CentER

Discussion Paper

No. 2005–26

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January 2005

ISSN 0924-7815

TILBURG UNIVERSITY

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We study a novel issue in the real-options-based technology innovation literature by means of double barrier contingent claims analysis. We show how much a firm with the monopoly over a project is willing to spend in investment technology innovation that softens the irreversible cost of accessing the project before its irreversible demise. The answer depends on the project's characteristics and on the effectiveness demanded from technology innovation.

*JEL-Classification:* G12, G13, G31.

*Keywords:* Double barrier options, cost irreversibility, demise irreversibility, technology innovation.

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## Abstract

We study a novel issue in the real-options-based technology innovation literature by means of double barrier contingent claims analysis. We show how much a firm with the monopoly over a project is willing to spend in investment technology innovation that softens the irreversible cost of accessing the project before its irreversible demise. The answer depends on the project's characteristics and on the effectiveness demanded from technology innovation.



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## 1 Introduction

Softening the irreversible cost to enter a project via technology innovation can be an issue for a firm with the monopoly over the project. This is more so if the firm faces the prospect of a second irreversibility, namely the project's demise. Irreversible demise amounts to the action of discarding the project for good when, given the *status-quo* level of the irreversible cost, the project's Present Value (PV) deteriorates down to a sufficient level of unattractiveness. This is the demise-adjusted problem considered by Ha-Duong and Morel (2003), who study the real options effects of a lower absorbing barrier for the project's PV. Softening the irreversible cost is a way to alleviate the value effects of an approaching demise. Some of the current problems faced by the oil industry are conducive evidence that technology innovation aimed at softening irreversible costs can restitute glamour to projects that are on their way to a dead end. The non-OPEC oil fields are ageing and

> ... the blizzard of breakthroughs that reduced the cost of finding and developing oil has now slowed to a trickle. ... [T]he majors cut research too much ... . Breakthroughs are clearly needed, though, if firms are to find oil economically in tricky places such as the Siberian tundra or the ultra-deep waters off Africa and Brazil[1].

The question is: How much is a firm willing to pay up front for having the chance of accessing a cheaper investment technology before the project's demise? That is, how

[1] "Oil companies' profits: Not exactly what they seem to be", *The Economist*, October 30th 2004, pp. 71-72.

valuable is investment technology innovation research that will achieve a softening of the irreversible cost before the project's irreversible demise? This is a fresh question in the real-options-based technology innovation literature. The existing literature has been considering the cost and the process of technology innovation as exogenous and independent from the value of the project to which the firm intends to apply the technical innovation. Examples are Grenadier and Weiss (1996) under monopoly and Farzin, Huisman, and Kort (1998) under perfect competition. Given a firm with the monopoly over the project, we provide a parsimonious answer to the question via an original use of barrier contingent claims analysis: the upper bound on the fair current expense in cost-softening innovation is the difference between the value of a demise-adjusted option to invest with a cost-softening barrier and the value of the plain demise-adjusted option to invest. Consider a firm that holds the rights on a project. Entering the project implies a *status-quo* irreversible cost of \$10 million. The project has a PV of \$10 million (the discount rate is 4%, the operating cashflows are 4% per PV unit, and the PV volatility is 20%). Given that the project will be discarded for good if its PV drops to \$5 million (the demise threshold), optimal delay of exercising the option to invest at the *status-quo* cost implies a Total Net PV of \$2.2235 million, which is the plain demise-adjusted option value. The firm is keen on funding a technology-innovation task force with the target of making the irreversible cost 50% cheaper than its *status-quo* level by the time the project's PV drops below \$6.5 million, that is, before the PV is corroded down to the demise threshold. How much funding should be allocated if the firm wants the target to be successfully achieved? We show that, given the effectiveness demanded from technology innovation and given the project's characteristics, the firm's expenditures in the required technology innovation should be no more than \$0.8512 million. Since a successful technology innovation implies a 50% softening of the irreversible cost at the \$6.5 million barrier, the \$2.53809 million figure is the difference between the value of a demise-adjusted option to invest with a \$6.5-million barrier that triggers cost softening and the value of the plain demise-adjusted option to invest. Following Sbuelz (2004a and 2004b), our analysis is carried out with a double barrier contingent claims technique[2] to handle the free upper boundary for immediate investment as well as the lower barrier at which cost softening is achieved-the demise threshold constitutes an additional barrier that lies further below.

