PUBLIC INVESTMENT IN A SMALL OPEN ECONOMY

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ABSTRACT

We study the effects of public investment in a dynamic overlapping-generations model of a small open economy. Boosting public investment stimulates private capital formation, output, employment, and wages in the long run. The impact effects depend critically on whether public capital is modeled as a stock or as a flow. The welfare benefits are unevenly distributed across generations since capital ownership, and the capital gain induced by the policy shock, rises with age, and because wages rise only gradually under the stock interpretation of public capital. A suitable egalitarian bond policy can be employed to ensure that everybody gains to the same extent. With this additional instrument the intergenerational externality can be neutralized and the resulting efficiency gain coincides with the one obtained in the corresponding representative agent model. A simple modified golden rule for public investment is derived which takes into account the time that is needed to build the public capital stock.

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1. Introduction

Macroeconomists have long felt that public investment is an important factor in enhancing the productive capacity of the economy. Yet, for a long time, it only played a very modest role in the literature. Recently this has changed quite drastically, however. The empirical research by Aschauer (1989, 1990) triggered a boom in the literature on public investment. Aschauer's results suggested that it had a strong positive effect on the productivity of private capital and that the slowdown in productivity growth in the United States since the early 1970s was due to a shortage in public infrastructure. These controversial results generated a substantial body of research directed at determining whether public investment had in fact been too low.[1] However, this question cannot be answered without a clear picture of the macroeconomic effects and the welfare effects of public investment. This paper is aimed to clarify this image.

In many cases it is the accumulated stock of public capital rather than the flow of public investment that is relevant for the productive capacity of the economy. Current decisions on public investment made by short-lived individuals therefore have long-lasting effects and thus influence the welfare of both present and future generations, i.e. there is both an *intra*temporal and an *inter*temporal external effect at work. Public investment policy thus must pay attention to both the efficiency question of 'how much to invest' and the distributional question of 'who pays for the public capital stock and how much.' Unfortunately, in much of the existing literature the flow of public investment rather than the stock of public capital is modeled as the source of contribution to productive capacity.[2] Moreover, attention has been unduly focused on only the efficiency effect of public investment. It is this void in the image of the effects of public investment that this paper is aimed to clarify.

Most theoretical papers on the productivity of public investment use a Ramsey framework with an infinitely-lived representative household.[3] In this type of models the optimal long-run level of public investment can easily be defined. This optimal level is derived either in a command economy (first best solution) or in a market economy where the government has a limited set of instruments to finance its expenditures and to correct external effects (second best solution). Some papers also study the transition from an initial state to this long-run optimum, either numerically (e.g. Baxter and King (1993)) or analytically (as in Fisher and Turnovsky (1996)). However, by assuming a Ramsey framework these papers do not allow for intergenerational redistribution effects. Indeed, to the extent that this fictional agent really constitutes a shortcut description of a dynasty of finitely-lived and altruistically linked generations, there is really no intergenerational external effect to worry about. By definition, altruistically linked generations view future generations as continuations of themselves, and therefore internalize intergenerational external

effects. In contrast to the Ramsey-framework, overlapping generations models, such as developed by Diamond (1965), Yaari (1965), and Blanchard (1985), allow for a simple demographic structure in which unlinked generations co-exist at any moment in time. In the Yaari-Blanchard formulation, agents face a constant exogenous probability of death and are unconnected to previous generations due to the absence of a bequest motive. Overlapping generations models of this type form a suitable framework to study the intergenerational effects of public investment.

In this paper both the efficiency and the intergenerational welfare effects of public investment are studied. To this end the small open economy version of the Yaari-Blanchard model is extended to include the productive effect of public investment. The model describes an economy that, apart from the foreign sector, consists of three sectors. Firstly, a household sector which comprises a large number of cohorts which differ with respect to age and thus the level and composition of their wealth. Labour is included in the utility function of the households along the lines set out by Greenwood et al. (1988) which implies that public investment affects labour supply through changes in the wage rate. Secondly, the model contains a perfectly competitive production sector with a standard adjustment cost function in order to guarantee smooth transition. And finally there is a government which invests in public capital. The model is flexible in the sense that both the stock and the flow interpretation of public capital can analyzed with it.

This paper is most closely related to the recent work of Fisher and Turnovsky (1996) who analyze the impact of the stock of public capital on macroeconomic performance. Their paper differs from ours at a number of points, however. In contrast to this paper, they allow for congestion. Congestion introduces an additional externality in the model, which is important if one wants to analyze the effects of different tax instruments to finance public investment, as they do. That is not the subject of this paper however. We restrict ourselves to lump sum taxes and government debt as the only sources for financing public investment. Our main contribution to the literature is that we use an overlapping generations model and analyze both efficiency and redistribution, whereas Fisher and Turnovsky concentrate on efficiency aspects in a Ramsey framework. Another important difference is that we assume a small open economy with endogenous labour supply instead of a closed economy facing an exogenously given supply of labour as they do. We regard this as a useful contribution to the literature because it is difficult to pin-point any truly closed economies in the global market at present. Moreover, the exogeneity of the interest rate substantially simplifies the analysis. This enables us to make another important contribution to the literature, namely the derivation of a simple 'modified golden rule' of public investment in a dynamic economy. This rule elegantly demonstrates the crucial interaction between the productivity of public capital, the degree of durability of public capital and the world interest

rate.

The rest of the paper is structured as follows. Section 2 sets out the overlapping generations model with public investment. It also describes the steady state of the model and its dynamic behaviour around an initial steady state. The macroeconomic effects of public investment are investigated in Section 3. This section describes the effect of an unanticipated permanent increase in public investment when no debt policy is used and the tax revenue needed to finance the government's investment plans is raised through an age-independent lump sum tax. Analytical expression for the impact, transitional, and long-run effects of the policy shock on the various macroeconomic variables are derived.

In Section 4 the intergenerational welfare effects of public investment are analyzed. This is done first for the case without debt policy. It is shown that, if we concentrate on the effects on utility of generations born in the steady state, the traditional 'golden rule' applies: utility is maximized if the share of public investment equals the elasticity of production with respect to public capital. However, this does not automatically imply that, if public investment is below this level, it should or will be raised. The reason for this is that the welfare effects of a boost in public investment are distributed very unevenly over generations. The welfare of very old generations, that possess a large amount of financial assets, will always rise due to the capital gain on their financial wealth. The effect on the welfare of younger existing generations may also be positive. This will be the case if human wealth increases on impact, that is, if the share of public investment is far below its 'golden-rule' level and the birth rate is not too high. In that case selfish current generations will vote in favour of a proposal to (marginally) increase public investment. If the level of public investment is larger, but still below the 'golden-rule' level, the welfare of younger existing generations will fall if the government increases investment. The reason for this is that these generations have no or little financial assets and thus do not capture a large capital gain, while they do not yet benefit from higher wages because it takes time for the public and the private capital stock to adjust. In that case, a supposedly 'efficiency improving' proposal to increase public investment may not be accepted in a vote. This finding demonstrates the need for a mechanism aimed at equalizing the distribution of welfare over generations. We show that a bond policy in combination with a once-off tax on capital owners can be used to neutralize the intergenerational welfare effects. This leads to a more ambitious stance regarding public investment. It does not lead to realization of the 'golden-rule' level of public investment, however. In fact, internalizing the intergenerational external effect brings the 'pure efficiency effects' of public investments to the fore. This leads to the 'modified golden rule' of public investment that was mentioned above. Finally, Section 5 presents some concluding remarks and directions for

further research.

2. A model of perpetual youth and public capital

2.1. Households

The utility functional at time t of the representative agent born at time v is denoted by $\Lambda(v,t)$ and has the following form:

$$\Lambda(v,t) \equiv \int_t^\infty \log X(v,\tau) \exp\left[(\alpha+\beta)(t-\tau)\right] d\tau, \tag{2.1}$$

where α is the pure rate of time preference ($\alpha>0$), β is the probability of death ($\beta\geq0$), and $X(v,\tau)$ is sub-utility or 'full consumption' which depends on labour supply, $L(v,\tau)$, and goods consumption, $C(v,\tau)$, respectively:

$$X(v,\tau) \equiv C(v,\tau) - \frac{L(v,\tau)^{1+1/\sigma_L}}{1+1/\sigma_L}, \tag{2.2}$$

with $\sigma_L>0$. Equation (2.2) was suggested by Greenwood et al. (1988) and is useful because it eliminates the intertemporal substitution effect in labour supply.[4]

The agent's budget restriction in terms of the world price of the good is equal to:

$$\dot{A}(v,\tau) = \left[r(\tau)+\beta\right]A(v,\tau) + W(\tau)L(v,\tau) - T(\tau) - C(v,\tau), \tag{2.3}$$

where $\dot{A}(\tau)\equiv dA(v,\tau)/d\tau$, $r(\tau)$ is the real rate of interest on government bonds, $W(\tau)$ is the real wage rate (assumed age-independent for convenience), $T(\tau)$ are net lump-sum taxes, and $A(v,\tau)$ are real tangible assets. All tangible assets are perfect substitutes:

$$A(v,\tau) \equiv B(v,\tau) + V(v,\tau) + F(v,\tau), \tag{2.4}$$

where $V(v,\tau)$ is the real value of shares in the hands of households of vintage v, $B(v,\tau)$ is real government bonds, and $F(v,\tau)$ denotes real net foreign assets. The domestic economy is small in world capital markets, so that the world real rate of interest r is fixed. In the absence of terms-of-trade effects, the domestic real interest rate is then determined by the familiar no-arbitrage condition:

$$r(\tau) = r. \tag{2.5}$$

Due to the separable structure of preferences, the choice problem for the representative agent can be solved in two steps. First, the dynamic problem is solved. This leads to an optimal

time profile for full consumption which is described by the agent's Euler equation:

$$\frac{\dot{X}(v,\tau)}{X(v,\tau)} = r - \alpha. \tag{2.6}$$

In the second step goods consumption and labour supply are decided:

$$C(v,\tau) = X(v,\tau) + \left(\frac{\sigma_L}{1+\sigma_L}\right) W(\tau)^{1+\sigma_L}, \quad L(v,\tau) = W(\tau)^{\sigma_L}. \tag{2.7}$$

