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ABSTRACT

This paper studies Pareto-optimal risk-sharing arrangements in a private information economy with aggregate uncertainty and ex ante heterogeneous agents. I show how to implement Pareto-optima as equilibria when agents can trade claims to consumption contingent on aggregate shocks in financial markets. The first result is that if aggregate and idiosyncratic shocks are independent, the implementation of optimal allocations does not require any interventions in financial markets. This result can be extended to dynamic settings in the sense that, in this case, only savings need to be distorted, but not trades in financial markets. Second, I characterize optimal trading distortions in financial markets when aggregate and idiosyncratic shocks are not independent. In this case, optimal asset taxes must be higher for those securities that pay out in aggregate states in which consumption is more volatile. For instance, this can provide an efficiency justification for the frequently observed differential tax treatment of different asset classes, such as debt and equity claims.

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1 Introduction

Individual households face substantial economic risk over their lifetimes in the form of both idiosyncratic and aggregate uncertainty. For instance, individuals' employment, income, health status and mortality are all subject to idiosyncratic shocks and to economy-wide shifts in unemployment rates, wages, technology and life-expectancies. The two kinds of shocks have very different implications for risk-sharing, though. Whereas individuals can influence their idiosyncratic uncertainty by taking unobservable actions, aggregate risk is not typically related to such private information problems. Moreover, aggregate uncertainty is harder to smooth by pooling risks than idiosyncratic uncertainty. Yet there do exist opportunities for smoothing even aggregate risk when aggregate shocks have different effects on different agents in the economy. For instance, country-specific aggregate shocks may only affect agents in one country, not those abroad. A recession may increase unemployment rates in some sectors of the economy more than in others, and changes in wages and mortality rates have different impacts on elderly, retired agents than on young workers.[1]

In this paper, I ask how idiosyncratic and aggregate risk should be shared optimally among different groups in the economy. I consider a model where ex ante heterogeneous agents are subject to both aggregate and idiosyncratic shocks. Individuals can influence their probability distribution over idiosyncratic shocks by choosing some hidden effort, leading to a standard moral hazard problem. Aggregate shocks, by contrast, are assumed to be exogenous and to affect all agents' outputs and probability distributions over idiosyncratic shocks, but in potentially different ways. If agents' preferences over consumption and effort are separable, any Pareto-optimal risk-sharing arrangement in this private information economy has to be such that the ratios of expected inverse marginal utilities between different agents are independent of aggregate shocks.

I use this efficiency condition to study the role of financial markets in my economy, where agents exchange claims to consumption contingent on aggregate shocks. In practice, agents are able to insure considerable parts of the aggregate risk that they are exposed to by trading such financial assets. For instance, agents can hedge country-specific risk by buying foreign assets, and workers in a given sector can buy shares of companies in other sectors to reduce their overall exposure to the effects of aggregate shocks on their own sector. In general, Pareto-optimal risk-sharing arrangements are inconsistent with agents having free access to such financial markets since trading in financial markets leads to

[1] Attanasio and Davis (1996) provide overwhelming empirical evidence for consumption insurance opportunities between birth cohorts and education groups in the US. Storesletten et al. (2004) also find that intergenerational sharing of aggregate risk is quantitatively important.

the equalization of ratios of expected marginal utilities across different agents rather than expected inverse marginal utilities.

However, I show that there exists an important benchmark case where this conflict disappears: If aggregate and idiosyncratic shocks are stochastically independent, so that aggregate shocks may affect individual outputs in arbitrary ways, but not the distributions of idiosyncratic risk, then any Pareto-optimum in my economy can be implemented without interventions in financial markets. Simple group-specific income transfers that condition on aggregate shocks and individual outputs are sufficient in this case. I also show that this result generalizes to a dynamic setting where agents can save in capital in addition to trading in financial markets. In this case, the result is that only savings need to be distorted in order to implement constrained-efficient allocations, but not trades in financial markets, whenever aggregate and idiosyncratic risk are independent.

The intuition relies on the fact that, in any Pareto optimum, marginal utilities depend on aggregate shocks through two channels. First, as in standard moral hazard models, it is optimal to allocate marginal utilities to agents according to likelihood ratios, which generally vary with aggregate shocks. However, if aggregate and idiosyncratic shocks are independent, aggregate uncertainty leaves the distribution of likelihood ratios unchanged. Second, aggregate shocks affect aggregate output in the economy. Variations in aggregate output, however, shift marginal utilities uniformly across agents at any Pareto-optimum. Hence, if aggregate and idiosyncratic shocks are independent, aggregate states are symmetric in terms of the marginal resource costs of providing incentives, and it is optimal to leave marginal rates of substitution between aggregate states undistorted. This in turn is consistent with free trading in financial markets.

Second, I characterize optimal distortions in financial markets when aggregate shocks do affect distributions of idiosyncratic shocks. In this case, taxes on transactions in financial markets are able to implement Pareto-optima. In particular, I show that the resulting marginal taxes must be higher for those financial assets that pay out in aggregate states in which likelihood ratios and hence consumption are more risky. With undistorted financial markets, agents would "self-insure" against this risk by buying additional consumption for these aggregate states in financial markets. The optimal distortions are designed to prevent agents from doing so.

In particular, the optimal asset tax schedule derived here can provide an efficiency based justification for a differential tax treatment of different asset classes, a feature shared by many real-world tax systems. For instance, I discuss an example in which equity claims pay out relatively more in aggregate states with more volatile consumption compared to fixed income securities, which pay out in states in which it is optimal to provide more insurance across idiosyncratic shocks. In this case, optimal asset taxes are higher on equity claims compared to debt claims. Indeed, the deductibility of interest on debt payments from the corporate tax base in many tax systems leads to a situation where equity claims are effectively taxed at a higher rate than debt claims, consistent with the optimal pattern derived here.

Related Literature. This paper builds on the literature studying and testing optimal risk-sharing arrangements in economies with heterogeneous agents but without private information, as pioneered by Borch (1962), Wilson (1968), Townsend (1994) and Attanasio and Davis (1996). I demonstrate how the first-best risk-sharing rules derived there have to be modified when idiosyncratic risk is subject to moral hazard, so that risk-sharing has to be traded-off against the provision of incentives. In that respect, this paper shares a common goal with the contribution by Demange (2008) who also considers a moral hazard model with aggregate uncertainty and discusses properties of risk-sharing rules under various assumptions on preferences and for a numerical example. However, financial markets are absent from her analysis, so that the implications of efficient risk-sharing for optimal tax policy in financial markets presented here are not considered.

My analysis of a moral hazard model with aggregate uncertainty and of its implications for tax policy in financial markets is also related to a large literature that studies the optimal taxation of capital income in dynamic private information economies with idiosyncratic shocks. In these models, the Inverse Euler equation is derived as an intertemporal optimality condition and used to obtain implications for optimal savings distortions.[2] I establish an analogous optimality condition with respect to sharing aggregate risk across heterogeneous agents in a completely static environment. More importantly, due to the absence of aggregate shocks, no financial markets as discussed here emerge in these models.

An important exception are the contributions by Kocherlakota (2005) and Kocherlakota and Pistaferri (2009). Kocherlakota (2005) generalizes the Inverse Euler equation to allow for aggregate shocks in a dynamic optimal taxation model. However, since all agents are ex ante identical in his model, no restriction similar to the purely intratemporal Pareto-optimality condition for the sharing of aggregate risk across heterogeneous groups derived here can be obtained in his framework. Also, agents in his model can

[2]See, for instance, Diamond and Mirrlees (1978), Rogerson (1985), Ligon (1998), Golosov et al. (2003), Farhi and Werning (2009) and Weinzierl (2011). With the exception of Rogerson (1985) and Ligon (1998), these contributions consider optimal tax models with private skill shocks rather than moral hazard models. While the Inverse Euler equation has been shown to emerge in both private information and hidden action models, the hidden action framework considered here allows to derive the implications of aggregate uncertainty for constrained-efficiency and financial markets in a particularly transparent way (see section 4.1 for a discussion).

only trade capital, so that implications for optimal tax policy in financial markets do not arise. Kocherlakota and Pistaferri (2009) consider incomplete markets equilibria in which individuals can trade securities contingent on aggregate shocks, but are interested in their asset pricing implications in comparison to Pareto optimal allocations. They do not characterize tax systems that implement Pareto optima as incomplete markets equilibria with such taxes, which is the focus of this paper.

Golosov et al. (2006) also consider optimal savings and labor wedges with aggregate uncertainty, but have to rely on numerical simulations to obtain results on how aggregate shocks affect wedges. In contrast, I analytically derive transparent conditions that allow me to characterize optimal distortions in financial markets with aggregate uncertainty. The asset taxes that I construct to implement Pareto-efficient allocations when aggregate and idiosyncratic shocks are not independent are inspired by Werning (2009), even though his implementation concerns capital taxes in a dynamic Mirrlees economy as opposed to the static environment with financial markets considered here.

For a Bewley economy with aggregate uncertainty and uninsured idiosyncratic risk, Krueger and Lustig (2010) show that, if aggregate risk is independent of idiosyncratic risk, individuals do not trade in bond markets, only in stock markets, and asset prices are unaffected by aggregate risk. However, their focus on equilibria in an incomplete markets model is quite different from the characterization of constrained-efficient allocations in the private information economy that I consider, where the tradeoff between incentives and insurance is central. Moreover, their results crucially depend on a utility function with constant relative risk aversion. In contrast, the results in the present paper do not rely on functional form restrictions of preferences other than separability between consumption and effort.

The paper is structured as follows. In section 2, I introduce the economic environment and characterize constrained-efficient allocations (Lemma 1). Section 3 begins with the definition of equilibria in financial markets and the observation that, in general, trading distortions in financial markets are required to implement constrained-efficient allocations. The first result in Theorem 1 is, however, that these are zero if aggregate shocks affect individual outputs only but are stochastically independent from idiosyncratic risk. In section 4, this result is first generalized to a dynamic setting (Theorem 2). Moreover, section 5 characterizes optimal trading distortions in financial markets when aggregate and idiosyncratic risk are not independent (Theorems 3 and 4). Finally, section 6 concludes.

2 Constrained-Efficient Allocations

In order to address the issues discussed in the introduction formally, I consider a continuum of agents of unit mass where each individual belongs to one of N groups. These groups may be thought of as workers in different sectors of the economy, individuals of different age (such as workers and retired agents) or even agents in different countries, so that the assignment of individuals to groups is public information. The mass of a given group $i \in I \equiv \{1, ..., N\}$ is given by n_i. Individuals within a group are ex ante identical in terms of both preferences and technology. In particular, let agents of group i be endowed with separable preferences $U_i(c, a) = u_i(c) - v_i(a)$ over consumption $c \in \mathbb{R}^+$ and an action (effort) $a \in A_i$, where A_i is a finite action set available to agents of group i. I assume $u_i(c)$ to be twice continuously differentiable with $u_i'(c) > 0, u_i''(c) < 0 \; \forall c \in \mathbb{R}^+$.