The work is organized as follows. Section 2 introduces the investment problem with irreversible demise. Section 3 looks at the dynamic programming formulation of the investment problem with irreversible demise under potential softening of the irreversible cost. Section 4 solves the problem by double barrier contingent claims analysis. Section 5 discusses the characteristics of the firm's maximal expenditure in the cost-softening technology innovation. Section 6 concludes.

## 2 Investment and demise

For a project with PV $V$, Ha-Duong and Morel (2003) consider the following problem: At what critical level $V^*$ of the project's value is it optimal to pay the *status-quo* irreversible cost $I$ in return for the project's value itself, given that the project will be discarded for good whenever $V$ drops to the irreversible-demise threshold? Such a threshold returns

[2]Sbuelz (2004a and 2004b) studies the real options effects of lower-barrier triggers in the absence of demise irreversibility.

a sufficiently negative Net PV to trigger the project's demise. We assume the demise threshold to be a percentage $\kappa$ ($\kappa \in [0,1]$) of the current irreversible cost to enter the project:

$$\text{Demise threshold} \quad = \quad \kappa I \qquad \text{(without cost softening)}.$$

The stochastic changes in $V$ are assumed to be spanned by existing traded assets of the economy. The spanning assumption will make the analysis akin to the treatment of perpetual American options. However, the entire analysis of the present work is valid even if the spanning assumption is removed: One must only replace the riskfree rate $r$ with an appropriate discount rate[3] and conceive the analysis under the physical probability measure. $V$ evolves according to the following geometric Brownian motion (under the equivalent martingale measure):

$$dV = (r - \delta)\, V dt + \sigma V dz,$$

where $r$ is the riskfree rate, $\delta$ is the payout rate, $\sigma$ is the volatility rate, and $dz$ is the increment of a Wiener process. The value of the demise-adjusted option to invest at the *status-quo* cost $I$ is $F(V)$. It is the opportunity cost of investing now rather than waiting in the presence of the demise threshold. Ha-Duong and Morel (2003) use differential

[3]Dixit and Pindyck (1994) denote such a discount rate with the parameter $\rho$.

equations techniques to show that

$$
\begin{aligned}
F(V) &= \frac{(V)^{\beta_1}(\kappa I)^{\beta_2} - (V)^{\beta_2}(\kappa I)^{\beta_1}}{(V^*)^{\beta_1}(\kappa I)^{\beta_2} - (V^*)^{\beta_2}(\kappa I)^{\beta_1}}(V^* - I)1_{\{V \leq V^*\}} \\
&\quad + (V - I)\, 1_{\{V^* < V\}}, \\
\beta_1 &= \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1, \\
\beta_2 &= \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0.
\end{aligned}
$$

We employ a double barrier contingent claims analysis of the Ha-Duong and Morel (2003) solution. Indeed, given the double barrier nature of the problems at hand, solution techniques based on double barrier contingent claims will prove to be neat and effective. $\mathcal{E}[\cdot \mid V]$ is the conditional expectation given the project's PV $V$. $T_a$ denotes $V$'s stopping time at the real non-negative number $a$ and $1_{\{A\}}$ is the indicator function of the event $A$. $\mathcal{E}\left[e^{-rT_{V^*}} 1_{\{T_{V^*} < T_{\kappa I}\}} \mid V\right]$ is the value of a perpetual double barrier cash-at-hit claim written on the project's value: One unit of numeraire is paid as soon as the project's value hits the upper barrier $V^*$ provided that the lower barrier $\kappa I$ is not reached first. Given the geometric Brownian motion assumption, it is well known that (see, for example, Geman and Yor (1996) and Sbuelz (2004c))

