

Campaign Advertising and Voter Welfare

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Abstract

This paper investigates the role of campaign advertising and the opportunity of legal restrictions on it. An electoral race is modeled as a signalling game with three classes of players: a continuum of voters, two candidates, and one interest group. The group has non-verifiable insider information on the candidates' valence and, on the basis of this information, offers a contribution to each candidate in exchange for a favorable policy position. Candidates spend the contributions they receive on non-directly informative advertising. This paper shows that: (1) A separating equilibrium exists in which the group contributes to a candidate only if the insider information about that candidate is positive; (2) Although voters are fully rational, a ban on campaign advertising can be welfare-improving; and (3) Split contributions may arise in equilibrium (and should be prohibited).

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1 Introduction

In electoral competitions throughout the world campaign advertising is becoming more and more common.¹ In the last US Senate election the average candidate made campaign expenditures of \$4.5 million. The need for new forms of regulation is widely felt. In the US Senate and House combined, more than 80 bills have been introduced to reform campaign finance. Notwithstanding the interest that surrounds the issue, we still lack a welfare analysis of campaign advertising that can form the basis for discussing alternative forms of regulation.

Three stylized facts are observed in campaign advertising:

- ² It Is Paid for by Groups whose Objectives Differ from the Median Voter's Objectives. Campaign contributions come from groups of voters whose preferences are often at odd with the preferences of the majority of voters.² For instance, in the US, agricultural interest groups are habitual donors. Their preferred policies { agricultural subsidies and other forms of protection to farmers { cause well documented welfare losses. Lopez and Pagoulatos [22] conduct a study on trade barriers in the US food and tobacco industry. They find that welfare losses can be up to 12.50% of domestic consumption and are positively associated with campaign contributions from agricultural interest groups.
- ² It Does Not Convey Hard Information. Casual observation suggests that campaign advertising contains little direct information. Political ads are not credible. In the US, the First Amendment protects campaign advertising as free speech. Voters have no legal recourse against a candidate who broadcasts ads with misleading statements or misrepresentation of reality (such a strong protection does not apply to commercial advertising).
- ² It Works. Despite the first two facts, campaign advertising is effective. Ansolabehere and Iyengar [1] have conducted a laboratory experiment with more than 3000 residents of the Los Angeles area. Their goal was

¹For a recent cross-country survey of campaign spending and campaign regulation, see The Economist [9].

²A survey of campaign giving patterns can be found in Lehman Schlozman and Tierney [19, Chapter 10]

to study how political advertising on mass media affects the voters' decisions. The experimenters produced several versions of thirty-second TV ads, showed them to the subjects, and then asked the subjects to fill a questionnaire. Subjects who viewed an ad from a candidate were much more likely to vote for that candidate (exposure to a single ad increased the candidate's share of the vote by 5%). Notice that the ads contained so little hard information that they could apply to a candidate as well as to her opponent (the ads were produced in two versions that differed solely in the name of the sponsoring candidate).

A theory of campaign advertising should explain the three stylized facts above. The literature has so far used two types of models of advertising: directly informative advertising and persuasive advertising. A model of the first type has been developed by Austen-Smith [3]. However, as we have argued above, direct information transmission does not seem to be the main component of advertising. Models of the second type are widely used.³ They assume a fraction of electors cast their vote according to an advertising influence function. This function is a mapping from advertising expenditures made by candidates into vote shares. These models can explain the three stylized facts above. The second and the third fact are assumed. The first fact can occur in equilibrium because a fraction of voters do not realize that a candidate who spends a lot on advertising is a candidate who has promised a lot to interest groups. Those voters { it must be concluded { are irrational.⁴

Although they explain observed patterns, models of the second type are not micro-founded. The advertising influence function is not derived from primitive assumptions on the preferences of voters. This is a drawback because those models cannot be used to make welfare comparisons. If our goal is to evaluate proposed regulation, we need a micro-founded model.^{5,6}

The contribution of this paper is twofold. First, we develop a general

³See, among others, Baron [5, 6], Morton and Cameron [26], and Grossman and Helpman [15, 14]

⁴This point is made in LaFont and Tirole [18, p. 634].

⁵Indeed, the need for a micro-founded model of campaign advertising is perceived in the field. See Morton and Cameron [26, p. 85], Baron [6, p. 45], and the above cited LaFont and Tirole.

⁶Of course, one could construct a micro-founded model of persuasive advertising. This has been done for commercial advertising by Dixit and Norman [8]. However, as Fisher and McGowan [11] point out, a model of advertising in which preferences are endogenous does not lend itself to general welfare comparisons.

model of campaign advertising in which voters are fully rational and advertising is not directly informative. The crucial assumption is that interest groups observe insider information about candidates. We show that this model explains the three stylized facts above. Thus, the advertising influence function commonly used in the literature can be seen as a reduced form of our model. Second, and more importantly, we use the model to make welfare comparisons. In particular, we evaluate the opportunity of banning campaign advertising or split contributions.

An electoral race is modeled as a signaling game with three classes of players: a continuum of voters, one interest group, and two candidates. Voters judge candidates on two dimensions: valence (e.g. ability, leadership, integrity) and policy. All voters agree on the valence dimension, but have heterogeneous preferences about policy. The interest group caters to the policy dimension of a subset of voters, but is not directly interested in the valence of candidates. The ideal policy of the median group member differs from the ideal policy of the median voter. Candidates maximize their chance of being elected.

The valence of a candidate is unknown, but there are imperfect signals about it. Some of these signals are public (candidates' records, TV debates, etc.) and some are observed by the interest group but not by voters (rumours, first-hand experience, etc.). The insider signals are non-verifiable. After observing the insider signals, the group makes an offer to each candidate. The offer consists of a monetary contribution to be spent on non-directly informative advertising and a policy to be implemented if the candidate is elected. Candidates accept or reject the group's offer. Each voter then observes the public signals, the policy choice, and the amount spent on advertising by each of the two candidates and casts a vote for one of the two candidates.

One may conjecture that in this model a ban on advertising cannot be optimal.⁷ Each candidate maximizes her chance of being elected. Thus, she will accept a contribution only if it increases her chance of being perceived in a positive way by voters. Voters observe policy positions. Thus, they see any promise that a candidate makes to the interest group and they can punish a candidate who promises too much by not electing her. Therefore, if advertising occurs in equilibrium, one might conclude that voters must get more benefit in terms of indirect information than they give up in terms

⁷Throughout this paper a ban on advertising and a ban on contributions produce the same effects. Thus we only refer to a ban on advertising. See Section 6 for a discussion.

of policies. As it turns out, this conjecture is incorrect.

The main results of the model are:

1. There exists a separating equilibrium in which the interest group contributes to a candidate if and only if the insider signal about that candidate is positive. A pooling equilibrium exists but does not survive the Intuitive Criterion. The insider signal is revealed to voters through the amount of campaign advertising. In exchange for a contribution, the group obtains from the candidate a policy position that is favorable to the group and detrimental to the median voter. Intuitively, the group sees contributions as an investment with stochastic return: the group gets the favorable policy only if the candidate is elected. Therefore, the group prefers to contribute to a likely winner and uses the insider signal to infer the chances of the candidate's victory. A group with a good insider signal can afford a contribution that a group with a bad insider signal cannot afford.
2. Under certain conditions, a ban on campaign advertising strictly increases the voters' welfare (which includes the group members' welfare). Campaign contributions represent a credible threat the interest group can use against a good candidate (i.e. a candidate with a positive insider signal): in equilibrium a good candidate who rejects the group's offer is perceived by voters as bad. Through this implicit threat, the group can obtain from a good candidate a policy position that makes the median voter indifferent between a good candidate with that policy position and a bad candidate with the median voter's ideal position. This represents the candidate's participation constraint. If the ideal policy of the median group member is distant from the ideal policy of the median voter, the group wants the participation constraint to be binding. In equilibrium, it is as if voters only encountered bad candidates with the median voter's ideal position. The presence of campaign advertising brings the median voter more cost in terms of biased policy than benefit in terms of information on candidate valence.
3. If the group receives equally good insider signals about the two candidates, it will contribute to both and will get favorable policies from both. If one candidate rejects the offer, he will be perceived as bad and the other candidate will be perceived as good. This situation is particularly negative for voters: they receive useless information and

they have to choose between two candidates who cater to the interest group. Indeed, it is proven that a ban on split contributions always increases the voters' welfare.

The problem is formulated in a general way. In particular, the probability distributions of signals are left in a generic form. Moreover, results are shown to be robust to modifications of the model such as the assumption that candidates make offers to the group or the assumption that policy positions are unobserved. However, results change dramatically if candidates do not receive contributions from groups but they finance campaigns out of their personal wealth. In that case a separating equilibrium need not exist.

The plan of the paper is as follows. Section 2 introduces the model. Then, for illustrative purposes, the core of the paper is divided into two main parts. Section 3 assumes that the interest group can only contribute to Candidate 1. With this restriction it is possible to prove results (1) and (2) in an intuitive way. However, the assumption that the groups can only contribute to a pre-specified candidate is unrealistic. Thus, Section 4 develops the full model in which the group can contribute to both candidates. Results (1) and (2) still hold, and, moreover, result (3) is proven. Section 5 discusses modifications of the model. Section 6 concludes.

Related Literature This paper is inspired by two strands of literature that are somewhat distant from each other: the political economy literature on campaign contributions and the industrial organization literature on advertising with rational consumers. In common with the first strand (See Morton and Cameron [26] for a survey), we model an electoral race as a game with three classes of players: voters, candidates, and interest groups. We adopt most of the definitions and the assumptions that are standard in the literature on campaign contributions, with three important differences: (1) all voters are rational; (2) candidates are judged on valence as well as policy; and (3) some non-verifiable signals about valence are only available to insiders. The second strand includes Milgrom and Roberts [25], Kihlstrom and Riordan [17], Hertzendorf [16], and Bagwell and Ramey [4], and others. In common with the second strand we assume that what matters in advertising is the amount spent on it, not its content. Under certain conditions, an agent with non-verifiable private information is able to reveal it through non-directly informative costly signalling. However, models of commercial advertising rely on concepts { such as quantity, price, and cost { that do not find a parallel in elections. Thus, while the spirit is similar, our model

is different from commercial advertising models.

Two other papers study models in which campaign advertising is non-directly informative

In Potters, Sloof, and Van Winden [27], a candidate can be of a high type or a low type. Both types benefit from being elected, but the high type benefits more (or finds advertising less expensive). Thus, the authors' rationale for campaign advertising is that good candidates have more to gain than bad candidates from being perceived as good. On the contrary, we take the agnostic viewpoint that candidates of different types benefit equally from election and face the same cost of advertising.

Gerber [12] argues that campaign advertising conveys information because it reveals the insider signals of groups. Thus, the rationale is similar to our model. However, in the separating equilibrium described by Gerber, both a group with a good candidate and a group with a bad candidate are indifferent between contributing or not contributing (we discuss this problem in Subsection 3.2.1). Thus, a separating equilibrium exists only when exogenous reasons guarantee that groups with good candidates contribute and groups with bad candidates do not. On the contrary, in our separating equilibrium a group with a good candidate has a strictly higher incentive to contribute than a group with a bad candidate.