A crucial feature of the Blanchard (1985) model (and all models deriving from it) is the simple demographic structure, which enables the aggregation over all currently alive households. Assuming that at each instance a large cohort of size βS is born and that βS agents die, and normalising S to unity, the size of the population is constant and equal to unity and the aggregated variables can be calculated as the weighted sum of the values for the different generations. For example, aggregate financial wealth is calculated as $A(\tau) \equiv \int_{-\infty}^{\tau} \beta A(v,\tau) e^{\beta(v-\tau)} dv$. The aggregated values for the other variables can be obtained in the same fashion. The main equations describing the behaviour of the aggregated household sector are:

$$\frac{\dot{X}(\tau)}{X(\tau)} = r - \alpha - \beta(\alpha+\beta)\left(\frac{A(\tau)}{X(\tau)}\right) = \frac{\dot{X}(v,\tau)}{X(v,\tau)} - \beta\left(\frac{X(\tau) - X(\tau,\tau)}{X(\tau)}\right), \tag{2.8}$$

plus the expressions for labour supply and aggregate consumption given in equations (T1.6) and (T1.10) in Table 1, respectively. In equation (2.8), $A(\tau) \equiv V(\tau) + F(\tau) + B(\tau)$ represents aggregate financial wealth. Throughout the paper we analyze the case in which *initially* both the government debt and the stock of foreign assets are zero, i.e. $B = F = 0$ initially. This not only ensures that the trade balance is zero in the initial steady state (see (2.12) below) but also that the capital stock is fully owned by domestic households and that financial wealth is strictly positive, i.e. $A = V > 0$ initially. The expression in (2.8) shows that this is consistent with a steady state provided the world interest rate exceeds the rate of time preference, i.e. $r > \alpha$ (see (2.6)). The rising full consumption profile that this implies ensures that financial wealth is transferred from old to young generations in the steady state (see Blanchard, 1985). Note that labour supply in (T1.6) only depends on the real wage rate.

2.2. Firms
The representative perfectly competitive firm has a Cobb-Douglas production function, reported in (T1.8), which is linearly homogenous in the two private production factors, private capital ($K(\tau)$) and labour ($L(\tau)$), where $Y(\tau)$ is gross output, $K_G(\tau)$ is the stock of public capital, and the

parameters satisfy $0 \leq \eta < \varepsilon_L < 1$. The restriction $\eta < \varepsilon_L$ ensures diminishing returns to broadly defined capital and thus excludes the possibility of endogenous growth in the model.

The firm faces convex adjustment costs defined on gross investment. We follow Uzawa (1969) by postulating an installation cost function $\Phi(.)$ which links gross to net capital accumulation. See equation (T1.3), where $I(\tau)$ is gross investment and δ is the depreciation rate. The firm maximizes the present value of its cash flow,

$$V(t) = \int_t^\infty \left[Y(\tau) - W(\tau)L(\tau) - I(\tau) \right] e^{r(t-\tau)} d\tau, \qquad (2.9)$$

subject to the production function (T1.8) and the installation cost function (T1.3). The resulting optimality conditions yield expressions for labour demand in (T1.5), investment demand (T1.7), and the shadow value of installed capital (viz. Tobin's q) in (T1.4). Since the installation cost function, $\Phi(.)$, is homogeneous of degree zero in $I(\tau)$ and $K(\tau)$ and the production technology is linear-homogeneous in private factors of production, Tobin's marginal and average q coincide, and the stock market value of the firm equals $V(t)=q(t)K(t)$ (see Hayashi (1982) and Heijdra and Meijdam (1997)).

2.3. The government and the foreign sector

The periodic budget identity of the government is:

$$\dot{B}(\tau) = rB(\tau) + I_G(\tau) - T(\tau), \qquad (2.10)$$

where $T(\tau)$ is the real lump-sum tax levied on the households and $I_G(\tau)$ is gross public investment. The stock of public capital evolves according to the expression in (T1.13), where δ_G represents the rate of depreciation of the stock of public capital. If $\delta_G \to \infty$, public capital evaporates instantaneously and the *flow* of government investment enters the private production function as in Turnovsky and Fisher (1995). Since the government is expected to remain solvent, the following NPG condition is relevant:

$$\lim_{\tau \to \infty} B(\tau) e^{r(t-\tau)} = 0. \qquad (2.11)$$

By combining (2.10) and (2.11), the intertemporal government budget restriction (T1.11) is obtained.

The trade balance is the difference between domestic production and absorption:

$$TB(\tau) \equiv Y(\tau) - C(\tau) - I_G(\tau) - I(\tau). \qquad (2.12)$$

The current account is then computed by using the time derivative of (T1.12) and simplifying:

$$\dot{F}(\tau) = rF(\tau) + TB(\tau). \qquad (2.13)$$

2.4. Equilibrium and stability

The complete dynamic model is given in aggregated form in Table 1. The dynamic part of the model is given by equations (T1.1)-(T1.4). Equation (T1.1) is obtained by using (2.7) in (2.3), aggregating over all generations, and noting that full consumption is proportional to total wealth, i.e. $X(t)=(\alpha+\beta)[A(t)+H(t)]$, where $H(t)$ is human wealth. Equation (T1.2) shows the time path for human wealth, where full income, $Y_F(t)$, represents the 'dividend payment' on human wealth, which equals after-tax labour income minus the utility cost of supplying the optimal amount of labour. Heijdra and Meijdam (1997) show that $Y_F(t)$ can be written as in (T1.9). Equations (T1.1)-(T1.2) represent the saving system of the model, whilst (T1.3)-(T1.4) represent the investment system (See also Bovenberg (1993, 1994) for this use of terminology in a similar model).

The static part of the model is given in equations (T1.5)-(T1.10). Labour demand and supply are given in (T1.5) and (T1.6), respectively, (T1.7) is investment demand, and (T1.8) is the aggregate production function. Goods consumption and full consumption are reported in (T1.10). Finally, equation (T1.11) is the intertemporal government budget restriction, (T1.12) is the definition of financial wealth, and (T1.13) shows the evolution of the stock of public capital.

In order to study the dynamical properties of the model, it is first log-linearized around an initial steady state. The main expressions are found in Table 2. A tilde ('~') above a variable denotes its rate of change around the initial steady-state, e.g., $\tilde{x}(t)\equiv \mathrm{d}x(t)/x$. A variable with a tilde and a dot is the time derivative expressed in terms of the initial steady-state, for example, $\dot{\tilde{x}}(t)\equiv\dot{x}(t)/x$. The only exceptions to that convention refer to the various wealth components, full income, and lump-sum taxes, i.e. $\tilde{x}(t)\equiv r\,\mathrm{d}x(t)/Y$ and $\dot{\tilde{x}}(t)\equiv r\dot{x}(t)/Y$ for $x\in(A,H,B,F)$, $\tilde{T}(t)\equiv \mathrm{d}T(t)/Y$, and $\tilde{Y}_F(t)\equiv \mathrm{d}Y_F(t)/Y$.

In order to solve the model, it is useful to first condense the static part of the model as much as possible. By using (T2.5), (T2.6), and (T2.8), we obtain 'quasi-reduced form' expressions for output ($\tilde{Y}(t)$), employment ($\tilde{L}(t)$), and the real wage ($\tilde{W}(t)$) in terms of the private and public capital stocks ($\tilde{K}(t)$ and $\tilde{K}_G(t)$):

$$\tilde{Y}(t) = \left(\frac{1+\sigma_L}{\sigma_L}\right)\tilde{L}(t) = (1+\sigma_L)\tilde{W}(t) = \frac{(1+\sigma_L)(1-\varepsilon_L)\tilde{K}(t) + \eta(1+\sigma_L)\tilde{K}_G(t)}{1+\sigma_L(1-\varepsilon_L)}. \qquad (2.14)$$

An increase in either one of the capital stocks boosts the demand for labour and consequently increases both the wage, equilibrium employment, and output. If $\sigma_L=0$, labour supply is exogenous and the employment effect vanishes.

The investment system can be summarized by using (2.14) and (T2.7) in (T2.3)-(T2.4) and writing the resulting expressions in a single matrix form:

$$\begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{q}}(t) \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{r\omega_I}{\sigma_A\omega_A} \\ \dfrac{r\varepsilon_L(\omega_A+\omega_I)}{\omega_A[1+\sigma_L(1-\varepsilon_L)]} & r \end{bmatrix} \begin{bmatrix} \tilde{K}(t) \\ \tilde{q}(t) \end{bmatrix} - \begin{bmatrix} 0 \\ \dfrac{r\eta(1+\sigma_L)(\omega_A+\omega_I)}{\omega_A[1+\sigma_L(1-\varepsilon_L)]} \end{bmatrix} \tilde{K}_G(t), \qquad (2.15)$$

In a similar fashion, the saving system can be summarized by writing (T2.1)-(T2.2) in the following matrix form:

$$\begin{bmatrix} \dot{\tilde{H}}(t) \\ \dot{\tilde{A}}(t) \end{bmatrix} = \begin{bmatrix} r+\beta & 0 \\ -(\alpha+\beta) & r-\alpha-\beta \end{bmatrix} \begin{bmatrix} \tilde{H}(t) \\ \tilde{A}(t) \end{bmatrix} - \begin{bmatrix} r\tilde{Y}_F(t) \\ -r\tilde{Y}_F(t) \end{bmatrix} \qquad (2.16)$$

We demonstrate in the Appendix that the investment system is always stable, and the saving system is stable provided the share of government spending is not too high. Proposition 1 summarizes the results that have been derived for the model.

