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Quantitative Forecast Model for the

Application of the Black-Litterman Approach

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Quantitative Forecast Model for the

Application of the Black-Litterman Approach

by Franziska Becker[*] and Marc Gürtler[**]

Abstract. The estimation of expected security returns is one of the major tasks for the practical imple-mentation of the Markowitz portfolio optimization. Against this background, in 1992 Black and Litter-man developed an approach based on (theoretically established) expected equili-brium returns which accounts for subjective investors' views as well. In contrast to historical estimated returns, which lead to extreme asset weights within the Markowitz optimization, the Black-Litterman model generally re-sults in balanced portfolio weights. However, the existence of investors' views is crucial for the Black-Litterman model and with absent views no active portfolio management is possible. Moreover, prob-lems with the implementation of the model arise, as analysts' forecasts are typically not available in the way they are needed for the Black-Litterman-approach. In this context we present how analysts' dividend forecasts can be used to determine an a-priori-estimation of the expected returns and how they can be integrated into the Black-Litterman model. For this purpose, confidences of the investors' views are determined from the number of analysts' forecasts as well as from a Monte-Carlo simulation. After introducing our two methods of view generation, we examine the effects of the Black-Litterman approach on portfolio weights in an empirical study. Finally, the perfor-mance of the Black-Litterman model is compared to alternative portfolio allocation strategies in an out-of-sample study.

Keywords: analysts' earnings forecasts, discount rate effect, equity premium puzzle, implied rate of return

JEL classification: G11, G12, G14

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The main problem of Markowitz' [1952, 1959] portfolio theory is the estimation of the required parameters: expected values, variances and covariances of individual asset returns. The input parameters are estimated and the optimization procedure assumes that they are true values of return moments. However, future returns are random variables and their true values differ from their expected values. Extreme short sale positions often result from optimization algorithms or, if portfolio weights are bounded between zero and one, only few assets will be incorporated into the optimal portfolio. If the parameters have been estimated correctly, the resulting portfolio weights obviously lead to the highest preference level. However, if the parameters deviate from the forecast, the poorly diversified portfolio could achieve a poor preference level, if the chosen assets develop suboptimally. Furthermore, the optimal weight vector is very sensitive towards input parameters. Marginal changes in expected returns can result in large variations of portfolio weights. Regarding changes in variances and covariances, the sensitivity of weights is not as pronounced as with changes in expected returns (Best and Grauer [1991], Chopra and Ziemba [1993]).

To mitigate these problems, Black and Litterman [1992] have developed a procedure that combines equilibrium expected returns with prior beliefs of investors. By applying their model more balanced portfolios result and only those asset weights vary from the associated market weights for which views are proposed. Due to the intuitive portfolio composition the Black-Litterman model has been widely accepted in practice, and further developments and specifications have been suggested in the literature.

Satchell and Scowcroft [2000] and Qian and Gorman [2001] extend the approach of Black-Litterman to the second moments of distribution – variances and covariances of asset returns. Meucci [2006], Beach and Orlov [2007], and Martellini and Ziemann [2007] include non-normally distributed returns and consider fat tails, which is essential for hedge funds and derivatives. The literature emphasizes that establishing an investor's view for the Black-Litterman model is rather difficult. For instance, several authors employ factor models for setting up subjective views. Fabozzi, Focardi, and Kolm [2006] combine equilibrium returns with a cross sectional momentum strate-

gy for application of the Black-Litterman model. Jones, Lim, and Zangari [2007] use Fama and French's [1992] factors HML and SMB and the momentum factor of Carhart [1997] to assess views. The potential disadvantage of applying a factor model is that identified relations of past data are projected into the future. For adequate views the assumption of a stable dependency between returns and selected factors has to be satisfied. Furthermore, most authors simply implement the historical confidence matrix of views, see, for example Herold [2003], Qian and Gorman [2001]. We detach ourselves from the past and utilize analysts' forecasts, as they are geared to the future per se. The literature of valuation models for the derivation of implied equity returns based on analysts' forecasts has made a rapid development during the last years. We implement such a model because an application of analysts' forecasts to the Black-Litterman model appears to be promising. Some contributions to the implementation of specific analysts' views in the Black-Litterman approach already exist, but analysts typically do not make predictions in the way they are presented in the literature. Herold [2003] is an exception. He utilizes qualitative forecasts to construct portfolios with the Black-Litterman model. In the framework of active portfolio management, the optimal portfolio is chosen such that it reaches a given tracking error. However, portfolio composition is based on a single analyst's forecast and not on a number of analysts' forecasts. Beyond that Herold [2003] employs historical confidences, as noted before.

If, furthermore, institutional investors are only familiar with particular market segments like "Stocks US", but have no expertise in "Stocks Europe", applying the Black-Litterman model would restrict them to hold the market in European stocks.[1] Private investors possibly have no views at all. They should implement the market portfolio, too – the application of the Black-Litterman model would be dispensable in this case. To benefit from the expertise of analysts and deviate from the market portfolio in rather unfamiliar market segments, we describe two possibilities of how views can be quantified for being used with the Black-Litterman procedure. We resort to the number of analysts' forecasts and a Monte-Carlo simulation for the generation of prior beliefs by implement-

ing a future-oriented valuation model. In our empirical examination, the out-of-sample performance of the methods is compared to several benchmark portfolios.

REVIEW OF THE BLACK-LITTERMAN MODEL

In their model, Black and Litterman [1992] combine equilibrium expected returns with investor views in order to calculate a new vector of expected returns μ_{BL}, which is then integrated into the Markowitz optimization. The purpose of optimization with these new input parameters is to gain relatively balanced portfolios without the implementation of long-only constraints or other restrictions.

