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**Optimal Unemployment Insurance in an Estimated Job Search  
Model with Savings**

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2004-10

# Optimal Unemployment Insurance in an Estimated Job Search Model with Savings\*

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March 30, 2003

## Abstract

This paper estimates a job search model with savings and determines optimal unemployment benefit policy for the estimated model. For observed and unobserved worker characteristics, the estimation strategy relates observed unemployment spell durations to the model implied unemployment hazard rate. The model is estimated on Danish unemployment spell data which include high quality wealth and income information. The estimation shows that Danish workers respond to changes in economic incentives in ways consistent with the model and that the magnitude of the effect of the responses on the unemployment hazard rate is small.

Optimal unemployment benefit level policy is determined as a trade-off between providing insurance against consumption fluctuation and the moral hazard of reducing the worker's incentives to search back into employment. Given the estimated low level of moral hazard, the optimal benefit level is quite high even though workers can self-insure via savings. Depending on the interest rate which is effectively the cost of using savings as self-insurance, the optimal replacement rate ranges between 43% and 82%. The policy analysis emphasizes the importance of including transitional dynamics to avoid a significant downward bias associated with a simple steady state comparison analysis.

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\*I would like to thank Dale T. Mortensen for his generous support and suggestions. The research has also benefited from conversations with Lawrence Christiano, Martin Eichenbaum, Christopher Taber, Victor Aguirregabiria, Joseph Altonji, Gadi Barlevy, Ariel Burstein, Russell Cooper, Simon Gilchrist, Kenneth Judd, Robert King, Kevin Lang, Bruce Meyer, Pedro Mira, Alex Monge, Karl Schmedders, Michael Svarer, and Torben Tranæs as well as from comments from seminar participants at the 2001 NBER Summer Institute, the 2001 Society for Economic Dynamics meetings, Boston University, Brown, University of Copenhagen (CAM), Georgetown, Stockholm University (IIES), Chicago GSB, University College London, UC San Diego, Minnesota, University of Southern California, and University of Toronto. All remaining errors are mine. Centre for Labour Market and Social Research in Århus, Denmark generously provided data access. Financial support for this research was awarded by the Danish Research Academy and the Northwestern University Graduate School Dissertation Year Fellowship.

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# 1 Introduction

This paper estimates a job search model with savings on Danish unemployment spell data and determines optimal unemployment benefit levels for the estimated model. The advantage of a structural estimation in this setting is that it allows the counterfactual experiment that is the optimal benefit policy analysis. Optimal unemployment benefit policy is determined as a trade-off between providing insurance against income loss and the moral hazard problem of reducing workers' incentives to search back into employment and thereby extending unemployment spell duration. Thus, optimal benefit policy will be sensitive to model parameters that affect the risk aversion in consumption as well as the extend of the moral hazard in the model. By relating observed unemployment durations to the model's unemployment hazard rate, the estimation determines these and other model parameters - and consequently optimal benefit levels.

Workers can insure themselves against income fluctuations by saving income from one period to another. Hence, the extend to which they demand additional insurance in the form of unemployment benefits depends on the access to and cost of savings as a self insurance instrument. In particular, for a given subjective discount rate the optimal replacement rate (that is, the benefit level relative to the expected wage) is highly sensitive to the interest rate, ranging from a 43% replacement rate for an interest rate almost equal to the subjective discount rate to an 82% replacement rate for a zero interest rate.

Conditioning on observed and unobserved worker characteristics, the identification strategy relates observed unemployment durations to the model implied unemployment hazard rate which is proportional to the search choice. To this end, data are needed where unemployment spell durations are linked to observed worker characteristics. In particular, since savings and earnings play an important part in the paper, high quality wealth and income data are needed. Data like this are hard to come by, however one such dataset is Statistics Denmark's 0.5 percent sample which was generously made available by Centre for Labour Market and Social Research in Århus, Denmark.

The model estimation successfully captures key relationships in data. Notably, wealthier individuals are observed to experience longer unemployment durations which is explained by the

model via a negative relationship between the choice of search intensity and savings.<sup>1</sup> Furthermore, data show a U-shaped relationship between unemployment duration and the expected wage of the worker. While not widely recognized, this type of relationship is quite natural in a sequential job search model with savings. Whereas, sequential search and directed search models without savings imply monotone relationships between unemployment duration and the wage. Both of these relationships are robust to conditioning on observed and unobserved worker characteristics. They are both important identifiers of the curvature of the utility of consumption function. The estimate implies a constant relative risk aversion coefficient of 2.21.

Thus, the workers in the data respond to changes in economic incentives in ways that are consistent with the sequential job search model with savings. However, the magnitude of the change in the unemployment hazard rate that results from these responses is quite small. The model estimation captures this fact via a high degree of curvature in the search cost function and/or offer arrival rate function. The two functions are not separately identified. Thus, the model estimation implies a low degree of moral hazard - the unemployment hazard rate is not very sensitive to changes in economic incentives.

The unemployment benefit policy is restricted to a constant level benefit path of infinite duration. The system is assumed to be financed by a proportional income tax and the benefit-tax scheme is restricted to satisfy inter-temporal budget balance. One may consider systems with finite duration benefits, however the results in Davidson and Woodbury (1997) suggest that it is optimal to extend the duration of a constant level benefit system indefinitely.<sup>2</sup> The policy analysis is ignoring the issue of optimal design over duration which has been analyzed in search models without savings in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). However, the more advanced benefit paths that vary continuously over unemployment duration are likely not politically implementable. Furthermore, Werning's (2002) and Kocherlakota's (2003) studies of optimal benefit design over unemployment duration in job search models with unobservable savings suggest that a constant level, infinite duration benefit path may in fact be the optimally designed path or at least not far from it.

Studies such as Acemoglu and Shimer (1999) show that there may be efficiency issues associated

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<sup>1</sup>This relationship has previously been established on Danish, Dutch and French data in Lentz and Tranæs (2002), Bloemen and Stancanelli (2001) and Algan, Chéron, Hairault, and Langot (2001), respectively.

<sup>2</sup>The result is shown via model simulations and is not established generally, though.

with providing unemployment benefits. However, in this study the role of unemployment benefits is purely one of providing insurance against consumption fluctuations at the cost of distorting search incentives. Papers such as Bailey (1978), Flemming (1978), Hansen and Imrohoroglu (1992), and Wang and Williamson (1999) have studied this question in models with savings. However, it is a common feature of these papers as well as the broader literature on optimal unemployment insurance that the use of savings as a self insurance instrument has been seriously curtailed.<sup>3</sup> I find that once these restrictions are lifted, the demand for additional insurance in the form of unemployment benefits is dramatically reduced.

The preferred method of determining the optimal level of unemployment benefits seems to be by comparing some social welfare criterion across steady states associated with different benefit-tax schemes. I show that once savings are included in the analysis, one must include transitional dynamics in order to avoid a serious downward bias in the optimal benefit results. The policy study in this paper includes full transitional dynamics.

The analysis of the job search model with savings is complicated by the inability to establish global concavity of the value functions. Danforth (1979) shows that in the special case where employment is an absorbing state, one can characterize the reservation wage choice in relation to the degree of absolute risk aversion of the utility function. In the case of decreasing absolute risk aversion, the reservation wage choice will be increasing in wealth. Flemming (1978) and Acemoglu and Shimer (1999) are examples of the constant absolute risk aversion case combined with the assumption that search costs are monetary, and that there is no lower bound on wealth.<sup>4</sup> In this case, the search choice is unaffected by wealth and the results for this special case do generalize to the case in which employment is not an absorbing state. However in general, once employment is no longer an absorbing state, the inability to establish global concavity of the value functions impedes construction of characterization theorems of the worker's search and savings choices. Lentz and Tranæs (2002) establish sufficient conditions to provide characterization theorems for the search intensity model for this more general case.

The paper is structured as follows. In section 2 the model is laid out and its key characteristics

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<sup>3</sup>In studies where savings are allowed, the return to savings is often set at such a low rate that holding savings becomes quite costly and consequently the option to use savings as a self-insurance instrument has low value

<sup>4</sup>In the reservation wage setup, search costs are necessarily monetary in that an increase in the hazard rate due to a lower reservation wage choice will be associated with lower expected future income. In the search intensity setup, the search costs are paid in the same period and enter via the budget constraint.

are explained. Section 3 presents the estimation strategy, data, and estimation results. Based on the results of the estimation, the paper proceeds by determining optimal benefit levels at an individual level in section 4.1 and section 4.2 considers optimal group wide insurance schemes. Finally, section 5 concludes.

## 2 The Model

Consider a utility maximizing worker who faces risk of job loss. When employed he receives a fixed wage  $w$  and during unemployment he receives unemployment benefits  $b$ , where  $b < w$ . The worker can smooth consumption over income states by use of savings that carry a return of  $r$ . There are no other insurance instruments available to the worker.

Generally, the worker faces two decision problems; how much to consume and how much to search. The objective is to maximize the discounted stream of future utility which is assumed to be separable in both time as well as consumption and search. During employment, the decision problem facing the worker is simply how much to save for the next period. Since the wage distribution is degenerate, on-the-job search is ruled out and issues such as effort choices on the job will also be ignored. It will simply be assumed that the worker faces an exogenous job separation rate  $\delta$ . During unemployment, the decision problem is more complex. Like in the state of employment, the worker will have to decide how much to save. Furthermore, the worker decides how much effort to put into job search. A higher search intensity will raise the probability of receiving a job offer but also implies a greater utility loss.

Workers are assumed to be infinitely lived. The estimation strategy in the following section relates observed unemployment duration to the model implied search choice while conditioning on observed wealth holdings. The alternative assumption of finitely lived workers will introduce life cycle savings motives but will not have a great impact on the search choice for a given wealth level. As such, for estimation purposes the issue of finitely versus infinitely lived agents is not of great concern.

It is assumed that all workers are eligible to receive  $b$  and that benefits are of infinite duration. This is motivated by the actual Danish unemployment system in which benefit duration in the 1980's and early 1990's was at times infinite and at no point shorter than 5 years. If a worker is no longer eligible to receive unemployment benefits he moves into the welfare system where he receives

payments roughly equal to  $2/3$  of the unemployment benefits. During the benefit period, the UI system would force the worker to accept brief employment spells or education offers. But the first of these ‘harassments’ was not forced on the worker until after more than two years of unemployment. The average unemployment spell in the data is 14 weeks. While there are restrictions on eligibility, they are generally quite easy to satisfy and the question of qualifying for benefits is not of great concern for the Danish worker. The system is voluntary and if the worker decides to participate he must pay an insurance premium. The system is heavily subsidized and the worker pays only about  $1/3$  of the actual premium. Thus, not surprisingly more than 80% of the labor force choose to participate in the system. Non-participants are generally very low wage workers and very high wage workers. For the very low wage workers, the welfare system will provide comparable insurance to the UI system. For the very high wage workers, the UI system provides very little insurance due to an upper bound on benefit payments. In effect, the upper bound on benefit payments is so restrictive that the Danish UI system can be characterized by a wage independent benefit scheme. In the data in this paper, everybody faces the same constant level of benefits except for the lowest 4 percent of the wage earners. The lowest 4 percent of wage earners receive benefits equal to 90% of their previous wage.

