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Inequality aversion and externalities

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Inequality Aversion and Externalities*

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Abstract

We conduct a general analysis of the effects of inequality aversion on decisions by homogeneous players in static and dynamic games. We distinguish between direct and indirect effects of inequality aversion. Direct effects are present when a player changes his action to affect disutility caused by inequality. Indirect effects occur when the own action is changed to affect other players' actions. We provide necessary and sufficient conditions for the occurrence of either effect. Moreover, we examine the direction of the effects. Whereas indirect effects induce players to internalize externalities they impose on others, direct effects act in the opposite direction.

Keywords: inequality aversion, externalities, direct effects, indirect effects
JEL classification: C72, D62, D63

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1 Introduction

In response to a vast amount of experimental and empirical evidence, economists have concluded that people are not only driven by selfish motives, but also care for the well-being of others. Formal theories have been developed to model such other-regarding preferences, with the theory of inequality aversion perhaps the most well known (Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000). According to this theory, players compare their payoffs with those of other players and dislike payoff inequality.

With a formal theory of inequality aversion at hand, research has started to examine how players' behavior in economic settings is affected by inequality aversion. However, attention has typically been focused on specific model types and analysis has been conducted on a case-by-case basis. For instance, Fehr and Schmidt (1999) analyze inequality-averse players' behavior in a variety of simple games (like the ultimatum game). Grund and Sliwka (2005) consider a tournament model with inequality-averse contestants. A dynamic team production model with inequality-averse players is dealt with in Mohnen et al. (2008).[1] Neilson and Stowe (2010) consider a model in which two inequality-averse agents are hired by a principal and offered piece-rate contracts.[2]

While the studies have provided important insights into the implications of inequality aversion in specific situations, it is still unclear under which circumstances inequality aversion affects players' behavior in general and in which direction the corresponding effects act. To deal with these issues, a general treatment of inequality aversion and its impact on players' behavior is necessary. The current paper provides such a general treatment by analyzing the effects caused by inequality aversion in static and dynamic games with homogeneous players.

Inequality aversion can have direct and indirect effects on a player's behavior. We say

[1] See also Rey-Biel (2008).

[2] Neilson and Stowe (2010) assume that each agent cares for the payoff of the other agent, but not for the principal's payoff. This kind of behavior is denoted as horizontal inequality aversion. Other models on horizontal inequality aversion include Kragl and Schmid (2009), Bartling and von Siemens (2010), Ederer and Patacconi (2010), von Siemens (2011), and Bartling (2011). For models on vertical inequality aversion between principal and agent see Itoh (2004), Dur and Glazer (2008), and Englmaier and Wambach (2010).

that there are *direct effects* of inequality aversion if an inequality-averse player behaves differently to a selfish one to *reduce the disutility he suffers from inequality.* We refer to *indirect effects* of inequality aversion if a player changes his decision to *affect another player's action.* Against this background the paper addresses two problems. First, we determine under which circumstances direct and indirect effects of inequality aversion are present. Second, we analyze the direction in which these effects act.

To isolate the single effects, we begin by considering a static model with simultaneous actions in which, by definition, inequality aversion can have only direct effects (because a player cannot react to the observation of a specific action by another player). We provide a necessary and sufficient condition for the occurrence of such direct effects. We then turn to a dynamic model in which the same game as before is played twice. We modify the model assumptions such that direct effects of inequality aversion are eliminated (taking into account the condition derived before) and only indirect effects can play a role. Again, we provide a necessary and a sufficient condition for the presence of such effects. In both models, the conditions indicate that inequality aversion can only affect behavior if there are externalities, i.e., if a player's action has an effect on another player's payoff. Interestingly, we find that direct and indirect effects act in opposite directions. Whereas indirect effects induce players to internalize the externalities they impose on others, direct effects cause players to reduce their actions when there are positive externalities and to increase them when externalities are negative. The reason is as follows. Direct effects aim at reducing disutility caused by inequality. Since players suffer more strongly from disadvantageous than from advantageous inequality (according to the theory of inequality aversion), players aspire to be better off than others. They achieve this aim by reducing actions that benefit others and increasing actions that lead to negative externalities. Indirect effects, by contrast, aim at affecting the actions of other players later in the game. If a player internalizes the externality imposed on others in the first period, he creates inequality by increasing other players' payoff in the first period. In doing so, he induces other players to change their second-period actions to reduce the inequality generated in the first period. This change in other players' actions is beneficial to the considered player.

At the beginning of this section, we have outlined several models that have analyzed the effects of inequality aversion on players' actions in specific economic settings. The current model is able to replicate all the effects that have been highlighted in these papers. Fehr and Schmidt (1999) as well as Mohnen et al. (2008) consider dynamic models in which players impose positive externalities on each other. Fehr and Schmidt (1999) find that proposers in the ultimatum game offer a higher amount of money to receivers, whereas Mohnen et al. (2008) show that team members increase their effort above the level that a selfish person would choose. Both observations are reminiscent of our finding that players internalize externalities they impose on others because of indirect effects of inequality aversion.[3] Grund and Sliwka (2005) find that inequality-averse participants in a (static) tournament choose higher effort than selfish ones do. This reflects our finding on direct effects of inequality aversion that induce players to increase their actions when externalities are negative. Neilson and Stowe (2010) find that decisions of selfish and inequality-averse agents do not differ if agents are offered a piece-rate contract and use their total payoff (i.e., wage payment minus effort costs) to determine inequality costs. This is in line with the current paper's finding that positive or negative externalities are necessary to observe any effects of inequality aversion. To sum up, one virtue of our paper is that it can reproduce the findings of many previous papers in a single model. The results of the current paper, however, go beyond that of the previous literature, because it allows for all kinds of externalities and both, direct and indirect effects of inequality aversion. It is therefore able to predict behavior in a variety of economic situations that have not been analyzed before. To give an example, consider two employees who work together in a firm for two periods and who receive compensation which depends on relative performance, i.e., an employee's compensation increases in his own performance, but decreases in the performance of the other employee. In the example, we have a dynamic

[3] Note that the ultimatum game is a sequential game, whereas we consider a repeated game. While the structure of the two kinds of games is thus different, the effects of inequality aversion on players' behavior are very similar.

situation with negative externalities and expect inequality-averse employees to lower their actions (e.g., efforts) and hence their performance because of the indirect effects of inequality aversion. Such kind of behavior can be interpreted as a collusion against the firm owner, a phenomenon that has been outlined as one of the main problems of relative-performance evaluation schemes,[4] but that can be explained with selfish players only in the context of an infinitely repeated game. A final virtue of the paper is the relatively general technology (i.e., the functional forms) that we consider. In this respect, previous studies – which often assume specific functional forms – are generalized as well.

The remainder of the paper is organized as follows. Section 2 presents the static model, while Section 3 contains the dynamic model. Section 4 concludes the paper. All proofs are relegated to the Appendix.