It is assumed that return vector r of N assets is multivariate normally distributed with $N \times 1$ expected return vector μ and $N \times N$ variance-covariance matrix Σ: $r \sim N(\mu, \Sigma)$. Variance-covariance matrix Σ is supposed to be known.[2] The vector of expected returns is a random vector that follows a multivariate normal distribution with known parameters Π, τ and Σ: $\mu \sim N(\Pi, \tau\Sigma)$. The variance-covariance matrix of expected returns is chosen to be a multiple of the variance-covariance matrix of returns r with scaling factor $\tau > 0$. This factor is not predetermined in the approach of Black-Litterman. Since uncertainty in expected returns is smaller than uncertainty in returns, Black-Litterman propose to implement a small τ. Π is the $N \times 1$ equilibrium expected return vector and serves as a neutral reference point. Black and Litterman use the market portfolio as a starting point for expected returns.[3] Thereby, the expected returns of the market portfolio are calculated via a reverse optimization. By maximizing the assumed preference function of an investor

$$\phi = \mu_P - \frac{\lambda}{2}\sigma_P^2 = X'\mu - \frac{\lambda}{2}X'\Sigma X \tag{1}$$

where μ_P is the expected portfolio return, σ_P^2 the portfolio variance, and λ the risk aversion parameter, the optimal weight vector X is determined by

$$X = \frac{1}{\lambda}\Sigma^{-1}\mu \tag{2}$$

Solving this equation for the expected return vector μ,

$$\Pi \equiv \mu = \lambda \cdot \Sigma \cdot X \tag{3}$$

is obtained. Thus, the vector of equilibrium returns Π is a multiple of the product of variance-covariance matrix Σ and asset weight vector X. The optimal asset weights x_j are given through the proportion of market capitalizations. Finally, the only missing component for the calculation of Π by using equation (3) is risk aversion λ. Taking the capital asset pricing model (CAPM) as a basis, the risk aversion parameter is determined as the market price of risk.[4]

As an additional opinion, investors can express k views or prior beliefs about returns in the following form:

$$Q = P\mu + \varepsilon, \quad \varepsilon \sim N(0, \Omega) \tag{4}$$

where Q is a $k \times 1$ vector of k forecast return expectations and P is a known $k \times N$ pick matrix of views. The $k \times 1$ error term ε follows a multivariate normal distribution with expected value zero and variance-covariance matrix Ω – the view confidence matrix. Following Black and Litterman [1992], Ω is a diagonal matrix, thus, views are independent. Entry ω_{ss} on the diagonal identifies the investor's confidence concerning the respective view: the bigger the entry, the less certain the investor is in view s. The beliefs can be indicated absolute as well as relative and there is no need for a view for every asset.

Applying the Bayesian rule to the equilibrium and investor's beliefs, Black-Litterman obtain the following expected return vector

$$\mu_{BL} = \Pi + \tau\Sigma P'\left(P\Sigma P'\tau + \Omega\right)^{-1}\left(Q - P\Pi\right) \tag{5}$$

In the literature, an alternative but equivalent notation is often used

$$\mu_{BL} = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1} \cdot \left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right] \tag{6}$$

Note that this expected return is a matrix weighted combination of equilibrium return vector Π and investor's views Q.

Problems with the Application of the Black-Litterman Model

As described above, the Black-Litterman model requires own views about return expectations of the investment horizon. The setting up of equation (4) could, for example, be deduced from the investor's beliefs (a) and (b):

(a) The investor is sure that the expected return of asset 1 will amount to 20 %.

(b) With a probability of 70 %, the investor believes that the difference of the expected returns

of asset 1 and an equally weighted portfolio of assets 3 and 4 will amount to 5 % to 7 %.

For an investment horizon of four stocks, these views are set up according to (4):

$$
\begin{pmatrix} 0.2 \\ 0.06 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & -0.5 & -0.5 \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}
$$

$$
\text{with } \varepsilon \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 9.317 \cdot 10^{-5} \end{pmatrix} \right)
$$

(7)

The view confidence is expressed through a probability that has to be converted to a variance. The more certain an investor is concerning his belief, the smaller is the variance at the respective position in matrix Ω. In reality analysts do not typically make predictions in the manner presented in the example, implying this possibility of employing forecasts for the application of the Black-Litterman model not to be given. Thus, private investors, who possibly do not have any information about stocks, would only realize the market portfolio. Moreover, institutional investors, who have no prior beliefs in certain market segments, would, by application of the Black-Litterman Model, hold the market portfolio in these particular fields, too, which turns the Black-Litterman model to be dispensable.

In order to make use of analysts' expertise for the formulation of views and thereby utilize future expectations in contrast to historical estimations, we describe methods to generate beliefs according to (a) and (b) for the Black-Litterman model with the help of analysts' dividend forecasts.

Moreover, our methods afford the possibility of diverging from the market portfolio in less conversant segments.

THE USE OF ANALYSTS' FORECASTS

Determining Views from the Number of Analysts' Forecasts

Historical expected equity returns have proved to be poor estimators for future returns.[5] Due to the necessity of finding better estimators for future returns, the literature on determination of implied equity returns on the basis of valuation models has rapidly developed during the last years. There are many empirical examinations of valuation models (e.g., dividend discount model, discounted cash flow model, residual income model, Ohlson and Jüttner-Nauroth [2005] model), which are based on analyst forecasts concerning several parameters, and the models achieve good results in comparison to historical estimations.[6] In the following, views based on the dividend discount model according to Williams [1938] and Gordon [1959, 1966] will be derived. Views can be deduced in almost the same manner on the basis of other valuation models.

In the dividend discount model, the expected stock return $\mu_i^{(t)}$ of company i is calculated at a given point in time t on the basis of the market value of equity $EK_i^{(t)}$ as the internal interest rate for the time series of expected dividend payments. Predominantly, a two-phase model is used. In the first phase of duration T, detailed estimations of the dividends $\hat{D}_i^{(t)}$ of company i are available. For the remaining time, a constant growth rate $g_i^{(t)}$ of dividends is assumed:

$$EK_i^{(t)} = \sum_{\tau=1}^{T} \frac{\hat{D}_i^{(t+\tau)}}{(1+\mu_i^{(t)})^{\tau}} + \frac{\hat{D}_i^{(t+T)} \cdot (1+g_i^{(t)})}{(\mu_i^{(t)} - g_i^{(t)}) \cdot (1+\mu_i^{(t)})^{T}} \tag{8}$$

Every month t, the following financial data concerning the expected dividends are provided: mean $\hat{D}_i^{(t+\tau)}$, standard deviation $\sigma_{D,i}^{(t+\tau)}$, highest and lowest estimation $D_{i,hi}^{(t+\tau)}$ and $D_{i,lo}^{(t+\tau)}$, and the number $NE_i^{(t+\tau)}$ of analysts' dividend forecasts $D_i^{(t+\tau)}$. These data are available for the next three years

$\tau = 1, 2, 3$ for every stock i. Furthermore, analysts estimate the long term growth of earnings $g_i^{(t)}$. The highest $g_{i,hi}^{(t)}$ and lowest $g_{i,lo}^{(t)}$ estimate is also available.[7] By application of these growth rates we assume a constant dividend payout ratio, since dividend and earnings growth are equal in this case. Now the question arises how the confidence interval and the confidence probability of the views can be determined using data of analysts' forecasts at hand.