$$
\mathcal{E}\left[e^{-rT_{V^*}} 1_{\{T_{V^*} < T_{\kappa I}\}} \mid V\right] = \frac{(V)^{\beta_1}(\kappa I)^{\beta_2} - (V)^{\beta_2}(\kappa I)^{\beta_1}}{(V^*)^{\beta_1}(\kappa I)^{\beta_2} - (V^*)^{\beta_2}(\kappa I)^{\beta_1}},
$$

which implies

$$
\begin{aligned}
F(V) &= \mathcal{E}\left[e^{-rT_{V^*}} 1_{\{T_{V^*} < T_{\kappa I}\}} \mid V\right](V^* - I)1_{\{V \leq V^*\}} \\
&\quad + (V - I)\, 1_{\{V^* < V\}}.
\end{aligned}
$$

Since the project's value is spanned, the value of claims written on it is also spanned and it must have no-arbitrage dynamics. Hence, the construction of $F(V)$ is such that

$F(V)$ satisfies the Black-Scholes differential equation in the continuation region, the Value Matching Condition at the free upper boundary $V^*$,

$$F(V^*) = V^* - I,$$

and the Demise Condition at the lower boundary $\kappa I$,

$$F(\kappa I) = 0.$$

The critical value $V^*$ is the solution of the restriction related to the Smooth Pasting Condition:

$$\frac{d}{dV} F|_{V=V^*} = \frac{d}{dV}(V - I)|_{V=V^*} = 1.$$

## 2.1 Investment and demise after cost softening

Irreversible-cost softening implies a cost saving of $(1-\omega) I$, $\omega \in (0,1)$. Once it is achieved, the demise threshold shifts down:

$$\text{Demise threshold} \quad = \quad \kappa\omega I \qquad \text{(after cost softening)}.$$

The value of the demise-adjusted option to invest at the novel irreversible cost $\omega I$ is $F_{\text{soft}}(V)$. Its expression is

$$F_{\text{soft}}(V) = \mathcal{E}\left[e^{-rT_{\overline{V^*}}} 1_{\{T_{\overline{V^*}} < T_{\kappa\omega I}\}} \mid V\right] (\overline{V^*} - \omega I) 1_{\{V \leq \overline{V^*}\}} + (V - \omega I)\, 1_{\{\overline{V^*} < V\}}.$$

The critical value $\overline{V^*}$ is the solution of the restriction from the Smooth Pasting Condition:

$$\frac{d}{dV} F_{\text{soft}}|_{V=\overline{V^*}} = \frac{d}{dV}(V - \omega I)|_{V=\overline{V^*}} = 1.$$

# 3 The dynamic programming problem

The chance of softening the irreversible cost via technology innovation changes the value of the demise-adjusted option to invest. I denote the changed option value with $J(V)$. Technology innovation is required to be successfully ignited by the time $V$ drops to $L$, the cost-softening barrier. At that PV level, entering the project becomes cheaper-the irreversible cost of it is cut down to $\omega I$. Technology innovation is required to be business-time effective, that is, to take effect before demise. Hence, the cost-softening barrier is a lower barrier that is stricly above the current demise threshold:

$$L > \kappa I.$$

The Bellman equation for $J(V)$ is

$$J(V) = \max\left\{ \quad \max\{V - I, 0\} \quad , \quad \exp(-r dt)\, \mathcal{E}\left[J(V) + dJ \mid V\right] \quad \right\},$$

with conditions,

$$\begin{aligned}
J(V^{**}) &= V^{**} - I && \text{(Value Matching Condition)},\\
\frac{d}{dV} J|_{V=V^{**}} &= \frac{d}{dV}(V - I)\,|_{V=V^{**}} = 1 && \text{(Smooth Pasting Condition)},\\
J(L) &= F_{\text{soft}}(L) && \text{(Cost Softening Condition)},
\end{aligned}$$

where $V^{**}$ is the critical value at which it is optimal for the firm to access the project by paying the *status-quo* irreversible cost $I$.

