Ageing and Pension Reform in a Small Open Economy: the Role of Savings Incentives

Theo Leers Lex Meijdam Harrie Verbon*

March 1998

Abstract

In this paper we analyse the effects of ageing in a small open economy with a representative government. More specific, we adress the question whether in case of ageing a transition from an unfunded to a more funded pension scheme is politically feasible. We show that the existence of a suitable subsidy on savings is crucial in this respect. Without a subsidy on savings, the economy is trapped at the pre-existing level of saving and ageing leads to an increase of the PAYG tax. However, if a subsidy exists which is linked to the tax rate in a non-linear way a conversion from PAYG to funded pensions is politically feasible.

JEL classification: J14, H55
Keywords: ageing, overlapping generations, pensions

*Department of Economics and CentER, Tilburg University, PO Box 90153, 5000 LE Tilburg, The Netherlands, e-mail: t.leers@kub.nl

1 Introduction

In the year 2030 more than 30 percent of the total population in the OECD countries will be over 60 years of age whereas in 1990 this was only 18 percent.[1] This ageing of the population will lead to higher costs in countries with a pay-as-you-go (PAYG) financed public pension scheme. As is well known, the rate of return of such schemes deteriorates in case of ageing. Some economists therefore advocate a complete (and fast) transition from unfunded to funded public pension schemes.[2] However, although such a transition increases utility in the long run, it is not a Pareto improvement as it harms at least one of the present generations.[3] As a consequence of this, the change from a PAYG-scheme to a capital reserve system may well be politically unfeasible. That is, it may not be implemented by politicians in a representative democracy who are accountable only to the current generations.

For a transition from unfunded to funded social security to be politically feasible, PAYG-taxes should initially be decreased only slightly in order not to harm the current old too much. At the same time, the current young should be encouraged to save more as this enables future governments to decrease PAYG-taxes. It may therefore be expected that the existence of a suitable subsidy on savings has important consequences for the political feasibility of a conversion of the method of financing of pensions. In this paper we analyse how politicians in a small open economy with a PAYG-scheme react to ageing under different subsidy schemes.

The model we use is a two-overlapping-generations (OLG) model with a representative government which for its political support depends on both the current young and the current old.[4] The government is formed by a sequence of drafts of politicians optimizing every period anew a weighted sum of lifetime utility of the living young and old. At any point in time the government decides on the level of the PAYG-tax rate.[5] Apart from this, the

[1] See e.g. The Economist, 1996.

[2] See e.g. Feldstein (1996)

[3] Moreover, it is not possible to turn it into a Pareto improvement by transferring wealth from future generations to present generations through debt policy (see Verbon (1988) or Breyer (1989)). We assume here that the elderly will bear the cost of the transition.

[4] For an overview of decision making in a representative democracy we refer to Gärtner (1994) and Verbon (1993).

[5] Notice that we restrict ourselves to balanced budget policies. Allowing for government debt as an additional instrument would make the analysis untractable. Moreover, in many countries like e.g. the EMU-countries, debt policy is constrained by exogenous institutional restrictions. For an analysis of decision making on public debt in a representative democracy see Meijdam, van de Ven en Verbon (1996).

government may promise the current young a subsidy on their savings when old which depends on the actual level of the PAYG-tax at that point in time. However, by then a new generation of politicains will be in power. Therefore such a promise has to be formalized in an institutional savings treatment (IST).

Our analysis learns that the existence and the form of the IST is indeed crucial for the political feasibility of a transition from unfunded to funded pensions. Firstly, we show that in a small open economy without a subsidy on savings ageing in the long run leads to a higher tax rate just as in a closed economy.[6] However, in this case, in contrast to a closed economy, savings will neither rise nor fall. So, without a subsidy the small open economy is trapped at the pre-existing level of savings and is not able to reduce the less efficient PAYG-scheme and increase savings as advocated by Feldstein et al. Secondly, we demonstrate that with a subsidy on savings which is linked to the tax rate, ageing in the long run leads to higher savings and a lower PAYG-benefit than before the decrease in population growth. Moreover, if the subsidy on savings is related to the PAYG-tax rate in a non-linear fashion, we show that the losses for the old are low enough to make a such a partial transition from unfunded to funded pensions politically feasible. In particular, we prove that for any set of parameters there exists a non-linear subsidy scheme such that the value of the government's target function (i.e. the weighted sum of lifetime utility of the currently young and old) at the time of the decrease in population growth as well as on all future points in time is higher than without a subsidy on savings. This implies that if such a subsidy scheme is institutionalized in the IST it will not be abolished in the face of ageing, neither by the current government nor by future governments. Consequently, there will be a partial transition from PAYG to funded pensions.

The paper is organised as follows. Section 2 introduces the well-known two-period OLG framework for the private sector of the small open economy. In section 3, we describe the target function an the instruments of the representative government. The effects of domestic ageing under different subsidy schemes are analysed in section 4 and section 5 concludes.

[6] See Meijdam and Verbon (1997) who analyse the consequences of ageing in a closed economy. They show that ageing will in the long run lead to lower savings and higher taxes to finance the PAYG-scheme. Notice that this is exactly opposite to what Feldstein and others champion.

