Sequential Common Agency *

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Abstract

In a common agency game a set of principals promises monetary transfers to an agent which depend on the action he will take. The agent then chooses the action, and is paid the corresponding transfers. Principals announce their transfers simultaneously. This game has many equilibria; Bernheim and Whinston ([1]) prove that the action chosen in the coalition-proof equilibrium is efficient. The coalition-proof equilibria have an alternative characterization as truthful equilibria. The other equilibria may be inefficient.

Here we study the sequential formulation of the common agency game: principals announce their transfers sequentially. We prove that the set of equilibria is different in many important ways. The outcome is efficient in all the equilibria. The truthful equilibria still exist, but are no longer coalition-proof. Focal equilibria are now a different type of equilibria, that we call thrifty. In thrifty equilibria of the sequential games, principals are better off (and the agent worse off) than in the truthful equilibria of the simultaneous common agency.

These results suggest that the sequential game is more desirable institution, because it does not have inefficient equilibrium outcomes; but it is less likely to emerge when agents have the power to design the institution.

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1 Introduction

In a common agency game there are several principals and one agent. The agent must choose one action. Each principal has preferences over the set of possible actions and tries to in°uence the agent by o®ering monetary contributions conditional on the action chosen. A principal can make o®ers on more than one action and she will only have to pay for the action that the agent chooses. The agent has preferences over the set of possible actions, but also derives utility from the total amount of monetary contributions he receives. After observing the contributions o®ered by the principals, the agent chooses the action that maximizes his expected utility.

The model of common agency has many applications (see Bernheim and Whinston ([1] for a discussion). It is of particular interest in the study of pressure group politics (See for instance Grossman and Helpman ([6], [7]). The agent is a politician who must make a decision over a particular issue. The principals are the various lobbies who are a®ected by the decision. Each lobby represents a subset of voters with preferences over the issue at stake. Lobbies may o®er monetary contributions to the politician conditional on the decision made. The contributions are bene¯cial to the politician either because he keeps them for personal use or - as is the case in the US and in other democracies - because he can use them to ¯nance his campaign expenditures.

Bernheim and Whinston [1] analyze the common agency game under the assumption that all lobbies choose their contributions simultaneously. In general this game has multiple equilibria. However, Bernheim and Whinston prove the striking result that in coalition-proof equilibria the agent chooses an e±cient action, that is an action which maximizes a weighted sum of the utilities of principals and agent (gross of monetary transfers). Moreover, principals can restrict without loss their attention to a particular contribution schedule, which Bernheim and Whinston call truthful. If all principals use truthful contribution schedules, the resulting equilibrium is coalition-proof.

Bernheim and Whinston's results are valid under the assumption that principals make their o®ers simultaneously and secretly. However, this assumption may not correspond to the practice of lobbying in the USA. According to a survey conducted by Schlozman and Tierney ([12]) on a sample of interest groups active in Washington, the most widely used lobbying technique is testifying at congressional hearings. Often legislators ask interest group representatives to provide their views on a particular bill under consideration. This is a highly structured process which is both public and sequential. Interest groups provide technical expertise, but also what Schlozman and Tierney call \political intelligence," that is, information about how various alternatives will a®ect the members of that particular interest group. Congressional hearings give lobbyists the opportunity to make implicit promises of contributions. Suppose an interest group has a reputation for rewarding sympathetic politicians in a generous way. Then, by stating its preferences over the proposed alternatives, the group signals its willingness to reward the politician for each of the alternatives. The fact that the implicit promise is made in a public and focal way adds credibility. If the same promise were made in secret, the group could renege without hurting its reputation.

With this motivation in mind, we turn our attention to the common agency game played in a sequential manner. In a pre-speci¯ed order, principals make o®ers to the agent. Each o®er is public, so that the principals who have not yet made their o®er can condition their strategy on the o®ers already made. After every principal has signaled her intentions, the agent makes his choice.

How do the results in the sequential game di®er from those in the simultaneous game? We ¯nd that the e±ciency property of common agency is strengthened in the sequential game: for any given ordering of principals, in all subgame-perfect equilibria the agent chooses an e±cient action. Thus, in the sequential version the e±ciency result does not depend on the use of equilibrium re¯nements. Since the choice of a re¯nement is always controversial, the sequential game seems preferable from the normative point of view.

