolling games, equilibrium and other stability concepts such as the core typically fail to exist. Ferejohn's early observation was echoed in later works (Schwartz (1977, 1981), Shubik and van der Heyden (1978), Weiss (1988), Philipson and Snyder (1996)), and with very few exceptions (Piketty (1994), Kultti and Salonen (2005)), the theoretical interest in voters trading votes among themselves effectively came to an end.

The goal of this paper is to make some progress in addressing these obstacles, both theoretically and experimentally. To do so, we build on two existing contributions, one based on general equilibrium theory and one based on mechanism design theory. Following a general equilibrium approach, Casella et al. (2011) has reopened the debate on competitive vote markets by proposing the concept of *Ex Ante Competitive Equilibrium:* a market price and demands such that each individual is maximizing his expected utility and the market clears *in expectation.* Ex ante competitive equilibrium overcomes the non-convexity of markets for votes and the discontinuity of demand caused by pivotality by allowing individuals to express mixed–i.e. probabilistic–demands. Realized demands will not clear the market ex post, but the equilibrium concept retains the discipline of competitive equilibrium by requiring that deviations from market clearing be unsystematic and unexpected: in the spirit of rational expectations, the expected deviation from market clearing is zero. Casella et al. show that an equilibrium exists when both the direction and the intensity of others' preferences are not known. In this paper we show, for the group sizes explored in the experimental design, that an ex ante equilibrium continues to exist when the size of the two opposing groups is known. The extension is not trivial: it has been argued, plausibly, that in markets for votes equilibrium existence problems should be worse when information about the direction of preferences is available (Piketty (1994)). The concept of ex ante equilibrium, however, extends to such settings. We find that in equilibrium the direction of preferences is revealed, and the competition for votes becomes a competition for dictatorship between the highest-intensity member of the majority and the highest-intensity member of the minority. The frequency of minority victories then reflects the relative intensity of the most intense minority member, without taking into account the smaller size of the minority and the aggregate group values. As a result, relative to utilitarian efficiency the minority wins too often. As in Casella et al., the bias can be strong enough that ex ante welfare is lower with a vote market than in the absence of trade.[3]

In the literature, an alternative response to Ferejohn's negative conclusion has been to model vote trading agreements as mediated either by a market-maker or by party leaders.[4] Koford (1982) and Philipson and Snyder (1996) both conclude that vote trading through a market-maker improves welfare. Their models, however, rely on auxiliary assumptions that play a large role in the result.[5] We argue in this paper that centralized vote trading closely resembles a bilateral bargain between the leaders of the two opposing groups. Viewed from this perspective, one can directly apply results from the mechanism design literature on bilateral bargaining. As long as the other group's preferences are not commonly known, in general vote trading through party leaders cannot be fully efficient. Absent trade, the majority "owns" the decision: the scenario is isomorphic to bilateral trading between a seller and a buyer when preferences are private information. Hence we know, from Myerson and Satterthwaite (1983)'s seminal theorem, that in the absence of outside subsidies, there is no incentive compatible mechanism that guarantees efficient trade and voluntary participation. There is too little trade, and the minority wins too rarely. However, because the minority *never* wins in the absence of trade, we confirm the conclusion in the literature: trading through party leaders has higher ex ante welfare than no trade.

There is one special case in which vote trading mediated by party-leaders can be fully efficient: when the two parties have equal size, and ties are broken by a fair coin toss. In the absence of vote trading, each group expects to win the decision with probability one-half. The environment is then isomorphic to the Cramton, et al. (1987) model of efficient dissolution of an equal partnership. Hence there exist optimal auctions guaranteed to transfer ownership of the partnership to the partner valuing it most and such that all agents will enter the auction voluntarily.

[3] Kultti and Salonen (2005) also conclude that the Walrasian approach to vote markets requires allowing for mixed demands. They do not impose any market clearing condition and study a model where trade is efficient by assumption.

[4] A different literature studies vote-buying by either candidates or lobbyists: for example, Myerson (1993), Groseclose and Snyder (1996), Dal Bò (2007), Dekel, Jackson and Wolinsky (2008) and (2009). We focus instead on vote-buying *within* the committee (or the electorate). The agents buying or selling votes are the voters themselves, acting either independently or through their leaders.

[5] Philipson and Snyder assume that only trades that are unanimously preferred to no-trade by all members of the two parties are allowed to take place. Koford assumes that the two party leaders cooperate in maximizing their members' surplus.

We conduct a series of laboratory experiments to explore both approaches - i.e., both a competitive market for votes, and vote trading through party leaders, and in this case with groups of equal and unequal size. For market trading, the experimental methodology on competitive markets for goods and assets has developed a widely accepted design: a continuous open-book double auction in which consumers and producers post prices and are matched (Smith, 1965,1982; Forsythe, Palfrey and Plott 1982; Gray and Plott, 1990, and Davis and Holt, 1992). In experiments, this market design has regularly been found to induce behavior that approximates closely the theoretical competitive market (for example, Friedman 1984, 1993). In a market for votes, the asymmetric position of consumers and producers is missing, and we simplify the design by allowing only bids for buying votes. To ensure that the results are comparable, we use the same design to study both trading through the market and trading with party leaders. In this latter case, the experimental design comes to approximate closely well-known auction games from bilateral bargaining theory.

Our main predictions can be summarized in three main points: (1) Relative to majority voting with no vote trading, efficiency gains exist only when trade is centralized, i.e. mediated through party leaders. (2) With market trading, there is too much trade: the minority party reaps significant benefits, but these are outweighed by the losses incurred by the majority party. (3) While centralized vote trading improves welfare, it can attain full efficiency only if the groups have equal size. If not, there is too little trade, and the minority benefits are too small.

In the experiment, centralized trade with equal size groups falls short of full efficiency: there are significant efficiency gains and the frequency of trade is higher than with different sizes, but the gains are not fully exploited. All other predictions, however, are satisfied in every experimental session. In particular, the opposite source of inefficiency when votes are traded in the market, as opposed to bargained between the group leaders emerges very clearly: in *every* experimental session with a majority group, the fraction of minority victories is too low when trade occurs through party leaders, and too high when trade occurs through the market, relative to the frequency that maximizes realized aggregate payoffs.

The next section describes the basic model, in the two specifications applying to groups leaders and to competitive trading, and derives the theoretical predictions; Section 3 describes the experimental design; Section 4 discusses the experimental results, starting with prices and allocations and concluding with voting outcomes and welfare; Section 5 concludes.

2 The Model

A committee of size n must decide between two alternatives, X and Y, and is divided into two groups with opposite preferences: it is publicly known that M individuals prefer alternative X, and m prefer alternative Y, with $m = n - M \leq M$. We will use M and m to indicate not only the size of the two groups, but also the groups' names. While the *direction* of each individual's preference is known, the *intensity* of such preference is private information. Intensity is summarized by a value v_i representing the utility that individual i attaches to obtaining his preferred alternative, relative to the competing one: individual i's utility is v_i if his preferred alternative is chosen, and 0 if it is not. Intensity v_i is private information, but it is common knowledge that each v_i is drawn independently for each individual from a distribution $F_i(v)$ atomless and will full support $[0, 1]$.

Each individual has one vote, and the group decision is taken through majority voting, with ties broken with a coin toss. Prior to voting, however, individuals can purchase or sell votes among themselves for money: a trade is an actual transfer of the vote and of all rights to its use. Individual utility u_i is given by:

$$u_i = v_i I + t_i \tag{1}$$

where I equals 1 if i's preferred decision is chosen and 0 otherwise, and t_i is i's net monetary transfer, which can be positive, if i is a net seller of votes, or negative, if he is a net buyer. Each individual makes his trading and voting choices so as to maximize his expectation of (1). In all that follows, we define as *efficient* the decision that maximizes the sum of realized utilities, or, equivalently, the decision preferred by the group of voters with higher total values.

With two alternatives and a single voting decision, voting sincerely is always a weakly dominant strategy, and we restrict our attention to sincere voting equilibria. Our focus is on the vote trading mechanism, and in particular, we are interested in two trading arrangements: a competitive spot market for votes, and a scenario where each group is represented by a leader and trade is restricted to the two leaders. We begin by studying the latter.

2.1 Trading through group leaders

The literature on vote markets stresses the externalities caused by individual vote trades on individuals who are not part of the transaction, and considers trade between party leaders a more promising route for efficiency gains (for example, Koford, 1982, Philipson and Snyder, 1996). Without specifying the details of the trading mechanism, the first question is whether *any* mechanism exists that always results in the efficient decision when trade occurs through the two party leaders.

In each group, the leader is the only member authorized to buy or sell votes; he knows and internalizes the total values of his group, and enforces any compensatory transfers within the group, if necessary. Suppose first that the groups have different sizes: $M > m$. If no vote is traded, the alternative preferred by the majority is chosen. Thus, lacking trade, the majority leader "owns" the decision. The model is then identical to Myerson and Satterthwaite's (1983) bargaining model, and the conclusion follows immediately: there is no mechanism that always guarantees ex post efficiency and satisfies incentive compatibility and interim individual rationality.[6] Myerson and Satterthwaite establish that the most efficient, incentive compatible and interim individually rational mechanism will have too little trade: without outside subsidies it is impossible to ensure the participation of both group leaders when the two group values are "too" similar. In our setting, too little trade–too few minority victories–are expected when the aggregate group values are similar. With specific trading institutions, vote trading by group leaders may well outperform simple majority voting, but Myerson and Satterthwaite's theorem tells us that it will fall short of full efficiency.

Suppose then that the groups have equal size: $M = m$. Without trade, the vote is tied, and given the random tie-break rule, each group leader expects to win with probability one-half. With risk-neutrality, each leader "owns" half of the decision. The problem is isomorphic to the dissolution of an equal share partnership in a private good, and we know from Cramton et al. (1987) that in this case an efficient, incentive compatible and interim individually rational bargaining mechanism does exist.[7] If the two groups have equal size, vote trading through the two group leaders can potentially be efficient.

We summarize these observations in the following remark:

[6] Note that our assumptions match Myerson and Satterthwaite's assumptions: the valuations draws are independent, and the supports of the distributions of valuations are full and overlap at least partially. With individual $F_i(v)$ distributed over $[0,1]$, the supports of the distributions of valuations for the group leaders, $G_p(\sum_{i\in p} v_i)$, $p \in \{M, m\}$, are full and must overlap for any size of the two groups (because both have 0 as lower bound).

[7] The tie-break rule is important. Cramton et al. show that efficient, incentive compatible and interim individually rational mechanisms exist for *all* distributions of valuations when the tie-break rule is a coin toss, as in our model. As long as there is *any* randomness in the tie-break rule, the result holds for *some* distributions of valuations. If one side wins for sure in case of ties, we are back to Myerson and Satterthwaite's model, and to their impossibility result.

Remark 1. (Myerson and Satterthwaite (1983), and Cramton et al. (1987)). *Suppose all vote trades occur through the two group leaders and the tie break rule is a coin toss. Then an efficient, incentive compatible, and interim individually rational trading mechanism exists if and only if $M = m$. If $M \neq m$, the most efficient, incentive compatible and individually rational mechanism has too little trade: the majority wins too often.*

In this paper our interest is not in optimal mechanisms, but in a specific institution, a market for votes, and in its experimental properties. We need to specify the details of the trading technology. Different trading rules are plausible, but the experimental focus of the paper helps us restrict the theoretical models. The classical experiments on competitive goods markets are designed as a continuous open-book auction between buyers and sellers (for example, Smith, 1982, Plott, 1982, Plott and Smith, 1978). Using this platform is then both desirable per se – because it provides an immediate comparison between goods and votes markets–and has the added advantage of supplying natural auction models for the case of trading between group leaders.

When trade occurs exclusively through the groups leaders, the model has two agents only. The natural unit of trade is the minimum number of votes necessary to acquire decision power.[8] In all cases, we can normalize the object of trade to one vote without loss of generality.

2.1.1 Two equal-sized groups.

In the case of equal-sized groups, in the absence of trade, either alternative is chosen with probability 1/2. Here, the continuous auction implemented in market experiments (and in our experimental design) approximates a second-price auction, where the individual submitting the highest bid acquires one vote from his opponent, but by barely overbidding him effectively pays the opponent's bid. A second price sealed-bid auction is then the appropriate reference for the experimental results.

Call b_i the bid submitted by individual i, and focus on symmetric bidding strategies, such that $b_i = B(v_i)$. Individual i' s utility is given by:

$$u_i = \begin{cases} v_i - b_j & \text{if } b_i > b_j \\ b_i & \text{if } b_j > b_i \end{cases} \tag{2}$$

[8] A trade of any number of votes either below or above such number has no value or no additional value.

and thus his problem is:

$$\max_{b_i}[E(v_i - B(v_j)|v_j < B^{-1}(b_i))F(B^{-1}(b_i)) + b_i(1 - F(B^{-1}(b_i)))]$$

where $B^{-1}(b_i) = v_i$. If F is Uniform, as it will be in the experiment, the problem becomes:

$$\max_{b_i}\left[\int_0^{B^{-1}(b_i)}(v_i - B(z))dz + b_i(1 - B^{-1}(b_i))\right]$$

with first order condition:

$$[v_i - 2B(v_i)]\frac{1}{B'(v_i)} = v_i - 1$$

The differential equation has solution:

$$B(v_i) = \frac{1}{(1-v_i)^2}\left(\frac{v_i^3}{3} - \frac{v_i^2}{2} + C\right)$$

The only solution for which the bid remains finite at $v_i = 1$ has $C = 1/6$. Hence the equilibrium bidding strategy must be given by:

$$B(v_i) = \frac{1 + 2v_i}{6} \tag{3}$$

The properties of the auction are immediate. First, $B(v_i)$ is increasing in v_i: the winning bid always belongs to the individual with the highest value, who will thus acquire the other's vote and impose his preferred decision - the mechanism is efficient. Second, the probability that individual i wins the auction is v_i, and thus increases in individual i's value - the mechanism is incentive compatible. Finally, i's expected utility from participating equals $\int_0^{v_i}[v_i - (1+2v_j)/6]dv_j + \int_{v_i}^1[(1+2v_i)/6]dv_j = 1/6 + v_i^2/2$, always larger than $v_i/2$ - the mechanism is interim individually rational. The auction is identical to one of the efficient mechanisms for dissolution of a partnership identified by Cramton, Gibbons and Klemperer.

2.1.2 Two unequal-sized groups.

When groups have different sizes, in the absence of trade, one group owns the decision power. With $M > m$, the majority leader assumes the role of the seller in a private market, and the minority leader the role of the buyer. In the continuous double auction implemented in the experiment, trade occurs only if the bid by group

m is higher than the value attributed to the decision by group M. The trading mechanism approximates a sealed buyer's bid double auction (Satterthwaite and Williams (1989)), with a single buyer and seller: the seller submits an offer, the buyer submits a bid, and trade occurs, at the buyer's bid, if the bid is higher than the offer. Indexing the seller by s and the buyer by b, utilities are given by:

$$u_s = \begin{cases} v_s & \text{if } b_s > b_b \\ b_b & \text{if } b_b > b_s \end{cases} \qquad u_b = \begin{cases} v_b - b_b & \text{if } b_s < b_b \\ 0 & \text{if } b_s > b_b \end{cases} \tag{4}$$

The seller's dominant strategy is to bid $b_s = v_s$. The buyer's problem then is:

$$\max_{b_b}[(v_b - b_b)F(b_b)] \tag{5}$$

If F is Uniform over $[0, 1]$, $F(b_b) = b_b$, and thus:

$$b_b(v_b) = \frac{v_b}{2} \tag{6}$$

The trading mechanism is incentive compatible and interim individually rational, but, as expected, ex post inefficient: trade occurs only if the buyer's value is at least *twice* the seller's value. With F Uniform, the probability that a trade is concluded is 25 percent, where it should be 50 percent with full efficiency; expected ex ante welfare is 94 percent of expected welfare under full efficiency (versus 75 percent without trade). In our application to vote trading between group leaders, the theoretical prediction is unambiguous: vote trading is not fully efficient because the majority wins too often. However, it dominates majority voting without vote trading because it allows for *some* minority victories, when the disparity in values is sufficiently high.

The mechanism has lower trade and lower expected efficiency than the optimal Myerson and Satterthwaite mechanism, but the difference is not large. With F Uniform over $[0, 1]$, the optimal incentive compatible and individually rational mechanism can sustain trade whenever $v_B \geq v_S + 1/4$; the probability that a trade is concluded is then 28 percent, and expected welfare 96 percent of full efficiency. Relative to the optimal mechanism, the buyer's bid auction supports more trade when the seller's value is less than $1/4$, and less trade otherwise. The net result is a welfare loss, but because the missed trading opportunities occur mostly when the buyer's and the seller's values are not too different, the quantitative impact is not large.

