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Investors' Direct Stock Holdings and Performance Evaluation for Mutual Funds

von Wolfgang Breuer* und Marc Gürtler**

Abstract. Investors need performance measures particularly as a means for funds selection in the process of ex-ante portfolio optimization. Unfortunately, there are various performance measures recommended for different decision situations. Since an investor may be uncertain which kind of decision problem is best apt to describe his personal situation the question arises up to which extent funds rankings react sensitive with respect to changes in performance measurement. To be more precise, an investor with mean-variance preferences is considered who is trying to identify the best fund f^* out of a set consisting of F funds and to combine this one optimally with the direct holding of a broadly diversified (reference) portfolio P of stocks as well as riskless lending or borrowing. For an investor just starting to acquire risky securities all three fractions of the various assets in question as part of his overall portfolio can be considered variable, while there also might be investors with already given direct holdings of stocks amounting to a certain fraction of their total wealth which cannot or shall not be altered. For both situations different adequate performance measures have been suggested by *Breuer/Gürtler* (1999, 2000) and *Scholz/Wilkens* (2003). We analyze theoretically as well as empirically possible deviations in resulting funds rankings for the two decision situations described previously. While there are indeed only loose theoretical relationships between the performance measures under consideration, empirical evidence suggests almost identical funds rankings. As a consequence, potential investors need not bother much about whether their situation is best described by an already fixed or a still variable amount of direct stock holdings. Moreover, traditional performance measures like the *Sharpe* ratio or the *Treynor* ratio will in general lead to reasonable funds selection in both situations.

Zusammenfassung. Performancemaße werden von Investoren insbesondere als Mittel zur Selektion von Investmentfonds im Rahmen von Ex-ante-Optimierungen verwandt. Unglücklicherweise existieren verschiedene Performancemaße für unterschiedliche Entscheidungsprobleme. Da ein Anleger im Unklaren darüber sein mag, welches Entscheidungsproblem am besten seine persönliche Situation beschreibt, drängt sich die Frage auf, in welchem Ausmaß Fondsrankings auf einen Wechsel des Performancemaßes reagieren. Präziser formuliert, wird ein Investor mit μ-σ-Präferenzen betrachtet, der versucht, den besten Fonds f^* aus einer Menge von F zur Auswahl stehenden zu bestimmen und diesen in optimaler Weise mit dem direkten Halten eines breit diversifizierten (Referenz-) Portfolios P aus Aktien sowie risikoloser Anlage bzw. Verschuldung zu kombinieren. Aus Sicht eines Investors, der gerade beginnt, seine Mittel in riskanten Aktiva anzulegen, können alle Anteile der verschiedenen Aktivaklassen an seinem Gesamtvermögen als variabel aufgefasst werden, während auch Anleger mit bereits vor Fondsselektion gegebenem positiven Aktienengagement existieren mögen, das nicht ohne weiteres geändert werden kann oder soll. Für beide Situationen wurden geeignete Performancemaße vorgeschlagen, und zwar von *Breuer/Gürtler* (1999, 2000) und *Scholz/Wilkens* (2003). Mögliche Unterschiede in den jeweiligen Fondsreihungen für die beiden genannten Entscheidungssituationen werden theoretisch wie empirisch untersucht. Während sich nur lockere theoretische Zusammenhänge belegen lassen, weist der empirische Befund auf tatsächlich fast identische Reihungen hin. Als Konsequenz hieraus müssen sich potentielle Anleger nicht allzu viele Gedanken darüber machen, ob ihre Situation besser durch ein fixes oder ein variables Aktienengagement beschrieben wird. Ferner führen traditionelle Performancemaße wie die *Sharpe* Ratio oder die *Treynor* Ratio in beiden Entscheidungssituationen zu akzeptablen Reihungen.

Stichworte: Performancemessung, Sharpe-Maß, Treynor-Maß, optimale Portfolioselektion

Keywords: Performance measurement, Sharpe ratio, Treynor ratio, optimal portaolio selection

JEL classification: G11

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I. Introduction

Issues of performance measurement for investment funds lie at the root of modern portfolio management research. Investors need performance measures as a means for the ex-post assessment of funds performance which in turn – under the assumption of stable return characteristics over time – can be utilized for adequate funds selection in the process of ex-ante portfolio optimization as well. Unfortunately, there are various performance measures recommended for different decision situations. Since an investor may be uncertain which kind of decision problem is best apt to describe his personal situation the question arises up to which extent funds rankings react sensitive with respect to changes in performance measurement. It is this issue to which we want to contribute.

To be more precise, an investor with mean-variance preferences is considered who is trying to identify the best fund f^* out of a set consisting of F funds f = 1, …, F and to combine this one optimally with the direct holding of a broadly diversified (reference) portfolio P of stocks as well as riskless lending or borrowing at a rate r_0.

For an investor just starting to acquire risky securities all three fractions of the various assets in question as part of his overall portfolio can be considered variable, while there also might be investors with already given direct holdings of stocks amounting to a fraction x_P^+ of their total wealth. If those investors are not able or not willing to change this part of their investment, we call such a situation the “exogenous case” since the allocation of a certain part of the investor’s initial wealth is fixed. Otherwise, we speak of the endogenous case because there are no restrictions (besides possible short-sales constraints) with respect to the allocation of the investor’s monetary wealth. Figure 1 visualizes the two different decision problems and introduces x_0, x_P, and x_f as symbols for fractions of initial wealth invested in riskless assets, equity portfolio P, and fund f eventually chosen by an individual.

>>> Insert Figure 1 about here <<<

In both the exogenous case and the endogenous one, we are able to identify central subportfolios of the investor’s respective overall portfolio which therefore should be explicitly characterized by adequate symbols. In the exogenous case, the investor only aims at optimizing his subportfolio of direct stock investments as represented by a combination of reference portfolio P and riskless lending or borrowing and thus we explicitly denote this subportfolio for chosen fund f as Q(f). In the endogenous case, as an application of the well-known two-funds separation theorem firstly identified by *Tobin* (1958) and later generalized by *Cass/Stiglitz* (1970), the structure of the risky subportfolio is independent of the investor's degree of risk aversion and thus this subportfolio for chosen fund f shall be characterized by R(f).

For the endogenous case, the application of so-called optimized *Sharpe* measures has been suggested by *Breuer/Gürtler* (1999, 2000), while for the exogenous case *Scholz/Wilkens* (2003) derived the investor specific performance measure as the adequate one. Both kinds of performance measures assure the selection of such a fund f and corresponding overall portfolio structure that an investor's mean-variance preference function is maximized. Thereby, in the endogenous case, the optimized unrestricted (restricted) *Sharpe* measure recommends that fund f^* that leads to the maximum *Sharpe* ratio of the corresponding best overall portfolio consisting of riskless assets, the reference portfolio P and just one fund f without (with) short-sales constraints. The investor specific performance measure fulfills the same task for the exogenous case. Contrary to the situation in the endogenous case, the two-funds separation theorem does not hold in the exogenous case so that the investor's degree of risk aversion has to be specified in order to identify the best fund f^* thus explaining the denomination of the performance measure as "investor specific". Funds rankings according to the optimized *Sharpe* measures are independent of an investor's degree of risk aversion.

Prima facie, we deem the endogenous case as well as the exogenous one as of equal practical importance. Hence, it seems to be interesting to analyze somewhat more in detail theoretical as well as empirical relationships between funds rankings in these two alternative settings in order to better assess the necessity of explicitly differentiating between these two decision problems. Thereby, the question should be answered whether the corresponding somewhat "new" performance measures could be replaced by a simple application of the well-established and widespread used "traditional" performance measures by *Treynor* (1965), *Sharpe* (1966), *Jensen* (1968), and *Treynor/Black* (1973).

Findings of this kind would be of immediate practical importance as they might support practitioners in the process of adequate funds selection for portfolio optimization by distinguishing more critical issues from less critical ones. Unfortunately, neither *Breuer/Gürtler* (1999,

2000) nor *Scholz/Wilkens* (2003) examine thoroughly theoretical and empirical connections between their proposed ways of performance measurement. In what follows we want to close this gap. To do so, in section II we briefly give an overview of the main findings in *Breuer/Gürtler* (1999, 2000) and in *Scholz/Wilkens* (2003). Section III is devoted to the theoretical analysis of the relationships among the optimized *Sharpe* measures of *Breuer/Gürtler* (1999, 2000), and the investor specific performance measure of *Scholz/Wilkens* (2003) as well as the "traditional" performance measures mentioned above. Section IV presents a possible empirical application of the findings of section III and thereby examines the practical relevance of the theoretically justified clear differentiation between the optimized *Sharpe* measures and the investor specific performance measure and thus the distinction between the endogenous case and the exogenous one. Section V concludes.

II. Performance Evaluation in the Endogenous Case and the Exogenous One Reconsidered

As a guidance, Table 1 gives an overview of all relevant mathematical symbols of the investor's portfolio selection problem. Moreover, while f stands for just one arbitrary fund out of all F accessible ones, we use the symbols g and h to distinguish between two different specific funds simultaneously considered when discussing evolving funds rankings as a consequence of the application of the various performance measures.

>>> Insert Table 1 about here <<<

In what follows, we only look at funds with expected excess return $\bar{u}_f > 0$, because otherwise even simple riskless lending will in general be preferred to an investment in fund f. With the background of Table 1, we are able to (re-) introduce formally the following "classical" or "traditional" measures for performance evaluation with respect to a fund f for reference portfolio P:

(1) $\varphi_f^{(S)} = \frac{\bar{u}_f}{\sigma_f}$, the *Sharpe* measure[1] of f,

(2) $\varphi_f^{(T)} = \frac{\bar{u}_f}{\beta_{fP}}$, the *Treynor* measure[2] of f,

(3) $\varphi_f^{(J)} = \bar{u}_f - \beta_{fP} \cdot \bar{u}_P$, the *Jensen* measure[3] of f,

(4) $\varphi_f^{(TB)} = \frac{\varphi_f^{(J)}}{\sigma_{\varepsilon_{fP}}}$, the *Treynor*/*Black* measure[4] of f.

Moreover, we need three additional performance measures, i.e.

(5) $\varphi_f^{(invJ)} = \bar{u}_P - \beta_{Pf} \cdot \bar{u}_f$, the "inverse" *Jensen* measure of f,

(6) $\varphi_f^{(S)*} = \frac{\bar{u}_{R^*(f)}}{\sigma_{R^*(f)}}$, the "optimized" *Sharpe* measure of f without short sales restrictions ("unrestricted optimized *Sharpe* measure"), and

(7) $\varphi_{f,restr.}^{(S)*} = \frac{\bar{u}_{R^*_{restr.}(f)}}{\sigma_{R^*_{restr.}(f)}}$, the "optimized" *Sharpe* measure of f with short sales restrictions ("restricted optimized *Sharpe* measure").[5]

The inverse *Jensen* measure of a fund f corresponds to the *Jensen* measure of reference portfolio P in the case of a linear regression of the excess return $\tilde{u}_P$ of portfolio P with respect to the excess return $\tilde{u}_f$ of fund f and thus reverses the "original" roles of fund f and reference portfolio P. The restricted optimized *Sharpe* measure of fund f refers to the *Sharpe* measure of the optimal risky subportfolio $R^*(f)$ of fund f and reference portfolio P when short sales restrictions regarding fund f and reference portfolio P are neglected.[6] Correspondingly, the restricted optimized *Sharpe* measure allows for the requirements $x_f \geq 0$ as well as $x_P \geq 0$ and thus is based on the optimal risky portfolio $R^*_{restr.}(f)$ in the case of short sales restrictions. In what follows, optimal solutions are generally characterized by an asterisk ("*").

