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COMPROMISING IN PARTITION FUNCTION FROM GAMES AND COOPERATION IN PERFECT EXTENSIVE FORM GAMES

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Compromising in partition function form games and cooperation in perfect extensive form games*

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Abstract

In this paper reasonable payoff intervals for players in a game in partition function form (p.f.f. game) are introduced and used to define the notion of compromisable p.f.f. game. For a compromisable p.f.f. game a compromise value is defined for which an axiomatic characterization is provided. Also a generic subclass of games in extensive form of perfect information without chance moves is introduced. For this class of perfect extensive form games there is a natural credible way to define a p.f.f. game if the players consider cooperation. It turns out that the p.f.f. games obtained in this way are compromisable.

1 Introduction

Games in partition function form (or p.f.f. games) are introduced in Thrall (1962), Thrall and Lucas (1963). In the last decade these games have received an increasing attention, especially in the environmental literature (Chander and Tulkens

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(1997), Funaki and Yamato (1999), Pham Do (2003)) because they are suitable for handling externality problems in cooperative situations. Especially various cores (Chander and Tulkens (1997), Funaki and Yamato (1999)) and Shapley values (Bolger (1983), Pham Do and Norde (2002), Potter (2000)) have been studied.

Inspired by the literature on reasonable outcomes (cf. Milnor (1952), Gerard-Varet and Zamir (1987), Tijs and Lipperts (1982)) and on compromise values for TU-games (cf. Tijs (1981), Tijs and Otten (1993), van den Brink (1994), Bergantinos and Masso (1996)) and for cooperative fuzzy games (Branzei et al. (2002)) we introduce in Section 2 of this paper for p.f.f. games reasonable payoff intervals for players cooperating in the grand coalition and use them to define compromisable p.f.f. games. For the subclass of compromisable p.f.f. games a compromise value is defined. Each coordinate of the compromise value lies in the reasonable payoff interval of the corresponding player.

In Section 3 of this paper a subclass of games in extensive form is considered, where subgame perfect equilibria (Selten (1965, 1975)) play an essential role to relate such games, when cooperation is considered, with a p.f.f. game in a natural way. For p.f.f. games obtained in this way the compromise value exists.

Section 4 concludes with some remarks.

2 Reasonable outcomes and a compromise value for games in p.f.f.

Let $N = \{1, 2, \ldots, n\}$ be the set of players and let $\Pi(N)$ be the set of possible partitions of N. So, each $\pi \in \Pi(N)$ is of the form $\{S_1, S_2, \ldots, S_k\}$, where the non-empty subcoalitions $S_1, S_2, \ldots, S_k$ in π are pairwise disjoint and $N = \cup_{r=1}^{k} S_r$.

If $\pi \in \Pi(N)$ and $S \in \pi$, then $(S \mid \pi)$ is called a *π-embedded coalition*. A game $\langle N, V \rangle$ in p.f.f. assigns to each π-embedded coalition $(S \mid \pi)$ a real number $V(S \mid \pi)$. This real number expresses the value of S given π, i.e. the amount the players in S can obtain given that the player set N splits up according to π. In the following we suppose that

$$V(N \mid \{N\}) = \max \left\{ \sum_{S \in \pi} V(S \mid \pi) \,\middle|\, \pi \in \Pi(N) \right\}.$$

Let us denote for each $i \in N$ by $\Pi^i(N)$ the set $\{\pi \in \Pi(N) \mid \{i\} \in \pi\}$. For a game $\langle N, V \rangle$ in p.f.f. we define now for each player $i \in N$ the (possibly empty) *reasonable payoff interval* $I_i = [\ell_i, u_i]$ as follows. The *lower value* ℓ_i for each $i \in N$ is given by

$$\ell_i = \min\{V(\{i\} \mid \pi) \mid \pi \in \Pi^i(N)\}$$

and the *upper value* u_i is given by

$$u_i = V(N \mid \{N\}) - \ell_{N\setminus\{i\}} ,$$

where $\ell_{N\setminus\{i\}} = \min\left\{ \sum_{S\in\pi\setminus\{i\}} V(S \mid \pi) \;\middle|\; \pi \in \Pi^i(N) \right\}$.