In validity tests for the evaluation of calculated expected returns from the above mentioned valuation models, a regression of different risk factors (e.g. beta, debt-equity ratio, market capitalization, information risk) on expected returns is often carried out.[8] Thereby, a certain relation between a specified risk factor and the expected returns is assumed and it is analyzed, whether this relation can be proved. Concerning the information risk, it is supposed that a larger amount of information available on a company reduces its cost of capital. As a measure for the information risk, Botosan and Plumlee [2005], for example, choose the distance between lowest and highest forecast of a stock. They emphasize that this distance reflects the uncertainty of forecasts. Brennan, Jegadeesh, and Swaminathan [1993] point out that prices of companies with larger analyst coverage react to market information more quickly. Gebhardt, Lee, and Swaminathan [2001] implement these examination results by also taking the number of analysts' forecasts as a measure for the information risk – the more analysts' forecasts at hand, the lower the cost of equity capital. In this contribution, we also assume that the uncertainty of analysts' forecasts is larger for fewer analysts' forecasts and vice versa.

In the following, the number of analysts' forecasts will be linked to the view confidence. In order to generate the return interval $\mu_{i,lo}^{(t)}$ and $\mu_{i,hi}^{(t)}$ of analysts' views, such as in the example, we compute these returns per point in time t and stock i by solving (9) and (10) implicitly for all other variables given:

$$EK_i^{(t)} = \sum_{\tau=1}^{3} \frac{D_{i,lo}^{(t+\tau)}}{(1+\mu_{i,lo}^{(t)})^{\tau}} + \frac{D_{i,lo}^{(t+3)} \cdot (1+g_{i,lo})}{(\mu_{i,lo}^{(t)} - g_{i,lo}) \cdot (1+\mu_{i,lo}^{(t)})^3} \qquad (9)$$

7

$$EK_i^{(t)} = \sum_{\tau=1}^{3} \frac{D_{i,hi}^{(t+\tau)}}{(1+\mu_{i,hi}^{(t)})^{\tau}} + \frac{D_{i,hi}^{(t+3)} \cdot (1+g_{i,hi})}{(\mu_{i,hi}^{(t)} - g_{i,hi}) \cdot (1+\mu_{i,hi}^{(t)})^3} \qquad (10)$$

The entries of view confidence matrix Ω_{NE} are calculated with the help of the number of analysts'

forecasts $NE_i^{(t+3)}$. For each stock i, point in time t and prediction variable $D_i^{(t+1)}, D_i^{(t+2)}$, and $D_i^{(t+3)}$,

there is a number of analysts who have made forecasts. Thus, for stock A in month t, there are mean

dividend forecasts $\hat{D}_A^{(t+1)}, \hat{D}_A^{(t+2)}, \hat{D}_A^{(t+3)}$ for the first, second, and third forthcoming year, and for each

of these forecasts, we have the number of analysts making these forecasts at hand:

$NE_A^{(t+1)}, NE_A^{(t+2)}, NE_A^{(t+3)}$. Abarbanell and Bernard [2000] and Courteau, Kao, and Richardson

[2001] measure a strong influence of the terminal value of the dividend discount model on the esti-

mation of expected return. Due to the great importance of the last term in (8), the number of ana-

lysts' forecasts for the last (third) year is taken as a basis for the calculation of the confidence prob-

ability. As a starting point, the maximum number of analysts' forecasts that are given over the

whole period of time in question for every single stock is marked with a confidence probability of

100 %.[9] Thus, if the maximum number of forecasts given for stock A is 40 and 40 analysts make

forecasts for the third year of stock A at one point in time, the confidence probability for the partic-

ular interval is 100 %. If no forecast was made for the third year of one asset at one point in time,

the confidence probability would be 0 %. The confidence probability is then linearly interpolated

between 0 % and 100 % based on the number of analysts' forecasts for the third forthcoming year

for every stock at every point in time. The confidence matrix determined this way is denoted by

Ω_{NE}. Consequently, forecasts according to (4) are:

$$
\begin{pmatrix}
\dfrac{\mu_{1,lo}^{(t)}+\mu_{1,hi}^{(t)}}{2} \\[2mm]
\dfrac{\mu_{2,lo}^{(t)}+\mu_{2,hi}^{(t)}}{2} \\[2mm]
\vdots \\[2mm]
\dfrac{\mu_{N,lo}^{(t)}+\mu_{N,hi}^{(t)}}{2}
\end{pmatrix}
=
\begin{pmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{pmatrix}
\cdot
\begin{pmatrix}
\mu_1 \\ \mu_2 \\ \vdots \\ \mu_N
\end{pmatrix}
+
\begin{pmatrix}
\varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N
\end{pmatrix}
\tag{11}
$$

$$
\text{with } \varepsilon \sim N\left(
\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix},
\begin{pmatrix}
\omega_{NE,11} & 0 & \cdots & 0 \\
0 & \omega_{NE,22} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \omega_{NE,NN}
\end{pmatrix}
\right)
$$

In this case, view matrix P is an identity matrix, as an absolute forecast is available for every stock. Equation (5) is then simplified and the respective Black-Litterman return is denoted by

$$
\mu_{BL,NE} = \Pi + \tau\Sigma\left(\Sigma\tau + \Omega_{NE}\right)^{-1}\left(Q - \Pi\right)
\tag{12}
$$

If analysts' forecasts are missing for some stocks in the portfolio, these can be ignored in the view matrix without further ado. At last, we get market weights as optimal weights for assets without views.