The agents' technology is affected by two kinds of shocks: aggregate shocks $s \in S$ and idiosyncratic shocks $\theta \in \Theta$, where both S and Θ are finite sets. An agent of group i who experiences an idiosyncratic shock θ in aggregate state s produces output $y_i(\theta, s)$. In addition, if an agent of group i chooses some action a and the aggregate shock is s, then the pdf over idiosyncratic shocks is given by $p_i(\theta|a, s)$. Idiosyncratic shocks θ within a given group i are iid across agents conditional on the aggregate shock s. Let the pdf over aggregate shocks $s \in S$ be given by $\pi(s)$. I assume both the realizations of idiosyncratic shocks θ and of aggregate shocks s to be publicly observable, but an agent's action a to be private information. Observe that the dependency of both outputs and probability distributions on aggregate shocks are indexed by i, so that aggregate shocks can affect different groups in the economy differently.

The timing of events is as follows. In a first stage, a social planner offers agents a consumption schedule $\{c_i(\theta, s)\}$ that specifies consumption levels for the agents in each group i contingent on the realization of both s and θ. Observing that, agents in each group privately choose an action $a_i \in A_i$. Next, an aggregate shock $s \in S$ is realized according to the distribution $\pi(s)$ and, conditional on this realization, idiosyncratic shocks $\theta \in \Theta$ are drawn from the distribution $p_i(\theta|a_i, s)$ for all agents and all groups $i \in I$. This determines outputs $y_i(\theta, s)$, which are used by the social planner to implement the promised consumption schedule $\{c_i(\theta, s)\}$.

I define an *allocation* $\{c_i(\theta, s), a_i\}$ in this economy as a consumption schedule $\{c_i(\theta, s)\}$ and an action profile $\{a_i\}$ that specifies an action a_i for each group $i \in I$. An allocation $\{c_i(\theta, s), a_i\}$ is *feasible* if it satisfies the resource constraint for each aggregate state s, i.e.

$$\sum_i n_i \sum_{\theta \in \Theta} c_i(\theta, s) p_i(\theta|a_i, s) \leq \sum_i n_i \sum_{\theta \in \Theta} y_i(\theta, s) p_i(\theta|a_i, s) \quad \forall s \in S. \tag{1}$$

It is *incentive compatible* if

$$\sum_{s\in S}\sum_{\theta\in\Theta} U_i(c_i(\theta,s),a_i)p_i(\theta|a_i,s)\pi(s) \geq \sum_{s\in S}\sum_{\theta\in\Theta} U_i(c_i(\theta,s),\tilde{a}_i)p_i(\theta|\tilde{a}_i,s)\pi(s) \tag{2}$$

$\forall i \in I, \tilde{a}_i \in A_i$. I will say that a consumption schedule $\{c_i(\theta,s)\}$ *implements* an effort profile $\{a_i\}$ if it satisfies the incentive compatibility constraints (2) given $\{a_i\}$. The following definition introduces a notion of optimality in this economy:

Definition 1. *An allocation $\{c_i^*(\theta,s), a_i^*\}$ is constrained-efficient if it solves*

$$\max_{\{c_i(\theta,s),a_i\}} \sum_i \psi_i \sum_{s\in S}\sum_{\theta\in\Theta} U_i(c_i(\theta,s),a_i)p_i(\theta|a_i,s)\pi(s) \tag{3}$$

subject to the feasibility constraints (1) and the incentive compatibility constraints (2) for some set of Pareto-weights $\{\psi_i\}$, $\psi_i \geq 0\ \forall i \in I$.

Hence, an allocation is constrained-efficient if it is Pareto-optimal within the class of feasible and incentive compatible allocations. I also refer to a consumption schedule $\{c_i^*(\theta,s)\}$ as constrained-efficient *given an effort profile* $\{a_i\}$ if it solves (3) subject to (1) and (2) for some given $\{a_i\}$ (which may not be the optimal one), i.e. it is feasible and implements a given action profile optimally.

I next ask how aggregate and idiosyncratic risk are shared optimally among the heterogeneous groups in the present moral hazard economy. The result is a restriction that any constrained-efficient allocation has to satisfy.

Lemma 1. *Any constrained-efficient consumption schedule $\{c_i^*(\theta,s)\}$ that solves problem (3) subject to (1) and (2) for a given action profile $\{a_i\}$ must be such that*

$$\frac{\mathbb{E}_i\left[1/u_i'(c_i^*(\theta,s))\middle|a_i,s\right]}{\mathbb{E}_i\left[1/u_i'(c_i^*(\theta,\tilde{s}))\middle|a_i,\tilde{s}\right]} = \frac{\mathbb{E}_j\left[1/u_j'(c_j^*(\theta,s))\middle|a_j,s\right]}{\mathbb{E}_j\left[1/u_j'(c_j^*(\theta,\tilde{s}))\middle|a_j,\tilde{s}\right]} \tag{4}$$

$\forall i,j \in I, s,\tilde{s} \in S$, where $\mathbb{E}_i\left[1/u_i'(c_i^*(\theta,s))\middle|a_i,s\right] \equiv \sum_{\theta\in\Theta} p_i(\theta|a_i,s)/u_i'(c_i^*(\theta,s))$.

Proof. Consider a constrained-efficient consumption schedule $\{c_i^*(\theta,s)\}$ that implements action profile $\{a_i\}$. Let me perform a change in variables from $\{c_i(\theta,s)\}$ to $\{u_i(\theta,s)\}$. Then $\{u_i^*(\theta,s)\}$ must solve

$$\max_{\{u_i(\theta,s)\}} \sum_i \psi_i \sum_{s\in S}\sum_{\theta\in\Theta} u_i(\theta,s)p_i(\theta|a_i,s)\pi(s) \tag{5}$$

subject to

$$\sum_i n_i \sum_{\theta\in\Theta} p_i(\theta|a_i,s)C_i(u_i(\theta,s)) \leq \sum_i n_i \sum_{\theta\in\Theta} p_i(\theta|a_i,s)y_i(\theta,s)\ \forall s \in S \tag{6}$$

and

$$\sum_{s\in S}\sum_{\theta\in\Theta} u_i(\theta,s)p_i(\theta|a_i,s)\pi(s) - v_i(a_i) \geq \sum_{s\in S}\sum_{\theta\in\Theta} u_i(\theta,s)p_i(\theta|\tilde{a}_i,s)\pi(s) - v_i(\tilde{a}_i) \tag{7}$$

$\forall i \in I, \tilde{a}_i \in A_i$, where I have defined the cost function $C_i(u) \equiv {u_i}^{-1}(u)$. The necessary first order condition for $u_i(\theta,s)$, integrated over θ, is given by

$$\frac{\pi(s)}{\xi(s)} = \frac{n_i}{\psi_i}\sum_{\theta\in\Theta}\frac{p_i(\theta|a_i,s)}{u_i'(c_i^*(\theta,s))} \; \forall i \in I,\, s \in S,$$

where I used $C_i'(.) = 1/u_i'(.)$ and $\xi(s)$ is the Lagrange multiplier on the resource constraint for state s in equation (6). Hence,

$$\frac{n_i}{\psi_i}\mathbb{E}_i\left[\frac{1}{u_i'(c_i^*(\theta,s))}\middle|a_i,s\right] = \frac{n_j}{\psi_j}\mathbb{E}_j\left[\frac{1}{u_j'(c_j^*(\theta,s))}\middle|a_j,s\right] \tag{8}$$

$\forall i,j \in I, s \in S$. Condition (4) in the theorem follows from the fact that equation (8) holds for all aggregate states $s \in S$. □

The risk-sharing condition (4) in the lemma requires any constrained-efficient consumption schedule $\{c_i^*(\theta,s)\}$ to be such that the ratios of expected inverse marginal utilities between different aggregate states are equalized across all agents (equivalently, the condition could also be written as requiring that the ratios of expected inverse marginal utilities between different agents be independent of aggregate shocks). Since this has to hold for arbitrary effort profiles $\{a_i\}$, it must notably hold for any constrained-efficient allocation $\{c_i^*(\theta,s), a_i^*\}$. The intuition behind the result is that $\mathbb{E}_i[1/u_i'(c_i^*(\theta,s))|a_i,s]$ is the expected marginal resource cost of providing additional utility to an agent in group i in aggregate state s without affecting incentives. In any given aggregate state s, the constrained-efficient consumption schedule must equalize this cost across agents, weighted by population shares n_i and Pareto-weights ψ_i. Taking ratios for different aggregate states, these weights cancel out and the condition in the lemma results.

Condition (4) is related to the Inverse Euler equation, which has received much attention in the framework of dynamic models with private information or hidden actions, where expected inverse marginal utilities are equalized across time periods, accounting for interest rates and discounting (see, for instance, Rogerson (1985) and Ligon (1998) for hidden action models and Diamond and Mirrlees (1978), Golosov et al. (2003) and Kocherlakota (2005) for private information models). In the context of age-dependent taxation, Weinzierl (2011) derives a similar condition labeled "symmetric Inverse Euler equation" for optimal consumption allocations across different age groups. Lemma 1 establishes an analogous optimality condition with respect to sharing aggregate risk across heterogeneous agents in a static environment.[3]

[3] See Demange (2008) for a related result in an economy in which the hidden action is taken *after* aggre-

3 Implementation with Financial Markets

3.1 Competitive Equilibria with Financial Markets

In this section, I ask under what conditions constrained-efficient allocations $\{c_i^*(\theta, s), a_i^*\}$ are consistent with agents having undistorted access to financial markets, in the sense that they can buy and sell a complete set of claims to consumption contingent on aggregate shocks s. In particular, let me consider the following modified timing. In the first stage, a transfer system $\{T_i(\theta, s)\}$ is announced that specifies group-specific transfers contingent on idiosyncratic and aggregate shocks. In stage 2, agents then simultaneously choose an action $a_i \in A_i$ and competitively trade s-contingent Arrow-Debreu securities among themselves, where a security for aggregate state s pays one unit of consumption if state s is realized and zero otherwise. Finally, risks are realized and consumption takes place as before, accounting for both transfers and traded financial assets.

Let $\{q(s)\}$ be the set of prices of the s-contingent claims to consumption. Then, in stage 2, agents in group $i \in I$ solve, taking transfers $\{T_i(\theta, s)\}$ and prices $\{q(s)\}$ as given,

$$\max_{\{c_i(\theta,s),\Delta_i(s),a_i\}} \sum_{s\in S} \sum_{\theta\in\Theta} U_i(c_i(\theta, s), a_i) p_i(\theta|a_i, s)\pi(s) \tag{9}$$

subject to

$$\sum_{s\in S} q(s)\Delta_i(s) \leq 0, \tag{10}$$

where

$$c_i(\theta, s) = y_i(\theta, s) - T_i(\theta, s) + \Delta_i(s) \ \forall\theta \in \Theta, s \in S, \tag{11}$$

and $\Delta_i(s)$ is the amount of securities for state s bought by agents in group i. I call $\{\Delta_i(s)\}$ a *trading profile*, which specifies a trading strategy $\Delta_i(s)$ for each group $i \in I$. Then an equilibrium without distortions in the financial markets is defined as follows.