2.2 Competitive vote trading

Characterizing the equilibrium and welfare properties of a competitive market for votes is more challenging. The crucial problem is well-known: there exist no price and allocation of votes such that the market clears. Call p the market price of a vote, and $d_i(p)$ individual i's net demand for votes in equilibrium (where $d_i = -1$ if i sells his vote). Then:

Remark 2. (Ferejohn (1974), Philipson and Snyder (1996), Piketty (1994)). *If $n > 2$, for any $M \geq m > 0$ and $\{v_1, ...v_n\}$, there is no price p such that $\sum_i d_i(p) = 0$.*

The logic is simple. For all $p > 0$, $\sum_{i\in m} d_i(p) \in \{-m, (M-m+1)/2\}$: if the aggregate demand of minority voters is positive, it must equal the minimum number of votes required to win; alternatively, at any positive price all losing votes must be offered for sale. But $\sum_{i\in M} d_i(p) \leq 0$: in equilibrium, the aggregate demand by majority voters cannot be positive. In addition, $\sum_{i\in M} d_i(p) \neq -(M-m+1)/2$: if $(M-m+1)/2$ votes were traded, the remaining $(M+m-1)/2$ votes collectively held by M voters would be worthless and thus offered for sale too. Thus for all $p > 0$, $\sum_{i\in m} d_i(p) + \sum_{i\in M} d_i(p) \neq 0$. If $p = 0$, $\sum_{i\in m} d_i(p) \geq (M-m+1)/2$[9], but $\sum_{i\in M} d_i(p) \geq -(M-m-1)/2$, because the only supply can come from M voters whose vote is not pivotal. Thus for all $p = 0$, $\sum_{i\in m} d_i(p) + \sum_{i\in M} d_i(p) > 0$.□

In response to this observation, Casella et al. (2011) develop the concept of *ex ante competitive equilibrium.* In an ex ante equilibrium, demand is allowed to be stochastic, the market clears in expectation only, and a rationing rule determines the ex post allocation of votes. Call δ_i i's mixed demand: δ_i is a discrete probability distribution over support $[-1, 0, 1, .., n-1]$. Call d_i the realization of δ_i, and R a rationing rule establishing how votes are allocated if $\sum_i d_i \neq 0$. A price p, a vector of demands $\boldsymbol{\delta}$, and a rationing rule R constitute an ex ante equilibrium if $Eu_i(\delta_i, \boldsymbol{\delta}_{-i}, R) \geq Eu_i(\widetilde{\delta}_i, \boldsymbol{\delta}_{-i}, R)$ for all $\widetilde{\delta}_i$, for all i, and $\sum \delta_i = 0$.[10]

Casella et al. discuss two anonymous rationing rules, but because the value of votes can be very discontinuous in the quantity purchased, they focus primarily on a rationing rule inspired by *All-Or-Nothing* orders in financial markets: either a voter fulfills his demand completely or is excluded from trade.[11] Because we adopt the

[9] We are assuming that at $p = 0$, voters on the losing side demand rather than sell votes. This is equivalent to the standard assumption that goods are in excess demand at 0 price.

[10] As in the analysis of competitive equilibrium with externalities (e.g., Arrow and Hahn, 1971, pp. 132-6), the definition of the equilibrium requires voters to best reply not only to the price but also to the demands of other voters. Optimal demands are interrelated.

[11] See the description of AON oders by the New York Stock Exchange

same rule below, we describe it more precisely: any individual with positive demand is considered with equal probability; in case his demand cannot be satisfied, the voter is left with his initial endowment, and the process goes on to another voter with positive demand. For such a rationing rule ($R1$ for ease of reference), Casella et al. show that an ex ante competitive equilibrium exists in a votes market where not only the intensity but the direction of voters' preferences is private information. As expected, in equilibrium votes concentrate in the hands of individuals with high intensities, but the result is extreme: after trade, it is always the case that either the highest-intensity voter or, less frequently, the second highest owns a majority of the votes. The equilibrium price depends on the realization of values, but is always such that the second highest-value voter is indifferent between buying a majority of the votes and selling his vote. The market equilibrium comes to resemble an auction for dictatorship.[12]

One question left open is whether the concept of ex ante competitive equilibrium can be applied to a market for votes in which the size of the two opposing groups is known. The literature conjectures informally that in such a case any notion of competitive equilibrium is doomed by the argument in Remark 2. But consider the following example:

Example. *Imagine a full information scenario where $v_i = 1$ for all $i \in M$, $v_i = 2$ for all $i \in m$, and $n = 5$. The rationing rule is $R1$. Then, there exists an ex ante competitive equilibrium such that one minority voter demands two votes, one majority voter randomizes between demanding two votes (with probability $2/3$) and selling his vote, and all other voters offer to sell. The equilibrium price is $p = 1/3$.*

The statement is verified in Appendix A. Note that with $n = 5$ controlling three votes amounts to having full decision power. Thus, not only does an equilibrium continue to exist, but it replicates closely the result in Casella et al., with the difference that the competition for dictatorship now takes place between two members of the opposite groups. Strikingly, equilibrium strategies and price do not depend on the size of the minority. As a result, there can be no presumption of efficiency: whether $m = 1$ and $M = 4$ (and efficiency dictates a majority victory), or $m = 2$ and $M = 3$ (and efficiency dictates a minority victory), with a market for votes the minority wins with probability $2/3$. Nor is the market necessarily superior to no trade: it is superior if $m = 2$, but inferior if $m = 1$.[13]

http://www.nyse.com/futuresoptions/nysearcaoptions/.

[12] Casella et al. identify sufficient conditions under which the equilibrium is robust to an alternative rationing rule where each vote on the long side of the market is rationed with equal probability.

[13] The example assumes that preferences are identical within each group, as in Kullti and Salonen

The example considers a full information scenario where values are constant within each group. We are interested in the private information case where the distribution of values is non-degenerate. In a related paper, we study the equilibrium for such a case and generic m and M (n odd).[14] Here, framing once again the theoretical analysis by the experimental design, we focus on the specific values of m and M employed in the experiment: $m = 2$, $M = 3$.

An equilibrium with no trade always exists: if all other voters are neither buying nor selling, being inactive is an individual's best response. Our interest is in an equilibrium with trade. We define:

Definition. *An equilibrium is* fully revealing *if, given others' strategies and price and the knowledge that the market is in a fully revealing equilibrium, each voter's strategy is identical to what it would be if he were fully informed about others' preferences.*

In the spirit of rational expectations models (Allen and Jordan, 1998), we call an equilibrium *fully revealing* if either: (1) the equilibrium price, together with the set of others' equilibrium strategies and market equilibrium, fully convey to voter i the direction of preferences associated to each demand; or (2) the information conveyed is partial but voter i has a unique best response, identical to his best response under full information.

Call $\overline{v}_g$ ($\underline{v}_g$) the maximal (minimal) realized value of a voter in group g, with $g = m, M$. Then:

Proposition 1. *Suppose $M = 3$ and $m = 2$. The rationing rule is $R1$. Then for all realizations of $\{v_1, .., v_5\}$ such that $\overline{v}_m \geq \underline{v}_M$, a fully revealing ex ante equilibrium with trade exists. The equilibrium price is always such that* $\min(\overline{v}_m, \overline{v}_M)$ *is indifferent between demanding two votes and selling his vote.*

The proposition states that unless *every* majority member has stronger preferences than *any* minority member, an equilibrium with trade exists.[15] We prove it by construction in Appendices A and B. The equilibrium recalls the example presented above, but to discuss it further, we need to describe it in more detail.

To simplify notation, call g' the group such that $\overline{v}_{g'} \geq \overline{v}_g$, and call $v_{(2)g'}$ the second highest value in group g'. Lemmas 1 to 3 in the Appendices show that the equilibrium has the following features: (1) If $\overline{v}_g > (2/7)v_{(2)g'}$, voters $\overline{v}_{g'}$ and $\overline{v}_g$ randomize between

(2005), but the requirement of expected market balance leads to very different equilibrium strategies and price.

[14] Casella and Turban (in preparation).

[15] An equilibrium with trade may exist when $\overline{v}_m < \underline{v}_M$, but the no-trade equilibrium seems focal in such a scenario.

demanding two votes and selling their vote, all other voters offer to sell their vote; (2) If $\overline{v}_g \in [(1/14)v_{(2)g'}, (2/7)v_{(2)g'}]$, voters $\overline{v}_{g'}$ and $\overline{v}_g$ randomize between demanding two votes and selling their vote, $v_{(2)g'}$ randomizes between demanding one vote and selling, all other voters offer to sell their vote; (3) Finally, if $\overline{v}_g < (1/14)v_{(2)g'}$, voters $\overline{v}_{g'}$ and $v_{(2)g'}$ demand one vote, voter $\overline{v}_g$ randomizes between demanding two votes and selling his vote, all other voters sell. The randomization probabilities and the equilibrium price depend on the realization of the vector $\{v_1, .., v_5\}$. Recall that neither one's value ranking in the distribution of values nor others' individual preferences are known ex ante, but both are revealed in equilibrium.[16]

We can rephrase the equilibrium strategies in more intuitive terms. For the great majority of value realizations, there is an equilibrium where $\overline{v}_m$ and $\overline{v}_M$ randomize between demanding two votes and selling their vote, and all other voters offer to sell their vote. The two top-value voters in each group compete for dictatorship, and all others sell. Such an equilibrium always exists when the two highest value voters in the committee disagree. When that is not the case, the result continues to hold as long as the distance, in value terms, between the two top voters in the committee and the highest value voter who disagrees with them is not too large. But even when this condition is violated, in equilibrium $\overline{v}_g$, or, with more transparent notation, $\min(\overline{v}_m, \overline{v}_M)$ must be indifferent between selling and bidding for dictatorship. If there is trade, the other members of his group, with weaker preferences, are selling; thus, if $\min(\overline{v}_m, \overline{v}_M)$ buys at all, then he must buy dictatorship.

The equilibrium has interesting implications. The induced competition between $\overline{v}_m$ and $\overline{v}_M$ implies that in equilibrium there is always a positive probability that a *majority* voter demands to buy votes. Thus both intra-group trade and a super-majority occur with positive probability for all profiles of values that support trade. The need to secure the cooperation of one's weakest allies is not surprising and routinely observed in political deals, but intra-group trades are absent from all vote-buying models we are familiar with.[17]

The description of the equilibrium strategies makes immediately clear that the equilibrium must fall short of full efficiency. As in the example, individual strategies reflect the relative ranking of individual values, but do not capture the aggregate

[16] As discussed in Appendix B, we have found multiple ex ante equilibria with trade for value realizations such that $\overline{v}_g \leq (1/4)v_{(2)g'}$. In all such equilibria $\min(\overline{v}_m, \overline{v}_M)$ is indifferent between demanding two votes and selling his vote. We do not know whether the equilibrium (with trade) is unique when $\overline{v}_g > (1/4)v_{(2)g'}$ but have not identified any other.

[17] Groseclose and Snyder's (1996) conclusion that vote-buying leads to supermajorities has the same flavor, but their paper studies vote-buying by two competing lobbyists, as opposed to vote trading in a market, where every voter is potentially a buyer and a seller.

values of the two groups. If the values are independent draws from a Uniform distribution, the expected frequency of minority victories in the equilibrium we characterize is 52.5 percent, while the efficient frequency is 22.5 percent. Relative to full efficiency, the minority wins too often. Nor is the market superior to no-trade: If values are distributed uniformly, simulations show that expected ex ante welfare in equilibrium corresponds to 84.2 percent of ex ante first best efficiency. In the absence of vote trading, expected ex ante welfare is 95 percent of full efficiency.[18] [19]

Our main theoretical result, thus, is that vote trading between a majority and a minority group cannot reach full efficiency, whether or not trading is coordinated by group leaders. In the presence of group leaders, the minority is expected to win too rarely, but in the absence of group leaders, the minority is expected to win too frequently. Because with simple majority voting the minority always loses, vote trading with group leaders, although not fully efficient, must dominate voting in the absence of trade. With our parametrization, the opposite holds for vote trading through the market: the market for votes is inferior to simple majority voting with no trading.

3 Experimental design

The experiment was run at the Center for Experimental Social Science at NYU (CESS), and at the Social Science Experimental Laboratory at the California Institute of Technology (SSEL) between June 2007 and February 2009, with enrolled students recruited from the whole campus through the laboratories' web sites. No subject participated in more than one session. After entering the computer laboratory, the students were seated randomly in booths separated by partitions and assigned ID numbers corresponding to their computer terminal; the experimenter then read aloud the instructions, projected views of the computer screens during

[18] Because the mixing probabilities depend on the exact realizations of the values, we calculated expected welfare in equilibrium by averaging the results of a 100,000 independent realizations of the vector of value draws. As stated in the previous footnote, in the market for votes there are multiple ex ante equilibria if $\overline{v}_g \leq (1/4)v_{(2)g'}$. But the frequency of such value realizations is so low that all the numbers are unchanged, up to the first decimal point, for any of the equilibria we have found. The observation holds for all numbers cited in the paper on the theoretical predictions of market trading, and we do not repeat it below.

[19] The numbers are not immediately comparable to those computed in the case of party leaders because in the auction the buyer and the seller are each assumed to draw a single value distributed uniformly. If the seller's value is instead the sum of three independent draws from a Uniform distribution, and the buyer's value the sum of two, then expected ex ante equilibrium welfare in the auction equals 96.5 percent of ex ante first best efficiency.

the experiment, and answered all questions publicly. Each session of the experiment amounted to 25 paid rounds, preceded by one unpaid practice round. Each experimental session consisted of a single treatment.

In the experiment, the two groups were called X and Y, from the name of the preferred alternative. In each round, subjects were matched randomly in committees and assigned either to group X or to group Y. The size of the two groups was commonly known. Each subject i was told by the computer whether he belonged to group X or group Y, and the value v_i he would win if his preferred policy prevailed. Values were expressed in experimental points, and subjects knew that values were drawn randomly by the computer, independently and privately for each subject, and could assume any integer value between 1 and 100, with equal probability.

After values were assigned, the market for votes opened. Each subject started the session with an initial endowment of 200 points, to be paid back at the end of the experiment. Any subject could post a bid specifying the price he was willing to pay for a vote; the bid appeared on all committee members' monitors, together with the name of the group the bidder belonged to and a running tally of the votes belonging to each group.[20] If anyone accepted the bid, the transaction was concluded; if not, anyone could post a new bid, higher than the previous one. After each trade, a new bid could be posted, at any value. The market for votes was open for three minutes, during which as many transactions were concluded as there were accepted bids.[21] Two observations on the experimental market should be added. First, we ran the experiment allowing only bids, as opposed to bids and asks. Second, in the experiment, the group identity of each bidder was public information. The fully revealing equilibrium is then the appropriate theoretical guide.

Once the market for votes closed, voting took place. All votes were automatically cast for the preferred option of the post-market owner of the vote, with ties broken randomly.[22] The session then proceeded to the following round, where subjects were randomly regrouped into new committees. Each experimental session lasted 25 rounds. At the end of each session, subjects were paid their cumulative earnings

[20]Posted prices had to be between 1 and 100, and no subject was allowed ot post a bid higher than his current endowment of points. If a subject had reached a 0 balance, he was excluded from bidding until the balance turned positive, either through selling his vote or through winning the committee decision.

[21]The market was open for two minutes in treatment $32C$ described below. In all treatments, the market closed early if there was no activity for 30 seconds.

[22]In one of the 11 sessions run at Caltech (session s5 in Table 1 below), in case of tie each subject received 50 percent of his value with probability 1. We changed the design to check whether the uncertainty of the outcome in case of a tie affected the results, but the session is indistinguishable from the others.

from all rounds, summing payoffs from obtaining their preferred committee decisions and net transfers from the market for votes, multiplied by a pre-announced exchange rate, plus a fixed show-up fee. Each session lasted about 90 minutes, and average earnings were around \$33. A sample of the instructions from one of the sessions is reproduced in Appendix C.[23]

Our treatment variables are the relative size of the two groups, m and M, and whether trade takes place through the group leaders or through the market. The first treatment, called $1, 1$, captures vote trading through group leaders when the groups have equal size: each group is represented by a single subject, with opposite preferences, and each subject enters the vote market with a single vote. The second treatment, called $3, 2C$, captures vote trading through group leaders when the groups have different sizes: each group is again represented by a single subject, with opposite preferences, but the two subjects enter the market endowed with three and two votes, respectively. We implemented this treatment by generating each subject's value as a single random draw form 1 to 100, assuming each integer value with equal probability.[24] The third treatment is the market treatment: the two groups, with opposite preferences, are formed by three and two subjects respectively, and each individual subject is free to trade, independently of the other members of his group. Each individual value is an independent random draw, assuming any integer value between 1 and 100 with equal probability, and each group value then is the sum of either two or three independent draws. We call this treatment $3, 2$. Table 1 reports the experimental design.