Determining Views by a Monte-Carlo Simulation

View confidence matrix Ω can also be determined by directly converting standard deviations of analysts' dividend forecasts into standard deviations for investors' expected returns. To this end, a Monte-Carlo simulation is carried out in order to compute the standard deviation of expected returns. For stock i at time t the number S = 1.000 of random variables are drawn in the following way: expected dividends $D_{i,s}^{(t+1)}$ are drawn from a normal distribution with mean $\hat{D}_i^{(t+1)}$ and standard deviation $\sigma_{D,i}^{(t+1)}$: $D_{i,s}^{(t+\tau)} \sim N\left(\hat{D}_i^{(t+\tau)}, \sigma_{D,i}^{(t+\tau)}\right)$, $\tau = 1, 2, 3$. However, growth rates $g_{i,s}^{(t)}$ are drawn from a uniform distribution on the interval $[g_{i,lo}^{(t)}, g_{i,hi}^{(t)}]$.[10] Mean $\hat{D}_i^{(t+\tau)}$ and standard deviation $\sigma_{D,i}^{(t+\tau)}$ of expected dividends as well as growth rates $g_{i,lo}^{(t)}$ and $g_{i,hi}^{(t)}$ are provided for example by Thomson Financial datastream.

Based on one respective simulated data set s: $D_{i,s}^{(t+1)}, D_{i,s}^{(t+2)}, D_{i,s}^{(t+3)}, g_{i,s}^{(t)}$, an expected return is numerically computed by solving the following equation for $\mu_{i,s}^{(t)}$:

$$EK_i^{(t)} = \sum_{\tau=1}^{3} \frac{D_{i,s}^{(t+\tau)}}{(1+\mu_{i,s}^{(t)})^{\tau}} + \frac{D_{i,s}^{(t+3)} \cdot (1+g_{i,s}^{(t)})}{(\mu_{i,s}^{(t)} - g_{i,s}^{(t)}) \cdot (1+\mu_{i,s}^{(t)})^3} \tag{13}$$

After the simulations, we get S expected returns, from which the variance of expected returns of stock i at time t is determined:

$$\omega_{MC,ii} = \frac{1}{S-1} \sum_{s=1}^{S} \left(\mu_{i,s}^{(t)} - \hat{\mu}_{i,s}^{(t)} \right)^2$$

$$\text{with } \hat{\mu}_{i,s}^{(t)} = \frac{1}{S} \sum_{s=1}^{S} \mu_{i,s}^{(t)} \tag{14}$$

Finally, the variances are inserted into the diagonal of matrix Ω_{MC}, and the Black-Litterman model can be applied to calculate the respective expected return vector $\mu_{BL,MC}$.

ADDITIONAL MODELS FOR THE ESTIMATION OF VIEW CONFIDENCES

Historical Variance-Covariance Matrix

A further possibility of determining the view confidence matrix Ω is to use the historical variance-covariance matrix of the specified views. Here, it is assumed that relations of stocks in the past are also valid for the future. In this case view confidence matrix Ω is calculated in the following way:

$$\Omega_{hist} = \text{diag}\left(P\Sigma P' \right) \tag{15}$$

If we look at a certain view portfolio, that is, a certain row k from view matrix P_k and at first multiply it by variance-covariance matrix Σ and then by column vector P_k', we get the portfolio variance of the view portfolio

$$\omega_{hist,kk} = P_k \Sigma P_k' \tag{16}$$

10

Since in our approach the view matrix P is an identity matrix, the confidence matrix is simplified to:

$$\Omega_{hist} = diag(\Sigma) \tag{17}$$

We denote the corresponding Black-Litterman return (BL-return) with $\mu_{BL,hist}$.

Implementation of He and Litterman [1999]

A different implementation is proposed by He and Litterman [1999]. The elements of view confidence matrix Ω for view k ω_{kk} are defined such that they equal the historical variances of view k multiplied by scaling factor τ:

$$\omega_{HL,kk} / \tau = diag(P_k \Sigma P_k ') \tag{18}$$

This definition has the advantage that parameter τ does not need to be specified in the equation for calculating the expected returns according to Black-Litterman (6). However, doing so we make an implicit assumption regarding the relation between confidence matrix Ω_{HL} and parameter τ. Subsequently, the entries in matrix Ω_{HL} will be much smaller than those in confidence matrix Ω_{hist}, since parameter τ is less than one. Hence, using the respective BL-return $\mu_{BL,HL}$, investors' views are given more importance.

EMPIRICAL EXAMINATION

Examination of Methods

In the following, the Black-Litterman model is applied to real capital market data. For this purpose, monthly data from 12/01/1993 to 01/01/2008 of all stocks of HDAX and DAX100, respectively, are available. The data is extracted from the Thomson Reuters Datastream database. Since DAX100 has been replaced by HDAX not until 03/24/2003, it will form the basis of our empirical examination in the beginning. Insofar, we are considering 100 stocks until 03/24/2003 and 110 stocks afterwards. Only stocks that are included in the index (either DAX100 before April 2003 or

HDAX from April 2003 on) at a specified point in time are considered in the optimization at this time. Furthermore, we examine whether all data are available for the estimation of input parameters for the Markowitz optimization – for example, stock returns of the last 36 points in time for estimation of historically expected returns. If, however, not all data that are required for the empirical examination are available, the respective stock is not used in the optimization for this point in time. This procedure allows for an optimization that, at a certain point in time, contains a stock which is no longer in the index after this point in time – thus, a survivorship bias does not exist. The number of stocks which are optimized over time varies between 31 and 59.

The period of portfolio optimization starts on 01/01/1997. The preceding data are, for example, used for the calculation of the variance-covariance matrix or the historical mean value of realized returns.[11] Variance-covariance matrix Σ is calculated according to the Single-Index-Model.[12]

The maximum number of analysts' forecasts for the whole period among all stocks amounts to 45, the minimum number to two.[13] Exhibit 1 presents an overview of input parameters for the calculation of the expected Black-Litterman returns $\mu_{BL,NE}$ of ten stocks on 12/01/2007.[14] For the purpose of clarity, only ten of 54 respective stocks from the optimization are depicted below.