Two recent papers do not tackle campaign advertising but are closely related to the present work. Grossman and Helpman [13] study political endorsements with rational voters.⁸ Lohmann [21] analyzes a model of retrospective voting in which a minority of voters is (endogenously) better informed than the majority. Both these two papers and the present paper show that in equilibrium candidates choose policy positions that are biased away from the median voter. This policy bias occurs despite the fact that voters can, at least partially, observe policy positions. The reason is that a minority of voters enjoy an informational advantage and use it to extract rent from candidates in the form of favorable policies. These models are in line with the emphasis that observers of interest groups politics put on the monitoring role of groups.

⁸In the present work, cheap-talk endorsements are never credible. This is because the insider signal is on valence and the group derives no direct utility from valence.

2 Model

2.1 Political Dimensions and Voters

A continuum of voters indexed with $i \in I$ must elect one of two candidates, indexed with $j \in \{1, 2\}$. The possibility of abstention is disregarded.

Each candidate is represented along two dimensions: his policy position $p_j \in \mathbb{R}$ and his valence $\mu_j \in \mathbb{R}$. The policy dimension can be interpreted both as ideological view (position on the left-right line) or as policy stance (e.g. position on the issue of subsidies to milk producers). The valence dimension captures a set of characteristics of the candidate that are unambiguously good for voters.

Voter i is described by his preferred policy p_i , which is strictly increasing in i . Let $e \in \{1, 2\}$ denote the candidate who wins the election. The utility function of Voter i is

$$u_i(\mu_e; p_e) = \mu_e + u(p_i - p_e)$$

where $u(\cdot)$ is continuous, symmetric, and strictly increasing in $|p_i - p_e|$. If Voter i knew μ_1 and μ_2 , he would vote for candidate 1 if and only if

$$\mu_1 + u(p_i - p_1) > \mu_2 + u(p_i - p_2)$$

Thus, if the two candidates have identical policy positions, Candidate 1 is elected if and only if he beats Candidate 2 on the valence dimension.

2.2 Information

Voters observe policy positions p_1 and p_2 perfectly.⁹ However, they cannot observe valences μ_1 and μ_2 directly. μ_1 and μ_2 are independent random variables, each of which has prior distribution $\hat{A}(\cdot)$ defined on \mathbb{R} . Priors are common knowledge.

Three signals about the valence of Candidate $j \in \{1, 2\}$ are received sequentially. First, all agents (voters, candidates, group) observe a public signal $x_j \in \mathbb{R}$, which represents the candidates' historical record (for instance, it may capture the well-documented incumbency advantage). Second, only interest groups observe an insider signal $y_j \in \mathbb{R}$ which can

⁹Section 5.1 will show that results do not change dramatically if voters cannot observe the policy dimension because, in equilibrium, they can infer it perfectly.

be thought of as impressions, word-of-mouth, unproven allegations, etc.¹⁰ The insider signal is non-verifiable. Third, all agents observe a public signal $z_j \in Z$ that derives from the candidate's performance during the campaign (e.g. pre-electoral TV debates). More complex signal sequences could be accommodated. The results of this paper depend uniquely on the assumption that the last public signal is received after the first insider signal.¹¹

The cumulative distributions of x_j , y_j , and z_j given μ_j are, respectively, $F_x(x_j | \mu_j)$, $F_y(y_j | \mu_j)$, and $F_z(z_j | \mu_j)$. $F_x(x_j | \mu_j)$ is strictly increasing in x_j for any $\mu_j \in \mathcal{E}$, and similarly for F_y and F_z . The random variables x_1 , y_1 , and z_1 are assumed to be stochastically independent from x_2 , y_2 , and z_2 . Furthermore,

Assumption 1 For $j = 1, 2$, x_j , y_j , and z_j are mutually independent given μ_j and satisfy the Monotone Likelihood Ratio Property (MLRP).¹²

The assumption implies that an increase in any of the three signals translates in an increase in the expected value of the valence.

Let

$$\hat{\mu}(x_j; y_j; z_j) = E(\mu_j | x_j; y_j; z_j)$$

and

$$\mu(x_j; z_j) = E(\mu_j | x_j; z_j)$$

$\hat{\mu}$ is the expected value of μ_j given the public signals and the insider signal, while μ is the expected value given the public signals only. Applying Milgrom [24, Proposition 2], if F_x , F_y , and F_z satisfy MLRP, then $\hat{\mu}(x_j; y_j; z_j)$ is strictly increasing for all $x_j \in X$, $y_j \in Y$, and $z_j \in Z$. Similarly for $\mu(x_j; z_j)$.

A simplifying assumption we make throughout the paper is that y_j is a binary signal: $Y = \{0, 1\}$. The insider signal is either good or bad. This as-

¹⁰It is assumed that candidates do not observe the insider signal. This assumption avoids the possibility that candidates signal their good type by adopting bad policies (choosing a position far away from the median voter is a costly signal). While this case may be interesting in its own, it lies outside the scope of this paper. Of course, in a separating equilibrium candidates infer the insider signal from the group's offer.

¹¹Indeed, the presence of x_j in this model is not necessary for most results.

¹²A cumulative distribution function $F(t; \epsilon)$ satisfies MLRP if its p.d.f. $f(t; \epsilon)$ is such that, for every $s^0 > s$ and every $t^0 > t$

$$\frac{f(s^0; t^0)}{f(s^0; t)} > \frac{f(s; t^0)}{f(s; t)}$$

sumption will allow us to characterize the incentive-compatibility constraints in a simple way.

2.3 Voters' Choice

Let μ_j represent the expected value of μ_j , conditional on the voters' information at the moment of the vote.¹³ Given μ_j , Voter i votes for Candidate 1 if and only if

$$\mu_{1i} - \mu_{2i} - u(p_i | p_1) + u(p_i | p_2) \geq 0$$

Let m denote the median voter and p_m the median voter's ideal policy: $m \in I$ is the unique solution to $\int_{p_i < p_m} di = \int_{p_i > p_m} di$. The proof of the following is immediate:

Lemma 1 Candidate 1 is elected if and only if

$$\mu_{1m} - \mu_{2m} - u(p_m | p_1) + u(p_m | p_2) \geq 0$$

A candidate is elected if and only if he is preferred by the median voter. The median voter evaluates candidates on how high their expected valences are and on how close to the median voter's ideal position their policies are.

2.4 Candidates

The only goal of a candidate is to win the election. He derives no direct utility from policy or valence. While his valence is given, Candidate j chooses his policy position p_j , which is publicly observable.

Consider the policy choice of Candidate j . Given Lemma 1, for any voters' belief on valence and for any policy chosen by the other candidate ($i \neq j$), j maximizes his election chances by choosing $p_j = p_m$:

Lemma 2 For any distribution of probability over μ_j and $\mu_{i,j}$ and for any $p_{i,j}$, $p_j = p_m$ is a best response.

Lacking any other influence, both candidates should choose the median voter's ideal policy.

¹³This paper abstracts from the problem of the heterogeneity of information among voters and its aggregation. All voters observe the same signals and hold the same beliefs. Section 6 discusses this assumption.

2.5 Interest Group

An interest group leader acts as the representative of a subset of the voters regarding the policy dimension. The subset has mass λ and median member $g > m$. The group leader, G , maximizes the policy component of the utility of the median group member. The interest group is therefore not directly interested in the valence of candidates.¹⁴ G can make contributions to candidates 1 and 2, denoted respectively with A_1 and A_2 . The group's payoff[®] is assumed to be separable in contributions and policy. The payoff[®] to G if e is elected is $\lambda u(p_g | p_e) + A_1 + A_2$.¹⁵

G announces a desired policy p^a and then she can make an offer A_1^a to 1 and an offer A_2^a to 2. Each candidate can accept or reject the offer. If he accepts, he receives a campaign contribution $A_j = A_j^a$ and commits to implementing p^a if elected. If he rejects, he receives $A_j = 0$ but he is free of choosing any policy position. If the candidate accepts the contribution, he can use it for not directly informative campaign advertising.¹⁶

3 When Only One Candidate Can Receive Contributions

This model takes into account interactions on three levels: (a) How a candidate influences voters' beliefs through advertising, (b) How the group exchanges contributions for favorable policies, and (c) How candidates compete

¹⁴If the group represents a subset of voters, one may think it should care about both policy and valence. However, there are two reasons to believe that the group should be more concerned about policy. The first is that there can exist an agency problem between the group members and the group leader. Suppose that, while outcomes on the policy dimension can be contracted upon, outcomes on the valence dimension are hard to measure and to verify. Then, the group leader only has an incentive to perform on the policy line. The second reason has to do with the free-riding problem. If voters have identical preferences over valence but disagree over policy, one can expect that subsets of policy-homogeneous voters will have more incentives to pool resources to influence policy rather than to enhance valence.

¹⁵ A_1 and A_2 do not enter the other players' utilities. Thus, the assumption that they enter G 's utility function in a linear way and with unitary coefficients is without loss of generality.

¹⁶This model assumes that a candidate can credibly commit to implement p^a if elected. It is mostly an open question (outside the scope of this paper) why the candidate should live up to its pre-electoral promises to interest groups (See however Austen-Smith [2] for self-enforcing agreements in which the candidate credibly promises his group 'access' to the policy-making process in exchange for a contribution).

with each other for contributions. For illustrative purposes, it is useful to fully explore (a) and (b) before including (c). This section makes the temporary assumption that the group can only contribute to Candidate 1 and that there is no uncertainty about the valence of Candidate 2.

Assumption 2 (i) G can only make offers to Candidate 1 and (ii) $\mu_2 \geq 0$.

Assumption 2 is maintained throughout this section and will be dropped in Section 4.

To summarize, the electoral race is represented as follows:

Game 1 The players are: voter $i \in I$, candidate $j \in \{1, 2\}$, and interest group G . The game consists of four stages:

1. Nature: Nature chooses $\mu_1 \in E$, which remains unknown to all players. $\mu_2 \geq 0$. $x_1 \in X$ is realized and becomes common knowledge among all players.
2. Insider Stage: G observes $y_1 \in \{0, 1\}$, selects p^a , and offers A_1^a to 1. 1 accepts or rejects. If he rejects, then he makes advertising expenditure $A_1 = 0$ but he is free to set p_1 . If he accepts, then $A_1 = A_1^a$ and $p_1 = p^a$. $A_2 \geq 0$ and 2 is free to set p_2 .¹⁷
3. Public Stage: $z_1 \in Z$ is realized. Voters observe p_1 , p_2 , A_1 , and z_1 . For $i \in I$, Voter i votes for either 1 or 2. Let e denote the candidate that receives the higher number of votes and let $j \neq e$ denote the other candidate.
4. Payoff Distribution: μ_1 is revealed. Voter i receives μ_e if i votes for e and μ_j if i votes for j . G receives $u(p_g$ if $e = g$ and 0 if $e \neq g$ and A_1 .

The players' strategies are: $p^a \in [0, 1]$ and $A_1^a \in [0, 1]$ for G ; "accept", "reject" for 1; $p_1 \in [0, 1]$ (if "reject") for 1; $p_2 \in [0, 1]$ for 2; and $e_i \in \{1, 2\}$ for i .

¹⁷Here we assume that G makes offers in the time window after she observes y_j but before she observes z_j . If she made offers before or after this time window, it can be shown that advertising is never credible. Therefore, even if G is free to choose when to make a contribution, she will only make it in the 'credible' time window. With a more complex information structure (for instance, one with a continuous sequence of both public and insider signals) the credible time window could cover the whole campaign.