Definition 2. *An equilibrium in financial markets with the transfer system $\{T_i(\theta, s)\}$ and without taxes is an allocation $\{c_i^e(\theta, s), a_i^e\}$, a trading profile $\{\Delta_i^e(s)\}$ and prices $\{q^e(s)\}$ such that $\{c_i^e(\theta, s), \Delta_i^e(s), a_i^e\}$ solves the agents' problem (9) to (10) taking prices $\{q^e(s)\}$ and transfers $\{T_i(\theta, s)\}$ as given, the financial markets clear, i.e.*

$$\sum_i n_i \Delta_i^e(s) = 0 \ \forall s \in S, \tag{12}$$

gate shocks are realized.

and the goods market clears, i.e.

$$\sum_i n_i \sum_{\theta\in\Theta} p_i(\theta|a_i^e,s)c_i^e(\theta,s) = \sum_i n_i \sum_{\theta\in\Theta} p_i(\theta|a_i^e,s)y_i(\theta,s)\ \forall s\in S. \tag{13}$$

Note that the market clearing conditions (12) and (13) imply that the social planner's budget constraints $\sum_i n_i \sum_{\theta\in\Theta} p_i(\theta|a_i^e,s)T_i(\theta,s) = 0\ \forall s\in S$ are satisfied. From this definition and the necessary conditions of the agents' problem (9) subject to (10), it immediately follows that any equilibrium without taxes in the financial markets $\{c_i^e(\theta,s),\Delta_i^e(s),a_i^e\}$ must be such that

$$\frac{\mathbb{E}_i\left[u_i'(c_i^e(\theta,s))|a_i^e,s\right]}{\mathbb{E}_i\left[u_i'(c_i^e(\theta,\tilde{s}))|a_i^e,\tilde{s}\right]} = \frac{\mathbb{E}_j\left[u_j'(c_j^e(\theta,s))|a_j^e,s\right]}{\mathbb{E}_j\left[u_j'(c_j^e(\theta,\tilde{s}))|a_j^e,\tilde{s}\right]}\ \forall i,j\in I, s,\tilde{s}\in S. \tag{14}$$

Undistorted trading in financial markets thus leads to the equalization of ratios of expected marginal utilities between aggregate states across agents of different groups. This generally conflicts with the condition for constrained-efficiency (4), which requires the equalization of ratios of expected *inverse* marginal utilities, as I will further explore in the following.[4]

3.2 Independent Aggregate and Idiosyncratic Shocks

The comparison between the optimality condition (4) and the equilibrium condition (14) indicates that, in general, distortions have to be introduced in financial markets in order to implement constrained-efficient allocations as competitive equilibria in the sense of Definition 2. In particular, taxes may be required such that the agents' trading in financial markets does not necessarily lead to an equalization of the marginal rates of substitution between aggregate states across groups, as in (14). However, it turns out that there exists an important benchmark case where such interventions are in fact not necessary and no distortions in financial markets are required to be consistent with constrained-efficiency:

Theorem 1. *Consider a constrained-efficient allocation $\{c_i^*(\theta,s),a_i^*\}$ and suppose that aggregate and idiosyncratic shocks are independent, so that aggregate shocks s affect outputs $y_i(\theta,s)$ only, but not probability distributions, i.e. $p_i(\theta|a_i,s) = p_i(\theta|a_i,\tilde{s})$ for all $i\in I$, $\theta\in\Theta$, $s,\tilde{s}\in S$ and*

[4] Kocherlakota and Pistaferri (2009) compare the asset pricing implications of the equivalents of (4) and (14) in a dynamic Mirrlees environment, comparing incomplete markets equilibria with Pareto efficient allocations. They do not consider tax systems that implement Pareto optima as incomplete markets equilibria, which will be my focus in the following.

$a_i \in A_i$. Then it can be implemented as an equilibrium in financial markets using the transfers $\{T_i(\theta,s)\}$ only and without tax interventions in financial markets.

Proof. Set transfers $T_i^*(\theta,s) = y_i(\theta,s) - c_i^*(\theta,s)$ for all $i \in I, \theta \in \Theta, s \in S$ and prices $q^*(s) = \xi(s)\,\forall s \in S$, where $\xi(s)$ is the multiplier on the resource constraint (1) for state $s \in S$ in the Pareto-problem (3) s.t. (1) and (2). Any constrained-efficient allocation $\{c_i^*(\theta,s), a_i^*\}$ is feasible, and hence by (1) the goods market clearing condition (13) is satisfied. Hence, all that remains to be shown is that, given the transfers $T_i^*(\theta,s)$ and prices $q^*(s)$, $\{c_i^*(\theta,s), a_i^*\}$ and $\Delta_i(s) = 0\,\forall i \in I, s \in S$ solve the agents' problem (9) to (11). Then the financial market clearing condition (12) is satisfied trivially.

To prove this, I proceed in two steps: First, I show that, given any effort choice $\tilde{a}_i \in A_i$, it is optimal for all agents to set $\Delta_i(s) = 0\ \forall i \in I, s \in S$. This implies by condition (11) and the design of the transfers $\{T_i^*(\theta,s)\}$ that agents choose the constrained-efficient consumption schedule $\{c_i^*(\theta,s)\}\ \forall i \in I, \theta \in \Theta, s \in S$ in equilibrium. The second step then involves demonstrating that agents also find it optimal to choose the constrained-efficient action $a_i^*\ \forall i \in I$.

Step 1. Fix some $\tilde{a}_i \in A_i$. Then the optimization problem for an agent of group $i \in I$ reduces to

$$\max_{\{\Delta_i(s)\}} \sum_s \sum_\theta u_i\Big(y_i(\theta,s) - T_i^*(\theta,s) + \Delta_i(s)\Big) p_i(\theta|\tilde{a}_i,s)\pi(s) \quad \text{s.t.} \quad \sum_s q^*(s)\Delta_i(s) \leq 0.$$

The objective function is strictly concave and the constraint is linear, so that first order conditions are necessary and sufficient. I therefore only need to show that $\Delta_i(s) = 0\ \forall s \in S$ satisfies the budget constraint (10) and first order conditions. The budget constraint is satisfied trivially. The first order conditions imply that, for all $s, \tilde{s} \in S$,

$$\frac{\pi(s)\mathbb{E}_i\left[u_i'\Big(y_i(\theta,s) - T_i^*(\theta,s) + \Delta_i(s)\Big)\Big|\tilde{a}_i,s\right]}{\pi(\tilde{s})\mathbb{E}_i\left[u_i'\Big(y_i(\theta,\tilde{s}) - T_i^*(\theta,\tilde{s}) + \Delta_i(\tilde{s})\Big)\Big|\tilde{a}_i,\tilde{s}\right]} = \frac{q^*(s)}{q^*(\tilde{s})}. \tag{15}$$

Setting $\Delta_i(s) = 0\ \forall s \in S$ and substituting $T_i^*(\theta,s) = y_i(\theta,s) - c_i^*(\theta,s)$ yields

$$\frac{\pi(s)\mathbb{E}_i\left[u_i'(c_i^*(\theta,s))\right|\tilde{a}_i,s]}{\pi(\tilde{s})\mathbb{E}_i\left[u_i'(c_i^*(\theta,\tilde{s}))\right|\tilde{a}_i,\tilde{s}]} = \frac{q^*(s)}{q^*(\tilde{s})}. \tag{16}$$

Note that the necessary first order condition of the Pareto-problem (3) subject to (1) and (2) for $c_i(\theta,s)$ can be rearranged to

$$\frac{1}{u_i'(c_i^*(\theta,s))} = \frac{\pi(s)}{n_i\xi(s)}\left[\psi_i + \sum_{\tilde{\tilde{a}}_i \in A_i} \mu_i(\tilde{\tilde{a}}_i)\left(1 - \frac{p_i(\theta|\tilde{\tilde{a}}_i,s)}{p_i(\theta|a_i^*,s)}\right)\right], \tag{17}$$

where $\xi(s)$ is the Lagrange-multiplier on the aggregate resource constraint in state s and $\mu_i(\tilde{\tilde{a}}_i)$ on the incentive constraint for group i and action $\tilde{\tilde{a}}_i \in A_i$. Inverting and taking expectations over θ on both sides yields

$$\mathbb{E}_i[u_i'(c_i^*(\theta,s))|\tilde{a}_i,s] = \Psi(s)\Phi_i(\tilde{a}_i,s)$$

with

$$\Psi(s) \equiv \frac{\xi(s)}{\pi(s)} \quad \text{and} \quad \Phi_i(\tilde{a}_i,s) \equiv \sum_{\theta\in\Theta} \frac{n_i p_i(\theta|\tilde{a}_i,s)}{\psi_i + \sum_{\tilde{\tilde{a}}_i \in A_i}\mu_i(\tilde{\tilde{a}}_i)\left(1 - p_i(\theta|\tilde{\tilde{a}}_i,s)/p_i(\theta|a_i^*,s)\right)}.$$

If $p_i(\theta|a_i,s) = p_i(\theta|a_i,\tilde{s})$ for all $i \in I, \theta \in \Theta, s, \tilde{s} \in S$ and $a_i \in A_i$, it is clear that $\Phi_i(\tilde{a}_i,s)$ is in fact independent of s. Let me therefore write $\Phi_i(\tilde{a}_i)$ in the following. Then (16) implies

$$\frac{\pi(s)\mathbb{E}_i\left[u_i'(c_i^*(\theta,s))\middle|\,\tilde{a}_i,s\right]}{\pi(\tilde{s})\mathbb{E}_i\left[u_i'(c_i^*(\theta,\tilde{s}))\middle|\,\tilde{a}_i,\tilde{s}\right]}=\frac{\pi(s)\Psi(s)\Phi_i(\tilde{a}_i)}{\pi(\tilde{s})\Psi(\tilde{s})\Phi_i(\tilde{a}_i)}=\frac{\xi(s)}{\xi(\tilde{s})}=\frac{q^*(s)}{q^*(\tilde{s})}.$$

But this is satisfied by having set prices $q^*(s)=\xi(s)\ \forall s\in S$.

Step 2. Step 1 implies that, for any action choice $\tilde{a}_i\in A_i$, agents find it optimal to set $\Delta_i(s)=0\ \forall s\in S$ and hence, by construction of the transfers $\{T_i^*(\theta,s)\}$, to choose the constrained-efficient consumption schedule $\{c_i^*(\theta,s)\}$. The fact that the constrained-efficient allocation $\{c_i^*(\theta,s),a_i^*\}$ is incentive-compatible and satisfies (2) for all $\tilde{a}_i\in A_i$ then implies that the action that agents choose is the constrained-efficient action $a_i^*\ \forall i\in I$. This completes the proof. □

Theorem 1 implies that if aggregate shocks affect outputs but not probability distributions over idiosyncratic risk, then all agents' marginal rates of substitution are equalized in any constrained-efficient allocation. No distortions in the financial markets are therefore required. Note that, for this result to hold, no restrictions on how aggregate shocks may affect outputs nor on individuals' preferences other than separability are required.