With these definitions in mind, *Breuer*/*Gürtler* (1999, 2000) were able to derive the following results for the endogenous case:

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[1] As already mentioned in section I, also known as the *Sharpe* ratio. See *Sharpe* (1966).
[2] Also known as the *Treynor* ratio. See *Treynor* (1965).
[3] Also known as *Jensen*'s Alpha. See *Jensen* (1968).
[4] Also known as the *Treynor/Black* appraisal ratio. See *Treynor*/*Black* (1973).
[5] For the unrestricted as well as the restricted optimized *Sharpe* measure see in particular *Breuer*/*Gürtler* (2000).
[6] It should be mentioned that the maximization of the optimized unrestricted (restricted) *Sharpe* measure as defined in (6) (in (7)) is equivalent to the maximization of the *Sharpe* measure of an investor's overall portfolio in the case without (with) short sales constraints. This equivalence was used in section I for the informal description of the optimized *Sharpe* measure.

(BG1) Funds, which should best be sold short, i.e. $x_f^* < 0$, or lead to $x_f^* = 0$ when combined with reference portfolio P, are characterized by a negative *Jensen* measure or a *Jensen* measure of 0, respectively, and might be called "inferior" funds.

(BG2) A ranking by the *Jensen* measure or the (negative inverse of the) *Treynor* measure can be justified (only) to rank inferior funds with the latter measure – in contrast to the former one – not being prone to manipulation by the variation of the amount of riskless lending or borrowing of the manager of a fund f. In the case of all beta coefficients being positive, a ranking according to the negative inverse of the *Treynor* measure is equivalent to a ranking according to the *Treynor* measure itself.

(BG3) Among several funds, that one should be chosen which offers the highest optimized (restricted or unrestricted) *Sharpe* measure.

(BG4) Funds which should best be combined with short sales of equity portfolio P ($x_P^* < 0$) coincide with a negative inverse *Jensen* measure of f.[7] In such a case short sales restrictions imply an evaluation of the respective fund f by its simple *Sharpe* measure.

(BG5) The optimized unrestricted *Sharpe* measure implies the same ranking as the square of the *Treynor*/*Black* measure.[8]

(BG6) All funds for which $R_{\text{restr.}}^*(f) = R^*(f)$, i.e. which do not lead to violations of short sales restrictions, can be ranked among each other according to their *Treynor*/*Black* measure. Thus, in this case the latter ranking is equivalent to that based on the (restricted or unrestricted) optimized *Sharpe* measure.

For the exogenous case, *Scholz*/*Wilkens* (2003) accomplished to derive a performance measure which differs in some respect from the ones introduced above.[9] They call it the "investor specific performance measure" and define it originally as

$$(8)\ \ ISM_{f,\text{orig.}}(x_P) := \left[\left(\frac{\bar{u}_{Q(f)}^{+}(x_P)}{\sigma_P}\right)^2 \cdot \left(-\frac{1}{(\varphi_f^{(S)})^2}\right) + 2 \cdot x_P \cdot \bar{u}_{Q(f)}^{+}(x_P) \cdot \left(-\frac{1}{\varphi_f^{(T)}}\right)\right] \cdot \frac{1}{1-x_P}.$$

Thereby, $\bar{u}_{Q(f)}^{+}(x_P)$ stands for the contribution of subportfolio Q(f) to an investor's overall achievable expected excess return, i.e., $\bar{u}_{Q(f)}^{+}(x_P)$ is the product of fraction $1-x_P$ of subportfolio Q(f) and its corresponding expected excess return $\bar{u}_{Q(f)}$. As pointed out earlier, contrary to

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[7] To be precise, this result is not mentioned in *Breuer*/*Gürtler* (1999, 2000), but it immediately follows from (BG1), if one reverses the roles of fund f and reference portfolio P.

[8] A rigorous portfolio-theoretical foundation for the application of the square of the *Treynor/Black* measure was first derived by *Jobson*/*Korkie* (1984).

[9] See also *Breuer*/*Gürtler* (1998) for a similar, but earlier approach.

the endogenous case the two-funds separation theorem does not apply in the exogenous case. Therefore the investor's degree of risk aversion must be at least implicitly taken into account. This can be done by fixing the desired expected return of the investor's overall portfolio at a certain value $\bar{u}^+ > 0$. With given fraction x_P of reference portfolio P and given overall expected excess return $\bar{u}^+$, the contribution $\bar{u}^+_{Q(f)}(x_P)$ of the complementary subportfolio Q(f) is necessarily given, too. In fact, we have $\bar{u}^+ = x_P \cdot \bar{u}_P + (1 - x_P) \cdot \bar{u}_{Q(f)} = x_P \cdot \bar{u}_P + \bar{u}^+_{Q(f)}(x_P)$ and thus $\bar{u}^+_{Q(f)}(x_P) = \bar{u}^+ - x_P \cdot \bar{u}_P =: \bar{u}^+_Q(x_P)$ as a function of x_P, but being independent of fund f under consideration. With this formal background, and under the additional assumptions of all regression coefficients $\beta_{fP} := \sigma_{fP} / \sigma_P^2$ being positive and short sales of funds being impossible the following results have been stated by *Scholz/Wilkens* (2003) for the exogenous case:

(SW1) For given fraction x_P with $0 \le x_P < 1$ and desired overall expected return $\bar{u}^+$, funds f should be ranked according to (8).

(SW2) If the *Sharpe* measure as well as the *Treynor* measure of a fund g is not smaller than that of a fund h, we have $ISM_{g,orig.}(x_P) \ge ISM_{h,orig.}(x_P)$.

(SW3) For the case $x_P = 0$ funds rankings according to the investor specific performance measure (8) reduce to rankings on the basis of the conventional *Sharpe* measure.

(SW4) For desired overall expected return $\bar{u}^+ = \bar{u}_P + \delta$, with $\delta > 0$ and $\delta \to 0$, funds rankings according to the investor specific performance measure (8) reduce to rankings on the basis of the conventional *Treynor* measure.

Unfortunately, as shown in the Appendix, result (SW1) can only be generalized to the case $1-x_P > 0$. Otherwise, i.e. for $1-x_P < 0$, a funds ranking according to (8) will lead to the best score for that fund f which minimizes the investor's mean-variance preference function. Situations with $1-x_P < 0$ occur, for example, when direct equity holdings are financed by riskless borrowing and thus cannot a priori be excluded. For the general case, i.e. for arbitrary signs of $1-x_P$ the following modified investor specific performance measure $ISM_f(x_P)$ has to be applied:

$$(9)\quad ISM_f(x_P) := ISM_{f,orig.}(x_P) \cdot (1 - x_P) = \left(\frac{\bar{u}^+_Q(x_P)}{\sigma_P} \right)^2 \cdot \left(-\frac{1}{(\varphi_f^{(S)})^2} \right) + 2 \cdot x_P \cdot \bar{u}^+_Q(x_P) \cdot \left(-\frac{1}{\varphi_f^{(T)}} \right).$$

In what follows we simply speak of the investor specific performance measure, although we mean $ISM_f(x_P)$ instead of the original one (8) as introduced by *Scholz/Wilkens* (2003).

We consider both the endogenous as well as the exogenous case to be of practical importance. Apparently, there must be some theoretical relationships between funds rankings in these both cases since the investor specific performance measure of a fund f is determined by its *Sharpe* measure and its *Treynor* measure. These relationships are examined in the next section.

III. Theoretical Relationships among Traditional Measures, Optimized Ones, and the Investor Specific Performance Measure

We start with the consideration of possible connections between the investor specific performance measure and the traditional ones. Thereby, contrary to *Scholz/Wilkens* (2003) we allow for negative regressions coefficients β_{fP} and short sales possibilities in order to formulate our results as generally as possible.

Result 1:

(R1.1) $\varphi_g^{(S)} \geq \varphi_h^{(S)} \;\wedge\; -1/\varphi_g^{(T)} \geq -1/\varphi_h^{(T)}$ *implies* $ISM_g(x_P) \geq ISM_h(x_P)$ *for all* $0 \leq x_P \leq \bar{u}^+/\bar{u}_P$.

(R1.2) $\varphi_g^{(S)} \geq \varphi_h^{(S)} \;\wedge\; -1/\varphi_g^{(T)} \leq -1/\varphi_h^{(T)}$ *implies* $ISM_g(x_P) \geq ISM_h(x_P)$ *for all* $x_P < 0 \vee x_P > \bar{u}^+/\bar{u}_P$.

(R1.3) $\varphi_g^{(S)} \geq \varphi_h^{(S)} \;\wedge\; \varphi_g^{(T)} \geq \varphi_h^{(T)} > 0$ *implies* $ISM_g(x_P) \geq ISM_h(x_P)$ *in the absence of short sales possibilities.*

(R1.4) For $x_P = 0$ *we have* $\varphi_g^{(S)} \geq \varphi_h^{(S)} \Leftrightarrow ISM_g(x_P) \geq ISM_h(x_P)$.

(R1.5) For desired overall expected return $\bar{u}^+ = \bar{u}_P + \delta$, *with* $\delta > 0$ *and* $\delta \to 0$, *we have* $-1/\varphi_g^{(T)} \leq -1/\varphi_h^{(T)} \Leftrightarrow ISM_g(x_P) \geq ISM_h(x_P)$ *which, in the case of positive Treynor measures* $\varphi_g^{(T)}$ *and* $\varphi_h^{(T)}$, *can be simplified to* $\varphi_g^{(T)} \geq \varphi_h^{(T)} \Leftrightarrow ISM_g(x_P) \geq ISM_h(x_P)$.

(R1.6) $\varphi_g^{(J)} \geq 0 \;\wedge\; \varphi_h^{(J)} \leq 0$ *does not imply* $ISM_g(x_P) \geq ISM_h(x_P)$ *for any* $x_P \in \mathbb{R} \setminus \{\bar{u}/\bar{u}_P\}$.

(R1.7) $ISM_g(x_P) \geq ISM_h(x_P)$ *for all* $x_P \in X \subseteq \mathbb{R} \setminus \{\bar{u}/\bar{u}_P\}$ *(with X being an arbitrary subset of* $\mathbb{R}$ *) does not imply* $\varphi_h^{(J)} \leq 0$.