2 The static model

2.1 Model description and notation

Consider a situation with two players ($i = 1, 2$), each of whom chooses some action $x_i \in [x_l, x_h] \subset \mathbb{R}$ ($x_h > x_l$) to produce some output $f(x_i)$ which accrues to player i himself. $f : [x_l, x_h] \to \mathbb{R}$ is an increasing and concave C^2 function. By choosing action x_i, player i also affects the payoff of player j ($j = 1, 2,\ j \neq i$). In particular, the payoff of player j is reduced by $e(x_i)$. $e : [x_l, x_h] \to \mathbb{R}$ is assumed to be linear and $e' := e'(x)$ denotes the constant first derivative of e. Depending on whether $e' > 0$, $e' = 0$, or $e' < 0$, players impose negative externalities, no externality or positive externalities on each other. Furthermore, choice of action x_i entails a cost for player i that (in monetary terms) is described by $c(x_i)$. $c : [x_l, x_h] \to \mathbb{R}$ is an increasing and convex C^2 function. We assume that $f(x_i) - c(x_i)$ is strictly concave, i.e., f is strictly concave or c is strictly convex. Finally, the payoff of player i is also affected by an individual random term, denoted by $\varepsilon_i \in [\varepsilon_l, \varepsilon_h] \subset \mathbb{R}$ ($\varepsilon_h > \varepsilon_l$), where ε_1 and ε_2 are identically distributed. To sum up, a player's payoff is given

[4] See Dye (1984).

by $u_i = f(x_i) - e(x_j) - c(x_i) + \varepsilon_i$. Moreover, a player's utility is $v_i = u_i - \alpha G(u_j - u_i)$. The function $G(\cdot)$ accounts for a player's disutility from inequality.[5] Following Englmaier and Wambach (2010), we suppose that G is a C^2 function satisfying $G(0) = 0$ and $sgn(G'(u)) = sgn(u)$, as well as $G''(u) > 0$ for all u. The parameter $\alpha \geq 0$ measures the strength of the inequality aversion of the players. If $\alpha = 0$, we have the standard model in which relative comparisons do not matter. For $\alpha > 0$, the players are inequality averse. Previous studies have argued that players suffer more strongly from disadvantageous than from advantageous inequality. To account for this, we assume $G(u) > G(-u)$ and $G'(u) > |G'(-u)|$ for all $u > 0$. Furthermore, we assume $\alpha G'(u) > -1$ for all u, which characterizes the fact that a player never suffers from advantageous inequality to such an extent that he wants to "burn money" to make the situation more even.

We search for a Nash equilibrium of the game and assume that the players choose their actions to maximize expected utility. In this respect, we presume that the model parameters are such that an interior solution to the players' maximization problems always exists.

Before we turn to the model solution, we briefly address the model with selfish players (i.e., $\alpha = 0$) as a benchmark case. Here the optimal action x_s maximizes a player's expected payoff. Because of our assumptions regarding the functions, we can use a first-order approach to characterize the solution; hence, we have $f'(x_s) = c'(x_s)$. Note that x_s is independent of e. Since a selfish player does not care about the payoff of the other player, he does not internalize the externality that he imposes on that player.

[5] Note that our specification of disutility from inequality differs somewhat from the original specification in Fehr and Schmidt (1999), who assume it to be piecewise linear. By considering a disutility function that is twice continuously differentiable, we do not need to introduce messy case distinctions and are able to characterize optimal actions just by the respective first-order conditions.

2.2 Solution to the model

We define $H(x) := f(x) + e(x) - c(x)$ and $\varepsilon_{ji} := \varepsilon_j - \varepsilon_i \in [\varepsilon_l - \varepsilon_h, \varepsilon_h - \varepsilon_l] \subset \mathbb{R}$ and denote the expectation operator by $E[\cdot]$. Then expected utility of player i can be written as

$$E[v_i] = f(x_i) - e(x_j) - c(x_i) + E[\varepsilon_i] - \alpha E[G(H(x_j) - H(x_i) + \varepsilon_{ji})].$$

The optimal actions are chosen such that expected utility is maximized (while taking the other player's equilibrium action into account). Under the assumptions imposed, we can again use a first-order approach to characterize the optimal solution. This means that optimal actions are characterized by the following pair of first-order conditions.

$$\begin{aligned} f'(x_1^*) - c'(x_1^*) + \alpha H'(x_1^*) E[G'(H(x_2^*) - H(x_1^*) + \varepsilon_{21})] &= 0, \\ f'(x_2^*) - c'(x_2^*) + \alpha H'(x_2^*) E[G'(H(x_1^*) - H(x_2^*) + \varepsilon_{12})] &= 0, \end{aligned}$$

or, alternatively,

$$H'(x_1^*)(1 + \alpha E[G'(H(x_2^*) - H(x_1^*) + \varepsilon_{21})]) = e', \tag{1}$$

$$H'(x_2^*)(1 + \alpha E[G'(H(x_1^*) - H(x_2^*) + \varepsilon_{12})]) = e'. \tag{2}$$

The next lemma shows that there is no asymmetric equilibrium.

Lemma 1 *In equilibrium we have* $x_1^* = x_2^* =: x^*$.

Because equilibrium is symmetric, the optimality conditions can be significantly simplified:

$$H'(x^*)(1 + \alpha E[G'(\varepsilon_{ji})]) = e'. \tag{3}$$

Our first objective in this section is to analyze whether inequality aversion has direct effects on players' decisions (recall that indirect effects of inequality aversion cannot arise in the static model since players are not able to react to the observation of a specific action of the co-player). The following proposition gives a sufficient and necessary condition for inequality aversion not to affect decisions. In this context, $P[\cdot]$ denotes the probability operator.

Proposition 1 *Let* $\alpha > 0$*. Then* $x^* = x_s$ *if and only if* ε_{ji} *is degenerate (i.e.,* $P[\varepsilon_{ji} = 0] = 1$*) or there are no externalities (i.e.,* $e' = 0$*).*

As noted above, an inequality-averse player suffers more strongly from disadvantageous than from advantageous inequality. His optimal action is thus determined by a trade-off between two effects. On one hand, he wants to maximize his payoff; on the other hand, he wants to reduce the probability of suffering disadvantageous inequality. Since the second effect is absent for selfish players, selfish and inequality-averse players typically choose different actions. As Proposition 1 indicates, however, there are two exceptions. First, when there are no externalities, the action that maximizes a player's payoff also minimizes his risk of receiving a lower payoff than that of the other player. Then there is no trade-off between the two effects and an inequality-averse player chooses x_s as well. Second, if there is no uncertainty in the sense that $P[\varepsilon_{ji} = 0] = 1$, then there is no inequality in equilibrium. In this case, the second effect disappears and again inequality-averse players choose the payoff-maximizing action.