## 4 The double barrier contingent claims solution

In the region $\{V : V \in [L, V^{**})\}$ there is no immediate investment. Thus, the Bellman equation boils down to the Black-Scholes differential equation:

$$\mathcal{E}[dJ \mid V] = J(V) \cdot r \cdot dt.$$

The value of any spanned contingent claim satisfies the Black-Scholes differential equation: In risk-adjusted expectation, the percentage increase in the spanned claim value must equal the claim's percentage cost of carry. Since the operator $\mathcal{E}[d \quad \bullet \mid V]$ is linear, any linear combination of spanned contingent claim values does satisfy the Black-Scholes differential equation. The boundary conditions introduce a new double barrier corridor, whose extrema are the free upper boundary $V^{**}$ and the lower boundary $L$. Thus, the candidate solution must be made of a linear combination of the values of two perpetual double barrier cash-at-hit claims. The values of the two relevant claims are:

$$\mathcal{E}\left[e^{-rT_{V^{**}}} 1_{\{T_{V^{**}}<T_L\}} \mid V\right] = \frac{(V)^{\beta_1}(L)^{\beta_2} - (V)^{\beta_2}(L)^{\beta_1}}{(V^{**})^{\beta_1}(L)^{\beta_2} - (V^{**})^{\beta_2}(L)^{\beta_1}},$$

$$\mathcal{E}\left[e^{-rT_L} 1_{\{T_L<T_{V^{**}}\}} \mid V\right] = \frac{(V)^{\beta_2}(V^{**})^{\beta_1} - (V)^{\beta_1}(V^{**})^{\beta_2}}{(V^{**})^{\beta_1}(L)^{\beta_2} - (V^{**})^{\beta_2}(L)^{\beta_1}}.$$

The solution $J(V)$,

$$J(V) = \mathcal{E}\left[e^{-rT_{V^{**}}}1_{\{T_{V^{**}}<T_L\}} \mid V\right](V^{**}-I)$$

$$+\mathcal{E}\left[e^{-rT_{V^{**}}}1_{\{T_L<T_{V^{**}}\}} \mid V\right]F_{\text{soft}}(L)$$

satisfies by construction the Black-Scholes differential equation, the Value Matching Condition at the free upper boundary $V^{**}$, and the Cost Softening Condition at the lower boundary $L$. $J(V)$ must also fulfill the Smooth Pasting Condition, which generates a restriction that pins down the critical value $V^{**}$:

$$\frac{d}{dV}J|_{V=V^{**}} = \frac{d}{dV}(V-I)|_{V=V^{**}} = 1.$$

The upper bound on the fair current expense in technology innovation research that will achieve the softening of the irreversible cost in the presence of irreversible demise is

$$J(V) - F(V),$$

that is, the value difference between an $L$-barrier demise-adjusted option to invest and a plain demise-adjusted option to invest.