PROPOSITION 1: *Let $\varepsilon_L/(1+\sigma_L)>\omega_G>0$. The loglinearized model of Table 2 implies the following results: (i) The full model is locally saddle-point stable; (ii) The characteristic roots of the investment system are distinct and satisfy $-h_I<0$ and $r_I>r$; (iii) the stable root satisfies $\partial h_I/\partial\sigma_A<0$ and $h_I\rightarrow\infty$ as $\sigma_A\rightarrow0$; (iv) The characteristic roots of the saving system are distinct and satisfy $r_S=r+\beta$ and $-h_S=r-(\alpha+\beta)<0$; (v) the stable root satisfies $\partial h_S/\partial\beta>0$ and $h_S\rightarrow\infty$ as $\beta\rightarrow\infty$. PROOF: See Appendix.*

3. The macroeconomic effects of public investment

In this section we study the allocation effects of an unanticipated and permanent increase in the level of public investment. The time at which the policy shock occurs is normalized to zero (hence, $\tilde{I}_G > 0$ for $t \geq 0$). We assume throughout this section that no debt policy is used and that the tax revenue needed to finance the government's investment plans is raised in a lump-sum fashion from households. This implies that the loglinearized version of the government budget restriction (T1.11) reduces to $\tilde{T}(t) = \omega_G \tilde{I}_G$.

3.1. The investment system

In order to explain the intuition behind the results, we use the diagrammatic apparatus of Figure 1, which is the graphical representation of the investment system given in (2.15). The $\dot{K}(t)=0$ locus represents (q,K)-combinations for which the capital stock is in equilibrium, i.e. for which net investment is zero. It is horizontal and q^* represents the unique value for Tobin's q for which $\Phi(.) = \delta$ (see (T1.7) and (T1.3)). For values of $q(t)$ larger (smaller) than q^*, net investment is positive (negative) as is shown with horizontal arrows in Figure 1.

The $\dot{q}(t)=0$ locus represents (q,K)-combinations for which Tobin's q is constant over time. It is downward sloping because a higher capital stock leads to fall in the marginal product of capital and thus to a lower dividend to the owners of shares. For points to the right (left) of the line the marginal product of capital is too low (high) so that part of the return on shares is explained by capital gains (losses). Hence, $\dot{q}(t) > 0$ (< 0) to the right (left) of the line, as has been shown with vertical arrows in Figure 1. Not surprisingly, in view of the discussion in section 2.4, the arrow configuration in Figure 1 confirms that the equilibrium at E_0 is saddle point stable.

The increase in public investment ultimately leads to a permanently higher stock of public capital (see (T1.13) or, equivalently, (T2.13)). This explains why the $\dot{q}(t)=0$ locus shifts to the right in the long run, so that the new steady state is at point E_1 in Figure 1. In the long run, there is no effect on Tobin's q so that the increase in the marginal product of private capital caused by the increase in public capital is exactly offset by an increase in the private capital stock:

$$\tilde{q}(\infty) = 0, \qquad \tilde{K}(\infty) = \left(\frac{\eta(1+\sigma_L)}{\varepsilon_L}\right)\tilde{K}_G(\infty), \qquad \tilde{K}_G(\infty) = \tilde{I}_G > 0. \tag{3.1}$$

By using (3.1) and (2.14), the long-run results on output, employment, and the real wage are obtained:

$$\tilde{Y}(\infty) \ = \ \left(\frac{1+\sigma_L}{\sigma_L} \right) \tilde{L}(\infty) = \ (1+\sigma_L)\,\tilde{W}(\infty) \ = \ \left(\frac{\eta\,(1+\sigma_L)}{\varepsilon_L} \right) \tilde{I}_G \ > \ 0. \tag{3.2}$$

The impact effect on Tobin's q is determined by the requirement that the economy must be on a stable trajectory leading to the steady-state equilibrium at E_1 in combination with the fact that the private capital stock is predetermined ($\tilde{K}(0)=0$). It is shown in Heijdra and Meijdam (1997) that Tobin's q and thus private investment rises unambiguously as a result of the boost in public investment:

$$\tilde{q}(0) \ = \ \sigma_A \tilde{I}(0) \ = \ \frac{r\eta\,(1+\sigma_L)(\omega_A+\omega_I)\delta_G \tilde{I}_G}{\omega_A r_I (r_I+\delta_G)\big[1+\sigma_L(1-\varepsilon_L)\big]} \ > \ 0. \tag{3.3}$$

Public investment thus causes a boom in private investment.

If the depreciation rate of public capital is finite ($0<\delta_G\ll\infty$), the public capital stock is predetermined in which case equation (2.14) implies that output, employment and the real wages are all unaffected at impact:

$$\tilde{Y}(0) \ = \ \tilde{L}(0) \ = \ \tilde{W}(0) \ = \ 0, \qquad (\text{for } 0<\delta_G\ll\infty). \tag{3.4}$$

The investment boom gives rise to a gradual increase in the private capital stock which in turn affects the marginal product of capital during transition and thus also Tobin's q. The transition paths for capital and Tobin's q are:

$$\tilde{K}(t) \ = \ \big[1-e^{-h_I t}\big]\tilde{K}(\infty) \ - \ \left(\frac{r_I h_I}{r_I+\delta_G} \right) \tilde{K}(\infty)\,\mathrm{T}(h_I,\delta_G,t), \tag{3.5}$$

$$\tilde{q}(t) \ = \ e^{-h_I t}\tilde{q}(0) \ + \ r_I\tilde{q}(0)\,\mathrm{T}(h_I,\delta_G,t), \tag{3.6}$$

where $\mathrm{T}(h_I,\delta_G,t)$ is a bell-shaped transition term, which is zero at impact and in the long run and positive during transition.[5] The second term on the right-hand sides of (3.5) and (3.6) represents the transitory effect of the public investment shock on the private capital stock and Tobin's q, respectively.

Under the *stock interpretation* of public investment, there are important anticipation effects despite the fact that the policy shock itself is unanticipated. This is because individual investors are aware of the accumulation identity (T1.13) and thus know that the stock of public capital will rise over time. This explains why Tobin's q follows an upward sloping time profile immediately after

the public investment shock takes place. Indeed, by differentiating (3.6) with respect to time, we obtain:

$$\dot{\tilde{q}}(0) = r\tilde{q}(0) > 0, \qquad (\text{for } 0<\delta_G\lll\infty). \tag{3.7}$$

In terms of Figure 1, the economy jumps at impact from point E_0 to point A directly above it, and the saddle path is upward sloping in point A. In Figure 1 the transition path is represented by the dashed line from A to the new equilibrium at E_1.

The impact and transition effects are somewhat different if a *flow concept* of public capital is used, as in Turnovsky and Fisher (1995). In our model this case is obtained by letting $\delta_G\to\infty$ which (by (T2.13)) implies that $\tilde{K}_G(t)=\tilde{I}_G$ for all $t\geq0$. The impact effect for Tobin's q is obtained from (3.3) by letting $\delta_G\to\infty$. In contrast to the earlier case, output, employment, and the real wage rise at impact,

$$\tilde{Y}(0) = \left(\frac{1+\sigma_L}{\sigma_L}\right)\tilde{L}(0)= (1+\sigma_L)\tilde{W}(0) = \frac{\eta(1+\sigma_L)\tilde{I}_G}{1+\sigma_L(1-\varepsilon_L)} > 0, \quad (\text{for } \delta_G\to\infty). \tag{3.8}$$

and the adjustment path for Tobin's q is monotonic:

$$\dot{\tilde{q}}(0) = -\frac{r\eta(1+\sigma_L)(\omega_A+\omega_I)(r_I-r)\tilde{I}_G}{\omega_A\left[1+\sigma_L(1-\varepsilon_L)\right]r_I} < 0, \qquad (\text{for } \delta_G\to\infty), \tag{3.9}$$

where the sign in (3.9) follows from Proposition 1(ii). In terms of Figure 1, the economy jumps at impact from point E_0 to point A′ directly above it, and the saddle path is downward sloping in that point. There are no anticipation effects and Tobin's q returns smoothly and monotonically towards its equilibrium value. In Figure 1 this is represented by the broken line from A′ to the new equilibrium at E_1. As is apparent from Figure 1, the adjustment path for the private capital stock is monotonic regardless of the magnitude of δ_G. Indeed, by differentiating (3.5) with respect to time, we obtain:

$$\dot{\tilde{K}}(t) = h_I\tilde{K}(\infty)\left[e^{-h_It} - \left(\frac{r_I}{r_I+\delta_G}\right)\frac{dT(h_I,\delta_G,t)}{dt}\right] > 0, \tag{3.10}$$

where the inequality follows from the properties of the transition function.[6]

We have thus demonstrated that the effects of productivity enhancing government spending depend in a crucial way on the nature of these public outlays. The prototypical examples of public investment concern infrastructural projects such as roads, bridges, railway tracks, harbours, airports,

etcetera, for which the stock interpretation is the most appropriate one. For this reason attention in the rest of the paper will be focused on the stock interpretation, for which $0<\delta_G\ll\infty$. On occasion the results under the flow interpretation will be referred to.

Given the transition path for the capital stock (3.5), the paths for employment, wages and output follow directly from (2.14) above, so that the transition patterns of these respective variables all display the same pattern as the private capital stock, viz. there is a zero impact effect followed by a gradual monotonic increase to a new steady state. Accumulation of private and public capital increases the marginal product of labour, shifts labour demand outward and results in higher wages and employment. Output rises both because of the increased capital stocks and the induced boost in employment.