Proof. *See the Appendix.*

Statements (R1.1) to (R1.3) indicate that it is possible to draw some general conclusions with respect to the relationship between traditional performance measures and the investor specific one. To be precise, under certain conditions the knowledge of *Sharpe* and (the negative inverse of) *Treynor* measures may be sufficient to deduce the resulting ranking of two funds according to the investor specific performance measure. Nevertheless, the preferability of simultaneously higher values of both the *Sharpe* and the *Treynor* measure cannot generally be concluded, as is indicated by the general relevance of the negative inverse of the *Treynor* measure as well as by (R1.2). Fortunately, (R1.3) describes one important special case, when indeed simultaneously higher *Sharpe* and *Treynor* measure imply a better investor specific performance measure. Thereby, (R1.3) obviously is a direct extension of the result (SW2) by *Scholz/ Wilkens* (2003), as the former one is based on the performance measure $ISM_f(x_P)$ instead of the originally by *Scholz/Wilkens* (2003) suggested one $IFM_{f,orig.}(x_P)$ and thus holds true even for the case $1-x_P < 0$. In fact, results analogous to (SW3) as well as (SW4) for the original investor specific performance measure (8) can be derived on the basis of the more general investor specific performance (9) as well. This is stated by (R1.4) and (R1.5) which both are valid even if short sales of risky assets are not prohibited. Moreover, (R1.4) and the general formulation of (R1.5) hold true regardless of the signs of the *Treynor* measures of funds g and h under consideration.

As has already been sketched in section II we might define an inferior fund in the endogenous case as such a one which leads to $x_f^* \le 0.$ In the exogenous case we might call a fund h inferior compared to a fund g for given possible exogenous fractions $x_P \in X$, if we have $ISM_g(x_P) \ge ISM_h(x_P)$ for all $x_P \in X$. Obviously, according to (R1.1) to (R1.3) inferior funds in the exogenous case cannot as generally be determined as in the endogenous case. In particular, it generally is not possible to identify an inferior fund in the exogenous case by simply looking at its *Sharpe* and *Treynor* measure because the relevance of the *Treynor* measure is ambiguous. However, for most practical purposes we may expect the assumptions underlying (R1.3) to hold and *then* an inferior fund h is in fact (sufficiently) characterized by a simultaneously smaller *Sharpe* and *Treynor* measure than a "superior" fund g. Finally, as statements (R1.6) and (R1.7) reveal, inferior funds in the endogenous case need not be inferior in the exogenous one et vice versa. Both cases therefore must be considered separately.

With Result 1 in mind, we are now able to take a closer look at the possible connections between the investor specific performance measure and the (restricted or unrestricted) optimized *Sharpe* measure.

Result 2:

(R2.1) Let $x^{*}_{P,f}$ describe the optimal investment in reference portfolio P if we combine this portfolio with portfolio Q(f) (consisting of a fund f and riskless lending or borrowing). Then $ISM_g(x^{*}_{P,h}) \geq ISM_h(x^{*}_{P,h})$ implies $\varphi^{(S)*}_g \geq \varphi^{(S)*}_h$.

(R2.2) Let $x^{*}_{P,f,restr.}$ describe the optimal investment in reference portfolio P if we combine this portfolio with Q(f) and have to consider short sales restrictions. Then $ISM_g(x^{*}_{P,h,restr.}) \geq ISM_h(x^{*}_{P,h,restr.})$ implies $\varphi^{(S)*}_{g,restr.} \geq \varphi^{(S)*}_{h,restr.}$.

(R2.3) $ISM_g(x_P) \geq ISM_h(x_P)$ either for all $0 \leq x_P \leq \overline{u}^{+}/\overline{u}_P$ or for all $x_P < 0 \vee x_P > \overline{u}^{+}/\overline{u}_P$ does not imply $(\varphi^{(TB)}_g)^2 \geq (\varphi^{(TB)}_h)^2$ nor $\varphi^{(S)*}_g \geq \varphi^{(S)*}_h$.

(R2.4) $\varphi^{(S)*}_g \geq \varphi^{(S)*}_h$ does not imply $ISM_g(x_P) \geq ISM_h(x_P)$ for any $x_P \in \mathbb{R} \setminus \{\overline{u}/\overline{u}_P\}$.

(R2.5) $\varphi^{(S)*}_{g,restr.} \geq \varphi^{(S)*}_{h,restr.}$ does not imply $ISM_g(x_P) \geq ISM_h(x_P)$ for any $x_P \in \mathbb{R} \setminus \{\overline{u}/\overline{u}_P\}$.

Proof. *See the Appendix.*

Summarizing, there are only rather loose general connections between the investor specific performance measure for the exogenous case and its counterparts for the endogenous one. In particular, it may be possible to draw some conclusions from the ranking according to the investor specific performance measure to the ranking according to the restricted or unrestricted optimized *Sharpe* measure if one knows optimal restricted or unrestricted fund investments. However, in such a situation optimized *Sharpe* measures can directly be calculated.

In *Breuer*/*Gürtler* (1999, 2000) several relations between the classical performance measures and the optimized ones have been derived as has already been described in section I. Rather interestingly, the introduction of the investor specific performance measure by *Scholz*/*Wilkens* (2003) enables us to add some more findings to the ones stated above.

Result 3:

(R3.1) $\varphi_g^{(S)} \geq \varphi_h^{(S)} \wedge sgn(\varphi_g^{(J)}) \cdot sgn(\varphi_g^{(invJ)}) \cdot (-1/\varphi_g^{(T)}) \geq sgn(\varphi_h^{(J)}) \cdot sgn(\varphi_h^{(invJ)}) \cdot (-1/\varphi_h^{(T)})$

implies $\varphi_g^{(S)*} \geq \varphi_h^{(S)*}$.

(R3.2) $\varphi_g^{(S)} \geq \varphi_h^{(S)} \wedge sgn(\varphi_g^{(J)}) \cdot sgn(\varphi_g^{(invJ)}) \cdot (-1/\varphi_g^{(T)}) \geq sgn(\varphi_h^{(J)}) \cdot sgn(\varphi_h^{(invJ)}) \cdot (-1/\varphi_h^{(T)})$

implies $\varphi_g^{(TB)2} \geq \varphi_h^{(TB)2}$.

(R3.3) $\varphi_g^{(S)} \geq \varphi_h^{(S)} \wedge -1/\varphi_g^{(T)} \geq -1/\varphi_h^{(T)}$ *implies* $\varphi_{g,restr.}^{(S)*} \geq \varphi_{h,restr.}^{(S)*}$

Proof. *See the Appendix.*

In particular, under certain conditions it now becomes possible to recognize the superiority of a fund g in relation to a fund h according to the optimized *Sharpe* measure by simply looking at its original *Sharpe*, *Treynor*, and *Jensen* measure. Thereby, from *Breuer/Gürtler* (2000) it is already known that conditions $\varphi_g^{(J)} \geq 0 \wedge \varphi_h^{(J)} \leq 0$ imply $\varphi_{g,restr.}^{(S)*} \geq \varphi_{h,restr.}^{(S)*}$, since (only) fund h is "inferior" in the endogenous case.[10] From (R3.3) we learn that a fund g with a better *Sharpe* measure and a better (negative inverse of the) *Treynor* measure is once again also characterized by a higher restricted optimized *Sharpe* measure.

Things become more complicated if we allow for two funds g and h with g exhibiting a higher *Sharpe* measure but h being characterized by a higher *Treynor* measure. In such a situation conclusions are only possible regarding the unrestricted optimized *Sharpe* measure and in addition we need some more information with respect to the optimality of short sales of funds or the reference portfolio P. As this information is given by the signs of the *Jensen* measure and the inverse *Jensen* measure they both are necessary in order to derive results with respect to the relation between the unrestricted optimized *Sharpe* measures of funds g and h. In fact, the "dense" formulations according to (R3.1) and (R3.2) are valid for $-1/\varphi_g^{(T)} \geq -1/\varphi_h^{(T)}$ as well as $-1/\varphi_g^{(T)} < -1/\varphi_h^{(T)}$. Once again, it is not possible to reverse the conclusions (R3.1) to (R3.3), i.e. for example $\varphi_{g,restr.}^{(S)*} > \varphi_{h,restr.}^{(S)*}$ does not imply $\varphi_g^{(S)} > \varphi_h^{(S)} \wedge -1/\varphi_g^{(T)} \geq -1/\varphi_h^{(T)}$ though this seems to be of only minor importance.

[10] See also (BG1) of section II.

After all, from a theoretical point of view the introduction of the exogenous case by *Scholz/Wilkens* (2003) seems to be an interesting extension of the endogenous one analyzed by *Breuer/Gürtler* (1999, 2000). Quite remarkably, by deriving the "investor specific performance measure" it also becomes possible to clarify somewhat more in depth relationships between classical performance measures and the optimized ones of the endogenous case.

The empirical analysis of the following section aims at identifying the usefulness of the relationships theoretically found in Results 1 to 3. Moreover, the practical relevance of the caveats of Results 1 to 3 with respect to implications which are not generally valid are examined.

IV. Empirical Analysis

Similarly to *Breuer/Gürtler* (2003) we consider (post tax) return data for 45 mutual funds investing in German equity shares[11] over a period from July 1996 to August 1999 which are calculated on the basis of the development of the respective monthly repurchase prices per share.[12] We assume that all earnings paid out to the investors by a fund f are reinvested in this fund. The riskless interest rate r_0 can be approximated by the expected return of German time deposit running for one month and covering the respective period of time to be observed. We use the DAX 100 as a broadly diversified reference portfolio P.[13] For all 45 funds f and the DAX 100 unbiased estimators for the relevant moments of one-monthly returns are calculated and listed in Table 2.[14] The expected excess returns as well as the beta coefficients of all funds under consideration are positive so that the negative inverse of a *Treynor* measure can be generally replaced by the (positive) *Treynor* measure itself.

>>> Insert Table 2 about here <<<

[11] In what follows we briefly speak of German funds, though we do not mean their country of origin but the geographical focus of their investments.
[12] This means that possible selling markups are not taken into account. In this respect, the performance of funds generally tends to be overestimated when compared to the performance of any reference index. However, the determination here (in accordance with many other approaches) of "gross" performance measures allows at least some conclusions to be made with regard to the sensitivity of rankings when different types of performance measures are observed. Exactly this aspect forms the central issue of this paper as pointed out in section I.
[13] The DAX 100 was an index (listed until 03/21/2003) that consisted of 100 continuously traded shares of German companies including the 30 "blue chips" of the DAX 30 and the (former) 70 midcap-stocks of the MDAX. For further information see e.g. *Deutsche Boerse Group* (2003), p. 6.
[14] See *Rohatgi* (1976) for the unbiased estimators of the expectation value and the second central moment.

On this basis, we are mainly interested in the question whether the theoretical distinction between the endogenous and the exogenous case carries over to significantly different funds rankings in both cases in practical applications. Thereby, we focus on a situation with short sales restrictions because at least short sales of mutual funds are not realizable by private investors.