Table 1: Experimental Design

[23] We used the Multistage Game software package developed jointly between the SSEL and CASSEL labs. This open-source software can be downloaded from http://software.ssel.caltech.edu/

[24] The alternative would have been to generate the majority leader's value as the sum of three independent draws, and the minority leader's as the sum of two independent draws. Our design generates more frequent opportunities for trade, an important consideration in an experiment in which subjects play no other role and could becomes bored and inattentive, and allows us a direct comparison between $1, 1$ and $3, 2C$ treatments.

Session	n	Treatment	Subject pool	# Subjects
s1	2	$1, 1$	NYU	12
s2	2	$1, 1$	NYU	8
s3	2	$1, 1$	NYU	10
s4	2	$1, 1$	NYU	16
s5	2	$1, 1$	CIT	12
s6	2	$1, 1$	CIT	10
s7	5	$3, 2$	NYU	10
s8	5	$3, 2$	NYU	15
s9	5	$3, 2$	NYU	20
s10	5	$3, 2$	NYU	10
s11	5	$3, 2$	CIT	15
s12	5	$3, 2$	CIT	15
s13	2	$3, 2C$	NYU	12
s14	2	$3, 2C$	NYU	12
s15	2	$3, 2C$	CIT	10
s16	2	$3, 2C$	CIT	10

Table 2 summarizes the theoretical predictions discussed in the previous section. Columns 2 and 3 report the expected frequency of minority victories in equilibrium and under full efficiency; columns 4 and 5 report, respectively, expected ex ante utility in equilibrium and in the absence of trade, expressed as share of expected ex ante utility with full efficiency

Table 2: Theoretical predictions. Uniform distribution[25]

	Min victs %	Eff. min victs %	E(eff share) %	E(eff share, maj) %
1,1			100	75
3,2C	25	50	94	75
3,2	52.5	22.5	84.2	95

4 Experimental Results.

4.1 Prices

For each of the three treatments, the theory has precise predictions about equilibrium prices. We evaluate our data through two main questions. First, is there evidence

[25]Recall that treatment $3, 2C$ is implemented with a single value for each group, drawn for both groups from a Uniform distribution over support $[0, 1]$.

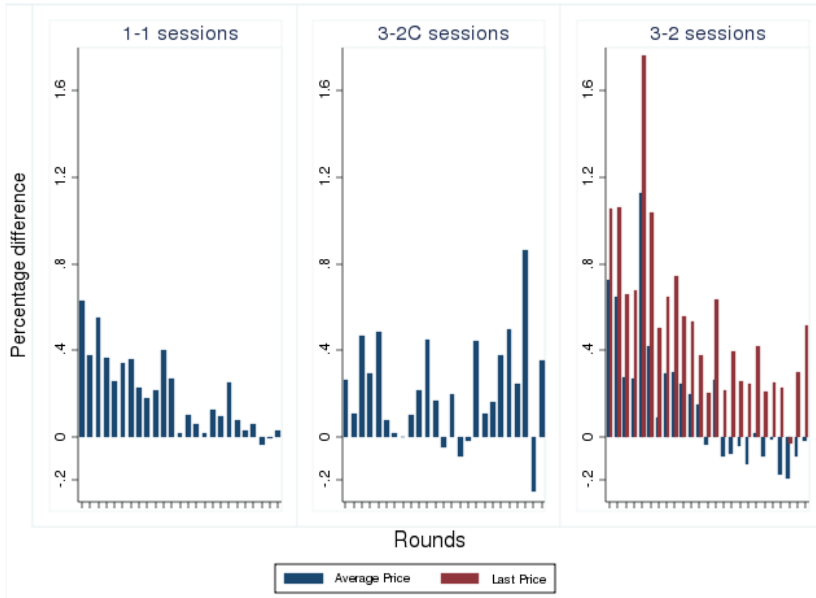

Figure 1: Average percentage difference between realized and equilibrium prices in each round. Blue columns: average realized price; red columns: last realized price

of convergence towards the equilibrium price over the length of the experimental sessions? Second, is the theory a good predictor of the prices realized in individual trades?

Figure 1 plots for each treatment the average percentage difference between realized and equilibrium prices in each round. For each group in each session, we calculate the equilibrium price given the realized values in each round; we then compare it to the price at which trade occurred in the experimental data. For each round, the resulting percentage difference is averaged over all groups and all sessions. If there are more than one trades, we calculate both the average and the last traded price. In $1, 1$ and $3, 2C$ treatments, the theory predicts a single trade; multiple trades are occasionally observed in the data but the results are indistinguishable whether we use average or last traded price; the figure reports the former. In the market treatment $3, 2$, multiple trades are expected and observed, and the results are sensitive to the price measure.

The figure shows three main regularities. First, in all treatments, there is tendency towards overpricing: the great majority of bars are above zero, indicating that experimental prices are above equilibrium prices. Second, in $3, 2$ sessions there must be more noise in the pricing of individual trades: the noise washes out when prices are averaged within the round, but is evident in the large and variable dispersion of last traded prices, relative to the equilibrium price. However, and this is the third remark, there is clear evidence of convergence to equilibrium in $1,1$ and $3,2$ sessions. Surprisingly, it is the relatively "easy" $3,2C$ treatment that most deviates from the theory. Here the figure shows no evidence of convergence.

A regression of the percentage difference between the realized price and the equilibrium price on the round number (and a constant) confirms what the figure shows and is reported in Table 3. The convergence over time in treatments $1,1$ and $3,2$, but not $3,2C$, as well as the lack of predictability of the $3,2C$ prices (notice the difference in the R^2's), and the common overpricing on average, all appear in the table.

	(1)	(2)	(3)	(4)
			3-2 sessions	3-2 sessions
VARIABLES	1-1 sessions	3-2C sessions	av. price	last price
Round	-0.0205***	0.0029	-0.0337***	-0.0377***
	(0.0025)	(0.0118)	(0.0049)	(0.0060)
Constant	0.466***	0.180	0.600***	1.028***
	(0.0325)	(0.132)	(0.0932)	(0.114)
Observations	25	25	25	25
R-squared	0.714	0.008	0.620	0.528

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3: Convergence Regressions

As Figure 1 shows, in treatments $1,1$ and $3,2$ overpricing falls sharply in the last rounds, suggesting that in earlier rounds it may be due mostly to inexperience. We have no explanation for the poor predictive power of the equilibrium bid in the $3,2C$ model; possibly subjects found the design confusing (two voters, each with multiple but unequal votes over a single decision), although an alternative explanation is risk-aversion.[26]. Much more striking is the convergence in the $3,2$ data towards the equilibrium price, because the equilibrium price is not only difficult to calculate but depends on the realized vectors of values, and thus changes across rounds. The result suggests that the underlying forces of competitive market exchange, where traders simply respond to the immediate gains and losses open to them, is indeed driving the

[26] Risk aversion seems particularly relevant for the $3,2C$ data, where overpricing persists even with experienced traders. In that case, CRRA utility is $u = \frac{(v_i - t_i)^{1-\rho}}{1-\rho}$, and the buyer (minority) equilibrium bid is $b(v) = v/(2-\rho)$ for $\rho \in [0,1)$ and $b(v) = v$ for $\rho \geq 1$.

trades, and the price converges to equilibrium without the conscious calculation of what the equilibrium price should be. Even in its application to a market for votes, the experimental data support the fundamental intuition at the heart of competitive market theory. Casella, et al. report similar convergence towards equilibrium price when the size of the two groups of voters is not publicly known.

Convergence is evaluated for the average realized price. But are disaggregated prices consistent with the theory?

Figure 2 presents scatter plots of observed traded prices for each transaction, on the vertical axis, plotted against the equilibrium price for that group and round, on the horizontal axis. Each panel in the figure corresponds to a different treatment, and in each, grey dots refer to early rounds (1-10) and black dots to late rounds (11-25). The panels also show two linear regression lines (grey for early rounds and black for late rounds) and the 45 degree line.

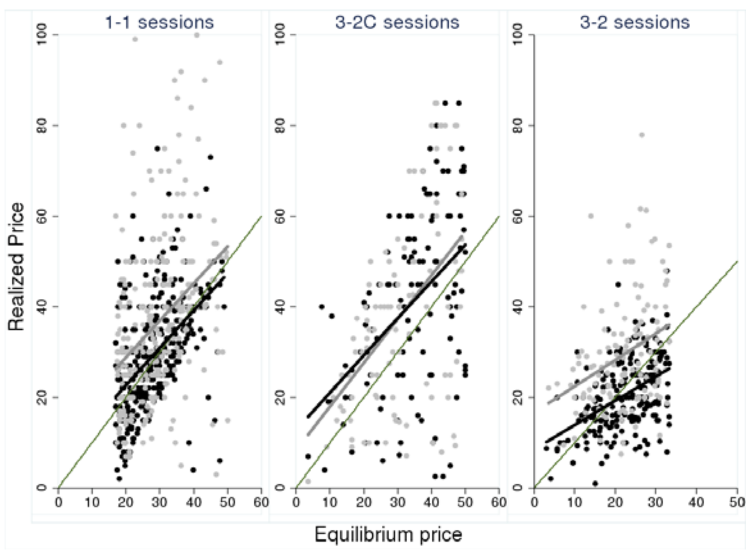

Figure 2: Traded prices versus equilibrium prices, per group and round, and linear regression lines. Rounds 1-10: grey; rounds 11-25: black.

The dispersion in realized prices is evident in the figure, but so is the positive correlation between realized prices and equilibrium prices, as well as the convergence towards equilibrium prices in the later rounds in $1,1$ and $3,2$ sessions. In both of these treatments, the regression line based on early data is shifted upward, relative to the regression line with late data, and the grey dots are both more dispersed and biased upward, relative to the black dots. Neither of these two observations applies to $3,2C$ sessions, where the two regression lines and the grey and black dots are effectively interchangeable. The figure makes visible another factor that may contribute to the observed overpricing: the upper bounds on equilibrium prices are a fraction of possible realized valuations. While valuations vary between 1 and 100, maximal equilibrium prices are 50, for $1, 1$ and $3, 2C$, and 33 for $3, 2$. If there is a diffuse random error in realized prices, with support over the full range of valuations, the result is systematic overpricing. And if there is more randomness in earlier rounds, the result is a correlation between overpricing and inexperience.[27]

Table 4 tests whether the regression lines are significantly different from the 45 degree line. The standard errors are clustered at the session level.

	1-1 sessions		3-2C sessions		3-2 sessions	
Rounds:	All	11-25	All	11-25	All	11-25
Eq. price	0.840***	0.825***	0.832***	0.791***	0.848**	0.865***
	(0.132)	(0.156)	(0.086)	(0.111)	(0.263)	(0.144)
Constant	8.226	6.182	9.442	12.04	13.06**	8.622**
	(4.641)	(4.681)	(4.724)	(5.174)	(5.043)	(3.230)
Obs.	687	409	262	135	337	208
R^2	0.185	0.265	0.191	0.188	0.131	0.187
p-val (coef=1)	0.277	0.315	0.146	0.157	0.589	0.390

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 4: Price Regressions

Whether over the full data set or the later rounds only, the slope is not significantly different from 1 at any conventional significance level, in all three treatments; in treatments $1, 1$ and $3, 2C$ we cannot reject a zero constant; we can, at the 5 percent level, in $3, 2$ treatments. The large unexplained noise in realized prices is reflected in the low R^2's, which predictably improve in the later rounds in treatments $1, 1$ and $3, 2$, but not $3, 2C$.

4.2 Trade and votes allocations

If the theory has some predictive power in explaining observed experimental prices, can it also explain the transactions observed in the experiment? The predictions differ across treatments, and we analyze them separately. In $1, 1$ sessions, the theory

[27] In $1, 1$ sessions, the equilibrium price is $p^* = (100 + 2v_s)/6$, and thus has a minimum at $100/6$. The figure also shows that, in later rounds especially, realized prices appear to lie above a linear function of the equilibrium price, with slope higher than 1 and negative intercept. The reason is that sellers only sell if $p \geq v_s/2$, since the default is a tie. With $p^* = (100 + 2v_s)/6$, $p \geq 3/2p^* - 25$.

states that all opportunities for trade should be exploited (and there is always an opportunity for trade), and that trade should always be efficient: the higher value voter should buy out the lower value voter. Figure 3 reports realized and unrealized gains from trade. In the first panel, the vertical axis is the buyer's value and the horizontal axis the seller's value, for each concluded trade: points above the diagonal are efficient trades, and points below the diagonal are inefficient trades. In the second panel, the vertical axis is the higher value and the horizontal axis the lower value for the two experimental subjects in all instances in which no trade took place; thus all points lie above the diagonal by construction, and each point represents unrealized gains from trade. As in Figure 2, grey dots refer to early rounds (1-10) and black dots to rounds 11-25.

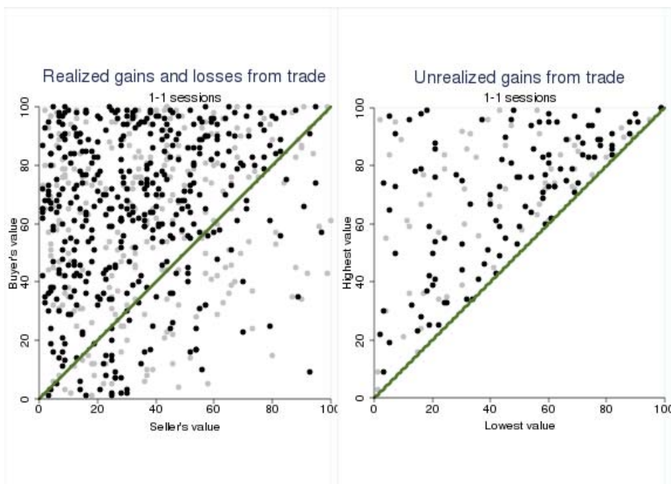

Figure 3: Realized and unrealized gains from trade; individual transactions; $1, 1$ sessions. Equilibrium predicts trade above the diagonal. Grey dots correspond to rounds 1-10; black dots to rounds 11-25.

The frequency of efficient trade–the fraction of *all* points that lie above the diagonal in the first panel–was 61 percent in early rounds and 66 percent in late rounds, a difference too small to be statistically significant.[28] Later rounds, however, had smaller mistaken trades: of all dots in the first panel, a greater share of the grey dots are below the diagonal, relative to black dots. Precisely, conditional on trade, 76 percent of transactions were efficient in early rounds, versus 84 percent in later rounds, percentages that according to a Pearson χ^2 test are significantly different

[28] In probit or logit regressions of the frequency of efficient trade per round on a constant and the round number, the round number coefficient is not statistically significant. Similarly, a Pearson χ^2 test cannot reject the hypothesis of no difference in the realized frequency of efficient trade between rounds 1-10 and rounds 11-25.

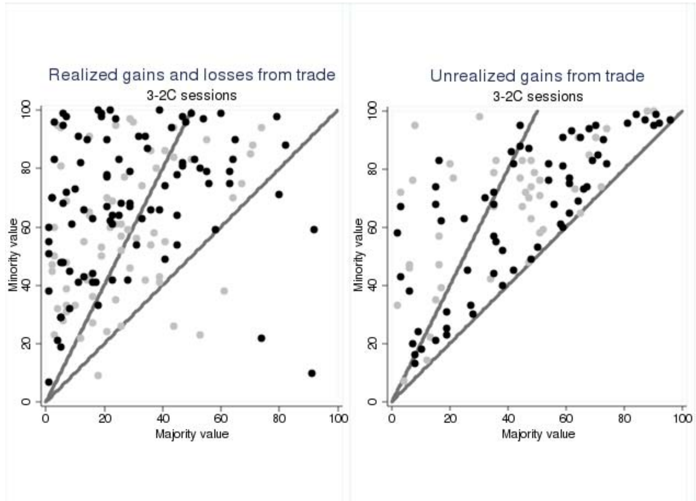

Figure 4: Realized and unrealized gains from trade; individual transactions; $3, 2C$ sessions. Equilibrium predicts trade above the steep line. Grey dots correspond to rounds 1-10; black dots to rounds 11-25.

at the 5 percent level. In later rounds, mistakes are also more predictable: inefficient trades were more likely at low voters' value, in the bottom left corner of the first panel, or when the two values were similar, along the diagonal. On the whole, then, transactions in $1, 1$ sessions were less efficient than the theory predicts, both because of lower trade and of mistakes in the direction of trade. There is some evidence that subjects learned to avoid mistaken trades, but not to exploit more trading opportunities.