Note that ℓ_i is the payoff guaranteed to player i if he stays alone; whatever the partition of $N\setminus\{i\}$ in subcoalitions his payoff is at least ℓ_i and there is a partition of $N\setminus\{i\}$ where he does not get more. Similarly, $\ell_{N\setminus\{i\}}$ is the payoff guaranteed to $N\setminus\{i\}$ if player i wants to stay alone. So, the marginal contribution of $\{i\}$ to the grand coalition is at most u_i, and this is a reasonable upper bound of the payoff interval of player i.

Definition 1. Let $\langle N, V\rangle$ be a p.f.f. game and for each $i \in N$ let $I_i = [\ell_i, u_i]$ be the corresponding reasonable payoff interval for player i. Then $\langle N, V\rangle$ is a *compromisable game* if

(C.1) $\ell_i \leq u_i$ for each $i \in N$;

(C.2) $\sum_{i\in N} \ell_i \leq V(N \mid \{N\}) \leq \sum_{i\in N} u_i$.

Definition 2. Let $\langle N, V\rangle$ be a compromisable game. The *compromise value* $\psi(N, V)$ is the convex combination $\alpha(\ell_1, \ell_2, \dots, \ell_n)+(1-\alpha)(u_1, u_2, \dots, u_n)$ of ℓ and u, where α is such that $\sum_{i\in N} \psi_i(N, V) = V(N \mid \{N\})$.

Note that for a compromisable game the reasonable payoff intervals are non-empty and that the lower vector $\ell = (\ell_1, \ell_2, \dots, \ell_n)$ lies 'below' the hyperplane

$$H = \left\{ x \in \mathbb{R}^N \;\middle|\; \sum_{i=1}^{n} x_i = V(N \mid \{N\}) \right\}$$

and the upper vector $u = (u_1, u_2, \dots, u_n)$ lies 'above' this hyperplane. $\psi(N, V)$ is the point in the intersection $H \cap L(\ell, u)$, where $L(\ell, u)$ is used for the line segment with end points ℓ and u.

Example 1. Let $N = \{1, 2\}$. Let V be given by

$$V(\{1\} \mid \{\{1\}, \{2\}\}) = 6, \qquad V(\{2\} \mid \{\{1\}, \{2\}\}) = 0, \qquad V(N \mid \{N\}) = 10.$$

Then $[\ell_1, u_1] = [6, 10], [\ell_2, u_2] = [0, 4]$, and $\psi(N, V) = \alpha(6, 0) + (1 - \alpha)(10, 4) = (8, 2)$, where $\alpha = \frac{1}{2}$.

Let us denote the set of compromisable n-person p.f.f. games by $\mathcal{CPG}^N$. Now we list some properties of the compromise value on $\mathcal{CPG}^N$.

(i) (*Individual Rationality*) $\psi_i(N, V) \geq \ell_i$ for each $\langle N, V\rangle \in \mathcal{CPG}^N$ and each $i \in N$.

(ii) (*Efficiency*) $\sum_{i\in N} \psi_i(N, V) = V(N \mid \{N\})$ for each $\langle N, V\rangle \in \mathcal{CPG}^N$.

(iii) (*Additive Game Property*) Let $a \in \mathbb{R}^N$ and let $\langle N, V_a\rangle$ be the additive game corresponding to $a \in \mathbb{R}^N$ with the property that for each $\pi \in \Pi(N)$ and $S \in \pi$: $V(S \mid \pi) = \sum_{i\in S} a_i$. Then $\langle N, V_a\rangle \in \mathcal{CPG}^N$ and $\psi(N, V_a) = a$.

(iv) (*Covariance Property*) Let $\langle N, V\rangle \in \mathcal{CPG}^N$ and $\langle N, V_a\rangle$ be the additive game corresponding to $a \in \mathbb{R}^N$. Then $\langle N, V - V_a\rangle \in \mathcal{CPG}^N$ and $\psi(N, V - V_a) = \psi(N, V) - a$.