It is assumed that each worker faces a wage distribution in which the second and higher moments are zero. Different workers may face different wage distribution means and as such the overall economy will display a full wage distribution. But each worker faces a single wage only. The assumption rules out reservation wage considerations and is a simplification relative to the fairly large literature on estimation of job search models. See Devine and Kiefer (1991), Wolpin (1995), and Van den Berg (1999) for a survey of the literature. I will be employing an estimation strategy that relates observed unemployed spell duration to the model implied unemployment hazard rate for given worker characteristics. It is well known that the search choice is sensitive to the moments of the wage offer distribution. I will be capturing individual variation in the first moment and its effect on the unemployment hazard rate. The decision to focus on the search intensity choice only is in part also driven by the desire for a relatively simple model in which one can carry out an optimal unemployment benefit analysis. The reservation wage model is considerably more complicated in this respect and the optimal unemployment benefit design literature has consequently almost exclusively focused on the search intensity model.

The entire decision problem can be written as:

$$\begin{aligned}
& \max_{\{c_t, s_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1 + \rho)^{-t} [u(c_t) - e(s_t)] \\
& \text{st} : k_{t+1} = (1 + r)k_t + n_t w + (1 - n_t)b - c_t \\
& c_t \geq 0 \\
& k_t \in [\underline{k}, \bar{k}] \\
& s_t \geq 0 \\
& \Pr(n_{t+1} = 1 | n_t = 1) = 1 - \delta, \Pr(n_{t+1} = 0 | n_t = 1) = \delta \\
& \Pr(n_{t+1} = 1 | n_t = 0) = \mu(\lambda s_t), \Pr(n_{t+1} = 0 | n_t = 0) = 1 - \mu(\lambda s_t),
\end{aligned}$$

where  $k_t$  is the worker's wealth at time  $t$ .  $r$  is the interest rate and  $\rho$  is the subjective discount rate.  $n_t \in \{0, 1\}$  denotes the state of employment at time  $t$ , where  $n = 1$  denotes employment and  $n = 0$  denotes unemployment. It is assumed that  $u(\cdot)$  is strictly increasing and strictly concave,  $e(\cdot)$  is strictly increasing and weakly convex with  $e(0) = 0$ , and  $\mu(\cdot)$  is strictly increasing and strictly concave with  $\mu(0) = 0$  and  $\lim_{x \rightarrow \infty} \mu(x) = 1$ .  $\lambda$  will be referred to as an offer arrival rate. One can equally well think of this simply as a parameter of the  $\mu(\cdot)$  function. In the structural estimation in the following sections it will be assumed that worker heterogeneity can enter via the  $\lambda$  parameter. The lower bound on wealth  $\underline{k}$  can be interpreted as a capital market imperfection. However, it can also simply be a lower bound that naturally arises from the assumption of non-negative consumption combined with asymptotic budget balance. See Aiyagari (1994) for more on this type of argument. The upper bound will be set so that it is not restrictive in equilibrium. This can be done as long as  $r < \rho$ .

The model can be formulated recursively. Let  $V_e(k)$  be the maximal present value of being employed with wealth  $k$ . Similarly,  $V_u(k)$  is the value function associated with unemployment at wealth  $k$ . Let the choice of next period's wealth level be denoted by  $k'$ . The Bellman-equations of the model can then be stated as:

$$V_e(k) = \max_{k' \in \Gamma_w(k)} \left\{ u((1+r)k + w - k') + \frac{(1-\delta)V_e(k') + \delta V_u(k')}{1+\rho} \right\} \quad (1)$$

$$V_u(k) = \max_{k' \in \Gamma_b(k), s \geq 0} \left\{ u((1+r)k + b - k') - e(s) + \frac{\mu(\lambda s)V_e(k') + (1-\mu(\lambda s))V_u(k')}{1+\rho} \right\}, \quad (2)$$

where  $\Gamma_y(k) = \{k' \in \mathbb{R} | \underline{k} \leq k' \leq (1+r)k + y\}$ . It will right away be assumed that:

$$\mu(\lambda s) = 1 - e^{-\lambda s}. \quad (3)$$



The policy functions of the model, savings when employed and unemployed as well as the choice of search intensity are denoted by  $k_e(k)$ ,  $k_u(k)$  and  $s(k)$ , respectively. The model is analytically quite intractable and it is hard to establish analytical characterizations of the model's value and policy functions. However, it is straightforward to solve the model numerically. The model estimation and the policy analysis in the following sections rely on numerical methods to solve the model.

Under a set of sufficient conditions, most notably the existence of a lottery in wealth, Lentz and Tranæs (2002) show that separability between consumption and search in the utility function will result in a decreasing search intensity choice in wealth. The sufficient conditions ensure concavity of the value functions. While the sufficient conditions have not been made in the model at hand, numerical model solutions in this paper always yield globally concave value functions and consequently given the separable utility function assumption that the search choice is decreasing in wealth. Intuitively, search intensity is decreasing in wealth because the gains to search  $V_e(k) - V_u(k)$  can be shown to diminish as wealth increases. Due to the separability in the utility function, the search costs will be unaffected by wealth holdings and the result follows directly.

Given  $r < \rho$ , the worker will always reduce wealth holdings during unemployment spells. There exists an upper wealth bound below which the worker will increase wealth holdings while employed. For wealth holdings above the upper wealth bound, the worker dissaves in both employment states but the worker always dissaves by more in the unemployed state.

In a regular search model without savings, it is well known that a right-shift of the wage offer distribution will increase the search intensity choice.<sup>5</sup> In the model at hand, an increase in the wage represents such a right-shift. However, once savings are included in the model the monotonicity result no longer holds. When the worker can smooth consumption via savings, a right-shift in the wage offer distribution is associated with two opposing effects analogous to the income and substitution effects associated with a wage increase in labor supply theory. A right-shift of the wage distribution implies a greater pay-off to search activity and the substitution effect dictates that the worker substitute into more search. However, when the worker can transfer income from the employed to the unemployed state via for example savings, a right-shift in the wage distribution is also associated with an income effect which dictates that the unemployed worker search less. In

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<sup>5</sup>See among others Mortensen (1986).

the case where the worker cannot transfer income from the employed state to the unemployed state either because she is at the lower wealth bound or because wealth cannot be stored, there is only a substitution effect. In the case where workers can save, model simulations suggest that the substitution effect dominates at low wage levels where the search choice is consequently increasing in the wage. If the utility of consumption has a sufficient amount of curvature, the income effect will eventually begin to dominate resulting in a decreasing relationship between the search intensity choice and the wage. It is tempting to extend these results to a decrease in the benefit level. However, in this case the substitution and income effects both point in the same direction. Policy function characterization is discussed in greater detail in the appendix.

### 3 Estimation of the Model

I will estimate the model by relating observed unemployment spell durations to the model implied unemployment hazard rate. Let  $h_i(\tau)$  be the model implied unemployment hazard rate in the  $\tau$ 'th period of the unemployment spell. The probability of observing a spell length  $t$  for worker  $i$  is then simply  $\Pr(T_i = t) = h_i(t) \prod_{\tau=1}^{t-1} h_i(\tau)$  which is the basis of the likelihood function for the data. The estimation strategy requires unemployment spell duration data where observed unemployment spell durations are linked with observed worker characteristics.

Denote by  $t_{ij}$  the length of worker  $i$ 's  $j$ 'th spell. Let  $z_{ij}$  denote whether the spell is right censored.  $\kappa_{ij}$  is worker  $i$ 's observed wealth level at the outset of the  $j$ 'th spell and  $w_{ij}$  is the wage that the worker is expecting to receive in the new job. Finally, denote by  $X_{ij}$  a set of other worker characteristics which includes education, age, gender, spouse's income, number of children, and occupation.

The search intensity choice will be characterized by positive duration dependence because wealth is being gradually reduced throughout the spell. Given an observed initial amount of wealth at the outset of the spell  $\kappa$ , one can infer the search intensity choice at any point during the spell. The wealth inference is made by iterating on  $k_u(\cdot)$  (i.e. the inference about the worker's wealth holdings one period into an unemployment spell is  $k_u(\kappa)$ ). Naturally, in the case where one directly observes wealth throughout the spell, this procedure is moot. Such wealth data is not available, though. Wealth is only observed at a yearly frequency. Therefore, one cannot observe the change

in wealth holdings during the spell which is observed at a weekly frequency.<sup>6</sup>

In the estimation of the model, worker heterogeneity can enter via wealth at the outset of the spell  $\kappa$ , the wage level  $w$  and the offer arrival rate  $\lambda$ . While the offer arrival rate is not itself observable, all of the remaining observed worker heterogeneity will enter via this parameter. Furthermore, the estimation will also allow unobserved worker heterogeneity to enter via the offer arrival rate. The unobserved heterogeneity is assumed to be uncorrelated with the observed worker characteristics and is analyzed in much the same way as in Heckman and Singer (1984). In particular, it is assumed that:

$$\lambda_{ij}(\beta, \sigma_i) = e^{X'_{ij}\beta + \sigma_i}, \quad \sigma_i \sim G(\cdot),$$

where  $\beta$  will be a set of parameters to be estimated.<sup>7</sup> The unobserved heterogeneity term  $\sigma_i$  will be assumed to be drawn from a common probability distribution with  $L$  support points. The distribution and the support points will all be estimated.

The unemployment hazard rate in the model is given by  $\mu(\lambda s)$  and is directly determined by the worker's search intensity choice. The search intensity choice will be a function of the worker's initial wealth, her wage, her offer arrival rate, and via the reduction in wealth over the unemployment spell, how long she has been unemployed. Finally, the search decision will also depend on all of the structural model parameters denoted by  $\theta$ . The search choice is given by  $s(\kappa, w, \lambda, t, \theta)$ . All in all, for a given unobserved heterogeneity term  $\sigma_i$ , the probability of observing the tuple  $\{t_{ij}, z_{ij}, \kappa_{ij}, w_{ij}, X_{ij}\}$  is simply the probability of an arrival of an offer in week  $t_{ij}$  and no arrivals prior to this:

$$L_{ij}(\theta, \beta, \sigma_i) = \mu(\lambda_{ij}(\beta, \sigma_i) \cdot s(\kappa_{ij}, w_{ij}, \lambda_{ij}(\beta, \sigma_i), t_{ij}, \theta))^{z_{ij}} \times \prod_{t=1}^{t_{ij}-1} [1 - \mu(\lambda_{ij}(\beta, \sigma_i) \cdot s(\kappa_{ij}, w_{ij}, \lambda_{ij}(\beta, \sigma_i), t, \theta))].$$

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<sup>6</sup>In the estimation, it is simply assumed that the wealth level at the outset of the unemployment spell is equal to the wealth level at the beginning of the year of the spell. Ideally, based on the model one can attempt to adjust the wealth holding according to the week of the year the spell actually starts in order to more precisely capture the wealth holdings at the beginning of the spell. But simulations of the model suggest that this will result in very minor adjustments to the wealth holdings and consequently the procedure is not likely to affect the results in any significant way.

<sup>7</sup>Both spouse's income as well as the number of children are included in  $X$ . Ideally, these characteristics should enter via the budget constraint but this will add another dimension of heterogeneity to the estimation. While this is not conceptually hard, it pushes the computational requirements beyond what is currently feasible. The characteristics are included in the offer arrival rate in an attempt to also control for this dimension of observed heterogeneity.