3.2. The saving system

The shock affecting the saving system is time-varying and fully determined by the time path of full income. Since the path of wages rises monotonically (see above), and the lump-sum tax exhibits a once-off rise at impact, the time path of full income is also monotonically increasing. At impact full income falls because of the increase in the lump-sum tax:

$$\tilde{Y}_F(0) = -\tilde{T}(0) = -\omega_G \tilde{I}_G < 0. \tag{3.11}$$

The long-run effect on full income can be computed by using (3.2) in (T2.9) and noting that $\tilde{T}(t)=\omega_G\tilde{I}_G$:

$$\tilde{Y}_F(\infty) = \varepsilon_L \tilde{W}(\infty) - \tilde{T}(\infty) = (\eta-\omega_G)\tilde{I}_G \overset{>}{\underset{<}{=}} 0, \quad \tilde{Y}_F(\infty) > \tilde{Y}_F(0). \tag{3.12}$$

The long-run effect on full income is determined by the interplay of two separate influences. On the one hand an increase in public investment raises the marginal product of labour and thus increases the wage and, since labour supply is upward sloping, gross labour income. This is represented by the positive term involving η on the right-hand side of (3.12). On the other hand, the additional public investment leads to higher lump-sum taxes which causes a reduction of full income. On balance, if $\eta>\omega_G$ (=,<) the gross labour-income effect dominates (equals, is dominated by) the tax effect, so that full income rises (stays the same, falls) in the long run. By invoking the steady state in (2.16) and substituting (3.12), the long-run effects on human and financial wealth and full consumption are obtained:

$$\tilde{H}(\infty) = \frac{r(\eta - \omega_G)\tilde{I}_G}{r+\beta}, \qquad \tilde{A}(\infty) = \omega_A \tilde{X}(\infty) = \left[\frac{r-\alpha}{\alpha+\beta-r}\right]\tilde{H}(\infty). \qquad (3.13)$$

Full consumption changes proportionally with both financial and human wealth. Equation (2.8) shows that full consumption equilibrium implies a unique (X/A)-ratio which only depends on the exogenous interest rate and taste parameters. But since full consumption is proportional to total wealth, this automatically implies that the (X/H)- and (A/H)-ratios are also constant in the steady state.

We now turn to the impact effects and the transitional dynamics implied by the saving system. As was demonstrated in section 2.4 above, the saving system is saddle point stable, with financial assets (net of capital gains or losses) acting as the predetermined variable and human wealth as the jumping variable. The initial condition for the saving system is provided by the requirement that at impact the jump in financial assets is equal to the capital gain enjoyed by shareholders due to the increase in Tobin's q. By using (3.3) and (T2.12) and noting that $\tilde{K}(0)=\tilde{F}(0)=\tilde{B}(0)=0$ this implies:

$$\tilde{A}(0) = \omega_A \tilde{q}(0) > 0. \qquad (3.14)$$

It is shown in Heijdra and Meijdam (1997) that the jump in the value of human wealth which takes place at impact is proportional to the present value of full income, using the unstable root of the saving system ($r_S \equiv r+\beta$) as the discount factor:

$$\tilde{H}(0) = r\int_0^\infty \tilde{Y}_F(t)\, e^{-(r+\beta)t}\mathrm{d}t \equiv r\mathcal{L}\{\tilde{Y}_F, r+\beta\}. \qquad (3.15)$$

where $\mathcal{L}\{\tilde{Y}_F, s\}$ denotes the Laplace transformation of the time path of $\tilde{Y}_F(t)$ using the discount factor s. The impact effect on human wealth thus depends on the entire path of full income and its sign is ambiguous. The downward jump of full income at impact ($\tilde{Y}_F(0)<0$) gives it a negative impulse which may or may not be offset by the long-run effect on full income. The following observations clarify what is going on. First, if public capital is relatively abundant ($\omega_G \geq \eta$), then $\tilde{Y}_F(\infty) \leq 0$ (see (3.12)) and human wealth must fall at impact, i.e. $\tilde{H}(0)<0$ in that case. Second, if public capital is relatively scarce ($\eta < \omega_G$), then $\tilde{Y}_F(\infty)>0$ and the sign of the impact effect on human wealth depends critically on the magnitude of the turnover rate of the population (β) relative to the adjustment speed of the investment system (h_I). If the turnover rate is low or physical capital is highly mobile, β/h_I is low and the impact effect on human wealth is dominated by the long-run effect on full income and thus tends to be positive. Intuitively, full income rises rapidly to its higher level, and agents live long enough to enjoy this. Conversely, if β/h_I is high, the impact

effect on human wealth is dominated by the impact effect on full income and thus tends to be negative.[7]

The time profile of real financial assets may be non-monotonic due to intergenerational distributional effects, and depends on the transition speeds of both the investment system (h_I) and the saving system (h_S). Since we shall have no need to refer to the time path of financial assets, we pay no further attention to it in the interest of brevity. Proposition 2 summarizes the most important macroeconomic effects of the policy shock that have been discussed in this section.

PROPOSITION 2: *Let $0<\delta_G\ll\infty$. Then (i) at impact Tobin's q, financial assets, and private investment rise; (ii) human wealth falls at impact if public capital is relatively scarce and the birth rate is relatively low; (iii) in the long run, the private capital stock, output, wages, and employment rise, and Tobin's q is unchanged; (iv) Full income, human wealth, and financial wealth rise (stay the same, fall) if $\omega_G<(=,>)\eta$; (v) the transition paths for private capital, wages, output, employment, full income, and human wealth are monotonically increasing; (vi) the transition path for Tobin's q is non-monotonic.* PROOF: *See text.*

4. The welfare effects of public investment

In order to evaluate the welfare effects during transition, we must take into account that different generations are affected differently by the public investment shock. Indeed, it was shown in the previous section that owners of the private capital stock enjoy a capital gain on their shares as a result of the shock (see (3.3) and (3.14)) whilst human wealth may fall at impact if public capital is already relatively abundant or if the birth rate is relatively low (see the discussion below (3.15)). Since the old existing generations are the owners of the capital stock and young existing generations rely mostly on human wealth, it is to be expected that the welfare effects are distributed unevenly across existing generations. The same holds for future generations. Because there exists a considerable amount of transitional dynamics in the model, as both private and public capital accumulate only gradually, generations born at different dates in the future will generally be affected differently by the investment shock. By using the Laplace transform techniques pioneered by Judd (1982) and Bovenberg (1993, 1994), it is nevertheless possible to evaluate the entire welfare profile across time (for future generations) and across ages (for generations alive at the time of the shock).

4.1. Intergenerational welfare effects without bond policy

To bring out the issues most clearly, we first study the case where no debt policy is used. The welfare effect on generations that exist at the time of the shock (t=0) is denoted by $d\Lambda(v,0)$, with $v \leq 0$. It is shown in Heijdra and Meijdam (1997) that this effect can be written as follows:

$$(\alpha + \beta) d\Lambda(v,0) = \tilde{X}(v,0) = \left(\tilde{A}(0)/\omega_A\right)\left[1 - e^{(r-\alpha)v}\right] + \left(\tilde{H}(0)/\omega_H\right)e^{(r-\alpha)v}, \qquad (4.1)$$

for $v \leq 0$, where $\omega_H \equiv rH/Y$, $\omega_A \equiv rA/Y$, and $\tilde{X}(v,0) \equiv dX(v,0)/X(v,0)$ is the jump at impact in the level of full consumption by a household of generation v. Note that $d\Lambda(v,0)$ in (4.1) can be also be written as a weighted average of the effect on an extremely old generation, $d\Lambda(-\infty,0) \equiv [(\alpha+\beta)\omega_A]^{-1}\tilde{A}(0)$, and the effect on a newborn, $d\Lambda(0,0) \equiv [(\alpha+\beta)\omega_H]^{-1}\tilde{H}(0)$:

$$d\Lambda(v,0) = \left[1 - e^{(r-\alpha)v}\right]d\Lambda(-\infty,0) + e^{(r-\alpha)v}d\Lambda(0,0), \quad v \leq 0. \qquad (4.2)$$

The interpretation of (4.2) is straightforward. Extremely old generations possess a large amount of financial assets so that what happens to their human wealth does not matter in the limit. Hence, the welfare effect on these generations consists of only the capital-gain effect on their financial wealth. Equation (3.14) thus ensures that extremely old generations gain as a result of the increase in public investment, i.e. $d\Lambda(-\infty,0)>0$.

The welfare effect on newly-born generations at the time of the shock, $d\Lambda(0,0)$, is fully explained by the impact effect on human wealth as this is the only kind of wealth these generations possess. Since the sign of the impact effect on human wealth is ambiguous (see section 3.2), the same holds for the wealth effect on newly-born generations. As was explained above, if $\eta>\omega_G$ and the birth rate is not too high, $\tilde{H}(0)$ is positive and the newly-born (like the extremely old) gain from the boost to public investment, i.e. $d\Lambda(0,0)>0$ in that case. In the next section it is shown with some numerical simulations that this result can be easily reversed however.

It is thus in principle possible that all existing generations gain as a result of the public investment boost. This prompts the question about which generations gain the most. It turns out that no unambiguous conclusion emerges from the model. If $d\Lambda(-\infty,0)>d\Lambda(0,0)$, the welfare effect on existing generations is monotonically increasing in age, i.e. $\partial d\Lambda(v,0)/\partial v<0$, and the extremely old generations gain the most. The reverse conclusion holds if $d\Lambda(-\infty,0)<d\Lambda(0,0)$.

Future generations are born in a world that is different from the initial steady state as a result of the shock. The change in welfare that future generations experience is evaluated at birth, i.e. the relevant indicator is $d\Lambda(t,t)$ for $v=t \geq 0$. It is shown in Heijdra and Meijdam (1997) that this welfare indicator is proportional to the path for human wealth:

$$(\alpha + \beta) d\Lambda(t,t) = \tilde{H}(t)/\omega_H, \quad t = v \geq 0. \tag{4.3}$$

We have already demonstrated (in section 3.2) that human wealth rises monotonically over time, so that the same holds for the welfare effect on future generations, i.e. $\partial d\Lambda(t,t)/\partial t > 0$. This also implies that steady-state generations are better off than newly-born generations, i.e. $d\Lambda(\infty,\infty) > d\Lambda(0,0)$. Furthermore, in view of the first expression in (3.13), steady-state generations are better off in absolute terms, $d\Lambda(\infty,\infty) > 0$, if public capital is relatively scarce, $\eta > \omega_G$.