Furthermore, the assumption of a higher number of analysts' forecasts involving a higher confidence implies that a higher number of analysts' forecasts also results in a lower standard deviation. The first chart shows return differences $\mu_{i,hi} - \mu_{i,lo}$, which enter the calculation of view standard deviations (see Appendix). The higher the return difference, the higher the view standard deviation. The second chart shows view confidence probabilities based on the number of analysts' estimates. A higher number of estimates involves a higher confidence probability of the view. Furthermore, a higher confidence probability leads to a smaller standard deviation, which is displayed in the third chart. Stock 7 has the lowest standard deviation, which results from the low return difference of $\mu_{7,hi}$ and $\mu_{7,lo}$ and a high confidence probability of 71 %. In contrast, stock 10 has the highest standard deviation, primarily as a consequence of a high return difference since the confidence

probability is moderate. The fourth chart in Exhibit 1 shows equilibrium returns Π_i, mean returns from the dividend forecasts $Q \equiv \mu_{i,mi} = \left(\mu_{i,hi} + \mu_{i,lo} \right) / 2$, and the expected BL-return $\mu_{BL,NE}$, which is calculated from the previous returns according to (5). There is evidence that expected BL-returns by trend are closer to the expected returns from analysts' dividend forecasts. Stock 10 features a relatively high standard deviation ω_{10}, which is also reflected in the higher deviation between the investors' view return Q and BL-return $\mu_{BL,NE}$.

Exhibit 1 about here

In Exhibit 2 confidences ω_i are compared between the two introduced approaches. Black bars represent standard deviations on the basis of the number of analysts' forecasts $\omega_{i,NE}$ and white bars display standard deviations calculated with Monte-Carlo simulation $\omega_{i,MC}$. The latter ones are clearly smaller. As noted before, views with lower standard deviations are assigned a higher weight in the calculation of the expected BL-return, thereby the resulting weights deviate stronger from market weights.

Exhibit 2 about here

As expected, optimal portfolio weights from the Monte-Carlo simulation $\mu_{BL,MC}$, which are depicted in Exhibit 3, are always larger compared to weights from the optimization with $\mu_{BL,NE}$ when comparing their absolute values.

Exhibit 3 about here

Exhibit 3 gives an overview of the resulting portfolio weights of all introduced Black-Litterman approaches for the ten shares on 12/01/2007. In absolute values, weights of the pure dividend discount model (white bars) are highest except for few values. Moreover, weights of the He-Litterman approach are closer to weights of the dividend discount model compared to weights of the historical BL-approach, as expected.

In order to assess all approaches, the performance of the four different methods for the determination of confidence matrix Ω and thereby calculation of BL-returns (number of analysts' fore-

casts (NE), Monte-Carlo simulation (MC), historical variance-covariance matrix (hist) , He-Litterman (HL)) is compared to several benchmark strategies in the following section.

Out-of-Sample Performance Study

In a rolling optimization procedure, monthly out-of-sample returns and the resulting performances are calculated from 02/01/1997 to 01/01/2008.[15] We assume that at a specific time t only past and current data of times $t-s,\ldots,t,$ with $s>0$ are known. Based on this information, the expected returns for time t and the optimal portfolios and benchmark portfolios for the following nine strategies are determined subsequently: expected returns of the dividend discount model μ_{mi}, BL-returns $\mu_{BL,NE}$, $\mu_{BL,MC}$, $\mu_{BL,hist}$, $\mu_{BL,HL}$, historical expected returns μ_{hist}, and a portfolio composed according to a Bayesian estimator.[16] The market portfolio and the equally weighted portfolio also serve as a benchmark.

Expected BL-returns are calculated at each point in time t with current information and analysts' forecasts of time t, as mentioned before. Subsequently, weight vectors are determined with these expected returns and the resulting portfolio compositions are kept fixed for one month from t to t+1. At time t+1, the actual stock returns of this month (period from t to t+1) are multiplied by the specified weight vectors to obtain the portfolio return of every strategy for this month. The riskless interest rate is subtracted from this value, in order to identify real excess out-of-sample portfolio returns for the period from t to t+1. We choose prevalent performance measures to compare and assess the different strategies: Sharpe ratio, Jensen's alpha, Treynor ratio, and certainty equivalent from (1).

The optimizations are accomplished with different restrictions, unconstrained as well as constrained. However, risky asset weights greater than one are allowed. The portion which is invested in the riskless asset amounts to $x_0 = 1 - \sum_{i=1}^{N} x_i$ in both optimizations. A negative x_0 indicates a debt position in the riskless interest rate. We analyze several parameter constellations for the risk aversion parameter λ and the scaling factor τ for the Black-Litterman model and apply common

risk aversions from 0.5 to 3.5 in steps of 0.3 (11 values). In order to establish small values for τ, we choose τ from 0.01 to 0.46 in steps of 0.05 (10 values). Thereby, 110 parameter constellations result. The results of the unconstrained optimization with $\tau = 0.21$ and $\lambda = 2$ are presented in Exhibit 4.

Exhibit 4 about here

Three BL-approaches achieve the highest Sharpe ratios. Of these, the introduced Black-Litterman approach $- \mu_{BL,NE} -$ leads to the highest Sharpe ratio with 27.03 % by far. No strategy attains a negative Sharpe ratio, this means that the realized portfolio return on average achieves at least the riskless interest rate.[17] According to the certainty equivalent, the He-Litterman BL-approach outperforms all other strategies and the historical implementation of Black-Litterman comes second. If the strategies are arranged on the basis of Jensen's alpha, the pure historical portfolio outperforms all other strategies and the pure dividend strategy is ranked second. However, only Jensen's alpha of the second strategy is significantly greater than zero at 1 % significance level. The respective Jensen's alpha for the pure dividend discount model and the He-Litterman implementation of the Black-Litterman model are greater than zero at 10 % significance level. According to the Treynor ratio, the portfolio with historical expected returns attains the greatest risk premium per accepted systematic risk. The BL-approach with the number of analysts' forecasts is again on rank two, above the pure dividend approach.