The likelihood of observing all of worker  $i$ 's  $J_i$  spells is then simply,

$$L_i(\theta, \beta, G) = \sum_{l=1}^L \Pr(\sigma_i = \sigma_l) \prod_{j=1}^{J_i} L_{ij}(\theta, \beta, \sigma_l).$$

The full likelihood for all the observations is given by:

$$L(\theta, \beta, G) = \prod_{i=1}^N L_i(\theta, \beta, G). \quad (4)$$

The estimates of  $(\theta, \beta)$  are found via basic maximum likelihood estimation, that is:

$$\left(\hat{\theta}, \hat{\beta}, \hat{G}\right) \in \arg \max_{\theta, \beta, G} L(\theta, \beta, G). \quad (5)$$

One cannot obtain a closed form solution for  $s(\cdot)$ . Thus, for each parameter choice one must numerically solve the model and evaluate the likelihood function based on the new policy functions. The details of the numerical issues are described in the appendix.

### 3.1 Data

The data used in the estimation below were generously made available by Centre for Labour Market and Social Research in Århus, Denmark. The dataset follows 0.5% of the Danish population on a weekly basis from 1980 through 1994 recording basic information on each individual's employment status. The data are then merged with individual specific information which includes financial data from the Danish tax authorities. In particular, this includes information on individual earnings and wealth holdings where the wealth measure includes all assets as well as liabilities except pension savings.<sup>8</sup> The tax filings also provide information on any income stemming from other family members. Other databases provide information on the level of education of the worker, the amount of work experience, age and gender of the worker, the number of children the worker cares for, etc.

A dataset is then constructed where the basic observation is an unemployment spell. For each spell, the length and the worker ID of the spell is recorded, whether the length is right censored, the level of wealth at the beginning of the spell, the year in which the spell started, the wage of the worker and other worker characteristics. In general, all worker characteristics are observed only on

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<sup>8</sup>Wealth data were collected as a basis for the Danish wealth tax that was in effect through 1996. The wealth tax only taxed wealth holdings above a very large threshold which happens to be well above the upper censoring bound on wealth. Thus, one need not worry that the wealth measure underestimates wealth holdings due to tax avoidance incentives. Gross asset and liability observations are censored above 1.1 mio DKK (inflation adjusted indexed to 1990 DKK) by Statistics Denmark due to anonymity considerations.

Table 1: Summary Statistics

| Variable               | Minimum       | Maximum      | Mean       | Standard Deviation |
|------------------------|---------------|--------------|------------|--------------------|
| Spell duration         | 3.00          | 52.00        | 16.16      | 13.94              |
| Spells per worker      | 1.00          | 32.00        | 7.05       | 5.02               |
| Years of education     | 9.00          | 18.00        | 11.99      | 2.64               |
| Female                 | 0.00          | 1.00         | 0.47       | 0.50               |
| Spouse's yearly income | 0.00          | 1,046,000.00 | 159,760.00 | 137,138.00         |
| Number of children     | 0.00          | 12.00        | 1.66       | 1.88               |
| Age                    | 18.00         | 66.00        | 42.51      | 8.86               |
| Upper management       | 0.00          | 1.00         | 0.15       | 0.35               |
| Lower management       | 0.00          | 1.00         | 0.21       | 0.40               |
| Salaried worker        | 0.00          | 1.00         | 0.29       | 0.45               |
| Skilled worker         | 0.00          | 1.00         | 0.12       | 0.32               |
| Hourly wage            | 85.36         | 443.99       | 144.32     | 49.29              |
| Owner of real estate   | 0.00          | 1.00         | 0.53       | 0.50               |
| Net wealth             | -1,046,000.00 | 1,046,000.00 | 115,313.00 | 307,580.00         |

N=12,865

All income and wealth amounts are in 1994 DKK.

a yearly basis while the employment status is observed weekly. Multiple worker spells are quite common which will be very useful in dealing with issues of unobserved heterogeneity. The data are summarized in table 1.

All spell lengths greater than 52 weeks have been censored at 52 weeks and all spells with durations below 3 weeks have been ignored in that they provide only noise. Most of these spells are either vacations or spells that do not represent unemployed search but rather a brief spell of non-employment associated with a job-to-job switch. Estimations were run in which the short spells were included. The point estimates did not change appreciably but variance increased. Seasonal and other types of temporary layoffs where the worker returns to her previous employer after a relatively brief period of time have also been eliminated from the data. The final data set consists of a total of 12,865 spells.

The Danish benefit system provides a constant 90% replacement rate up to an absolute upper bound. The upper bound happens to be set so that benefits are constrained at the upper bound for all wage earners except for those at the very lower end of the wage distribution. Thus, the benefit level observation is constant for the vast majority of the observations. However, for observations with wages below the 4th percentile of the wage distribution in the data, benefits are 90% of the

wage observation. The estimation of the model does not allow for heterogeneity in the benefit level observation. Lifting this restriction implies that one must allow for yet another dimension of heterogeneity in the estimation and is currently not computationally feasible. To maintain the correct replacement rate for the observations with wages below the 4th percentile, the wage observations are censored at the 4th percentile. The wage measure has been censored at the upper 99.5 percentile level to deal with possible measurement error as well as for practical reasons to limit the wage span that the model must be solved for.

The model allows for a full wage distribution for the economy but assumes that each worker faces a single wage. All higher moments of the wage distribution are assumed to be zero. The first moment of the individual's wage offer distribution can be identified in various ways. In the estimation below, the wage measure is simply the realized wage of the year prior to the year of the unemployment spell. Observationally, it is clear that there is a noise component to the realized wage. As such, one can attempt to reduce the noise by taking an average over several wage observations around the year of the unemployment spell in question. Alternatively, one can adopt the approach that the worker's expectation about the future wage is best approximated by the realized wage in the new job. Furthermore, one can attempt to predict the wage via a wage regression. This approach has the disadvantage that wage regressions typically only explain roughly 25-30% of the observed wage variation. All of these choices are consistent with the model in that it is assumed that the wage of the particular worker is fixed. Reduced form estimations were performed on all of the different wage measures mentioned and they all yield the same qualitative results.

### 3.2 Estimation Results

The estimation is performed given a constant relative risk aversion specification of consumption,  $u(c) = c^{1-\alpha}/(1-\alpha)$ . Search cost are assumed to be exponential,  $e(s) = As^\gamma$ , where  $A$  is not separately identified from the level of the offer arrival rate  $\lambda$  and is set to be numerically convenient,  $A = 100$ . The period length in the simulation is set at one week. This is in part driven by the weekly observation frequency in data. But also, it allows a less restricted use of the search intensity decision because it is not assumed to be fixed for long periods of time. The interest rate  $r$  is set an annual rate of 5%, that is  $r = (1.05)^{1/52} - 1$ . The subjective discount rate is set at the slightly higher annual rate of 5.1%, which ensures the existence of an upper bound on the ergodic wealth

Table 2: Results of Structural Estimation

|                                 | Estimate | Std. Dev. |
|---------------------------------|----------|-----------|
| $\alpha$                        | 2.2081*  | 0.0053    |
| $\gamma$                        | 13.3610* | 0.0113    |
| $\beta$ -estimates:             |          |           |
| Years of Education              | -0.0052  | 0.0078    |
| 1=Female                        | 0.0277   | 0.0466    |
| Spouse's Income                 | 0.0486*  | 0.0187    |
| (1=Female)*(Spouse's Income)    | -0.0763* | 0.0230    |
| Number of Children              | 0.0052   | 0.0110    |
| (1=Female)*(Number of Children) | -0.0138  | 0.0159    |
| Age                             | -0.0846* | 0.0196    |
| 1=Owner of Real Estate          | -0.0132  | 0.0291    |
| 1=Upper Management              | -0.0074  | 0.0629    |
| 1=Lower Management              | 0.0605   | 0.0528    |
| 1=Salaried Worker               | 0.0174   | 0.0446    |
| 1=Skilled Worker                | 0.0091   | 0.0597    |
| Constant                        | -1.8173* | 0.1496    |
| $G$ -estimates:                 |          |           |
| $\sigma_1$                      | -0.6560* | 0.0547    |
| $\sigma_2$                      | 0.0000   | —         |
| $\sigma_3$                      | 0.8257*  | 0.0717    |
| $\sigma_4$                      | 1.6126*  | 0.1372    |
| Pr( $\sigma_1$ )                | 0.2780*  | 0.0709    |
| Pr( $\sigma_2$ )                | 0.4362*  | 0.0615    |
| Pr( $\sigma_3$ )                | 0.2383*  | 0.0298    |
| Pr( $\sigma_4$ )                | 0.0474*  | 0.0173    |

\*Significantly different from zero at the 1% level.

distribution. The job destruction rate is set at  $\delta = 1/243$  which fits the average employment spell in the data of 243 weeks. The estimate is taken from Rosholm and Svarer (2000) who estimate the parameter on the same basic data as in this paper. The benefit level is normalized at  $b = .1$ . All income and wealth observations are adjusted according to this normalization. Finally, the lower wealth bound is set at the minimum observed wealth level  $\underline{k} = -41.3$ . The upper bound is set above the upper bound on the ergodic wealth distribution which is also well above the maximum observed wealth distribution.

The identification strategy does not identify the difference between  $r$  and  $\rho$ . This parameter relationship primarily affects savings behavior and has little impact on search behavior. Theoretically, one would expect that the the  $(r, \rho)$  relationship will affect the duration dependence of the search choice because the rate of dissaving during unemployment is sensitive to changes in  $(r, \rho)$ .

But given that most workers are not liquidity constrained, the changes in rates of dissaving are not large enough to generate noticeable effects on duration dependence.

It is very reasonable to expect that there is heterogeneity in  $\delta$ . Identifying heterogeneity in  $\delta$  can be done in much the same way as it is done for  $\lambda$ . However, the extension requires that data are expanded to include employment spells.

Identification of a common lower wealth bound has not been feasible. The wealth bound must lie at or below the minimum wealth observation in the data. Thus, identification will be driven by the very few observations with very low wealth levels. The vast majority of the spell observations are characterized by wealth levels such that a change in the lower wealth bound below the minimum wealth observation has no effect on worker behavior. Consequently, the sensitivity of the search decision with respect to wealth is purely driven by the utility function parameters.

All in all, the structural estimation determines  $(\alpha, \gamma, \beta, G)$  where  $\beta$  includes 26 parameters. The estimates of all parameters except the yearly dummy parameters are given in table 2. The yearly dummy estimates are displayed in figure 1. The number of support points in  $G$  is set at  $L = 4$ . Ideally, one would prefer to continue to add support points until additional points no longer improve the estimation. However, estimating the model is computationally very expensive and experimentation with the number of support points in  $G$  is not currently feasible.<sup>9</sup>

Generally, the signs and statistical significance of the  $\beta$  coefficients are similar to the reduced form estimates in Lentz and Tranæs (2002). It is seen that the signs of the age and education effects are negative. Similar results are found in the literature such as Meyer (1990).

While the risk aversion estimate is well within the range of previous such estimates, it is worth noting that the estimate does imply significantly more risk averse workers than the typical assumption of log-utility in the optimal unemployment insurance literature. Furthermore, the search cost function estimate implies that the search behavior of Danish workers is relatively insensitive to changes in incentives. This is consistent with the findings on Danish labor supply elasticities in Frederiksen, Graversen, and Smith (2001).