(v) (*Weak Proportionality Property*) Let $\langle N, V\rangle \in \mathcal{CPG}^N$ and let the lower vector ℓ of $\langle N, V\rangle$ be equal to $0 \in \mathbb{R}^N$. Then $\psi(N, V)$ is a multiple of the upper vector u.

We leave the proofs of the properties (i) – (v) to the reader.

The following theorem shows that the properties (ii), (iv) and (v) are characterizing properties for the compromise value ψ (cf. Tijs (1987)).

Theorem 1. *There is a unique solution $\varphi : \mathcal{CPG}^N \to \mathbb{R}^N$ with the properties Efficiency, Covariance and Weak Proportionality, and it is the compromise value ψ.*

Proof. We know already that ψ possesses the three properties. Take $\varphi : \mathcal{CPG}^N \to \mathbb{R}^N$ satisfying the three properties. We have to prove that $\varphi(N, V) = \psi(N, V)$ for each $\langle N, V\rangle \in \mathcal{CPG}^N$. Take $\langle N, V\rangle \in \mathcal{CPG}^N$. Let $a \in \mathbb{R}^N$ be the vector with a_i equal to the lower value ℓ_i for each $i \in N$. Then for $\langle N, V - V_a\rangle \in \mathcal{CPG}^N$ the lower value is 0. By the Efficiency Property and the Weak Proportionality Property for ψ and φ we obtain

(1) $\psi(N, V - V_a) = \varphi(N, V - V_a)$.

From (1) and the Covariance Property then it follows $\psi(N, V) = \varphi(N, V)$. □

In Section 3 we consider a class of compromisable games arising from a subclass of extensive form games, where the players consider cooperation possibilities.

3 Cooperation in perfect extensive form games

In this section we pay attention to a generic subclass of games in extensive form of perfect information without chance moves. We adopt as much as possible notation from Selten (1975), cf. Varoufakis (2001) and we refer also to this source for background information. So, we denote such a game with player set $N = \{1, 2, \ldots, n\}$ by $\Gamma = \langle N, K, P, h \rangle$. Here K is the game tree with origin or root 0. With Z we denote the set of vertices which are end points in the tree K, and with X we denote the set of other vertices. $P = \{X_1, X_2, \ldots, X_n\}$ is the player partition of X, where X_i is the set of decision vertices of player $i \in N$. The payoff function is $h : Z \to \mathbb{R}^N$ and the i-th coordinate $h_i(z)$ of $h(z)$ is the payoff resulting for player i if the point z is reached as a result of the decisions of the players. For each partition $\pi = \{S_1, S_2, \ldots, S_k\}$ of $\Pi(N)$ we consider the extensive form game $\Gamma(\pi) = \langle \pi, K(\pi), P(\pi), h^\pi \rangle$ which corresponds to the situation where the player set N splits up in cooperative player collectives $S_1, S_2, \ldots, S_k$ who decide (jointly) in all their decision vertices with the aim to maximize the sum of their payoffs.

Formally, given $\Gamma = \langle N, K, P, h \rangle$ and $\pi \in \Pi(N)$ we introduce the game in extensive form $\Gamma(\pi) = \langle \pi, K(\pi), P(\pi), h^\pi \rangle$ as follows:

(i) The players are collectives $S_1, S_2, \ldots, S_k$, subsets of N, such that $\pi = \{S_1, S_2, \ldots, S_k\}$;

(ii) $K(\pi) = K$;

(iii) $P(\pi) = \{X_{S_1}, X_{S_2}, \ldots, X_{S_k}\}$ is the player partition of X, where for each $r \in \{1, 2, \ldots, k\}$, $X_{S_r} = \cup_{i \in S_r} X_i$;

(iv) $h^\pi = (h^\pi_{S_1}, h^\pi_{S_2}, \ldots, h^\pi_{S_k})$ is the payoff function, where for each $r \in \{1, 2, \ldots, k\}$, $h^\pi_{S_r}(z) = \sum_{i \in S_r} h_i(z)$ for each $z \in Z$.