3.1 Equilibrium under Advertising Ban

As a benchmark, consider the case in which advertising (or campaign giving) is prohibited by law ($A_1 = 0$). By Lemma 2, candidates will set $p_1 = p_2 = p_m$. By Lemma 1, Candidate 1 will be elected if and only if x and z are such that $\mu(x; z) \geq 0$. Let $\hat{z}^P(x)$ be the unique solution to $\mu(x; \hat{z}^P(x)) = 0$ for all $x \in X$. \hat{z}^P is strictly decreasing in x .

Proposition 1 Under an advertising ban, $p_j = p_m$ for $j = 1, 2$ and $e = 1$ if and only if x and z are such that $z > \hat{z}^P(x)$.

Under an advertising ban, candidates cannot do anything to influence the voters' beliefs over the valence dimension. Their optimal strategy is to cater to the median voter on the policy dimension.

3.2 Equilibrium with Advertising

In a separating equilibrium, three conditions must be satisfied: (1) A_1 must reveal the value of y . This translates into an incentive-compatibility constraint on A_1^a ; (2) given the ICC, 1 must be better off accepting the offer than rejecting the offer. This translates into a participation constraint on p^a ; and (3) Given the ICC and the PC, G must select the level of p^a that maximizes her benefit. This determines an optimal p^a . In general, the PC may or may not bind according to the value of the optimal p^a . Let us consider the three conditions one at a time:

3.2.1 Incentive-Compatibility Constraint for G

Let $\hat{z}(x_1; y_1; p_1)$ denote the unique value of z_1 for which

$$\hat{\mu}(x_1; y_1; z_1) + u(p_m | p_1) - u(0) = 0$$

If we suppose that x_1 has been realized, that voters believe y_1 , that 1 chooses p_1 , and that 2 chooses $p_2 = p_m$ (a consequence of Lemma 2), then $\hat{z}(x_1; y_1; p_1)$ represents the lowest realization of z_1 at which 1 is elected. \hat{z} is strictly decreasing in x_1 , z_1 and strictly increasing in p_1 for $p_1 > p_m$.¹⁸

If both candidates choose p_m , G 's payoff is certainly $\frac{1}{2}u(p_g | p_m)$. Let

$$V_y(p^a) = \frac{1}{2} \Pr^h \left[\hat{\mu}(x; 1; z) - u(p^a | p_m) - u(0) \geq 0 \right] \int_0^{\hat{z}(x; 1; p^a)} [u(p_g | p_m) - u(p_g | p^a)] dz \quad (1)$$

¹⁸As we have assumed that $p_g > p_m$, it is obvious that 1 will never choose $p_1 < p_m$.

$v_y(p)$ is the expected payoff of G net of $\int u(p_g | p_m)$, given that 1 has accepted policy p^* . $v_y(p)$ is gross of the contribution A_1^* , which will be determined shortly. To avoid confusion between candidate subscripts and realizations of y , let $v_H = v_1$ and $v_L = v_0$ as in 'high type' and 'low type'.

$v_H(p^*)$ represents the gross expected payoff for G if her candidate is a high-type and voters believe her candidate is a high type. On the other hand, $v_L(p^*)$ represents the gross expected payoff for G if her candidate is a low-type and voters believe her candidate is a high type. In a separating equilibrium, given p^* , the contribution A_1 must be such that

$$v_L(p^*) \cdot A_1 < v_H(p^*) \quad (2)$$

Then, a group with a high-type candidate can make an offer that a group with a low-type candidate cannot afford.

Lemma 3 For any $p^* \in (0; 1)$, there exists an $A^*(p^*) \in (0; 1)$ such that

$$v_H(p^*) > A^*(p^*) \cdot v_L(p^*)$$

Condition (2) is standard in models of uninformative advertising (See for instance Milgrom and Roberts [25]). It is a necessary condition for the existence of a separating equilibrium. In this model it is always satisfied. It is always true that the group has more to gain by contributing to a high-type candidate than to a low-type candidate. This is because y_1 can be used to predict z_1 . The conditional distribution of z_1 given $y_1 = 1$ dominates (in a first-order stochastic sense) the conditional distribution of z_1 given $y_1 = 0$. Thus, for any given policy offer, a group with a high-type has a higher expected payoff than a group with a low-type.¹⁹

Remark 1 If we assumed that y_1 is perfectly informative, then Condition (2) would not hold.

Suppose that y_1 is perfectly informative. Then, if voters know y_1 , they can infer μ_1 . Then, in a separating equilibrium, voters decide based only on y_1 and they do not look at z_1 . If voters believe $y_1 = 1$, a group with a high-type candidate has the same expected payoff as a group with a low-type candidate. Thus, Condition (2) does not hold.²⁰

¹⁹It is important to stress that x_1 and z_1 are not positively correlated: this would amount to assuming the result. x_1 and z_1 are mutually independent given any value of μ_1 . The reason why y_1 can be used to predict z_1 is because μ_1 is unknown.

²⁰Still, when y_1 is perfectly informative, a separating equilibrium can exist. However, this equilibrium is both arbitrary and brittle, as Section 5.1 of the Appendix shows.

3.2.2 Participation Constraint for 1:

Candidate 1 wants a high p_1 and a p_1 close to p_m . If he does not accept a contribution, he sets $p_1 = p_m$. If he accepts a contribution, he must set $p_1 = p^a$, which, in itself, decreases his chance of being elected. If he spends less than $\frac{1}{L}(p^a)$ on advertising, voters will believe $\mu_1 = \hat{\mu}(x; 0; z)$. Thus, he can only lose by accepting a contribution of less than $\frac{1}{L}(p^a)$. If, on the contrary, he spends at least $\frac{1}{L}(p^a)$ on advertising, voters will believe $\mu_1 = \hat{\mu}(x; 1; z)$. He will accept such a contribution only if the policy he is asked to implement is not too far from the median voter's ideal policy.

Lemma 4 In a separating equilibrium, 1 rejects any offer $A_1^a < \frac{1}{L}(p^a)$. He accepts an offer $A_1^a \geq \frac{1}{L}(p^a)$ if and only if $p^a \cdot \beta$ where $\beta(x_1)$ is the largest p such that

$$z(x_1; 1; p) \cdot z(x_1; 0; p_m)$$

3.2.3 Optimal Offer for G

Lemma 3 and Lemma 4 restrict the set of offers G can make. A_1^a cannot be lower than $\frac{1}{L}(p^a)$ and p^a cannot be higher than β . Obviously, G will not make an offer strictly above $\frac{1}{L}(p^a)$. Still, G faces a continuum of possible offers corresponding to $0 \leq p^a \leq \beta$. What is the optimal choice from the point of view of G?

Lemma 5 In a separating equilibrium, G sets $p^a = p_{\max}$, where

$$p_{\max} = \operatorname{argmax}_p \frac{1}{L}(p) \mid \frac{1}{L}(p)$$

subject to $p^a \leq \beta$.

G chooses p^a subject to the constraint that $p^a \leq \beta$. One may wonder whether it could be the case that the Participation Constraint is always binding or never binding. The following is proven in Section 7.2 by means of a numerical example:

Remark 2 Both the case $p_{\max} < \beta$ and the case $p_{\max} = \beta$ are possible.

The participation constraint is not binding when p_g is close to p_m . It is obvious that G will never ask for $p^a > p_g$. However, β is independent of p_g . So for any β , if p_g is small enough, $p^a < \beta$. On the other hand, the participation constraint is binding when p_g is high and z_j is more informative

than y_j . Hold fixed the precision of y and increase the precision of z : voters become less interested in learning the signal y and Candidate 1 has less to gain from being revealed as $y_1 = 1$ rather than $y_1 = 0$. However, G would still like to have a policy position close to p_g .

3.2.4 Equilibrium

After having examined the incentive-compatibility constraint, the participation constraint, and the solution to G 's constrained-maximization problem, it is now possible to characterize the separating equilibrium with advertising:

Proposition 2 A sequential equilibrium of Game 1 is

(i) Voters' beliefs: For any $p_1 \in (0, 1)$

$$\hat{\mu} = \begin{cases} \hat{\mu}(x; 1; z) & \text{if } A_1 \leq \hat{\mu}_L(p_1) > 0 \\ \hat{\mu}(x; 0; z) & \text{otherwise} \end{cases}$$

(ii) Voters' choice: $e_i = 1$ if and only if $\hat{\mu}_i u(p_i | p_1) \geq u(p_i | p_2) > 0$. 1 is elected if and only if $\hat{\mu}_i u(p_m | p_1) \geq u(p_m | p_2) > 0$

(iii) Group's offer: $p^a = p_{\max}$ and

$$A_1^a = \begin{cases} \hat{\mu}_L(p^a) & \text{if } y_1 = 1 \\ 0 & \text{if } y_1 = 0 \end{cases}$$

(iv) Candidate 1 accepts A_1^a if and only if $A_1^a \leq \hat{\mu}_L(p^a)$ and $p^a \cdot \beta$. If he rejects, $p_1 = p_m$.

(v) Candidate 2 sets $p_2 = p_m$.

Proof: (i) $\hat{\mu}$ is consistent with (iii) and (iv). (ii) By Lemma 1. (iii) p_{\max} is optimal by Lemma 5 and A_1^a is consistent with (i), (iv), and Lemma 3. (iv) Best response given (i), (ii), and Lemma 4. In case of rejection, Lemma 2 applies. (v) By Lemma 2. \square

The intuition for the existence of a separating equilibrium has to do with the confidence G has in the election chances of Candidate 1. When G makes a contribution to 1, she undertakes a risky investment. She forgoes A_1^a for sure: if 1 is elected, she benefits of a favorable policy; if 2 is elected, she

gets the same policy she would have gotten if she had made no contribution. Thus, G is more willing to invest in Candidate 1 if 1 has a higher chance of producing a high z_1 . G is more confident that z_1 will be high if she has observed $y_1 = 1$ than if she has observed $y_1 = 0$. If $y_1 = 1$, G can make a contribution that she could not afford if $y_1 = 0$. If voters observe such a contribution, they must infer $y_1 = 1$.

The separating equilibrium takes two forms according to whether the participation constraint of Candidate 1 is binding or not. If the preferences of G are close to the preferences of the median voter or if the insider signal is highly informative, the participation constraint is likely to be nonbinding. Then, G chooses $p^* = p_{\max}$, which maximizes the expected payoff subject to the constraint that $A^*(p^*)$ be separating. If, on the other hand, G 's preferences are distant from the median voter's preferences and the insider signal is not very informative, then the participation constraint tends to be binding. G sets $p^* = \beta$.

3.3 Pooling Equilibrium

Let us investigate whether a pooling equilibrium can exist. By definition, in a pooling equilibrium, p_1 and A_1 do not depend on y_1 . Pooling equilibria with $p_1 \leq p_m$ can be ruled out by a deviation from candidate 1. On the other hand it is easy to see that the pooling equilibrium in which 1 rejects all offers from G and $p_1 = p_m$ is a sequential equilibrium of Game 1. Such a pooling equilibrium, in which G plays no role, is identical to the equilibrium under Advertising Ban of Proposition 1.