Figure 4 reports the corresponding data in $3, 2C$ treatments. The first panel reports realized gains (and losses) from trade: the vertical axis is the minority value and the horizontal axis the majority value in each instance in which the minority won. The second panel plots the same values in those instances in which a minority victory would have been efficient but did not take place.[29] Again by construction, all points above the diagonal represent efficient trades. The steeper line is the theoretical boundary for trade: our auction model predicts that trade will occur for points above the line, but not below.

The theory predicts that trade, when it occurs, should be efficient, and indeed the fraction of points below the diagonal in the first panel is very small, whether in early or late rounds.[30] It also predicts that a large fraction of trading opportunities

[29] The figures do not report values for which the absence of trade was efficient and was observed.
[30] The numbers are 4 of 68 total trades in early rounds (5.9 percent), and 4 of 91 in rounds 11-25 (4.4. percent); the difference is statistically insignificant.

should remain unexploited: with two independent draws from the same Uniform distribution, the frequency of trade should be 25 percent, or 50 percent if we condition on the buyer's value being higher than the seller's. In the figure, of all points above the diagonal, the fraction in the first panel is 57 percent in late rounds and 59 percent in rounds 1-10, values that are not statistically different either from one another or from 50 percent.[31] In line with the evidence on prices, we see no clear sign of learning, but our experimental subjects did indeed conclude fewer trades in the $3, 2C$ treatment, relative to the $1, 1$ treatment. More precisely, the disparity between the two treatments should be concentrated on realized pairs of valuations for which the theory predicts no trade in $3, 2C$ treatments–realizations such that the buyer's (the minority's) value is higher than the seller's (the majority's) but less than twice as high. The figures show quite clearly that in both treatments a large fraction of missed trading opportunities corresponded to realized draws close to the diagonal, as intuition suggests. But is the concentration significantly higher in $3, 2C$ sessions?

In Table 5 we regress the fraction of efficient trades realized in the two treatments on a constant, the round number, and three indicator variables. The first indicator captures whether data were generated in $1, 1$ or $3, 2C$ sessions; the second whether the valuation draws were in the critical area $v_b \in [v_s, 2v_s]$; the third indicator is the interaction term selecting instances where the valuation draws were in the critical area in $1, 1$ sessions. We report both logit and probit estimations.

As expected, when the valuations are in the critical area, the frequency of realized trades is smaller; both estimations however show that the effect is significantly larger in $3, 2C$ sessions, in line with theoretical predictions. In fact, again as predicted, all the difference between $1, 1$ and $3, 2C$ sessions is concentrated in this area: over the remaining range of value realizations, the coefficient of the indicator variable for $1, 1$ sessions is not significantly different from zero. The constant, capturing the frequency of trade common to both treatments when the buyer's value is more than twice the seller's, is predictably positive and large. The estimates show no significant evidence of learning.[32]

In the $3, 2$ treatment, the market design induces multiple trades within each

[31] The relevant Pearson χ^2 values are 0.08 for early versus late rounds, 1.45 for the late rounds versus 50 percent, and 1.94 for early rounds versus 50 percent. None of these values is significant at the 5 percent significance level.

[32] An unexpected finding in $3, 2C$ sessions is the presence of redundant trades: trades that do not change ownership of the decision power. Redundant trades are pure transfers with no allocative or efficiency effect; they are not plotted in Figure 4 and do not change any of our substantive results. They are however a sign of subjects' confusion and confirm the lack of ease with which subjects managed their multiple votes in the $3, 2C$ treatment.

VARIABLES	(1) probit	(2) logit
$1, 1$-sessions	-0.0474	-0.0812
	(0.136)	(0.231)
$v_b \in [v_s, 2v_s]$	-1.012***	-1.643***
	(0.164)	(0.273)
$v_b \in [v_s, 2v_s]$ interacted with $1, 1$-sessions	0.442**	0.713**
	(0.186)	(0.310)
Experience (Round)	0.00670	0.0108
	(0.00539)	(0.00887)
Constant	0.627***	1.025***
	(0.136)	(0.230)
Observations	1,124	1,124

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 5: Probability of Realizing an Efficient Trade

group. The validity of the theory should be tested on the basis of the final allocation of votes, when the market closes. The equilibrium price and the probabilities with which individuals with different valuations randomize between demanding one or two votes, or offering their vote for sale, depend on the realized vector of valuations. Figure 5 distinguishes the five possible scenarios. Ordering values as $v_5 \geq v_4 \geq ... \geq v_1$, the four panels correspond to (from left to right, starting on the top row): (1) $v_5 = \overline{v}_m$, $v_4 = \overline{v}_M$; (2) $v_5 = \overline{v}_M$, $v_4 = \overline{v}_m$; (3) $v_5 = \overline{v}_M$, $v_4 = v_{(2)M}$, $v_3 = \overline{v}_m$; (4) $v_5 = \overline{v}_m$, $v_4 = v_{(2)m}$; (5) $v_5 = \overline{v}_M$, $v_4 = v_{(2)M}$, $v_3 = \underline{v}_M$, the scenario where we expect the no-trade equilibrium to be focal. The red columns are the expected theoretical allocations of votes when the market closes, given the experimental valuation draws; the black columns report the number of votes in the data in rounds 11-25, and the grey columns in rounds 1-10.

The red columns reflect the equilibrium strategies described earlier: in all scenarios with trade, the largest purchases of votes should come from $\overline{v}_M$ and $\overline{v}_m$, regardless of the exact position that those two voters occupy in the ranking of valuations. The prediction is supported by the data: in all four of the top panels, the highest black columns correspond to $\overline{v}_M$ and $\overline{v}_m$. The result is remarkable because neither others' realized valuations nor one's own position in the ranking of valuations were known to the subjects, and the experimental design was such that valuations changed every

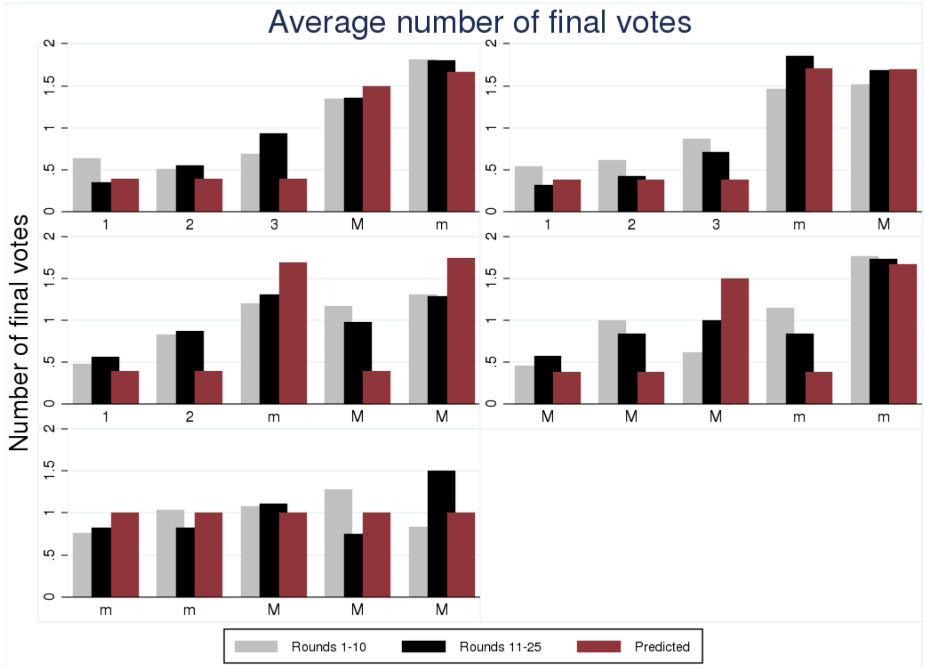

Figure 5: Average votes allocations by value ranking. Experimental results (grey: rounds 1-10; black: rounds 11-25) versus equilibrium (red columns).

round. And yet apparently, as theory predicts, the voters' relative rankings were revealed through trading. In the data, the voter whose valuation was closest to $\overline{v}_M$ and $\overline{v}_m$ tended to demand more votes than predicted: the black column corresponding to v_3 in the top two panels and to v_4 in the second two is consistently higher than the red column. In all four cases, however, the number of votes remains below the number acquired by $\overline{v}_M$ and $\overline{v}_m$, even when, as in the second two panels, the value of the voter in question was intermediate between $\overline{v}_M$ and $\overline{v}_m$. There is some weak appearance of learning: in three of the four cases, the black columns are closer to equilibrium than the grey columns. It is noticeable too that the concentration of votes in the hands of $\overline{v}_M$ and $\overline{v}_m$ is always observed in the data, except when $\overline{v}_m < \underline{v}_M$ and the no-trade equilibrium seems particularly plausible. Although the data show that some trade did take place, the dispersion in votes holdings across members of the group is much less pronounced than in all other scenarios, and this is true both in early and late rounds.

Summarizing then, the data show convergence towards equilibrium prices in $1,1$ and $3,2$ treatments, but not in $3,2C$ treatments. We find more trade in $1,1$ than in $3,2C$ treatments, as expected, and again as expected, the difference arises mostly from lack of trade when the traders' values are close to one another. However the data also show that the higher propensity to trade in $1,1$ treatments is accompanied by a higher propensity for inefficient trades. Finally, the votes allocations in the $3,2$ market appear in line with theory: votes are bought by the two top value members in each group, regardless of their ranking over the full profile of valuations, unless all majority voters have higher values than any minority voter. We find this last result particularly striking because relative voters' values, within each group and over the full committee, were not revealed in the experiment and in general changed with each round.

If the theory does reasonably well in explaining the trade data, can it also predict voting outcomes and realized payoffs?

4.3 Outcomes

Because price data show evidence of learning (in $1,1$ and $3,2$ sessions), later rounds may be more representative of subjects' intended behavior, and the results we report below are based on rounds 11 to 25. But because allocations data show little if any learning, the results are in fact qualitatively identical over the full data set.

How much surplus were the experimental subjects able to appropriate, in the different treatments? Figure 6 shows the aggregate payoff in each experimental session, on the vertical axis, versus the corresponding payoff if all subjects had played the equilibrium strategies, on the horizontal axis. Both payoffs are expressed as share of the first-efficient payoff (the maximal aggregate payoff) and all numbers are calculated on the basis of the realized experimental valuation draws. The symbols distinguish the different treatments: circles are $1,1$ sessions; squares are sessions with groups of unequal size, with solid squares corresponding to $3,2C$ sessions and empty squares to $3,2$ sessions. The diagonal line is the 45 degree line; thus points above the line indicate experimental payoffs in excess of the theoretical prediction, and points below payoffs that fall short of equilibrium payoffs.

Experimental payoffs are lower than equilibrium predictions in $1,1$ sessions–it could hardly be otherwise, since the theory predicts full efficiency, but the result also reflects the unexpected frequency of inefficient trades. They are comparable to equilibrium payoffs in $3,2C$ sessions, and higher in the $3,2$ market sessions. The more noticeable feature of the figure is the disparity between the clear efficiency rankings of the theory and the more uniform level of the experimental payoffs. While the different treatments are distinctly organized along the horizontal axis, experimental payoffs as share of efficiency, on the vertical axis, are similar across treatments.

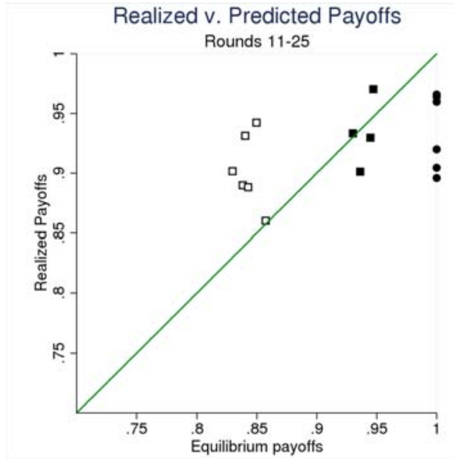

Figure 6: Experimental payoff per session versus equilibrium (as share of efficiency). Rounds 11-25. Solid white dots correspond to $1, 1$ sessions; solid squares to $3, 2C$ sessions, and empty squares to $3, 2$.

The similarity hides, however, systematically different mistakes. Consider in particular treatments $3, 2C$ and $3, 2$. Recall that the theory predicts too little trade and thus too few minority victories in treatment $3, 2C$, and too much trade, and too many minority victories, in treatment $3, 2$. Figure 7 disaggregates the source of efficiency losses for these treatments. The diagonal lines in the figure are iso-efficiency loss lines. The origin, at 0,0, denotes full efficiency, or the maximum possible aggregate payoff; moving up and to the right, the efficiency losses increase, with the three iso-loss lines corresponding to losses of 2, 6 and 10 percent respectively. The horizontal axis measures losses from missed trading opportunities: instances where the aggregate experimental values of the minority group were higher than the majority's and yet the majority won. The vertical axis measures losses from inefficient trades: instances where the aggregate experimental values of the minority group were lower than the majority's and yet the minority won. The symbols are as in Figure 6: solid squares are $3, 2C$ treatments, and empty squares $3, 2$.

In $3, 2C$ sessions, the source of losses is almost exclusively missed trading opportunities, or inefficient majority victories. Two of the sessions sit exactly on the horizontal axis: there is not a single instance where the minority wins with lower aggregate values. Nevertheless, those two sessions show an average loss of 5 percent of efficiency because of unrealized gains from trade. In $3, 2$ treatments, the result is the opposite: efficiency losses come primarily from too much trade and inefficient minority victories. Again, in two of the sessions there is not a single instance when the majority wins with lower aggregate values. But substantial losses are realized

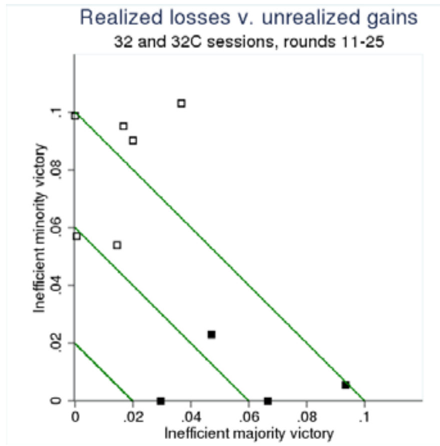

Figure 7: Realized losses versus unrealized gains, both as share of full efficiency. Rounds 11-25. The diagonal lines are iso-efficiency loss lines. Solid squares are $3, 2C$ sessions; empty squares $3, 2$.

because the minority wins too often.

Figure 8 tests directly how the fraction of minority victories in $3, 2C$ and $3, 2$ data compares to equilibrium predictions and to the efficient fraction of victories. The first panel plots the realized frequency of minority victories in each experimental session, on the vertical axis, and the frequency predicted by the theoretical models, given the realized experimental draws, on the horizontal axis. In $3, 2C$ sessions, the minority won slightly more than the theory predicts.[33] In $3, 2$ sessions, on the other hand, the minority systematically won less frequently than theory predicts. Nevertheless, in *all* $3, 2$ sessions, realized minority victories were more frequent than in *any* $3, 2C$ session.[34] The second panel compares the fraction of minority victories in each experimental session to the efficient fraction, again calculated for each session according to the realized experimental draws. Again, the predictive power of the theory is strongly supported. *Every* point representing a $3, 2$ session is above the 45 degree line, and *every* point representing a $3, 2C$ session is below: as expected, the minority wins too much in $3, 2$ sessions, and too little in $3, 2C$ sessions. (In the

[33] A possible conjecture is that the continuous auction format of the experimental design allowed more communication than the sealed bid auction of the theoretical model. In other experiments, direct communication has been shown to increase trade and in fact induce more trade than predicted by Myerson and Satterthwaite's optimal contract (Valley et al., 2002).

[34] The two treatments are not immediately comparable because in $3, 2C$ sessions each leader's value in the experiment was a single draw, as opposed to multiple draws in $3, 2$ sessions. Had leaders' values in $3, 2C$ sessions resulted from summing multiple draws, on average the ratio of minority to majority value would have been lower, leading to fewer theoretical and most likely experimental minority victories. The different design thus makes our conclusions stronger. It does not influence the comparison of the data to the theory.

second panel, the solid square located most to the right hides a second solid square with almost identical values).

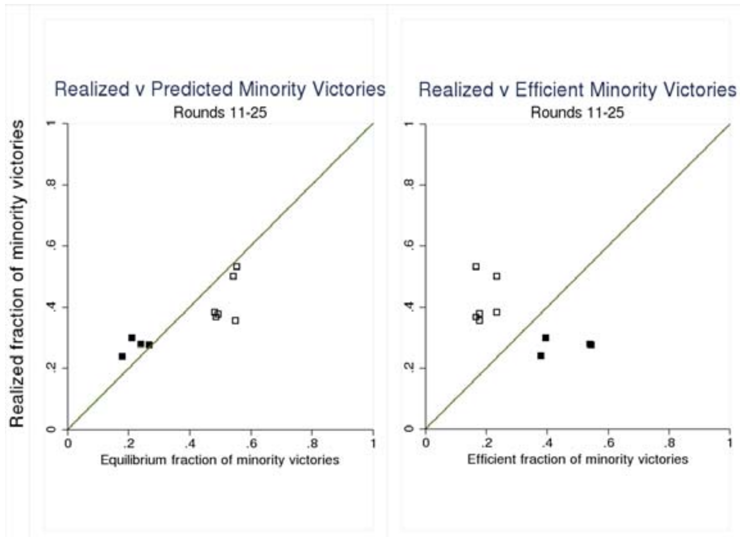

Figure 8: Realized frequency of minority victories, versus equilibrium (panel 1) and effient frequency (panel 2). Rounds 11-25. Solid squares are $3, 2C$ sessions; empty squares $3, 2$.