Although there are only two situations in which inequality-averse players and selfish players make the same decisions, it is not hard to find real-world examples that match these situations. For instance, consider two employees who work together in a firm and suppose that each employee chooses some effort to produce output that accrues to the firm. Typically, there are three possible types of compensation. Employees could be paid individually, on the basis of team output or on the basis of relative performance. If they are paid individually (i.e., on the basis of their own output only), then there are obviously no externalities. If pay depends on aggregate output only, employees receive the same compensation and there is no inequality in equilibrium (as long as both employees receive the same wage contract). Finally, consider relative performance pay in the sense that an employee's compensation increases with his own output, but decreases as a function of the other employee's output. Moreover, let ε_1 and ε_2 be perfectly positively correlated. Then in equilibrium there is again no inequality. In all these cases, the conditions of Proposition 1 apply and optimal actions of inequality-averse employees and selfish employees are the same.

Having seen the circumstances under which inequality aversion has direct effects on players' behavior, we now analyze the direction in which these effects act.

Proposition 2 *Let $\alpha > 0$ and $P[\varepsilon_{ji} = 0] < 1$. Then $sgn(x^* - x_s) = sgn(e')$.*

As Proposition 2 indicates, direct effects of inequality aversion do not induce players to internalize externalities to a greater degree than that of selfish players. On the contrary, if there are negative externalities (i.e., $e' > 0$), inequality-averse players choose even higher actions than selfish players, while they choose lower actions in the case of positive externalities. This is because inequality-averse players want to reduce the risk of suffering from disadvantageous inequality. Accordingly, they decide to "hurt" the co-player more strongly than a selfish player would want to do.

3 The dynamic model

In this section, we consider a model in which the same game as in the previous section is played twice. To focus on the indirect effects of inequality aversion, we eliminate direct effects by assuming that payoffs are deterministic (Proposition 1). We index the period by $t = 1, 2$ and denote actions by x_{it}, payoffs by $u_{it} = f(x_{it}) - e(x_{jt}) - c(x_{it})$ and overall utility by $v_i = u_{i1} + u_{i2} - \alpha G(u_{j1} + u_{j2} - u_{i1} - u_{i2})$.[6] All other assumptions are the same as in the previous section. We use subgame-perfect equilibrium as the solution concept.

3.1 The second period

The model is solved by backward induction and we start by analyzing actions in period 2. We define $\Delta U := u_{11} - u_{21}$. Derivation of the optimality conditions is very similar to the analysis

[6] Implicit in this specification is that players compare their total payoffs and each player dislikes obtaining a different total payoff than the coplayer. Note that the qualitative model results hold under alternative assumptions as well. All that is required is missing additive separability of overall utility in $u_{i1} - u_{j1}$ and $u_{i2} - u_{j2}$ which in turn implies interrelation across periods. See also Oechssler (2011).

in Section 2 and the optimal second-period actions satisfy the following two conditions:

$$H'\left(x_{12}^*\right)\left[1+\alpha G'\left(H\left(x_{22}^*\right)-H\left(x_{12}^*\right)-\Delta U\right)\right] = e', \tag{4}$$

$$H'\left(x_{22}^*\right)\left[1+\alpha G'\left(H\left(x_{12}^*\right)-H\left(x_{22}^*\right)+\Delta U\right)\right] = e'. \tag{5}$$

Inspection of these conditions yields Lemma 2.

Lemma 2 *Let* $\alpha > 0$.

$$(i) \quad sgn\left(H\left(x_{22}^*\right)-H\left(x_{12}^*\right)-\Delta U\right) = sgn\left(H\left(x_{12}^*\right)-H\left(x_{22}^*\right)\right).$$

$$(ii) \quad \Delta U > 0 \Rightarrow H\left(x_{22}^*\right)-H\left(x_{12}^*\right) \in (0, \Delta U),$$

$$\Delta U = 0 \Rightarrow H\left(x_{22}^*\right)-H\left(x_{12}^*\right) = 0,$$

$$\Delta U < 0 \Rightarrow H\left(x_{22}^*\right)-H\left(x_{12}^*\right) \in (\Delta U, 0).$$

$$(iii) \quad x_{11}^* = x_{21}^* \Rightarrow x_{12}^* = x_{22}^*.$$

If there is inequality in the first period in the sense that $u_{11} \neq u_{21}$, Lemma 2 indicates how the players deal with this inequality in the second period. In particular, it shows that the player suffering from disadvantageous inequality after the first period chooses a different action in the second period than his co-player. In this way, the initial inequality is reduced but it does not completely disappear. These observations are important for the analysis of the first period, because they imply that first-period and second-period actions are interconnected. As a result, there may be indirect effects of inequality aversion and a player may want to change his first-period action to affect the other player's second-period action in a favorable way.

3.2 The first period

According to (4) and (5) the optimal second-period actions depend on ΔU and thus on x_{11} and x_{21}, respectively. This dependence is analyzed in more detail in the following. Using the definitions

$$\Phi_1(x_{11},x_{21},x_{12},x_{22}) := \quad H'(x_{12})\left[1+\alpha G'(H(x_{22})-H(x_{12})-(H(x_{11})-H(x_{21})))\right]-e'$$
$$\Phi_2(x_{11},x_{21},x_{12},x_{22}) := \quad H'(x_{22})\left[1+\alpha G'(H(x_{12})-H(x_{22})+(H(x_{11})-H(x_{21})))\right]-e'$$

and considering $\Delta U = H(x_{11}^*) - H(x_{21}^*)$ in the case of optimal first-period actions, (4) and (5) immediately lead to:

$$\Phi_1(x_{11}^*,x_{21}^*,x_{12}^*,x_{22}^*) = \Phi_2(x_{11}^*,x_{21}^*,x_{12}^*,x_{22}^*) = 0.$$

Note that (Φ_1,Φ_2) is a C^1 function. To analyze player reactions in the second period we use the implicit-function theorem, for which the essential requirement is examined in the following lemma.