## 5 Value characteristics of cost softening with demise

In picking the parameter values, I rely on the classic real options calibration in Dixit and Pindyck (1994), p. 153. Unless otherwise noted, I set the *status-quo* cost of the investment, $I$, equal to 1, $r = 4\%$, $\delta = 4\%$, and $\sigma = 20\%$ (at annual rates). All the figures[4] (Figures 1-5) plot the project's Total Net PVs $F(V)$, $J(V)$, and $F_{\text{soft}}(V)$ against the project's value $V$. Figure 1 shows that technology innovation must come along with a percentage cost saving $1-\omega$ that is sizeable enough (25% is perhaps a good starting point) before the difference $J(V)-F(V)$ can reach a level of notice. In Figure 1, the ratio of the cost-softening barrier $L$ to the investment $I$ is set at 65% and the ratio of the demise threshold to $I$ is set at $\kappa = 50\%$. In Figure 2, $L/I$ is set at 75%, the percentage cost saving $1-\omega$ is set at 50%, and the ratio $\kappa$ takes four different values, from 70% to 0%. As the ratio $\kappa$ decreases, demise looms less large and the upper bound on the fair current expense to soften the irreversible cost decreases but does not vanish: Even with a zero chance of the project's demise event, cost softening remains valuable. In Figures 3-5, $L/I$ is set at 75%, the ratio $\kappa$ is set at 50%, and the percentage cost saving $1-\omega$ is set at 75% to better visualize the value effects. Figure 3 shows that an increasing riskfree rate greatly shrinks the difference $J(V)-F(V)$, because $V$'s rising drift inflates the call option value $F(V)$ even in the proximity of the demise threshold $\kappa I = 50\%$. Demise and drift-driven inflation of option values imply that the option values exhibit an initial concavity. They regain their usual convex pattern for higher PVs. Figure 4 shows that an increasing payout rate strongly boosts the difference $J(V)-F(V)$ in the region $[L, L+(1-\omega)I]$, because $V$'s declining drift deflates the option value $F(V)$ but inflates the probability of hitting the cost-softening barrier $L$. The project looks much unappealing under the *status quo*, but cost-softening innovation that is predictably quick in coming along restitutes glamour to the project. For $\delta \to \infty$, the project's PV tends to fall at an extraordinary speed and,

[4] The figures are based on *Mathematica* software calculations. Critical values are determined by built-in numerical routines (the maximum number of iterations is set at 200).

when $V$ is above $L + (1-\omega) I$, the option values $J(V)$ and $F(V)$ coincide:

$$J(V) = F(V) = \max\{V - I, 0\} = V - I > [L + (1-\omega) I] - I = L - \omega I,$$

that is, immediate investment pays more than the value of waiting to invest. Figure 5 shows that, as usual, more risk (an high $\sigma$) inflates the value of any option. No local value concavity arises because option value inflation comes about by means of current-value propagation of a riskier convex payoff. Hence, the upper bound on the fair current expense to soften the irreversible cost remains non-trivial for a wide range of the project's PV.

## 6 Conclusions

For a firm with the monopoly over a project that has a PV subject to uncertainty and that necessitates an irreversible investment to be entered, we study the maximum amount the firm is willing to spend up front in technology innovation to soften the irreversible investment cost-cost softening is sought in deteriorating business conditions to undo the negative value effects of the project's approaching demise. Demise is associated with a given lower PV barrier that triggers irreversible discarding of the project. This is a novel question in the real-options-based technology innovation literature and we answer it by means of a novel application of double barrier contingent claims analysis. The upper bound on the fair current expense to soften the irreversible cost is positive in a wide range of the project's PV when PV volatility is high. When the project enjoys a high payout rate or when the riskfree rate moves downward to zero, the upper bound is positive and sizeable in a selected area of PVs - those values that are above the cost-softening barrier and imply small investment-option values under the *status-quo* investment technology. An interesting extension of our analysis can come from removing the monopoly assumption that is behind the exogeneity of the cost softening barrier and of the demise barrier.

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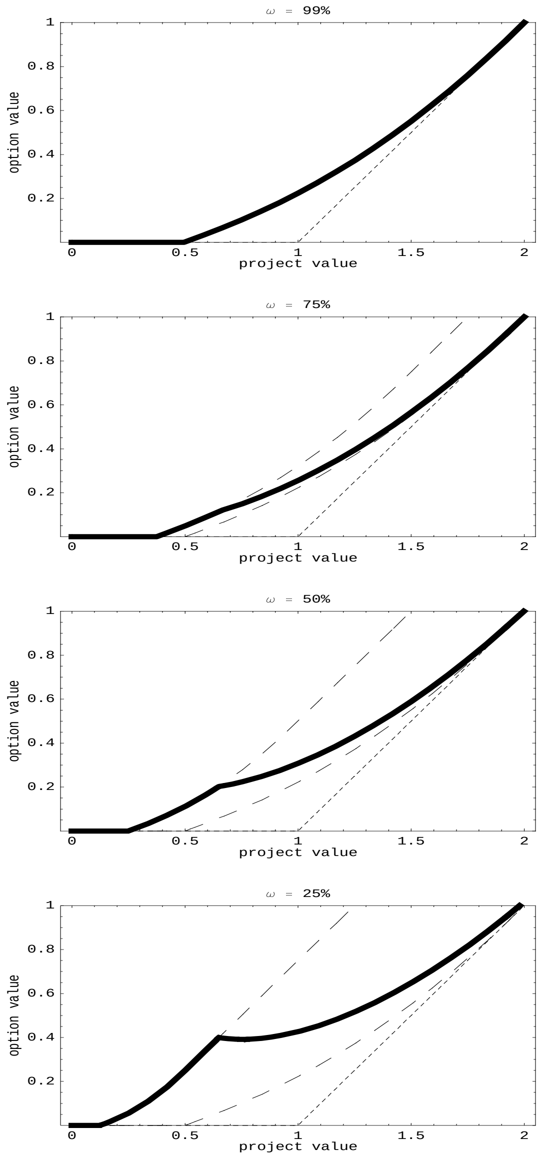