The conclusion is very robust and remains true at the level of the individual groups, even with the inevitable added noise. In Figure 9, each dot corresponds to the fraction of minority victories for each group label, in the two treatments. As in Figure 8, the fraction realized in the experiment is compared to the theoretical prediction, in the first panel, and to the efficient prediction, in the second panel. Both equilibrium and efficient minority victories are calculated on the basis of the realized experimental values draws for each group. In the first panel, solid squares $(3, 2C)$ remain mostly above the 45 degree line and empty squares $(3, 2)$ below, with some tendency to align themselves along the 45 degree line; in the second panel, as expected all empty squares are above (or on) the 45 degree line, and all solid squares but one are below.

We want to test whether the frequencies of minority victories observed in the experiment are significantly different across the two treatments. We can use the efficient frequency of minority victories as a means of normalizing the value draws, and test whether, across groups, the ratios of realized to efficient minority victories observed in the two treatments could be drawn from the same sample. The hypothesis is strongly rejected by a two-sided Kolmogorov-Smirnov test: the D statistic is 0.96, with p-value 0.000.

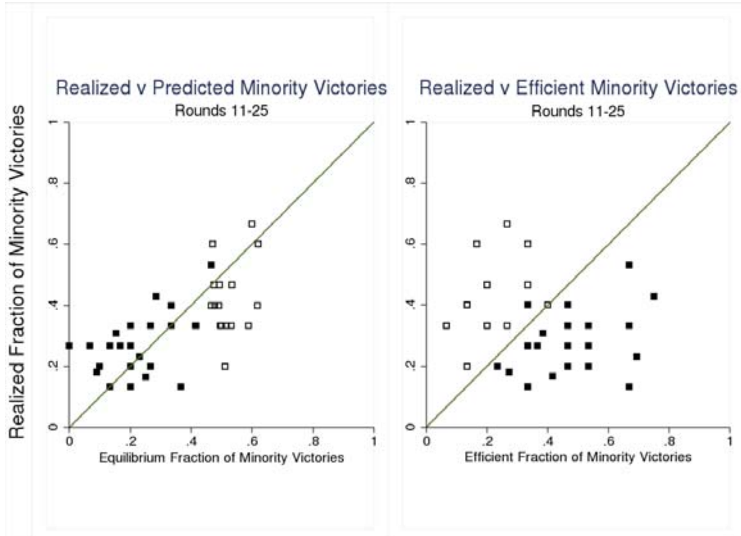

Figure 9: Realized frequency of minority victories by group versus equilibrium (panel 1) and efficient frequency (panel 2). Rounds 11-25. Solid squares are $3, 2C$ sessions, empty squares $3, 2$.

The theory has sharp predictions on the efficiency of vote trading relative to majority voting with no trade in the different treatments. Figure 10 plots aggregate experimental payoffs per session, on the vertical axis, versus aggregate session payoffs in the absence of trade, on the horizontal axis. Both measures are expressed as share of maximal possible payoffs.

The results strongly support the theory. As expected, payoffs were consistently higher than in the absence of trade in the two treatments with leaders' trading, and consistently lower in the treatment with market trading. Here too the theoretical prediction is confirmed in every single experimental session.[35]

5 Conclusions

The objective of this paper is a better understanding of vote trading in committees and legislature that operate under simple majority rule.

On the theoretical side, we show that standard economic models of bargaining and exchange can be appropriately modified or reinterpreted to provide tractable

[35] If the design of the $32C$ treatment had included multiple value draws for each side, the payoffs with majority voting in the absence of trade would have been higher. The scarcity of mistaken minority victories reported in Figure 7 suggests that the experimental payoffs would also have been higher. The theoretical prediction about the superiority of payoffs with vote trading, and thus in the experiment, would not have changed.

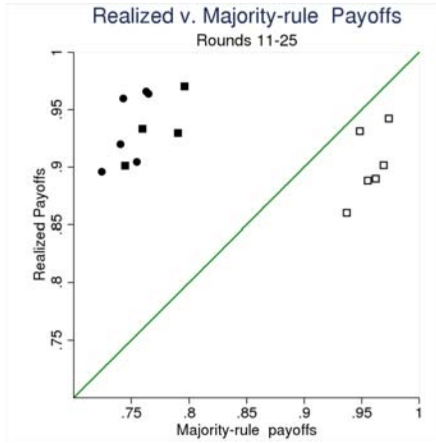

Figure 10: Realized payoff per session versus no-trade payoff (as share of efficiency). Rounds 11-25. Solid dots are $1,1$ sessions; solid squares $3,2C$ sessions, and empty squares $3,2$.

equilibrium models of vote trading. If vote trading is centralized, in the sense that there is a single representative of the interests of each side of an issue - for example, a party leader - then results from the mechanism design approach to bargaining theory translate directly to voting environments. Two different results emerge in this regard, depending on the relative sizes of the majority and minority parties. If the size of the two parties is exactly equal, then vote trading can theoretically lead to a first best outcome. This follows from an application of the main result in Cramton et al. (1989). If the size of the two parties is not exactly equal, then vote trading improves over majority rule without trade, but cannot lead to a first best outcome. There is too little vote trading: the majority wins too often. This follows from an application of the main result in Myerson and Satterthwaite (1982).

If vote trading is decentralized, in the sense that all trading takes place between individual party members rather than being coordinated by party leaders, then the standard general equilibrium model of competitive markets can be adapted to these voting environments. For this purpose, we extend the notion of ex ante competitive equilibrium developed by Casella et al. (2011) to an environment where the size of the two opposing groups is known. For the parameterization used in the experiment, we prove that an ex ante equilibrium exists and exhibits a significant volume of trade. In fact, because the competition for votes between the two groups depends on the relative intensities of preferences of the highest intensity majority voter and highest intensity minority voter, regardless of the size of the two groups, there is too much trade. Relative to utilitarian welfare or ex ante efficiency, the minority wins too often, and the theory predicts efficiency *losses* relative to the no-trade voting outcome.

We conduct laboratory experiments to explore the extent to which the actual outcomes in committees correspond to the equilibrium outcomes of the theoretical models of exchange. In line with the theoretical predictions, we observe efficiency gains to vote trading only in the case where it is centralized through party leaders. However, the efficiency gains with equal sized committees fall short of the first best. We observe efficiency losses in the experimental committees that engage in decentralized trade. Again in line with theory, in every single experimental session we observe too few minority victories, relative to first best efficiency, if trading occurs through party leaders, and too many if it occurs through the market. The transaction prices in our vote markets always start out above the equilibrium prices. With experience, prices converge to the theoretical equilibrium prices when there is decentralized trade and when there is centralized trading between equal sized parties. In the markets with centralized trading between unequal sized parties, prices persist above the theoretical equilibrium.

Our theoretical results can be extended in a number of directions. First, the main conclusions of the ex ante equilibrium in the competitive model–the competition for dictatorship between the highest value voters in the two groups, the high ratio of intra-group trade, the excessive frequency of minority victories–need to be verified for arbitrary committee sizes and arbitrary partitions in majority and minority voters. Preliminary results suggest that the generalization is likely to hold. Second, it is clearly important to allow committees to consider more than one issue, and thus introduce the possibility of log-rolling, or vote trading *across* issues. This variety of vote trading is believed to be common practice in real committees, and could be accomplished with or without the use of a numeraire commodity. Third, it would be interesting to study more general specifications of preferences, in particular the spatial representation of preferences that has become the standard model for theoretical and empirical work in political science. Finally, our model does not address the complex strategic issues related to agenda setting and proposal power. We have taken as exogenous the proposal to be voted upon. In practice, votes are taken only after a proposal has been made, and proposal-making itself would need to consider the possibility of vote trading that can take place between the proposal stage and the voting stage. One might conjecture that vote trading could dilute the proposal power of the agenda setter.

If our theoretical results are robust to these generalizations, they suggest interesting lines of thought for empirical work. If party discipline translates into more control by party leaders and more centralized trading, according to our analysis stronger party discipline will also imply fewer vote trades and fewer minority victories. In principle at least this could be tested.

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Appendix A

Example. *Imagine a full information scenario where* $v_i = 1$ *for all* $i \in M$*,* $v_i = 2$ *for all* $i \in m$*, and* $n = 5$*. The rationing rule is* $R1$*. Then, there exists an ex ante competitive equilibrium such that one minority voter demands two votes, one majority voter randomizes between demanding two votes (with probability* $2/3$*) and selling his vote, and all other voters offer to sell. The equilibrium price is* $p = 1/3$*.*

For consistency with notation used later in the analysis, call $\overline{v}_m$ the minority voter whose candidate equilibrium strategy is to demand two votes, and $\overline{v}_M$ the majority voter whose candidate strategy is to randomize between demanding two votes and selling ($v_{i\in M}$ and $v_{i\in M}$ will then be the members of each group whose candidate strategy is to sell). First, note that the strategies described satisfy expected market balance: $\sum \delta_i = 2 + (2/3)(2) - 3 - (1/3)(1) = 0$. Second, consider each voter in turn and verify that at $p = 1/3$, no-one has an incentive to deviate. Consider for example $\overline{v}_M$. Given others' demands, demanding four or more votes guarantees being rationed and is equivalent to staying out of the market, an action dominated by selling. Demanding three votes has equal probability of being rationed as demanding two votes, but higher expenditure and equal probability of victory if not rationed; it is dominated by demanding two votes. A demand of one vote is never rationed, but the voter always loses and pays p; it is dominated by selling. Expected utility from selling equals $p/2$; expected utility from demanding two votes equals $(1/2)(\overline{v}_M - 2p)$. At $p = 1/3$ and $\overline{v}_M = 1$, $p/2 = (1/2)(\overline{v}_M - 2p) = 1/6$. Voter $\overline{v}_M$ is indeed indifferent between offering to sell and demanding two votes, and any other action is dominated. The same reasoning establishes that demanding three or more votes or staying out of the market are both dominated by selling for all voters. As for the other actions, it is trivial to compute the corresponding expected utilities and verify the claim above. Call $EU_{v_i}A$ the expected utility of voter with value v_i from action $A \in \{S, D1, D2\}$, with obvious notation. Given $v_i = 1$ for all $i \in M$, $v_i = 2$ for all $i \in m$, we find: $EU_{\overline{v}_m}D2 = 8/9$; $EU_{\overline{v}_m}D1 = -1/3$ if $m = 1$ and $1/6$ if $m = 2$; $EU_{\overline{v}_m}S = 1/9$; $EU_{v_{i\in M}}D2 = 19/54$; $EU_{v_{i\in M}}D1 = 14/54$; $EU_{v_{i\in M}}S = 29/54$; and finally, if $m = 2$, $EU_{v_{i\in m}}D2 = 52/54$; $EU_{v_{i\in M}}D1 = 74/54$; $EU_{v_{i\in M}}S = 83/54$. The claim in the example is established.

Proposition 1. *Suppose* $M = 3$ *and* $m = 2$*. Then for all realizations of* $\{v_1, .., v_5\}$ *such that* $\overline{v}_m \geq \underline{v}_M$*, a fully revealing ex ante equilibrium exists. The equilibrium price is always such that* $\min(\overline{v}_m, \overline{v}_M)$ *is indifferent between demanding two votes and selling his vote.*

We prove the Proposition by constructing an equilibrium. As in the text, call g' the group such that $\overline{v}_{g'} \geq \overline{v}_g$, and call $v_{(2)g'}$ the second highest value in group g'. The equilibrium is characterized in the following three lemmas.

Lemma 1 *If $\overline{v}_g \geq (2/7)v_{(2)g'}$, then there exists a fully revealing ex ante equilibrium such that voters $\overline{v}_{g'}$ and $\overline{v}_g$ randomize between demanding two votes and selling their vote (with probabilities $\sigma_{\overline{v}_{g'}}$,and $\sigma_{\overline{v}_g}$) and all other voters offer to sell their vote. The randomization probabilities and the equilibrium price depend on the value realizations. In particular: (a) If $\overline{v}_m \in [(2/7)v_{(2)M}, (3/5)\overline{v}_M]$, then $\sigma_{\overline{v}_M} = 0$, $\sigma_{\overline{v}_m} = 1/3$, and $p = \overline{v}_m/3$; (b) If $\overline{v}_m \in (3/5\overline{v}_M,\ 5/6\overline{v}_M)$, then $\sigma_{\overline{v}_M}$, $\sigma_{\overline{v}_m}$, and p are solutions to the system:*

$$\begin{aligned} 1 &= 3(\sigma_{\overline{v}_M} + \sigma_{\overline{v}_m}) \\ p &= \overline{v}_M \left(\frac{1 - \sigma_{\overline{v}_m}}{3 + \sigma_{\overline{v}_m}} \right) \\ p &= \overline{v}_m \left(\frac{1 + \sigma_{\overline{v}_M}}{3 + \sigma_{\overline{v}_M}} \right) \end{aligned} \qquad \text{(S1)}$$

(iv) If $\overline{v}_M \in [(2/7)v_{(2)m}, (6/5)\overline{v}_m]$, then $\sigma_{\overline{v}_M} = 1/3$, $\sigma_{\overline{v}_m} = 0$, and $p = \overline{v}_M/3$.

Lemma 2 *If $\overline{v}_g \in [(1/14)v_{(2)g'}, (2/7)v_{(2)g'}]$, then there exists a fully revealing ex ante equilibrium such that voter $\overline{v}_{g'}$ demands two votes, $v_{(2)g'}$ randomizes between demanding one vote and offering his vote for sale (with probability $\sigma_{v_{(2)g'}}$),$\overline{v}_g$ randomizes between demanding two votes and selling his vote (with probability $\sigma_{\overline{v}_g}$), and all other voters offer to sell their vote. The randomization probabilities and the equilibrium price are solutions to the system:*

$$\begin{aligned} 3 &= 2\sigma_{v_{(2)g'}} + 3\sigma_{\overline{v}_g} \\ p &= v_{(2)g'} \left(\frac{1 - \sigma_{\overline{v}_g}}{6 + 3\sigma_{\overline{v}_g}} \right) \\ p &= \overline{v}_g \left(\frac{2 + \sigma_{v_{(2)g'}}}{10 - \sigma_{v_{(2)g'}}} \right) \end{aligned} \qquad \text{(S2)}$$

Lemma 3 *If $\overline{v}_g \leq (1/14)v_{(2)g'}$, then for all realizations of $\{v_1, .., v_5\}$ such that $\overline{v}_m \geq \underline{v}_M$ there exists a fully revealing ex ante equilibrium such that voters $\overline{v}_{g'}$ and $v_{(2)g'}$ demand one vote, $\overline{v}_g$ randomizes between demanding two votes and selling his vote (with probability $\sigma_{\overline{v}_g} = 2/3$) and all other voters offer to sell their vote. The equilibrium price is $\overline{v}_g/4$.*

We reproduce below the proof of Lemma 1. The proofs of Lemmas 2 and 3 are similar and the details are available as an online supplementary material. (See Appendix B of this manuscript version).