Exhibit 5 about here

Exhibit 5 shows the significances of the Sharpe (upper diagonal) and Treynor (lower diagonal) measure according to the test of Jobson and Korkie [1981].[18] As we can see, particularly measures of Black-Litterman approaches and the pure dividend discount model differ significantly from each other in comparison to measures of other strategies. Most likely, this is due to the fact that the first five strategies focus on the future, while the other strategies rely on historical data or neither past nor future data. Again Strategy 2 outperforms other strategies at a 5 % significance level.

Exhibit 6 about here

Exhibit 4 shows the performance of strategies for a specified parameter constellation. Of course, with modified parameter constellations the results could be different. For this reason, Exhibit 6 contains direct comparisons of how many times strategy A wins against strategy B for all possible 110 parameter constellations according to the Sharpe ratio. The table has to be read in the following way: the strategy of a certain row wins against the strategy in a certain column in x out of 110 parameter constellations. Thus the BL-approach with the number of analysts' forecasts wins in all parameter constellations against all other strategies. The BL-approach with a Monte-Carlo simulation is the second best approach, as it wins in more than half of all constellations against other strategies (except for strategy 2). In 59 of 110 parameter constellations, it achieves a better performance than the BL-approach according to He-Litterman. The pure historical strategy performs worst for all parameter constellations. The Bayesian and market portfolio follow directly. The performances of the nine strategies with long-only constraints for the former specified parameter constellation are presented in Exhibit 7.

Exhibit 7 about here

With short-sale restriction lower Sharpe ratios are achieved on average. As expected, all performance measures are distributed in a smaller interval. Our application of the BL-approach with number of analysts' estimates again outperforms all other strategies according to the Sharpe ratio. Two other Black-Litterman applications (strategy 3 and 4) come second and third. The historical implementation of the Black-Litterman model reaches the highest certainty equivalent. Regarding Jensen's alpha and the Treynor ratio again the pure historical strategy performs best. Interestingly, this strategy performs worst according to the certainty equivalent. As can be seen in Exhibit 8 there are not as many significant outperformances according to the Sharpe and Treynor ratio as without short sale constraints.

Exhibit 8 about here

Exhibit 9 shows comparisons of the strategies for all parameter constellations for the optimization with long-only constraints. Again, on average our application of the Black-Litterman model

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with the number of analysts' forecasts ($\mu_{BL,NE}$) performs best. In 100 of 110 constellations this strategy attains a higher Sharpe ratio than the historical BL-approach, which is the second best approach. All in all, the BL-approach with the historical variance-covariance matrix comes second and the BL-approach according to He-Litterman is ranked on position three. Thus, the good performance of the Black-Litterman approaches is confirmed for constrained optimization.

Exhibit 9 about here

CONCLUSION

For the application of the Black-Litterman model, own views about the expected returns of assets are necessary in order to deviate from market weights. If there are no views, the procedure does not provide an opportunity for active portfolio management. Furthermore, analysts do not make forecasts in the way they are required for the implementation of the Black-Litterman model. In this contribution, views for the Black-Litterman model are generated on the basis of a future-oriented valuation model – in our case the dividend discount model – with the help of analysts' forecasts. We examine four possibilities to compute expected returns with the Black-Litterman model. Two of these methods are described and examined for the first time. Confidences in specified views are determined firstly on the basis of number of analysts' forecasts and secondly by applying a Monte-Carlo simulation on the basis of distribution of analysts' forecasts. Thus, we contribute to the literature on a quantitative forecast model for the application of the Black-Litterman approach.

The effect of different views on portfolio weights was analyzed. In our out-of-sample analysis over a period of 132 months, performance measures of the described Black-Litterman approaches and several benchmark portfolios were calculated with real capital market data.

Our implementation of the Black-Litterman model based on the number of analysts' forecasts outperforms all other strategies regarding the Sharpe ratio, in both constrained and unconstrained case. Furthermore, all applications of Black-Litterman achieve good rankings in all performance measures. Thus, we recommend using the Black-Litterman model and give valuable advice on how to implement it.

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ENDNOTES

[1] See Fabozzi, Focardi, and Kolm [2006], p. 29.

[2] Cf. Black and Litterman [1992], p. 43.

[3] Strictly speaking a proxy for the market portfolio is chosen, as the market portfolio could not be reproduced. See Roll [1977]. For the selection of an adequate reference return, Black and Litterman also discuss historical expected returns of the individual securities, equal expected returns within the asset classes as well as risk adjusted expected returns. However, they arrive at the conclusion that market returns in contrast to the other strategies generate the most balanced and intuitive portfolios.

[4] See Fabozzi, Focardi, and Kolm [2006], pp. 29.

[5] See Elton [1999], Jorion [1986].

[6] See for instance Botosan and Plumlee [2005], Courteau, Kao, and Richardson [2001], Francis, Olsson, and Oswald [2000], Gebhardt, Lee, and Swaminathan [2001], Gode and Mohanram [2003], Easton [2004], Daske, Gebhardt, and Klein [2006].

[7] The data are from providers of financial data – in our case Thomson Reuters Datastream.

[8] See Botosan and Plumlee [2005], Gebhardt, Lee, and Swaminathan [2001].

[9] The maximum number is determined for each stock separately.

[10] For robustness, we also tested more simulation runs but the results were stable.

[11] The variance-covariance matrix is calculated on the basis of the last 36 points in time (three years) in every point in time. For the computation of the historical means 36 months are used, too.

[12] Although a historical variance-covariance matrix on the basis of the last 36 monthly returns could be calculated, the number of shares in the optimization exceeds 36. Thus, the resulting variance-covariance matrix would not be invertible and the optimization could not be applied. Furthermore the estimation error for the estimation of the variance-covariance matrix within the single index model is lower up to a time period of five years compared to the historical variance-covariance matrix. See Briner and Connor [2008], p. 12.

[13] If there is only one analyst forecast, the stock is not contained in the optimization, as there is no standard deviation of the estimations.

[14] For the exhibits 1 to 3 τ is set to 0.21 and λ to 2.

[15] The first optimization already proceeds on 01/01/1997. The optimized portfolios are kept fixed for one month and the first excess return is determined on 02/01/1997.