Turning to the effect of occupation, it is seen none of these estimates are significantly different from zero. The left out occupational category is unskilled workers. Thus, no occupational category

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<sup>9</sup>Lentz and Tranæs (2002) estimate a proportional hazard model with unobserved heterogeneity on the same data as in this paper. In this estimation,  $G$  was estimated to have 4 support points. Adding more points beyond this did not improve the estimation.



has significantly different offer arrival rates relative to unskilled workers. Of course, once one controls for wages it is indeed not clear from a theoretical perspective what the effect of occupation should be on the hazard rate.

It is interesting to note the gender difference associated with spousal income. A woman seems to experience longer unemployment spells the higher her spouse's income. This is consistent with the argument that the household insures the worker and further emphasizes the point that one should ideally include spouse's income in the budget constraint rather than in the offer arrival rate. However, the effect on men is directly the opposite! Furthermore, the effects are statistically very strong.

A dummy for whether the worker owns real estate has also been included in the analysis. One might suspect that if a significant portion of the worker's wealth is tied up in real estate and if the credit markets are imperfect in the sense that workers cannot borrow against real estate holdings, then this should reduce the insurance value of the wealth holdings. If this is the case, one should expect a positive sign on the dummy variable. Alternatively, one might suspect that moving costs are significantly higher if the worker is a homeowner. As such, homeowners may face lower offer arrival rates due to less geographical mobility. As it turns out, the sign is negative suggesting that the latter effect dominates. The effect is not significantly different from zero, though.<sup>10</sup>

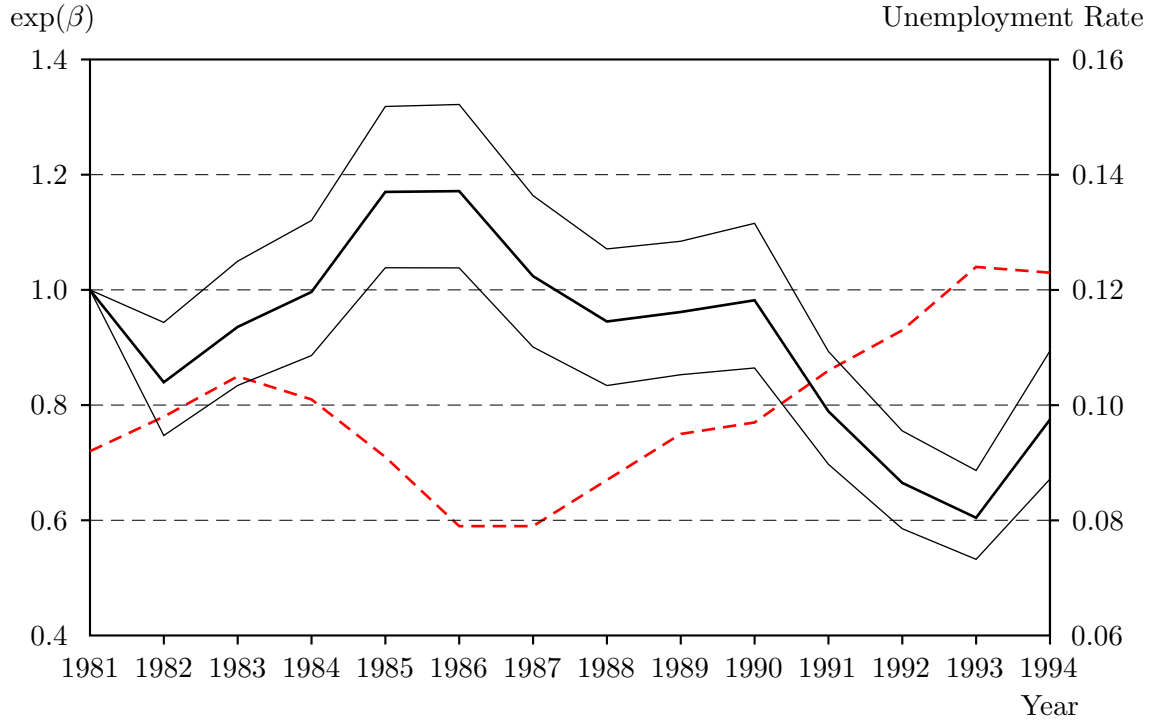
The yearly dummy effects in figure 1 very clearly capture the effect of the business cycle on the offer arrival rate. In times of recession, the offer arrival rate is low and the unemployment rate is high and vice versa. Furthermore, one can clearly detect a lead effect in the offer arrival rate of about one year. This is consistent with findings in Abowd, Corbel, and Kramarz (1999) that employers primarily adjust firm size via the hiring decision. Thus, in the face of diminished demand and therefore lower labor demand, firms lower the hiring rate which results in a lower offer arrival rate which with a lag is then eventually reflected in more unemployment as the relatively unaffected flow into the unemployment pool is now exceeding the lower flow out.

The utility function estimates primarily affect the wealth and wage effects on the hazard rate. In figure 2, the estimated unemployment hazard is shown for an average offer arrival rate ( $\lambda = .09826$ ).

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<sup>10</sup>Again, like in the case of spousal income and the number of children variables, one may have reservations about including the real estate dummy in the offer arrival rate given that the effect on the hazard rate should be a behavior driven phenomenon and consequently it should be included elsewhere in the model. However, given that this is infeasible it is on the other hand desirable to control for as much observed heterogeneity as possible in order to capture the wealth and wage effects as precisely as possible.

Figure 1: Yearly Dummy Estimates and the Unemployment Rate.



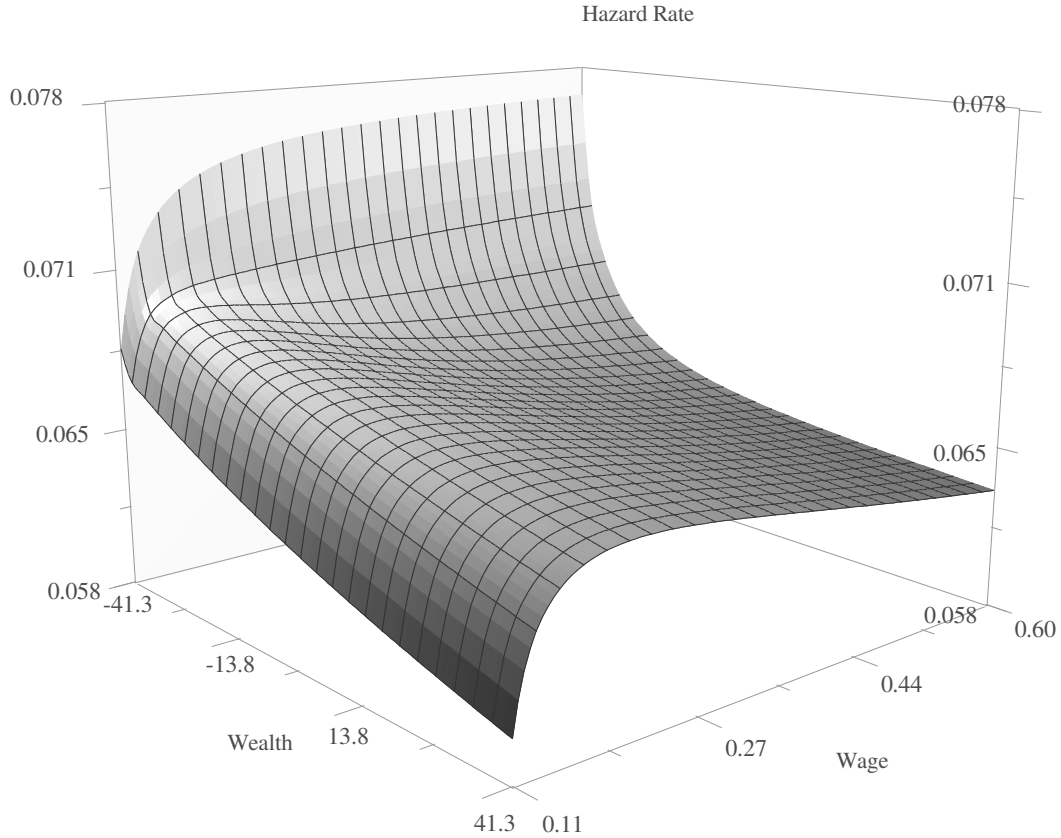
**Note:** Yearly dummy estimates and 95% confidence bounds drawn on left axis with solid lines. The unemployment rate is drawn on the right axis with a dashed line.

The wealth and wage effects are shown in isolation in figure 3 where reduced form estimates of the wealth and wage effects are also included with dashed lines. The reduced form estimates are based on a standard proportional hazard model and are discussed in detail in Lentz and Tranæs (2002).

The model imposes a negative wealth effect on the hazard. Thus, if a positive relationship exists in the data the best the model can do is to eliminate the wealth effect altogether. However, from the reduced form estimation in Lentz and Tranæs (2002), it is known that there is an overall positive relationship between duration and wealth in the data and the structural model successfully captures this relationship. The fundamental positive relationship between wealth and unemployment duration is also found on Dutch and French data in Bloemen and Stancaelli (2001) and Algan, Chéron, Hairault, and Langot (2001), respectively.

Reduced form estimations in Lentz and Tranæs (2002) confirm that the relationship between unemployment duration and wages is non-monotone. This is done under a wide variety of wage measures. It is seen that the structural estimation captures this relationship as well: For workers

Figure 2: Estimated Unemployment Hazard Rate.

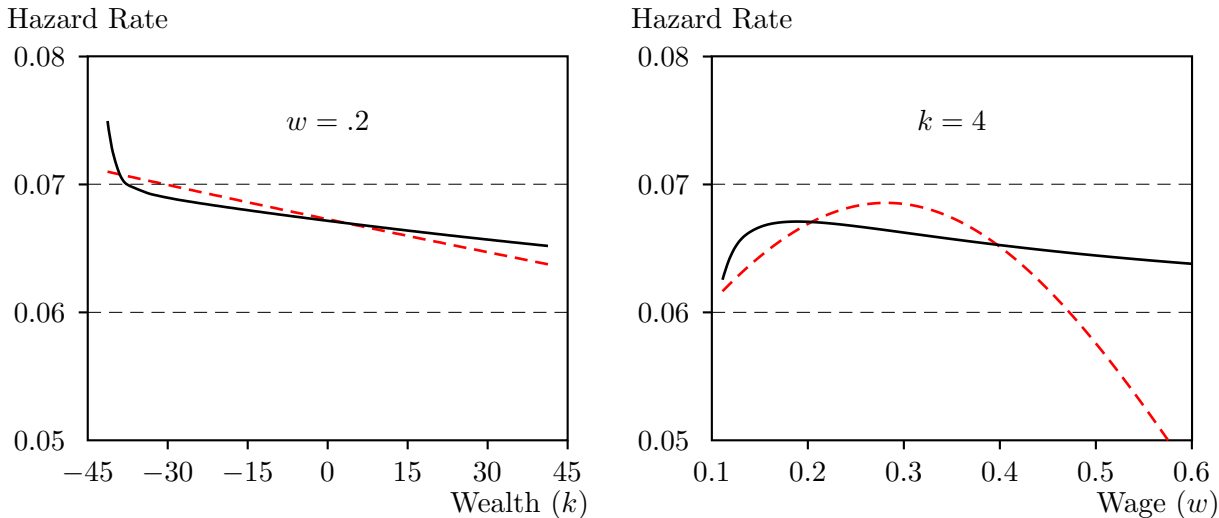


with sufficiently high wealth levels, the income effect associated with an increase in the permanent wage eventually drives down the search choice. At low wealth levels, the estimated unemployment hazard rate is monotonically increasing, though. This is driven by the model that dictates a monotone wage effect for the very low wealth levels.