2 The private sector

Consider a small open economy with a large number of identical individual agents, who only differ in age. Each generation of agents lives for two periods, and a new generation is born each period. For simplicity, labour supply is assumed to be inelastic and normalized to one.[7] A representative agent born on date t is assumed to have the utility function

$$U(c_t^y, c_{t+1}^o) = \log(c_t^y) + \beta \log(c_{t+1}^o). \tag{1}$$

In this equation c_t^y denotes consumption during youth of someone born on date t and c_{t+1}^o denotes the consumption of the same individual while old in period $t+1$. The parameter β indicates the private discount factor. For simplicity we have assumed that the utility function is logarithmic, though none of the main results that follow depends on this assumption.[8] Furthermore the agents are assumed not to be altruistic.

Let w denote wage income. A part τw of this income is taxed away by the government to be transferred to the currently living old. The remaining wage income is used for savings for old age (s_t) and for consumption, viz., $c_t^y = (1 - \tau_t)w - s_t$.

When old, the agent consumes the return on his savings and the transfer payment Δ from the government which is given for the (atomistic) agent. The consumption at time $t+1$ of an old agent born on date t is given by $c_{t+1}^o = (1 + \xi_{t+1} + r)s_t + \Delta_{t+1}$ where r denotes the exogenous and constant interest rate on the world capital market and ξ denotes the (possibly negative) subsidy on savings. Maximizing lifetime utility function (1) subject to the single-period budget constraints yields

$$c_{t+1}^o = (1 + \xi_{t+1} + r)\beta c_t^y. \tag{2}$$

Production is described by a standard constant-returns-to-scale production function. The open economy is closed by the conventional equilibrium conditions on the factor and good markets (see Appendix A).

[7] Allowing for endogenous labour supply considerably complicates the analysis, but does not affect the main results. The reason for this is that the supply of labour can be eliminated from the model using the first-order conditions. The dynamical system that results in that case is equivalent to the one in case of exogenous labour supply. The analysis for this case is available upon request from the authors.

[8] It would be sufficient for most of our results to assume that $U(c_t^y, c_{t+1}^o)$ is *homothetic* in first- and second-period consumption so that the optimal ratio $\frac{c_t^y}{c_{t+1}^o}$ depends only on the interest rate.

3 The representative government

In the economy described in the former section a representative government is operating. Every period, this government is formed by a new draft of politicians that is unrelated to past or future generations of politicians. These politicians take account only of the interests of both the current young and the current old. That is, the government maximizes the following function[9]

$$W_t = \lambda\beta \log(c_t^o) + (1 + n_t)[\log(c_t^y) + \beta \log(c_{t+1}^o)]. \tag{3}$$

In (3) λ can be interpreted as a measure for the political power of an old agent relative to a young one, which is determined exogenously, for example through an implicit process of lobbying and/or rentseeking of different generations. Because we are interested in public pension schemes, we can restrict λ to $[0, \infty)$ without loss of generality.

If we abstract from administrative costs, the (balanced) budget constraint for the goverment at date t is given by

$$(1 + n_t)\tau_t w = \Delta_t + \xi_t s_{t-1}, \tag{4}$$

where n_t stands for the rate of population growth at time t. Throughout the paper it is assumed that the economy is dynamically efficient ($r > n_t,\ \forall t$).

The government in period t determines the level of the tax rate τ_t. We assume that each generation of politicians adopts Nash behaviour towards the private sector. Because the current and future tax rates are linked through current savings, this assumption implies Nash behaviour towards future governments. So, each period the current, forward-looking government decides on the current tax rate, taking future tax rates as given. The optimal tax rate in period t is follows from maximizing the target function (3) subject to the relevant budget constraints. This gives the following first-order condition

$$c_t^o = \lambda\beta c_t^y. \tag{5}$$

It follows directly from (2) and (5) that in the steady state $\xi = \lambda - 1 - r$ has to hold.

As a benchmark, we start from the case where there is no subsidy scheme, i.e. $\lambda = 1 + r$ and $\xi_t = 0\ \forall t$.[10] As alternative institutional settings, we

[9]Notice that this target function is a truncated version of a Benthamite social welfare function. Bernheim (1989) calls this “the most natural class of welfare functions for a representative government” (p.124).

[10]All results can be generalized to the case where the benchmark is a constant subsidy $\xi_t = \lambda - 1 - r \neq 0\,\forall t$.

analyse situations where the constitution contains an article that describes the subsidy that will be paid to the old in period t as a function of the tax rate τ_t in that period:

$$\xi_t = \xi(\tau_t) = \phi + \tau_t^m, \ m \geq 1. \tag{6}$$

In order to allow for a comparison of the different institutional settings we start from identical steady states. That is, we assume that $\phi = -\tau^m$ where τ is the tax rate in the initial steady state. In case of an IST as described in equation (6) the incentive for the young to save depends on the level of the social security tax rate when old. As a consequence of this, the intertemporal allocation of consumption varies over time with τ (see equation 2). As we will show below, this allows for a partial transition from PAYG to a funded system in case of ageing. However, at each point in time the government has the possibility to abolish this IST and go back to the benchmark case without a subsidy and it will do so if this increases the value of the government's target function. Of course, the present goverment is not able to commit the future government that is formed by a new generation of politicians. Therefore, the promise of a certain level of subsidy to the young through an IST is only credible when future governments do not have an incentive to abolish this IST.