Lemma 3

$$\det\begin{pmatrix} \frac{\partial\Phi_1}{\partial x_{12}} & \frac{\partial\Phi_1}{\partial x_{22}} \\ \frac{\partial\Phi_2}{\partial x_{12}} & \frac{\partial\Phi_2}{\partial x_{22}} \end{pmatrix}\Bigg|_{\left(x_{11}^*,x_{21}^*,x_{12}^*,x_{22}^*\right)} > 0.$$

Using this lemma, we can apply the implicit-function theorem, according to which there exist C^1 functions x_{12}^* and x_{22}^* defined in a neighborhood of (x_{11}^*,x_{21}^*) so that for all (x_{11},x_{21}) in this neighborhood we have $\Phi_i(x_{11},x_{21},x_{12}^*(x_{11},x_{21}),x_{22}^*(x_{11},x_{21}))=0$ and

$$\begin{pmatrix} \frac{\partial x_{12}^*}{\partial x_{11}} & \frac{\partial x_{12}^*}{\partial x_{21}} \\ \frac{\partial x_{22}^*}{\partial x_{11}} & \frac{\partial x_{22}^*}{\partial x_{21}} \end{pmatrix} = -\begin{pmatrix} \frac{\partial\Phi_1}{\partial x_{12}} & \frac{\partial\Phi_1}{\partial x_{22}} \\ \frac{\partial\Phi_2}{\partial x_{12}} & \frac{\partial\Phi_2}{\partial x_{22}} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial\Phi_1}{\partial x_{11}} & \frac{\partial\Phi_1}{\partial x_{21}} \\ \frac{\partial\Phi_2}{\partial x_{11}} & \frac{\partial\Phi_2}{\partial x_{21}} \end{pmatrix}. \tag{6}$$

Against this background we can analyze the optimal behavior of the players in the first period. On the basis of the first-period action (x_{11},x_{21}) of the two players and using the abbreviation $x_{i2}^* = x_{i2}^*(x_{11},x_{21})$, the overall utility of player i can be written as:

$$\begin{aligned} v_i(x_{11},x_{21}) &= H(x_{i1}) + H(x_{i2}^*) - (e(x_{i1}) + e(x_{i2}^*)) \\ &\quad - \left(e(x_{j1}) + e\left(x_{j2}^*\right)\right) - \alpha G\left(H\left(x_{j2}^*\right) - H(x_{i2}^*) + H(x_{j1}) - H(x_{i1})\right). \end{aligned}$$

Differentiating v_i with respect to x_{i1} and considering (4) and (5), we obtain:

$$\begin{aligned}
&\frac{dv_i\left(x_{11}, x_{21}\right)}{dx_{i1}} \\
&= H'\left(x_{i1}\right) + H'\left(x_{i2}^*\right)\frac{\partial x_{i2}^*}{\partial x_{i1}} - e' - e'\frac{\partial x_{i2}^*}{\partial x_{i1}} - e'\frac{\partial x_{j2}^*}{\partial x_{i1}} \\
&\quad -\alpha G'\left(H\left(x_{j2}^*\right) - H\left(x_{i2}^*\right) + H\left(x_{j1}\right) - H\left(x_{i1}\right)\right) \\
&\quad \cdot \left(H'\left(x_{j2}^*\right)\frac{\partial x_{j2}^*}{\partial x_{i1}} - H'\left(x_{i2}^*\right)\frac{\partial x_{i2}^*}{\partial x_{i1}} - H'\left(x_{i1}\right)\right) \\
&= H'\left(x_{i1}\right)\left[1 + \alpha G'\left(H\left(x_{j2}^*\right) - H\left(x_{i2}^*\right) + H\left(x_{j1}\right) - H\left(x_{i1}\right)\right)\right] - e' \\
&\quad -\frac{\partial x_{j2}^*}{\partial x_{i1}} \cdot \left[e' + H'\left(x_{j2}^*\right)\alpha G'\left(H\left(x_{j2}^*\right) - H\left(x_{i2}^*\right) + H\left(x_{j1}\right) - H\left(x_{i1}\right)\right)\right]. \qquad (7)
\end{aligned}$$

Again, a symmetric equilibrium in which both players choose the same action always exists. Without imposing further structure on the model, however, it is not possible to rule out asymmetric equilibria. The following lemma gives a sufficient condition under which asymmetric equilibria do not exist.

Lemma 4 *Let $H'''(x)\,H'(x) \geq 2\left(H''(x)\right)^2$ for all $x \in [x_l, x_h]$. Then, in equilibrium we have $x_{11}^* = x_{21}^* =: x^{1*}$ and $x_{12}^* = x_{22}^* =: x^{2*}$.*

For example, the functions $f(x) = ln(x)$, $c(x) = x^2$ and $e(x) = 10x$ (i.e., $H(x) = ln(x) - x^2 + 10x$) fulfill the condition specified in Lemma 4 with $x_l > 0$, $x_l \to 0$, and $x_h = 1$. Referring to the lemma, we focus on symmetric equilibria in the following. A symmetric equilibrium is characterized by the condition $\partial v_i / \partial x_{i1}\left(x^{1*}, x^{1*}\right) = 0$. Considering

$$G'\left(H\left(x_{j2}^*\left(x^{1*}, x^{1*}\right)\right) - H\left(x_{i2}^*\left(x^{1*}, x^{1*}\right)\right) + H\left(x^{1*}\right) - H\left(x^{1*}\right)\right) = G'(0) = 0$$

and according to (7) condition $\partial v_i / \partial x_{i1}\left(x^{1*}, x^{1*}\right) = 0$ simplifies to

$$H'\left(x^{1*}\right) - e' - \frac{\partial x_{j2}^*}{\partial x_{i1}}\left(x^{1*}, x^{1*}\right)e' = 0. \qquad (8)$$

Condition (8) helps us to determine under which circumstances inequality aversion has indirect effects on players' behavior.

Proposition 3 *Let $\alpha > 0$. Then we have $x^{1*} = x_s$ if and only if there are no externalities (i.e., $e' = 0$). It is always the case that $x^{2*} = x_s$.*

Proposition 3 states that indirect effects of inequality aversion are present even if output is deterministic and there is no inequality in equilibrium. Indirect effects of inequality aversion result from the interrelation of actions across periods. Because of this interrelation, a player changes his first-period action to affect the other player's second-period action in a favorable manner. Note, however, that indirect effects disappear when there are no externalities because a player has no effect on the other player's payoff in this case. Hence, a player is not interested in changing his co-player's action.

In the following we investigate in which direction the indirect effects of inequality aversion act.

Proposition 4 *Let* $\alpha > 0$*. Then* $sgn(x_s - x_1^*) = sgn(e')$*.*

From Proposition 4 we observe that indirect effects of inequality aversion act in the opposite direction compared to direct effects. Because of indirect effects, a player internalizes the externality he imposes on the other player; in other words, a player increases his action in the case of positive externalities, while he decreases his action otherwise. The aim of this behavior is to create inequality by increasing the other player's payoff in the first period. This induces the other player to change his second-period action to increase the payoff of the first player and to reduce the inequality generated in the first period.

Note that in equilibrium both players have the same incentive to change the first-period action and inequality actually does not occur. Still, a player would not want to deviate to x_s because he would then be "punished" by his co-player in the second period. In this respect, the result is similar to the main result in the career-concerns model of Holmström (1999). In this model, players change their actions to pretend to have certain characteristics (e.g., high ability). The market, however, anticipates players' behavior and correctly infers their characteristics. Although players cannot fool the market in equilibrium, they do not want to deviate from equilibrium actions because they would then affect market perceptions of their characteristics in an unfavorable way.