It is important to note that the estimated wage effect need not be non-monotone. This is a result of the basic relationships in the data. In order to get this result, one must have a sufficient degree of risk aversion. For a lower degree of risk aversion one will find that the wage effect becomes monotone for all wealth levels. Consequently the wage relationship is an important identifier of the degree of risk aversion in the model.

In general,  $\alpha$  and  $\gamma$  are identified by both the wage and the wealth effects. A higher  $\alpha$  will tend to introduce a non-monotone wage relationship but also a stronger wealth effect. The estimate in the analysis is determined by both a positive relationship between spell duration and wealth and the non-monotone relationship between spell duration and the wage of the worker.  $\gamma$  primarily affects

Figure 3: Estimated Unemployment Hazard Rate (other 2 dimensions fixed)



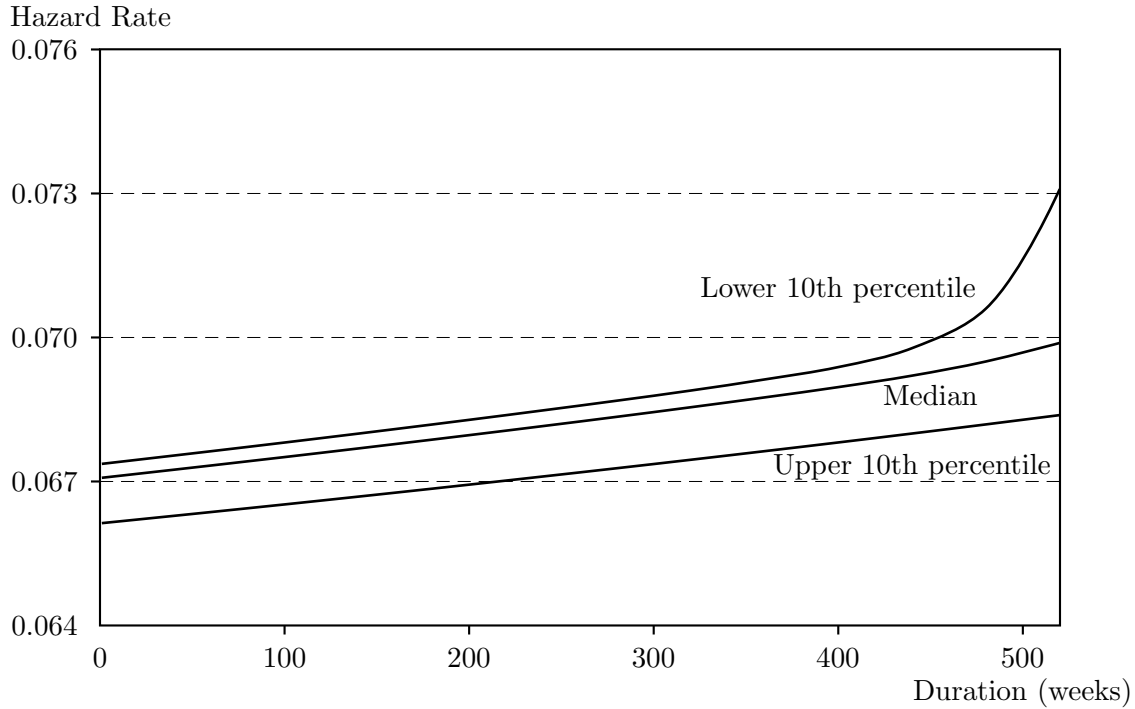
**Note:** Model estimated hazard rate drawn in solid pen. Reduced form estimates based on a standard proportional hazard model are drawn in dashed line.

the magnitude of the response to changes in wealth and wages. Data show that the magnitude of the change in the observed unemployment hazard rate over the wealth and wage dimensions is relatively small which consequently yields a high  $\gamma$  estimate.

Turning to the duration dependence of the search decision, it is seen in figure 4, that the estimated effect is rather small. For an average individual who holds wealth corresponding to the median of the wealth distribution, the hazard rate changes only from 6.70% to 6.98% over a 10 year long unemployment spell. In the first year, the hazard rate rises only to 6.73% implying a change of only 0.03 percentage points. This reflects an estimated wealth change that is not very big as well as the high  $\gamma$  estimate. Had one assumed a lower interest rate, the dissaving would be stronger and consequently the duration dependence would be stronger. However, the high  $\gamma$  estimate does greatly limit how large the effect can be.

One can of course choose  $(k, w, \lambda)$  combinations where the duration dependence is more pronounced but the basic message here is that one should not expect the wealth effect to play a dominant role in empirical duration dependence studies. And in fact, the general result in these studies is that the overall duration dependence (as seen in the baseline hazard) is either zero or negative suggesting the impact of other stronger effects such as loss of skills, discouragement, exhausting the pool of potential jobs and the like. A similar result is shown in Lentz and Tranæs

Figure 4: Estimated Hazard Rate at Different Positions in the Wealth Distribution.



(2002), where the wealth effect on spell duration is shown to be statistically significantly negative but small in absolute terms.

## 4 Optimal Unemployment Benefit Insurance

This section will study the optimal provision of unemployment benefit insurance in the estimated model. The benefit policy is constrained to a fixed level of benefits  $b$  and a fixed proportional income tax  $\tau$ . Benefits are assumed to last indefinitely and the worker is always eligible to receive them. Thus, I will be disregarding the issue of the optimal design of a benefit profile over unemployment duration. These much more complicated benefit profiles are quite likely not politically implementable. Furthermore, the intuition emerging from Werning's (2002) and Kocherlakota's (2003) studies suggests that constraining the benefit path to be constant in a model with unobservable savings may in fact not be far from the optimally designed path. The isolated question of optimal benefit design over duration has been studied in job search models without savings in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). Here, it is shown that the cost minimizing profile is decreasing in unemployment duration. Werning (2002) and Kocherlakota (2003)

study the same optimal design problem with savings and establish that once the direct link between income and consumption is broken, the optimal income path during unemployment may very well be constant, which is the result in Kocherlakota (2003), or even in some cases upward sloping as is shown in Werning (2002).<sup>11</sup>

Throughout the optimal policy analysis in this section, it will be assumed that  $\underline{k}$  is independent of the unemployment insurance system. The lower bound has to be consistent with non-negativity of consumption for all UI schemes. The strictest condition on the lower wealth bound is therefore imposed by the  $b = 0$  UI scheme which implies that  $\underline{k} = 0$ . The independence condition on the lower wealth bound is a useful simplification in what follows. But it also allows a focus on the effects of the UI system in isolation from any changes in the credit conditions. In future research one can contrast the results below to results in which the lower wealth bound is UI dependent to quantify the effects of UI via this channel.

#### 4.1 Individual Unemployment Benefit Insurance Schemes

In this section, I will study optimal unemployment benefit provision from an individual level taking the current state as well as the type of the worker as an initial condition. The income tax rate is set such that the worker's expected future stream of discounted tax payments exactly balance her expected future stream of discounted benefit receipts. In this sense, the system can be said to be actuarially fair. It is assumed that the worker can commit to this scheme and that it will remain fixed even as the state of the worker changes. This, in spite of the fact that there are cases where both parties of the contract would happily dissolve it to sign a new one. However, the design of a more sophisticated contract is beyond the scope of this paper.

The individually optimal unemployment benefit system is found by taking the state of an individual worker as given and determine the preferred replacement rate subject to the constraint that the discounted tax and benefit streams balance each other. The design problem can be stated

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<sup>11</sup>The discussion in Kocherlakota (2003) shows that the optimal design problem with unobservable savings is still very much an open question and that current methodology may in fact not allow computationally feasible strategies to compute optimal benefit paths even in the case of no state or type heterogeneity. Needless to say, once one allows for state or type heterogeneity, the problem does not become any easier.

Table 3: Optimal Individual Benefit Schemes ( $\lambda = .0983$ ).

|           | Employed         |          |           |                 |          |           |
|-----------|------------------|----------|-----------|-----------------|----------|-----------|
|           | Replacement Rate |          |           | Income Tax Rate |          |           |
|           | $k = 0$          | $k = 10$ | $k = 100$ | $k = 0$         | $k = 10$ | $k = 100$ |
| $w = .20$ | 68.0             | 47.1     | 28.7      | 4.4             | 3.0      | 1.9       |
| $w = .40$ | 68.2             | 56.1     | 32.9      | 4.6             | 3.8      | 2.3       |

|           | Unemployed       |          |           |                 |          |           |
|-----------|------------------|----------|-----------|-----------------|----------|-----------|
|           | Replacement Rate |          |           | Income Tax Rate |          |           |
|           | $k = 0$          | $k = 10$ | $k = 100$ | $k = 0$         | $k = 10$ | $k = 100$ |
| $w = .20$ | 75.5             | 48.1     | 27.3      | 6.0             | 3.8      | 2.3       |
| $w = .40$ | 75.4             | 58.0     | 32.0      | 6.4             | 4.8      | 2.6       |

as follows:

$$\max_{b, \tau} V_i(k), i \in \{e, u\}$$

$$st : V_e(k) = \max_{k' \in \Gamma_w(k)} \left\{ u \left( (1+r)k + (1-\tau)w - k' \right) + \frac{(1-\delta)V_e(k') + \delta V_u(k')}{1+\rho} \right\} \quad (6)$$

$$V_u(k) = \max_{k' \in \Gamma_b(k), s \geq 0} \left\{ u \left( (1+r)k + (1-\tau)b - k' \right) - e(s) + \frac{\mu(\lambda s)V_e(k') + (1-\mu(\lambda s))V_u(k')}{1+\rho} \right\} \quad (7)$$

$$B_i(k) = 0$$

$$B_e(k) = \tau w + \frac{(1-\delta)B_e(k_e(k)) + \delta B_u(k_e(k))}{1+r}$$

$$B_u(k) = -(1-\tau)b + \frac{\mu(\lambda s(k))B_e(k_u(k)) + (1-\mu(\lambda s(k)))B_u(k_u(k))}{1+r},$$

where  $k_e(k)$ ,  $k_u(k)$  and  $s(k)$  are the policy functions associated with the maximization problems. The budget constraint,  $B_i(k) = 0$ , associated with the unemployment benefit scheme is non-trivial. It takes into account the worker's behavior under the given insurance scheme and therefore makes the worker internalize the moral hazard problem.

The optimal unemployment benefit schemes for the estimated model are shown in table 3 as a function of state and type. The scheme is represented by a replacement rate  $b/w$  and the associated income tax rate that balances the budget. In general, the preferred replacement rate is a negative function of wealth. This reflects the fact that a wealthier worker is already well insured against income fluctuations via her wealth. Therefore, a wealthy worker can avoid the distortive effects of unemployment benefits without giving up much insurance.