4 The economic effects of ageing

This section deals with the economic effects of domestic ageing relative to the rest of the world. Formally, ageing is interpreted as a once-and-for-all decrease in the rate of population growth (n). Given some IST the long-run consequences of changes in the rate of population growth are derived analytically by comparative statics. The short-run effects are traced by comparative dynamics.[11] The latter analysis is based on the linearization of the model around the steady state.

Let $n_t = n + \pi h_t$, where h_t describes the time pattern of a peturbation of the steady state value of the rate of population growth and π is a measure for the magnitude. It is assumed that at some time $t = 0$ ageing unexpectedly occurs, i.e., $h_0 = h_1 = \cdots = h < 0$. The linearized model can then be condensed to a system describing the changes in the level of private savings (s_t) and in the social security tax rate (τ_t) for a given level of m (and thus of ϕ).

[11]The method of comparative dynamics was first introduced by Judd (1982) for a continuous-time model. It can easily be transformed to discrete time, however. The method is explained in Appendix B.

$$\begin{bmatrix} \frac{\partial s_t}{\partial \pi} \\ \\ \frac{\partial \tau_{t+1}}{\partial \pi} \end{bmatrix} = J \begin{bmatrix} \frac{\partial s_{t-1}}{\partial \pi} \\ \\ \frac{\partial \tau_t}{\partial \pi} \end{bmatrix} + Q \begin{bmatrix} h_t \\ \\ h_{t+1} \end{bmatrix}, \qquad t = 0, 1, \ldots \tag{7}$$

where J is the Jacobian matrix and Q is a matrix describing the effects of the current and next-period change in the rate of population growth. This system can be used to describe the evolution of savings and the PAYG-tax in reaction to ageing given the predetermined level of savings of the old in period $t = 0$.[12]

In order to learn the effects of the IST, given by (6), we first describe the reactions of the representative government and the private sector for the case where a subsidy on savings exists. Then we derive what happens if there is no subsidy. Finally we analyse the question whether a partial transition from unfunded to funded pensions is politically feasible. For a detailed exposition of the algebra underlying the following graphical analysis the reader is referred to Appendix B.

Subsidy

The effects of an unexpected decrease in the rate of population growth with a subsidy on savings are displayed in Figure 1.

[12]This implies that $\frac{\partial s_{t-1}}{\partial \pi} = 0$.

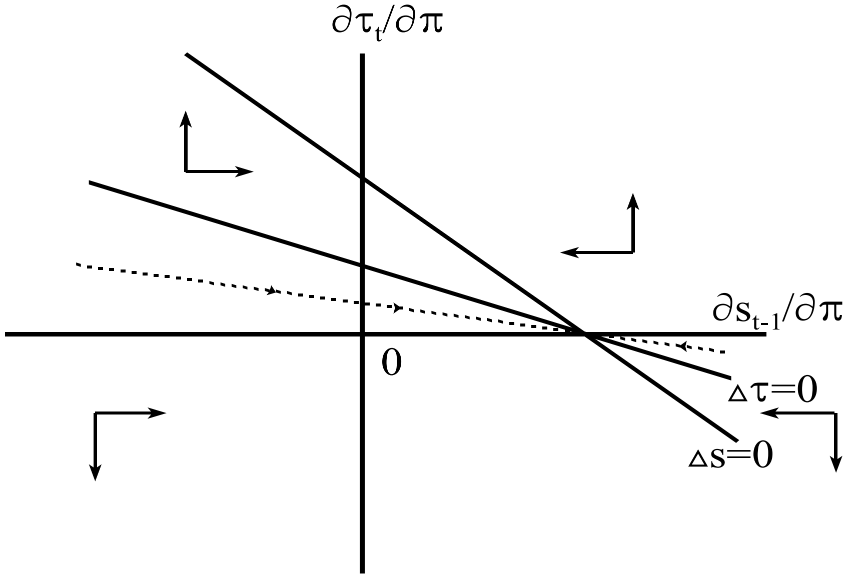

Figure 1: Effects of ageing with a subsidy on savings.