Proof of Lemma 1. If voters' preferred alternative is known, establishing that the candidate strategies and price are an equilibrium follows immediately from comparing the expected utilities of different actions, given others' strategies. Call $EU_{v_{ig}}A$ the expected utility of voter with value v_i belonging to group g from action $A \in \{S, 0, D1, D2\}$, with obvious notation. In this case, allowing for both $\sigma_{\overline{v}_M} > 0$ *and* $\sigma_{\overline{v}_m} > 0$:

$$\begin{aligned}
EU_{\overline{v}_M}D2 &= (\overline{v}_M - 2p)(1 + \sigma_{\overline{v}_m})/2 \\
EU_{\overline{v}_M}D1 &= \sigma_{\overline{v}_m}\overline{v}_M - p \\
EU_{\overline{v}_M}0 &= \sigma_{\overline{v}_m}\overline{v}_M \\
EU_{\overline{v}_M}S &= \sigma_{\overline{v}_m}\overline{v}_M + (1 - \sigma_{\overline{v}_m})p/2
\end{aligned}$$

$$\begin{aligned}
EU_{\overline{v}_m}D2 &= (\overline{v}_m - 2p)(1 + \sigma_{\overline{v}_M})/2 \\
EU_{\overline{v}_m}D1 &= \sigma_{\overline{v}_M}3\overline{v}_m/4 - p \\
EU_{\overline{v}_m}0 &= 0 \\
EU_{\overline{v}_m}S &= (1 - \sigma_{\overline{v}_M})p/2
\end{aligned}$$

$$\begin{aligned}
EU_{v_{iM}}D2 &= \sigma_{\overline{v}_M}[\sigma_{\overline{v}_m}(v_{iM} - 2p) + (1 - \sigma_{\overline{v}_m})(v_{iM}/2 - p)] + \\
&\quad +(1 - \sigma_{\overline{v}_M})[\sigma_{\overline{v}_m}(v_{iM} - p) + 2(1 - \sigma_{\overline{v}_m})(v_{iM} - p)/3] \\
EU_{v_{iM}}D1 &= \sigma_{\overline{v}_M}[\sigma_{\overline{v}_m}(v_{iM} - p) - (1 - \sigma_{\overline{v}_m})p] + \\
&\quad +(1 - \sigma_{\overline{v}_M})[\sigma_{\overline{v}_m}(v_{iM} - p) + (1 - \sigma_{\overline{v}_m})(2v_{iM} - p)/3] \\
EU_{v_{iM}}0 &= \sigma_{\overline{v}_m}[\sigma_{\overline{v}_M}v_{iM} + (1 - \sigma_{\overline{v}_M})v_{iM}] + (1 - \sigma_{\overline{v}_m})(1 - \sigma_{\overline{v}_M})v_{iM}/2 \\
EU_{v_{iM}}S &= \sigma_{\overline{v}_M}[\sigma_{\overline{v}_m}v_{iM} + (1 - \sigma_{\overline{v}_m})p/2] + \\
&\quad +(1 - \sigma_{\overline{v}_M})[\sigma_{\overline{v}_m}(v_{iM} + p/2) + (1 - \sigma_{\overline{v}_m})(v_{iM}/2 + 2p/3)]
\end{aligned} \tag{S3}$$

$$\begin{aligned}
EU_{v_{im}}D2 &= \sigma_{\overline{v}_M}[\sigma_{\overline{v}_m}(v_i - 2p) + (1 - \sigma_{\overline{v}_m})(v_{im} - p)] + \\
&\quad +(1 - \sigma_{\overline{v}_M})[\sigma_{\overline{v}_m}(v_{im}/2 - p) + 2(1 - \sigma_{\overline{v}_m})(v_{im} - p)/3] \\
EU_{v_{im}}D1 &= \sigma_{\overline{v}_M}[\sigma_{\overline{v}_m}(3v_{im}/4 - p) + (1 - \sigma_{\overline{v}_m})(v_{im} - p)] + \\
&\quad +(1 - \sigma_{\overline{v}_M})[\sigma_{\overline{v}_m}(-p) + (1 - \sigma_{\overline{v}_m})(2v_{im} - p)/3] \\
EU_{v_{im}}0 &= (1 - \sigma_{\overline{v}_m})[\sigma_{\overline{v}_M}v_{im} + (1 - \sigma_{\overline{v}_M})v_{im}/2] \\
EU_{v_{im}}S &= (1 - \sigma_{\overline{v}_m})\sigma_{\overline{v}_M}(v_{im} + p/2) + (1 - \sigma_{\overline{v}_M})[\sigma_{\overline{v}_m}p/2 + (1 - \sigma_{\overline{v}_m})(v_{im}/2 + 2p/3)]
\end{aligned}$$

where $v_{iM} \leq \overline{v}_M$, and $v_{im} \leq \overline{v}_m$. Expected market balance requires $\sum \delta_i = 0$, where δ_i is individual i's expected demand, or $2(1-\sigma_{\overline{v}_M}) + 2(1-\sigma_{\overline{v}_m}) = 3 + \sigma_{\overline{v}_M} + \sigma_{\overline{v}_m}$, or $\sigma_{\overline{v}_M} + \sigma_{\overline{v}_m} = 1/3$. Given the equations in (S2), it follows immediately that $\overline{v}_M$ and $\overline{v}_m$ are both indifferent between $D2$ and S if:

$$\begin{aligned} p &= \overline{v}_M \left(\frac{1-\sigma_{\overline{v}_m}}{3+\sigma_{\overline{v}_m}} \right) \\ p &= \overline{v}_m \left(\frac{1+\sigma_{\overline{v}_M}}{3+\sigma_{\overline{v}_M}} \right) \end{aligned}$$

It is not difficult to verify that expected market balance and the two indifference conditions can be satisfied simultaneously at $\sigma_{\overline{v}_M} \in (0,1]$, $\sigma_{\overline{v}_m} \in (0,1]$ only if $\overline{v}_m \in (3/5\overline{v}_M,\ 5/6\overline{v}_M)$. If $\overline{v}_m \geq (5/6)\overline{v}_M$, the price that makes $\overline{v}_M$ indifferent between $D2$ and S is too low to induce $\overline{v}_m$ to sell with positive probability: the equilibrium must then have $\sigma_{\overline{v}_m} = 0$, $\sigma_{\overline{v}_M} = 1/3$, and $p = \overline{v}_M/3$. If $\overline{v}_M \geq (5/3)\overline{v}_m$, the price that makes $\overline{v}_m$ indifferent between $D2$ and S is too low to induce $\overline{v}_M$ to sell with positive probability: the equilibrium must have $\sigma_{\overline{v}_M} = 0$, $\sigma_{\overline{v}_m} = 1/3$, and $p = \overline{v}_m/3$. Establishing that the stated strategies are best responses to each other is trivial, given p, $\sigma_{\overline{v}_m}$ and $\sigma_{\overline{v}_M}$ and the equations in (S3). In addition, if $\overline{v}_m > \overline{v}_M$, the condition $v_{im} \leq (7/2)\overline{v}_M$ is required to prevent the profitable deviation of voter v_{im} to demanding a positive number of votes; similarly if $\overline{v}_M > \overline{v}_m$, the condition $v_{iM} \leq (7/2)\overline{v}_m$ is required to guarantee that selling is a best response for voter v_{iM}. The intuition is straightforward: if $v_{im} > (7/2)\overline{v}_M$, the price that makes $\overline{v}_M$ indifferent between selling and demanding two votes is too low to induce v_{im} to sell, as this equilibrium prescribes; and similarly if $v_{iM} > (7/2)\overline{v}_m$.

Finally, we need to show that the equilibrium is fully revealing. First notice that there can be no equilibrium with trade where both m members offer to sell their vote with probability one–because no M member would have an incentive to buy. Hence $\overline{v}_M$ knows that in equilibrium the other voter with positive expected demand belongs to group m; of the sellers, two must belong to M and one to m. Consider now the problem from the point of view of $\overline{v}_m$. Given others' equilibrium strategies, expected market balance requires $\overline{v}_m$ to demand a positive number of votes. It is not difficult to verify that at p if the voter mixing between $D2$ and S with probability $\sigma_{\overline{v}_M}$ belonged to m, $\overline{v}_m$'s best response is S. However, S does not satisfy expected market balance. Hence $\overline{v}_m$ knows that in equilibrium the other voter with positive expected demand belongs to group M; again, of the sellers, two must belong to M and one to m. As for the sellers, market balance requires each of them to sell with probability one. Among them, each M member knows that the two voters with positive expected demand cannot both belong to group m, by the argument above; not can they both belong to group M, because in equilibrium at least one m member must demand votes with positive probability. Similarly, the m seller knows that the other m member cannot also be selling with probability one. Hence, each seller knows that one but not both of the voters with positive demands must belong to his own group; the seller cannot know which one, but is indifferent: the unique best response is to sell. Thus the equilibrium is indeed fully revealing.$\square$ $\blacksquare$

Appendix B (Online supplementary material. Not intended for printed publication.)

Lemma 2. *If $\overline{v}_g \in [(1/14)v_{(2)g'}, (2/7)v_{(2)g'}]$, then there exists a fully revealing ex ante equilibrium such that voter $\overline{v}_{g'}$ demands two votes, $v_{(2)g'}$ randomizes between demanding one vote and offering his vote for sale (with probability $\sigma_{v_{(2)g'}}$),$\overline{v}_g$ randomizes between demanding two votes and selling his vote (with probability $\sigma_{\overline{v}_g}$), and all other voters offer to sell their vote. The randomization probabilities and the equilibrium price are solutions to the system:*

$$\begin{aligned} 3 &= 2\sigma_{v_{(2)g'}} + 3\sigma_{\overline{v}_g} \\ p &= v_{(2)g'}\left(\frac{1-\sigma_{\overline{v}_g}}{6+3\sigma_{\overline{v}_g}}\right) \\ p &= \overline{v}_g\left(\frac{2+\sigma_{v_{(2)g'}}}{10-\sigma_{v_{(2)g'}}}\right) \end{aligned} \tag{S2}$$

Proof of Lemma 2.We need to distinguish the two possible cases: $g' = m$, and $g' = M$. (1) If $g' = m$, each voter's preferred alternative is known, and each expects the others to follow the strategies described in the lemma, the expected utilities of different actions are given by:

$$\begin{aligned} EU_{\overline{v}_m}D2 &= \sigma_{\overline{v}_M}\sigma_{v_{(2)m}}(\overline{v}_m - 2p) + \sigma_{\overline{v}_M}(1-\sigma_{v_{(2)m}})(\overline{v}_m - 2p) + \\ &\quad +(1-\sigma_{\overline{v}_M})\sigma_{v_{(2)m}}(\overline{v}_m/2 - p) + 2(1-\sigma_{\overline{v}_M})(1-\sigma_{v_{(2)m}})(\overline{v}_m - p)/3 \\ EU_{\overline{v}_m}D1 &= \sigma_{\overline{v}_M}\sigma_{v_{(2)m}}(3\overline{v}_m/4 - p) + \sigma_{\overline{v}_M}(1-\sigma_{v_{(2)m}})(\overline{v}_m - p) + \\ &\quad +(1-\sigma_{\overline{v}_M})\sigma_{v_{(2)m}}(-p) + 2(1-\sigma_{\overline{v}_M})(1-\sigma_{v_{(2)m}})(\overline{v}_m - p)/3 \\ EU_{\overline{v}_m}0 &= (1-\sigma_{v_{(2)m}})\sigma_{\overline{v}_M}\overline{v}_m + (1-\sigma_{\overline{v}_M})(1-\sigma_{v_{(2)m}})\overline{v}_m/2 \\ EU_{\overline{v}_m}S &= (1-\sigma_{v_{(2)m}})\sigma_{\overline{v}_M}(3\overline{v}_m/4 + p/4) + (1-\sigma_{\overline{v}_M})\sigma_{v_{(2)m}}p/2 \\ &\quad +(1-\sigma_{\overline{v}_M})(1-\sigma_{v_{(2)m}})p \end{aligned}$$

$$\begin{aligned} EU_{v_{(2)m}}D2 &= \sigma_{\overline{v}_M}(v_{(2)m} - p) + 2(1-\sigma_{\overline{v}_M})(v_{(2)m} - p)/3 \\ EU_{v_{(2)m}}D1 &= \sigma_{\overline{v}_M}(v_{(2)m} - p) + 2(1-\sigma_{\overline{v}_M})(v_{(2)m} - p/2)/3 \\ EU_{v_{(2)m}}0 &= \sigma_{\overline{v}_M}v_{(2)m} + (1-\sigma_{\overline{v}_M})v_{(2)m}/2 \\ EU_{v_{(2)m}}S &= \sigma_{\overline{v}_M}(v_{(2)m} + p/2) + (1-\sigma_{\overline{v}_M})(v_{(2)m}/2 + 2p/3) \end{aligned} \tag{S4}$$

$$\begin{aligned}
EU_{\overline{v}_M}D2 &= \sigma_{v_{(2)m}}(\overline{v}_M/2-p)+(1-\sigma_{v_{(2)m}})(\overline{v}_M/3-2p/3) \\
EU_{\overline{v}_M}D1 &= \sigma_{v_{(2)m}}(-p)+(1-\sigma_{v_{(2)m}})(-2p/3) \\
EU_{\overline{v}_M}0 &= 0 \\
EU_{\overline{v}_M}S &= \sigma_{v_{(2)m}}p/2+(1-\sigma_{v_{(2)m}})p
\end{aligned}$$

$$\begin{aligned}
EU_{v_{iM}}D2 &= \sigma_{\overline{v}_M}\sigma_{v_{(2)m}}(v_{iM}/2-p)+\sigma_{\overline{v}_M}(1-\sigma_{v_{(2)m}})(v_{iM}-2p)/3+ \\
&\quad +(1-\sigma_{\overline{v}_M})2\sigma_{v_{(2)m}}(v_{iM}-p)/3 \\
EU_{v_{iM}}D1 &= \sigma_{\overline{v}_M}\sigma_{v_{(2)m}}(-p)+\sigma_{\overline{v}_M}(1-\sigma_{v_{(2)m}})(-2p/3)+ \\
&\quad +(1-\sigma_{\overline{v}_M})\sigma_{v_{(2)m}}(2v_{iM}-p)/3+(1-\sigma_{\overline{v}_M})(1-\sigma_{v_{(2)m}})(v_{iM}-p)/2 \\
EU_{v_{iM}}0 &= (1-\sigma_{\overline{v}_M})\sigma_{v_{(2)m}}v_{iM}/2 \\
EU_{v_{iM}}S &= \sigma_{\overline{v}_M}\sigma_{v_{(2)m}}p/2+\sigma_{\overline{v}_M}(1-\sigma_{v_{(2)m}})p+(1-\sigma_{\overline{v}_M})\sigma_{v_{(2)m}}(v_{iM}/2+2p/3)+ \\
&\quad +(1-\sigma_{\overline{v}_M})(1-\sigma_{v_{(2)m}})(v_{iM}/3+5p/6)
\end{aligned}$$

with $v_{iM}\leq\overline{v}_M<v_{(2)m}$. Expected market balance requires $2+(1-\sigma_{v_{(2)m}})+2(1-\sigma_{\overline{v}_M})=2+\sigma_{v_{(2)m}}+\sigma_{\overline{v}_M}$, or $3=2\sigma_{v_{(2)m}}+3\sigma_{\overline{v}_M}$, which implies $\sigma_{\overline{v}_M}\in[1/3,1]$. Given system (S4), it is cumbersome but not difficult to verify that the strategies assigned to each voter are best responses to others' stated strategies and to the price if, conditional on expected market balance, p, $\sigma_{v_{(2)m}}$, and $\sigma_{\overline{v}_M}$ satisfy:

$$\begin{aligned}
p &= v_{(2)m}\left(\frac{1-\sigma_{\overline{v}_M}}{6+3\sigma_{\overline{v}_M}}\right) \\
p &= \overline{v}_M\left(\frac{2+\sigma_{v_{(2)m}}}{10-\sigma_{v_{(2)m}}}\right)
\end{aligned}$$

and $\overline{v}_M\in[(1/14),v_{(2)m},(2/7)v_{(2)m}]$.[36]

To conclude the proof, we thus need to show that when voters preferences are not known ex ante, the equilibrium is fully revealing. We proceed in steps. Consider first $\overline{v}_m$. The question is whether in equilibrium $\overline{v}_m$ can identify that the other m member must be the individual randomizing between $D1$ and S. The possible alternative scenarios are described in Table A1. The first row identifies the group of the individual whose perspective we are taking, and the constraint that expected

[36] The lower bound on acceptable $\overline{v}_M$ values is not necessary: the equilibrium exists for all $\overline{v}_M\leq(2/7)v_{(2)m}$. A corresponding restriction however is required when $g'=M$. We impose it in this case too to maintain the symmetry of the notation in the lemma. As discussed at the end of the proof, there are ranges of value realizations for which multiple equilibria exist.

market balance imposes on his strategy.

Table A1

	1	2	3
$D2$	m	m	m
$D2/S$	m	M	M
$D1/S$	M	M	m
S	M	m	M
S	M	M	M

Case 1 cannot be an equilibrium: for any p, $\overline{v}_m$ would gain by deviating from $D2$ to $D1$, violating expected market balance. Case 2 cannot be a fully revealing equilibrium either: consider the M member with assigned strategy $D1/S$; expected market balance requires the voter assigned strategy $D2/S$ to sell with some probability $\sigma \geq 1/3$–the probability labeled $\sigma_{\overline{v}_M}$–but for any $\sigma \geq 1/3$ in a fully revealing equilibrium there exists no value v such that the M member with assigned strategy $D1/S$ could be indifferent between $D1$ and S and prefer $D1$ to $D2$. Thus if a fully revealing equilibrium exists, preferences and strategies must be described by Case 3: the other m member must be the individual randomizing between $D1$ and S. Consider now the problem from the perspective of $v_{(2)m}$. Again the question is whether he can identify that the other m member must be the individual demanding two votes. Expected market balance constrains $v_{(2)m}$ to sell his vote with positive probability, and to demand votes with positive probability. We indicate this constraint by the notation D/S, because it could be satisfied by randomizing over the full set of possible actions, as long as positive probability is assigned to selling and to demanding. The possible cases are described in Table A2.