An unemployed worker generally prefers more insurance than an employed worker which is not

surprising since the contingency that the insurance is supposed to cover has already occurred in this case. However, for a given wealth level and replacement rate an unemployed worker faces a higher tax rate than an employed worker because benefits are being paid immediately. Thus, once the importance of the insurance aspect fades for higher wealth levels, the cost effect may actually drive the unemployed worker to demand slightly less insurance than the employed. It is important to keep in mind that a given wealth level corresponds to fewer weeks worth of consumption for a high wage type than a low wage type. Thus, a given wealth level has less insurance value for a higher wage. This explains why high wage types want more insurance than low wage types for a given wealth level.

Given the obvious link between the degree of risk aversion and the need for insurance against income fluctuations, one may wonder about the sensitivity of the results to the particular estimate in the paper. Table 4 presents a sensitivity study that varies the curvature of the utility of consumption function and the curvature of the search cost function. The replacement rates are calculated for a wage  $w = 0.2$  which is the median wage. The results in table 4 are found by setting the utility function parameters at a particular set of values and then re-estimate the model subject to the utility function parameterization in question. The optimal replacement rates are then calculated for the average offer arrival rate of the new estimation. This way the unemployment risk is held constant over changes in the utility function parameters and the changes in the optimal replacement rates consequently only reflect the changes in the utility function parameterization.

The middle row of table 4 displays the sensitivity of the results in table 3 to a change in the curvature of the search cost function. In this case, the search cost function has been assumed to be quadratic and as such, the moral hazard problem is now significantly more pronounced. And indeed, the optimal replacement rates are significantly lower because an increase in the unemployment benefits has a greater impact on the worker's search choice. The sensitivity of optimal UI to the moral hazard dimension of the problem is also analyzed in Hansen and Imrohorglu (1992) where a similar type of sensitivity is found. The top and bottom rows then vary the degree of risk aversion. Generally, one finds the natural result that less risk averse workers prefer less insurance.

Overall, one finds high optimal replacement rates for the estimated model. In fact, the optimal replacement rates are within the range of the high actual replacement rates in the Danish unemployment benefit system. This is primarily due to the high estimate of the curvature of the



Table 4: Sensitivity Study ( $w = 0.2$ ).

|  | Employed         |          |           |                 |          |           |
|--|------------------|----------|-----------|-----------------|----------|-----------|
|  | Replacement Rate |          |           | Income Tax Rate |          |           |
|  | $k = 0$          | $k = 10$ | $k = 100$ | $k = 0$         | $k = 10$ | $k = 100$ |
| $\begin{pmatrix} \alpha = 1.5 \\ \gamma = 2.0 \\ \lambda = 0.83 \end{pmatrix}$ | 30.95            | 11.61    | 3.94      | 1.28            | 0.46     | 0.19      |
| $\begin{pmatrix} \alpha = 2.2 \\ \gamma = 2.0 \\ \lambda = 0.45 \end{pmatrix}$ | 36.71            | 14.74    | 6.23      | 1.64            | 0.63     | 0.37      |
| $\begin{pmatrix} \alpha = 2.5 \\ \gamma = 2.0 \\ \lambda = 0.33 \end{pmatrix}$ | 39.19            | 14.93    | 7.42      | 1.80            | 0.65     | 0.48      |

|  | Unemployed       |          |           |                 |          |           |
|--|------------------|----------|-----------|-----------------|----------|-----------|
|  | Replacement Rate |          |           | Income Tax Rate |          |           |
|  | $k = 0$          | $k = 10$ | $k = 100$ | $k = 0$         | $k = 10$ | $k = 100$ |
| $\begin{pmatrix} \alpha = 1.5 \\ \gamma = 2.0 \\ \lambda = 0.83 \end{pmatrix}$ | 45.89            | 12.60    | 3.40      | 2.54            | 0.62     | 0.21      |
| $\begin{pmatrix} \alpha = 2.2 \\ \gamma = 2.0 \\ \lambda = 0.45 \end{pmatrix}$ | 51.84            | 16.20    | 5.50      | 3.11            | 0.85     | 0.40      |
| $\begin{pmatrix} \alpha = 2.5 \\ \gamma = 2.0 \\ \lambda = 0.33 \end{pmatrix}$ | 54.28            | 17.55    | 6.70      | 3.56            | 0.95     | 0.53      |

search cost function. The high degree of curvature implies that the worker's search choice is quite insensitive to changes in incentives. Thus, one can offer a high level of insurance without worrying that search choices are being distorted. As was seen in table 4, the optimal replacement rates drop dramatically for higher levels of moral hazard.

It is also seen that the preferred replacement rates are insensitive to the wage of the worker. This result suggests that an optimal benefit system should offer a constant replacement rate to all wage types. This is in stark contrast to the current Danish system in which the replacement rate is highly wage sensitive. Thus at a replacement rate of 90%, Danish low wage workers seem to be over-insured and with replacement rates as low or even lower than 10%, high wage workers look to be under-insured.

## 4.2 Group Wide Unemployment Benefit Insurance Schemes

After the exploration of optimal individual unemployment benefit insurance schemes in the previous section, the analysis now turns to optimal group wide unemployment benefit insurance. I determine optimal replacement rates for groups of identical type workers according to a utilitarian social welfare criterion. Thus, the wage and offer arrival rate are identical for all workers in the insurance scheme but the state varies according to each individual's employment history. The benefit scheme is constrained to satisfy inter-temporal budget balance.

An important aspect of the analysis is that it includes full transitional dynamics from the initial state distribution of the group to the steady state of the new benefit scheme. Once wealth is part of the analysis, transitional dynamics must be included so as to avoid a serious downward bias in the optimal replacement rate results that would otherwise occur if one were to simply maximize social welfare across steady states. It is assumed that the social planner can commit to the benefit scheme which is assumed to be constant even as the group's state distribution is changing.

Consider a group of identical type workers who face an unemployment insurance scheme characterized by a constant benefit level  $b$  and a proportional income tax  $\tau$ . The worker's problem is represented by the Bellman equations (6)-(7). Denote by  $\varphi(\cdot) : \{e, u\} \times [\underline{k}, \bar{k}] \rightarrow \mathbb{R}_+$  the state distribution for the group. (6) and (7) together induce a mapping  $\Psi_{b,\tau}$  that maps a given state distribution into next period's state distribution according to the policy functions and parameters of the model. Given an initial state distribution  $\varphi$ , the state distribution density for state  $y \in \{e, u\} \times [\underline{k}, \bar{k}]$  in the next period can then be written as,

$$(\Psi_{b,\tau}\varphi)(y) = \int_{x \in \{e,u\} \times [\underline{k}, \bar{k}]} P(x, y|b, \tau) \varphi(x) dx,$$

where  $P(x, y|b, \tau)$  is the transition function probability of moving from state  $x$  into state  $y$  given the unemployment benefit scheme  $(b, \tau)$ . The fact that  $\Psi_{b,\tau}$  depends on other parameters than  $(b, \tau)$  has been suppressed for notational convenience.

Denote by  $\varphi^0(\cdot)$  the initial state distribution of the group. Employing a simple utilitarian welfare criterion, the basic design problem can then be stated as,

$$\begin{aligned} \max_{b,\tau} \quad & \int_k [V_e(k|b, \tau) \varphi^0(e, k) + V_u(k|b, \tau) \varphi^0(u, k)] dk \\ \text{s.t. : } & (b, \tau) \in \Theta, \end{aligned} \tag{8}$$

where  $\Theta$  is a set that restricts the choice of benefit scheme. By stating the welfare criterion in terms of value functions, one correctly takes into account the dynamic adjustment that each individual will face given the current state of the individual and subject to the new benefit-tax scheme.

In the following,  $\Theta$  will be defined as the set of inter-temporally balanced benefit-tax schemes,

$$\Theta = \left\{ (b, \tau) \in [0, w) \times [0, 1] : \sum_{i=0}^{\infty} (1+r)^{-i} \int_k (\tau w \varphi_{b,\tau}^i(e, k) dk - (1-\tau) b \varphi_{b,\tau}^i(u, k)) dk = 0 \right\}, \quad (9)$$

where

$$\begin{aligned} \varphi_{b,\tau}^{i+1} &= \Psi_{b,\tau} \varphi_{b,\tau}^i, \quad i = 1, 2, \dots \\ \varphi_{b,\tau}^1 &= \Psi_{b,\tau} \varphi^0. \end{aligned}$$

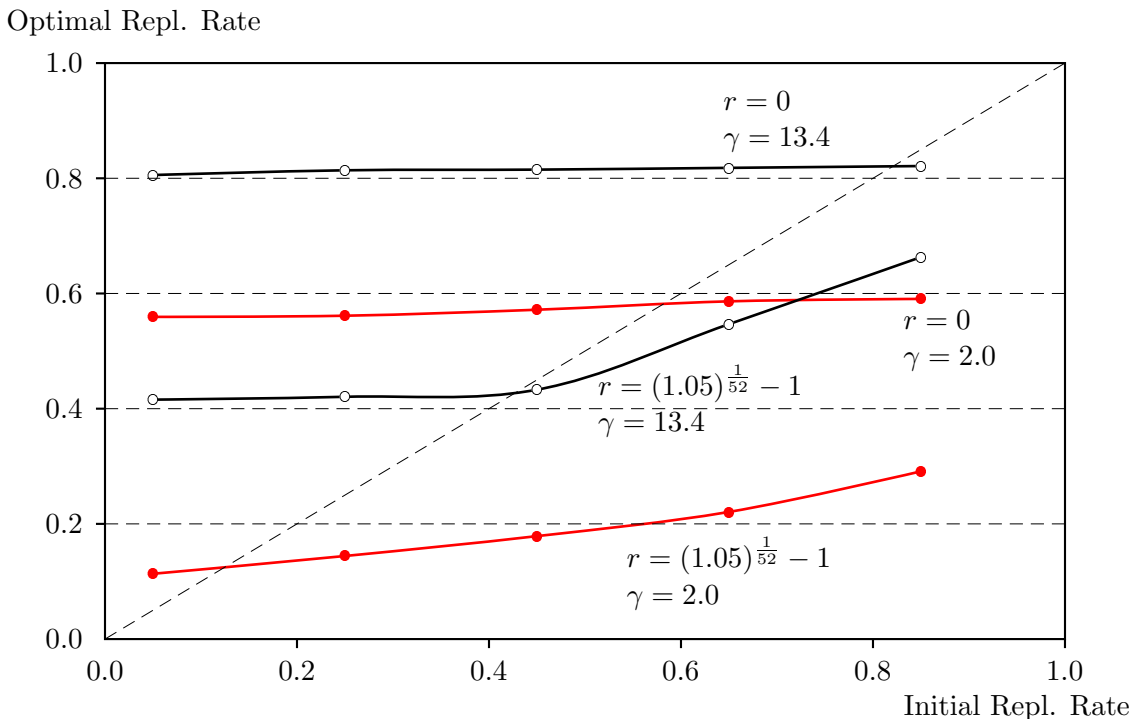
The set of balanced budgets is found by Monte Carlo methods. Clearly, for a given benefit level there may be multiple tax rates that solve the budget problem. The one to maximize welfare is always the minimum tax rate.