Initially, ageing leads to a jump in the tax rate from the origin to the intersection of the stable manifold of the system (which is displayed by the dotted line in Figure 1) and the $\frac{\partial\tau}{\partial\pi}$-axis (this jump is described by (B.6) in part B of the Appendix). That is, the representative government initially reacts to the unexpected decrease in n by an enlargement of the PAYG-scheme to partly compensate the currently living old. This increase in the tax rate has a negative effect on savings by the current young. In spite of this savings increase, however. This is partly due to the fact that the PAYG benefit for the current young will be lower as the increase in τ_1 is not large enough to offset the increase in the dependency ratio. The increase in savings is, however, also partly caused by the higher subsidy on savings which results from the increase in the future tax rate. In the following periods the tax rate and savings evolve along the stable manifold until a new steady state (at the intersection of the stable manifold with the $\frac{\partial s}{\partial\pi}$-axis) is reached. During this adjustment process the PAYG-tax rate gradually decreases to its original level before the shock and savings gradually increase. Note that the rise in savings is tempered by the fact that the subsidy on savings falls over time because of the fact that, after the initial upward jump, the PAYG-tax rate is gradually brought down to its old level. Note further that the adjustment path implies that the initially enlarged PAYG scheme is gradually and partly replaced by a capital reserve system where old-age consumption is to a larger degree financed out of private savings. The long-run effect on the level of savings follows from (5) evaluated in the steady state, i.e., $\frac{\partial s}{\partial \pi} = \frac{-\tau w h}{1+r+\lambda\beta} > 0$.

No subsidy

Without a subsidy on savings the economy appears to be trapped at its attained level of savings before ageing. To understand why the level of private savings will not change, consider what would happen if the government changed the intergenerational relation between young age consumption c_t^y and old age consumption c_t^o by choosing an appropriate social security tax rate. Say, the representative government decreases the ratio $\frac{c_t^y}{c_t^o}$ to favour the currently living old by an appropriate choice of its instruments. At the same time the ratio $\frac{c_t^y}{c_{t+1}^o}$ given by first-order-condition of the private sector remains unaltered. So, the (relatively) lower young-age consumption leads to a lower old-age consumption c_{t+2}^o. The next government (in period $t+1$) will then again have the incentive to decrease the ratio $\frac{c_{t+1}^y}{c_{t+2}^o}$ favouring the old-aged individuals at time $t+2$. Continuing this reasoning we get the following imploding path of old age consumption $c_t^o > c_{t+1}^o > c_{t+2}^o > \ldots$. It follows directly that this consumption path is infeasible because it bankrupts the private sector at some moment in time. Therefore, the only possible consumption paths are those which do not alter the intergenerational link between currently living young and old, or alternatively, paths with a constant ratio $\frac{c_t^y}{c_t^o},\ t = 0, 1, \ldots$. As we will explain below, the government reacts to ageing in this case by setting a tax rate such that the young generation has no incentive to change its savings.

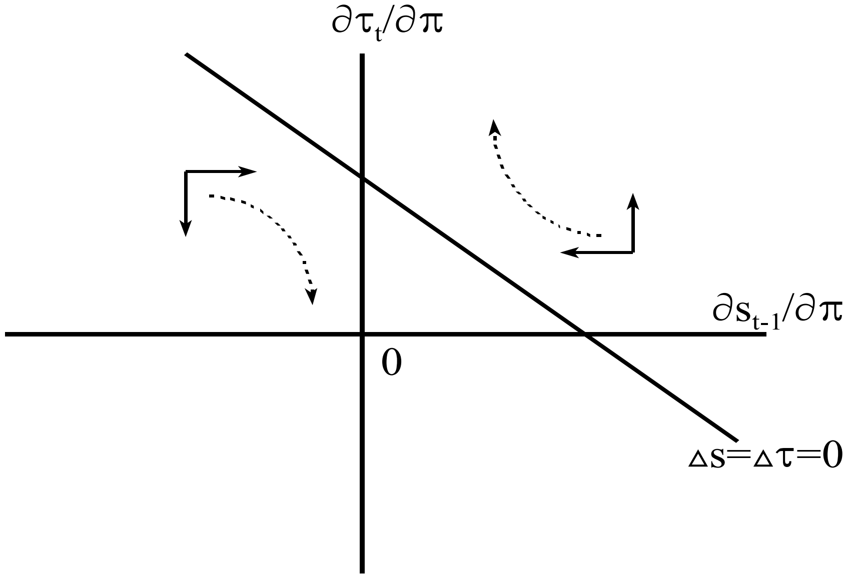

Figure 2: Effects of ageing without a subsidy on savings.

Figure 2 is a flinched version of Figure 1. In Figure 1 the stable manifold and the two phaselines belonging to the state variables are all three distinguishable and only intersect in the new steady state. Here the phaselines are given by the same equations and no saddlepoint dynamics exist. There is an infinite number of steady states and the steady state that is actually attained after the shock is completely determined by the level of savings of the old living at the time of the shock. So the change in the level of private savings due to an unexpected decrease in n is equal to zero in the short run as well as in the long run. Consequently, it is smaller than the initial change in savings in case a subsidy exists,

$$\left(\frac{\partial s_0}{\partial \pi}\right)_{\text{subsidy}} > \left(\frac{\partial s_t}{\partial \pi}\right)_{\text{no subsidy}} = 0, \ \forall t \geq 0$$

Given this savings trap the government reacts to ageing by an increase in the tax rate which is displayed in Figure 2 by the intersection of the phaselines with the $\frac{\partial \tau}{\partial \pi}$-axis. Any other intial jump in the tax rate would lead to implosive or explosive timepaths of consumption.[13] Notice that without a subsidy,