Next, we characterize the equilibrium transfers and we ¯nd that SPE can in general be supported by several transfer pro¯les. There always exists a SPE transfer pro¯le which corresponds to one of the truthful equilibria of the simultaneous game. There also exists another SPE transfer pro¯le, which we call thrifty, in which the total amount of transfers on the equilibrium action is lower than in the truthful case. We show that if there exists a coalition-proof equilibrium, it must entail thrifty transfers. Hence, the connection between truthfulness and coalition-proofness does not carry over to the sequential case.

Finally we note that the agent may get more than his reservation value, even in the case in which he is indi®erent among the actions. It is su±cient that principals have a divergence of interests, that is they do not all prefer the same action to all the other actions. In this case the attempt of a principal to implement his preferred action produces a positive transfer at equilibrium to the agent.

It may be useful to compare the equilibria of the common agency game, as we move from simultaneous to sequential setup, with the equilibria in the Cournot duopoly, when we move from the simultaneous move to the Stackelberg. It is well known that (i) in the Stackelberg oligopoly the ¯rst mover has a higher payo®, but (ii) the sum of payo®s of the two ¯rms may decrease, (iii) the welfare of the consumer increases, and ¯nally (iv) the total welfare increases.

The fact that the competition is in quantities rather than in prices is very important: this issue, and the comparison of Bertrand and Cournot models, are thoroughly discussed by van Damme and Hurkens in a sequence of papers ([8], [9], [10]). Let us take these four points in detail.

The last point is similar in common agency and oligopoly: although it is not true in general that the move to sequential games improves e±ciency. As one moves from a simultaneous-move game to its sequential version, the set of equilibria can be radically modi¯ed. However, it is not possible to say in general how the equilibrium outcome and payo®s change. For instance, consider the simple game with two players:

	L	C	R
T	3; 3	0; 0	0; 0
M	0; 0	4; 1	0; 0
B	0; 0	0; 0	1; 4

(1.1)

The simultaneous version of the game has three pure-strategy Nash equilibria. The outcome which maximizes the players' joint payo® is MC and it corresponds to one of the three equilibria. In the sequential version of the game, if the row player goes ¯rst, the equilibrium is TL, while if the column player goes ¯rst, it is BR. Thus, in this particular game, passing from simultaneous to sequential: (i) bene¯ts the ¯rst-mover; (ii) hurts the second-mover; (iii) reduces the total payo®. As we shall see, the conclusions of this simple game tend to be reversed in the common agency game.

But, roughly, the opposite holds on all the other three counts in common agency. The ¯rst principal does not necessarily do better (for instance, if the agent has no preferences among actions, then the last mover pays zero in the thrifty equilibrium). The sum of the net payo®s of the principals increases as we pass from the truthful equilibrium in the simultaneous common agency to the thrifty in the sequential. And the agent does worse in the simultaneous common agency.

The plan of the work is as follows. The next two subsections use a simple example to provide intuition on the di®erences between simultaneous common agency and sequential common agency. Section 2 states the problem of sequential common agency. Section 3 characterizes the set of subgame-perfect equilibria. Section 4 de¯nes and studies the thrifty equilibrium. Section 5 focuses on the truthful equilibrium of the sequential game and on its relation with the truthful equilibria of the simultaneous game. Section 6 applies two game-theoretical re¯nements, namely, trembling-hand perfection and coalition-proofness. Section 7 concludes.

A simple example

Bernheim and Whinston (1986) introduce their discussion of the common agency game with a simple problem, which we also take as our starting point. In the game there are two principals and one agent. The agent can choose one out of four di®erent actions; the payo®s of the principals are monetary, and are determined by the action chosen by the agent. The payo® to the agent only depends on the transfers of the principals, and he is otherwise indi®erent among the actions. The two principals can promise non-negative monetary transfers to the agent, one for each action. The payment of these transfers will be made only for the action that is chosen by the agent.