4 Conclusion

The current paper has provided a general analysis of direct and indirect effects caused by inequality aversion on decisions by homogeneous players in static and dynamic games. We determined necessary and sufficient conditions for which direct and indirect effects play a role. Moreover, we observed that direct and indirect effects act in opposite directions when they are present. Indirect effects cause players to internalize the externalities they impose on others, while direct effects cause players to decrease their actions when there are positive externalities and to increase them when externalities are negative.

A possible extension of the analysis would be to consider heterogeneous players. If heterogeneity is introduced, there is typically inequality in equilibrium. Therefore, there are fewer situations in which inequality aversion does not affect behavior. For instance, even if output is deterministic (and there are no direct effects of inequality aversion when players are homogeneous), heterogeneous players would adapt their decisions to take the inequality generated by their different characteristics into account. While the introduction of heterogeneity thus leads to additional effects, the direct and indirect effects we have highlighted in this paper should continue to be of importance.

Appendix

Proof of Lemma 1. Obviously, the left-hand sides (LHS) of (1) and (2) are the same. Assume first $e' \geq 0$ implying $H'(x_i^*) \geq 0$ since $1 + \alpha G' > 0$ by assumption. Because of the strict concavity of H, we have $H'(x) > H'(x_1^*) \geq 0$ for all $x < x_1^*$. Consequently, $x_1^* > x_2^*$ immediately leads to $H'(x_1^*) < H'(x_2^*)$ and $H(x_1^*) > H(x_2^*)$. This in turn implies

$$E[G'(H(x_2^*) - H(x_1^*) + \varepsilon_{21})] < E[G'(H(x_1^*) - H(x_2^*) + \varepsilon_{12})]$$

and thus the LHS of (1) to be strictly smaller than the LHS of (2) which contradicts the optimality of x_1^* and x_2^*. An analogous argument shows that we cannot have $x_1^* < x_2^*$.

Assume second $e' < 0$ which is equivalent to $H'(x_i^*) < 0$. In this case $x_1^* > x_2^*$ leads to $H'(x_1^*) < H'(x_2^*) < 0$ and $H(x_1^*) < H(x_2^*)$. It follows that

$$E[G'(H(x_2^*) - H(x_1^*) + \varepsilon_{21})] > E[G'(H(x_1^*) - H(x_2^*) + \varepsilon_{12})]$$

implying the LHS of (1) to be strictly smaller than the LHS of (2) which again contradicts the optimality of x_1^* and x_2^*. Analogously, it can be shown that $x_1^* < x_2^*$ leads to a contradiction.
■

Proof of Proposition 1. (i) Suppose $x^* = x_s$ which implies $f'(x^*) = c'(x^*)$ and condition (3) becomes

$$e'(1 + \alpha E[G'(\varepsilon_{ji})]) = e' \quad \Leftrightarrow \quad e'\alpha E[G'(\varepsilon_{ji})] = 0.$$

This condition can only be fulfilled if $e' = 0$ or $E[G'(\varepsilon_{ji})] = 0$. The latter condition can be transformed as follows:

$$0 = E[G'(\varepsilon_{ji})] = P[\varepsilon_{ji} > 0]E[G'(\varepsilon_{ji})|\varepsilon_{ji} > 0] + P[\varepsilon_{ji} < 0]E[G'(\varepsilon_{ji})|\varepsilon_{ji} < 0].$$

Because ε_{ji} is symmetrically distributed around zero and $G'(|\varepsilon_{ji}|) > |G'(-|\varepsilon_{ji}|)|$ for $\varepsilon_{ji} \neq 0$, the latter equation immediately leads to $P[\varepsilon_{ji} > 0] = P[\varepsilon_{ji} < 0] = 0$.

(ii) The assumption $P[\varepsilon_{ji} = 0] = 1$ corresponds to $P[\varepsilon_{ji} > 0] = P[\varepsilon_{ji} < 0] = 0$ and (under consideration of the last equation in the first part of the proof) leads to $E[G'(\varepsilon_{ji})] = 0$.

Against this background condition (3) becomes $H'(x^*) = e' \Leftrightarrow f'(x^*) - c'(x^*) = 0$, hence $x^* = x_s$.

Under the assumption of $e' = 0$ condition (3) becomes

$$(f'(x^*) - c'(x^*))(1 + \alpha E[G'(\varepsilon_{ji})]) = 0 \quad \Leftrightarrow \quad f'(x^*) - c'(x^*) = 0 \quad \Leftrightarrow \quad x^* = x_s$$

since $1 + \alpha G' > 0$. ■

Proof of Proposition 2. As already mentioned and according to (3) we have $sgn(H'(x^*)) = sgn(e')$. The assumptions on G imply that $\alpha E[G'(\varepsilon_{ji})] > 0$ if $P[\varepsilon_{ji} = 0] < 1$. Thus, (3) leads to $|H'(x^*)| < |e'| = |H'(x_s)|$. This in turn means that $0 < H'(x^*) < H'(x_s)$ if $e' > 0$ and $0 > H'(x^*) > H'(x_s)$ if $e' < 0$. Since $H'' < 0$ the statement of the proposition is proven. ■

Proof of Lemma 2. Conditions (4) and (5) indicate that $sgn(e') = sgn(H'(x_{i2}^*))$ since $1 + \alpha G' > 0$ by assumption. Note further that H is strictly concave. We also obtain from (4) and (5)

$$H'(x_{12}^*)[1 + \alpha G'(H(x_{22}^*) - H(x_{12}^*) - \Delta U)] = H'(x_{22}^*)[1 + \alpha G'(H(x_{12}^*) - H(x_{22}^*) + \Delta U)],$$

which is equivalent to

$$\begin{aligned} & \frac{H'(x_{22}^*) - H'(x_{12}^*)}{H'(x_{12}^*)} \\ = \; & \alpha \frac{G'(H(x_{22}^*) - H(x_{12}^*) - \Delta U) - G'(-(H(x_{22}^*) - H(x_{12}^*) - \Delta U))}{1 + \alpha G'(-(H(x_{22}^*) - H(x_{12}^*) - \Delta U))}. \end{aligned} \tag{9}$$

(i) In consequence of the properties of G, we immediately have

$$sgn(H(x_{22}^*) - H(x_{12}^*) - \Delta U) = sgn(G'(H(x_{22}^*) - H(x_{12}^*) - \Delta U)).$$

Condition (9) leads to

$$sgn(G'(H(x_{22}^*) - H(x_{12}^*) - \Delta U)) = sgn(H'(x_{22}^*) - H'(x_{12}^*))\, sgn(H'(x_{12}^*)).$$

Since $H'' < 0$, we also get

$$\begin{aligned} & sgn(H'(x_{22}^*) - H'(x_{12}^*))\, sgn(H'(x_{12}^*)) \\ = \; & sgn(x_{12}^* - x_{22}^*)\, sgn(H'(x_{12}^*)) = sgn(H(x_{12}^*) - H(x_{22}^*)) \end{aligned}$$

and thus the statement of part (i).