Note that the original game Γ coincides with $\Gamma(\pi^d)$, where π^d is the discrete partition of N.

Example 2. Let Γ be the (perfect) extensive form game depicted in Figure 1, where the game tree K has origin v_1 and where $N = \{1, 2\}, Z = \{z_1, z_2, z_3\}, X = \{v_1, v_2\}, P = \{\{v_1\}, \{v_2\}\}$ and $h : Z \to \mathbb{R}^2$ is given by $h(z_1) = (6, 0), h(z_2) = (2, 1)$ and $h(z_3) = (5, 5)$.

The corresponding extensive form game $\Gamma(\{N\})$, where the players cooperate, is depicted in Figure 2. The game tree is the same but now we have a one-player game with player N, so $P(\{N\}) = \{v_1, v_2\}, h^{\{N\}}_N(z_1) = 6, h^{\{N\}}_N(z_2) = 3$ and $h^{\{N\}}_N(z_3) = 10$.

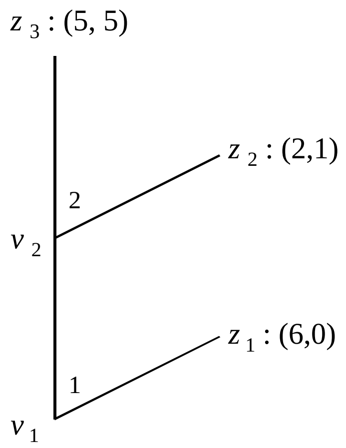

Figure 1 : $\Gamma(\{\{1\}, \{2\}\})$

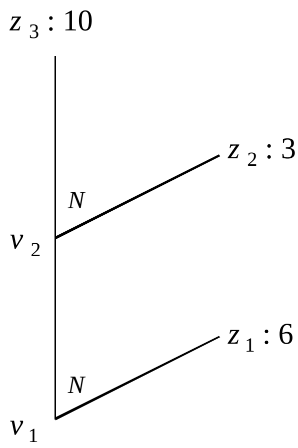

Figure 2 : $\Gamma(\{\{1, 2\}\})$

We are only interested in subgame perfect equilibria for the family of games in extensive form $\{\Gamma(\pi) \mid \pi \in \Pi(N)\}$. We will restrict our attention to perfect extensive form games. We will call a game Γ a *perfect extensive form game* if two conditions are satisfied:

(P.1) (*Perfect Information Condition*) Γ is a game in extensive form of perfect information without chance moves;

(P.2) (*Genericity Condition*) In each game $\Gamma(\pi)$ with $\pi \in \Pi(N)$ and also in each of its subgames there is a unique subgame perfect equilibrium (SP-equilibrium).

We call property (P.2) the genericity condition because (a) for each game satisfying (P.1) by stochastically perturbing the payoffs we get with probability 1 a game satisfying (P.1) and (P.2); (b) for each game satisfying (P.1) and each $\epsilon > 0$ there is a game satisfying (P.1) and (P.2) for which the payoffs do not differ with more than ϵ than the corresponding payoffs in the original game.

Remark 1. The extensive form games in Example 2 and Example 3 are perfect extensive form games.

Now, in a natural credible way we can define a p.f.f. game $\langle N, V\rangle$ corresponding to a perfect extensive form game $\Gamma = \langle N, K, P, h\rangle$: for each partition $\pi \in \Pi(N)$

and $S \in \pi$ we define $V(S \mid \pi)$ as the payoff to the (collective) player S in the unique subgame perfect equilibrium of $\Gamma(\pi)$.

Remark 2. The p.f.f. game $\langle N, V\rangle$ corresponding to the perfect extensive form game in Example 2 is the game described in Example 1. See also the p.f.f. game $\langle N, V\rangle$ ontained from the perfect extensive form game in Example 3.