The equilibrium is supported by the following voters' belief: $\mu = \mu(x; z)$ independently of the value of A_1 . In a pooling equilibrium, voters do not listen to campaign advertising and candidates, anticipating that advertising is useless, reject all contributions. Thus, the pooling equilibrium relies on the out-of-equilibrium belief $\mu = \mu(x; z)$ if $A_1^* > 0$. Is this plausible?

The signalling game literature has developed several refinement concepts with the goal of ruling out equilibria based on implausible out-of-equilibrium beliefs. One of the most widely used is the Intuitive Criterion introduced by Cho and Kreps [7]. In the present model, a sequential equilibrium fails the Intuitive Criterion if there exists a deviation that is profitable for both G and 1 if and only if G has observed $y_1 = 1$.

Proposition 3 The pooling equilibrium does not survive the Intuitive Criterion.

Game 1 can be seen as a signalling game with two types of senders: G with private information $y_1 = 1$ and G with private information $y_1 = 0$. The first type is strictly better off in a separating equilibrium than in a pooling equilibrium. The second type is indifferent (she gets a zero-payoff in both equilibria). The first type can offer Candidate 1 a contribution that the second type cannot afford. By observing such a contribution, voters must infer that the sender is of the first type. Provided the participation constraint is satisfied, a candidate should accept that contribution. For this reason, a sender of the first type can deviate from the pooling equilibrium in a way that is both credible and profitable. This shows that the pooling equilibrium is implausible.

3.4 Welfare

The ex-post voter welfare is²¹

$$\begin{aligned} W(e; p_e) &= \int_{i \in I} [\mu_e \int_{i \in I} u(p_i | p_e)] di \\ &= \mu_e \int_{i \in I} u(p_i | p_e) di \end{aligned}$$

Let us assume that, for $i \in I$, p_i is symmetrically distributed around p_m . Recall that $u(t)$ is symmetric around p_i . Then, as is well known, W is maximized when the ex-post welfare of the median voter is maximized. Thus, from now on we focus on $w = \mu_e \int_{i \in I} u(p_i | p_e)$, which can be expressed as a function of e and p_1 :

$$w(e; p_1) = \begin{cases} \mu_1 \int_{i \in I} u(p_m | p_1) & \text{if } e = 1 \\ \int_{i \in I} u(0) & \text{if } e = 2 \end{cases}$$

In the separating equilibrium of Proposition 2,

$$w(e; p_1) = w_S(e; y_1; p^a) = \begin{cases} \mu_1 \int_{i \in I} u(p_m | p^a) & \text{if } e = 1 \text{ and } y_1 = 1 \\ \int_{i \in I} u(0) & \text{otherwise} \end{cases} \quad (3)$$

while in the equilibrium under advertising ban as in Proposition 1,

$$w(e; p_1) = w_P(e) = \begin{cases} \mu_1 \int_{i \in I} u(0) & \text{if } e = 1 \\ \int_{i \in I} u(0) & \text{if } e = 2 \end{cases}$$

²¹As G is made of a subset of voters, the payoff of group members is already included in the voter welfare.

Let the expected voter welfare be the expected payoff[®] for m after x is realized but before y and z are observed.²² Under a separating equilibrium, the expected voter welfare is $\bar{w}_S(p^a) = E_\mu(w_S(e; y_1; p^a) | x)$, while under the advertising ban, $\bar{w}_P = E_\mu(w_P(e) | x)$. Let us first consider the case in which the Participation Constraint binds, that is $p^a = \bar{p}$.

Lemma 6 if $p^a = \bar{p}$, the separating equilibrium yields the same voting outcome as an equilibrium under advertising ban in which voters use the rule

$$e = 1, \quad \mu(x; z) \geq b(x)$$

where $b(x) > 0$ for all $x \in X$, instead of using the rule

$$e = 1, \quad \mu(x; z) \geq 0$$

Thus, $\bar{w}_S(\bar{p}) < \bar{w}_P$.

Lemma 6 examines the case in which Candidate 1's participation constraint is binding. In that case, the expected voter welfare under an advertising ban is strictly higher than the expected voter welfare with advertising. What is the intuition behind this result?

If the participation constraint is binding, Candidate 1 is exactly indifferent between: (a) being revealed as $y_1 = 1$ and choosing p^a ; or (b) being revealed as $y_0 = 0$ and choosing p_m . He is indifferent because he is elected under (a) if and only if he is elected under (b). If the candidate were more likely to be elected under (a) than under (b), the participation constraint would not be binding, while if he were more likely to be elected under (b) than under (a), he would reject G 's offer. In equilibrium, if the insider signal is good, case (a) occurs, while, if the insider signal is bad, case (b) occurs. Notice however that case (b) is equivalent to the following scenario: G does not exist; voters observe the insider signal directly; the insider signal happens to be bad. As Candidate 1 is elected under (a) if and only if he is elected under (b), voters behave as if they always had a 'bad' Candidate 1.

In general G uses its insider signal to extract rent from voters in the form of biased policy. If the participation constraint is binding, it means that G has pushed the policy bias to the point at which, in the eyes of voters, there is no difference between a good candidate with a biased policy and a bad candidate with an unbiased policy. Thus, it is as if voters encountered

²²The welfare analysis holds a fortiori if the expected voter welfare is defined as the expected payoff[®] before x is realized.

only bad candidates with unbiased policies. However, in an equilibrium under advertising ban, voters encounter only 'average' candidates (that is, candidates for whom the insider signal can be good or bad). Under an advertising ban, candidates always adopt an unbiased policy. Thus, voters are strictly better off under an advertising ban than under a separating equilibrium.

Let us now characterize the general case in which the participation constraint may or may not be binding:

Proposition 4 for any $x \in X$, there exists a $k \in (0; \beta)$ such that $v_S(p_{\max}) < v_P$ when $p_{\max} \in (k; 1)$, $v_S(p_{\max}) = v_P$ when $p_{\max} = k$, and $v_S(p_{\max}) > v_P$ when $p_{\max} \in (0; k)$.

If the participation constraint is not binding, G still extracts all the rent she can extract, but, in doing so, she leaves some informational rent to voters. Now it is not anymore as if voters encountered only bad candidates with unbiased policies. By the fact that the expected voter welfare is continuous and strictly decreasing in p^* , there exists a policy $k \in (0; \beta)$ such that if $p^* = k$ voters are indifferent between prohibiting advertising and allowing advertising. If the goal of G or the information structure is such that $p^* > k$, voters would like advertising to be banned. If, on the other hand, $p^* < k$, advertising is beneficial.

4 When Both Candidates Can Receive Contributions

The previous section relied on the assumptions that the valence of Candidate 2 is known and that G can only contribute to Candidate 1. This section removes both assumptions.

4.1 Modifications to the Model

Assumption 2 is substituted with

Assumption 3 G can make an offer to each candidate. Offers are simultaneous and secret.

G can try to win the favors of both candidates. The assumption that offers are simultaneous excludes the possibility that G makes an offer to

one candidate, waits for his reply, and then makes an offer to the other candidate. While it may be more realistic, this possibility is outside the scope of this paper. The assumption that offers are secret means that A_j^a is not observed by Candidate $i \neq j$. This is to avoid the possibility that G could pre-commit to financing only one of the two candidates.

Let us make the following simplifying assumptions on the primitives of the model:

Assumption 4 (i) $x_1 \sim x_2 \sim 0$; (ii) $\hat{A}(\mu_j)$ is symmetric around the mean; (iv) voters do not observe z_1 and z_2 but only $z = z_1 \wedge z_2$; (v) G does not observe y_1 and y_2 but only $y = y_1 \wedge y_2$.

Part (i) of the assumption eliminates the incumbent advantage and makes candidates equal before until the insider signal is observed. Parts (ii) guarantees the symmetry of the problem. Parts (iv) and (v) assume that voters and groups can only observe the differences between signals and not the absolute value of signals. Although Assumption 4 does not appear to be central to the results that are going to be presented, it is useful because it leads to a simple characterization of the participation constraints for candidates.

With Assumption 4, it is possible to rewrite the problem in terms of differences rather than absolute values. The domains of y and z are respectively $Y = \{y \in \mathbb{R} \mid y \geq 0\}$ and $Z = \{z \in \mathbb{R} \mid z \geq 0\}$.

Taking into account Assumptions 3, Assumptions 4, and the new definitions, Game 1 becomes:

Game 2 The players are: voter $i \in I$, candidate $j \in \{1, 2\}$, and interest group G . The game consists of four stages:

1. Nature: Nature chooses $\mu_1 \in \mathbb{R}$ and $\mu_2 \in \mathbb{R}$, which remain unknown to all players.
2. Insider Stage: G observes $y \in \mathbb{R}_+$; announces p^a ; and makes offers $A_1^a \geq 0$ and $A_2^a \geq 0$. Candidate j does not observe $A_{i \neq j}^a$. If candidate j accepts the offer, then he must set $p_j = p^a$. If he rejects, he is free to decide p_i .
3. Public Stage: $z \in Z$ is realized. Voters observe p_1, p_2, A_1, A_2 , and z . For $i \in I$, Voter i votes for either 1 or 2. Let e denote the candidate that receives the higher number of votes.

4. Payoff Distribution: μ_1 and μ_2 are revealed. Voter i receives $\mu_{e_i} u(p_{e_i}; p_1)$. e receives 1 and i receives 0. G receives $\int u(p_g; p_e) dA_1; A_2$.

The players' strategies are: For G , $p^a \in [0, 1]$ and $A_j^a \in [0, 1]$ with $j = 1, 2$; For Candidate $j \in \{1, 2\}$, f^a , f^r and $p_j \in [0, 1]$ (if $f^r > 0$); For Voter $i \in I$, $e_i \in \{1, 2\}$.

Let H indicate Candidate 1 and L indicate Candidate 2 if $y = 1$ and viceversa if $y = -1$. Let M (as in 'medium quality') indicate both candidates if $y = 0$.

4.2 Equilibrium

As in the previous section, we need to take into account three factors: (1) the incentive-compatibility constraints for G ; (2) the participation constraints for the candidates; and (3) the optimal choice of p^a for G . Factors (1) and (2) will differ from the previous section. Instead of one constraint, there will be multiple constraints for each candidate.

4.2.1 Incentive-Compatibility Constraints

In a fully separating equilibrium, G must contribute only to H if $y \neq 0$ and to neither or both candidates if $y = 0$. It turns out that there are two separating equilibria according to whether a G with $y = 0$ contributes to both candidates or contributes to neither candidates. The former is labeled "Split-Contribution Equilibrium" and is analyzed here. The second is labeled "No-Split-Contribution Equilibrium" and is analyzed in Section 7.3. That section will argue that a no-split-contribution equilibrium is defeated (in the sense of Mailath, Okuno-Fujiwara, and Postlewaite [23]) by a split-contribution equilibrium and therefore is unlikely to arise.

Let us redefine $\hat{\mu}$ as

$$\hat{\mu}(y; z) = E(\mu_1; \mu_2 | y; z)$$

$\hat{\mu}(y; z)$ is strictly increasing in y and z . Notice that, as the voters' utility is separable in valence and policy, voters gain nothing by basing their decisions on both $E(\mu_1 | y; z)$ and $E(\mu_2 | y; z)$ rather than only on $E(\mu_1; \mu_2 | y; z)$.