Most importantly, we know from (R3.3) and (R1.3), respectively, that a fund g with a higher *Sharpe* measure and a higher *Treynor* measure than a fund h simultaneously exhibits a greater optimized restricted *Sharpe* measure and a greater investor specific performance measure ISM in the case of short sales restrictions. With this result, it is possible to identify 28 of our 45 funds for which the ranking according to their *Sharpe* measure and their *Treynor* measure, respectively, is identical so that for them rankings in the exogenous case (with short sales restrictions) will always coincide with the corresponding ranking for the endogenous case. For these funds numbered from # 1 to # 28 in Table 2 and separately listed in Table 3, investors may not bother whether the endogenous or the exogenous case is of more practical importance. We therefore restrict our remaining analyses to the 17 funds for which rankings according to their *Sharpe* and their *Treynor* measure differ. For such funds the application of the investor specific performance measure will lead to rankings which are not necessarily identical to that of the endogenous one and even may vary for different exogenous fractions x_P of the reference portfolio P and desired overall expected excess returns $\overline{u}^+$.

>>> Insert Table 3 about here <<<

In order to better assess resulting differences in rankings we calculate *Spearman* ranking correlation coefficients between rankings according to ISM (in what follows: "ISM-rankings") for given identical desired overall expected excess returns $\overline{u}^{+(1)} = \overline{u}^{+(2)} = \overline{u}^+$ with $\overline{u}^+ \in$ {1.7719 %, 1.9 %, 2.0 %, …, 2.7 %, 10 %[15]} and different values $x_P^{(1)}$ and $x_P^{(2)}$ with $x_P^{(1)}$, $x_P^{(2)}$ $\in$ {0, 5 %, …., 100 %}. We find that correlation coefficients between two ISM-rankings are very similar for given difference $\Delta x_P := \left| x_P^{(1)} - x_P^{(2)} \right|$. For illustrative purposes, Table 4 presents all resulting different correlation coefficients for the special case of a desired expected return $\overline{u}^+$ = 2.3 % and varying $x_P^{(1)}$ and $x_P^{(2)}$. For example, with $\overline{u}^+$ = 2.3 % rankings for $x_P^{(1)}$ = 10

[15] We add $\overline{u}^+ = 10$ % as an extreme value in order to better assess the stability of our results.

% and $x_P^{(2)}$ = 30 % exhibit a correlation coefficient of 99.0196 % while ISM-rankings for $\overline{u}^+$ = 2.3 %, $x_P^{(1)}$ = 30 % and $x_P^{(2)}$ = 50 % lead to a correlation coefficient of 99.2647 %.

>>> Insert Table 4 about here <<<

Table 4 is based on 21 different funds rankings as this is the number of exogenous values $x_P^{(1)}$ and $x_P^{(2)}$ taken into account. 10 more tables of this kind based on 210 additional funds rankings could be presented for all other overall desired expected excess returns $\overline{u}^+$ under consideration. Certainly, because of space constraints all these data should be presented in a somewhat more condensed way. In order to so, we summarize our findings in Table 5 by presenting average correlation coefficients between ISM-rankings for different identical values of desired expected returns $\overline{u}^+$ and varying differences Δx_P between exogenously given holdings of the reference portfolio P. For example, according to the shaded "cell" in Table 5 the average ranking correlation coefficient for the pair $(\overline{u}^+, \Delta x_P) = (2.3\ \%, 20\ \%)$ amounts to about 99.36563 % and is computed as the average value of all ranking correlation coefficients in Table 4 which are shaded as well.

As can easily be learnt from Table 5, correlations are rather high even if we restrict ourselves to funds which cannot be unambiguously ranked according to the *Sharpe* and the *Treynor* measure. Moreover, average ranking correlation coefficients are decreasing with falling value for $\overline{u}^+$. Nevertheless, since we refrain from considering situations with short sales of stocks or funds, the minimum accessible value for $\overline{u}^+$ amounts to 1.7719 % because $\overline{u}_Q^+(x_P) = \overline{u}^+ - x_P \cdot \overline{u}_P > 0$ (and thus $x_f > 0$) is only fulfilled for all $0 \le x_P \le 1$ if $\overline{u}^+ > \overline{u}_P$ ≈ 1.77189 %.

>>> Insert Table 5 about here <<<

Finally, ranking correlation coefficients in Table 5 are smallest for high differences Δx_P which is intuitively appealing. Since $\Delta x_P = 1$ implies either $x_P^{(1)} = 1$ or $x_P^{(2)} = 1$, according to (R1.4) and (R1.5), for $\Delta x_P = 1$ and $\overline{u}^+ = \overline{u}_P + \delta$ (δ positive and small) the corresponding (average) ranking correlation coefficient is identical to the correlation coefficient between the rankings according to the *Sharpe* and the (positive) *Treynor* measure. For correlations being increasing in $\overline{u}^{+}$ and decreasing in Δx_P, a high correlation between rankings according to *Sharpe* measure and *Treynor* measure thus implies only minor importance of ISM.[16] The limited independent relevance of the exogenous case is also underpinned by ranking correlation coefficients between funds rankings according to the optimized restricted *Sharpe* measure and ISM for different values $\overline{u}^{+}$ and x_P as Table 6 points out. Moreover, Table 6 indicates that two ISM-rankings with identical equity holdings as described by x_P, but different values $\overline{u}^{+(1)}$ and $\overline{u}^{+(2)}$ for desired overall expected excess return will generally be very similar since ranking correlation coefficients between ISM-rankings and the (given) restricted optimized *Sharpe* measure do not change much for varying expected excess returns $\overline{u}^{+}$. In fact, Table 6 suggests that variations of $\overline{u}^{+}$ affect the ISM-ranking even less than changes in x_P.

>>> Insert Table 6 about here <<<

Table 7 explicitly presents the ranking of the 17 funds which underlie Tables 5 and 6 for x_P = 0 in the exogenous case (just leading to a funds ranking according to the *Sharpe* measure and thus being independent of $\overline{u}^{+}$[17]) and for the endogenous case. As expected, differences in rankings seem to be almost negligible which is underlined by a high ranking correlation coefficient of approximately 93.14 %.

>>> Insert Table 7 about here <<<

Summarizing, at least for our empirical example there seems to be no need to explicitly distinguish between the exogenous case and the endogenous one.[18] In fact, we repeated all calculations underlying Tables 2 to 7 for the period from June 1993 to July 1996 for all but four[19]

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[16] In fact, high correlations between *Sharpe* and *Treynor* measure seem to be typical for practical decision problems as *Scholz/Wilkens* (2003), p. 4, point out. See also, for example *Möhlmann* (1993), pp. 178-179, or *Reilly/Brown* (1997), p. 1010.

[17] For this last result see also *Breuer/Gürtler* (1999), pp. 275-276.

[18] It should be mentioned that results would be quite different if one allows for short sales of risky assets as even inferior funds may become very attractive when sold short thus possibly leading to an almost perfectly negative correlation between the funds ranking based on the unrestricted optimized *Sharpe* measure and the restricted optimized one. Nevertheless, as mentioned earlier, we do not deem such short sales possibilities to be of practical importance.

[19] Three funds (# 30, # 32, # 34) were opened at a later date and one fund (# 45) realized a negative average excess return thus violating our basic assumptions. For the latter reason it was not possible to analyze as a third subperiod during the nineties the time interval from May 1991 to June 1993 as for these years actually none of the funds under consideration realized a positive average excess return. Since this paper is not primarily empirically oriented we refrain from discussing issues regarding the adequate estimation of a priori unknown return moments. See e.g. *Breuer/Gürtler/Schuhmacher* (2004), pp. 240-293.

funds under consideration. Essentially, our empirical findings are the same as for the period from July 1996 to August 1999.[20]

Both the endogenous case and the exogenous one seem to be of practical importance and from a theoretical point of view there might be significantly varying funds rankings depending on the situation under consideration. Yet, there is empirical evidence that resulting overall funds rankings are *in general* almost identical. In fact, investors may restrict themselves to funds rankings according to the *Sharpe* measure (or even the *Treynor* measure) and will probably arrive at outcomes very similar to those by application of the optimized (restricted) *Sharpe* measure or ISM. Thus, from an empirical point of view, the use of the optimized (restricted) *Sharpe* measure or the ISM seems to make no difference.

Nevertheless, we recommend the use of the optimized *Sharpe* measure because it represents the correct solution for the endogenous case and there is no more information required than for the calculation of the traditional performance measures so that there is no need to apply just an "approximation" of the correct funds ranking in the endogenous case. On the contrary, ISM can only be computed for given desired expected overall excess return $\overline{u}^{+}$ though the influence of this variable on funds rankings seems to be only limited. Moreover, as a by-product of considering the endogenous case one can determine optimal investments in fund f and reference portfolio P, i.e the (preference-independent) optimal structure of risky subportfolio R(f).

V. Conclusion

This paper examined theoretically relationships between funds rankings for given exogenous investor's holdings of a certain reference portfolio P of stocks ("exogenous case") and for decision situations where purchases of funds, stocks and riskless assets can simultaneously be optimized by investors ("endogenous case"). For the exogenous case *Scholz/Wilkens* (2003) recommend a so-called investor specific performance measure while for the endogenous case *Breuer/Gürtler* (1999, 2000) derived the adequacy of the application of an "optimized *Sharpe* measure". From a theoretical point of view the concept of the investor specific performance measure in particular enabled us to draw new conclusions regarding the relationship between

[20] Details are available from the authors upon request.

classical performance measures and the optimized *Sharpe* measure. Most importantly, in a situation with short sales restrictions, a fund g with both a higher *Sharpe* and a higher (positive) *Treynor* measure than a fund h will be better than fund h in the endogenous as well as the exogenous case regardless of the specific parameters of the investor's portfolio selection problem (i.e. for any desired overall expected excess return and any exogenously given holding of reference portfolio P of direct equity holding). This theoretical finding contributes particularly to our understanding of the relevance of the traditional performance measures by *Sharpe* (1966) and *Treynor* (1965).

Nevertheless, theoretically, rankings in the exogenous case and in the endogenous one may differ considerably since there are only a few (loose) connections between them. For this reason, we analyzed empirically differences in rankings for both cases. After all, we did not find sufficient evidence that a distinction between the endogenous and the exogenous case is of real practical importance. Certainly, *this* result is practically important, since it indicates possibilities for the simplification of funds selection problems. In particular, investors need not care much about the question whether their given holding of a portfolio P of stocks can be altered or not when searching for a good fund investment. Optimal fund selections thus seem to be quite robust for mean-variance preferences and in general may be approximated rather well by a simple application of the traditional *Sharpe* or *Treynor* measure. This result is another indicator for the usefulness of these classical performance measures even in complicated settings. Despite this we recommend the application of the restricted optimized *Sharpe* measure as developed in *Breuer/Gürtler* (1999, 2000) since it is based on the same information as the traditional performance measures and gives the correct solution for the "endogenous" case so that there is no need to use an "approximation" of that funds ranking as supplied by the simple *Sharpe* or *Treynor* measure.