[13]This is due to the fact that with an infinite value for m, the stable manifold coincides

ageing leads to a rise of the PAYG tax rate, while the long-run effect on the tax rate equals the short-run effect. This short-run effect is larger than in case a subsidy exists,

$$\left(\frac{\partial \tau_0}{\partial \pi}\right)_{\text{no subsidy}} > \left(\frac{\partial \tau_0}{\partial \pi}\right)_{\text{subsidy}}$$

Political feasibility of transition

What we have learned by now is that with a subsidy government policy results in a partial replacement of the intergenerational transfer system by an intertemporal transfer system (private savings) which promises a higher rate of return. This immediately implies higher steady-state lifetime utility with a savings subsidy than without a subsidy. This, however, does not mean that such a policy is politically feasible. Cases may exist where abolishing the subsidy increases the political target function of the government. This should be clear given the fact that the transition process harms the currently living old at the moment of occurrence of the shock relative to a situation without a subsidy. So, with a representative government consisting of politicians only under the influence of currently living generations, such a transition is politically feasible if the harm to the elderly is (more than) compensated by the gain from this policy for the currently living young. From our numerical computations[14] it appears that is only the case if $m >> 1$. In that case the government initially reacts to ageing through an increase in the tax rate that is almost equal to the increase under the treatment with a fixed subsidy. So the consumption of the currently living old almost equals the level in case no subsidy exists. With a subsidy the increase in the tax rate also enhances the incentive to save of the currently living young, but this enhancement is muffled down by the choice of $m >> 1$. The initial increase in the tax rate as well as the higher savings rate decrease current young age consumption. This is compensated, however, by a higher old age consumption for them as a result of the increased savings which outweigh the decrease in the tax rate in the next period.

with the phaselines, so a jump which is not onto the phaselines is attracted by explosive vector fields.

[14] Available upon request.

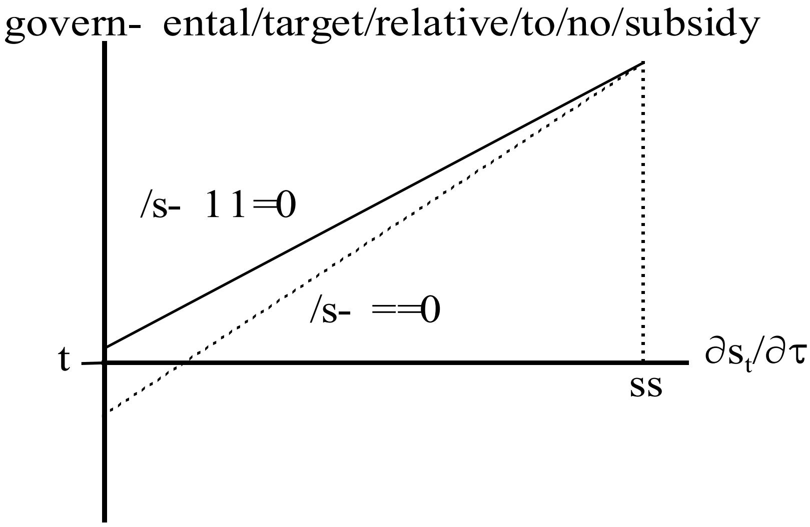

Figure 3: Effect of m on the governmental target function.

Figure 3 illustrates the effect of the parameter m on the governmental target function. On the horizontal axis the change in saving in reaction to ageing is given while on the vertical axis the effect on the objective function of the government as compared to the effect in case no subsidy exists is represented. We distinguish two cases. First, if $m = 1$ the initial loss in political support from the old is larger than the gain in support from the young. So it is advantageous to politicians to abolish the subsidy and a partial conversion of the PAYG-scheme is not politically feasible. Second, if m is chosen at a sufficiently high level so that both taxes and savings change in a very gradual way, the effect on the lifetime utilities of the currently living generations is such that a (partial) transition to funded pensions initially increases the value of the political target function and therefore is politically feasible. So, for the specific values of the parameters chosen, the subsidy on savings should be highly non-linear in order to make a conversion of the method of financing pensions politically feasible. More general, the following proposition[15] shows that there always exists a subsidy scheme such that a partial transition from PAYG to funded pensions is politically feasible.

[15]For proof of this proposition see Appendix C.

Proposition 1 *For every tuple of exogenous parameters* $(r, n, \beta, \lambda, \tau)$ *there exists a coefficient* $m < \infty$ *such that a transition to a more funded pension scheme is politically feasible. For this coefficient it holds that* $\frac{\partial m}{\partial r} < 0, \frac{\partial m}{\partial n} > 0, \frac{\partial m}{\partial \lambda} > 0$ *and* $\frac{\partial m}{\partial \tau} > 0$.

Figure 3 also shows that if at some initial point in time the government does not abolish the IST future governments will not have an incentive to change the IST either. This can be seen as follows. Under a transition savings rise, i.e. a movement to the right on the horizontal axis occurs. But, for every point on the horizontal axis, it holds that for a given high value of m the values of the government's objective function is higher than without a subsidy as can be seen from inspection of the figure.