Unlike in the previous section, the individual preference for insurance under this scheme will be affected by subsidization issues. It is clear that depending on the current state of the worker, the individual worker's discounted stream of benefits may not exactly balance her discounted stream of taxes. In particular, currently employed workers will expect to be subsidizing the currently unemployed and perhaps more surprising, the currently poor workers will be subsidizing the currently wealthy because the wealthy workers have lower unemployment hazard rates.

Figure 5 shows a set of optimal replacement rates calculated for the group of  $w = 0.2$  type workers. The horizontal axis represents the initial state of the economy. It is stated in terms of a replacement rate. The initial state is the steady state associated with the replacement rate and the tax rate that balances the steady state budget for the given replacement rate. The vertical axis is the optimal replacement rate given the initial state distribution.

Replacement rates are found for the estimated curvature of the search cost function as well as the case of quadratic search costs ( $\gamma = 2$ ). The offer arrival rate  $\lambda$  is set at the average estimated level for each given search cost specification. Thus, for the case where  $\gamma = 13.36$  the base offer arrival rate is set at  $\lambda = .098$  whereas the case where  $\gamma = 2$ , the base offer arrival rate is set at  $\lambda = .45$ . Furthermore, the sensitivity of the replacement rates to the interest rate is also highlighted by calculating optimal replacement rates for annual interest rates of five and zero percent. The subjective discount rate is constant at an annual rate of 5.1%.

Figure 5: Optimal Group Wide Replacement Rates.



**Note:** The curves show the solution to the optimal utilitarian social planner problem set out in (8) subject to the inter-temporal budget constraint (9). The initial state  $\varphi^0$  is the balanced budget steady state distribution associated with the initial replacement rate. All curves are determined for the group of  $w = 0.2$  type workers.

Generally, the optimal replacement rate is increasing in the initial replacement rate which is due to the fact that workers hold less wealth in higher replacement rate initial states. Thus, given the lower initial wealth holdings, workers have a higher preference for unemployment benefit insurance.

The gap between the interest rate and the subjective discount rate is effectively the cost of using savings as a self-insurance instrument. While previous studies of optimal unemployment insurance have noted the importance of whether or not workers have access to savings as a self-insurance instrument, the cost of using savings as insurance has not been noted. It is seen that the optimal replacement rates are highly sensitive to the interest rate assumption. At zero percent, self-insurance is expensive and workers substitute into unemployment benefits. As the interest rate is increased to almost equal the subjective discount rate, optimal replacement rates drop dramatically.

The intersect between the replacement rate curve and the diagonal has special significance. Here, the social planner does not wish to change the initial state. Based on this point, the optimal replacement rate for the estimated model is 43%. If the interest rate were zero, the estimated

replacement rate would be 82%. It is worth noting that these replacement rates are in fact not too different from the observed replacement rates in Denmark. For a quadratic search cost function, the estimated replacement rate is as low as 14%. In this particular case, the use of self-insurance is sufficiently cheap and there is enough moral hazard associated with the use of unemployment benefits that workers choose to rely almost exclusively on self-insurance.

Previous literature on optimal unemployment insurance typically determines optimal benefit policy by maximizing the social welfare criterion across steady states associated with different balanced benefit schemes. The problem can be written as:

$$\begin{aligned} \max_b \int_k & \left[ V_e(k|b, \tau^*(b)) \varphi_{b, \tau^*(b)}^*(e, k) + V_u(k|b, \tau^*(b)) \varphi_{b, \tau^*(b)}^*(u, k) \right] dk & (10) \\ \text{s.t. : } \tau^*(b) &= \min_{\tau} \left\{ \tau \in [0, 1] : \int_k (\tau w \varphi_{b, \tau}^*(e, k) dk - (1 - \tau) b \varphi_{b, \tau}^*(u, k)) dk = 0 \right\} \\ \varphi_{b, \tau}^* &= \Psi_{b, \tau} \varphi_{b, \tau}^* \end{aligned}$$

where  $\varphi_{b, \tau}^*$  is the ergodic state distribution for the benefit scheme characterized by  $(b, \tau)$ .  $\tau^*(b)$  is the minimum tax rate that balances the steady state budget.

In the case where savings are included in the analysis, simple maximization across steady states will introduce a significant downward bias in the results. To illustrate the bias, the replacement rate that maximizes (10) is found to be 45% in the quadratic search cost case and given a zero interest rate. As seen in figure 5, this compares to a 57% optimal replacement rate when transitional dynamics are included. In the 5% interest rate case, the replacement rate that maximizes (10) was found to be less than 0.5%. Including transitional dynamics for this case yields an optimal replacement rate of 14%.

The bias arises because the steady state wealth distribution is a function of the benefit scheme. Lower benefits result in higher wealth holdings. For interest rates close to the subjective discount rate, the sensitivity of the wealth distribution to the benefit level can be so high that lowering benefits while holding taxes fixed can even increase the steady state social welfare criterion! While any worker is obviously worse off for a given wealth level, the steady state wealth distribution can increase so much that the evaluation of worker welfare at the higher wealth levels more than offsets the poorer insurance. The erroneous conclusion that overall worker welfare has increased as a result of the lower benefit level is based on the simple fact that the consumption cost of building up the higher wealth levels in the new steady state has been ignored. These results emphasize the

importance of including transitional dynamics in the analysis.<sup>12</sup>

One can employ other aggregation mechanisms. A majority voting game is one such mechanism. Here, the median voter decides on the replacement rate for the group subject to the budget constraint (9). The median voter preference is determined by establishing a replacement rate preference for each possible worker state and then based on the given state distribution one obtains the replacement rate preference distribution for the group. A steady state majority voting equilibrium is defined as a steady state in which the median voter prefers exactly the existing replacement rate. One can appeal to the median voter theorem because the individual replacement rate preference turns out to be single peaked. The analysis is an interesting complementary analysis to Wright (1986). Here, exogenously given heterogeneity over re-employment probabilities generates a preference distribution over unemployment benefit levels. This is similar to the model at hand but in this case the heterogeneity is generated endogenously.

The results of this type of analysis are quite similar to the outcome of the utilitarian aggregation scheme.<sup>13</sup> The latter does yield somewhat higher replacement rate results since it places more weight on the poorly insured due to their very low levels of utility. However, it is important to keep in mind that the majority voting scheme as it has been presented here suffers from the important limitation that it assumes commitment to the chosen unemployment benefit level. A more satisfactory but also considerably more complicated analysis will allow for repeated voting.

## 5 Conclusion

This paper has estimated a job search model with savings and subsequently determined optimal benefit policy for the estimated model. Depending on the cost of self-insuring via savings, the optimal replacement rate ranges between 43% and 82%. If savings carry a low return relative to the subjective discount rate, self-insurance via savings will be expensive and consequently workers will want to rely more on unemployment benefit insurance to guard against income fluctuations.

The estimation strategy relates observed unemployment durations to the model implied unemployment hazard rate which the worker can affect by the choice of search intensity. Data show a positive relationship between wealth holdings and unemployment duration which the model suc-

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<sup>12</sup>A similar point is emphasized in Joseph and Weitzenblum (2000)

<sup>13</sup>The details of the analysis and the results are not presented here but can be obtained from the author by request.

cessfully captures as a result of worker search behavior; wealthier workers search less. The U-shaped relationship between unemployment duration and the worker's wage is also successfully captured as a straightforward search choice response to variation in wage expectations in a search model with savings. The savings aspect is important in this respect since search models without savings cannot generate a non-monotone relationship between the search choice and the wage. The relationships between unemployment duration and other worker characteristics are explained as a combination of a market effect captured in the individual offer arrival function parameter  $\lambda$  and the worker's search response to the market effect.

The estimated utility of consumption function implies a constant relative risk aversion coefficient of 2.21 and the estimated search cost function displays a high degree of curvature. While Danish workers respond to changes in economic incentives in ways that are consistent with the model, the high degree of curvature in the estimated search cost function reflects that the unemployment hazard rate response is small in magnitude. As a consequence, while the model does imply positive duration dependence of the unemployment hazard rate, the effect will be so small in magnitude that it will likely be dominated by other effects. Indeed, most duration dependence analyses show a slightly negative or zero trend in the baseline hazard rate over unemployment duration.

The determination of optimal unemployment benefit level policy relies purely on a trade-off between providing insurance against consumption fluctuation and the moral hazard problem associated with reducing the worker's incentives to search back into employment. The high degree of curvature in the estimated search cost function implies a low level of moral hazard in the model and consequently yields relatively high replacement rate results even though workers have access to self-insurance via savings. For higher levels of moral hazard, the paper showed that the optimal replacement rate drops as low as 14% as workers switch into savings as their main insurance vehicle.

The analysis has disregarded life cycle issues. In particular, it is possible that steady state wealth holdings could be greater if the model were to allow for life cycle savings in addition to the precautionary savings considered so far, and consequently that optimal unemployment benefit levels would be lower. Allowing for positive higher order moments in the wage offer distribution could have a similar type of effect in that workers may increase their savings in an attempt to insure against the added income uncertainty. However, in contrast to the very temporary nature of the income shock associated with unemployment, the income shock associated with uncertainty about

the wage earned on the job is much more permanent in nature. Savings are not a particularly effective insurance instrument against permanent income shocks and as such, it is not obvious whether this source of added income uncertainty would affect precautionary savings significantly.

The model analysis is partial. In particular, each individual's wage offer distribution is exogenously given which closes a potentially important equilibrium feedback channel for policies such as those pertaining to unemployment benefits. Extensions of the analysis to general equilibrium job search models with wage dispersion is a particularly interesting avenue of future research. One such example is Acemoglu and Shimer (1999) where efficiency aspects of unemployment benefit policy that arise in this setup are highlighted.

The policy analysis emphasized the importance of including transitional dynamics in the analysis. Failure to do so, will introduce a significant downward bias in the results because the wealth distribution in the economy depends on the benefit scheme. A lower level of benefits will result in greater wealth holdings in steady state. By not including transitional dynamics in the analysis one will be ignoring the consumption cost associated with building these greater wealth holdings and consequently lower benefit levels look more attractive. The analysis showed that the downward bias can be quite large even in the case where the interest rate is zero and the steady state wealth distribution is less sensitive to the benefit scheme.



# A Appendix

## A.1 Policy Function Characterization

In the model (1)-(2), wealth effects can enter via two sources; the utility function specification and the lower wealth bound. The lower wealth bound will in isolation unambiguously imply more search the closer a worker gets to the bound since the worker is running out of insurance and the state of unemployment is getting harder and harder to face. Turning to the utility function specification, special attention should be given to the assumptions made about the relationship between search costs and the consumption level. If one assumes separability as is the case in this paper, wealth will affect the search intensity choice negatively for any concave choice of  $u(\cdot)$  and any convex choice of  $e(\cdot)$ . If one allows the level of consumption to affect the marginal utility cost of search, depending on the exact utility specification one can get non-monotone relationships between wealth and the search decision or one can entirely eliminate any wealth effect whatsoever. For an example of the latter in a search intensity model, see Flemming (1978). When the search choice is a reservation wage, the marginal utility costs and gains associated with changing the search decision are automatically tied to the consumption level and the degree of absolute risk aversion becomes the deciding factor. Danforth (1979) and Acemoglu and Shimer (1999) are examples of this.