Appendix

Proof of the statement "result (SW1) only holds true in the case $1-x_P > 0$":

Denote with $\sigma_{Q(f)P}$ the covariance between the excess returns of subportfolio Q(f) and of reference portfolio P. In the case of a risk-averse investor with mean-variance preferences, for given overall expected return $\bar{u}^+$, fund g is better than fund h if the overall variance resulting from the choice of fund g is lower than the overall variance when selecting fund h, i.e.,

$$\mathrm{Var}(x_P \cdot \tilde{u}_P + (1-x_P)\cdot \tilde{u}_{Q(g)}) < \mathrm{Var}(x_P \cdot \tilde{u}_P + (1-x_P)\cdot \tilde{u}_{Q(h)})$$

$$\Leftrightarrow \; (1-x_P)^2 \cdot \sigma^2_{Q(g)} + 2\cdot x_P \cdot (1-x_P)\cdot \sigma_{Q(g)P} + x_P^2 \cdot \sigma_P^2$$
$$< \; (1-x_P)^2 \cdot \sigma^2_{Q(h)} + 2\cdot x_P \cdot (1-x_P)\cdot \sigma_{Q(h)P} + x_P^2 \cdot \sigma_P^2$$

$$\Leftrightarrow \; \left(\frac{\bar{u}^+_Q(x_P)}{\bar{u}_{Q(g)}}\right)^2 \cdot \sigma^2_{Q(g)} + 2\cdot x_P \cdot \left(\frac{\bar{u}^+_Q(x_P)}{\bar{u}_{Q(g)}}\right)\cdot \sigma_{Q(g)P}$$

$$\text{(A1)} \qquad < \left(\frac{\bar{u}^+_Q(x_P)}{\bar{u}_{Q(h)}}\right)^2 \cdot \sigma^2_{Q(h)} + 2\cdot x_P \cdot \left(\frac{\bar{u}^+_Q(x_P)}{\bar{u}_{Q(h)}}\right)\cdot \sigma_{Q(h)P}$$

$$\Leftrightarrow \; \left(\frac{\bar{u}^+_Q(x_P)}{\sigma_P}\right)^2 \cdot \frac{-1}{\frac{\bar{u}^2_{Q(g)}}{\sigma^2_{Q(g)}}} + 2\cdot x_P \cdot \bar{u}^+_Q(x_P)\cdot \frac{-1}{\frac{\bar{u}_{Q(g)}}{\beta_{Q(g)P}}} > \left(\frac{\bar{u}^+_Q(x_P)}{\sigma_P}\right)^2 \cdot \frac{-1}{\frac{\bar{u}^2_{Q(h)}}{\sigma^2_{Q(h)}}} + 2\cdot x_P \cdot \bar{u}^+_Q(x_P)\cdot \frac{-1}{\frac{\bar{u}_{Q(h)}}{\beta_{Q(h)P}}}$$

$$\underset{(8)}{\Leftrightarrow} \; \mathrm{ISM}_{g,\mathrm{orig.}}(x_P)\cdot(1-x_P) \; > \; \mathrm{ISM}_{h,\mathrm{orig.}}(x_P)\cdot(1-x_P).$$

For the last equivalence we use the fact that the *Sharpe* measure and the *Treynor* measure cannot be influenced by riskless lending or borrowing and thus $\varphi_f^{(S)} = \varphi_{Q(f)}^{(S)}$ and $\varphi_f^{(T)} = \varphi_{Q(f)}^{(T)}$. (A1) immediately implies the postulated statement.

Proof of Result 1:

(R1.1) and (R1.2). Results (R1.1) and (R1.2) are obvious since the product $x_P \cdot \bar{u}^+_Q(x_P) = x_P \cdot (\bar{u}^+ - x_P \cdot \bar{u}_P)$ is positive for all $0 < x_P < \bar{u}^+ / \bar{u}_P$ and negative for all $x_P \in \mathbb{R} \setminus [0, \bar{u}^+ / \bar{u}_P]$, respectively.

(R1.3). Since $\varphi_g^{(T)}$ and $\varphi_h^{(T)}$ are both positive we have $-1/\varphi_g^{(T)} \geq -1/\varphi_h^{(T)} \Leftrightarrow \varphi_g^{(T)} \geq \varphi_h^{(T)}$. Moreover, $\bar{u}^+ / \bar{u}_P = (x_P \cdot \bar{u}_P + \bar{u}^+_Q(x_P))/\bar{u}_P = x_P + (\bar{u}^+_Q(x_P)/\bar{u}_P) > x_P$ if we refrain from short sales possibilities. This directly yields $0 \leq x_P \leq \bar{u}^+ / \bar{u}_P$ and (R1.3) then immediately follows from part (R1.1).

(R1.4). The facts $\bar{u}^+_Q(x_P = 0) = \bar{u}^+ > 0$ and $x_P = 0$ directly imply the postulated statement under consideration of the definition of $\mathrm{ISM}_f(x_P)$.

(R1.5). An overall expected return $\bar{u}^+ = \bar{u}_P + \delta = x_P \cdot \bar{u}_P + (1-x_P)\cdot \bar{u}_{Q(f)}$ with $\delta > 0$ is only achievable in the case of $\bar{u}_{Q(f)} \neq \bar{u}_P$. Thus, $\bar{u}^+ = \bar{u}_P \Leftrightarrow x_P = 1$. This in turn implies $\mathrm{ISM}_g(x_P = 1) = 0 = \mathrm{ISM}_h(x_P = 1)$ for arbitrary funds g and h. Because of

$$\left.\frac{\partial \mathrm{ISM}_g(x_P)}{\partial \bar{u}^+}\right|_{\substack{x_P=1,\\ \bar{u}^+=\bar{u}_P}} \geq \left.\frac{\partial \mathrm{ISM}_h(x_P)}{\partial \bar{u}^+}\right|_{\substack{x_P=1,\\ \bar{u}^+=\bar{u}_P}}$$

(A2) $$\Leftrightarrow \; 2\cdot\underbrace{\bar{u}_Q^+(1)}_{=0}\cdot\left(-\frac{1}{(\varphi_S^{(g)})^2}\right)+2\cdot\sigma_P^2\cdot 1\cdot\left(-\frac{1}{\varphi_T^{(g)}}\right)\geq 2\cdot\bar{u}_Q^+(1)\cdot\left(-\frac{1}{(\varphi_S^{(h)})^2}\right)+2\cdot\sigma_P^2\cdot 1\cdot\left(-\frac{1}{\varphi_T^{(h)}}\right)$$

$$\Leftrightarrow \; -\frac{1}{\varphi_T^{(g)}}\geq -\frac{1}{\varphi_T^{(h)}}$$

result (R1.5) is obvious.

(R1.6). First of all we show that a situation with $\varphi_g^{(J)} \geq 0$ and $\varphi_h^{(J)} \leq 0$ may coincide simultaneously with $\varphi_g^{(S)} \to 0$ as well as $\varphi_h^{(S)} > 0$. To this end, for each fund f we use a linear regression according to the ordinary-least-squares method to determine parameters α_{fP} and β_{fP} with $\tilde{u}_f = \alpha_{fP} + \beta_{fP}\cdot\tilde{u}_P + \tilde{\varepsilon}_{fP}$, $E(\tilde{\varepsilon}_{fP}) = 0$, and $\mathrm{Cov}(\tilde{u}_P, \tilde{\varepsilon}_{fP}) = 0$. In particular, the parameters for funds g and h may exhibit the properties $0 < \beta_{gP} < \frac{\bar{u}_g}{\bar{u}_P}$ and $\beta_{hP} \geq \frac{\bar{u}_h}{\bar{u}_P}$ as well as $\mathrm{Var}(\tilde{\varepsilon}_{gP}) \to \infty$ and $\mathrm{Var}(\tilde{\varepsilon}_{hP}) \ll \infty$ and consequentially $\varphi_g^{(J)} = \bar{u}_g - \beta_{gP}\cdot\bar{u}_P > 0$ and $\varphi_h^{(J)} = \bar{u}_h - \beta_{hP}\cdot\bar{u}_P \leq 0$. In addition, we have

(A3) $\mathrm{Var}(\tilde{u}_g) = \beta_{gp}^2\cdot\mathrm{Var}(\tilde{u}_P) + \mathrm{Var}(\tilde{\varepsilon}_{gP}) \to \infty$ and $\mathrm{Var}(\tilde{u}_h) = \beta_{hp}^2\cdot\mathrm{Var}(\tilde{u}_P) + \mathrm{Var}(\tilde{\varepsilon}_{hP}) \ll \infty.$

Thus, the corresponding *Sharpe* measures are

(A4) $$\varphi_g^{(S)} = \frac{\bar{u}_g}{\sigma_g} = \frac{\bar{u}_g}{\sqrt{\beta_{gp}^2\cdot\mathrm{Var}(\tilde{u}_P) + \mathrm{Var}(\tilde{\varepsilon}_{gP})}} \to 0,\; \varphi_h^{(S)} = \frac{\bar{u}_h}{\sigma_h} = \frac{\bar{u}_h}{\sqrt{\beta_{hp}^2\cdot\mathrm{Var}(\tilde{u}_P) + \mathrm{Var}(\tilde{\varepsilon}_{hP})}} > 0.$$

Since the *Treynor* measures of both funds are finite, we get $\mathrm{ISM}_g(x_P) \to -\infty$ and $\mathrm{ISM}_h(x_P) \gg -\infty$ for all $x_P \neq \bar{u}^+/\bar{u}_P$ which immediately implies (R1.6).

(R1.7). Result (R1.7) follows directly from the situation presented in the proof of (R1.6). If we reverse the roles of funds g and h, it results $\varphi_h^{(J)} > 0$ and $\mathrm{ISM}_h(x_P) \ll \mathrm{ISM}_g(x_P)$ for all $x_P \neq \bar{u}^+/\bar{u}_P$.

Proof of Result 2:

(R2.1). From the derivation of the "investor specific performance measure" ISM at the beginning of the Appendix and the assumption $\mathrm{ISM}_g(x_{P,h}^*) \geq \mathrm{ISM}_h(x_{P,h}^*)$ underlying statement

(R2.1), we know that an investment in fund g with $x_P = x^*_{P,h}$ implies lower overall portfolio risk than an investment in fund h (with $x_P = x^*_{P,h}$) since we have a given overall expected excess return $\bar{u}^+$ regardless of the fund actually chosen by the investor. Therefore, better funds are characterized by a lower variance of overall portfolio return. As a consequence, for $x_P = x^*_{P,h}$ the *Sharpe* measure of the overall portfolio including fund g is higher than the *Sharpe* measure of the overall portfolio including fund h, i.e.

(A5) $\varphi_g^{(S)}(x^*_{P,h}) \geq \varphi_h^{(S)}(x^*_{P,h})$.