The main characteristics of the path of (the change of) utility have been summarised in Proposition 3.

PROPOSITION 3. *Let* $\eta > 0$. *The solution paths for utility given in* (4.2) *and* (4.3) *satisfy the following properties: (i) Old existing generations experience a welfare gain, i.e.,* $d\Lambda(-\infty,0) > 0$; *(ii) Steady-state generations gain more than newborns,* $d\Lambda(0,0) < d\Lambda(\infty,\infty)$.; *(iii) Steady-state generations gain (lose) in absolute terms if* $\eta > \omega_G$ ($\eta < \omega_G$). PROOF: See text.

4.2. Some numerical simulations

We now further illustrate the welfare properties of the model with the aid of some numerical simulations. The objective is to study the effects on the intergenerational welfare distribution of the initial share of government investment (ω_G), the efficiency of public capital (as parameterized by η), the substitution elasticity of labour supply (σ_L), the degree of capital mobility (as summarized by σ_A), and the degree of durability of public capital (as summarized by δ_G). The model is calibrated in such a way that these parameters can be freely varied. The parameters that are held fixed throughout the simulations are the rate of pure time preference ($\alpha = 0.02$), the foreign interest rate ($r = 0.05$), the depreciation rate on private capital ($\delta = 0.1$), the share of labour income ($\varepsilon_L = 0.7$), and the national income share of consumption ($\omega_C = 0.7$). The birth rate (β) is used as a calibration parameter. For given values of ω_G, η, σ_L, σ_A and δ_G, it is then possible to compute all relevant remaining information.[8]

Table 3 presents a number of welfare indicators for different values of ω_G along the columns. Panel (a) of Table 3 is devoted to investigating the effect of the efficiency parameter η on the distribution of welfare, whilst panels (b) through (d) do the same for σ_A, δ_G, and σ_L, respectively. Since the production function is Cobb-Douglas (see (T1.8)) all production factors are essential, and ω_G must thus be strictly positive for the model to yield a meaningful solution.

A number of conclusions can be drawn from Table 3, panel (a). First, and rather obviously, if public capital is completely unproductive ($\eta = 0$), then no generations gain from an introduction

(or further increase in the level) of public investment. Indeed, as the column for $\eta=0$ reveals, only extremely old generations are unaffected by the shock, i.e. $d\Lambda(-\infty,0)=0$. There is no effect on Tobin's q (see (3.3)) and thus no capital gain either in that case (see (3.14)). Newborns and all future generations lose out by the same amount, i.e. $d\Lambda(t,t)<0$ and $\partial d\Lambda(t,t)/\partial t=0$ for all $t\geq0$. The shock does not give rise to transitional dynamics in the economy, and all these generations are affected equally by the once-off decline in the value of their human wealth which is caused by the increase in the lump-sum tax needed to pay for the additional public investment.

The more interesting cases are of course those associated with strictly positive values for η. Table 3 demonstrates that the intergenerational distribution of welfare is very uneven. Consider first the case with $\omega_G=0.01$ and $\eta=0.1$ (scarce productive public capital) in the second column. All generations gain if more public capital is put in place. Steady-state generations gain the most, followed by newly-born agents. Hence, $d\Lambda(\infty,\infty)>d\Lambda(0,0)>d\Lambda(-\infty,0)$ and $\partial d\Lambda(v,0)/\partial v>0$. This pattern is preserved for all values of η considered if $\omega_G=0.01$.

If there is more pre-existing public capital ($\omega_G\geq0.05$), the pattern changes because capital gains on financial assets start to dominate the human wealth effect so that $d\Lambda(0,0)<d\Lambda(-\infty,0)$ and $\partial d\Lambda(v,0)/\partial v<0$. Throughout Table 3(a), the identity of the best-off generations depends on the sign of $\eta-\omega_G$. If $\eta<\omega_G$ then the best-off generations are those born in the new steady state, i.e. $d\Lambda(\infty,\infty)>d\Lambda(-\infty,0)$. The reverse holds if public capital is relatively abundant, i.e. if $\omega_G>\eta$.

Another important feature of the simulation results is that the golden-rule solution for public investment that is implied by the model, i.e. $\omega_G=\eta$, is not Pareto optimal as it ignores the generational distributional effects. Indeed, $\omega_G=\eta$ is only optimal for steady-state generations. Extremely old generations still benefit from a further expansion of public investment, whilst new-born generations lose out. We shall return to this issue in the next section.

The simulations discussed so far demonstrate that, for a wide range of values of η and ω_G, very old generations gain and very young generations lose as a result of an increase in the level of public investment. But how does the policy shock affect the population alive at the time of the shock? In order to answer that question we compute $\sigma(\%)$, which represents the percentage of the population (alive at the time of the shock) which is no worse-off as a result of the policy shock. In view of equation (4.2), $\sigma(\%)$ can be written as:

$$\sigma(\%) \equiv \begin{cases} \Omega & \text{if } d\Lambda(-\infty,0) > 0 > d\Lambda(0,0) \\ 100 & \text{if } d\Lambda(-\infty,0) > 0 \text{ and } d\Lambda(0,0) > 0. \end{cases} \tag{4.4}$$

where Ω is defined as:

$$\Omega \equiv 100 \left[\frac{d\Lambda(-\infty,0)}{d\Lambda(-\infty,0) - d\Lambda(0,0)} \right]^{\beta/(r-\alpha)}. \qquad (4.5)$$

This variable can be interpreted as the degree of political support that exists for a marginal increase in the level of public investment. Indeed, if $\sigma(\%)$ exceeds fifty percent one would expect the existing population to vote in favour of increasing public investment.

The information in panel (a) of Table 3 suggests that the degree of political support increases with the efficiency parameter η. For example, if $\omega_G=0.1$ initially, political support is below fifty percent for $\eta \leq 0.2$ but is above fifty percent for $\eta \geq 0.3$. This shows that, provided public capital is sufficiently productive, further increases in public investment can occur under majority rule despite the uneven intergenerational welfare burden associated with such a measure.

In panel (b) of Table 3 the effect of σ_A on the intergenerational welfare distribution is studied. As was pointed out by Bovenberg (1993, p. 13), σ_A measures the degree of physical capital mobility; the lower is σ_A, the lower is the degree of concavity of the adjustment cost function, and the higher is the mobility of physical capital. As $\sigma_A \to 0$, there are no adjustment costs, physical capital is perfectly mobile internationally, and Proposition 1(iii) suggests that $h_I \to \infty$. See also Barro and Sala-i-Martin (1995, p. 98). The main conclusion which emerges from these simulation results refers to the welfare effect on extremely old generations. For all cases considered in Table 3(b), $d\Lambda(-\infty,0)$ is increasing in σ_A, i.e. decreasing in the degree of physical capital mobility. The intuition behind this result is as follows. With a high degree of capital mobility, installed and new capital goods are close substitutes. As a result, a boost in public investment only gives rise to a small change in Tobin's q (see (3.3)), and hence a small capital gain on financial assets (see (3.14)). Overall, the results of panel (b) are very similar in qualitative terms to those reported in panel (a).

In panel (c) of Table 3, the effect of the intratemporal labour supply elasticity (σ_L) on the intergenerational distribution is studied. Note that the first column reports the case with exogenous labour supply ($\sigma_L=0$). The results are qualitatively very similar to those for the benchmark case reported in the third column. Even though the welfare effects on different generations depend in absolute terms on the labour supply elasticity, the welfare ranking across generations is not affected. This confirms that the results reported in panel (a) are not very sensitive to the assumed labour supply elasticity.

In panel (d) of Table 3 the effect of the degree of durability of public capital, δ_G, is investigated. The benchmark case ($\delta_G=0.05$) is reported in the third column. The degree of durability of public capital has an important effect on the welfare change for newly-born generations. Indeed, panel (d) suggests that $d\Lambda(0,0)$ is increasing in δ_G. The intuition is as follows.

If δ_G is low, public capital is very durable, and a given public investment shock only slowly leads to an increase in the stock of public capital. As a result, labour productivity and before-tax wages are only slowly affected by the policy shock. Since the path of lump-sum taxes is unaffected, the path of after-tax wages lies below the benchmark path, so that the impact effect on human wealth and welfare is smaller. Indeed, for $\omega_G = 0.01$ and $\delta_G = 0.01$, newly-born generations are worse off as a result of the boost in public investment. The qualitative picture that emerges from panel (d) is, however, similar to that from panel (a).

4.3. Intergenerational redistribution

In the previous sub-section it has been demonstrated that the welfare effect of a supposedly 'efficiency-improving' policy measure is generation-dependent, and may be very uneven across generations. This finding demonstrates the need for a mechanism aimed at equalizing the distribution of welfare over generations. In many of the simulations reported in Table 3, generations born close to the time of the shock lose out. These generations cannot capture the full benefits of public investment, as some of these benefits spill over to old existing generations in the form of capital gains on their financial assets, and to future generations in the form of higher full income. Young existing generations have no or little financial assets and thus do not capture a large capital gain, whilst future generations born soon after the shock do not yet benefit from the higher wages because it takes time for the public and private capital stocks to be increased.

In this section we endow the policy maker with the ability to use bond policy in order to neutralize the intergenerational welfare effects. In doing so the pure 'efficiency effects' of public investment are brought to the fore. We assume that the policy maker is able to choose a path of debt that is parameterized as follows:

$$\tilde{B}(t) = b_0 + b_1 e^{-\xi_1 t} + b_2 e^{-\xi_2 t}, \tag{4.6}$$

with $\xi_j > 0$ ($j=1,2$) and b_i all finite ($i=0,1,2$), so that the bond path is stable and converges in the long run to $\tilde{B}(\infty) = b_0$. By choosing the policy parameters (ξ_j and b_i) appropriately, the policy maker can smooth the intergenerational welfare distribution.