The p.f.f. games corresponding to perfect extensive form games have a special superadditivity property for root-connected coalitions, which is described in Theorem 2. Here a coalition S of players is called a *root-connected coalition* if for each player $i \in S$ and each $v \in P_i$, the path $[0, v]$ from origin to v contains only vertices in $X_S = \cup_{j\in S} X_j$. Note that N is always a root-connected coalition. In Example 3 the root-connected coalitions are $\{1\}, \{1, 2\}$ and $\{1, 2, 3\}$, but the coalition $S = \{1, 3\}$ is not root-connected.

Theorem 2. *Let $\langle N, V\rangle$ be the p.f.f. game corresponding to a perfect game in extensive form $\langle N, K, P, h\rangle$. Let $S \in 2^N \backslash \{\emptyset\}$ be a root-connected coalition and let π be a partition of N with $S \in \pi$. Let $T_1, T_2, \dots, T_m$ be a partition of S and $\pi' = \{T_1, T_2, \dots, T_m\} \cup \pi \setminus \{S\}$. Then*

$$V(S \mid \pi) \geq \sum_{r=1}^{m} V(T_r \mid \pi').$$

Proof. Note that the root $0 \in X_S$. Let $[v_0, v_1, v_2, \dots, v_t]$ be the path in the tree corresponding to the unique SP-equilibrium of $\Gamma(\pi')$. Since S is root-connected, there is an $i \in \{0, \dots, t-1\}$ such that $0 = v_0, v_1, \dots, v_i \in X_S$ and $v_{i+1}, v_{i+2}, \dots, v_t \notin X_S$. This implies that by choosing suitable actions in the decision points $v_0, v_1, \dots, v_i$ the coalition S can assure the same path resulting in a total payoff for S of $\sum_{r=1}^{m} V(T_r \mid \pi')$. If this path is not optimal for S given π,
then $V(S \mid \pi) \geq \sum_{r=1}^{m} V(T_r \mid \pi')$. □

Example 3. Consider the perfect extensive form game depicted in Figure 3 with player set $N = \{1, 2, 3\}$.

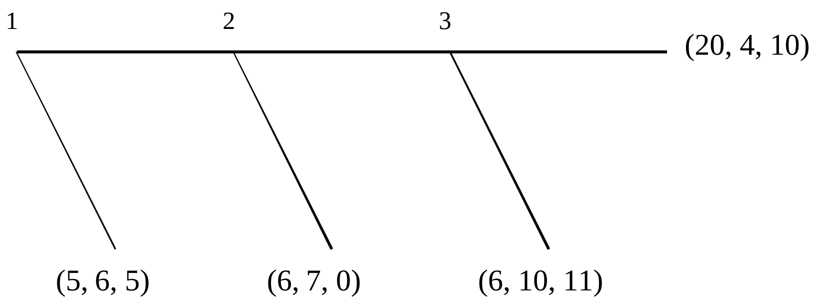

Figure 3

The corresponding p.f.f. game $\langle N, V\rangle$ is described by

$$\begin{array}{ll}
V(\{1\}\,|\,\{\{1\},\{2\},\{3\}\}) = 6, & V(\{2\}\,|\,\{\{1\},\{2\},\{3\}\}) = 10,\\
V(\{3\}\,|\,\{\{1\},\{2\},\{3\}\}) = 11, & V(\{1,2\}\,|\,\{\{1,2\},\{3\}\}) = 16,\\
V(\{3\}\,|\,\{\{1,2\},\{3\}\}) = 11, & V(\{1,3\}\,|\,\{\{1,3\},\{2\}\}) = 10,\\
V(\{2\}\,|\,\{\{1,3\},\{2\}\}) = 6, & V(\{1\}\,|\,\{\{1\},\{2,3\}\}) = 6,\\
V(\{2,3\}\,|\,\{\{1\},\{2,3\}\}) = 21, & V(N\,|\,\{N\}) = 34.
\end{array}$$