**Figure 1: The investment-option values: The effect of the ratio $\omega$ of the softened cost to the *status-quo* cost**

The values $F_{\text{soft}}(V)$ (upper dashed line), $J(V)$ (bold solid line), $F(V)$ (lower dashed line), and $\max\{V-I,0\}$ (dotted line) are plotted against the project's value $V$. The parameter values are $r=4\%$, $\delta=4\%$, and $\sigma=20\%$. The *status-quo* cost is $I=1$, the cost-softening barrier is $L=65\%$, and the demise threshold is $\kappa I=50\%$.

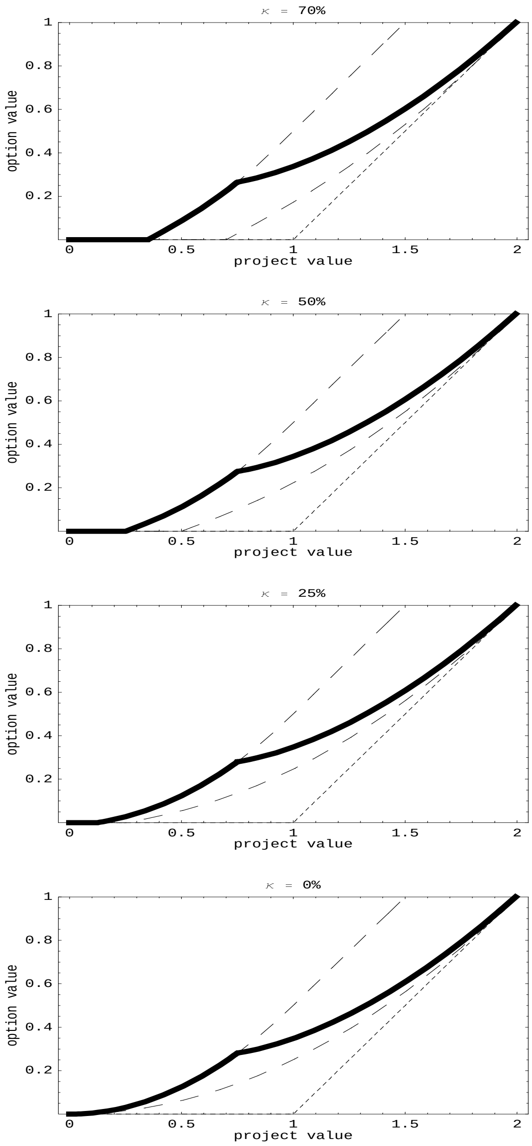


**Figure 2: The investment-option values: The effect of the ratio $\kappa$ of the demise threshold to the *status-quo* cost**

The values $F_{\text{soft}}(V)$ (upper dashed line), $J(V)$ (bold solid line), $F(V)$ (lower dashed line), and $\max\{V - I, 0\}$ (dotted line) are plotted against the project's value $V$. The parameter values are $r = 4\%$, $\delta = 4\%$, and $\sigma = 20\%$. The *status-quo* cost is $I = 1$, the cost-softening barrier is $L = 75\%$, and the ratio of the softened cost to the *status-quo* cost is $\omega = 50\%$.