The payo® of the principal i when action s is chosen is written as G^i_s, and are as follows:

$$G^1 = (8; 0; 6; 5); \; G^2 = (0; 7; 6; 5): \qquad (1.2)$$

The two principals move simultaneously, announce their transfers, then the agent observes the transfers and chooses the action.

This game has a large set of equilibria. To organize their description we can classify them in three types.

In the ¯rst type, each agent bids on his most favorite action, that is the ¯rst and the second action respectively for the ¯rst and second principal. The second principal is willing to bid at most 7 for the second action, so the ¯rst principal gets his favorite action by bidding the same amount. If we denote the transfer of principal i for action s by t^i_s, then the equilibrium is:

$$t^1 = (7; 0; 0; 0); \; t^2 = (0; 7; 0; 0);$$

and the agent chooses the ¯rst action if indi®erent.

In the second type of equilibrium, the principals try to coordinate on a better action, but choose the "wrong" action, namely the fourth. One of these equilibria is, for example:

$$t^1 = (6; 0; 0; 3); \; t^2 = (0; 6; 0; 3);$$

Here the agent chooses the ¯rst action if indi®erent. Note that neither of the two principals can unilaterally move to the better action (the third), because of the 6 o®ered by the other principal on his favorite action.

Finally, in the third type of equilibria, the action chosen at equilibrium is the third action, which gives the highest total payo®. A particularly interesting case of equilibrium is the following. The transfers are:

$$t^1 = (3; 0; 1; 0); \; t^2 = (0; 3; 2; 1):$$

The agent chooses the third action if indi®erent. In this equilibrium the di®erence between the payo® from an action and the transfer for that action is the same for each principal, when the transfer is positive. When the transfer is zero, the di®erence must be smaller. Bernheim and Whinston call any equilibrium with this property truthful. One of their main, striking results is that this equilibrium is also robust to coalitional, self-enforcing, deviations of the principals: there is no other equilibrium that gives a higher payo® to both.

The same, e±cient, outcome is achieved in this other equilibrium, e±cient but expensive, where principals coordinate on the good action, but still bid one's favorite action:

$$t^1 = (7; 0; 5; 0); \; t^2 = (0; 7; 2; 0):$$

Here each of the two principals is forcing the other to counter with a high transfer for the third action the existing bid on the other's favorite action.

These are not the only equilibria. Bernheim and Whinston describe six of them in their discussion. Of these equilibria, the ones giving the fourth action as outcome seem arti¯cial. The two principals fail to coordinate on the third action, which is similar in many respects, and gives a higher payo® to both. It is harder to say that also the equilibrium yielding the ¯rst action is arti¯cial.

A sequential game

Consider the common game discussed in the previous section (with the same players, and same payo®s) but assume that principals move sequentially: the principal 1 moves ¯rst, and announces his transfers; then the principal 2 does the same, and ¯nally the agent chooses his actions. We prove later (see theorem 3.4) that the only equilibrium outcome action is the e±cient action, in this example the third one.

If we use this information, the equilibrium transfers are easy to ¯nd. We look for an equilibrium where the ¯rst mover is trying to implement the third action. The result we have mentioned insures that no equilibrium is lost when we add this constraint.

Now we can use backward induction, looking ahead and considering the best response of the second mover. For any vector of transfers of the ¯rst principal, the second may induce the agent to choose any action s he wants, by paying the di®erence between the maximum amount o®ered by the other principal on any action, and the amount o®ered for s. So he has to make the third, e±cient, action more appealing than the second action. The di®erence in payo® between these two actions for the second principal is 1, so the ¯rst principal must o®er a transfer on the third action at least equal to the transfer on the second, plus 1. The least expensive way for him to do this is to set $t^1_3 = 1; t^1_2 = 0$ on these two actions.

He has now to decide the transfers on the other actions. Here, any transfer with 0 t^1_1 6 and 0 t^1_4 2 will still induce the second principal to implement the e±cient action. So these transfers are just di®erent ways of supporting the same outcome, and they give the same payo® to the ¯rst principal. They do change, however, the amount that principals pay at equilibrium to the agent, as it was happening in the e±cient but expensive equilibrium. For instance, by o®ering 6 on the ¯rst action (his favorite) the ¯rst principal is forcing the second to counteract with a transfer of 5 on the third action.