Let $\hat{p}_y(p^a)$ be defined analogously to (1) except that now $y \in \{-1, 0, 1\}$:

$$\hat{p}_y(p^a) = \int \Pr^h \left[\hat{\mu}(1; z) \geq u(p^a; p_m) \geq u(0) \right] dF^i \left[u(p_g; p_m) \geq u(p_g; p^a) \right]$$

Let $v_H(p^x) = v_1(p^x)$, $v_M(p^x) = v_1(p^x)$, and $v_L(p^x) = v_1(p^x)$. For instance, $v_H(p^x)$ denotes the expected gross payoff to G if exactly one candidate has accepted an offer and that candidate is of type H (given the symmetry of the problem, it does not matter whether $H = 1$ or $H = 2$). Also, let

$$v_{all}(p^x) = \frac{1}{2}[u(p_g | p_m) + u(p_g | p^x)]$$

$v_{all}(p^x)$ is the expected (and ex-post) gross payoff of G if both candidates have accepted G's offer. Let us also define $v(p^x) = v_{all}(p^x) + v_H(p^x)$.

Lemma 7 Suppose that candidates always accept an offer of $v(p^x)$. If $y \in (0, 1)$, then G is willing to offer $v(p^x)$ to H but not to L. If $y = 0$, then G is willing to offer $v(p^x)$ to both candidates.

If candidates are equally good, G makes an offer to both. If one is better than the other, G makes an offer only to the better one. Under this strategy, G's insider information is fully revealed to voters. If $A_1 \geq v(p^x)$ and $A_2 \geq v(p^x)$, voters infer that $y = 0$. If $A_1 \geq v(p^x)$ and $A_2 < v(p^x)$, voters infer that $y = 1$. If $A_1 < v(p^x)$ and $A_2 \geq v(p^x)$, voters infer that $y = \frac{1}{2}$.

4.2.2 Participation Constraints

Let $z(y; p_1; p_2)$ be the unique solution to

$$\hat{u}(y; z) + u(p_m | p_1) + u(p_m | p_2) = 0$$

z is continuous and increasing in $y, p_1, -p_2$.

Recall that candidates do not observe y . However, in equilibrium, candidates of type L are not made offers. Thus, if a candidate is made an offer, he knows he must be H or M. Let us, for the moment assume that the candidate knows if he is H or M and let us write down the participation constraints for the two cases.

If $y \in (0, 1)$, let us assume without loss of generality that $y = 1$. Then, only Candidate 1 is made an offer. If he accepts, voters believe $y = 1$. If he rejects, voters believe $y = 0$. Thus, 1 accepts if he prefers $(y = 1; p_1 = p^x; p_2 = p_m)$ to $(y = 0; p_1 = p_m; p_2 = p_m)$, that is

$$z(1; p^x; p_m) \geq z(0; p_m; p_m) \tag{4}$$

If $y = 0$, both candidates are made offers. The subgame between candidates is as follows: if they both accept or both reject, voters believe $y = 0$;

if j accepts and i, j rejects, voters believe $y \in 0$ and $j = H$ and $i, j = L$. It is optimal for 1 to accept if he prefers $(y = 0; p_1 = p^a; p_2 = p^a)$ to $(y = i, 1; p_1 = p_m; p_2 = p^a)$, that is

$$z(0; p^a; p^a) \geq z(i, 1; p_m; p^a) \quad (5)$$

Let \underline{p} be the unique p for which $z(1; p; p_m) = 0$ (\underline{p} may be higher than, equal to, or lower than p). Then,

Lemma 8 Candidate j accepts $A_j^a \geq \theta(p^a)$ if and only if $p^a \geq \underline{p}$ and rejects any offer below $\theta(p^a)$.

Lemma 8 has a simple form because it turns out that constraints (4) and (5) are equivalent. This is due to Assumption 4 and greatly simplifies the problem at hand. Lemma 8 is analogous to Lemma 4 except that \underline{p} is in general different from \bar{p} (without further assumptions, it is not possible to say whether \underline{p} is greater or smaller than \bar{p}).

4.2.3 Optimal Offer for G

Given the incentive-compatibility constraint in Lemma 7 and the participation constraint in Lemma 8, G chooses p^a in order to maximize the net expected payoff. Analogously to the previous section,²³

Lemma 9 In a separating equilibrium, G sets $p^a = \underline{p}_{\max}$, where

$$\underline{p}_{\max} = \operatorname{argmax}_p 2 \int_H(p) i \int_L(p)$$

subject to $p^a \geq \underline{p}$.

If the participation constraint is binding, $\underline{p}_{\max} = \underline{p}$. If the participation constraint is not binding, $\underline{p}_{\max} < \underline{p}$.

4.2.4 Equilibrium

Proposition 5 A sequential equilibrium of Game 2 is

(i) Voters' beliefs:

$$\hat{\mu} = \begin{cases} \hat{\mu}(1; z) & \text{if } A_1 \geq \theta(p^a) \text{ and } A_2 < \theta(p^a) \\ \hat{\mu}(0; z) & \text{if } \max(A_1; A_2) < \theta(p^a) \text{ or } \min(A_1; A_2) \geq \theta(p^a) \\ \hat{\mu}(i, 1; z) & \text{if } A_1 < \theta(p^a) \text{ and } A_2 \geq \theta(p^a) \end{cases}$$

²³In general $\underline{p}_{\max} \neq p_{\max}$.

- (ii) Voters' choice: $e_i = 1$ if and only if $\mu_j u(p_i | p_1) > u(p_i | p_2)$. 1 is elected if and only if $\mu_j u(p_m | p_1) > u(p_m | p_2)$.
- (iii) Group's offer: For $y \in \{0, 1\}$, $p^* = p_{\max}$, and
 - (a) If $y = 1$, $A_1^* = 0$ and $A_2^* = \mu_j(p^*)$;
 - (b) If $y = 0$, $A_1^* = A_2^* = \mu_j(p^*)$;
 - (c) If $y = 1$, $A_1^* = \mu_j(p^*)$ and $A_2^* = 0$.
- (iv) Candidates' acceptance: for $j = 1, 2$, Candidate j accepts A_j^* if and only if $A_j^* \geq \mu_j(p^*)$ and $p^* \leq \underline{p}$. If j rejects, $p_j = p_m$.

Two cases are possible: $y = 0$ and $y \in \{0, 1\}$. In the first case, G makes an offer $\mu_j(p^*)$ to both candidates and both candidates accept. In the second case, G makes an offer only to H and H accepts. If voters observe that both candidates advertise, they infer $y = 0$. If voters observe that 1 advertises and 2 does not, they infer $y = 1$, and viceversa if only 2 advertises. Thus, the equilibrium fully reveals y . The offer is set at $\mu_j(p^*)$, which corresponds to the difference between the gross expected payoff when G contributes to both H and L and the gross expected payoff when G contributes to H only. In equilibrium, an advertising level of at least $A_j = \mu_j(p^*)$ guarantees voters that $j \in L$.²⁴

4.3 Split Contributions

It is worth spelling out the following:

Corollary 1 In the Split-Contribution Equilibrium, if $y = 0$, G offers a contribution to both candidates, both candidates accept, and \underline{p} is implemented for sure. If Candidate j rejected the contribution, voters would believe $j = L$ and $j = H$.

²⁴One may wonder if there could be a separating equilibrium in which, if $y \in \{0, 1\}$, G makes an infinitesimal contribution to H and a zero-contribution to L . This equilibrium could be seen as an endorsement à la Grossman and Helpman [13]. If G could commit to contribute to exactly one candidate, such equilibrium would indeed exist. However, this possibility is excluded in the present model by the assumption that offers are secret. G has no way of committing to contribute to only one candidate. The only way to ensure that G does not make two contributions when $y \in \{0, 1\}$ is the respect of the incentive-compatibility constraint $A_j \geq \mu_j(p^*)$.

When $y = 0$, G has an implicit threat against both candidates. If one of the candidates rejects the contribution, only the other candidate will advertise and voters will perceive the candidate who advertises as H and the candidate who does not advertise as L .²⁵

Split contributions are a tool G uses to extract rent from voters. Would a ban on split contributions (assuming that it is feasible) be optimal from the point of view of voters?

Proposition 6 A ban on split contributions always increases the ex-ante voter welfare.

If split contributions are banned, full revelation of y will still occur but, when $y = 0$, neither candidate will advertise and candidates will select p_m rather than \underline{p} . Voters will be better off and G will be worse off. Thus, a ban on split contributions always increases the voter welfare.

4.4 Voter Welfare

What happens to voter welfare if advertising is banned altogether? The answer is analogous to the answer in the case in which G can only contribute to Candidate 1. A formal statement is superfluous. If the participation constraint of the candidates is binding, that is if $\underline{p} = \underline{p}_{max}$, then by an argument analogous to Lemma 6, a ban on advertising certainly increases voter welfare. If the participation constraint is not binding, then a ban on advertising may or may not be optimal according to how close \underline{p} is to p_m .

5 Discussion and Extensions

This section tackles some important aspects of campaign advertising that were disregarded in the previous sections. For ease of exposition, we refer to the simpler model of Section 3.

²⁵In the present model, when $y = 0$, candidates are indifferent between the situation in which both advertise and the hypothetical situation in which neither advertises. However, we could assume that each candidate derives a small, but positive, utility from catering to voters. Then, candidates would strictly prefer the situation in which neither advertise to the situation in which both advertise. However, in equilibrium, it would still be a dominant strategy for candidate j to accept G 's offer. This equilibrium is Pareto inefficient from the point of view of candidates. If candidates could commit not to accept contributions, they would be better off. This situation is known in the political literature as the "Candidates' Prisoner Dilemma."

5.1 Unobservable Policy Choice

The model has assumed that, before the election, voters observe policies p_1 and p_2 that candidates are going to adopt if elected. Suppose on the contrary that p_1 and p_2 are unobservable. For the rest, let us consider an electoral race as in Game 1.

There is, however, one problem with a model in which p_j is unobservable and candidates maximize only their election chances: candidates are indifferent among policy positions. To sidestep this problem, let us assume that candidates pursue two goals: election and the maximization of the median voter's welfare. However, the second goal is infinitely less important than the first. Thus, if a candidate does not receive a contribution, he will choose p_m .²⁶

Then,

Proposition 7 If $p_g > \bar{p}$, there exists no separating equilibrium. If $p_g \leq \bar{p}$, there exists a separating equilibrium which is identical to the separating equilibrium in Proposition 2 except that $p^* = p_g$ and $A_1^* = \bar{L}(p_g)$.

If the candidate's policy choice is unobservable, G will take all the advantage she can from making an offer. Thus, she will ask the candidate to implement p_g if he is elected. Voters realize that a candidate who advertises is going to implement p_g . Thus, even if voters do not observe p_1 , they can anticipate it perfectly. If p_g is not too high, the median voter prefers (for the same z) a candidate who advertises to a candidate who does not advertise. If p_g is very high, then the median voter prefers a candidate who does not advertise. In that case, candidates would reject a contribution from G .

The case $p_g > \bar{p}$ corresponds to a notoriously extremist group. The median voter punishes anyone who associates with such group. An example may be represented by the tobacco industry in the US, whose ideal policies seem to be hated by the median voter. A candidate who is caught receiving money from the tobacco industry is likely to be stigmatized by the media and by his opponents. Thus, most candidates reject any contribution from tobacco interests.