(ii) Let $\Delta U > 0$. Because of (i) we cannot have $H(x_{22}^*) - H(x_{12}^*) \geq \Delta U$. Hence, we must have $H(x_{22}^*) - H(x_{12}^*) < \Delta U$ and, again because of (i), $H(x_{12}^*) - H(x_{22}^*) < 0$. The proof is very similar for the cases $\Delta U = 0$ and $\Delta U < 0$ and therefore omitted.

(iii) From the second line of part (ii) identity $x_{11}^* = x_{21}^*$ immediately implies $H(x_{12}^*) = H(x_{22}^*)$. Since $sgn(H'(x_{12}^*)) = sgn(H'(x_{22}^*))$ and $H'' < 0$, function H is injective in the relevant range and consequently $x_{12}^* = x_{22}^*$. ■

Proof of Lemma 3. The determinant can be written as

$$\begin{aligned}
&\det \begin{pmatrix} \frac{\partial\Phi_1}{\partial x_{12}} \frac{\partial\Phi_1}{\partial x_{22}} \frac{\partial\Phi_2}{\partial x_{12}} \frac{\partial\Phi_2}{\partial x_{22}} \end{pmatrix}\Bigg|_{\left(x_{11}^*, x_{21}^*, x_{12}^*, x_{22}^*\right)} \\
= \;& [H''(x_{12}^*)\,[1 + \alpha G'(H(x_{22}^*) - H(x_{12}^*) - (H(x_{11}^*) - H(x_{21}^*)))] \\
& - (H'(x_{12}^*))^2\, \alpha G''(H(x_{22}^*) - H(x_{12}^*) - (H(x_{11}^*) - H(x_{21}^*)))] \\
& \cdot [H''(x_{22}^*)\,[1 + \alpha G'(H(x_{12}^*) - H(x_{22}^*) - (H(x_{21}^*) - H(x_{11}^*)))] \\
& - (H'(x_{22}^*))^2\, \alpha G''(H(x_{12}^*) - H(x_{22}^*) - (H(x_{21}^*) - H(x_{11}^*)))] \\
& - H'(x_{12}^*)\, H'(x_{22}^*)\, \alpha G''(H(x_{22}^*) - H(x_{12}^*) - (H(x_{11}^*) - H(x_{21}^*))) \\
& \cdot H'(x_{12}^*)\, H'(x_{22}^*)\, \alpha G''(H(x_{12}^*) - H(x_{22}^*) - (H(x_{21}^*) - H(x_{11}^*)))\,.
\end{aligned}$$

The expression can be simplified into

$$\begin{aligned}
& H''(x_{12}^*)\,[1 + \alpha G'(H(x_{22}^*) - H(x_{12}^*) - (H(x_{11}^*) - H(x_{21}^*)))] \\
& \cdot H''(x_{22}^*)\,[1 + \alpha G'(H(x_{12}^*) - H(x_{22}^*) - (H(x_{21}^*) - H(x_{11}^*)))] \\
& - H''(x_{12}^*)\,[1 + \alpha G'(H(x_{22}^*) - H(x_{12}^*) - (H(x_{11}^*) - H(x_{21}^*)))] \\
& \cdot\, (H'(x_{22}^*))^2\, \alpha G''(H(x_{12}^*) - H(x_{22}^*) - (H(x_{21}^*) - H(x_{11}^*))) \\
& - (H'(x_{12}^*))^2\, \alpha G''(H(x_{22}^*) - H(x_{12}^*) - (H(x_{11}^*) - H(x_{21}^*))) \\
& \cdot H''(x_{22}^*)\,[1 + \alpha G'(H(x_{12}^*) - H(x_{22}^*) - (H(x_{21}^*) - H(x_{11}^*)))]\,.
\end{aligned}$$

This expression is strictly positive since $H'' < 0$ and $1 + \alpha G', G'' > 0$. ■

Proof of Lemma 4. Because of Lemma 2 it suffices to show that $x_{11}^* = x_{21}^*$. Suppose that $(x_{11}^*, x_{12}^*, x_{21}^*, x_{22}^*)$ with $x_{11}^* \neq x_{21}^*$ is an equilibrium. Furthermore, assume without loss of generality that $H(x_{11}^*) < H(x_{21}^*)$. If player 2 deviates from x_{21}^*, we immediately have a contradiction against the equilibrium assumption. Thus, we assume player 2 not to deviate. Against this background we analyze a deviation of player 1 from x_{11}^* to x_{21}^* implying both players also to choose the same action in period 2, i.e., $x_{12}^* = x_{22}^* =: x^{2*}$. Note from (4) and (5) that x^{2*} is characterized by $H'(x^{2*}) = e'$ (since $G'(0) = 0$), hence $x^{2*} = x_s$. Consequently, the first player's utility changes by

$$\begin{aligned}
\Delta v_1 &:= v_1(x_{21}^*, x_s, x_{21}^*, x_s) - v_1(x_{11}^*, x_{12}^*, x_{21}^*, x_{22}^*) \\
&= [H(x_{21}^*) - 2e(x_{21}^*) + H(x_s) - 2e(x_s)] \\
&\quad -[H(x_{11}^*) - e(x_{11}^*) - e(x_{21}^*) + H(x_{12}^*) - e(x_{12}^*) - e(x_{22}^*) \\
&\quad -\alpha G(H(x_{21}^*) + H(x_{22}^*) - H(x_{11}^*) - H(x_{12}^*))] \\
&= e(x_{11}^*) - e(x_{21}^*) + H(x_{21}^*) + H(x_s) - 2e(x_s) - H(x_{11}^*) \\
&\quad -H(x_{12}^*) + e(x_{12}^*) + e(x_{22}^*) + \alpha G(H(x_{21}^*) + H(x_{22}^*) - H(x_{11}^*) - H(x_{12}^*)).
\end{aligned}$$

Since player 2 does not deviate from x_{21}^* to x_{11}^*, the corresponding second player's utility change is not allowed to be positive, i.e.

$$\begin{aligned}
\Delta v_2 &:= v_2(x_{11}^*, x_s, x_{11}^*, x_s) - v_2(x_{11}^*, x_{12}^*, x_{21}^*, x_{22}^*) \\
&= [H(x_{11}^*) - 2e(x_{11}^*) + H(x_s) - 2e(x_s)] \\
&\quad -[H(x_{21}^*) - e(x_{21}^*) - e(x_{11}^*) + H(x_{22}^*) - e(x_{22}^*) - e(x_{12}^*) \\
&\quad -\alpha G(H(x_{11}^*) + H(x_{12}^*) - H(x_{21}^*) - H(x_{22}^*))] \quad \leq 0 \\
\Leftrightarrow e(x_{11}^*) - e(x_{21}^*) &\geq H(x_{11}^*) + H(x_s) - 2e(x_s) - H(x_{21}^*) - H(x_{22}^*) \\
&\quad + e(x_{22}^*) + e(x_{12}^*) + \alpha G(H(x_{11}^*) + H(x_{12}^*) - H(x_{21}^*) - H(x_{22}^*)).
\end{aligned}$$