Generally, the difference between  $u(c_e(k))$  and  $u(c_u(k))$  is reduced as wealth increases. The utility difference falls because at higher wealth levels the worker consumes more and smooths consumption better. Consequently, there are fewer incentives to search at higher wealth levels. If search costs are unaffected by wealth holdings it directly follows that the search choice must depend negatively on wealth because gains to search are decreasing and the costs are unaffected. If costs are also affected by wealth holdings, the search choice can become a non-monotone function of wealth.

One cannot directly establish concavity of the value functions in the model which complicates the analysis considerably. Lentz and Tranæs (2002) show that concavity can be obtained by the introduction of a simple wealth lottery.<sup>14</sup> Once concavity has been established one can show that it must be that  $c_e(k) > c_u(k)$  for all  $k$  which provides the fundamental part of the characterization

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<sup>14</sup>The wealth lottery in question has zero expected value and will only be accepted if there is a convexity in the value function. Intuitively, the lottery has the effect of “smoothing” out any convexities in the value function by making it linear over the intervals where it would otherwise be convex.

of the relationship between search and wealth. In the paper, any such sufficient condition have not been made. However, the possibility of convexities in the value functions never seems to be an issue in numerical simulations of the model. All simulations have produced globally concave value functions.

Another relationship of interest is the one between the search decision and the income of the worker. In a job search model with no consumption smoothing, the effect is trivial: A higher wage or a lower benefit level will either increase consumption when employed or lower the consumption when unemployed while the other consumption level remains unchanged. This will unambiguously make the worker search harder. However, when the worker has access to savings, matters are more complex. In this case, one must consider both a substitution and income effect. The substitution effect dictates that the worker search harder when the worker's permanent wage increases. This is due to the fact that the difference between consumption when unemployed and consumption when employed has increased. However, the income effect of an increased permanent wage implies that both consumption levels increase which has the isolated effect of lowering the utility difference between the two paths and consequently a lower search choice. Thus, the effect on the search choice from an increase in future wages is ambiguous except when wealth is at the lower bound and there cannot be an income effect on the consumption level when unemployed. In this case, no consumption smoothing is possible and a higher wage will unambiguously lead to a higher search choice.

It is tempting to extend the above ambiguity to the case of changes in benefits. However in this case, the two effects mentioned point in the same direction. An unemployed worker who faces lower benefits will see both an increased difference between  $c_e$  and  $c_u$  as well as a lower  $c_u$ . The intuitive arguments above are summarized in corollary 1.

**Proposition 1** *In the model described by (1) and (2), given concavity of  $V_e(\cdot)$  and  $V_u(\cdot)$  and that  $r \leq \rho$ , it must be that  $\partial s(\underline{k}) / \partial w > 0$ . Furthermore,  $\partial s(k) / \partial b < 0, \forall k \in [\underline{k}, \bar{k}]$ .*

**Proof.** Using first order and envelope conditions, one can show the following comparative

statics:

$$\frac{\partial s(k)}{\partial w} = \frac{[V'_{e,w}(k_u(k)) - V'_{u,w}(k_u(k))] + [V'_{e,k}(k_u(k)) - V'_{u,k}(k_u(k))] k'_{u,w}(k)}{(1 + \rho) e''(s(k)) / \lambda e^{-\lambda s(k)} + \lambda [V_e(k_u(k)) - V_u(k_u(k))]} \quad (11)$$

$$\frac{\partial s(k)}{\partial b} = \frac{[V'_{e,b}(k_u(k)) - V'_{u,b}(k_u(k))] + [V'_{e,k}(k_u(k)) - V'_{u,k}(k_u(k))] k'_{u,b}(k)}{(1 + \rho) e''(s(k)) / \lambda e^{-\lambda s(k)} + \lambda [V_e(k_u(k)) - V_u(k_u(k))]} \quad (12)$$

Given concavity of  $V_e(\cdot)$  and  $V_u(\cdot)$  one can show that  $V'_{e,k}(k_u(k)) - V'_{u,k}(k_u(k)) < 0$  for all  $k$  and that  $k'_{u,w}(k) < 0$  and  $k'_{u,b}(k) > 0$  for all  $k > \underline{k}$ . And finally that  $k'_{u,w}(\underline{k}) = k'_{u,b}(\underline{k}) = 0$ . See Lentz and Tranæs (2002) for details. By the envelope theorem, it follows that:

$$\begin{aligned} V'_{u,w}(\underline{k}) &= \frac{\mu(\lambda s(\underline{k})) V'_{e,w}(\underline{k}) + (1 - \mu(\lambda s(\underline{k}))) V'_{u,w}(\underline{k})}{1 + \rho} \\ &\Downarrow \\ V'_{e,w}(\underline{k}) - V'_{u,w}(\underline{k}) &= V'_{u,w}(\underline{k}) \frac{\rho}{(1 + \rho) \mu(\lambda s(\underline{k}))} > 0. \end{aligned}$$

Combining this result with (11), one immediately gets that  $\partial s(\underline{k}) / \partial w > 0$ . For  $k > \underline{k}$ , the sign of  $V'_{e,w}(k_u(k)) - V'_{u,w}(k_u(k))$  becomes ambiguous. Simulations suggest that with enough curvature in  $u(\cdot)$  this term can for sufficiently high wealth and wage levels become negative and imply an overall negative impact on the search choice from increases in the wage. Turning to the effect of changes in the benefit level, using the envelope conditions it follows that:

$$V'_{e,b}(k) = \frac{(1 - \delta) V'_{e,b}(k_e(k)) + \delta V'_{u,b}(k_e(k))}{1 + \rho} \quad (13)$$

$$V'_{u,b}(k) = u'(c_u(k)) + \frac{\mu(\lambda s(k)) V'_{e,b}(k_u(k)) + (1 - \mu(\lambda s(k))) V'_{u,b}(k_u(k))}{1 + \rho}. \quad (14)$$

Since  $V''_{u,kb} = u''(c_u(k))(1 + r) c'_{u,b}(k) < 0$  as well as  $V''_{e,kb} = u''(c_e(k))(1 + r) c'_{e,b}(k) < 0$  and that  $k_u(k) < k_e(k)$ , it follows that  $V'_{u,b}(k_u(k)) > V'_{u,b}(k_e(k))$  and  $V'_{e,b}(k_u(k)) > V'_{e,b}(k_e(k))$ . Since, (1)-(2) forms a contraction mapping over a compact state space, it is known that a unique fix point must exist and that if one can show that the mapping maps a set into a subset of itself, then the fix point must lie within that subset. This fact is used to show that the fix point must be characterized by  $V'_{e,b}(k) - V'_{u,b}(k) < 0$ . Assume  $V'_{e,b}(k) - V'_{u,b}(k) \leq 0$ . From (13) and (14) it follows that:

$$\begin{aligned} V'_{e,b}(k) - V'_{u,b}(k) &< (1 - \delta - \mu(\lambda s(k))) \frac{V'_{e,b}(k_e(k)) - V'_{u,b}(k_e(k))}{1 + \rho} - u'(c_u(k)) \\ &< 0. \end{aligned}$$

Thus, the contraction mapping maps the set of function characterized by  $V'_{e,b}(k) - V'_{u,b}(k) \leq 0$  into a set of functions characterized by  $V'_{e,b}(k) - V'_{u,b}(k) < 0$ . Thus, it is proven that it must be that the

fix point is characterized by  $V'_{e,b}(k) - V'_{u,b}(k) < 0$ . Combining this with (12), one gets the result that  $\partial s(k)/\partial b < 0, \forall k \in [\underline{k}, \bar{k}]$ . ■

## A.2 Numerical Model Solution

The model is solved via value function iteration where a cubic spline projection is used to approximate the value functions.<sup>15</sup> The value function iteration is accelerated by performing a number of value function iterations for fixed policy functions between policy function updates.<sup>16</sup> For more on this see for example Judd (1998).

Denote by  $\tilde{V}_e(k)$  and  $\tilde{V}_u(k)$  the cubic spline projections that approximate the fix point of the mappings in (1) and (2). Based on the spline projections, one can then evaluate the two relevant policy function  $s(k)$  and  $k_u(k)$  for any  $k$  by simply solving the maximization problem in (2) using  $\tilde{V}_e(k)$  and  $\tilde{V}_u(k)$ .

In order to evaluate the likelihood function (4) for a given  $\theta$ , one must be able to evaluate the search intensity choice over 4 dimensions: The wealth level at the outset of the spell, the wage, the offer arrival rate and the duration of the spell to date. To this end, the model is first solved for each point in the grid  $\{w_i, \lambda_j\}_{i,j=1}^M$ . This amounts to  $M^2$  times that one must solve the model. For each grid point  $(w, \lambda)$ , the policy functions are then evaluated on a wealth grid  $\{k_l\}_{l=1}^M$ . The procedure leads to a full evaluation of the search and savings policy functions on the 3-dimensional grid  $\{k_i, w_j, \lambda_l\}_{i,j,l=1}^M$ . Interpolation is subsequently performed between the grid points which yields the continuous approximations to the policy functions denoted by  $\tilde{s}(k, w, \lambda)$  and  $\tilde{k}_u(k, w, \lambda)$ .

The duration dependence of the search choice is determined via the evolution of the worker's wealth over the unemployment spell. The wealth level at a particular time during the unemployment spell is inferred via the observed wealth level at the outset of the spell and then by simple iteration on  $\tilde{k}_u(\cdot)$ . Denote the wealth of a worker with initial wealth  $k$ , wage  $w$  and offer arrival rate  $\lambda$  at

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<sup>15</sup>Discretization methods are in this case impractical given the potentially very wide support of the state space. The upper bound on the ergodic wealth distribution can be very high when the interest rate is close to the subjective discount rate. Experimentation suggests that to properly capture the savings decisions via discretization methods, one will need such a large number of points in the state space that computation times become unacceptable. On the other hand, projection methods based on cubic splines turn out to be extremely effective.

<sup>16</sup>The expensive part of the value function iteration step is to solve the maximization problem on the right hand side of (1) and (2). Even though one may need more iteration steps, one can still greatly accelerate the solution of the model by not solving for new policy functions in every iteration.

spell duration  $t$  by  $\hat{k}(k, w, \lambda, t)$ . This is naturally defined by:

$$\begin{aligned}\hat{k}(k, w, \lambda, 1) &= k \\ \hat{k}(k, w, \lambda, t) &= \tilde{k}_u\left(\hat{k}(k, w, \lambda, t-1), w, \lambda\right), t = 2, 3, \dots\end{aligned}$$

Finally, denote the search intensity for a worker with initial wealth  $k$ , wage  $w$  and offer arrival rate  $\lambda$  at duration  $t$  by  $\hat{s}(k, w, \lambda, t)$ . This is defined by:

$$\hat{s}(k, w, \lambda, t) = \tilde{s}\left(\hat{k}(k, w, \lambda, t), w, \lambda\right), t = 1, 2, \dots$$

The method of approximating the policy functions over not only wealth and wages but also over the offer arrival rate allows for easy estimation of many  $\beta$  and unobserved heterogeneity parameters because one does not have to simulate the model for any new choice of  $\beta$  and  $G$ . The only time one must re-simulate the model is when a new  $\theta$  is tried. Simulating the model is quite computation intensive and it imposes a natural limit on how many structural parameters one can include in  $\theta$ .

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