In particular the equilibrium with the transfer $t^1 = (0; 0; 1; 0)$ is robust to coalitional, self-enforcing deviations. As we discuss later in more detail, (see section (6.2) in this game all equilibria are Pareto-undominated if we say that a payo® vector Pareto-dominates another one when all the players are strictly better o®. But if we simply require it to give a higher payo® to both, and strictly larger to at least one, then this is the only equilibrium which is robust to coalitional deviations. Since this equilibrium gives the highest aggregate payo® to the principals, and the gross payo® in the same in all equilibria, this is also the equilibrium that gives the minimum payo® to the agent.

We can now spend a few words on the intuition for the result that the e±cient action is the only chosen action.

Consider for simplicity the case of two principals. The general case is slightly more complicated, but is based on the iteration of the argument for this simple case. It is easy to see that the problem the principal who moves ¯rst is equivalent to the following problem. Choose the action that maximizes his payo®, net of the transfer on that action, and subject to the constraint that this action is in fact going to be chosen in the subgame that begins after this transfer. This is similar to a reduction which is typical in agency problems: the problem of the principal can be reformulated as the problem of choosing the compensation as well as the action of the agent, provided the choice is the best choice of the agent given that compensation.

In the new, equivalent, problem the principal is choosing the action with the highest payoff to him, minus the minimum cost to implement that action. How does the payoff of the other principal enter into this cost? The more the second principal likes this action, the less the first principal has to pay to induce him to implement it. So the payoff of the second principal for the chosen action reduces the cost of the first principal, and therefore increases his net payoff. So the first principal directly takes into account the payoff of the second principal in his choice of action. Hence the chosen action is efficient.

In other words, the sequential setup automatically forces the players to internalize the externality on the others. A reader who is familiar with the Groves-Clark mechanisms will recognize that this is the very effects that those mechnisms intend to produce. This effect is internalized only because there is an agent which is acting as intermediary. In general, the effect of the conflict among principals might dominate, as it does in the simple game (1.1) where the first mover advantage is there, and in fact dominates, and forces an inefficient action.

2 The game

There is one agent and a set $M = \{1, \ldots, m\}$ of principals. The agent has a finite action set $S = \{s_1, \ldots, s_n\}$: Principals can offer monetary transfers to the agent conditional on the action he eventually chooses. Their strategy space is the product R^n_+ of the non-negative reals. For each principal, $t^j \in R^n_+$ denotes the vector of transfers.

The game has $m + 1$-stages. In stage 1 principal m announces his transfers publicly, in stage 2 principal $m - 1$ makes a similar announcement, and so on. So we may interpret the index j of the principal as "j principals from the last." In the last stage the agent chooses the action.

Note that we have excluded, by the definition of the strategy space, the possibility that a principal can make his transfers conditional on the tranfers of the other principals. This choice has several reasons. One is, of course, simplicity. The second is that in many applications (like the political influence game) this seems an institution too complex and fragile, and is hardly observed.

The payoff of the agent depends on the action chosen, according to a vector $G^0 \in R^n$, and on the amount of money received from the principals, and is additively separable in the two components. So the utility from choosing the action s when the transfers of the principals are $(t^j)_{j \in M}$ is $\sum_{j \in M} t^j_s + G^0_s$. The payoff to the principal j is $G^j_s - t^j_s$, where t^j_s is the transfer he has promised on the actions s, and s is the action chosen by the agent.

A strategy for the principal j is a function σ^j from the vector of transfers announced by the principals who precede him into his vector of transfers. A strategy for the agent is a function from the m-tuple of transfers announced by the principals into actions. Each principal $j \in M$ has a payoff which depends on the action of the agent, and described by a vector $G^j \in R^n$.

In this paper we do not consider mixed strategies. Equilibria exist in pure strategies. It may be interesting to note that the common agency game with multiple agents may not have pure strategy equilibria (see [11]).

3 The equilibrium set

For any $k \in \{0; \ldots; m\}$ and any vector of transfers $(t^k; \ldots; t^m)$, we denote the subgame beginning after that vector of transfers has been announced by $\Gamma(t^k; \ldots; t^m)$. A subgame-perfect equilibrium (SPE) of the game induces for any such vector of transfers an action chosen at equilibrium in that subgame, that we denote $\eta(t^k; \ldots; t^m)$. To lighten the notation we do not make the dependence of the function η on the equilibrium explicit.