Since an optimal investment $x^*_{P,g}$ in fund g does not lead to a lower *Sharpe* measure in comparison to the *Sharpe* measure of an investment $x^*_{P,h}$ in fund g, (R2.1) is obvious:

(A6) $\varphi_g^{(S)*} = \varphi_g^{(S)}(x^*_{P,g}) \geq \varphi_g^{(S)}(x^*_{P,h}) \underset{(A5)}{\geq} \varphi_h^{(S)}(x^*_{P,h}) = \varphi_h^{(S)*}$.

(R2.2). The proof of this part is analogue to the proof of (R2.1), if we replace $x^*_{P,f}$ with $x^*_{P,f,restr.}$ and $\varphi_f^{(S)*}$ with $\varphi_{f,restr.}^{(S)*}$.

(R2.3). It is sufficient to give just one numerical example in which $ISM_g(x_P) \geq ISM_h(x_P)$ for all $x_P < 0 \vee x_P > \bar{u}^+/\bar{u}_P$ coincides with $(\varphi_h^{(TB)})^2 > (\varphi_g^{(TB)})^2$ and $\varphi_h^{(S)*} > \varphi_g^{(S)*}$. Under consideration of the linear regressions presented in the proof of (R1.6) we therefore look at two funds g and h with $\alpha_{gP} = 0.04$, $\beta_{gP} = 0.8$, and $Var(\tilde{\varepsilon}_{gP}) = 0.004$ as well as $\alpha_{hP} = -0.02$, $\beta_{hP} = 1.7$, and $Var(\tilde{\varepsilon}_{hP}) = 0.0005$. In addition, we assume $\bar{u}_P = 1.77\ \%$ as well as $\sigma_P^2 = 0.004$ and thus have $-1/\varphi_h^{(T)} \approx -168.48 < -14.77 \approx -1/\varphi_g^{(T)}$ and $\varphi_h^{(S)} \approx 0.09 < 0.67 \approx \varphi_g^{(S)}$. From (R1.1) we know that $ISM_g(x_P) \geq ISM_h(x_P)$ for all $0 \leq x_P \leq \bar{u}^+/\bar{u}_P$. Since $(\varphi_h^{(TB)})^2 = 0.8 > 0.4 = (\varphi_g^{(TB)})^2$ and thus $\varphi_h^{(S)*} > \varphi_g^{(S)*}$ the first part of (R2.3) is proven. If we change the parameters to $\alpha_{hP} = 0.02$ and $\beta_{hP} = -0.8$ we get $-1/\varphi_h^{(T)} \approx 136.99 > -14.77 \approx -1/\varphi_g^{(T)}$ and $\varphi_h^{(S)} \approx 0.11 < 0.67 \approx \varphi_g^{(S)}$. From (R1.2), it follows $ISM_g(x_P) \geq ISM_h(x_P)$ for all $x_P < 0 \vee x_P > \bar{u}^+/\bar{u}_P$. In this situation again we have $(\varphi_h^{(TB)})^2 = 0.8 > 0.4 = (\varphi_g^{(TB)})^2$ and $\varphi_h^{(S)*} > \varphi_g^{(S)*}$. Thus, the second part of (R2.3) is verified, too.

(R2.4). Consider two funds g and h with both $\varphi_g^{(J)} > 0$ and $\varphi_h^{(J)} = 0$. According to (BG1) of section II we have $x_g^* > 0$ as well as $x_h^* = 0$ and thus $\varphi_g^{(S)*} > \varphi_P^{(S)} = \varphi_h^{(S)*}$. The proof of (R1.6). points out that the conditions $\varphi_g^{(J)} > 0$ and $\varphi_h^{(J)} \geq 0$ do not necessarily lead to $ISM_g(x_P) \geq ISM_h(x_P)$ for any $x_P \neq \bar{u}^+ / \bar{u}_P$, so that (R2.4) is proven.

(R2.5). Again, we look at the two funds g and h of the proof of (R2.4). On the one hand we have $\varphi_{h,restr.}^{(S)*} = \varphi_h^{(S)*} = \varphi_P^{(S)}$. On the other hand we know from (BG4) that $\varphi_{g,restr.}^{(S)*} \in \{\varphi_g^{(S)*}, \varphi_g^{(S)}\}$. Particularly, this implies $\varphi_{g,restr.}^{(S)*} \geq \varphi_P^{(S)}$ and (R2.5) is proven by the same arguments as (R2.4).

Proof of Result 3:

(R3.1). From (R2.1) we know we can restrict ourselves to show $ISM_g(x_{P,h}^*) \geq ISM_h(x_{P,h}^*)$. This statement is equivalent to

(A7)
$$\begin{aligned} &\bar{u}_Q^+(x_{P,h}^*)^2 \cdot \left(-\frac{1}{(\varphi_S^{(g)})^2} \right) + 2 \cdot x_{P,h}^* \cdot \sigma_P^2 \cdot \bar{u}_Q^+(x_{P,h}^*) \cdot \left(-\frac{1}{\varphi_T^{(g)}} \right) \\ &\geq \bar{u}_Q^+(x_{P,h}^*)^2 \cdot \left(-\frac{1}{(\varphi_S^{(h)})^2} \right) + 2 \cdot x_{P,h}^* \cdot \sigma_P^2 \cdot \bar{u}_Q^+(x_{P,h}^*) \cdot \left(-\frac{1}{\varphi_T^{(h)}} \right). \end{aligned}$$

Under consideration of the assumptions of this part as well as (A7) it is sufficient to prove that

(A8) $\operatorname{sgn}(x_{P,h}^* \cdot \bar{u}_Q^+(x_{P,h}^*)) = \operatorname{sgn}(\varphi_h^{(invJ)}) \cdot \operatorname{sgn}(\varphi_h^{(J)})$.

From $x_f^* \cdot \bar{u}_f + x_P^* \cdot \bar{u}_P = \bar{u}^+$ and $x_P^* \cdot \bar{u}_P + \bar{u}_Q^+(x_{P,f}^*) = \bar{u}^+$ we get $x_f^* = \bar{u}_Q^+(x_{P,f}^*) / \bar{u}_f$. Thus, x_f^* and $\bar{u}_Q^+(x_{P,f}^*)$ have the same sign. Together with (BG1) and (BG4) of section II this implies

(A9)
$$\begin{aligned} &\operatorname{sgn}(\varphi_f^{(J)}) \underset{(BG1)}{=} \operatorname{sgn}(\bar{u}_Q^+(x_{P,f}^*)), \\ &\operatorname{sgn}(\varphi_f^{(invJ)}) \underset{(BG4)}{=} \operatorname{sgn}(x_{P,f}^*). \end{aligned}$$

(A9) immediately leads to the asserted result (A8).

(R3.2). Result (R3.2) is a direct consequence of (R3.1) and (BG5).

(R3.3). In accordance with (R3.1) and under consideration of (R2.2), we only have to show:

$$\text{(A10)}\quad \begin{aligned}&\left(\frac{\overline{u}_Q^{+}(x^*_{P,h,restr.})}{\sigma_P}\right)^2\cdot\left(-\frac{1}{(\varphi_g^{(S)})^2}\right)+2\cdot x^*_{P,h,restr.}\cdot\overline{u}_Q^{+}(x^*_{P,h,restr.})\cdot\left(-\frac{1}{\varphi_g^{(T)}}\right)\\ &\geq\left(\frac{\overline{u}_Q^{+}(x^*_{P,h,restr.})}{\sigma_P}\right)^2\cdot\left(-\frac{1}{(\varphi_h^{(S)})^2}\right)+2\cdot x^*_{P,h,restr.}\cdot\overline{u}_Q^{+}(x^*_{P,h,restr.})\cdot\left(-\frac{1}{\varphi_h^{(T)}}\right).\end{aligned}$$

Since this result is obvious for $x^*_{P,h,restr.}=0$ (because of $\varphi_g^{(S)}\geq\varphi_h^{(S)}$), we only have to treat the case $x^*_{P,h,restr.}>0$. Even in such a situation we immediately get (A10), since $-1/\varphi_g^{(T)}\geq-1/\varphi_h^{(T)}$ and $\mathrm{sgn}(x^*_{P,h,restr.}\cdot\overline{u}_Q^{+}(x^*_{P,h,restr.}))=1$.

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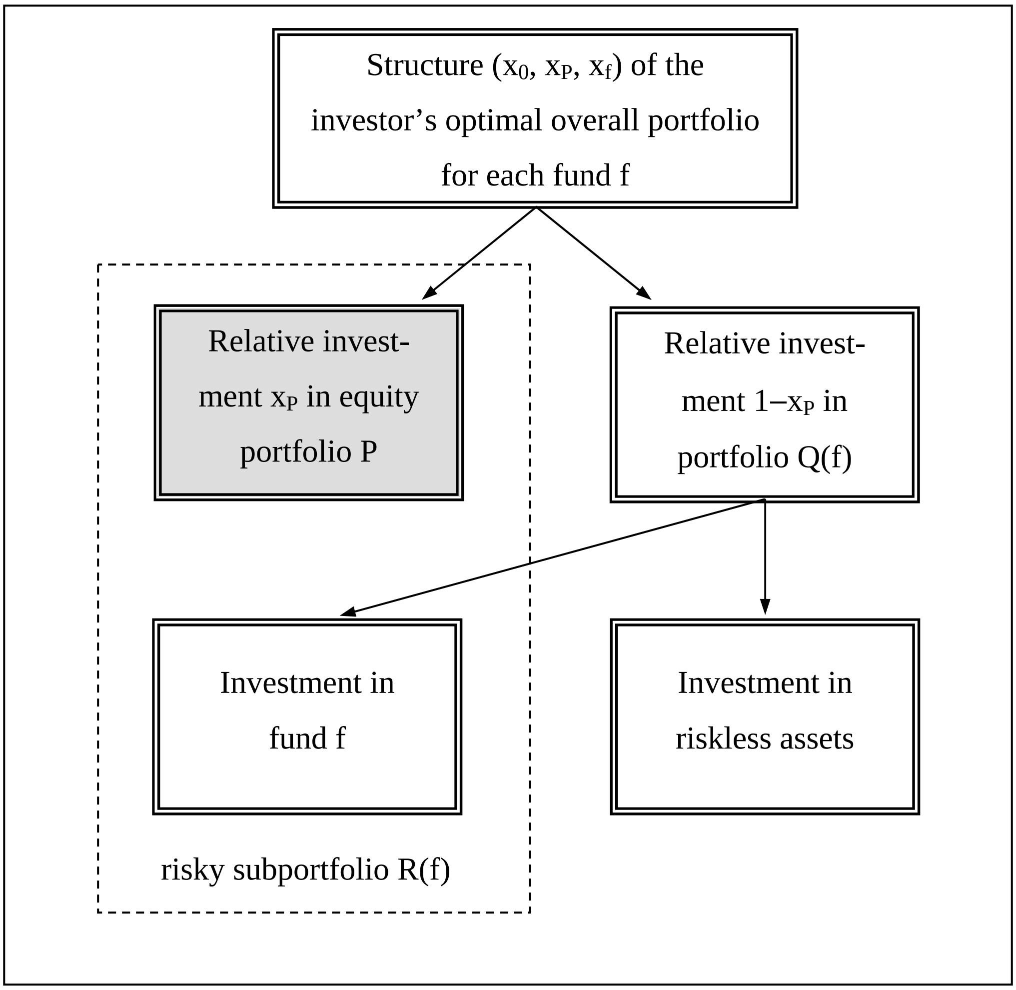
Figure 1

Structure of the Investor's Optimal Overall Portfolio



In the endogenous case, for any fund f under consideration the investor simultaneously optimizes relative shares x_0 (of riskless assets), x_P (of the equity portfolio P), and x_f (of the funds f). In the exogenous case, only x_f and x_0 (subportfolio Q(f)) can be optimized, since $x_P = x_P^+ = \text{const.}$ (i.e. the "shaded" component in Figure 1 is given).