On the welfare side, the results that were proven for the case in which policy is observed carry on to the present case. It is immediate that Proposition 4 holds as stated, except that p_{\max} is substituted with p_g .

²⁶The same candidate behavior would be achieved in a model in which p_j is observed with an infinitesimal probability.

5.2 Candidates Make Offers

The model has given all the bargaining power to G by assuming that G can make candidates a take-it-or-leave-it offer. Let us consider the opposite case. It will be shown that results change dramatically if policy is observable and are almost unchanged if policy is unobservable.

Let modify Game 1 by assuming that 1 asks G for contribution A_1^a in exchange for policy p^a and A_1^a . G accepts or rejects. The following is immediate.

Proposition 8 If candidates make offers and p_1 is observable, there exists a separating equilibrium in which 1 offers $p^a = p_m + \epsilon$, where ϵ is positive and infinitesimal, and asks for $A_1^a = \frac{1}{2}(p_m + \epsilon)$. G accepts if and only if $y_1 = 1$.

For any p^a , 1 can ask G for a contribution that G can afford only if $y = 1$. To maximize the chance of election, 1 sets p^a as low as possible. Revelation occurs at an infinitesimal cost. Thus,

Corollary 2 If candidates make offers and p_1 is observable, an advertising ban is never optimal.

However, this result relies on the perfect observability of p_1 . If, on the contrary, we assume that p_1 is unobservable, we have

Proposition 9 Suppose candidates make offers and p_1 is unobservable. Let P be the lowest solution to $\frac{1}{2}H(P) = \frac{1}{2}L(p_g)$. If $P > \bar{p}$, there exists no separating equilibrium. If $P < \bar{p}$, there exists a separating equilibrium in which 1 offers $p^a = P$ and asks for $A_1^a = \frac{1}{2}L(p_g)$. G accepts if and only if $y = 1$.

The equilibrium in Proposition 9 is identical to the equilibrium in Proposition 7 except that $P < p_g$. As the candidate, rather than G, has the bargaining power, he will choose the lowest p^a that satisfies the incentive-compatibility constraint. However, in general, P is not infinitesimal: P can take any value in $(0; \bar{p}]$. Thus, it is easy to see that Proposition 4 holds as stated, except that p_{\max} is replaced with P . Therefore:

Corollary 3 If candidates make offers and p_1 is unobservable, a ban on advertising can be optimal.

5.3 Self-Financing Candidates

This model has assumed the only source of campaign funds for candidates are group contributions. Sometimes, however, candidates dispose of large personal wealth they can use for campaign advertising.²⁷ One may conjecture that the results for group-financed campaigns extend readily to self-financed campaigns. As it will be seen, this conjecture is not granted. In particular, with self-financed candidates, a separating equilibrium may not exist.

Once again, reconsider Game 1. Let G be removed from the model. Candidate 1 is risk-neutral and can borrow freely. If 1 is elected, he receives \$1. If he is not elected, he receives 0. As before, 1 derives no direct utility from policy. Candidate 1 chooses an amount $A_1 \in [0, 1)$ to be spent on uninformative advertising.

Suppose a separating equilibrium exists. Let voters' beliefs on y_1 be denoted by $y_1 \in [0, 1]$. Let $R_{y_1, y_1} = \Pr[\hat{\mu}(y_1; z_1) \leq 0 | y_1]$. The incentive-compatibility constraint for 1 is

$$R_{1,0} \geq R_{0,0} \cdot A_1 < R_{1,1} \geq R_{0,1} \quad (6)$$

which corresponds to

$$f_y(z(x; 0) | 1) \geq f_y(z(x; 1) | 1) \leq f_y(z(x; 0) | 0) \geq f_y(z(x; 1) | 0)$$

(6) may or may not be satisfied. It depends on the form of f_y and f_z . It is always the case that $R_{1,1} > R_{1,0}$. However, $R_{0,1} > R_{0,0}$, and it is not possible to say in general if the difference between $R_{0,1}$ and $R_{0,0}$ is greater than the difference between $R_{1,1}$ and $R_{1,0}$. Thus, (6) may or may not hold. Then,

Proposition 10 With self-financing candidates, a separating equilibrium may not exist.

What is the difference between a self-financed campaign and a group-financed campaign? In a group-financed campaign, if G does not contribute, she gets 0 independently of y . In a self-financed campaign (and assuming a separating equilibrium existed), if the candidate does not advertise, he still has a positive probability of winning the election, and this probability is higher if $y = 1$ than if $y = 0$. This reduces the incentive for a candidate with $y = 1$ to signal his type.

²⁷Recent examples of large-scale self-financed campaigns include Ross Perot in the US, Silvio Berlusconi in Italy, and Bernard Tapie in France. In 1995, a candidate to the US Senate, Michael Huffington, spent over 30 million dollars of mostly personal funds on his campaign. For the record, he lost.

5.4 Existence of Optimal Mechanisms

Voters face a tradeoff between renouncing the insider information (advertising ban) and putting up with bad policies (equilibrium with advertising). Can voters avoid this tradeoff altogether? Is there a mechanism through which G reveals her information at a minimum cost for voters?

It is easy to see that many such mechanisms exist. For instance, suppose campaign contributions are banned and G is asked to name one of the two candidates. If the named candidate is actually elected, then G receives \$1 (or an infinitesimal shift on the policy line). If the named candidate is not elected, G receives nothing. With this mechanism, G will name the candidate which has given her the highest insider signal (if $x \leq 0$, the odds of the bet need to be modified in order to offset the ex-ante advantage). Such mechanism induces truthful revelation of y at an infinitesimal cost.

However, mechanisms of this type are unrealistic for two reasons. First, they are not robust to collusion between G and the candidates: G could offer a candidate to name him in exchange for a favorable policy. In order to achieve collusion-proofness, G should be promised at least as much as she would get under the equilibrium with advertising, but that defeats the purpose of those mechanisms. Second, these mechanisms assume that voters and interest groups can make agreements. It is difficult to see how an unorganized mass of voters can be so organized to coordinate on a mechanism choice and to make credible commitments.

6 Conclusion

An electoral race with campaign advertising has been modeled as a signaling game with one interest group, two candidates, and a continuum of fully rational voters. Two versions of the model have been developed. In the first version, the interest group can only contribute to a pre-specified candidate. The main results are that: (1) a separating equilibrium exists and (2) under certain conditions, the voters' welfare is higher under an advertising ban than under the separating equilibrium. In the second version of the model, the group can contribute to both candidates. The main results are similar to the first version except that, if the insider signals about the two candidates are of equal quality, then the group will make split contributions. Prohibiting split contributions strictly increases the voter welfare.

Campaign advertising is a complex issue. Many aspects that have been left out by the present may, in the future, be addressed within a similar

framework:

First, the model has assumed that only one interest group is active. It would be important to extend the model to several groups in competition with each other. This could be done in a common agency framework (See for instance Grossman and Helpman [14]). A conjecture is that the negative welfare effects of campaign advertising disappear if interest groups are symmetrically distributed around the median voter.²⁸

Second, the model has assumed that the amount spent on advertising is perfectly observable by all voters. In a more realistic framework (like Hertzendorf [16]), advertising expenditures translate in a probability distribution over the number of TV ads each voter will watch.

Third, in this model voters have heterogeneous preferences but they are assumed to have homogeneous information: x and z are the same for all voters. The model could be extended to include heterogeneous voters information, which will provide a link with the literature on information aggregation in elections (e.g. Lohmann [20] or Feddersen and Pesendorfer [10]).

Lastly, in this model a ban on advertising produces the same effect as a ban on contributions. In practice, there are important differences.²⁹ First, while advertising restrictions can be enforced, the experience of several countries shows that restrictions on campaign contributions are often disregarded or dodged. Second, contributions can be spent in a variety of ways, which give different signals to different voters. Thus, a ban on advertising does not necessarily make contributions useless to candidates. Third, campaign advertising is an expression of political opinion. Thus, restrictions on it can be seen as restrictions on free expression and may be unconstitutional. The first argument supports restrictions on advertising, the last two arguments point in favor of restrictions on contributions. More detailed models should be developed with the goal of comparing the effects of the two types of restrictions.

²⁸However, in reality interest groups do not seem to be symmetrically distributed around the median voter, but their median member appears to be more affluent and advantaged than the median voter. Lehman Schlozman and Tierney [19, p. 87] conduct a comprehensive survey on US groups and conclude that: "In terms of skew, organization members are drawn disproportionately from the ranks of upper-status individuals { those with high levels of income, education, and occupational prestige.[...] Surprisingly, in spite of the appearance of new groups representing the previously underrepresented, the imbalance of the pressure community seems to have become more pronounced in the recent years."

²⁹The US has chosen the road of regulating contributions but letting candidates spend freely. European countries, instead, tend to focus on spending. For instance, in Britain individual candidates are not allowed to run TV ads.

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7 Appendix

7.1 Proofs

Proof of Lemma 3: It is sufficient to show that $v_H(p^a) > v_L(p^a)$ for any $p^a > 0$. Let $v_i(t|x; y) = \Pr(z = t|x; y)$. Then

$$v_y(p^a) = (1 - v_i(z(x; 1; p^a)|x; y))^{1-u(p_g | p_m)} u(p_g | p^a) \quad (7)$$

Let $K(\mu_1|x_1; y_1)$ be the posterior distribution of μ_1 given x_1 and y_1 . $K(t|t; t)$ satisfies FOSD. Thus, for every strictly increasing function $\phi(t)$, $\int \phi(\mu) dH(\mu_1|x_1; y_1)$ is strictly increasing in x_1 and y_1 . But, because MLRP implies that $F_z(z_1|\mu_1)$ is a strictly decreasing function of μ_1 , then

$$v_i(z_1|x_1; y_1) = \int_Z F_z(z_1|\mu_1) dH(\mu_1|x_1; y_1)$$

is a strictly decreasing function on x_1 and y_1 (that is, v_i satisfies FOSD). Thus, for any $z_1 \in Z$, $v_i(z_1|x_1; 1) < v_i(z_1|x_1; 0)$. Therefore, $v_H(p^a) > v_L(p^a)$ for any $p^a > 0$.

Proof of Lemma 4: Candidate 1 minimizes $\hat{z}(x_1; y_1; p)$, where y_1 denotes the voters' belief on y_1 . In a separating equilibrium, $y_1 = 0$ if $A_1^a < v_L(p^a)$ and $y_1 = 1$ if $A_1^a \geq v_L(p^a)$. The first part of the lemma is immediate. If $A_1^a < v_L(p^a)$, then G accepts the contribution if and only if $\hat{z}(x_1; 1; p^a) \leq \hat{z}(x_1; 0; p_m)$.

Proof of Lemma 5: If $y = 1$, G solves

$$\max_{p^a, A_1^a} v_H(p^a) - A_1^a$$

subject to

$$A_1^a \geq v_L(p^a) \\ p^a \leq \beta$$

with $A_1^a \geq 0$. This is equivalent to

$$\max_{p^a} v_H(p^a) - v_L(p^a)$$

subject to $p^a \leq \beta$. As $v_H(p^a) - v_L(p^a)$ is continuous and strictly positive in p^a except when $p^a = 0$ or $p^a = +1$, the problem has at least one maximum on $(0; 1)$.