The intuition for the result can be understood by considering the inverse of equation (17), which characterizes marginal utilities in any constrained-efficient allocation:

$$u_i'(c_i^*(\theta,s))=\frac{n_i\xi(s)}{\pi(s)}\left[\psi_i+\sum_{\tilde{a}_i\in A_i}\mu_i(\tilde{a}_i)\left(1-\frac{p_i(\theta|\tilde{a}_i,s)}{p_i(\theta|a_i^*,s)}\right)\right]^{-1}. \tag{18}$$

As can be seen from (18), aggregate shocks affect marginal utilities through two channels. First, they influence individual outputs $y_i(\theta,s)$ and thus aggregate output $Y(s)\equiv\sum_i\sum_\theta n_i p_i(\theta|a_i^*,s)y_i(\theta,s)$. Moreover, since the only place where individual outputs enter the Pareto-problem is in the feasibility constraints (1), individual outputs $y_i(\theta,s)$ cannot affect the solution other than through aggregate output $Y(s)$ in state s. This is reflected in (18) in the term $\xi(s)/\pi(s)$, the (normalized) shadow price of the resource constraint in state s. However, (18) makes clear that variations in aggregate output and thus $\xi(s)/\pi(s)$ only scale marginal utilities up and down uniformly across agents in a constrained-efficient allocation, even in the present moral hazard economy. The second channel results from the fact that, as in standard moral hazard models, it is optimal to allocate marginal utilities to agents according to the likelihood ratios $p_i(\theta|\tilde{a}_i,s)/\ p_i(\theta|a_i^*,s)$ (see, for instance, Holmström (1979) and Milgrom (1981)). By the results in Milgrom (1981), the likelihood ratio is a measure of the "favorableness" of the information that output provides about the hidden effort choice. An output realization with a low likelihood ratio is "good news" about hidden effort choice and hence leads to higher optimal consumption. These likelihood ratios also generally depend on aggregate states. However, if aggregate and idiosyncratic shocks are independent, aggregate shocks leave the distribution of likelihood ratios unchanged, and thus the ratios of expected marginal utilities between different agents must be independent of aggregate states, which is the result in Theorem 1. In other words, if the distributions of likelihood ratios do not depend on s, aggregate states are symmetric in terms of the marginal resource costs of providing incentives, and it is optimal to provide incentives without distortions in the marginal rates of substitution between aggregate states.

As the proof of Theorem 1 shows, this property not only holds when agents choose the constrained-efficient action a_i^*, but for any action $a_i \in A_i$ they may choose, so that profitable double-deviations (where agents deviate to some action $\tilde{a}_i \neq a_i^*$ *and* trade in financial markets) do not exist. No interventions whatsoever are therefore required in financial markets, and any constrained optimum is consistent with agents freely trading financial securities. This is particularly interesting in comparison to dynamic contracting models. Not only are non-zero wedges between the return to saving and marginal rates of intertemporal substitution required in order to implement constrained-efficient allocations,[5] but also double-deviations (where agents deviate to a suboptimal action and save at the same time) are typically profitable, so that linear savings taxes equal to optimal wedges cannot implement the optimum (see, for instance, Kocherlakota (2005), Albanesi and Sleet (2006), and Golosov and Tsyvinski (2006)).

Moreover, Theorem 1 implies that unobservability of individual trades in financial markets does not put a further restriction on Pareto-optimal risk-sharing arrangements: constrained-efficient allocations with observable and unobservable trades fall together if aggregate shocks affect outputs only. This contrasts with a large literature on unobservable side trades in other settings. For instance, Acemoglu and Simsek (2009) consider a moral hazard model with anonymous side trades and show that the planner does not distort these trades if preferences between effort and consumption are separable. Even with separable preferences, double-deviations are generally binding in my framework, however, unless Theorem 1 applies.[6] Note also that the result in Theorem 1 is very different from those in dynamic models with unobservable savings such as in Golosov and Tsyvinski (2007). They show that, if the social planner cannot observe individual trades in a bond market, it is generally optimal to introduce a non-zero capital tax. An analogy to the result in Theorem 1 therefore does not exist in their framework. In order to be able to understand the driving forces behind these differences, I will consider a dynamic

[5]See Rogerson (1985) and Ligon (1998) for moral hazard models and Diamond and Mirrlees (1978) and Golosov et al. (2003) for private skill shock models with this property.

[6]The main difference is that there is no aggregate uncertainty in Acemoglu and Simsek (2009) and they assume that all trades take place *after* all (idiosyncratic) uncertainty is realized.

extension of the model in the next section.

4 A Dynamic Extension

In this subsection, I explore to what degree the result in Theorem 1 extends to dynamic settings where agents can save in addition to trade in financial markets. It will turn out that Theorem 1 generalizes to such economies in the sense that, whenever aggregate and idiosyncratic risk are independent, then while savings need to be distorted in order to implement constrained-efficient allocations, tax interventions in financial markets continue to be unnecessary. To formalize this claim, I consider the following dynamic extension of the model. There are two periods $t = 0, 1$. In the first period, individuals in group i consume c_i^0 and exert effort a_i. There is an aggregate endowment of capital k^0, and aggregate savings at the end of period 0 are denoted k^1. A linear savings technology transforms k^1 units of capital in period 0 into Rk^1 units in period 1. Both aggregate and idiosyncratic shocks are realized in period 1, so that outputs $y_i(\theta, s)$ are produced with probability distributions $\pi(s)$ and $p_i(\theta|a_i, s)$. Consumption in period 1 is denoted by $c_i^1(\theta, s)$. With a discount factor β, preferences are given by $u_i(c_i^0) - v_i(a_i) + \beta \sum_{s \in S} \sum_{\theta \in \Theta} u_i(c_i^1(\theta, s)) p_i(\theta|a_i, s)\pi(s)$ and the Pareto-problem becomes

$$\max_{\{a_i, c_i^0, c_i^1(\theta,s), k^1\}} \sum_{i \in I} \psi_i \left[u_i(c_i^0) - v_i(a_i) + \beta \sum_{s \in S} \sum_{\theta \in \Theta} u_i(c_i^1(\theta, s)) p_i(\theta|a_i, s)\pi(s) \right] \tag{19}$$

subject to

$$\sum_{i \in I} n_i c_i^0 + k^1 \leq k^0, \tag{20}$$

$$\sum_{i \in I} n_i \sum_{\theta \in \Theta} c_i^1(\theta, s) p_i(\theta|a_i, s) \leq \sum_{i \in I} n_i \sum_{\theta \in \Theta} y_i(\theta, s) p_i(\theta|a_i, s) + Rk^1 \quad \forall s \in S, \tag{21}$$

$$\begin{aligned} & \beta \sum_{s \in S} \sum_{\theta \in \Theta} u_i(c_i^1(\theta, s)) p_i(\theta|a_i, s)\pi(s) - v_i(a_i) \\ \geq \quad & \beta \sum_{s \in S} \sum_{\theta \in \Theta} u_i(c_i^1(\theta, s)) p_i(\theta|\tilde{a}_i, s)\pi(s) - v_i(\tilde{a}_i) \quad \forall \tilde{a}_i \in A_i. \end{aligned} \tag{22}$$

(20) and (21) are the resource constraints in periods 0 and 1, respectively, and (22) is the set of incentive constraints. The resulting constrained-efficient allocation is denoted $\{c_i^{0*}, c_i^{1*}(\theta, s), k^{1*}, a_i^*\}$.

The purpose of this subsection is to show that, if θ and s are stochastically independent, then any Pareto optimal allocation can be implemented as an equilibrium with financial markets, where agents face transfers T_i^0 in period 0 and $T_i^1(\theta, s)$ in period 1, and only savings in capital are distorted by taxes, but not trades in financial markets. In particular, let agents now make two trading decisions in period 0: First, they decide about trades in financial securities, which pay out $\Delta_i(s)$ in state s in period 1 and have prices $q(s)$ as before. Second, they choose how much to save in capital, whereby saving a unit k_i^1 in period 0 yields $R(1 - t_i(\theta, s))k_i^1$ in period 1. Here, $t_i(\theta, s)$ is a linear but shock-contingent tax on the return to saving imposed in period 1. Given this, agents of group $i \in I$ solve

$$\max_{\{a_i, c_i^0, c_i^1(\theta,s), k_i^1, \Delta_i(s)\}} u_i(c_i^0) - v_i(a_i) + \beta \sum_{s \in S} \sum_{\theta \in \Theta} u_i(c_i(\theta, s)) p_i(\theta | a_i, s) \pi(s) \tag{23}$$

subject to

$$c_i^0 = k_i^0 - T_i^0 - k_i^1, \tag{24}$$

$$c_i^1(\theta, s) = y_i(\theta, s) - T_i(\theta, s) + \Delta_i(s) + R(1 - t_i(\theta, s))k_i^1, \tag{25}$$

$$\sum_{s \in S} q(s)\Delta_i(s) = 0. \tag{26}$$

Here, k_i^0 denotes the capital endowment of agents in group i in period 0, with $\sum_{i \in I} n_i k_i^0 = k^0$. Note that this asset structure transparently separates intertemporal from intratemporal trading: Agents move resources across periods by saving in capital, and across aggregate states within periods by trading in financial markets. Then the definition of an equilibrium with financial markets is a straightforward extension of Definition 2:

Definition 3. *An equilibrium in financial markets with transfers $\{T_i^0, T_i^1(\theta, s)\}$ and savings taxes $\{t_i(\theta, s)\}$ is an allocation $\{c_i^{0e}, c_i^{1e}(\theta, s), k_i^{1e}, a_i^e\}$, a trading profile $\{\Delta_i^e(s)\}$ and prices $\{q^e(s)\}$ such that $\{c_i^{0e}, c_i^{1e}(\theta, s), k_i^{1e}, a_i^e, \Delta_i^e(s)\}$ solves the agent's problem (23) to (26) taking prices $\{q^e(s)\}$, transfers $\{T_i^0, T_i^1(\theta, s)\}$ and saving taxes $\{t_i(\theta, s)\}$ as given, financial markets clear*

$$\sum_{i \in I} n_i \Delta_i^e(s) = 0 \; \forall s \in S, \tag{27}$$

and the goods market clears in both periods

$$\sum_{i \in I} n_i c_i^{0e} = \sum_{i \in I} n_i (k_i^0 - k_i^{1e}), \tag{28}$$

and

$$\sum_{i \in I} n_i \sum_{\theta \in \Theta} c_i^{1e}(\theta, s) p_i(\theta | a_i^e, s) = \sum_{i \in I} \sum_{\theta \in \Theta} y_i(\theta, s) p_i(\theta | a_i^e, s) + \sum_{i \in I} n_i R k_i^{1e} \; \forall s \in S. \tag{29}$$

Note that this again implies budget balance for the government by Walras' law. This leads to the following result:

Theorem 2. *Consider a constrained-efficient allocation* $\{c_i^{0*}, c_i^{1*}(\theta, s), k^{1*}, a_i^*\}$ *and suppose that aggregate shocks and idiosyncratic shocks are independent. Then it can be implemented as an equilibrium in financial markets using transfers* $\{T_i^0, T_i^1(\theta, s)\}$ *and savings taxes* $\{t_i(\theta, s)\}$ *only and without tax interventions in financial markets.*

Proof. Fix some $\{k_i^{1*}\}$ such that $\sum_{i\in I} n_i k_i^{1*} = k^{1*}$ and set the savings taxes such that

$$1 - t_i^*(\theta, s) = u_i'(c_i^{0*})/(\beta R u_i'(c_i^{1*}(\theta, s))), \tag{30}$$

and transfers $T_i^{0*} = k_i^0 - c_i^{0*} - k_i^{1*}$ and $T_i^{1*}(\theta, s) = y_i(\theta, s) - c_i^{1*}(\theta, s) + R(1 - t_i^*(\theta, s))k_i^{1*}$ for all $i \in I, s \in S, \theta \in \Theta$. To see that we can implement the Pareto-optimal allocation $\{c_i^{0*}, c_i^{1*}(\theta, s), k^{1*}, a_i^*\}$ as an equilibrium with these taxes and transfers, let prices be $q^*(s) = \xi^1(s)$, where $\xi^1(s)$ is the shadow price of the resource constraint (21) in period 1 of the Pareto-problem (19) to (22), and rewrite the agents' problem for a given effort $\tilde{a}_i \in A_i$ substituting out consumption as follows:

$$\begin{aligned}\max_{\{k_i^1, \Delta_i(s)\}} \quad & \Gamma_i\left(\{k_i^1, \Delta_i(s)\}\right) \\ \equiv \quad & u_i(k_i^0 - T_i^{0*} - k_i^1) + \beta \sum_{s\in S}\sum_{\theta\in\Theta} u_i\left(y_i(\theta, s) - T_i^{1*}(\theta, s) + \Delta_i(s) + R(1 - t_i^*(\theta, s))k_i^1\right) p_i(\theta|\tilde{a}_i)\pi(s)\end{aligned}$$

such that $\sum_{s\in S} q^*(s)\Delta_i(s) = 0$. Note that I already used the fact that θ and s are stochastically independent by writing $p_i(\theta|\tilde{a}_i)$, dropping the dependence on s.