5 Concluding remarks

In this paper we analyzed the effects of domestic ageing in a positive general equilibrium model of a small open economy. More precisely, we compared economies which have the same steady state and are hit by the same ageing shock but have different subsidy schemes for savings. Our main conclusion is that, in case of ageing in a small open economy with a PAYG system, the existence and the form of the institutional savings treatment is crucial for the political feasibility of a transition to a more savings based pension system, which is favourable for future generations. In particular we show that without a subsidy on savings the economy is trapped at the existing level of savings. We also prove, however, that for any economy a non-linear subsidy scheme exists which enables a transition to a new steady state where the tax rate is the same as in the original steady state, but where individual savings are higher. So, it is possible to get the desired result, but it needs a lot of 'fine-tuning'. This demonstrates that the political-economy of conversion policies is not such a straightforward matter as some economists suggest.

References

Bernheim, B.D. (1989) 'Intergenerational altruism, dynastic equilibria and social welfare', *Review of Economic Studies*, **56**, 119-128.

Blanchard, O.J. and C.M. Kahn (1980) 'The solution of linear difference models under rational expectation', *Econometrica*, **48**, 1305-1311.

Breyer, F. (1989) 'On the intergenerational Pareto efficiency of pay-as-you-go financed pension systems', *Journal of Institutional and Theoretical Economics*, **145**, 643-658.

The Economist (1996) 'All our tomorrows: a survey of the economics of ageing', *The Economist*, January 27th 1996.

Feldstein, M (1996), 'The Missing Piece in Policy Analysis: Social Security Reform',*American Economic Review*, **86(2)**, 1-14.

Gärtner, M. (1994) 'Democracy, elections and macroeconomic policy: Two decades of progress', *European Journal of Political Economy*, **10**, 85-109.

Judd, K.L. (1982) 'An alternative to steady-state comparisons in perfect foresight models', *Economics Letters*, **10**, 55-59.

Meijdam, A.C., M. van de Ven and H.A.A. Verbon (1996) 'The dynamics of government debt', *European Journal of Political Economy*, **12**, 67-90.

Meijdam, A.C. and H.A.A. Verbon (1997) 'Aging and public pensions in an overlapping-generations model', *Oxford Economic Papers*, **49**, 29-42.

Verbon, H.A.A. (1988) *The evolution of public pension schemes*, Berlin: Springer Verlag .

Verbon, H.A.A. (1993) 'Public pensions. The role of public choice and expectations', *Journal of Population Economics*, **6**, 123-135.

Appendix

The Appendix contains three sections. The first section is devoted to an overview of the economic relations of the small open economy considered. The second part describes the method of comparative dynamics, while the last section proves Proposition 1.

A Overview of the (per capita) economic relations

To study the short-run as well as the long-run effects of a change in the population growth rate through a change in the state variables s and τ on the other variables of the model we introduce the following expressions[16]. Firstly we assume a standard CRS production technology i.e.

$$y_t = \frac{Y_t}{N_t} = F(\frac{K_t}{N_t}, 1) = f(k_t), \; f' > 0, \; f'' < 0. \tag{A.1}$$

Equilibrium on the factor market requires that $f'(k_t) = r$. Given this expression the per capita wage rate (or first-period endowment) is given by $w = y_t - rk_t$. The per capita net PAYG benefit is given by $\eta_t = \Delta_t + \xi(\tau_t) r s_{t-1} = (1 + n_t) w \tau_t$. The per capita investment is given by $i_t = \frac{I_t}{N_t} = (1 + n_{t+1})k_{t+1} - k_t$. The per capita net export is given by $b_t = \frac{B_t}{N_t} = y_t - c_t^y - \frac{c_t^o}{1+n_t} - i_t$. The per capita current and capital account balance levels are given by

$$cab_t = \frac{CAB_t}{N_t} = b_t + r\left(\frac{s_{t-1}}{1+n_t} - k_t\right), \tag{A.2}$$
$$kab_t = \frac{KAB_t}{N_t} = -cab_t.$$

Finally, the per capita foreign debt is given by $d_t = k_t - \frac{s_{t-1}}{1+n_t}$.

Given the above expressions, the deviations of these variables from their steady state values due to a marginal change in the population growth rate is given by the following expressions

[16] Omitting the time subscripts denotes the steady state values of the respective variables.