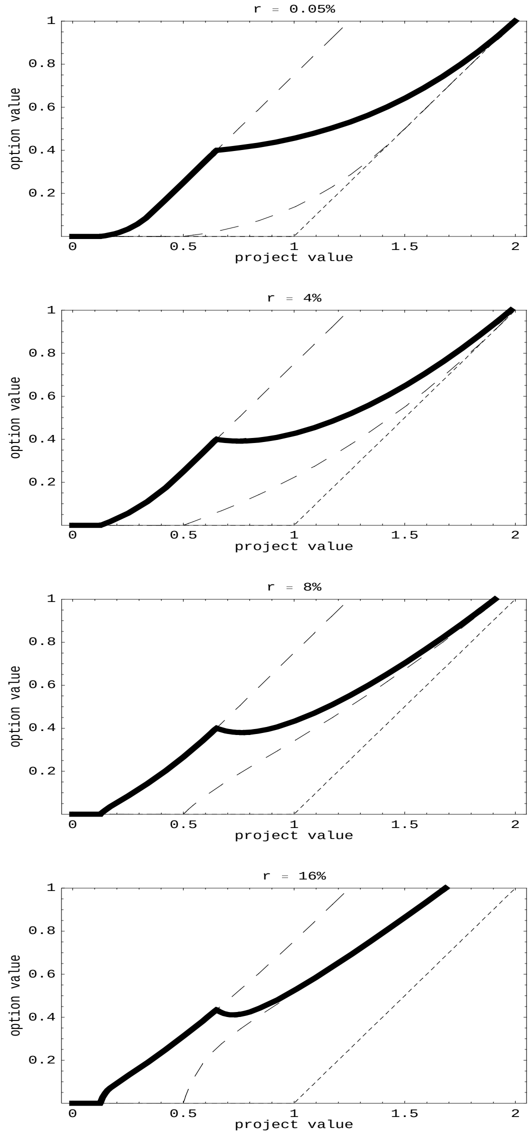


**Figure 3: The investment-option values: The effect of the riskfree rate $r$**

The values $F_{\text{soft}}(V)$ (upper dashed line), $J(V)$ (bold solid line), $F(V)$ (lower dashed line), and $\max\{V - I, 0\}$ (dotted line) are plotted against the project's value $V$. The parameter values are $\delta = 4\%$ and $\sigma = 20\%$. The *status-quo* cost is $I = 1$, the cost-softening barrier is $L = 75\%$, the demise threshold is $\kappa I = 50\%$, and the ratio of the softened cost to the *status-quo* cost is $\omega = 25\%$.