Table A2

	1	2	3
D/S	m	m	m
$D2/S$	m	M	M
$D2$	M	M	m
S	M	m	M
S	M	M	M

It is not difficult to verify that under both Case 1 and Case 2, for any $\sigma_{\overline{v}_M} \geq 1/3$, $v_{(2)m}$'s unique best response is $D2$. But $D2$ would violate expected budget balance. Hence in equilibrium the only possible case is 3: it must be that the other m member is demanding two votes. Finally, we need to establish that all majority members, $\overline{v}_M$ and the two members labeled v_{iM}, in equilibrium assign the correct corresponding strategies to the two minority members. Consider first individual $\overline{v}_M$, who in equilibrium is assigned strategy $D2/S$. The relevant cases are in Table A3:

Table A3

	1	2	3	4
D/S	M	M	M	M
S	m	m	m	M
S	m	M	M	M
$D2$	M	m	M	m
$D1/S$	M	M	m	m

Case 1 is ruled out because there can be no fully revealing equilibrium where both minority members sell with probability one. In Case 2, $D2$ always dominates $D1$ for $\overline{v}_M$. If $\sigma_{v_{(2)m}} > 1/2$ (recall that $\sigma_{v_{(2)m}}$ is the selling probability of the individual with strategy $D1/S$), then for $\overline{v}_M$, S strictly dominates not entering the market. Hence if $\sigma_{v_{(2)m}} > 1/2$, $\overline{v}_M$ must play strategy $D2/S$. But expected market balance then constrains $\sigma_{\overline{v}_M} > 1/3$, in which case strategy $D1/S$ cannot be a best response for an M member because for any value v both $D1$ and S are dominated by $D2$**.** If $\sigma_{v_{(2)m}} < 1/2$, then for $\overline{v}_M$, not entering the market dominates S. But then $\overline{v}_M$'s expected demand must be positive, and with $\sigma_{v_{(2)m}} < 1/2$, expected market balance is violated.[37] Hence Case 2 is excluded. In Case 3, it can be verified easily that S is $\overline{v}_M$'s best response to the others' strategies; but for all $\sigma_{v_{(2)m}} > 0$ selling with probability one violates expected market balance.[38] Hence Case 3 can be excluded too. The relevant scenario must be Case 4. Consider now majority member v_{iM}, who is constrained by expected market balance to sell his vote. The possible scenarios

[37]If $\sigma_{v_{(2)m}} = 1/2$, $D2$ is $\overline{v}_M$ best response, which again violates market balance.

[38]If $\sigma_{v_{(2)m}} = 0$ and $\overline{v}_M$ sells with probability one, expected market balance is satisfied but $v_{(2)m}$'s strategy is clearly suboptimal: $EU_{v_{(2)m}}D1 = -p$, dominated for example by not entering the market.

are described in Table A4 below:

Table A4

	1	2	3	4	5	6
S	M	M	M	M	M	M
$D2$	m	m	M	M	M	m
$D2/S$	m	M	M	m	m	M
$D1/S$	M	M	m	m	M	m
S	M	m	m	M	m	M

Taking into account the constraints on the randomization probabilities, the arguments described above exclude Case 1 (because the m voter assigned strategy $D2$ would have a profitable deviation to $D1$), Case 2 (because the M voter assigned strategy $D1/S$ would have a profitable deviation to $D2$), and Cases 3 and 4 (because in both the m voter assigned strategy $D1/S$ would have a profitable deviation to $D2$). From the perspective of majority member v_{iM}, Cases 5 and 6 cannot be distinguished. But the distinction is irrelevant: v_{iM}'s unique best response is S in both scenarios.

(2). If $g' = M$, each voter's preferred alternative is known, and each voter expects the others to follow the strategies described in the lemma, the expected utilities of different actions are given by:

$$\begin{aligned}
EU_{\overline{v}_M}D2 &= \sigma_{v_{(2)M}}[\sigma_{\overline{v}_m}(\overline{v}_M - 2p) + (1-\sigma_{\overline{v}_m})(\overline{v}_M/2 - p)] + \\
&\quad +(1-\sigma_{v_{(2)M}})[\sigma_{\overline{v}_m}(\overline{v}_M - 2p) + 2(1-\sigma_{\overline{v}_m})(\overline{v}_M - p)/3] \\
EU_{\overline{v}_M}D1 &= \sigma_{v_{(2)M}}[\sigma_{\overline{v}_m}(\overline{v}_M - p) + (1-\sigma_{\overline{v}_m})(-p)] + \\
&\quad +(1-\sigma_{v_{(2)M}})[\sigma_{\overline{v}_m}(\overline{v}_M - p) + 2(1-\sigma_{\overline{v}_m})(\overline{v}_M - p)/3] \\
EU_{\overline{v}_M}0 &= \sigma_{\overline{v}_m}\overline{v}_M + (1-\sigma_{\overline{v}_m})(1-\sigma_{v_{(2)M}})\overline{v}_M/2 \\
EU_{\overline{v}_M}S &= \sigma_{v_{(2)M}}[\sigma_{\overline{v}_m}\overline{v}_M + (1-\sigma_{\overline{v}_m})p/2] + \\
&\quad +(1-\sigma_{v_{(2)M}})[\sigma_{\overline{v}_m}(\overline{v}_M + p/4) + (1-\sigma_{\overline{v}_m})p]
\end{aligned}$$

$$\begin{aligned}
EU_{v_{(2)M}}D2 &= \sigma_{\overline{v}_m}(v_{(2)M} - p) + (1-\sigma_{\overline{v}_m})(2v_{(2)M} - 2p)/3 \\
EU_{v_{(2)M}}D1 &= \sigma_{\overline{v}_m}(v_{(2)M} - p) + (1-\sigma_{\overline{v}_m})(2v_{(2)M} - p)/3 \\
EU_{v_{(2)M}}0 &= \sigma_{\overline{v}_m}v_{(2)M} + (1-\sigma_{\overline{v}_m})v_{(2)M}/2 \\
EU_{v_{(2)M}}S &= \sigma_{\overline{v}_m}(v_{(2)M} + p/2) + (1-\sigma_{\overline{v}_m})(v_{(2)M}/2 + 2p/3)
\end{aligned}$$

$$
\begin{aligned}
EU_{\overline{v}_m}D2 &= \sigma_{v_{(2)M}}(v_{\overline{v}_m}/2-p)+(1-\sigma_{v_{(2)M}})(v_{\overline{v}_m}/3-2p/3) \\
EU_{\overline{v}_m}D1 &= \sigma_{v_{(2)M}}(-p)+(1-\sigma_{v_{(2)M}})(-2p/3) \qquad\qquad \text{(S5)} \\
EU_{\overline{v}_m}0 &= 0 \\
EU_{\overline{v}_m}S &= \sigma_{v_{(2)M}}(p/2)+(1-\sigma_{v_{(2)M}})p
\end{aligned}
$$

$$
\begin{aligned}
EU_{\underline{v}_M}D2 &= \sigma_{v_{(2)M}}[\sigma_{\overline{v}_m}(\underline{v}_M-p)+(1-\sigma_{\overline{v}_m})2(\underline{v}_M-p)/3+ \\
&\quad +(1-\sigma_{v_{(2)M}})[\sigma_{\overline{v}_m}(\underline{v}_M-2p/3)+(1-\sigma_{\overline{v}_m})\underline{v}_M] \\
EU_{\underline{v}_M}D1 &= \sigma_{\overline{v}_m}[\sigma_{v_{(2)M}}(\underline{v}_M-p)+(1-\sigma_{v_{(2)M}})(\underline{v}_M-2p/3)]+ \\
&\quad +(1-\sigma_{\overline{v}_m})[\sigma_{v_{(2)M}}(2\underline{v}_M-p)/3+(1-\sigma_{v_{(2)M}})(\underline{v}_M-p/2)] \\
EU_{\underline{v}_M}0 &= \sigma_{\overline{v}_m}\underline{v}_M+(1-\sigma_{\overline{v}_m})[(1-\sigma_{v_{(2)M}})\underline{v}_M+\sigma_{v_{(2)M}}\underline{v}_M/2] \\
EU_{\underline{v}_M}S &= \sigma_{\overline{v}_m}[\sigma_{v_{(2)M}}(\underline{v}_M+p/2)+(1-\sigma_{v_{(2)M}})(\underline{v}_M+p)]+ \\
&\quad +(1-\sigma_{\overline{v}_m})[\sigma_{v_{(2)M}}(\underline{v}_M/2+2p/3)+(1-\sigma_{v_{(2)M}})(2\underline{v}_M/3+5p/6)]
\end{aligned}
$$

$$
\begin{aligned}
EU_{v_{im}}D2 &= \sigma_{v_{(2)M}}[\sigma_{\overline{v}_m}(v_{im}/2-p)+2(1-\sigma_{\overline{v}_m})(v_{im}-p)/3]+(1-\sigma_{v_{(2)M}})\sigma_{\overline{v}_m}(v_{im}-2p)/3 \\
EU_{v_{im}}D1 &= \sigma_{v_{(2)M}}[\sigma_{\overline{v}_m}(-p)+(1-\sigma_{\overline{v}_m})(2v_{im}-p)/3]+ \\
&\quad +(1-\sigma_{v_{(2)M}})[\sigma_{\overline{v}_m}(-2p/3)+(1-\sigma_{\overline{v}_m})(v_{im}-p)/2] \\
EU_{v_{im}}0 &= \sigma_{v_{(2)M}}(1-\sigma_{\overline{v}_m})v_{im}/2 \\
EU_{v_{im}}S &= \sigma_{v_{(2)M}}[\sigma_{\overline{v}_m}p/2+(1-\sigma_{\overline{v}_m})(v_{im}/2+2p/3)]+ \\
&\quad +(1-\sigma_{v_{(2)M}})[\sigma_{\overline{v}_m}p+(1-\sigma_{\overline{v}_m})(v_{im}/3+5p/6)]
\end{aligned}
$$

Exactly as in (1) above, if voters' preferences are known, the strategies and price in the lemma can be shown to be an equilibrium by verifying that expected market balance is satisfied and no profitable deviation exists for any voter, given system (S5).[39] The logical arguments followed in (1) can also be used here to show that the equilibrium is fully revealing. Recall that expected market balance imposes $\sigma_{\overline{v}_m} \in [1/3, 1]$.Consider first voter $\overline{v}_M$, constrained by expected market balance to

[39]To prevent deviation by $\overline{v}_M$, there is a necessary constraint on the lower bound of $\overline{v}_m$: $\overline{v}_m \geq 592/(3(1381+51\sqrt{817}))v_{(2)M}$, or $\overline{v}_m \geq 0.0695v_{(2)M}$, satisfied by $\overline{v}_m \geq (1/14)v_{(2)M}$. The condition is required because at low enough $\overline{v}_m/v_{(2)M}$, $\sigma_{\overline{v}_m}$ approaches 1, and $\sigma_{v_{(2)M}}$ approaches 0, and thus $\overline{v}_M$ can profitable deviate to demanding a single vote. The profitable deviation only exists because $\overline{v}_M$ is a majority voter.

strategy $D2$. From his perspective, the possible cases are depicted in Table A5:

Table A5

	1	2	3	4
$D2$	M	M	M	M
$D2/S$	m	m	M	M
$D1/S$	m	M	m	M
S	M	m	m	m
S	M	M	M	m

Case 4 is excluded because in equilibrium both minority members cannot be selling with probability one. Case 1 and Case 3 are identical to Case 4 and Case 3 in Table A4 above, and can be excluded through the same arguments. Hence the correct scenario must be Case 2. Consider then voter $v_{(2)M}$, who in equilibrium must, by market balance, demand votes with some positive probability. Table A6 reports the possible cases, from his perspective:

Table A6

	1	2	3	4
D/S	M	M	M	M
$D2/S$	m	m	M	M
$D2$	m	M	m	M
S	M	m	m	m
S	M	M	M	m

Case 4 is excluded because there cannot be an ex ante equilibrium with trade in which both m members sell with probability one. Cases 1 and 2 can also be excluded because for all $\sigma_{\overline{v}_m} \in [1/3, 1]$, p must be such that voter $v_{(2)M}$ would have a profitable deviation to $D2$, violating expected market balance. Hence the correct scenario is Case 2. From the point of view of voter $v_{iM} \leq v_{(2)M}$, with equilibrium strategy S, the possible cases are represented in Table A4 above. As argued there, all scenarios can be excluded but Case 6 (the true state when $g' = m$) and Case 5 (the true state here). But the inability to distinguish the two scenarios is irrelevant: in both cases, the unique best response for v_{iM} is S. Finally, for each of the two m members, the strategy of the other can be identified easily. From the point of view of $\overline{v}_m$, with equilibrium strategy $D2/S$, the other m member cannot play either strategy $D2$ (because $\overline{v}_m$'s best response would then be S, violating expected market balance), or strategy $D1/S$ (because for any $\sigma_{v_{(2)M}} > 0$, $\overline{v}_m$'s best response would then be $D2$, again violating expected market balance); hence he must be playing strategy S. From the point of view of $v_{im} \leq \overline{v}_m$, with equilibrium strategy S, the other m member cannot play strategy S (because none of the M voters would then be buying with positive probability), or $D2$ (because the M voter with assigned strategy $D1/S$ would have a profitable deviation to $D2$), or $D1/S$ (because, again as argued earlier, the m voter with assigned strategy $D1/S$ would have a profitable deviation to $D2$); hence he must be playing strategy $D2/S$.□

Lemma 3. *If $\overline{v}_g \leq (1/14)v_{(2)g'}$, then for all realizations of $\{v_1, .., v_5\}$ such that $\overline{v}_m \geq \underline{v}_M$ there exists a fully revealing ex ante equilibrium such that voters $\overline{v}_{g'}$ and $v_{(2)g'}$ demand one vote, $\overline{v}_g$ randomizes between demanding two votes and selling his vote (with probability $\sigma_{\overline{v}_g} = 2/3$), and all other voters offer to sell their vote. The equilibrium price is $\overline{v}_g/4$.*

Proof of Lemma 3. Again, we need to distinguish the two cases: $g' = m$, and $g' = M$. (1) If $g' = m$, each voter's preferred alternative is known, and each expects the others to follow the strategies described in the lemma, the expected utilities of different actions are given by:

$$\begin{aligned}
EU_{v_{jm}}D2 &= \sigma_{\overline{v}_M}(v_{jm} - 2p) + (1 - \sigma_{\overline{v}_M})2(v_{jm} - p)/3 \\
EU_{v_{jm}}D1 &= \sigma_{\overline{v}_M}(v_{jm} - p) + (1 - \sigma_{\overline{v}_M})2(v_{jm} - p)/3 \\
EU_{v_{jm}}0 &= \sigma_{\overline{v}_M}v_{jm} + (1 - \sigma_{\overline{v}_M})v_{jm}/2 \\
EU_{v_{jm}}S &= \sigma_{\overline{v}_M}(3v_{jm}/4 + p/4) + (1 - \sigma_{\overline{v}_M})p
\end{aligned}$$

$$\begin{aligned}
EU_{\overline{v}_M}B2 &= (\overline{v}_M - 2p)/3 \\
EU_{\overline{v}_M}B1 &= -2p/3 \\
EU_{\overline{v}_M}0 &= 0 \\
EU_{\overline{v}_M}S &= 2p/3
\end{aligned} \tag{S6}$$

$$\begin{aligned}
EU_{v_{iM}}B2 &= \sigma_{\overline{v}_M}(v_{iM}/3 - 2p/3) \\
EU_{v_{iM}}B1 &= \sigma_{\overline{v}_M}2(-p)/3 + (1 - \sigma_{\overline{v}_M})(v_{iM}/3 - p/3) \\
EU_{v_{iM}}0 &= 0 \\
EU_{v_{iM}}S &= \sigma_{\overline{v}_M}(2p/3) + (1 - \sigma_{\overline{v}_M})(v_{iM}/3 + p)
\end{aligned}$$

where $v_{jm} = \overline{v}_m, v_{(2)m}$, and $v_{iM} \leq \overline{v}_M$. Expected market balance requires $2 + \sigma_{\overline{v}_M} = 2+2(1-\sigma_{\overline{v}_M})$, or $\sigma_{\overline{v}_M} = 2/3$, and $\overline{v}_M$ is indifferent between demanding two votes and offering to sell his vote if $p = \overline{v}_M/4$. It can then be verified immediately that if $\overline{v}_M < v_{(2)m}/4$ the strategies in the lemma are an equilibrium; thus they are an equilibrium if $\overline{v}_M < (1/14)v_{(2)m}$. We then need to verify that the equilibrium is fully revealing. Consider first the perspective of minority voter $v_{jm} = \overline{v}_m, v_{(2)m}$, constrained by expected market equilibrium to demand one vote. The possible scenarios are in Table A7:

Table A7

	1	2	3
$D1$	m	m	m
$D1$	M	M	m
$D2/S$	M	m	M
S	m	M	M
S	M	M	M

In both Case 1 and Case 2, the voter assigned strategy $D2/S$ could deviate to 0, satisfying expected market balance and increasing his expected utility; thus the equilibrium must be Case 3. Consider now majority voter $\overline{v}_M$. The two voters offering their vote for sale cannot both be minority voters. Hence one majority voter must have strategy S. If the other majority voter had strategy $D1$, then $\overline{v}_M$'s best response would be to do nothing; this would satisfy expected market balance but would mean that the voter assigned strategy $D1$, a minority member, would be playing a strategy that cannot be a best response–for example it would be clearly dominated by doing nothing. Hence the assigned strategies could not be an equilibrium. Thus, from the perspective of voter $\overline{v}_M$, both voters who offer their vote for sale must be majority members, and both voters demanding one vote must be minorities. Finally, consider majority voter v_{iM}, constrained by expected market balance to selling his vote. From his perspective, the possible cases are as in Table A8:

Table A8

	1	2	3	4	5	6
S	M	M	M	M	M	M
$D1$	m	m	m	m	m	M
$D1$	m	M	M	M	M	M
$D2/S$	M	m	M	m	M	m
S	M	M	m	M	m	m

But in Cases 2, 3, 4, and 5, the voter assigned strategy $D2/S$ would have a profitable deviation to 0, compatible with expected market equilibrium; hence those cases are excluded. Cases 1 and 6 cannot be distinguished, but the distinction is irrelevant: v_{iM}'s unique best response is S in both scenarios.