The equilibrium action

We begin with a characterizaton of the action chosen at the equilibrium outcome.

Proposition 3.1 For any k and any $(t^{k+1}; \ldots; t^m)$, the action s^k is a solution of the problem:

$$\max_{s^k}\left(\sum_{j=0}^{k} G^j_{s^k} + \sum_{j=k+1}^{m} t^j_{s^k}\right) \tag{3.3}$$

if and only if $s^k = \eta(t^{k+1}; \ldots; t^m)$ for a SPE of the game $\Gamma(t^{k+1}; \ldots; t^m)$.

Proof of proposition (3.1). The basic idea of the proof is the one used in the solution of a principal agent problem. In the solution of the backwards induction problem we may think that the principal k is choosing his transfer and the action of the agent, provided this choice satisfies the incentive constraint that the chosen action is an equilibrium in the subgame beginning at $(t^k; \ldots; t^m)$. We are then going to use, in an induction argument, that this incentive constraint has the special form described in the statement of the proposition.

We first define the auxiliary problem:

$$\max_{(s^k; t^k)} G^k_{s^k} - t^k_{s^k}; \tag{3.4}$$

$$\text{subject to } s^k \in \operatorname{argmax} \sum_{j=0}^{k-1} G^j_s + \sum_{j=k}^{m} t^j_s.$$

This is the reformulation of the backwards induction problem, with the incentive constraint written in the form (3.3). In lemma (3.3) we prove that these two different ways of writing the incentive constraint are equivalent. To do this, we first observe that the problem (3.4) can be reformulated as:

$$\max_{s^k} G^k_{s^k} - t^k(s^k)_{s^k} \tag{3.5}$$

where for each action s^k the vector $t^k(s^k)$ is a solution of the cost minimization problem for the principal k to implement the action k, that is the problem:

$$\min_{t^k} t^k_{s^k}; \tag{3.6}$$

$$\text{subject to } \sum_{j=0}^{k-1} G^j_{s^k} + \sum_{j=k}^{m} t^j_{s^k} \geq \sum_{j=0}^{k-1} G^j_s + \sum_{j=k}^{m} t^j_s; \text{ for every } s.$$

The cost minimization problem has a simple solution. In the next lemma, the vector F replaces the term $\sum_{j=0}^{k-1} G^j_s + \sum_{j=k+1}^{m} t^j_s$.

Lemma 3.2 For any $F \in R^n$ and any $s_0 \in S$, the solution of:

$$\min_{t_{s_0} \geq 0} t_{s_0}; \text{ subject to: } F_{s_0} + t_{s_0} \geq \max_{s \in S} \{F_s + t_s\}$$

has value $\max_s F_s - F_{s_0}$, and solution any t such that

$$t_s \in [0; \max_s F_s - F_s]; \text{ for } s \neq s_0; t_{s_0} = \max_s F_s - F_{s_0}.$$

Proof of lemma (3.2). Note that the constraint of the problem is satisfied if and only if the constraint: $F_{s_0} + t_{s_0} \geq \max_{s \in S; s \neq s_0} \{F_s + t_s\}$ is satisfied. So an optimal solution is to set $t_s = 0$ for any $s \neq s_0$; so our problem has the same value as: $\min_{t \geq 0} t_{s_0}$; subject to: $t_{s_0} \geq \max_{s \in S} F_s - F_{s_0}$. Now it is easy to check that any t is the set of the statement is feasible; and that any other vector is not optimal. ■

Now we show that the two formulations of the incentive constraint are equivalent:

Lemma 3.3 For any k and any $(t^{k+1}, \ldots, t^m)$, an action s^k is an optimal action in a solution of (3.4) if and only if it solves (3.3).