[16] Since some estimators require independent and identically distributed returns, we tested the returns for normality, stationarity, and autocorrelation. Returns which not satisfy the assumptions are removed from the database.

[17] The riskless interest rate is represented through the interest rate of government bonds with a maturity of one year. The data are available in the internet at http://www.bundesbank.de/statistik/-statistik_zinsen.php#geldmarkt.

[18] The test was rectified by Cadsby [1986] and Memmel [2003].

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EXHIBITS

Exhibit 1
Overview of Inputs for the Black-Litterman Approach with Number of Analysts' Forecasts
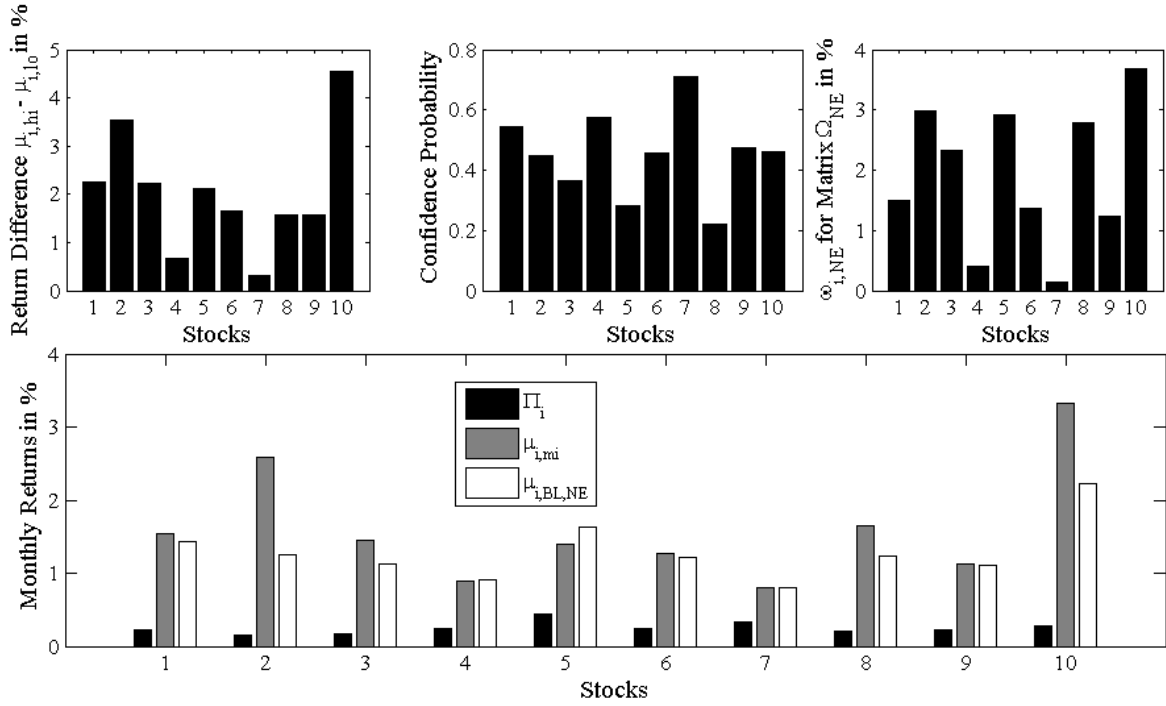

Exhibit 2
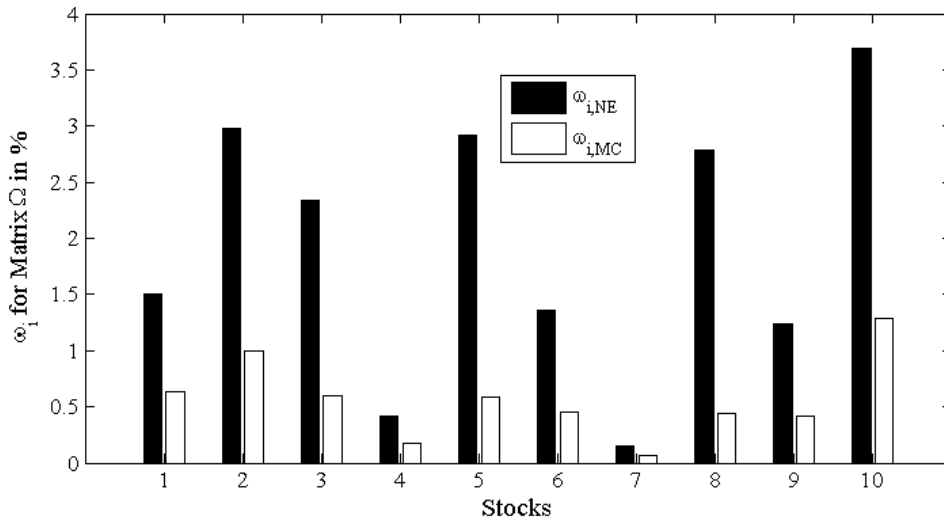Standard Deviations of Analysts' Views

Exhibit 3
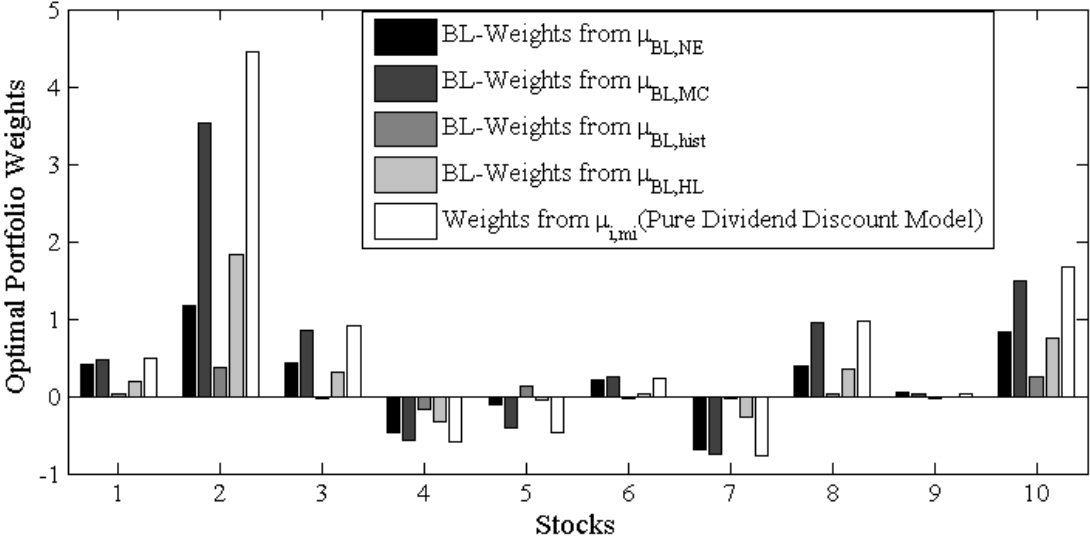Overview of Portfolio Weights of Different Implementations of Black-Litterman