Table 1

Synopsis of Relevant Symbols

Assets:
f,g,h: investment funds,
F: total number of funds,
f^*: "best" fund out of all funds f = 1, …, F,
P: portfolio of direct stock holdings (serving as the "reference portfolio").

Investor's subportfolios (being part of the investor's total asset holdings):
R(f): risky subportfolio, i.e. (only) investment in fund f and in reference portfolio P,
Q(f): subportfolio which – in the exogenous case – is not already fixed, i.e. (only) investment in fund f and riskless lending or borrowing.

Return characteristics:
r_0: riskless interest rate,
$\tilde{r}_f$: return of fund f,
$\tilde{r}_P$: return of reference portfolio P,
$\tilde{u}_f$: excess return $\tilde{r}_f - r_0$ of f with expectation value $\bar{u}_f$ and standard deviation σ_f,
$\tilde{u}_P$: excess return $\tilde{r}_P - r_0$ of P with expectation value $\bar{u}_P$ and standard deviation σ_P,
$\tilde{u}_{Q(f)}$: excess return $\tilde{r}_{Q(f)} - r_0$ of Q(f) with expectation value $\bar{u}_{Q(f)}$ and standard deviation $\sigma_{Q(f)}$,
$\tilde{u}_{R(f)}$: excess return $\tilde{r}_{R(f)} - r_0$ of R(f) with expectation value $\bar{u}_{R(f)}$ and standard deviation $\sigma_{R(f)}$,
σ_{fP} : covariance between $\tilde{u}_f$ and $\tilde{u}_P$,
$\beta_{fP} := \sigma_{fP} / \sigma_P^2$ (regression coefficient of a linear regression of $\tilde{u}_f$ with respect to $\tilde{u}_P$),
$\beta_{Pf} := \sigma_{fP} / \sigma_f^2$ (regression coefficient of a linear regression of $\tilde{u}_P$ with respect to $\tilde{u}_f$),
$\tilde{\varepsilon}_{fP}$: error term of a linear regression of $\tilde{u}_f$ with respect to $\tilde{u}_P$,
$\sigma_{\varepsilon_{fP}}$: standard deviation of error term $\tilde{\varepsilon}_{fP}$.

Decision variables:
x_0: fraction of monetary wealth risklessly invested ($x_0 < 0$: borrowing of money),
x_P: fraction of monetary wealth invested in reference portfolio P,
x_f: fraction of monetary wealth invested in shares of fund f.

Specific parameters for the exogenous case:
$\bar{u}^+$: overall expected excess return desired by the investor,
x_P^+ : percentage of initial wealth already fixed by an investment in the reference portfolio P,
$u_{Q(f)}^+(x_P) = u_Q^+(x_P)$: contribution of subportfolio Q(f) to an investor's overall achievable expected excess return (independent of f).

Performance measures:
$\varphi_f^{(S)}$: *Sharpe* measure of f,
$\varphi_f^{(T)}$: *Treynor* measure of f,
$\varphi_f^{(J)}$: *Jensen* measure of f,
$\varphi_f^{(TB)}$: *Treynor*/*Black* measure of f,
$\varphi_f^{(invJ)}$: "inverse" *Jensen* measure of f,
$\varphi_f^{(S)*}$: unrestricted optimized *Sharpe* measure of f,
$\varphi_{f,restr.}^{(S)*}$: restricted optimized *Sharpe* measure of f,
$ISM_{f,orig.}(x_P)$: original investor specific performance measure of f,
$ISM_f(x_P)$: modified investor specific performance measure of f.

Optimal values are generally characterized by an asterisk ("*") and in the endogenous case with short sales restrictions additionally by an index "restr.". Tildes ("~") denote random variables.

Table 2

Unbiased Estimators for Expectation Values $\bar{u}_f$, Standard Deviations σ_f , and Covariances σ_{fP} of Excess Returns of German Funds and Reference Portfolio P

No.	name of fund	$\bar{u}_f$	σ_f	σ_{fP}
1	Aberdeen Global German Eq	0.46351 %	5.77708 %	0.33096 %
2	ABN AMRO Germany Equity	2.42189 %	7.09676 %	0.42209 %
3	ADIFONDS	2.16243 %	7.22614 %	0.44304 %
4	Baring German Growth	2.85000 %	7.05836 %	0.33608 %
5	CB Lux Portfolio Euro Aktien	1.79676 %	6.77890 %	0.42088 %
6	Concentra	1.85919 %	6.71783 %	0.41575 %
7	CS EF (Lux) Germany	1.58297 %	6.66003 %	0.40816 %
8	DekaFonds	1.91459 %	6.81638 %	0.42138 %
9	DELBRÜCK Aktien UNION-Fonds	1.42919 %	6.25222 %	0.38175 %
10	Dexia Eq L Allemagne C	1.67865 %	6.23957 %	0.38700 %
11	DIT Wachstumsfonds	1.88919 %	6.28905 %	0.37674 %
12	DVG Fonds SELECT INVEST	2.07243 %	6.61112 %	0.40792 %
13	EMIF Germany Index plus B	1.57108 %	6.45667 %	0.40139 %
14	Flex Fonds	1.39730 %	5.98888 %	0.36524 %
15	Frankfurter Sparinvest Deka	1.81324 %	6.41583 %	0.39600 %
16	FT Deutschland Dynamik Fonds	1.79459 %	6.59269 %	0.40786 %
17	Hauck Main I Universal Fonds	1.45865 %	6.58482 %	0.40521 %
18	Incofonds	2.13865 %	6.04074 %	0.34912 %
19	Interselex Equity Germany B	1.72514 %	6.60614 %	0.40989 %
20	Lux Linea	1.71378 %	7.60317 %	0.46976 %
21	Metallbank Aktienfonds DWS	2.07324 %	5.14655 %	0.26836 %
22	MK Alfakapital	1.98243 %	7.41669 %	0.45851 %
23	MMWI PROGRESS Fonds	1.76081 %	6.71760 %	0.41379 %
24	Parvest Germany C	1.60108 %	6.31697 %	0.39222 %
25	Plusfonds	2.40324 %	6.83304 %	0.40050 %
26	Portfolio Partner Universal G	1.09946 %	6.08717 %	0.32420 %
27	SMH Special UBS Fonds 1	1.90811 %	6.60503 %	0.40739 %
28	Thesaurus	1.72811 %	6.36330 %	0.39459 %
29	AC Deutschland	1.86378 %	7.09276 %	0.41137 %
30	Baer Multistock German Stk A	1.77270 %	5.48620 %	0.32287 %
31	BBV Invest Union	1.90946 %	6.30927 %	0.38537 %
32	Berlinwerte Weberbank OP	1.57595 %	5.68085 %	0.33807 %
33	DIT Fonds für Vermögensbildung	1.32405 %	5.79650 %	0.34777 %
34	DWS Deutschland	1.60784 %	6.08441 %	0.36909 %
35	Fidelity Fds Germany	1.72892 %	6.24931 %	0.37989 %
36	Gerling Deutschland Fonds	1.41054 %	5.19347 %	0.31236 %
37	HANSAeffekt	1.73973 %	6.49867 %	0.40096 %
38	INVESCO GT German Growth C	1.71649 %	5.67770 %	0.24657 %
39	Investa	2.11541 %	6.92485 %	0.42699 %
40	Köln Aktienfonds DEKA	1.83865 %	6.54772 %	0.40355 %
41	Oppenheim Select	1.69757 %	6.47148 %	0.39475 %
42	Ring Aktienfonds DWS	1.86784 %	6.15453 %	0.37430 %
43	Trinkaus Capital Fonds INKA	1.71541 %	6.49609 %	0.40013 %
44	UniFonds	1.74784 %	6.42735 %	0.39665 %
45	Universal Effect Fonds	1.74568 %	6.27421 %	0.38306 %
P	DAX 100	1.77189 %	6.24936 %	0.39055 %

Only the ranking of funds # 1 to # 28 is the same for the *Sharpe* measure as well as the *Treynor* measure (see Table 3) and thus they are separated by a horizontal line from funds # 29 to # 45.

Table 3

Sharpe ($\varphi_f^{(S)}$) and _Treynor_ ($\varphi_f^{(T)}$) Measures of German Funds

with Identical Resulting Rankings

No.	name of fund	$\varphi_f^{(S)}$	$\varphi_f^{(T)}$
4	Baring German Growth	40.37767 %	3.31185 %
21	Metallbank Aktienfonds DWS	40.28415 %	3.01718 %
18	Incofonds	35.40378 %	2.39243 %
25	Plusfonds	35.17095 %	2.34349 %
2	ABN AMRO Germany Equity	34.12671 %	2.24092 %
12	DVG Fonds SELECT INVEST	31.34768 %	1.98418 %
11	DIT Wachstumsfonds	30.03932 %	1.95841 %
3	ADIFONDS	29.92514 %	1.90622 %
27	SMH Special UBS Fonds 1	28.88873 %	1.82921 %
15	Frankfurter Sparinvest Deka	28.26202 %	1.78827 %
8	DekaFonds	28.08814 %	1.77449 %
6	Concentra	27.67543 %	1.74649 %
16	FT Deutschland Dynamik Fonds	27.22096 %	1.71840 %
28	Thesaurus	27.15740 %	1.71039 %
10	Dexia Eq L Allemagne C	26.90326 %	1.69401 %
22	MK Alfakapital	26.72935 %	1.68859 %
5	CB Lux Portfolio Euro Aktien	26.50514 %	1.66726 %
23	MMWI PROGRESS Fonds	26.21190 %	1.66191 %
19	Interselex Equity Germany B	26.11410 %	1.64370 %
24	Parvest Germany C	25.34571 %	1.59426 %
13	EMIF Germany Index plus B	24.33268 %	1.52863 %
7	CS EF (Lux) Germany	23.76824 %	1.51466 %
14	Flex Fonds	23.33154 %	1.49411 %
9	DELBRÜCK Aktien UNION-Fonds	22.85890 %	1.46214 %
20	Lux Linea	22.54039 %	1.42479 %
17	Hauck Main I Universal Fonds	22.15170 %	1.40585 %
26	Portfolio Partner Universal G	18.06190 %	1.32444 %
1	Aberdeen Global German Eq	8.02332 %	0.54696 %