The latter inequality and $H(x_{21}^*) + H(x_{22}^*) - H(x_{11}^*) - H(x_{12}^*) > 0$ (according to Lemma 2) lead to

$$\begin{aligned}\Delta v_1 &> H\left(x_{11}^*\right)+H\left(x_s\right)-2e\left(x_s\right)-H\left(x_{21}^*\right)-H\left(x_{22}^*\right)+e\left(x_{22}^*\right)+e\left(x_{12}^*\right)\\ &\quad +H\left(x_{21}^*\right)+H\left(x_s\right)-2e\left(x_s\right)-H\left(x_{11}^*\right)-H\left(x_{12}^*\right)+e\left(x_{12}^*\right)+e\left(x_{22}^*\right)\\ &= 2\left(f\left(x_s\right)-c\left(x_s\right)\right)-\left(f\left(x_{12}^*\right)-c\left(x_{12}^*\right)\right)-\left(f\left(x_{22}^*\right)-c\left(x_{22}^*\right)\right)\\ &\quad +e\left(x_{12}^*\right)+e\left(x_{22}^*\right)-2e\left(x_s\right).\end{aligned}$$

Since x_s maximizes $f(x)-c(x)$, the deviation of player 1 would be profitable if $e\left(x_{12}^*\right)-e\left(x_s\right)\geq e\left(x_s\right)-e\left(x_{22}^*\right)$.

We now demonstrate that the latter condition is fulfilled if $H'''(x)\,H'(x)\geq 2\left(H''(x)\right)^2$. $H\left(x_{21}^*\right)+H\left(x_{22}^*\right)-H\left(x_{11}^*\right)-H\left(x_{12}^*\right)>0$ implies $G'\left(H\left(x_{21}^*\right)+H\left(x_{22}^*\right)-H\left(x_{11}^*\right)-H\left(x_{12}^*\right)\right)>0$ and $G'\left(-H\left(x_{21}^*\right)-H\left(x_{22}^*\right)+H\left(x_{11}^*\right)+H\left(x_{12}^*\right)\right)<0$. We first deal with the case $e'>0$. Then from (4), (5), and the definition of x_s as well as $H''<0$ it follows

$$0<H'\left(x_{12}^*\right)<H'\left(x_s\right)<H'\left(x_{22}^*\right)\quad\Leftrightarrow\quad x_{12}^*>x_s>x_{22}^*.$$

Equations (4) and (5) also imply

$$\begin{aligned}&\frac{1}{H'\left(x_{12}^*\right)}-\frac{1}{H'\left(x_s\right)}\\ &= \frac{\alpha}{e'}G'\left(H\left(x_{21}^*\right)+H\left(x_{22}^*\right)-H\left(x_{11}^*\right)-H\left(x_{12}^*\right)\right)\\ &> -\frac{\alpha}{e'}G'\left(-H\left(x_{21}^*\right)-H\left(x_{22}^*\right)+H\left(x_{11}^*\right)+H\left(x_{12}^*\right)\right)=\frac{1}{H'\left(x_s\right)}-\frac{1}{H'\left(x_{22}^*\right)}.\end{aligned}$$

The inequality between the second and third line results from the properties of G. As a result of the mean value theorem there exist $y_1\in\left(x_s,x_{12}^*\right)$ and $y_2\in\left(x_{22}^*,x_s\right)$ such that

$$\left(x_{12}^*-x_s\right)\left(\frac{1}{H'(y_1)}\right)'>\left(x_s-x_{22}^*\right)\left(\frac{1}{H'(y_2)}\right)'.$$

Furthermore, the function $(1/H')'$ is decreasing since

$$(1/H')''=-\frac{H'''H'-2\left(H''\right)^2}{\left(H'\right)^3},$$

$H'''(x)\,H'(x)\geq 2\left(H''(x)\right)^2$, and $H'(x)>0$. As a result, we obtain from the latter inequality (observe that $y_1>y_2$)

$$\left(x_{12}^*-x_s\right)>\left(x_s-x_{22}^*\right)\quad\overset{e'>0}{\Leftrightarrow}\quad e\left(x_{12}^*\right)-e\left(x_s\right)>e\left(x_s\right)-e\left(x_{22}^*\right).$$

In the second case $e' < 0$ we obtain

$$0 > H'(x_{12}^*) > H'(x_s) > H'(x_{22}^*) \quad \Leftrightarrow \quad x_{12}^* < x_s < x_{22}^*$$

and

$$\begin{aligned} & \frac{1}{H'(x_s)} - \frac{1}{H'(x_{12}^*)} \\ = \; & -\frac{\alpha}{e'} G'(H(x_{21}^*) + H(x_{22}^*) - H(x_{11}^*) - H(x_{12}^*)) \\ > \; & \frac{\alpha}{e'} G'(-H(x_{21}^*) - H(x_{22}^*) + H(x_{11}^*) + H(x_{12}^*)) = \frac{1}{H'(x_{22}^*)} - \frac{1}{H'(x_s)}. \end{aligned}$$

The renewed application of the mean value theorem leads to the existence of $y_1 \in (x_{12}^*, x_s)$ and $y_2 \in (x_s, x_{22}^*)$ such that

$$(x_s - x_{12}^*) \left(\frac{1}{H'(y_1)} \right)' > (x_{22}^* - x_s) \left(\frac{1}{H'(y_2)} \right)'.$$

Note that $H'(x) < 0$ for all $x \in [x_{12}^*, x_{22}^*]$ and in particular for all $x \in [y_1, y_2]$. Together with the condition $H'''(x) H'(x) \geq 2 (H''(x))^2$, this implies that $(1/H')'$ is increasing in the relevant range so that we obtain (observe now that $y_1 < y_2$)

$$(x_s - x_{12}^*) > (x_{22}^* - x_s) \quad \overset{e'<0}{\Leftrightarrow} \quad e(x_{12}^*) - e(x_s) > e(x_s) - e(x_{22}^*).$$

In the third case $e' = 0$ we obtain $x_{12}^* = x_s = x_{22}^*$, and the condition $e(x_{12}^*) - e(x_s) \geq e(x_s) - e(x_{22}^*)$ is obviously fulfilled.