Exhibit 4

Performances of the Strategies in the Unconstrained Optimization with $\tau=0.21$ and $\lambda=2$

#	Strategy	Sharpe-Ratio	Rank	Certainty Equivalent	Rank	Jensen's Alpha	Rank	Treynor-Ratio	Rank
1	Dividend Discount Model	0.2117	4	- 0.1642	8	0.0742 *	2	0.0282	3
2	BL: Number of Estimates	0.2703	1	- 0.0120	5	0.0625 ***	4	0.0350	2
3	BL: Historical	0.1411	6	0.0076	2	0.0037	7	0.0141	7
4	BL: He-Litterman	0.2203	3	0.0081	1	0.0243 *	5	0.0222	5
5	BL: Monte-Carlo	0.2209	2	- 0.1377	7	0.0695	3	0.0276	4
6	Historical	0.1031	9	- 1.6341	9	0.1107	1	0.0490	1
7	Bayes	0.1297	8	- 0.0746	6	0.0242	6	0.0203	6
8	Market	0.1402	7	0.0042	4	0.0000	9	0.0095	9
9	Equally Weighted	0.1742	5	0.0070	3	0.0026	8	0.0128	8

* 10 % significance, ** 5 % significance, *** 1 % significance

Exhibit 5

Significance of the Sharpe and Treynor Ratio in the Unconstrained Optimization with $\tau=0.21$ and $\lambda=2$

#	Strategy	1	2	3	4	5	6	7	8	9
1	Dividend Discount Model	-	*							
2	BL: Number of Estimates		-			*		**		
3	BL: Historical			-						
4	BL: He-Litterman		**		-					
5	BL: Monte-Carlo					-				
6	Historical						-			
7	Bayes							-		
8	Market	*	**		*	*			-	
9	Equally Weighted	*	**			*				-

* 10 % significance, ** 5 % significance, *** 1 % significance

Exhibit 6
Comparison of all Parameter Constellations in the Unconstrained Optimization

#	↓ wins against →	1	2	3	4	5	6	7	8	9
1	Dividend Discount Model	-	0	95	0	0	110	110	110	110
2	BL: Number of Estimates	110	-	110	110	110	110	110	110	110
3	BL: Historical	15	0	-	5	5	71	63	61	47
4	BL: He-Litterman	110	0	105	-	51	110	110	110	110
5	BL: Monte-Carlo	110	0	105	59	-	110	110	110	110
6	Historical	0	0	39	0	0	-	0	0	0
7	Bayes	0	0	47	0	0	110	-	0	0
8	Market	0	0	49	0	0	110	110	-	0
9	Equally Weighted	0	0	63	0	0	110	110	110	-

Exhibit 7

Performances of the Strategies in the Constrained Optimization with $\tau=0.21$ and $\lambda=2$

#	Strategy	Sharpe-Ratio	Rank	Certainty Equivalent	Rank	Jensen's Alpha	Rank	Treynor-Ratio	Rank
1	Dividend Discount Model	0.1658	7	- 0.0561	8	0.0614 *	2	0.0212	4
2	BL: Number of Estimates	0.2085	1	0.0010	5	0.0533 **	3	0.0226	2
3	BL: Historical	0.1951	2	0.0119	1	0.0251	6	0.0180	7
4	BL: He-Litterman	0.1784	3	0.0029	4	0.0408 *	5	0.0202	6
5	BL: Monte-Carlo	0.1716	6	- 0.0471	7	0.0056	7	0.0222	3
6	Historical	0.1756	4	- 0.2514	9	0.0934 *	1	0.0234	1
7	Bayes	0.0403	9	- 0.0418	6	0.0448	4	0.0206	5
8	Market	0.1402	8	0.0042	3	0.0000	9	0.0095	9
9	Equally Weighted	0.1742	5	0.0070	2	0.0026	8	0.0128	8

* 10 % significance, ** 5 % significance, *** 1 % significance

Exhibit 8

Significance of the Sharpe and Treynor Ratio in the Unconstrained Optimization with $\tau=0.21$ and $\lambda=2$

#	Strategy	1	2	3	4	5	6	7	8	9
1	Dividend Discount Model	-								
2	BL: Number of Estimates		-			*				
3	BL: Historical			-						
4	BL: He-Litterman				-					
5	BL: Monte-Carlo					-				
6	Historical						-			
7	Bayes							-		
8	Market		*	*					-	
9	Equally Weighted									-

* 10 % significance, ** 5 % significance, *** 1 % significance

Exhibit 9
Comparison of all Parameter Constellations in the Constrained Optimization

#	↓ wins against →	1	2	3	4	5	6	7	8	9
1	Dividend Discount Model	-	0	12	0	11	0	110	110	0
2	BL: Number of Estimates	110	-	100	110	110	110	110	110	110
3	BL: Historical	98	10	-	91	98	93	110	110	94
4	BL: He-Litterman	110	0	19	-	110	110	110	110	110
5	BL: Monte-Carlo	99	0	12	0	-	0	110	110	0
6	Historical	110	0	17	0	110	-	110	110	110
7	Bayes	0	0	0	0	0	0	-	0	0
8	Market	0	0	0	0	0	0	110	-	0
9	Equally Weighted	110	0	16	0	110	0	110	110	-