Proof of Proposition 3: Let \hat{p}^d be the unique $p > 0$ such that $\hat{z}(x; 1; p) = \hat{z}^P(x; p_m)$. Consider the deviation $p^a = \hat{p}^d$ with $0 < \hat{p}^d < \beta$ and $v_L(\hat{p}^d) < A_1^a < v_H(\hat{p}^d)$. Such deviation is profitable to G if and only if $y = 1$. 1 accepts the offer because it strictly increases his chances of elections.

Proof of Lemma 6: In the separating equilibrium, if $y_1 = 0$, Candidate 1 is elected if and only if $z_1 \geq \hat{z}(x_1; 0; p_m)$. If $y_1 = 1$, Candidate 1 is elected if and only if $z_1 \geq \hat{z}(x_1; 1; p^a)$. If $p^a = \hat{p}$, Lemma 4 implies $\hat{z}(x_1; 1; p^a) = \hat{z}(x_1; 0; p_m)$. Thus, $e=1$ if and only if $z \geq \hat{z}(x; 0; p_m)$, irrespective of whether $y = 1$ or $y = 0$. Thus, $e=1$ if and only if $\hat{\mu}(x; 0; z) \geq 0$. Let $b(x) = \mu(x; \hat{z}(x; 0; p_m)) \wedge \hat{\mu}(x; 0; \hat{z}(x; 0; p_m))$. Then, $e = 1$, $\mu(x; z) \geq b(x)$.

As the rule $e = 1$, $\mu(x; z) \geq 0$, is optimal by definition within the set of rules which use only x and z , it follows that $w_S(\hat{p}) < w_P$.

Proof of Proposition 4: Given Lemma 6 and the fixed-point theorem, it is sufficient to prove the two following claims:

1. $w_S(p^a)$ is strictly decreasing and continuous in p^a ;
2. Let $w_S(p_m) = \lim_{p^a \downarrow p_m^+} w_S(p^a)$. Then, $w_S(p_m) > w_P$.

To prove Claim 1, recall (3) and notice that $w_S(1; 1; p^a)$ is strictly decreasing and continuous in p^a and $w_S(0; 0; p^a) = w_S(0; 1; p^a) = w_S(1; 0; p^a)$ is constant in p^a . Suppose $y_1 = 1$: then the expected voter welfare is $w_S(p^a | y_1 = 1) \wedge \max\{w_S(1; 1; p^a); w_S(0; 1; p^a)\}g$, which is strictly decreasing in p^a . Suppose $y_1 = 0$: then the expected voter welfare is $w_S(p^a | y_1 = 0) \wedge \max\{w_S(1; 0; p^a); w_S(0; 1; p^a)\}g$, which is constant in p^a . As

$$w_S(p^a) = \Pr(y_1 = 1)w_S(p^a | y_1 = 1) + \Pr(y_1 = 0)w_S(p^a | y_1 = 0)$$

and the event $y_1 = 1$ has positive probability which does not depend on p^a , the claim is proven.

To prove Claim 2, suppose that, if $p^a \neq p_m^+$. Then, under the separating equilibrium, when $y = 1$, $p_1 \neq p^m$. As $p_1 = p_m$ when $y_1 = 0$ in all cases, the ex post welfare under a separating equilibrium becomes

$$w(e; p_m) = \begin{cases} \frac{1}{2} & \text{if } e = 1 \\ 0 & \text{if } e = 2 \end{cases}$$

which is identical to the ex post welfare under the pooling equilibrium. However, under the separating equilibrium voters know x , y , and z while under a pooling equilibrium voters only know x and z . Because y is informative,

$$E_\mu[\max_{e \in \{1, 2\}} E(w(e; p_m) | x; y; z)] > E_\mu[\max_{e \in \{1, 2\}} E(w(e; p_m) | x; z)]$$

which proves Claim 2.

Statement and Proof of Lemma 10 The following technical result is useful:

Lemma 10 For any $y \in [0, 1]$ and any $z \in \mathcal{Z}$, $\hat{\mu}(y; z) = \int \hat{\mu}(y; z) dz$.

Proof: Let

$$g_y(y; \mu_1; \mu_2) = \int_{y_1 + y_2 = y} f(y_1; \mu_1) f(y_2; \mu_2)$$

and

$$g_z(z; \mu_1; \mu_2) = \int_z f_z(z_1; \mu_1) f_y(z - z_1; \mu_2) dz_1$$

It is easy to see that the p.d.f.'s g_y and g_z are antisymmetric in μ_1 and μ_2 , that is

$$g_y(y; \mu^0; \mu^0) = g_y(i; y; \mu^0; \mu^0) \quad (8)$$

for any $y \in Y$ and any $\mu^0; \mu^0 \in \mathcal{E}$ (and similarly for z). Also, it follows immediately from Assumption 1 that g_y and g_z satisfy the Monotone Likelihood Ratio Property with respect to μ_1 and μ_2 .

Thus,

$$\hat{\mu}(y; z) = \frac{\int_{\mathcal{E}} \int_{\mathcal{E}} (\mu_1; \mu_2) f_y(y; \mu_1; \mu_2) f_z(z; \mu_1; \mu_2) \hat{A}(\mu_1) \hat{A}(\mu_2) d\mu_1 d\mu_2}{\int_{\mathcal{E}} \int_{\mathcal{E}} f_y(y; \mu_1; \mu_2) f_z(z; \mu_1; \mu_2) \hat{A}(\mu_1) \hat{A}(\mu_2) d\mu_1 d\mu_2}$$

and

$$i \hat{\mu}(i; y; i; z) = \frac{\int_{\mathcal{E}} \int_{\mathcal{E}} (\mu_1; \mu_2) f_y(i; y; \mu_1; \mu_2) f_z(i; z; \mu_1; \mu_2) \hat{A}(\mu_1) \hat{A}(\mu_2) d\mu_1 d\mu_2}{\int_{\mathcal{E}} \int_{\mathcal{E}} f_y(i; y; \mu_1; \mu_2) f_z(i; z; \mu_1; \mu_2) \hat{A}(\mu_1) \hat{A}(\mu_2) d\mu_1 d\mu_2}$$

By recalling Part (ii) of Assumption 4, applying (8), and switching μ_1 with μ_2 , we have $i \hat{\mu}(i; y; i; z) = \hat{\mu}(y; z)$. \square

Proof of Lemma 7: It is easy to see that $v_{all}(p) > v_H(p) > v_M(p) > v_L(p)$ and that $v_{all}(p) - v_M(p) > v_{all}(p) - v_H(p)$. Also, $v_H(p) + v_L(p) > v_{all}(p)$, which implies $v_L(p) > v^*(p)$. If $y = 0$, $v_{all}(p) - v^*(p) > v_M(p) - v^*(p) > 0$: G is willing to make an offer to both candidates. If $y \neq 0$, $v_{all}(p) - v^*(p) = v_H(p) - v^*(p) > 0$ and G is exactly indifferent between making an offer to H only or to both H and L .

Proof of Lemma 8: Notice that $z(0; p; p) = 0$ for any p and, by Lemma 10,

$$z(y; p^0; p^0) = i z(i; y; p^0; p^0)$$

for any p^0, p^0 and y . Then, (5) is equivalent to $z(1; p^m; p_m) \leq 0$. Both (5) and (4) are satisfied if and only if $z(1; p^m; p_m) \leq 0$, which is true if $p^m \leq p$.

Proof of Proposition 5: Proof of (i): Beliefs are consistent with (iii) and (iv). Proof of (ii): Lemma 1. Proof of (iii): Lemma 7. Proof of (iv): There are two cases: $j = H$ or $j = M$. By (i) and Lemma 8, given the strategy of i, j , the strategy of j is optimal in both cases.

Proof of Lemma 9: The net expected payoff[@] for G is, in the case of $y \neq 0$,

$$v_H(p^m) - v^*(p^m) = 2v_H(p^m) - v_{all}(p^m)$$

and, in the case of $y = 0$ is

$$v_{all}(p^m) - v^*(p^m) = 2v_H(p^m) - v_{all}(p^m)$$

Thus, the net expected payoff[@] is equal in both cases (another useful consequence of Assumption 4). Let

$$p_{max} = \operatorname{argmax}_p 2v_H(p) - v_{all}(p)$$

subject to $p_{max} \leq p$. Analogously to Lemma 5, G sets $p^m = p_{max}$.

Proof of Proposition 6: Under a ban on split contributions, the No-Split-Contribution Equilibrium (NSCE) of Proposition 11 in Section 7.3 will still be a sequential equilibrium of Game 2. Let us compare SCE with NSCE. If $y \neq 0$, the outcome for voters is identical in both equilibria. If $y = 0$, 1 is elected in one equilibrium if and only he is elected in the other (when $z > 0$) but in the SCE $p_1 = p_2 = \underline{p}$ while in the NSCE $p_1 = p_2 = p_m$. Thus, if $y = 0$, the voter welfare is strictly higher under NSCE. Therefore, the ex-ante welfare (before y is realized) is higher in NSCE than in SCE.

Proof of Proposition 7: Recall that p_1 is unobserved and Candidate 1 cares about policy only in infinitesimally. Then, if he accepts an offer p_1 , he will accept it independently of p^a . Thus G will always ask for her preferred policy: $p^a = p_g$. In equilibrium voters know that $p^a = p_g$. Thus, if they observe an offer $p_1 > 0$, they infer $p_1 = p_g$. If $A_1^a > \int_H(p_g)$, the incentive-compatibility condition is satisfied. The Participation constraint is satisfied if and only if $p_g \geq \beta$. Thus, Candidate 1 accepts G's offer if and only if $p_g \geq \beta$. Thus, if $p_g > \beta$, a separating equilibrium cannot exist. If $p_g \geq \beta$, the existence of a separating equilibrium is proven analogously to Proposition 2.

Proof of Proposition 9: In a separating equilibrium, the following constraints must be satisfied:

1. PC for 1: $p^a \geq \beta$;
2. ICC for G: $A_1^a \geq \int_L(p_g)$ (because $\int_L(p_g)$ is the maximum G is willing to offer, under any p^a , if $y_1 = 0$).
3. PC for G: $\int_H(p^a) \geq A^a$.

Considering the last two constraints, Candidate 1 minimizes p^a by choosing $p^a = P$ where P is the lowest solution to $\int_H(P) = \int_L(p_g)$. The PC for 1 is satisfied if and only if $P \geq \beta$. If $P < \beta$, the PC for 1 and the ICC cannot be both satisfied and a separating equilibrium does not exist.

7.2 Example for Remark 2

Let us assume that $u(t) = \frac{1}{2}t^2$ where $\frac{1}{2} > 0$. Thus, $u(p_i | p_e)$ is decreasing at a constant rate up to p_i and increasing at a constant rate after p_i . Of course, the same applies to $u(p_g | p_e)$. Let us also assume that $p_m = 0$.