Observe that the objective function $\Gamma_i\left(\{k_i^1, \Delta_i(s)\}\right)$ is strictly concave and the constraint is linear, so that first order conditions are necessary and sufficient for an optimum.[7] First, the first order condition for k_i^1 evaluated at k_i^{1*} and $\Delta_i(s) = 0\ \forall s \in S$ is

$$u_i'(c_i^{0*}) = \beta R \sum_{s\in S}\sum_{\theta\in\Theta}(1 - t_i^*(\theta, s))u_i'(c_i^{1*}(\theta, s))p_i(\theta|\tilde{a}_i)\pi(s)$$

and substituting $t_i^*(\theta, s)$ from (30) shows that it is satisfied for all $i \in I$. Next, the first order condition for $\Delta_i(s)$, evaluated at $\Delta_i(s) = 0\ \forall s \in S$ and k_i^{1*}, implies

$$\frac{\pi(s)\mathbb{E}_i[u_i'(c_i^{1*}(\theta, s))|\tilde{a}_i, s]}{\pi(\tilde{s})\mathbb{E}_i[u_i'(c_i^{1*}(\theta, \tilde{s}))|\tilde{a}_i, \tilde{s}]} = \frac{q^*(s)}{q^*(\tilde{s})}. \tag{31}$$

Note that, from the necessary first order condition for $c_i^{1*}(\theta, s)$ from the Pareto-problem (19) to (22),

$$\mathbb{E}_i[u_i'(c_i^{1*}(\theta, s))|\tilde{a}_i, s] = \frac{\xi^1(s)}{\beta\pi(s)}\sum_{\theta\in\Theta}\frac{n_i p_i(\theta|\tilde{a}_i)}{\psi_i + \sum_{\tilde{\tilde{a}}_i\in A_i}\mu_i(\tilde{\tilde{a}}_i)\left(1 - p_i(\theta|\tilde{\tilde{a}}_i)/p_i(\theta|a_i^*)\right)},$$

where $\xi^1(s)$ is the multiplier on the resource constraint in period 1 for state $s \in S$ (21) and $\mu_i(\tilde{a}_i)$ the

[7] To see that $\Gamma_i\left(\{k_i^1, \Delta_i(s)\}\right)$ is strictly concave, note that for any $\{k_i^1, \Delta_i(s)\}$ and $\{\hat{k}_i^1, \hat{\Delta}_i(s)\}$, the function $H_i(\lambda) \equiv \Gamma_i\left(\{\lambda k_i^1 + (1-\lambda)\hat{k}_i^1, \lambda\Delta_i(s) + (1-\lambda)\hat{\Delta}_i(s)\}\right)$ is strictly concave.

multiplier on the incentive constraint for action $\tilde{a}_i \in A_i$ (22). Hence,

$$\frac{\pi(s)\mathbb{E}_i[u_i'(c_i^{1*}(\theta,s))|\tilde{a}_i,s]}{\pi(\tilde{s})\mathbb{E}_i[u_i'(c_i^{1*}(\theta,\tilde{s}))|\tilde{a}_i,\tilde{s}]} = \frac{\xi^1(s)}{\xi^1(\tilde{s})}$$

whenever θ and s are stochastically independent. Substituting this in (31) reveals that (31) is always satisfied when prices are set such that $q^*(s)/q^*(\tilde{s}) = \xi^1(s)/\xi^1(\tilde{s})$ for all $s, \tilde{s} \in S$.

This shows that, for any given $\tilde{a}_i \in A_i$, agents find it optimal to set $\Delta_i(s) = 0\ \forall\, s \in S$ and $k_i^1 = k_i^{1*}\ \forall i \in I$. Finally, incentive compatibility of the constrained-efficient allocation implies that agents also find it optimal to set $a_i = a_i^*$ for all $i \in I$, which completes the proof. □

As is common for a large class of dynamic incentive problems, any Pareto-optimal allocation in the present model satisfies the Inverse Euler equation. Indeed, the necessary first order conditions for the Pareto-problem (19) to (22) can be combined to

$$\sum_{s\in S} \pi(s) \frac{1/u_i'(c_i^{0*})}{1/(\beta R) \times \mathbb{E}_i\left[1/u_i'(c_i^{1*}(\theta,s)) \,\middle|\, a_i^*, s\right]} = 1,$$

which is the generalized version of the Inverse Euler equation accounting for aggregate uncertainty derived by Kocherlakota (2005). It immediately implies that a wedge between the marginal rate of intertemporal substitution $u_i'(c_i^{0*})/\left(\beta \sum_s \pi(s)\mathbb{E}_i\left[u_i'(c_i^{1*}(\theta,s)) \,\middle|\, a_i^*, s\right]\right)$ and the return to saving R needs to be introduced in the agent's Euler equation. As was pointed out by Kocherlakota (2005), due to double-deviations, simple linear taxes on the return to savings that only depend on the group $i \in I$ and the aggregate state $s \in S$ do not implement a constrained-efficient allocation, however. Extending his insights to the present setting reveals that any Pareto-optimum is implementable as an equilibrium with group-specific linear savings taxes

$$t_i^*(\theta,s) = 1 - \frac{u_i'(c_i^{0*})}{\beta R u_i'(c_i^{1*}(\theta,s))} \tag{32}$$

that depend on both the aggregate and the idiosyncratic shock for each agent. The key additional result in Theorem 2 is that these are in fact sufficient even when agents can trade in financial markets rather than just save, and further tax interventions in these financial markets are not required if aggregate and idiosyncratic shocks are independent.

Even though the aggregate savings technology is assumed to be risk-free for simplicity, the savings taxes $t_i^*(\theta,s)$ in (32) generally depend on the aggregate shock $s \in S$.[8] However, in the special case where there exists a risk-neutral group, it can be shown that

[8] It is easy to see that the setup and the result in Theorem 2 are straightforward to generalize to a savings technology that is nonlinear and/or contingent on $s \in S$, or to more than two periods.

they are in fact independent of s.

Proposition 1. *Suppose there exists a risk-neutral group $i \in I$ and aggregate shocks and idiosyncratic shocks are independent. Then any constrained-efficient allocation can be implemented as an equilibrium in financial markets without tax interventions, using transfers $\{T_i^0, T_i^1(\theta, s)\}$ and savings taxes $\{t_i(\theta)\}$ only, where the latter do not depend on aggregate shocks.*

Proof. The necessary first order conditions of the Pareto-problem (19) to (22) imply that $u_i'(c_i^{0*}) = \xi^0 n_i/\psi_i$, where ξ^0 is the multiplier on the period 0 resource constraint, and

$$\frac{1}{u_i'(c_i^{1*}(\theta,s))} = \frac{\beta\pi(s)}{n_i\xi^1(s)}\phi_i(\theta) \quad \text{with} \quad \phi_i(\theta) \equiv \psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i)\left(1 - \frac{p_i(\theta|\tilde{a}_i)}{p_i(\theta|a_i^*)}\right). \tag{33}$$

Substituting this in the construction of the saving taxes in (30) yields

$$1 - t_i^*(\theta, s) = \frac{u_i'(c_i^{0*})}{\beta R u_i'(c_i^{1*}(\theta,s))} = \frac{\pi(s)}{\xi^1(s)}\frac{\xi^0\phi_i(\theta)}{\psi_i R}. \tag{34}$$

Assuming that group $i = 1 \in I$ is risk neutral, integrating equation (33) for group 1 over $\theta \in \Theta$ implies

$$\text{const.} = \sum_{\theta\in\Theta} p_1(\theta|a_1^*)\frac{\beta\pi(s)}{n_1\xi^1(s)}\phi_1(\theta) = \frac{\beta\pi(s)\psi_1}{\xi^1(s)n_1} \quad \forall s \in S,$$

so that $\pi(s)/\xi^1(s)$ in is in fact independent of s in this case. Using this in (34) completes the proof. □

If there exists a group of risk-neutral investors in the financial market, then in addition to the previous results, the taxes $t_i^*(\theta)$ that are required to discourage savings at the optimum do not need to condition on the realized aggregate state $s \in S$. Hence, in this case, not only do financial markets not put an additional constraint on the social planner, but in addition aggregate uncertainty does not even affect the way in which the social planner provides intertemporal incentives.

This is in contrast to Golosov et al. (2006) who find that optimal savings taxes move with aggregate government spending shocks, even though they do not affect the distribution of private skill shocks. The difference is a result of the moral hazard model studied here rather than the skill shock model that they consider. Here, the unobservable action is chosen before shocks are realized. In skill shock models, agents choose their actions, such as labor supply, after all shocks have been realized and optimal consumption is not determined by likelihood ratios, so that results similar to Theorems 1 and 2 have not been derived in this case.

5 Non-independent Aggregate and Idiosyncratic Shocks

5.1 Implementation with Non-linear Asset Taxes

Let me next turn to the case where aggregate and idiosyncratic shocks are not stochastically independent, so that aggregate shocks do affect probability distributions over idiosyncratic risk. In this case, it is necessary to introduce tax distortions in financial markets in order to implement constrained-efficient allocations as competitive equilibria in the sense of Definition 2. Moreover, similar to the observation for savings taxes in the previous subsection, simple linear taxes on transactions in the financial markets that only depend on the group $i \in I$ and the aggregate state $s \in S$ do not implement Pareto-optima. Building upon the insights derived by Werning (2009) for capital taxation in a dynamic Mirrlees model, I construct a non-linear asset tax system for the present static moral hazard model with aggregate uncertainty that implements constrained efficient allocations. The resulting asset taxes on financial trades also exhibit interesting properties that further clarify the underlying economics of the implementation problem with aggregate uncertainty, particularly in comparison to earlier results on capital taxation in dynamic models, as I demonstrate in the following.