(2). If $g' = M$, each voter's preferred alternative is known, and each voter expects the others to follow the strategies described in the lemma, the expected utilities of different actions are given by:

$$\begin{aligned}
EU_{v_{jM}}D2 &= \sigma_{\overline{v}_m}(v_{jM} - 2p) + (1-\sigma_{\overline{v}_m})2(v_{jM} - p)/3 \\
EU_{v_{jM}}D1 &= \sigma_{\overline{v}_m}(v_{jM} - p) + (1-\sigma_{\overline{v}_m})2(v_{jM} - p)/3 \\
EU_{v_{jM}}0 &= \sigma_{\overline{v}_m}v_{jM} + (1-\sigma_{\overline{v}_m})v_{jM}/2 \\
EU_{v_{jM}}S &= \sigma_{\overline{v}_m}(v_{jM} + p/4) + (1-\sigma_{\overline{v}_m})p
\end{aligned}$$

$$\begin{aligned}
EU_{\overline{v}_m}D2 &= (\overline{v}_m - 2p)/3 \\
EU_{\overline{v}_m}D1 &= 2(-p)/3 \\
EU_{\overline{v}_m}0 &= 0 \\
EU_{\overline{v}_m}S &= 2p/3
\end{aligned} \tag{S7}$$

$$\begin{aligned}
EU_{v_{im}}D2 &= \sigma_{\overline{v}_m}(v_{im}/3 - 2p/3) \\
EU_{v_{im}}D1 &= \sigma_{\overline{v}_m}(-2p/3) + (1-\sigma_{\overline{v}_m})(v_{im}/3 - p/3) \\
EU_{v_{im}}0 &= 0 \\
EU_{v_{im}}S &= \sigma_{\overline{v}_m}(2p/3) + (1-\sigma_{\overline{v}_m})(v_{im}/3 + p)
\end{aligned}$$

$$\begin{aligned}
EU_{\underline{v}_M}D2 &= \sigma_{\overline{v}_m}(\underline{v}_M - 2p/3) + (1-\sigma_{\overline{v}_m})\underline{v}_M \\
EU_{\underline{v}_M}D1 &= \sigma_{\overline{v}_m}(\underline{v}_M - 2p/3) + (1-\sigma_{\overline{v}_m})(\underline{v}_M - p/3) \\
EU_{\underline{v}_M}0 &= \underline{v}_M \\
EU_{\underline{v}_M}S &= \sigma_{\overline{v}_m}(\underline{v}_M + 2p/3) + (1-\sigma_{\overline{v}_m})(2\underline{v}_M/3 + p)
\end{aligned}$$

where $v_{jM} = \overline{v}_M, v_{(2)M}$; $v_{im} \leq \overline{v}_m \leq v_{(2)M}$, and $\underline{v}_M \leq v_{im}$. As in (1) above, expected market balance requires $2+\sigma_{\overline{v}_m} = 2+2(1-\sigma_{\overline{v}_m})$, or $\sigma_{\overline{v}_m} = 2/3$, and $\overline{v}_m$ is indifferent between demanding two votes and offering to sell his vote if $p = \overline{v}_m/4$. For the value realizations considered here, when $g' = M$, $\underline{v}_M$'s best response is selling as long as $\underline{v}_M < 7/4\overline{v}_m$, a condition guaranteed by $\underline{v}_M \leq \overline{v}_m$. It can then be verified immediately that if $\overline{v}_m \leq v_{(2)M}/4$ the strategies in the lemma are an equilibrium; thus they are an equilibrium if $\overline{v}_m \leq (1/14)v_{(2)M}$.

We then need to verify that the equilibrium is fully revealing. Consider first the perspective of voter v_{jM}, assigned strategy $D1$ by expected market balance. At least one of the voters selling with probability one must belong to M. But if the other M member played either strategy $D2/S$ or strategy S, then, since $p < v_{jM}/4$, voter v_{jM} would gain from deviating to $D2$, violating expected market balance. Hence the other M member must be playing strategy $D1$; and thus the two m members must be playing strategies $D2/S$ and S. Consider then voter $\overline{v}_m$, who, by expected market balance must be demanding a positive number of votes with positive probability. If the other m member played strategy $D1$, at $p < \overline{v}_m$, $\overline{v}_m$'s best response would be doing nothing, which would satisfy expected market balance. But if a fully revealing ex ante equilibrium exists it cannot have $\overline{v}_m$ staying out of the market and the other voters playing the strategies conjectured here: the M voter playing $D1$ would be sure to buy a vote and lose the election, a strategy that cannot be a best response for any positive p. Hence the other m voter must be selling, and voter $\overline{v}_m$ can deduce the direction of preferences of all other voters. Consider then voter $v_{im} \leq \overline{v}_m$, who must sell to satisfy expected market balance. From his perspective, the other m voter cannot be selling. If he played $D1$, an M voter would have strategy $D2/S$, but would then gain by staying out of the market. Hence the other m voter must be playing strategy $D2/S$. Finally consider voter $\underline{v}_M$, assigned strategy S by expected market balance. The possible scenarios are in Table A.8 above, and as argued earlier, the only possibilities are Cases 1 and 6. These two cases cannot be distinguished, but the distinction is irrelevant: $\underline{v}_M$'s unique best response is S in both scenarios. We have thus shown that the equilibrium is fully revealing.$\square$

By proving Lemma 1, 2 and 3, we have shown that a fully revealing ex-ante equilibrium exists. The proposition is proven. Note that as the proofs of Lemma 2 and Lemma 3 make clear, for $\overline{v}_g \in [0.0695v_{(2)g'}, (1/4)v_{(2)g'}]$, both the equilibrium described in Lemma 2 and the equilibrium described in Lemma 3 exist.

Appendix C (Online supplementary material. Not intended for printed publication.)

This appendix contains a sample of the instructions given to subjects. It corresponds to the $3, 2$ market treatment.

INSTRUCTIONS

[SCREEN 0]

Thank you for agreeing to participate in this decision making experiment. During the experiment we require your complete, undistracted attention, and ask that you follow instructions carefully. You may not open other applications on your computer, chat with other students, or engage in other distracting activities, such as using your phone, reading books, etc.

You will be paid for your participation in cash, at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance.

The entire experiment will take place through computer terminals, and all interaction between you will take place through the computers. It is important that you not talk or in any way try to communicate with other participants during the experiments.

We will start with a brief instruction period. During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. If you have any questions during the instruction period, raise your hand and your question will be answered out loud so everyone can hear. If you have any questions after the experiment has begun, raise your hand, and an experimenter will come and assist you.

The experiment you are participating in is a committee voting experiment, where you will have an opportunity to buy and sell votes before voting on an outcome.

At the end of the experiment you will be paid the sum of what you have earned, plus a show-up fee of $10.00. Everyone will be paid in private and you are under no obligation to tell others how much you earned. Your earnings during the experiment are denominated in POINTS. Your DOLLAR earnings are determined by multiplying your earnings in POINTS by a conversion rate. For this experiment the conversion rate is 0.03, meaning that 100 POINTS equal 3 DOLLARS.

The experiment will have 25 matches. At the beginning of the first match, you will be randomly assigned with 4 other persons in the room to form a 5-member committee, which will vote to decide on outcome X or outcome Y. Of the 5 members of your committee, 3 members are in favor of X; the other 2 members are in favor of Y. Whether you are in favor of X or Y is decided randomly by the computer and will be displayed on your computer monitor.

You are also randomly assigned an "outcome value", which is a payoff you get if your preferred outcome is the committee decision. Your assigned "outcome value" is equally likely to be any integer from 1 to 100 points. Different members are randomly assigned different outcome values. If you are in favor of X, you will earn your value if X is the committee decision and zero if Y wins. Similarly, you will earn your value if you are in favor of Y and Y is the committee decision, and zero otherwise. Outcome X is the committee decision if there are more votes for X than for Y and vice versa.

Each committee member starts the match with one vote. After being told your outcome value, but before voting, there will be a 3 minute trading period, during which you and the other members of your committee will have an opportunity to buy or sell each other's votes. We will describe how trading occurs momentarily.

After the trading period ends, we proceed to the voting stage. In this stage you do not really have any choice. You will simply be asked to click a button to cast all your votes, if you have any, for your preferred outcome. When all committees have finished, the first match is complete and we will go to the next match. You will be randomly re-matched into new 5-person committees, and will repeat the procedure described above.

This will continue for a total of 25 matches.

[SCREEN 1]

When we begin a match, you will see a screen like this. Your Subject ID# is printed at the very top left of your screen, and remains the same through the whole experiment.

The current match number, your outcome value and your group (outcome preference) are displayed below your subject ID in the left part of the screen. Notice that this is an example of a committee member in favor of X with value 80. The numbers on this slide are for illustration only. Your outcome values will always be between 1 and 100. Below your value is a table that clarifies how many votes each member of the committee currently has, the group each member is in, and assigns you a temporary committee member number for this match. Your information is highlighted in green and the other members' information is not highlighted. Notice that you do not see the values of the other members.

The middle panel is the trading window. The right panel is the voting panel, which is inactive now because we are in the trading stage. Just above the table, is your cash holdings. At the beginning of the experiment, you will be loaned an initial amount of cash of 200 points, which will not be included in your final earnings. Trading occurs in the following way. At any time during this trading period, any member may post a bid for any amount between 1 and 100 points, which indicates an amount that you are willing to pay to buy the other member's vote. This bid will be posted on the trading board on the computer screens of all committee members. If some other member has already posted a bid to buy, you may post your own bid, but your bid must be an improvement, i.e., a higher bid. Whenever a new bid is entered, it cancels any outstanding bid if there is one. If another member has an active bid, then you may accept that bid, in which case a trade has occurred. You have traded your vote to the other member, and in exchange the other member pays you his bid. Similarly, if you have an active bid and the other member accepts it, you receive their vote, but you must pay the other member the amount of your bid. Trade is entirely voluntary. If a trade occurs, then the trading board is open once again for new bids. Because there is no longer an active bid, any new bid between 1 and 100 is acceptable. Once a new bid has been made, additional bids must be improvements, until another trade occurs, if it does. Bids may not be canceled. The trading period ends after 2 minutes. There are two additional trading rules. First, if your cash holdings ever become 0 or negative, you may not place any bids until it becomes positive again. Second, you may not sell votes if you do not have any.

As the experiment proceeds, your cash holdings will be updated to reflect any earnings you make. It increases when you sell your vote or when you earn your outcome value as a result of the voting. It decreases when you buy a vote. At the top right, above the voting window is a countdown timer that tells you how much time is left in the trading period. The timer will turn red when there are 10 seconds left in the trading period. There is a history panel in the lower part of the screen which will keep track of the history of the current and all past matches.

At the bottom of the middle trading panel there is an area where you can type in your bid. When you do so, it will look like this: [SCREEN 2, 54 entered]. Of course the bid you see here is just for illustration. Your bids must always be between 1 and 100. After you type a bid in, you click the "bid" button just to the right, and your bid will appear in middle column of the trading table above, labeled "Bids", and your temporary committee member number will be displayed in the left column, labeled "Buyer ID". All members in your committee see this information [SCREEN 3, WITH INFO IN TABLE] You may revise your bid at any time simply by entering a new bid, which must be greater than the current bid listed in the table. [SCREEN 4]. This member entered a new bid of 57, so now 57 is the current bid, canceling his earlier bid. Other members of your committee may, at any time, also submit improving bids. [SCREEN 5 WITH A NEW BID OF 91]. If one of the other members' bid is the current bid, you may accept it by clicking on the "Sell at Current Bid" button. This seals a trade between you and that other member, where you have now traded away your vote and the other member pays you his or her bid. [SCREEN 6]. Notice that when you do this your temporary committee member number is entered under the "Seller #" column, and the left panel is updated to show that you now have 0 votes and the other member has 2 votes. Your cash holdings are also updated. [point out on screen] The screen of the other person who just traded will look like this. [SCREEN 7] The trading period is 2 minutes, however, if there is no activity for 30 seconds, the trading period will terminate early. This ends the trading phase of the match.

Now that the trading phase of the match is over, we proceed to the voting stage. Your screen would now look like [SCREEN 8]. Member 1's screen would look like [SCREEN 9]. In the voting stage, the trading window is deactivated and the voting window to the right is activated. At this stage, you simply cast your votes by clicking on the vote button. These votes are automatically cast as votes for outcome X if you are in the X group, and are automatically cast as votes for outcome Y if you are in the Y group.

After you and the other members of your committee have voted, the results are displayed in the right hand panel, and summarized in the history screen. [SCREEN 10] [Go over the columns of the history screen and explain the "Payoff" column that summarizes what you earned this match: in this case, payoff is the same as net profit from trading.] The other member's screen will look like this. [SCREEN 11. Explain payoff screen.]

After all committees are finished, we proceed to the next match, where you are randomly grouped into new committees, and randomly assigned new outcome values. Are there any questions before we proceed to the first practice match? You are not paid for the practice match, so it has no effect on your final earnings. The only purpose of the practice match is to help you understand how the computer interface works.

[SCREEN 13: Summary slide] [START SERVER]

We will now proceed to the practice match. Remember that you are randomly assigned to a committee in this match. Similarly, your outcome value, your committee member number, the other participants in your committee, and your group are assigned randomly. Please click on the icon marked VM. Then enter your first and last name and click on submit. Then wait. Please don't use the computer while waiting.

[CONNECT EVERYONE AND START]

Please complete the practice match on your own. Remember, you are not paid for these practice matches. Feel free to raise your hand if you have any questions.

[WAIT FOR SUBJECTS TO COMPLETE PRACTICE MATCH]

The practice match is now over. Remember, you will not be paid the earnings from this practice match. Any questions?

If you have any questions from now on, raise your hand, and an experimenter will come and assist you. We will now begin the 25 paid matches.

[GO TO NEXT MATCH]

Please notice that your cash holdings are reinitialized to 200. This 200 is a loan that will be subtracted your cash holdings in the last match.

(Play 25 real matches) [After last MATCH, read:]

This is the end of the experiment. You should now see a popup window, which displays your total earnings in the experiment. Please record this on your payment receipt sheet, rounding up to the nearest quarter. After you are done, please, click OK to close the window. Do not close any other windows on your computer and do not use your computer for anything else. Also enter $10.00 on the showup fee row. Add the two numbers and enter the sum as the total.

[Write output]

We will pay each of you in private in the next room in the order of your Subject ID numbers. Remember you are under no obligation to reveal your earnings to the other players.

Please put the mouse behind the computer screen and do not use either the mouse or the keyboard at all. Please be patient and remain seated until we call you to be paid. Do not converse with the other participants or use your cell phone. Thank you for your cooperation.

Could the person with subject ID number 0 please go to the next room to be paid. Please bring all your belongings with you, including your payment receipt sheet.