Also, $\mu_1 \in [0, 1]$ with $\Pr(\mu_1 = 0) = \Pr(\mu_1 = 1)$; $X \in [0, 1]$ and $Z \in [0, 1]$ with

$$f_X(x_1 | \mu) = \begin{cases} \frac{1}{2} 2x & \text{if } \mu_1 = 1 \\ \frac{1}{2} 2(1 - x_1) & \text{if } \mu_1 = 0 \end{cases}$$

and

$$f_Z(z_1 | \mu) = \begin{cases} \frac{1}{2} 2z & \text{if } \mu_1 = 1 \\ \frac{1}{2} 2(1 - z_1) & \text{if } \mu_1 = 0 \end{cases}$$

Instead of the usual $\mu_2 = 0$, let $\mu_2 = \frac{1}{2}$.³⁰ As usual, $Y \in [0, 1]$. Let

$$f_Y(y_1 | \mu_1) = \begin{cases} \frac{1}{2} [1 + \frac{1}{2}(2y_1 - 1)] & \text{if } \mu_1 = 1 \\ \frac{1}{2} [1 - \frac{1}{2}(2y_1 - 1)] & \text{if } \mu_1 = 0 \end{cases}$$

³⁰Instead, one could assume $\mu_2 = 0$ and $\mu_1 \in [0, 1]$. The results would not change.

where $\frac{1}{2} \in (0; 1)$ represents the precision of y_1 . Then,

$$\begin{aligned}\hat{\mu}(x_1; y_1; z_1) &= \Pr(\mu_1 = 1 | x_1; y_1; z_1) \\ &= \frac{f_x(x_1 | \mu_1 = 1) f_y(y_1 | \mu_1 = 1) f_z(z_1 | \mu_1 = 1)}{f_x(x_1 | \mu_1 = 0) f_y(y_1 | \mu_1 = 0) f_z(z_1 | \mu_1 = 0) + f_x(x_1 | \mu_1 = 1) f_y(y_1 | \mu_1 = 1) f_z(z_1 | \mu_1 = 1)} \\ &= \frac{x_1 z_1 [1 + \frac{1}{2}(2y_1 - 1)]}{(1 - x_1)(1 - z_1)[1 - \frac{1}{2}(2y_1 - 1)] + x_1 z_1 [1 + \frac{1}{2}(2y_1 - 1)]}\end{aligned}$$

To find $\hat{\mu}$, let

$$\begin{aligned}\phi(z_1 | x_1; y_1) &= \frac{1}{2} f_z(z_1 | \mu_1 = 1) \Pr(\mu_1 = 1 | x_1; y_1) + \frac{1}{2} f_z(z_1 | \mu_1 = 0) \Pr(\mu_1 = 0 | x_1; y_1) \\ &= \frac{z_1 x_1 [1 + \frac{1}{2}(2y_1 - 1)] + (1 - z_1)(1 - x_1)[1 - \frac{1}{2}(2y_1 - 1)]}{(1 - x_1)[1 - \frac{1}{2}(2y_1 - 1)] + x_1 [1 + \frac{1}{2}(2y_1 - 1)]}\end{aligned}$$

Then,

$$\begin{aligned}\hat{\mu}(z_1 | x_1; y_1) &= \int_0^{z_1} \phi(t | x_1; y_1) dt \\ &= \frac{2(1 - x_1)z_1 [1 - \frac{1}{2}(2y_1 - 1)] + z_1^2 (2x_1 - 1 - \frac{1}{2} + 2y_1 \frac{1}{2})}{1 + \frac{1}{2} [2x_1 \frac{1}{2} - 2y_1 \frac{1}{2} + 4x_1 y_1 \frac{1}{2}]}\end{aligned}$$

Recall that $\hat{z}(x_1; y_1; p^a)$ is the unique z_1 that solves

$$\hat{\mu}(\hat{z}(x_1; y_1; p^a) | x_1; y_1) - \frac{1}{2} = 0$$

Then,

$$\hat{z}(x_1; y_1; p^a) = \frac{[1 + x_1 - 2p^a - 2p^a x_1 + \frac{1}{2} (x_1 \frac{1}{2} + 2p^a \frac{1}{2} - 2p^a x_1 \frac{1}{2})]}{[1 - 2p^a + 4p^a x_1 + \frac{1}{2} (2x_1 \frac{1}{2} + 2p^a \frac{1}{2})]} \quad (9)$$

Let \hat{p} be the unique p^a that solves

$$\hat{z}(x_1; 1; \hat{p}) = \hat{z}(x_1; 0; 0)$$

Then,

$$\hat{p} = \frac{\frac{1}{2}}{1 + \frac{1}{2} x_1^2}$$

(the fact \hat{p} is independent of x_1 is a feature of this example, but is not true in general).

By (7),

$$\hat{y}(p^a) = (1 - \hat{z}(x; 1; p^a) j(x; y))^{-1} [u(p_g | p_m) - u(p_g | p^a)]$$

Thus,

$$\hat{y}(p^a) - \hat{y}(p^a) = \frac{8p^a (x_1 - 1)^2 x_1^2 (4(p^a)^2 - 2 - 1)^{1/2} (\frac{1}{2} - 1)}{[1 - (1 - 2x_1)^2 \frac{1}{2}] [1 + \frac{1}{2} (2x_1 \frac{1}{2} - 2x_1 \frac{1}{2} + 2 \frac{1}{2} (2x_1 - 1 + \frac{1}{2}))^2]}$$

p_{\max} cannot be found analytically, but, given some parameter values, it can be computed. For instance, let us assume that $p_g = 1$, $p_m = 1$, and $\frac{1}{2} = 1$. Suppose $\frac{1}{2} = 0.3$. Then,

$$\begin{aligned}\hat{p} &= 0.275 \\ p_{\max} &= 0.326\end{aligned}$$

Suppose instead that $\frac{1}{2} = 0.5$. Then,

$$\begin{aligned} \bar{p} &= 0.4 \\ p_{\max} &= 0.358 \end{aligned}$$

Thus, we have shown that both the case $\bar{p} > p_{\max}$ and the case $\bar{p} < p_{\max}$ are possible.

7.3 No-Split-Contribution Equilibrium

The electoral game of Section 4 has another separating equilibrium besides the one characterized in Proposition 5:

Proposition 11 The following is a sequential equilibrium of Game 2:

(i) Voters' beliefs:

$$\mu = \begin{cases} \hat{\mu}(1; z) & \text{if } A_1 \geq \frac{1}{2}M(p^*) \text{ and } A_2 < \frac{1}{2}M(p^*) \\ \hat{\mu}(0; z) = \mu(z) & \text{if } \max(A_1; A_2) < \frac{1}{2}M(p^*) \text{ or } \min(A_1; A_2) \geq \frac{1}{2}M(p^*) \\ \hat{\mu}(i; z) & \text{if } A_1 < \frac{1}{2}M(p^*) \text{ and } A_2 \geq \frac{1}{2}M(p^*) \end{cases}$$

(ii) Voters' choice: $e_i = 1$ if and only if $\mu_j u(p_i | p_1) + (1 - \mu_j) u(p_i | p_2) > 0$. 1 is elected if and only if $\mu_j u(p_m | p_1) + (1 - \mu_j) u(p_m | p_2) > 0$

(iii) Group's offer: For $y \in \{1, 0\}$, $p^* = \min(p_{\max}; \underline{p})$.

(a) If $y = 1$, $A_2^* = \frac{1}{2}M(p^*)$ and $A_1^* = 0$.

(b) If $y = 0$, $A_1^* = A_2^* = 0$.

(c) If $y = 1$, $A_1^* = \frac{1}{2}M(p^*)$ and $A_2^* = 0$.

(iv) Candidates' acceptance: for $j = 1, 2$, Candidate j accepts A_j^* if and only if $A_j^* \geq \frac{1}{2}M(p^*)$ and $p^* \leq P$. If j rejects, $p_j = p_m$.

Which of the two separating equilibria is more plausible? Let us adapt to our problem the belief-based refinement due to Mailath, Okuno-Fujiwara, and Postlewaite [23]. Consider a signaling game with a Sender with two types and a Receiver. The framework of Mailath, Okuno-Fujiwara, and Postlewaite is different from this framework. They consider a signaling game with one Sender and one Receiver. In the present game, the role of the Sender is exercised by G and by the candidates. Thus, the definition of Mailath, Okuno-Fujiwara, and Postlewaite needs to be adapted as follows

Definition 1 Suppose $E1$ and $E2$ are two sequential equilibria of Game 2. It is said that $E1$ defeats $E2$ if the expected payoff for G given y is strictly higher under $E1$ than under $E2$.

There are two 'types' of G , the one with $y = 0$ and the one with $y \in \{1, 0\}$. If both types are strictly better off under a Split-Contribution Equilibrium, then it is hard to see why G (the sender) should play the No-Split-Contribution Equilibrium. Voters and candidates (the receivers) should expect groups to play the Split-Contribution Equilibrium and should select their strategies accordingly.

Proposition 12 The Split-Contribution Equilibrium defeats the No-Split-Contribution Equilibrium.

Proof: The expected payoff of G under NSCE is 0 if $y = 0$ and $\frac{1}{2}(\mu_H + \mu_M)$ if $y \neq 0$. The expected payoff under SCE is $2\frac{1}{2}(\mu_H + \mu_M)$ for any y (see Subsection 4.2.3). $\frac{1}{2}(\mu_H + \mu_M) > \frac{1}{2}(\mu_H + \mu_M)$ implies $\frac{1}{2}(\mu_H + \mu_M) > \frac{1}{2}(\mu_H + \mu_M)$, and thus $\frac{1}{2}(\mu_H + \mu_M) < 2\frac{1}{2}(\mu_H + \mu_M)$. The expected payoff for G given y is strictly higher under SCE than under NSCE. \square

G always prefers the Split-Contribution Equilibrium. If voters and candidates know that, they should anticipate that G will play according to the Split-Contribution Equilibrium. Then the No-Split-Contribution Equilibrium is unlikely to arise.

7.4 Perfectly Informative y_1 .

In this model y_1 is not completely informative. Suppose on the contrary that y_1 were perfectly informative (or that G observes μ_1 as in Gerber [12]). For instance: $\mu_1 \geq \mu_0$; $1g$ and $y_1 = \mu_j$. Then, $\hat{\mu}(x_1; y_1; z_1) = y_j$ and, from (1), one can see that $\frac{1}{2}(\mu_H(p)) = \frac{1}{2}(\mu_L(p))$. There would exist a separating equilibrium.³¹ However, it would be quite arbitrary. In equilibrium, by (7), $A^*(p^*) = \frac{1}{2}(\mu_H(p^*)) = \frac{1}{2}(\mu_L(p^*))$, so that G would be indifferent between contributing and not contributing both when she has observed $y_1 = 0$ and when she has observed $y_1 = 1$.

To illustrate the brittleness of such a separating equilibrium, assume that G must pay an infinitesimal amount ϵ to observe y_1 . Because ϵ is a sunk cost, it does not influence $\frac{1}{2}(\mu(p^*))$. G's net expected payoff would be $r(p^*) - [\frac{1}{2}(\mu_H(p^*)) + \frac{1}{2}(\mu_L(p^*))] \Pr(y_1 = 1) - \epsilon$. If y_1 is perfectly informative, $\frac{1}{2}(\mu_H(p^*)) = \frac{1}{2}(\mu_L(p^*))$ and $r(p^*) = \frac{1}{2}(\mu_H(p^*))$. G would not observe y_1 and the separating equilibrium would not exist. If on the contrary y_1 is not perfectly informative, then $\frac{1}{2}(\mu_H(p^*)) > \frac{1}{2}(\mu_L(p^*))$ and $r(p^*) > 0$, so that G would pay ϵ to observe y_1 .

³¹I thank Randolph Sloof for pointing out the existence of a separating equilibrium in this case.