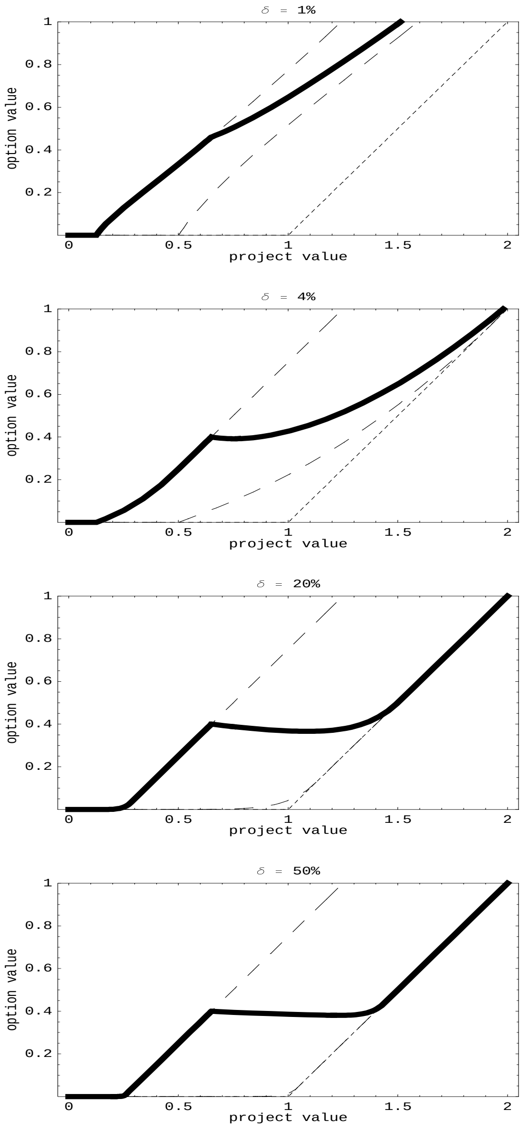


**Figure 4: The investment-option values: The effect of the payout rate $\delta$**

The values $F_{\text{soft}}(V)$ (upper dashed line), $J(V)$ (bold solid line), $F(V)$ (lower dashed line), and $\max\{V - I, 0\}$ (dotted line) are plotted against the project's value $V$. The parameter values are $r = 4\%$ and $\sigma = 20\%$. The *status-quo* cost is $I = 1$, the cost-softening barrier is $L = 75\%$, the demise threshold is $\kappa I = 50\%$, and the ratio of the softened cost to the *status-quo* cost is $\omega = 25\%$.

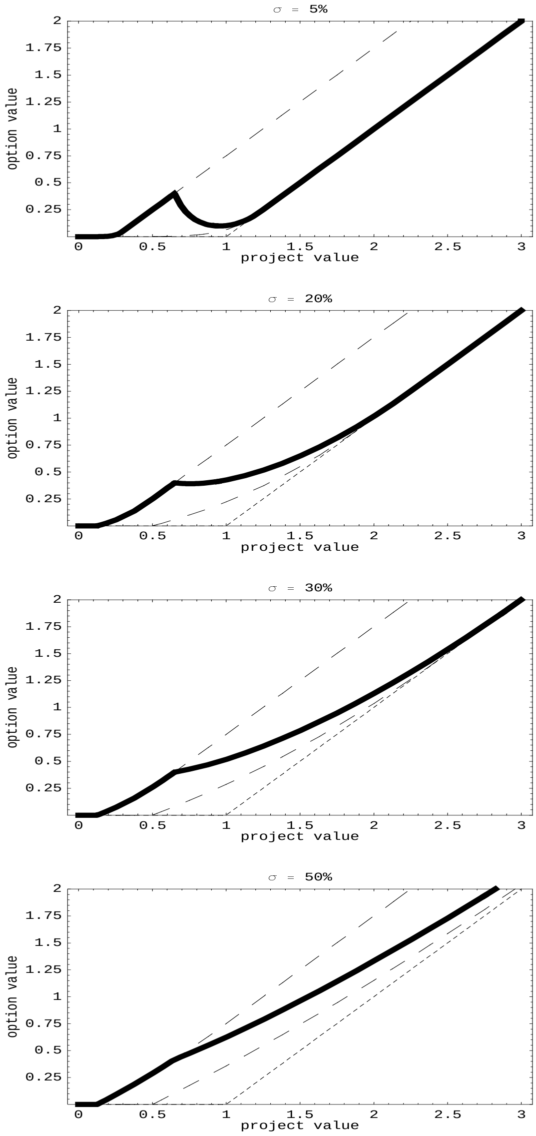


**Figure 5: The investment-option values: The effect of the volatility rate $\sigma$**

The values $F_{\text{soft}}(V)$ (upper dashed line), $J(V)$ (bold solid line), $F(V)$ (lower dashed line), and $\max\{V - I, 0\}$ (dotted line) are plotted against the project's value $V$. The parameter values are $r = 4\%$ and $\delta = 4\%$. The *status-quo* cost is $I = 1$, the cost-softening barrier is $L = 75\%$, the demise threshold is $\kappa I = 50\%$, and the ratio of the softened cost to the *status-quo* cost is $\omega = 25\%$.