We focus on an egalitarian policy, according to which the policy maker engineers the bond path in such a way that all generations share a common gain π (or loss, if π is negative), i.e. $d\Lambda(v,0) = d\Lambda(t,t) = \pi$ for $v \le 0$ and $t \ge 0$. By thus spreading the benefits equally over all generations, π can be interpreted as the pure efficiency gain of the public investment boost.[9]

In order to get all existing generations to be affected equally, $d\Lambda(v,0) = \pi$, the generation-specific term appearing in (4.1) must be eliminated. This is done by levying a once-off tax, τ_K, on

the owners of the capital stock. It is shown in Heijdra and Meijdam (1997) that the suitable tax equals:

$$\tau_K = \tilde{q}(0) - (\alpha + \beta)\pi, \tag{4.7}$$

where $\tilde{q}(0)$ is given in (3.3) above. To the extent that the capital gain on financial assets exceeds the common (flow) gain to all generations, the policy maker must levy a once-off tax on capital owners to equalize the welfare profile for existing generations. The revenue of this once-off levy gives rise to a discrete adjustment in the level of public debt at impact:

$$\tilde{B}(0) \equiv b_0 + b_1 + b_2 = -\omega_A \tau_K. \tag{4.8}$$

All future generations are affected equally by the policy shock, $d\Lambda(t,t)=\pi$, provided the path of human wealth is smoothed (see (4.3)). This requires that the path for bonds and thus lump-sum taxes is set in such a way that all transitional dynamics is eliminated from the path of full income. It is shown in Heijdra and Meijdam (1997) that this requires the following settings for the policy parameters:[10]

$$\xi_1 = h_I, \quad \xi_2 = \delta_G, \quad b_0 = \omega_A(\alpha + \beta)\pi + \left(\frac{r\eta}{r + \delta_G}\right)\tilde{I}_G,$$

$$b_1 = \left(\frac{r\varepsilon_L \zeta_H}{r + h_I}\right)\tilde{I}_G, \quad b_2 = \left(\frac{r\varepsilon_L \zeta_G}{r + \delta_G}\right)\tilde{I}_G, \tag{4.9}$$

where ζ_0, ζ_H, and ζ_G are constants:

$$\zeta_0 \equiv \eta/\varepsilon_L, \quad \zeta_H \equiv -\frac{\eta\delta_G(1 + \sigma_L)(1 - \varepsilon_L)(r + \delta_G)}{\varepsilon_L[1 + \sigma_L(1 - \varepsilon_L)](r_I + \delta_G)(\delta_G - h_I)}, \quad \zeta_G \equiv -(\zeta_0 + \zeta_H). \tag{4.10}$$

The intuition behind (4.9) is straightforward. As was pointed out above, the path of full income contains dynamic effects due to adjustment in the private capital stock (which occurs at the adjustment speed h_I) and due to the accumulation of public capital (the speed of which is δ_G). In order to smooth full income the path of debt (and lump-sum taxes) must contain both exponential adjustment speeds.[11] The parameters b_1 and b_2 ensure that the exponential terms receive the correct weight.

The common welfare gain implied by (4.7)-(4.10) can be written as follows:

$$(\alpha + \beta)\pi = (1/\omega_X)\left[\frac{\eta\delta_G}{r + \delta_G} - \omega_G\right]\tilde{I}_G, \tag{4.11}$$

where $\omega_X \equiv X/Y$ is the initial full consumption share in output. The sign of the common gain is fully determined by the term in square brackets on the right-hand side. If public capital is relative scarce (ω_G small), this term is positive and all generations can be made better off by increasing the level of public investment. This conclusion is confirmed numerically in Table 3, which reports values for π as the first entry in each cell.

An interesting conclusion that emerges from Table 3 is that the prudent use of debt policy indeed undoes society's bias towards the provision of too little public capital by spreading the costs and benefits equally over all generations. Take, for example, the last column in panel (a) of Table 3. The efficiency of public capital is high, $\eta = 0.4$, and present generations gain sufficiently to vote in favour of further public investment for $\omega_G \leq 0.1$. There is insufficient support to push the public investment share to $\omega_G = 0.15$, however, as only 36.4 percent of the population would end up better off as a result. With an egalitarian policy, however, the common gain to all generations is in fact positive for $\omega_G = 0.15$, i.e. $\pi = 2.178$, suggesting that the level of public investment should be higher than $\omega_G = 0.1$. By internalizing the intergenerational external effect by means of bond policy, the egalitarian policy leads to a more ambitious policy stance regarding public infrastructure.

The optimal egalitarian policy, ω_G^*, fully internalizes the intergenerational externalities. By setting ω_G such that all present and future generations are unaffected by a marginal increase in the level of public investment (i.e. $\pi = 0$), the expression for the optimal public investment share is obtained from equation (4.11):

$$\omega_G^* \equiv \frac{\eta \delta_G}{r + \delta_G}. \tag{4.12}$$

Equation (4.12) is a simple 'modified golden rule' of public investment in a dynamic economy and demonstrates the crucial interaction between the efficiency of public capital (as parameterized by η), the degree of durability of public capital (as regulated by δ_G), and the world interest rate (r). With non-durable public capital ($\delta_G \to \infty$), (4.12) collapses to the golden rule result, i.e. $\omega_G^* = \eta$ (see above). With durable public capital, on the other hand, the golden rule is modified by taking into account the societal costs associated with installing the stock of public capital. These costs are affected by both the degree of durability of public capital and by the rate of interest. Note finally that (4.12) gives rise to sensible comparative static results regarding the optimal public investment share, i.e. $\partial \omega_G^* / \partial \eta > 0$, $\partial \omega_G^* / \partial r < 0$, and $\partial \omega_G^* / \partial \delta_G > 0$.

5. Conclusions

This paper could be usefully extended in a number of directions. First, in the present paper all production factors including public capital enter the production function with a substitution elasticity of unity. This yields convenient simplifications but may be untenable from an empirical point of view. A richer pattern of substitution possibilities would potentially reverse some of our conclusions regarding the macroeconomic and distributional effects of public investment. Second, in this paper we have restricted attention to the case where lump-sum taxes and bonds constitute the only governmental financing instruments. Though this is a useful simplification and allows us to focus on efficiency and distribution in the cleanest possible setting, it is too restrictive as very few non-distorting taxes appear to be available in actual economies. In this context, a fruitful extension to our analysis would make use of labour-income taxation as a means of financing the public infrastructural investments. We hope to return to these issues in future research.

Appendix

The Jacobian matrix on the right-hand side of (2.15) is denoted by Δ_I with typical element δ_{ij}^I. Its characteristic polynomial is given by:

$$p_I(x) = x(r-x) + \frac{r^2 \varepsilon_L \omega_I (\omega_A + \omega_I)}{\sigma_A \omega_A^2 \left[1 + \sigma_L (1 - \varepsilon_L) \right]},$$ (A.1)

which has distinct roots $-h_I < 0$ and $r_I = r + h_I > 0$. By noting $p_I(-h_I) = 0$ and implicitly differentiating (A.1) with respect to h_I and σ_A we obtain:

$$-(r + 2h_I) \frac{\partial h_I}{\partial \sigma_A} = \frac{p_I(0)}{\sigma_A} > 0,$$ (A.2)

where $p_I(0) \equiv -|\Delta_I| > 0$. Hence, the adjustment speed of the investment system is negatively related to σ_A as stated in Proposition 1.

The Jacobian matrix on the right-hand side of (2.16) is denoted by Δ_S with typical element δ_{ij}^S and its characteristic polynomial is:

$$p_S(x) = \left[r - (\alpha + \beta) - x \right] (r + \beta - x),$$ (A.3)

which has distinct roots $-h_S = r - (\alpha + \beta)$ and $r_S = r + \beta$. Since $r > \alpha$ follows from our earlier discussion surrounding (2.8), stability for the saving system holds if $r < \alpha + \beta$ or, equivalently, if $\omega_X > \omega_A$. The stability condition thus holds provided the share of government spending in national income is not too high. This follows from the fact that $(\omega_X - \omega_A)$ equals $\varepsilon_L / (1 + \sigma_L) - \omega_G$ which is positive provided ω_G is not too large (see the information on shares at the bottom of Table 2).

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Footnotes

1.   For a comprehensive review of this literature see Gramlich (1994).

2.   See for example Aschauer (1988), Aschauer and Greenwood (1985), Barro (1981, 1990), Turnovsky and Fisher (1995) and Turnovsky (1996).

3.   Exceptions are formed by Glomm and Ravikumar (1994) and Van de Ven (1996).

4.   This not only simplifies the analysis substantially but also appears to be more empirically relevant. Indeed, to the present day, the empirical literature has not been able to demonstrate a strong intertemporal substitution effect in labour supply. See Card (1994) for an assessment.

5.   This term is defined as:

$$T(h_I, \delta_G, t) \equiv \frac{e^{-h_I t} - e^{-\delta_G t}}{\delta_G - h_I},$$

which satisfies $T > 0$ for $t \in (0, \infty)$, $T = 0$ for $t = 0$ and as $t \to \infty$, $dT/dt > (=, <) 0$ for $t < (=, >) \hat{t} \equiv \log(h_I/\delta_G)/(h_I - \delta_G)$, $dT/dt = 0$ as $t \to \infty$, $dT/dt = 1$ for $t = 0$, and $d^2T/dt^2 = 0$ for $t = 2\hat{t}$. If h_I happens to equal δ_G, the term becomes:

$$T(h_I, h_I, t) \equiv t e^{-h_I t},$$

which has the same properties.

6.   Specifically, we use the result that:

$$e^{-h_I t} - \frac{dT(h_I, \delta_G, t)}{dt} = \delta_G T(h_I, \delta_G, t) \geq 0.$$

This proves that the term in square brackets on the right-hand side of (3.10) is non-negative.