Finally, we demonstrate that in equilibrium we cannot have $x_{11}^* \neq x_{21}^*$ and $H(x_{11}^*) = H(x_{21}^*)$. If both conditions would hold, we would either have (i) $f(x_{11}^*) - c(x_{11}^*) > f(x_{21}^*) - c(x_{21}^*)$ or (ii) $f(x_{11}^*) - c(x_{11}^*) < f(x_{21}^*) - c(x_{21}^*)$. If in case (i) player 2 would deviate to x_{11}^*, he would increase his first-period payoff, while we would still have $\Delta U = 0$, and the second-period solution as well as inequality costs would not be affected. Hence, player 2 would find it profitable to deviate. Similarly, in case (ii) player 1 would benefit from deviating to x_{21}^*. ■

Proof of Proposition 3. From the analysis of Section 2 (Proposition 1) it is obvious that $x^{2*} = x_s$ because we have a symmetric equilibrium (so that $\Delta U = 0$) and direct effects of inequality aversion have been eliminated by the assumption that output is deterministic. Thus, we are able to focus on x^{1*}.

(i) If $x^{1*} = x_s$, we know that $H'(x^{1*}) - e' = f'(x^{1*}) - c'(x^{1*}) = 0$. Condition (8) becomes $\partial x^*_{j2}/\partial x_{i1}(x^{1*}, x^{1*})\, e' = 0$. It remains to show that $\partial x^*_{j2}/\partial x_{i1}(x^{1*}, x^{1*}) \neq 0$ for $e' \neq 0$. Without loss of generality we show this statement for the case $i = 1$ and $j = 2$. From (6) and Cramer's rule, we obtain in the symmetric solution

$$\frac{\partial x^*_{22}}{\partial x_{11}} = \frac{\det\begin{pmatrix} \frac{\partial \Phi_1}{\partial x_{12}} & -\frac{\partial \Phi_1}{\partial x_{11}} \\ \frac{\partial \Phi_2}{\partial x_{12}} & -\frac{\partial \Phi_2}{\partial x_{11}} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial \Phi_1}{\partial x_{12}} & \frac{\partial \Phi_1}{\partial x_{22}} \\ \frac{\partial \Phi_2}{\partial x_{12}} & \frac{\partial \Phi_2}{\partial x_{22}} \end{pmatrix}} = \frac{-H'(x^{1*})\, H'(x^{2*})\, \alpha G''(0)}{H''(x^{2*}) - 2\left(H'(x^{2*})\right)^2 \alpha G''(0)} \tag{10}$$

since in the symmetric equilibrium we have

$$\begin{aligned} &\det\begin{pmatrix} \frac{\partial \Phi_1}{\partial x_{12}} & \frac{\partial \Phi_1}{\partial x_{22}} \\ \frac{\partial \Phi_2}{\partial x_{12}} & \frac{\partial \Phi_2}{\partial x_{22}} \end{pmatrix} \\ =\;& [H''\left(x^{2*}\right) - \left(H'\left(x^{2*}\right)\right)^2 \alpha G''(0)]^2 - \left[\left(H'\left(x^{2*}\right)\right)^2 \alpha G''(0)\right]^2 \\ =\;& \left(H''\left(x^{2*}\right)\right)^2 - 2H''\left(x^{2*}\right)\left(H'\left(x^{2*}\right)\right)^2 \alpha G''(0) \end{aligned}$$

and

$$\begin{aligned} &\det\begin{pmatrix} \frac{\partial \Phi_1}{\partial x_{12}} & -\frac{\partial \Phi_1}{\partial x_{11}} \\ \frac{\partial \Phi_2}{\partial x_{12}} & -\frac{\partial \Phi_2}{\partial x_{11}} \end{pmatrix} \\ =\;& -\left[H''\left(x^{2*}\right) - \left(H'\left(x^{2*}\right)\right)^2 \alpha G''(0)\right] H'\left(x^{1*}\right) H'\left(x^{2*}\right) \alpha G''(0) \\ & - \left(H'\left(x^{2*}\right)\right)^2 \alpha G''(0)\, H'\left(x^{1*}\right) H'\left(x^{2*}\right) \alpha G''(0) \\ =\;& -H'\left(x^{1*}\right) H'\left(x^{2*}\right) \alpha G''(0)\, H''\left(x^{2*}\right). \end{aligned}$$

Note that the denominator of the fraction in (10) is strictly negative and $G''(0) > 0$. Hence, we just need to verify that $H'(x^{1*}), H'(x^{2*}) \neq 0$ for $e' \neq 0$. From the second-period optimality conditions it is straightforward to see that in the symmetric equilibrium we have $H'(x^{2*}) = e'$, thus we can focus on x^{1*}. Rewriting (8) under consideration of (10), we obtain

$$H'\left(x^{1*}\right) - e' + \frac{H'(x^{1*})\, H'(x^{2*})\, \alpha G''(0)}{H''(x^{2*}) - 2\left(H'(x^{2*})\right)^2 \alpha G''(0)}\, e' = 0.$$

Because of $H'(x^{2*}) = e'$ the condition is equivalent to

$$H'\left(x^{1*}\right) - e' + \frac{H'(x^{1*})\,\alpha G''(0)}{\frac{H''(x^{2*})}{(e')^2} - 2\alpha G''(0)} = 0$$

$$\Leftrightarrow H'\left(x^{1*}\right)\left(\frac{H''\left(x^{2*}\right)}{(e')^2} - \alpha G''(0)\right) = e'\left(\frac{H''\left(x^{2*}\right)}{(e')^2} - 2\alpha G''(0)\right).$$

Since the terms in parentheses are strictly negative, $H'(x^{1*})$ and e' have the same sign.

(ii) Suppose that $e' = 0$. Then condition (8) becomes

$$f'\left(x^{1*}\right) - c'\left(x^{1*}\right) = 0$$

which in turn implies $x^{1*} = x_s$. ■

Proof of Proposition 4. Because of (8) and under consideration of $H'(x^{2*}) - e' = 0$ we obtain

$$H'\left(x^{1*}\right) - e' - \frac{\partial x^*_{j2}}{\partial x_{i1}}\left(x^{1*}, x^{1*}\right) e' = H'\left(x^{2*}\right) - e' \quad \Leftrightarrow \quad H'\left(x^{1*}\right) - H'\left(x^{2*}\right) = \frac{\partial x^*_{j2}}{\partial x_{i1}}\left(x^{1*}, x^{1*}\right) e'.$$

Since $H'' < 0$ the statement of the proposition is shown if $\partial x^*_{j2}/\partial x_{i1}(x^{1*}, x^{1*}) > 0$. Again we verify this for the case $i = 1$ and $j = 2$. From the proof of Proposition 3 we know that $sgn(H'(x^*_1)) = sgn(H'(x^*_2))$ (since both, $H'(x^*_1)$ and $H'(x^*_2)$, have the same sign as e'). $\partial x^*_{22}/\partial x_{11}(x^{1*}, x^{1*}) > 0$ then follows immediately from (10). ■

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