For the analysis in this subsection, it is useful to return to the static model of section 3 and to index the set S of aggregate states with $h = 0, ..., H$, where $H = \|S\|$. Let me also use the Arrow-Debreu security for state $s_0 \in S$ as the numeraire asset and normalize $q(s_0) = 1$. Then the idea is that, when an agent chooses some trading strategy $\Delta_i(s)$ in the financial markets, she is required to pay an asset tax $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ in terms of the numeraire asset for state s_0 (and the tax schedule does not condition on $\Delta_i(s_0)$ without loss of generality). Agents in group $i \in I$ take the asset tax κ_i and the transfers $\{T_i(\theta, s)\}$ as well as the prices $\{q(s)\}$ of the Arrow-Debreu securities as given and solve

$$\max_{\{c_i(\theta,s),\Delta_i(s),a_i\}} \sum_s \sum_\theta u_i(c_i(\theta, s))p_i(\theta|a_i, s)\pi(s) - v_i(a_i) \tag{35}$$

subject to their budget constraint in the financial market

$$\Delta_i(s_0) + \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) + \sum_{s \neq s_0} q(s)\Delta_i(s) \leq 0, \tag{36}$$

where $c_i(\theta, s)$ is given by

$$c_i(\theta, s) = y_i(\theta, s) - T_i(\theta, s) + \Delta_i(s) \tag{37}$$

for all $\theta \in \Theta, s \in S$. An equilibrium with the asset tax $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ in the financial market and transfers $\{T_i(\theta,s)\}$ can then be defined analogously to the previous subsection, namely as an allocation $\{c_i^e(\theta,s), a_i^e\}$, a trading profile $\{\Delta_i^e(s)\}$ and prices $\{q^e(s)\}$ such that $\{c_i^e(\theta,s),\ \Delta_i^e(s),\ a_i^e\}$ solve the agents' problem (35) to (37) given prices $\{q^e(s)\}$, the asset tax $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ and transfers $\{T_i(\theta,s)\}$, financial markets clear for each aggregate state (equation (12)), and the the goods market clears in each state (equation (13)). The following theorem constructs a tax system with a non-linear asset tax schedule $\kappa_i(\Delta_i(s_1), ..., \ \Delta_i(s_H))$ and transfers $\{T_i(\theta,s)\}$ that implements a constrained-efficient allocation $\{c_i^*(\theta,s),\ a_i^*\}$ as an equilibrium with financial markets.

Theorem 3. *Consider any constrained-efficient allocation $\{c_i^*(\theta,s), a_i^*\}$ that solves (3) subject to (1) and (2). Let*

$$W_i^* \equiv \sum_{s\in S}\sum_{\theta\in\Theta} u_i(c_i^*(\theta,s))p_i(\theta|a_i^*,s)\pi(s) - v_i(a_i^*)\ \forall i \in I \tag{38}$$

and fix prices $\{q^*(s)\}$. For each $i \in I$ and each trading profile $\{\Delta_i(s)\}$, let the asset tax schedule $\kappa_i\ (\Delta_i(s_1), ..., \Delta_i(s_H))$ be implicitly defined by

$$\begin{aligned} W_i^* \ = \ & \max_{a_i\in A_i}\Bigg\{\sum_{\theta} u_i\left(c_i^*(\theta,s_0) - \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) - \sum_{s\neq s_0} q^*(s)\Delta_i(s)\right) p_i(\theta|a_i,s_0)\pi(s_0) \\ & + \sum_{s\neq s_0}\sum_{\theta} u_i(c_i^*(\theta,s)+\Delta_i(s))p_i(\theta|a_i,s)\pi(s) - v_i(a_i)\Bigg\}. \end{aligned} \tag{39}$$

If $u_i(c)$ is unbounded, strictly increasing and continuous, then there exists a unique and continuous asset tax schedule $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ solving (39). Set transfers $\{T_i(\theta,s)\}$ such that

$$T_i^*(\theta,s) = y_i(\theta,s) - c_i^*(\theta,s)\ \ \forall i \in I, \theta \in \Theta, s \in S. \tag{40}$$

Then the allocation $\{c_i^*(\theta,s), a_i^*\}$, the trading profile $\{\Delta_i(s) = 0\}$ and prices $\{q^*(s)\}$ are an equilibrium given the asset tax $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ and transfers $\{T_i^*(\theta,s)\}$.

Proof. See the Appendix. □

The result in Theorem 3 is that, for any constrained efficient allocation, there exists a continuous asset tax schedule that conditions only on an individual's trades, not on idiosyncratic shocks, and implements it as an equilibrium. Thus, when asset taxes are not constrained to be linear, information about individual outputs is not necessary to impose asset taxes. Moreover, the implementation does not rely on sharp penalties in the form of discontinuous taxes. This is in contrast to a direct mechanism that would completely prevent agents from trading in financial markets, which can be thought of imposing an infinite tax whenever agents deviate from $\Delta_i(s) = 0 \; \forall s \in S$.

The construction of the asset tax in equation (39) makes clear how the implementation works: For any trading strategy $\Delta_i(s)$, the asset tax schedule $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ is such that, when choosing the optimal action given this trading strategy, the agent is just indifferent to the constrained efficient allocation $\{c_i^*(\theta, s), a_i^*\}$. There therefore exists no deviation that could make the agent better off.

5.2 Properties of Optimal Asset Taxes

Let me finally emphasize some properties of the asset tax schedule $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ that are particularly useful to further illuminate the intuition behind the previous results and to relate it to the literature on optimal dynamic taxation. Notably, the marginal asset taxes are quite closely related to what has been termed "wedges" there (see e.g. Golosov et al. (2006)). To illustrate this, let me set prices such that $q^*(s)/q^*(\tilde{s}) = \xi(s)/\xi(\tilde{s}) \; \forall s, \tilde{s} \in S$, i.e. let relative prices equal the social marginal rate of substitution between aggregate states s and $\tilde{s}$ given by the ratio of the shadow costs of the aggregate resource constraints in the two states at the constrained-efficient allocation to be implemented[9]. For any trading strategy $\Delta_i(s)$, let the solution of the maximization in (39) be given by $\alpha_i^*(\Delta_i(s_1), ..., \Delta_i(s_H))$, so that α_i^* is the set of actions $a_i \in A_i$ maximizing the RHS of (39) given $\Delta_i(s)$. If $\alpha_i^*(\Delta_i(s_1), ..., \Delta_i(s_H))$ is single-valued, then a standard Envelope Theorem implies that for $h \in \{1, ..., H\}$,

$$\frac{\partial \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))}{\partial \Delta_i(s_h)} = \frac{\pi(s_h)\mathbb{E}_i[u_i'(c_i^*(\theta, s_h) + \Delta_i(s_h))|\alpha_i^*, s_h]}{\pi(s_0)\mathbb{E}_i\left[u_i'\left(c_i^*(\theta, s_0) - \kappa_i - \sum_{s \neq s_0} \Delta_i(s)\right)\middle|\, \alpha_i^*, s_0\right]} - \frac{\xi(s_h)}{\xi(s_0)},$$

where κ_i stands short for $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ and α_i^* for $\alpha_i^*(\Delta_i(s_1), ..., \Delta_i(s_H))$.[10] The marginal tax is thus equal to the "wedge," namely the difference between the private and social marginal rate of substitution. In particular, at the implemented trading profile

[9] Note that Theorem 3 allows us to fix prices in this way.

[10] This would be typically the case when a_i is chosen from a continuum of possible actions and α_i^* satisfies a first-order condition. However, when A_i is a discrete set, this is not the case since the optimal allocation $\{c_i^*(\theta, s), a_i^*\}$ is such that an agent of group i is just indifferent between a_i^* and a deviation $\tilde{a}_i \in A_i$. In this case, the asset tax schedule actually has a kink and is not differentiable at the equilibrium trading strategy $\Delta_i(s) = 0$. Nevertheless, left and right derivatives can still be computed using Envelope Theorems (see e.g. Milgrom and Segal (2002)), leading to the same results on (directional) marginal asset taxes as derived below.

$\{\Delta_i(s) = 0\}$ and with the transfers $\{T_i(\theta, s)\}$ defined in (40), this reduces to

$$\frac{\partial \kappa_i^*}{\partial \Delta_i(s_h)} = \frac{\pi(s_h)\mathbb{E}_i[u_i'(c_i^*(\theta, s_h))|a_i^*, s_h]}{\pi(s_0)\mathbb{E}_i\left[u_i'\left(c_i^*(\theta, s_0)\right)\middle| a_i^*, s_0\right]} - \frac{\xi(s_h)}{\xi(s_0)}. \tag{41}$$

In the following, I demonstrate how these marginal asset taxes can be signed. In particular, it turns out that the asset taxes impose higher marginal tax rates on those assets that pay out in aggregate states in which optimal consumption is more volatile due to a more severe moral hazard problem: If for group i consumption at the constrained efficient allocation is more risky in state s_h compared to state s_0 (as will be formalized below), then $\partial \kappa_i^* / \partial \Delta_i(s_h) > 0$.

Recall that optimal consumption is determined by likelihood ratios according to equation (18). For later use, I therefore denote the likelihood ratio of group $i \in I$ for a given constrained-efficient allocation $\{c_i^*(\theta, s), a_i^*\}$, a deviation $\tilde{a}_i \in A_i$ and for all $\theta \in \Theta$ and $s \in S$ by

$$l_i(\theta|\tilde{a}_i, s) \equiv \frac{p_i(\theta|\tilde{a}_i, s)}{p_i(\theta|a_i^*, s)}. \tag{42}$$

Let me denote the cumulative distribution function of $l_i(\theta|\tilde{a}_i, s)$ given the constrained-efficient action a_i^* by $G_i(l|\tilde{a}_i, s) \equiv \mathrm{Pr}_i(l_i(\theta|\tilde{a}_i, s) \leq l|a_i^*, s)$ and the corresponding probability density function by $g_i(l|\tilde{a}_i, s)$. Note that the mean of $l_i(\theta|\tilde{a}_i, s)$ given the constrained-efficient action is one for all groups $i \in I$, states $s \in S$ and deviations $\tilde{a}_i \in A_i$ because

$$\sum_l l g_i(l|\tilde{a}_i, s) = \mathbb{E}_i[l_i(\theta|\tilde{a}_i, s)|a_i^*, s] = \sum_{\theta \in \Theta} \frac{p_i(\theta|\tilde{a}_i, s)}{p_i(\theta|a_i^*, s)} p_i(\theta|a_i^*, s) = 1.$$

Aggregate shocks therefore cannot shift the mean of the distribution of likelihood ratios, but only change higher moments. The following general result shows that it is the *volatility* of this distribution that is crucial for optimal asset taxes in financial markets.

Theorem 4. *Consider a constrained-efficient allocation $\{c_i^*(\theta, s), a_i^*\}$. Suppose that $G_i(l|\tilde{a}_i, s_h)$ is a mean-preserving spread of $G_i(l|\tilde{a}_i, s_0)$, i.e. it is more risky in the sense of second-order stochastic dominance (SOSD), for all $\tilde{a}_i$ for which group i's incentive constraint (2) is binding. Then $\partial \kappa_i^* / \partial \Delta_i(s_h) \geq 0$.*

Proof. See the Appendix. □

To understand the claim in Theorem 4, consider for simplicity the typical case where the incentive constraint is binding for only a single deviation $\tilde{a}_i \in A_i$, so that only one distribution of likelihood ratios needs to be considered. Then the claim is that group i

faces a positive marginal asset tax on assets that pay out in state s_h if state s_h leads to a *riskier* distribution of likelihood ratios than state s_0 for group i in the sense of a mean-preserving spread.