Note that (see Theorem 2)

$$V(\{1,2\}\,|\,\{\{1,2\},\{3\}\}) = 16 \geq \sum_{i=1}^{2} V(\{i\}\,|\,\{\{1\},\{2\},\{3\}\}),$$

but for the non root-connected coalition $\{1,3\}$ we have

$$\begin{aligned}
V(\{1,3\}\,|\,\{\{1,3\},\{2\}\}) &= 10 < 6 + 11\\
&= V(\{1\}\,|\,\{\{1\},\{2\},\{3\}\}) + V(\{3\}\,|\,\{\{1\},\{2\},\{3\}\}).
\end{aligned}$$

Theorem 3. *Let $\langle N, V\rangle$ be the p.f.f. game corresponding to the perfect extensive form game $\Gamma = \langle N, K, P, h\rangle$. Then $\langle N, V\rangle$ is compromisable.*

Proof. Let $\pi^d = \{\{i\}\,|\,i \in N\}$ be the discrete partition of N. From the definition of $\ell_i, \ell_{N\setminus\{i\}}, u_i$ we obtain for each $i \in N$

(1) $\ell_i \leq V(\{i\}\,|\,\pi^d)$, $\quad \ell_{N\setminus\{i\}} \leq \sum_{j\in N\setminus\{i\}} V(\{j\}\,|\,\pi^d)$;

(2) $u_i \geq V(N \mid \{N\}) - \sum_{j \in N \setminus \{i\}} V(\{j\} \mid \pi^d)$.

Since N is a root-connected coalition by Theorem 2 we have

(3) $V(N \mid \{N\}) \geq \sum_{j \in N} V(\{j\} \mid \pi^d)$.

To prove that $u_i \geq \ell_i$ for each $i \in N$ note that applying (2), (3) and (1)

$$u_i \geq V(N \mid \{N\}) - \sum_{j \in N \setminus \{i\}} V(\{j\} \mid \pi^d) \geq V(\{i\} \mid \pi^d) \geq \ell_i.$$

That $\sum_{i \in N} \ell_i \leq V(N \mid \{N\})$ follows from (1) and (3):

$$\sum_{i \in N} \ell_i \leq \sum_{i \in N} V(\{i\} \mid \pi^d) \leq v(N \mid \{N\}).$$

Finally $\sum_{i \in N} u_i \geq V(N \mid \{N\})$ follows from

$$\begin{aligned}
\sum_{i \in N} u_i &\geq \sum_{i \in N} (V(N \mid \{N\}) - \sum_{j \in N \setminus \{i\}} V(\{j\} \mid \pi^d)) \\
&= nV(N \mid \{N\}) - \sum_{i \in N} \sum_{j \in N \setminus \{i\}} V(\{j\} \mid \pi^d) \\
&= nV(N \mid \{N\}) - (n-1) \sum_{j \in N} V(\{j\} \mid \pi^d) \\
&\geq V(N \mid \{N\}).
\end{aligned}$$

by applying (2) in the first inequality, and (3) in the second inequality. Hence $\langle N, V \rangle$ is compromisable. □

Example 4. Consider the compromisable game $\langle N, V \rangle$ in Example 3. Then $\ell = (6, 6, 11), u = (13, 24, 18)$ and $\psi(N, V) = (8.40625, 12.1875, 13.40625)$, where $\alpha = 0.65625$.

4 Concluding remarks

Remark 3. In the paper of Tijs (1981) two compromise values were introduced:

(i) the τ-value on the cone of quasi-balanced games

(ii) the σ-value on the larger cone consisting of n-person games with

(a) $v(\{i\}) \leq v(N) - v(N \setminus \{i\})$ for all $i \in N$;

(b) $\sum_{i\in N} v(\{i\}) \le v(N) \le \sum_{i\in N} (v(N) - v(N\backslash\{i\}))$.

In this paper we have defined a compromise value on the set $\mathcal{CPG}^N$ of compromisable n-person p.f.f. games. The next example shows that the set $\mathcal{CPG}^N$ is not a cone.