$$
\begin{aligned}
\frac{\partial r}{\partial \pi} &= \frac{\partial k_t}{\partial \pi} = \frac{\partial y_t}{\partial \pi} = \frac{\partial w}{\partial \pi} = 0, \\
\frac{\partial \eta_t}{\partial \pi} &= (1+n) w \frac{\partial \tau_t}{\partial \pi} + \tau w h_t, \\
\frac{\partial c_t^y}{\partial \pi} &= -w \frac{\partial \tau_t}{\partial \pi} - \frac{\partial s_t}{\partial \pi}, \\
\frac{\partial c_t^o}{\partial \pi} &= (1+r) \frac{\partial s_{t-1}}{\partial \pi} + \frac{\partial \eta_t}{\partial \pi}, \\
\frac{\partial U(c_t^y, c_{t+1}^o)}{\partial \pi} &= \frac{1}{c_t^y} \frac{\partial c_t^y}{\partial \pi} + \frac{\beta}{c_{t+1}^o} \frac{\partial c_{t+1}^o}{\partial \pi}, \\
\frac{\partial b_t}{\partial \pi} &= -\frac{\partial c_t^y}{\partial \pi} - \frac{1}{1+n} \frac{\partial c_t^o}{\partial \pi} + \frac{c^o}{(1+n)^2} h_t - k h_{t+1}, \\
-\frac{\partial kab_t}{\partial \pi} &= \frac{\partial cab_t}{\partial \pi} = \frac{\partial b_t}{\partial \pi} + r \left(\frac{1}{1+n} \frac{\partial s_{t-1}}{\partial \pi} - \frac{s}{(1+n)^2} h_t \right), \\
\frac{\partial i_t}{\partial \pi} &= k h_{t+1}, \\
\frac{\partial d_t}{\partial \pi} &= -\frac{1}{1+n} \frac{\partial s_{t-1}}{\partial \pi} + \frac{s}{(1+n)^2} h_t.
\end{aligned}
\tag{A.3}
$$

B Comparative dynamics

This appendix describes the method of comparative dynamics. Let h_t describe the time pattern of the peturbation of the steady state value of the population growth rate n and π the magnitude of the shock. As mentioned in section 4.1 we can write $n_t = n + \pi h_t,\ t = 0, 1, \ldots$. The effects of a marginal decrease or increase in n can be traced by differentiation of the intertemporal allocation equation (2) and the first-order-condition for the tax rate (5) subject to the single-period budget constraints with respect to π around the steady state. This differentiation yields the following linear system in the state variables s (savings per capita) and τ (the social security tax rate)

$$
\begin{bmatrix} \frac{\partial s_t}{\partial \pi} \\ \frac{\partial \tau_{t+1}}{\partial \pi} \end{bmatrix} = J \begin{bmatrix} \frac{\partial s_{t-1}}{\partial \pi} \\ \frac{\partial \tau_t}{\partial \pi} \end{bmatrix} + Q \begin{bmatrix} h_t \\ h_{t+1} \end{bmatrix}, \qquad t = 0, 1, \ldots \tag{B.1}
$$

where J is a the Jacobian matrix

$$J = \begin{bmatrix} \left[-\frac{(1+r)}{\lambda\beta}\right] & \left[-\frac{w((1+n)+\lambda\beta)}{\lambda\beta}\right] \\ \left[\frac{(1+r+\lambda\beta)(1+r)}{\lambda\beta((1+n)w-\xi'(\tau)r\beta c^y)}\right] & \left[\frac{(1+r+\lambda\beta)w((1+n)+\lambda\beta)-(\lambda\beta)^2 w}{\lambda\beta((1+n)w-\xi'(\tau)r\beta c^y)}\right] \end{bmatrix}, \tag{B.2}$$

and Q is a matrix describing the effects of the current and next-period changes in the exogenous population growth rate n on the state variables

$$Q = \begin{bmatrix} \left[-\frac{\tau w}{\lambda\beta}\right] & 0 \\ \left[\frac{(1+r+\lambda\beta)\tau w}{\lambda\beta((1+n)w-\xi'(\tau)r\beta c^y)}\right] & \left[-\frac{\beta\tau w}{\lambda\beta((1+n)w-\xi'(\tau)r\beta c^y)}\right] \end{bmatrix}. \tag{B.3}$$

This system comprises one predetermined or backward looking variable (s) and one jump or forward looking variable (τ). The eigenvalues of the Jacobian matrix J are

$$\varepsilon_{1,2} = \frac{\Psi}{\Omega} \pm \frac{\sqrt{4w(\lambda\beta)^2(1+r)(\xi'(\tau)\beta r c^y - (1+n)w) + \Psi^2}}{\Omega}. \tag{B.4}$$

with

$$\Psi = (1+r)\xi'(\tau)\beta r c^y + (2+r+n)\lambda\beta w,$$
$$\Omega = 2\lambda\beta((1+n)w - \xi'(\tau)\beta r c^y).$$

For the case where a subsidy on savings exists some tedious algebra shows that one of the eigenvalues lies inside while the other lies outside the unit circle.[17] So the system is saddlepoint stable and can be solved to find the initial effect of changes in n on the social security tax (see Blanchard and Kahn 1980)[18]

$$\frac{\partial \tau_0}{\partial \pi} = \frac{\varepsilon_1 - j_{11}}{j_{12}} \left[\frac{\partial s_{-1}}{\partial \pi} + q_{11}\sum_{i=0}^{\infty} \varepsilon_2^{-i-1} h_i + q_{12}\sum_{i=0}^{\infty} \varepsilon_2^{-i-1} h_{i+1}\right] \tag{B.5}$$
$$- q_{21}\sum_{i=0}^{\infty} \varepsilon_2^{-i-1} h_i - q_{22}\sum_{i=0}^{\infty} \varepsilon_2^{-i-1} h_{i+1},$$

[17] Without a subsidy on savings the eigenvalues given by (B.4) boil down to respectively $\varepsilon_1 = 1$ and $\varepsilon_2 = \frac{1+r}{1+n}$. So the system is not saddlepoint stable and the steady state can be characterized as a non-hyperbolic fixed point.