Table 4

Correlation Coefficients between ISM-Rankings for Desired Expected Excess Return $\bar{u}^{+}$ = 2.3 % and different values $x_P^{(1)}$ and $x_P^{(2)}$ (in %)

$x_P^{(2)}$ \ $x_P^{(1)}$	0 %	5 %	10 %	15 %	20 %	25 %	30 %	35 %	40 %	45 %	50 %	55 %	60 %	65 %	70 %	75 %	80 %	85 %	90 %	95 %	100 %
0 %	100.0000	98.5294	98.2843	98.0392	97.3039	97.3039	97.3039	96.3235	96.3235	95.5882	95.0980	95.0980	94.8529	94.8529	94.8529	93.6275	93.6275	93.6275	93.6275	93.1373	93.1373
5 %	98.5294	100.0000	99.7549	99.5098	98.7745	98.7745	98.7745	98.5294	98.5294	97.7941	97.3039	97.3039	97.0588	97.0588	97.0588	95.8333	95.8333	95.8333	95.8333	95.3431	95.3431
10 %	98.2843	99.7549	100.0000	99.7549	99.0196	99.0196	99.0196	98.7745	98.7745	98.0392	97.5490	97.5490	97.3039	97.3039	97.3039	96.3235	96.3235	96.3235	96.3235	95.8333	95.8333
15 %	98.0392	99.5098	99.7549	100.0000	99.5098	99.5098	99.5098	99.2647	99.2647	98.7745	98.2843	98.2843	98.0392	98.0392	98.0392	97.3039	97.3039	97.3039	97.3039	97.0588	97.0588
20 %	97.3039	98.7745	99.0196	99.5098	100.0000	100.0000	100.0000	99.7549	99.7549	99.5098	99.2647	99.2647	99.0196	99.0196	99.0196	98.5294	98.5294	98.5294	98.5294	98.2843	98.2843
25 %	97.3039	98.7745	99.0196	99.5098	100.0000	100.0000	100.0000	99.7549	99.7549	99.5098	99.2647	99.2647	99.0196	99.0196	99.0196	98.5294	98.5294	98.5294	98.5294	98.2843	98.2843
30 %	97.3039	98.7745	99.0196	99.5098	100.0000	100.0000	100.0000	99.7549	99.7549	99.5098	99.2647	99.2647	99.0196	99.0196	99.0196	98.5294	98.5294	98.5294	98.5294	98.2843	98.2843
35 %	96.3235	98.5294	98.7745	99.2647	99.7549	99.7549	99.7549	100.0000	100.0000	99.7549	99.5098	99.5098	99.2647	99.2647	99.2647	98.7745	98.7745	98.7745	98.7745	98.5294	98.5294
40 %	96.3235	98.5294	98.7745	99.2647	99.7549	99.7549	99.7549	100.0000	100.0000	99.7549	99.5098	99.5098	99.2647	99.2647	99.2647	98.7745	98.7745	98.7745	98.7745	98.5294	98.5294
45 %	95.5882	97.7941	98.0392	98.7745	99.5098	99.5098	99.5098	99.7549	99.7549	100.0000	99.7549	99.7549	99.5098	99.5098	99.5098	99.2647	99.2647	99.2647	99.2647	99.0196	99.0196
50 %	95.0980	97.3039	97.5490	98.2843	99.2647	99.2647	99.2647	99.5098	99.5098	99.7549	100.0000	100.0000	99.7549	99.7549	99.7549	99.5098	99.5098	99.5098	99.5098	99.2647	99.2647
55 %	95.0980	97.3039	97.5490	98.2843	99.2647	99.2647	99.2647	99.5098	99.5098	99.7549	100.0000	100.0000	99.7549	99.7549	99.7549	99.5098	99.5098	99.5098	99.5098	99.2647	99.2647
60 %	94.8529	97.0588	97.3039	98.0392	99.0196	99.0196	99.0196	99.2647	99.2647	99.5098	99.7549	99.7549	100.0000	100.0000	100.0000	99.7549	99.7549	99.7549	99.7549	99.5098	99.5098
65 %	94.8529	97.0588	97.3039	98.0392	99.0196	99.0196	99.0196	99.2647	99.2647	99.5098	99.7549	99.7549	100.0000	100.0000	100.0000	99.7549	99.7549	99.7549	99.7549	99.5098	99.5098
70 %	94.8529	97.0588	97.3039	98.0392	99.0196	99.0196	99.0196	99.2647	99.2647	99.5098	99.7549	99.7549	100.0000	100.0000	100.0000	99.7549	99.7549	99.7549	99.7549	99.5098	99.5098
75 %	93.6275	95.8333	96.3235	97.3039	98.5294	98.5294	98.5294	98.7745	98.7745	99.2647	99.5098	99.5098	99.7549	99.7549	99.7549	100.0000	100.0000	100.0000	100.0000	99.7549	99.7549
80 %	93.6275	95.8333	96.3235	97.3039	98.5294	98.5294	98.5294	98.7745	98.7745	99.2647	99.5098	99.5098	99.7549	99.7549	99.7549	100.0000	100.0000	100.0000	100.0000	99.7549	99.7549
85 %	93.6275	95.8333	96.3235	97.3039	98.5294	98.5294	98.5294	98.7745	98.7745	99.2647	99.5098	99.5098	99.7549	99.7549	99.7549	100.0000	100.0000	100.0000	100.0000	99.7549	99.7549
90 %	93.6275	95.8333	96.3235	97.3039	98.5294	98.5294	98.5294	98.7745	98.7745	99.2647	99.5098	99.5098	99.7549	99.7549	99.7549	100.0000	100.0000	100.0000	100.0000	99.7549	99.7549
95 %	93.1373	95.3431	95.8333	97.0588	98.2843	98.2843	98.2843	98.5294	98.5294	99.0196	99.2647	99.2647	99.5098	99.5098	99.5098	99.7549	99.7549	99.7549	99.7549	100.0000	100.0000
100 %	93.1373	95.3431	95.8333	97.0588	98.2843	98.2843	98.2843	98.5294	98.5294	99.0196	99.2647	99.2647	99.5098	99.5098	99.5098	99.7549	99.7549	99.7549	99.7549	100.0000	100.0000

Table 5

Average Correlation Coefficients between ISM-Rankings for Varying Identical Values of Desired Expected Excess Return $\bar{u}^{+}$ and Identical Differences $\Delta x_P = | x_P^{(1)} - x_P^{(2)} |$ between Exogenous Investments in Reference Portfolio P

Δx_P \ $\bar{u}^{+}$	1.7719 %	1.90 %	2.00 %	2.10 %	2.20 %	2.30 %	2.40 %	2.50 %	2.60 %	2.70 %	10.00 %
0 %	100.00000 %	100.00000 %	100.00000 %	100.00000 %	100.00000 %	100.00000 %	100.00000 %	100.00000 %	100.00000 %	100.00000 %	100.00000 %
5 %	99.75490 %	99.77941 %	99.79167 %	99.79167 %	99.79167 %	99.80392 %	99.80392 %	99.80392 %	99.81618 %	99.84069 %	99.91422 %
10 %	99.57430 %	99.61300 %	99.62590 %	99.62590 %	99.63880 %	99.65170 %	99.66460 %	99.66460 %	99.69040 %	99.67750 %	99.81940 %
15 %	99.41449 %	99.45534 %	99.45534 %	99.46895 %	99.48257 %	99.50980 %	99.52342 %	99.52342 %	99.55065 %	99.53704 %	99.72767 %
20 %	99.27912 %	99.32238 %	99.30796 %	99.32238 %	99.35121 %	99.36563 %	99.38005 %	99.38005 %	99.40888 %	99.40888 %	99.61073 %
25 %	99.06556 %	99.17279 %	99.15748 %	99.18811 %	99.23407 %	99.24939 %	99.26471 %	99.28002 %	99.28002 %	99.28002 %	99.52512 %
30 %	98.85621 %	98.93791 %	98.93791 %	98.97059 %	99.03595 %	99.10131 %	99.13399 %	99.16667 %	99.15033 %	99.15033 %	99.42810 %
35 %	98.63445 %	98.72199 %	98.72199 %	98.75700 %	98.84454 %	98.84454 %	98.89706 %	98.96709 %	99.00210 %	99.00210 %	99.33473 %
40 %	98.41629 %	98.51056 %	98.49170 %	98.52941 %	98.62368 %	98.62368 %	98.68024 %	98.77451 %	98.73680 %	98.75566 %	99.24585 %
45 %	98.20261 %	98.34559 %	98.28431 %	98.32516 %	98.36601 %	98.36601 %	98.44771 %	98.54984 %	98.50899 %	98.52941 %	99.14216 %
50 %	97.95009 %	98.15062 %	98.08378 %	98.12834 %	98.19519 %	98.15062 %	98.23975 %	98.28431 %	98.23975 %	98.26203 %	99.01961 %
55 %	97.59804 %	97.89216 %	97.81863 %	97.89216 %	97.99020 %	97.94118 %	98.03922 %	98.06373 %	98.01471 %	98.03922 %	98.92157 %
60 %	97.22222 %	97.54902 %	97.49455 %	97.57625 %	97.68519 %	97.68519 %	97.82135 %	97.90305 %	97.84858 %	97.87582 %	98.82898 %
65 %	96.84436 %	97.12010 %	97.08946 %	97.18137 %	97.42647 %	97.42647 %	97.57966 %	97.70221 %	97.61029 %	97.67157 %	98.71324 %
70 %	96.42857 %	96.74370 %	96.63866 %	96.77871 %	97.05882 %	97.05882 %	97.26891 %	97.40896 %	97.33894 %	97.37395 %	98.56443 %
75 %	96.03758 %	96.40523 %	96.28268 %	96.40523 %	96.56863 %	96.60948 %	96.85458 %	97.05882 %	96.97712 %	97.01797 %	98.40686 %
80 %	95.58824 %	95.98039 %	95.83333 %	95.98039 %	96.12745 %	96.22549 %	96.27451 %	96.51961 %	96.42157 %	96.51961 %	98.23529 %
85 %	94.97549 %	95.46569 %	95.28186 %	95.34314 %	95.52696 %	95.58824 %	95.71078 %	95.71078 %	95.77206 %	95.89461 %	98.10049 %
90 %	94.03595 %	94.60784 %	94.52614 %	94.60784 %	94.68954 %	94.93464 %	95.09804 %	95.09804 %	95.26144 %	95.01634 %	97.95752 %
95 %	92.64706 %	93.50490 %	93.99510 %	93.99510 %	94.11765 %	94.24020 %	94.48529 %	94.48529 %	94.73039 %	94.36275 %	97.67157 %
100 %	91.17647 %	91.91176 %	92.89216 %	92.89216 %	92.89216 %	93.13725 %	93.13725 %	93.13725 %	93.62745 %	93.62745 %	97.30392 %