Example 5. Let $N = \{1, 2, 3\}$ and let V and W be defined as following.
$V(N \,|\, \{N\}) = 5, W(N \,|\, \{N\}) = 2$, and for $i, j, k \in N$ with $i \neq j, j \neq k, k \neq i$,

$$\begin{array}{lll} V(\{i\} \,|\, \{\{i\}, \{j\}, \{k\}\}) = 3, & V(\{i\} \,|\, \{\{i\}, \{j, k\}\}) = 0, & V(\{j, k\} \,|\, \{\{i\}, \{j, k\}\}) = 3 \\ W(\{i\} \,|\, \{\{i\}, \{j\}, \{k\}\}) = 0, & W(\{i\} \,|\, \{\{i\}, \{j, k\}\}) = 1, & W(\{j, k\} \,|\, \{\{i\}, \{j, k\}\}) = 2. \end{array}$$

Then $\ell_i^V = 0$ and $u_i^V = 5 - \min\{6, 3\} = 2$, and $\ell_i^W = 0$ and $u_i^W = 2 - \min\{0, 2\} = 2$. Now for $V + W$, we have $\ell_i^{V+W} = 1$ and $u_i^{V+W} = 7 - \min\{6, 5\} = 2$. (C.2) does not hold, i.e. $\sum_{i\in N} u_i^{V+W} = 6 < 7 = V(N \,|\, \{N\})$. Thus $\langle N, V + W\rangle \notin \mathcal{CPG}^N$.

Remark 4. An interesting topic for further research might be the introduction of other reasonable payoff intervals based on which new compromise values can be defined for the corresponding compromisable p.f.f.games.

Remark 5. In the paper of Tijs (1981) it is proved that the core of a game is included in the hypercube $[a^v, b^v]$ determined by the lower vector (minimum right vector) and the upper vector (marginal contribution vector). For a game $\langle N, V\rangle \in \mathcal{CPG}^N$ it turns out that the hypercube $[\ell, u] = \{x \in \mathbb{R}^N \,|\, \ell_i \le x_i \le u_i$ for each $i \in N\}$ induced by the reasonable payoff intervals $I_i = [\ell_i, u_i]$ for each $i \in N$, contains the pessimistic core of $\langle N, V\rangle$ as Theorem 4 shows.

Recall that the pessimistic core of $\langle N, V\rangle$ is the core $C(v)$ of the pessimistic TU-game $\langle N, v\rangle$ obtained from $\langle N, V\rangle$ by

$$v(S) = \min\{V(S \,|\, \pi) \,|\, \pi \ni S\}, \qquad S \subseteq N$$

(see Theorem 3 in Funaki and Yamato (1999)).

Theorem 4. *Let $\langle N, V\rangle \in \mathcal{CPG}^N$ and let $\langle N, v\rangle$ be the corresponding pessimistic TU-game. Suppose for any $\pi \neq \{N\}$, $V(N \,|\, \{N\}) > \sum_{S\in\pi} V(S \,|\, \pi)$. Then the hypercube $[\ell, u]$ catches the pessimistic core.*

Proof. First we prove that $[a^v, b^v] \subseteq [\ell, u]$.

$$\begin{aligned}
a_i^v &= v(\{i\}) = \min_{\pi \ni \{i\}} V(\{i\} \,|\, \pi) = \ell_i; \\
b_i^v &= v(N) - v(N\backslash\{i\}) \\
&= v(N) - \min\{V(N\backslash\{i\} \,|\, \pi) \,|\, \pi \ni N\backslash\{i\}\} \\
&= V(N \,|\, \{N\}) - V(N\backslash\{i\} \,|\, \{N\backslash\{i\}, \{i\}\}) \\
&\leq V(N \,|\, \{N\}) - \min\{V(S \,|\, \pi) \,|\, \pi \in \Pi^i(N)\} = u_i.
\end{aligned}$$

Now note that $[\ell, u] \supseteq C(v)$ follows from $[a^v, b^v] \supseteq C(v)$ (Tijs (1981)). □

Remark 6. We have shown that for the generic class of perfect games in extensive form the corresponding credible games in p.f.f. based on subgame perfect equilibria are compromisable. A topic for further research might be the study of p.f.f. games arising when relaxing the perfectness conditions (P.1) and (P.2) for games in extensive form.

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