[18] In the following expression ε_1 denotes the eigenvalue inside the unit circle and ε_2 denotes the one outside the unit circle.

where j_{ij} and q_{ij} $(i, j = 1, 2)$ denote elements of J and Q. To distinguish between current and future effects, the terms of (B.5) can be rearranged as follows

$$\frac{\partial \tau_0}{\partial \pi} = \frac{\varepsilon_1 - j_{11}}{j_{12}} \frac{\partial s_{-1}}{\partial \pi} + \left[\frac{\varepsilon_1 - j_{11}}{j_{12}} q_{11} - q_{21} \right] \frac{h_0}{\varepsilon_2} \tag{B.6}$$
$$+ \left[\frac{\varepsilon_1 - j_{11}}{j_{12}} \left(\frac{q_{11}}{\varepsilon_2} + q_{12} \right) - \frac{q_{21}}{\varepsilon_2} - q_{22} \right] \sum_{j=1}^{\infty} \varepsilon_2^{-j} h_j.$$

Using the facts that $\frac{\partial s_{-1}}{\partial \pi} = q_{12} = 0$, $\sum_{j=1}^{\infty} \varepsilon_2^{-j} = \frac{1}{\varepsilon_2 - 1}$ and assuming that the once-and-for-all decrease in n is unexpected i.e. $h_0 = h_1 = \cdots = h = -1$, (B.6) boils down to

$$\frac{\partial \tau_0}{\partial \pi} = \frac{1}{\varepsilon_2 - 1} \left[q_{21} + q_{22} - \frac{\varepsilon_1 - j_{11}}{j_{12}} q_{11} \right]. \tag{B.7}$$

The evolution of both s and τ can then be derived from the system equations. Given the evolution of these state variables we can calculate the time paths of the variables such as young- and old-age consumption, lifetime utility, the net PAYG benefit, the per capita investment, net export, the current and capital account balance levels and the per capita debt according to the relations given in part A of this Appendix.

C Proof of proposition 1

The roots of the saddlepoint stable system (B.1) can be written as a parameterized functions δ, ρ of m for large but countable m, i.e.

$$\varepsilon_1 = 1 - \delta(m; r, n, \beta, \lambda, \tau), \qquad \varepsilon_2 = \frac{1 + r}{1 + n} + \rho(m; r, n, \beta, \lambda, \tau). \tag{C.1}$$

The roots given in (C.1) can be derived from (B.4) by straightforward but tedious algebra. Given dynamic efficiency, $\beta > 0$, $\lambda > 0$ and $\tau \in (0, 1)$, the following holds for the functions $f = \delta, \rho$

$$\frac{\partial f}{\partial m} < 0, \qquad \lim_{m \to \infty} f(m; r, n, \beta, \lambda, \tau) = 0. \tag{C.2}$$

From (B.4) the following relations for the parameters can be derived

$$\frac{\partial f_{m \text{ fixed}}}{\partial r} < 0, \qquad (C.3)$$
$$\frac{\partial f_{m \text{ fixed}}}{\partial n} > 0,$$
$$\frac{\partial f_{m \text{ fixed}}}{\partial \lambda} > 0,$$
$$\frac{\partial f_{m \text{ fixed}}}{\partial \tau} > 0.$$

Given (C.1) the initial jump in τ given by (B.7) can be written as a function of m, $\delta(m)$ and $\rho(m)$ with

$$\frac{\frac{\partial \tau_0}{\partial \pi}(m, \delta(m), \rho(m))}{\partial \delta} < 0, \; \frac{\frac{\partial \tau_0}{\partial \pi}(m, \delta(m), \rho(m))}{\partial \rho} < 0, \; \frac{\frac{\partial \tau_0}{\partial \pi}(m, \delta(m), \rho(m))}{\partial m} > 0. \qquad (C.4)$$

Given $\frac{\partial s_{-1}}{\partial \pi} = 0$, $\frac{\partial s_0}{\partial \pi}$ and $\frac{\partial \tau_1}{\partial \pi}$ can be derived from substituting $\frac{\partial \tau_0}{\partial \pi}(m, \delta, \rho)$ into the the system (7). Given $\frac{\partial \tau_0}{\partial \pi}$, $\frac{\partial s_0}{\partial \pi}$ and $\frac{\partial \tau_1}{\partial \pi}$ the governmental target function (3) can be written as $W(m, \delta(m), \rho(m))$. Given the parameters above tedious algebra gives for m large enough (and thus δ, ρ small but unequal to zero)

$$W\left(\frac{\partial \tau_0}{\partial \pi}(m, \delta, \rho), \frac{\partial s_0}{\partial \pi}(m, \delta, \rho), \frac{\partial \tau_1}{\partial \pi}(m, \delta, \rho)\right) \qquad (C.5)$$
$$> W\left(\frac{\partial \tau_0}{\partial \pi}(m = \infty, \delta = 0, \rho = 0), 0, \frac{\partial \tau_1}{\partial \pi}(m = \infty, \delta = 0, \rho = 0)\right),$$

which completes the proof ■