Intuitively, since the social planner varies consumption according to likelihood ratios at the optimum, consumption will be more volatile in those aggregate states that involve a more volatile likelihood ratio. Whereas the planner spreads consumption in all aggregate states such that ratios of expected inverse marginal utilities are the same across states by Theorem 1, individuals' trading incentives in the financial market are determined by expected marginal utilities. By the convexity of the function $f(x) = 1/x$, individuals have a higher expected marginal utility and thus an incentive to buy additional consumption in financial markets for those aggregate states in which the likelihood ratio and hence consumption vary more. One may think of this as individuals buying additional consumption to 'self-insure' against their more volatile consumption in those states. To prevent this, the social planner needs to introduce a positive marginal asset tax on the corresponding security.

In particular, Theorem 4 has the following immediate Corollary:

Corollary 1. *Consider a constrained-efficient allocation $\{c_i^*(\theta, s), a_i^*\}$. Suppose there is a group $i \in S$ for which consumption in state s_0 is deterministic, i.e. $c_i^*(\theta, s_0) = c_i^*(s_0)$ for all $\theta \in \Theta$. Then $\partial\kappa_i^*/\partial\Delta_i(s_h) \geq 0 \ \ \forall s_h \neq s_0 \in S$, with strict inequality whenever consumption in state s_h is not deterministic for group i.*

There are two important cases in which optimal consumption is deterministic in state s_0: The first arises when the agents' action does not affect probability distributions in state s_0, i.e. $p_i(\theta|a_i^*, s_0) = p_i(\theta|\tilde{a}_i, s_0) \ \forall \tilde{a}_i \in A_i, \theta \in \Theta$ (no moral hazard state). Then the likelihood ratios are flat with $p_i(\theta|\tilde{a}_i, s_0)/p_i(\theta|a_i^*, s_0) = 1 \ \forall \tilde{a}_i \in A_i, \theta \in \Theta$, and full insurance is optimal since output contains no information about agents' effort. The second case results when output is deterministic for the optimal action in state s_0, i.e. $p_i(\theta|a_i^*, s_0) = 1$ for some $\theta \in \Theta$, which immediately implies full insurance. In either case, Corollary 1 implies that all the securities for the other states that involve stochastic consumption must be taxed at the margin.

The special case with an aggregate state that leads to deterministic consumption is particularly interesting because it illustrates the relationship of the results about wedges in the present model to those that arise in the dynamic models discussed in the preceding subsection. There, it was noted that the Inverse Euler equation

$$\frac{1}{u'(c_t)} = \frac{1}{\beta R_t}\mathbb{E}_t\left[\frac{1}{u'(c_{t+1})}\right]$$

implies the equalization of expected inverse marginal utilities across time periods. Similarly to financial markets here, agents equalize expected marginal utilities over time, however, when they can freely save, as implied by a standard Euler equation

$$u'(c_t) = \beta R_t \mathbb{E}_t[u'(c_{t+1})].$$

This conflict also generates a wedge between agents' intertemporal marginal rate of substitution and the marginal return to saving R_t, which can be thought of as an (implicit) tax on the return to saving that has to be introduced to implement the optimum. It is straightforward to see (based on Jensen's inequality) that this implicit tax is always positive in these models.[11] The reason can be understood from Corollary 1: Current consumption c_t is deterministic from the point of view of period t, whereas future consumption c_{t+1} is typically stochastic. Agents would buy too much consumption for the risky state (or the future) by buying securities in the financial market (or saving) if there were no distortions. The social planner therefore needs to tax the Arrow-Debreu securities for the risky state (the return to saving).

5.3 An Example and Implications for Differential Asset Taxation

Consider a situation with two aggregate states, "good times" s_0 and "bad times" s_1 (for instance a recession). Let the idiosyncratic uncertainty of group i be unemployment risk, described by two states as well with θ_u for the state in which the worker is unemployed and θ_e when he is employed. Finally, let there be two possible effort levels $a \in \{a_l, a_h\}$, where low effort a_l corresponds to shirking, which raises the risk of unemployment (both in good and bad times). For example, one could think of this as the unobservable component of some ex ante human capital investment decision that is undertaken before the aggregate state of the economy is realized. Suppose that, in good times, the unemployment risk when putting high effort is very low, and becomes high only when the agent has shirked, i.e.

$$p_i(\theta_u|a_h, s_0) \ll p_i(\theta_u|a_l, s_0),$$

capturing the idea that, in good times, essentially only people who shirked face layoff risk. In contrast, suppose that, in a recession, even agents who put high effort a_h face considerable layoff risk, so that

$$p_i(\theta_u|a_l, s_1) \approx p_i(\theta_u|a_h, s_1).$$

[11] See, for instance, Diamond and Mirrlees (1978), Golosov et al. (2003), Farhi and Werning (2009).

From Theorem 4, we know that what matters for asset taxes is the likelihood ratio, which by the above assumptions is very volatile in good times because

$$l_i(\theta_e|a_l,s_0) = \frac{1-p_i(\theta_u|a_l,s_0)}{1-p_i(\theta_u|a_l,s_0)} \ll \frac{p_i(\theta_u|a_l,s_0)}{p_i(\theta_u|a_l,s_0)} = l_i(\theta_u|a_l,s_0).$$

Accordingly, at a constrained-efficient allocation $\{c_i^*(\theta,s), a_h\}$ (assuming that it is optimal to implement high effort a_h), the social planner varies consumption considerably in good times: $c_i^*(\theta_u,s_0) \ll c_i^*(\theta_e,s_0)$ by equation (18). This is because, in good times, being unemployed is a strong signal that the worker shirked, which is punished by low consumption. However, in bad times, the likelihood ratio is much less informative about effort, so that $l_i(\theta_e|a_l,s_1) \approx l_i(\theta_e|a_h,s_1)$ and the planner provides much more insurance with $c_i^*(\theta_u,s_1) \approx c_i^*(\theta_e,s_1)$. Intuitively, during a recession, even many agents who put high effort end up unemployed, so it becomes optimal for the planner to provide more generous unemployment benefits than in good times.[12]

If the objective is to implement such an optimal insurance system over the business cycle while allowing agents to trade in financial markets, by the above arguments they would want to self-insure against their more risky consumption profile during good times by purchasing assets that pay out in good rather than bad times, unless the planner imposes taxes on these assets. For instance, one could think of the asset that pays out relatively more in state s_0 as a stock market index as opposed to fixed income securities, which would pay out relatively more in state s_1. Then, by Theorem 4, the optimal asset tax schedule would involve taxing the purchase of such equity claims relative to fixed income assets at the margin.

Hence, the present framework and the derived properties of the optimal asset tax schedule provide an efficiency based justification for a differential tax treatment of different asset classes that is indeed observed in many countries. For instance, the deductibility of interest on debt payments from the corporate tax base in many tax systems effectively leads to a situation where equity claims are taxed at a higher rate than debt claims, consistent with the optimal pattern derived in Theorem 4.

6 Conclusion

I have derived optimality conditions for allocations in a moral hazard economy with heterogeneous agents and aggregate shocks and characterized their implications for optimal

[12]See for instance Landais et al. (2011) for a less stylized model that generates such a pattern of optimal unemployment insurance over the business cycle.

tax policy in financial markets. As a benchmark result, I have first shown that financial markets should be undistorted at the optimum if aggregate and idiosyncratic shocks are independent. I have also demonstrated that this result generalizes in a natural way to a dynamic setting where agents can save in addition to trade in financial markets. When aggregate and idiosyncratic risk are correlated, then in order to decentralize Pareto-optimal allocations as competitive equilibria with financial markets, the government may impose nonlinear asset taxes in financial markets. They are such that financial assets that pay out in aggregate states with more risky consumption are taxed, preventing agents from self-insuring against these shocks.

Given that consumption is taken to be observable, the tax decentralization developed here is one of many possible implementations. For instance, Pareto-optima could also be implemented by private insurance companies that competitively provide insurance contracts, prohibiting their customers from trading in financial markets. Alternatively, the government could provide all the insurance and completely shut down financial markets. The implementation considered here, with agents trading in financial markets, but subject to tax distortions, is an intermediate case that may be more closely related to real-world tax systems. For instance, I have shown that a differential tax treatment of debt and equity claims can be justified, a feature shared by many real-world corporate tax systems. In addition, in a multi-country setting, the tax implementation discussed here would share features of transaction taxes on international financial flows that have been discussed recurrently.

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A Appendix

A.1 Proof of Theorem 3

I start with showing that the asset tax schedule defined in (39) is unique and continuous if $u_i(c)$ is unbounded, strictly increasing and continuous. To do so, note that, for any given trading strategy $\Delta_i(s)$, (39) is equivalent to requiring that

$$\begin{aligned} & \sum_\theta u_i \left(c_i^*(\theta, s_0) - \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) - \sum_{s \neq s_0} q^*(s)\Delta_i(s) \right) p_i(\theta|a_i, s_0)\pi(s_0) \\ \leq \quad & W_i^* - \sum_{s \neq s_0} \sum_\theta u_i(c_i^*(\theta, s) + \Delta_i(s)) p_i(\theta|a_i, s)\pi(s) + v_i(a_i) \end{aligned} \tag{43}$$

for all $a_i \in A_i$, with equality for some $a_i \in A_i$. Given the consumption schedule $\{c_i^*(\theta, s_0)\}$ for state s_0, I define

$$\tilde{W}_i(\Delta_i(s_0), a_i) \equiv \sum_\theta u_i \left(c_i^*(\theta, s_0) + \Delta_i(s_0) \right) p_i(\theta|a_i, s_0)\pi(s_0).$$

The function $\tilde{W}_i(\Delta_i(s_0), a_i)$ is unbounded, continuous and strictly increasing in $\Delta_i(s_0)$ by the assumed properties of $u_i(c)$. It is therefore invertible w.r.t. its first argument and the inverse function $\tilde{W}_i^{-1}(W_i, a_i)$ is continuous and strictly increasing in W_i on an unbounded domain. Using this to rewrite (43) yields the following explicit expression for the asset tax schedule:

$$\begin{aligned} & \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) \\ = \quad & \max_{a_i \in A_i} \left\{ -\tilde{W}_i^{-1} \left(W_i^* - \sum_{s \neq s_0} \sum_\theta u_i(c_i^*(\theta, s) + \Delta_i(s)) p_i(\theta|a_i, s)\pi(s) + v_i(a_i), a_i \right) \right. \\ & \left. - \sum_{s \neq s_0} q^*(s)\Delta_i(s) \right\}. \end{aligned} \tag{44}$$

This proves that there is a unique solution $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ to (39) for each trading strategy $\Delta_i(s)$. Moreover, $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ is defined as a maximization and the RHS of (44) is continuous in both $\Delta_i(s_1), ..., \Delta_i(s_H)$ (by continuity of $u_i(c)$ and $\tilde{W}_i^{-1}$ in its first argument) and in a_i (since $a_i \in A_i$ and A_i is a finite and thus discrete set).[13] Berge's Maximum Theorem therefore implies that $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ is a continuous function.

I next prove the second part of the theorem. The market clearing conditions (12) and (13) are satisfied by feasibility of $\{c_i^*(\theta, s), a_i^*\}$ and construction of $\{\Delta_i(s) = 0\}$. It thus remains to be shown that given the prices $\{q^*(s)\}$, the asset tax schedule $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ defined in (39) and the transfers $\{T_i(\theta, s)\}$ in (40), the solution to the agents' problem (35) to (37) is given by $\{c_i^*(\theta, s), a_i^*\}$ and $\Delta_i(s) = 0 \ \forall i \in I, s \in S$.