7.   The same results hold if β/δ_G is varied instead of β/h_I. A relatively high value for δ_G implies that full income rises rapidly towards its steady-state value.

8.   Specifically, $\omega_I = 1 - \omega_C - \omega_G$ and $\omega_A = \omega_C + \omega_G - \varepsilon_L$.

9.   This egalitarian policy approach is not unlike the one adopted by Auerbach and Kotlikoff (1987, p. 56). They use the construct of a Lump Sum Redistribution Agency to compute efficiency gains in a numerical overlapping generations model.

10.   In deriving (4.9)-(4.10), we have also incorporated the restrictions implied by (4.7)-(4.8).

11.   In Heijdra and Meijdam (1997) we demonstrate the case for which the two adjustment speeds coincide ($\delta_G = h_I$).

Table 1: Short-run version of the model

$$\dot{A}(t) = (r - \alpha - \beta)A(t) - (\alpha + \beta)H(t) + Y_F(t) \tag{T4.1}$$

$$\dot{H}(t) = (r + \beta)H(t) - Y_F(t) \tag{T4.2}$$

$$\dot{K}(t) = \left[\Phi\left(\frac{I(t)}{K(t)}\right) - \delta\right]K(t) \tag{T4.3}$$

$$\frac{\dot{q}(t)}{q(t)} = r + \delta - \Phi\left(\frac{I(t)}{K(t)}\right) + \frac{I(t)}{q(t)K(t)} - \frac{F_K\left[L(t),K(t),K_G(t)\right]}{q(t)} \tag{T4.4}$$

$$F_L\left[L(t),K(t),K_G(t)\right] = W(t) \tag{T4.5}$$

$$L(t) = W(t)^{\sigma_L} \tag{T4.6}$$

$$q(t)\Phi'\left(\frac{I(t)}{K(t)}\right) = 1, \tag{T4.7}$$

$$Y(t) = F\left[L(t),K(t),K_G(t)\right] = L(t)^{\varepsilon_L}K(t)^{1-\varepsilon_L}K_G(t)^{\eta} \tag{T4.8}$$

$$Y_F(t) = (1+\sigma_L)^{-1}W(t)^{(1+\sigma_L)} - T(t) \tag{T4.9}$$

$$C(t) = X(t) + \sigma_L\left[Y_F(t) + T(t)\right], \quad X(t) = (\alpha + \beta)\left[A(t)+H(t)\right] \tag{T4.10}$$

$$B(t) = \int_t^{\infty}\left[T(\tau) - I_G(\tau)\right]e^{r(t-\tau)}d\tau \tag{T4.11}$$

$$A(t) = q(t)K(t) + B(t) + F(t) \tag{T4.12}$$

$$\dot{K}_G(t) = I_G(t) - \delta_G K_G(t). \tag{T4.13}$$

Table 2: Log-linearized version of the model

$$\dot{\tilde{A}}(t) = (r - \alpha - \beta)\tilde{A}(t) - (\alpha + \beta)\tilde{H}(t) + r\tilde{Y}_F(t) \qquad \text{(T2.1)}$$

$$\dot{\tilde{H}}(t) = (r + \beta)\tilde{H}(t) - r\tilde{Y}_F(t) \qquad \text{(T2.2)}$$

$$\dot{\tilde{K}}(t) = (r\omega_I/\omega_A)\left[\tilde{I}(t) - \tilde{K}(t)\right] \qquad \text{(T2.3)}$$

$$\dot{\tilde{q}}(t) = r\tilde{q}(t) - \left[r(\omega_A + \omega_I)/\omega_A\right]\left[\tilde{Y}(t) - \tilde{K}(t)\right] \qquad \text{(T2.4)}$$

$$\tilde{Y}(t) - \tilde{L}(t) = \tilde{W}(t) \qquad \text{(T2.5)}$$

$$\tilde{L}(t) = \sigma_L \tilde{W}(t) \qquad \text{(T2.6)}$$

$$\tilde{q}(t) = \sigma_A\left[\tilde{I}(t) - \tilde{K}(t)\right] \qquad \text{(T2.7)}$$

$$\tilde{Y}(t) = \varepsilon_L \tilde{L}(t) + (1 - \varepsilon_L)\tilde{K}(t) + \eta \tilde{K}_G(t) \qquad \text{(T2.8)}$$

$$\tilde{Y}_F(t) = \varepsilon_L \tilde{W}(t) - \tilde{T}(t) \qquad \text{(T2.9)}$$

$$\omega_C \tilde{C}(t) = \omega_X \tilde{X}(t) + \sigma_L\left[\tilde{Y}_F(t) + \tilde{T}(t)\right], \quad \omega_X \tilde{X}(t) = \left((\alpha + \beta)/r\right)\left[\tilde{A}(t) + \tilde{H}(t)\right] \qquad \text{(T2.10)}$$

$$\tilde{B}(0) = r\left[\mathcal{L}\{\tilde{T}, r\} - \omega_G \mathcal{L}\{\tilde{I}_G, r\}\right] \qquad \text{(T2.11)}$$

$$\tilde{A}(t) = \omega_A\left[\tilde{K}(t) + \tilde{q}(t)\right] + \tilde{B}(t) + \tilde{F}(t) \qquad \text{(T2.12)}$$

$$\dot{\tilde{K}}_G(t) = \delta_G\left[\tilde{I}_G(t) - \tilde{K}_G(t)\right] \qquad \text{(T2.13)}$$

Shares:

ω_I	I/Y	Share of firm investment in national income.
ω_X	X/Y	Share of full consumption in national income.
ω_A	$rA/Y = rqK/Y$	Share of asset income in national income.
ω_C	C/Y	Share of consumption in national income.
ω_G	I_G/Y	Share of public investment in national income.

Relationships between shares and parameters:

$$\omega_A = \omega_C + \omega_G - \varepsilon_L$$
$$\omega_C = \omega_X + \sigma_L \varepsilon_L / (1 + \sigma_L)$$
$$1 = \omega_C + \omega_G + \omega_I$$
$$r(r - \alpha)\omega_X = \beta(\alpha + \beta)\omega_A$$
$$\omega_A + \varepsilon_L / (1 + \sigma_L) = \omega_G + \omega_X$$

Notes: (a)   We have used the normalization $B = F = 0$ initially.

(b) $\sigma_A \equiv -(I/K)(\Phi''/\Phi') \geq 0$, represents the degree of concavity of the installation cost function. A low value for σ_A implies that physical capital is highly mobile, with the limiting case of $\sigma_A = 0$ (no adjustment costs) representing perfect mobility of capital.

Table 3. The efficiency and intergenerational distribution effects of public investment

Panel (a): The effect of η

		η=0	η=0.1	η=0.2	η=0.3	η=0.4
$\omega_G=0.01$	π	-0.141	0.564	1.269	1.974	2.678
	$d\Lambda(-\infty,0)$	0.000	0.023	0.046	0.068	0.091
	$d\Lambda(0,0)$	-0.145	0.120	0.385	0.650	0.915
	$d\Lambda(\infty,\infty)$	-0.145	1.306	2.757	4.208	5.659
	$\sigma(\%)$	0.0	100.0	100.0	100.0	100.0
$\omega_G=0.05$	π	-1.429	0.000	1.429	2.857	4.286
	$d\Lambda(-\infty,0)$	0.000	0.206	0.413	0.619	0.826
	$d\Lambda(0,0)$	-1.667	-0.726	0.215	1.156	2.096
	$d\Lambda(\infty,\infty)$	-1.667	1.667	5.000	8.333	11.667
	$\sigma(\%)$	0.0	0.5	100.0	100.0	100.0
$\omega_G=0.10$	π	-3.758	-1.879	0.000	1.879	3.758
	$d\Lambda(-\infty,0)$	0.000	0.480	0.960	1.440	1.921
	$d\Lambda(0,0)$	-5.261	-3.654	-2.048	-0.441	1.166
	$d\Lambda(\infty,\infty)$	-5.261	0.000	5.261	10.522	15.783
	$\sigma(\%)$	0.0	0.7	7.2	54.1	100.0
$\omega_G=0.15$	π	-6.535	-4.357	-2.178	0.000	2.178
	$d\Lambda(-\infty,0)$	0.000	0.750	1.500	2.250	3.300
	$d\Lambda(0,0)$	-11.436	-9.152	-6.867	-4.583	-2.299
	$d\Lambda(\infty,\infty)$	-11.436	-3.812	3.812	11.436	19.061
	$\sigma(\%)$	0.0	1.0	4.7	13.9	36.4
$\omega_G=0.20$	π	-9.613	-7.210	-4.807	-2.403	0.000
	$d\Lambda(-\infty,0)$	0.000	1.005	2.009	3.014	4.018
	$d\Lambda(0,0)$	-22.431	-19.286	-16.140	-12.995	-9.849
	$d\Lambda(\infty,\infty)$	-22.431	-11.216	0.000	11.216	22.431
	$\sigma(\%)$	0.0	1.2	3.9	8.6	16.1
$\omega_G=0.25$	π	-12.914	-10.331	-7.748	-5.166	-2.583
	$d\Lambda(-\infty,0)$	0.000	1.243	2.487	3.730	4.973
	$d\Lambda(0,0)$	-45.199	-40.621	-36.043	-31.465	-26.888
	$d\Lambda(\infty,\infty)$	-45.199	-27.119	-9.040	9.040	27.119
	$\sigma(\%)$	0.0	1.2	3.1	5.8	9.5

Note: Parameter values are $\alpha=0.03$, $r=0.05$, $\sigma_L=1.0$, $\delta=0.1$, $\delta_G=0.05$, $\varepsilon_L=0.70$, $\omega_C=0.7$, and $\sigma_A=0.5$. $\sigma(\%)$ is the percentage of the population (alive at the time of the shock) that does not lose as a result of a marginal increase in the level of public investment. The efficiency gain under egalitarian redistributive bond policy is given by π.

		σ_A=0.1	σ_A=0.3	σ_A=0.5	σ_A=0.7	σ_A=0.9
ω_G=0.01	π	1.269	1.269	1.269	1.269	1.269
	$d\Lambda(-\infty,0)$	0.009	0.028	0.046	0.063	0.080
	$d\Lambda(0,0)$	0.387	0.386	0.385	0.384	0.384
	$d\Lambda(\infty,\infty)$	2.757	2.757	2.757	2.757	2.757
	$\sigma(\%)$	100.0	100.0	100.0	100.0	100.0
ω_G=0.05	π	1.429	1.429	1.429	1.429	1.429
	$d\Lambda(-\infty,0)$	0.098	0.265	0.413	0.547	0.672
	$d\Lambda(0,0)$	0.273	0.241	0.215	0.192	0.172
	$d\Lambda(\infty,\infty)$	5.000	5.000	5.000	5.000	5.000
	$\sigma(\%)$	100.0	100.0	100.0	100.0	100.0
ω_G=0.10	π	0.000	0.000	0.000	0.000	0.000
	$d\Lambda(-\infty,0)$	0.271	0.660	0.960	1.211	1.427
	$d\Lambda(0,0)$	-1.783	-1.938	-2.048	-2.134	-2.205
	$d\Lambda(\infty,\infty)$	5.261	5.261	5.261	5.261	5.261
	$\sigma(\%)$	1.0	4.3	7.2	9.6	11.6
ω_G=0.15	π	-2.178	-2.178	-2.178	-2.178	-2.178
	$d\Lambda(-\infty,0)$	0.510	1.103	1.500	1.800	2.039
	$d\Lambda(0,0)$	-6.220	-6.623	-6.867	-7.041	-7.175
	$d\Lambda(\infty,\infty)$	3.812	3.812	3.812	3.812	3.812
	$\sigma(\%)$	1.0	3.1	4.7	5.9	6.8
ω_G=0.20	π	-4.807	-4.807	-4.807	-4.807	-4.807
	$d\Lambda(-\infty,0)$	0.846	1.593	2.009	2.287	2.488
	$d\Lambda(0,0)$	-14.893	-15.722	-16.140	-16.406	-16.593
	$d\Lambda(\infty,\infty)$	0.000	0.000	0.000	0.000	0.000
	$\sigma(\%)$	1.4	3.0	3.9	4.5	5.0
ω_G=0.25	π	-7.748	-7.748	-7.748	-7.748	-7.748
	$d\Lambda(-\infty,0)$	1.398	2.167	2.487	2.667	2.783
	$d\Lambda(0,0)$	-34.025	-35.482	-36.043	-36.349	-36.543
	$d\Lambda(\infty,\infty)$	-9.040	-9.040	-9.040	-9.040	-9.040
	$\sigma(\%)$	1.7	2.7	3.1	3.4	3.5

Note: Parameter values are α=0.03, r=0.05, σ_L=1.0, δ=0.1, δ_G=0.05, ε_L=0.70, ω_C=0.7, and η=0.2. $\sigma(\%)$ is the percentage of the population (alive at the time of the shock) that does not lose as a result of a marginal increase in the level of public investment. The efficiency gain under egalitarian redistributive bond policy is given by π.

Panel (c): The effect of σ_L

		$\sigma_L=0$	$\sigma_L=0.5$	$\sigma_L=1$	$\sigma_L=2$	$\sigma_L=5$
$\omega_G=0.01$	π	0.459	0.833	1.269	2.289	6.219
	$d\Lambda(-\infty,0)$	0.017	0.030	0.046	0.082	0.218
	$d\Lambda(0,0)$	0.096	0.218	0.385	0.843	3.111
	$d\Lambda(\infty,\infty)$	0.983	1.797	2.757	5.049	14.359
	$\sigma(\%)$	100.0	100.0	100.0	100.0	100.0
$\omega_G=0.05$	π	0.532	0.950	1.429	2.523	6.535
	$d\Lambda(-\infty,0)$	0.160	0.280	0.413	0.706	1.685
	$d\Lambda(0,0)$	-0.026	0.061	0.215	0.716	4.051
	$d\Lambda(\infty,\infty)$	1.719	3.192	5.000	9.632	34.309
	$\sigma(\%)$	45.2	100.0	100.0	100.0	100.0
$\omega_G=0.10$	π	0.000	0.000	0.000	0.000	0.000
	$d\Lambda(-\infty,0)$	0.393	0.668	0.960	1.568	3.374
	$d\Lambda(0,0)$	-0.752	-1.323	-2.048	-4.189	-41.104
	$d\Lambda(\infty,\infty)$	1.667	3.211	5.261	11.436	114.712
	$\sigma(\%)$	2.4	5.0	7.2	9.9	5.6
$\omega_G=0.15$	π	-0.841	-1.473	-2.178	-3.746	
	$d\Lambda(-\infty,0)$	0.646	1.068	1.500	2.351	
	$d\Lambda(0,0)$	-1.989	-3.940	-6.867	-18.975	unstable
	$d\Lambda(\infty,\infty)$	1.070	2.170	3.812	10.490	
	$\sigma(\%)$	2.1	3.7	4.7	5.0	
$\omega_G=0.20$	π	-1.879	-3.268	-4.807	-8.194	
	$d\Lambda(-\infty,0)$	0.911	1.466	2.009	3.021	
	$d\Lambda(0,0)$	-3.767	-8.176	-16.140	-84.209	unstable
	$d\Lambda(\infty,\infty)$	0.000	0.000	0.000	0.000	
	$\sigma(\%)$	2.3	3.5	3.9	2.4	
$\omega_G=0.25$	π	-3.061	-5.292	-7.748		
	$d\Lambda(-\infty,0)$	1.196	1.864	2.487		
	$d\Lambda(0,0)$	-6.192	-14.960	-36.043	unstable	unstable
	$d\Lambda(\infty,\infty)$	-1.587	-3.800	-9.040		
	$\sigma(\%)$	2.6	3.4	3.1		

Note: Parameter values are $\alpha=0.03$, $r=0.05$, $\delta=0.1$, $\delta_G=0.05$, $\varepsilon_L=0.70$, $\omega_C=0.7$, $\sigma_A=0.5$, and $\eta=0.2$. $\sigma(\%)$ is the percentage of the population (alive at the time of the shock) that does not lose as a result of a marginal increase in the level of public investment. The efficiency gain under egalitarian redistributive bond policy is given by π. For some combinations of ω_G and σ_L the calibration results in a negative value for ω_H which renders the saving system unstable.

Panel (d): The effect of δ_G

		$\delta_G=0.01$	$\delta_G=0.03$	$\delta_G=0.05$	$\delta_G=0.1$	$\delta_G=0.2$
$\omega_G=0.01$	π	0.329	0.916	1.269	1.739	2.114
	$d\Lambda(-\infty,0)$	0.009	0.028	0.046	0.088	0.167
	$d\Lambda(0,0)$	-0.021	0.198	0.385	0.749	1.217
	$d\Lambda(\infty,\infty)$	2.757	2.757	2.757	2.757	2.757
	$\sigma(\%)$	0.0	100.0	100.0	100.0	100.0
$\omega_G=0.05$	π	-0.476	0.714	1.429	2.381	3.143
	$d\Lambda(-\infty,0)$	0.093	0.262	0.413	0.725	1.166
	$d\Lambda(0,0)$	-1.168	-0.379	0.215	1.202	2.203
	$d\Lambda(\infty,\infty)$	5.000	5.000	5.000	5.000	5.000
	$\sigma(\%)$	0.0	4.4	100.0	100.0	100.0
$\omega_G=0.10$	π	-2.505	-0.939	0.000	1.253	2.255
	$d\Lambda(-\infty,0)$	0.239	0.637	0.960	1.543	2.216
	$d\Lambda(0,0)$	-4.349	-2.996	-2.048	-0.593	0.740
	$d\Lambda(\infty,\infty)$	5.261	5.261	5.261	5.261	5.261
	$\sigma(\%)$	0.1	1.8	7.2	47.3	100.0
$\omega_G=0.15$	π	-5.083	-3.268	-2.178	-0.726	0.436
	$d\Lambda(-\infty,0)$	0.407	1.036	1.500	2.260	3.026
	$d\Lambda(0,0)$	-10.084	-8.162	-6.867	-4.960	-3.281
	$d\Lambda(\infty,\infty)$	3.812	3.812	3.812	3.812	3.812
	$\sigma(\%)$	0.3	2.1	4.7	12.7	27.1
$\omega_G=0.20$	π	-8.011	-6.008	-4.807	-3.204	-1.923
	$d\Lambda(-\infty,0)$	0.588	1.432	2.009	2.880	3.676
	$d\Lambda(0,0)$	-20.526	-17.883	-16.140	-13.612	-11.406
	$d\Lambda(\infty,\infty)$	0.000	0.000	0.000	0.000	0.000
	$\sigma(\%)$	0.5	2.2	3.9	7.7	12.5
$\omega_G=0.25$	π	-11.192	-9.040	-7.748	-6.027	-4.649
	$d\Lambda(-\infty,0)$	0.779	1.821	2.487	3.425	4.222
	$d\Lambda(0,0)$	-42.416	-38.574	-36.043	-32.360	-29.118
	$d\Lambda(\infty,\infty)$	-9.040	-9.040	-9.040	-9.040	-9.040
	$\sigma(\%)$	0.6	2.0	3.1	5.1	7.3

Note: Parameter values are $\alpha=0.03$, $r=0.05$, $\delta=0.1$, $\sigma_L=1$, $\varepsilon_L=0.70$, $\omega_C=0.7$, $\sigma_A=0.5$, and $\eta=0.2$. $\sigma(\%)$ is the percentage of the population (alive at the time of the shock) that does not lose as a result of a marginal increase in the level of public investment. The efficiency gain under egalitarian redistributive bond policy is given by π.