To see this, observe that, by construction of the transfers $\{T_i(\theta, s)\}$ in (40) and the fact that the budget constraint (36) is binding at the optimum, the agent's problem given prices $\{q^*(s)\}$ can be written as

$$\max_{\Delta_i(s), a_i} \sum_s \sum_\theta u_i(c_i^*(\theta, s) + \Delta_i(s)) p_i(\theta | a_i, s) \pi(s) - v_i(a_i) \tag{45}$$

subject to

$$\Delta_i(s_0) + \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) + \sum_{s \neq s_0} q^*(s) \Delta_i(s) = 0. \tag{46}$$

Substituting $\Delta_i(s_0)$ from (46) yields the following problem that agents solve:

$$\begin{aligned} \max_{\Delta_i(s), a_i} \Bigg\{ & \sum_\theta u_i \left(c_i^*(\theta, s_0) - \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) - \sum_{s \neq s_0} q^*(s) \Delta_i(s) \right) p_i(\theta | a_i, s_0) \pi(s_0) \\ & + \sum_{s \neq s_0} \sum_\theta u_i(c_i^*(\theta, s) + \Delta_i(s)) p_i(\theta | a_i, s) \pi(s) - v_i(a_i) \Bigg\}. \end{aligned} \tag{47}$$

The construction of the asset tax schedule $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ in (39) implies that all agents are indifferent between any trading strategy $\Delta_i(s)$ when they are able to choose their optimal action given $\Delta_i(s)$. By incentive compatibility of the allocation $\{c_i^*(\theta, s), a_i^*\}$, the maximum in (47) is therefore attained for all $i \in I$ by setting $\Delta_i(s) = 0 \ \forall s \in S$ and $a_i = a_i^*$, which produces expected utility W_i^* as defined in (38) and completes the proof.

A.2 Proof of Theorem 4

By the definition of optimal marginal taxes in (41), $\partial \kappa_i^* / \partial \Delta_i(s_h) \geq 0$ if

$$\frac{\mathbb{E}_i[u_i'(c_i^*(\theta, s_h)) | a_i^*, s_h]}{\mathbb{E}_i[u_i'(c_i^*(\theta, s_0)) | a_i^*, s_0]} \geq \frac{\xi(s_h)/\pi(s_h)}{\xi(s_0)/\pi(s_0)}. \tag{48}$$

Substituting from (18) yields

$$\frac{\mathbb{E}_i[u_i'(c_i^*(\theta, s_h)) | a_i^*, s_h]}{\mathbb{E}_i[u_i'(c_i^*(\theta, s_0)) | a_i^*, s_0]} = \frac{\dfrac{\xi(s_h)}{\pi(s_h)} \displaystyle\sum_{\theta \in \Theta} \frac{p_i(\theta | a_i^*, s_h)}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) \left(1 - p_i(\theta | \tilde{a}_i, s_h) / p_i(\theta | a_i^*, s_h)\right)}}{\dfrac{\xi(s_0)}{\pi(s_0)} \displaystyle\sum_{\theta \in \Theta} \frac{p_i(\theta | a_i^*, s_0)}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) \left(1 - p_i(\theta | \tilde{a}_i, s_0) / p_i(\theta | a_i^*, s_0)\right)}}. \tag{49}$$

[13] Otherwise, continuity in a_i could be guaranteed by imposing continuity of $p_i(\theta | a_i, s)$ and $v_i(a_i)$ in a_i $\forall \theta \in \Theta, s \in S$.

Hence, (48) is satisfied if

$$\sum_{\theta \in \Theta} \frac{p_i(\theta|a_i^*, s_h)}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) \left(1 - \frac{p_i(\theta|\tilde{a}_i, s_h)}{p_i(\theta|a_i^*, s_h)}\right)} \geq \sum_{\theta \in \Theta} \frac{p_i(\theta|a_i^*, s_0)}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) \left(1 - \frac{p_i(\theta|\tilde{a}_i, s_0)}{p_i(\theta|a_i^*, s_0)}\right)}. \tag{50}$$

By the definition of $l_i(\theta|\tilde{a}_i, s)$ in (42), I can write

$$\sum_{\theta \in \Theta} \frac{p_i(\theta|a_i^*, s)}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) \left(1 - \frac{p_i(\theta|\tilde{a}_i, s)}{p_i(\theta|a_i^*, s)}\right)} = \sum_{\theta \in \Theta} \frac{p_i(\theta|a_i^*, s)}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i)(1 - l_i(\theta|\tilde{a}_i, s))}. \tag{51}$$

Let me define the new random variable

$$\mathcal{L}_i(\theta|s) \equiv \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) l_i(\theta|\tilde{a}_i, s), \tag{52}$$

which, for each $\theta \in \Theta$, is a weighted sum of the likelihood ratios of the actions $\tilde{a}_i$ for which the incentive constraint of group i binds.[14] Its cumulative distribution function given the constrained-efficient action a_i^* is denoted by $\Gamma_i(\mathcal{L}|s) \equiv \Pr_i(\mathcal{L}_i(\theta|s) \leq \mathcal{L}|a_i^*, s)$ and the corresponding probability density function by $\gamma_i(\mathcal{L}|s)$. The following result will be useful.

Lemma 2. *Suppose $G_i(l|\tilde{a}_i, s_h)$ is riskier than $G_i(l|\tilde{a}_i, s_0)$ in terms of SOSD for all $\tilde{a}_i \in A_i$ for which (2) is binding. Then $\Gamma_i(\mathcal{L}|s_h)$ is riskier than $\Gamma_i(\mathcal{L}|s_0)$ in terms of SOSD.*

Proof. Since $G_i(l|\tilde{a}_i, s_h)$ is a mean-preserving spread of $G_i(l|\tilde{a}_i, s_0)$, it can be constructed as a compound lottery, where in a first stage, l is drawn from $G_i(l|\tilde{a}_i, s_0)$ and, subsequently, each possible outcome of l is further randomized so that the final likelihood ratio is $l + z_{\tilde{a}_i}$ where $z_{\tilde{a}_i}$ has a cumulative distribution function $H_i^l(z_{\tilde{a}_i}|\tilde{a}_i)$ and a corresponding probability density function $h_i^l(z_{\tilde{a}_i}|\tilde{a}_i)$ with mean zero for all l (i.e. $\sum_{z_{\tilde{a}_i}} z_{\tilde{a}_i} h_i^l(z_{\tilde{a}_i}|\tilde{a}_i) = 0 \, \forall l$). I want to show that $\Gamma_i(\mathcal{L}|s_h)$ is a mean-preserving spread of $\Gamma_i(\mathcal{L}|s_0)$, where $\Gamma_i(\mathcal{L}|s_0)$ is the cumulative distribution function of $\mathcal{L}_i(\theta|s_0) = \sum_{\tilde{a}_i} \mu_i(\tilde{a}_i) l_i(\theta|\tilde{a}_i, s_0)$ by (52). To see this, note that $\Gamma_i(\mathcal{L}|s_h)$ can be constructed as a compound lottery where, first, $\mathcal{L}$ is drawn from $\Gamma_i(\mathcal{L}|s_0)$, and, in a second stage, each possible realization of $\mathcal{L}$ is further randomized so that the final outcome is $\mathcal{L} + \mathcal{Z}$ with $\mathcal{Z} = \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) z_{\tilde{a}_i}$. The mean of $\mathcal{Z}$ is $\sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) \sum_{z_{\tilde{a}_i}} z_{\tilde{a}_i} h_i^l(z_{\tilde{a}_i}|\tilde{a}_i) = 0 \, \forall \mathcal{L}$, so that $\Gamma_i(\mathcal{L}|s_h)$ is a mean-preserving spread of $\Gamma_i(\mathcal{L}|s_0)$, as claimed in the lemma. □

Substituting the definitions of $\mathcal{L}_i(\theta|s)$ and $\gamma_i(\mathcal{L}|s)$ in (51), I can write

$$\sum_{\theta \in \Theta} \frac{p_i(\theta|a_i^*, s)}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) \left(1 - \frac{p_i(\theta|\tilde{a}_i, s)}{p_i(\theta|a_i^*, s)}\right)} = \sum_{\mathcal{L} \in L_i} \frac{\gamma_i(\mathcal{L}|s)}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) - \mathcal{L}} = \sum_{\mathcal{L} \in L_i} \Lambda_i(\mathcal{L}) \gamma_i(\mathcal{L}|s) \tag{53}$$

with

$$\Lambda_i(\mathcal{L}) \equiv \frac{1}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) - \mathcal{L}}.$$

Note first that $\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) - \mathcal{L} > 0$ because of (18) and $u_i'(c_i^*(\theta, s)) > 0$ for all $i \in I, \theta \in \Theta, s \in S$. Hence $\mathcal{L}$ must always lie in the interval $L_i \equiv [0, \psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i))$. It is then straightforward to verify that $\Lambda_i(\mathcal{L})$ is strictly convex in this domain.

[14] In the generic case where the incentive constraint (2) only binds for one action $\tilde{a}_i \in A_i$, $\mathcal{L}_i$ is just a rescaling of l_i with $\mathcal{L}_i(\theta|s) = \mu_i(\tilde{a}_i) l_i(\theta|\tilde{a}_i, s) \, \forall \theta \in \Theta$.

Since $\Gamma_i(\mathcal{L}|s_h)$ is a mean preserving spread of $\Gamma_i(\mathcal{L}|s_0)$ by Lemma 2, it can be constructed from a compound lottery where, in the first stage, $\mathcal{L}$ is drawn from $\Gamma_i(\mathcal{L}|s_0)$ and, in the second stage, each possible outcome of $\mathcal{L}$ is further randomized so that the final likelihood ratio is $\mathcal{L} + \mathcal{Z}$, where $\mathcal{Z}$ has a cumulative distribution function $H_i^{\mathcal{L}}(\mathcal{Z})$ and a corresponding probability density function $h_i^{\mathcal{L}}(\mathcal{Z})$ with mean zero for all $\mathcal{L}$ (i.e. $\sum_{\mathcal{Z}} \mathcal{Z} h_i^{\mathcal{L}}(\mathcal{Z}) = 0 \,\forall \mathcal{L}$). Then convexity of $\Lambda_i(\mathcal{L})$ and Jensen's inequality imply that

$$\begin{aligned}\sum_{\mathcal{L}\in L_i} \Lambda_i(\mathcal{L})\gamma_i(\mathcal{L}|s_h) &= \sum_{\mathcal{L}\in L_i} \Big(\sum_{\mathcal{Z}} \Lambda_i(\mathcal{L}+\mathcal{Z}) h_i^{\mathcal{L}}(\mathcal{Z})\Big)\gamma_i(\mathcal{L}|s_0) \\ &\geq \sum_{\mathcal{L}\in L_i} \Lambda_i\Big(\sum_{\mathcal{Z}} (\mathcal{L}+\mathcal{Z}) h_i^{\mathcal{L}}(\mathcal{Z})\Big)\gamma_i(\mathcal{L}|s_0) \\ &= \sum_{\mathcal{L}\in L_i} \Lambda_i(\mathcal{L})\gamma_i(\mathcal{L}|s_0)\end{aligned}$$

since $\sum_{\mathcal{Z}} \mathcal{Z} h_i^{\mathcal{L}}(\mathcal{Z}) = 0 \,\forall \mathcal{L} \in L_i$. Using this together with (53) yields the desired inequality (50) for group i and thus completes the proof.