Proof of lemma (3.3). Substitute the optimal value in the cost minimization problem (3.6) determined in lemma (3.2) into the problem (3.5) to conclude that the optimal action in the problem (3.4) is the solution of

$$\max_{s^k} \Big(\sum_{j=0}^{k} G^j_{s^k} + \sum_{j=k+1}^{m} t^j_{s^k}\Big) - \max_{s \in S} \Big(\sum_{j=0}^{k} G^j_s + \sum_{j=k+1}^{m} t^j_s\Big):$$

Since the second term is a constant, the choice of the optimal action in (3.4) is the solution of (3.3) as claimed. ■

Conclusion of the proof of proposition. The proof is by induction. For $k = 0$, the proposition simply states that the agent chooses an action maximizing the sum of his payoff and the total transfers.

Assume now that the statement holds for $k - 1$; we claim the statement holds for k. Take any vector of transfers $(t^{k+1}, \ldots, t^m)$ of the principals who have moved before k. Consider the backwards induction problem of k: he chooses his transfer t^k to solve:

$$\max_{t^k} G^k_{s^k} - t^k_{s^k} \tag{3.7}$$

where $s^k = \sigma(t^k, t^{k+1}, \ldots, t^m)$, that is the action s^k is the SPE action outcome in the game $\Gamma(t^k, t^{k+1}, \ldots, t^m)$. If the equilibrium outcome is a probability distribution on the action set, then (3.7) has to be understood as the expectation of the net payoff with respect to this distribution. The argument below shows that equilibria are in pure strategies.

Now we claim that $\hat{t}^k$ is the solution of the problem (3.7), if and only if the pair $(\hat{s}^k, \hat{t}^k)$ of the equilibrium action and transfer is the solution of the problem (3.3). The proof is a standard argument in principal-agent problems, that we spell out for completeness. For the "if" part, proceed by contradiction. If the pair $(\hat{s}^k, \hat{t}^k)$ is not a solution of (3.3), then for some pair (s^*, t^*) we have:

$$G^k_{s^*} - t^*_{s^*} > G^k_{\hat{s}^k} - \hat{t}^k_{\hat{s}^k}$$

and s^* satisfies the constraint in (3.3) with $t^k = t^*$. But then for some ϵ small enough s^* is the unique element in $\arg\max \sum_{j=0}^{k-1} G^j_s + (t^*_s + \epsilon) + \sum_{j=k+1}^{m} t^j_s$, and therefore the unique action outcome in the subgame $\Gamma(t^*, t^{k+1}, \ldots, t^m)$.

Since for ϵ small enough $G^k_{s^*} - t^*_{s^*} - \epsilon > G^k_{\hat{s}^k} - \hat{t}^k_{\hat{s}^k}$, we have contradicted the assumption that $\hat{t}^k$ is the SPE choice of k. The "only if" part is immediate. ∎

The following is an immediate corollary of the proposition, obtained by considering the $k = m$ case. We assume that the action that solves:

$$\max_{s \in S} \sum_{j=0}^{m} G^j_s \tag{3.8}$$

is unique, we call it the efficient action, and we denote it by $\hat{s}$. Of course, this action only depends on the payoffs of the players, and not on the order in which they move. Then

Theorem 3.4 In any SPE the agent chooses the efficient action.

Of course the efficient action is independent of the order of move of the principals. So in any of the games that can be obtained by choosing a different order of moves of the principal, and in any subgame perfect equilibrium, the outcome is the same, and is the efficient outcome.

The equilibrium transfers

In addition the lemma (3.2) gives an explicit expression for the sequence of equilibrium transfers of the principals. Recall that the the only equilibrium action is $\hat{s}$.

Proposition 3.5 The sequence of equilibrium transfers is any sequence where

i. the transfer of the principal who moves first is any vector:

$$t^m_s \in [0, \max_{s \in S} \sum_{j=0}^{m-1} G^j_s - \sum_{j=0}^{m-1} G^j_s] \text{ for } s \neq \hat{s}; \tag{3.9}$$

$$t^m_{\hat{s}} = \max_{s \in S} \sum_{j=0}^{m-1} G^j_s - \sum_{j=0}^{m-1} G^j_{\hat{s}}; \tag{3.10}$$

ii. and for any other principal k:

$$t^k_s \in [0, \max_{s \in S}\Big(\sum_{j=0}^{k-1} G^j_s + \sum_{j=k+1}^{m} t^j_s\Big) - \Big(\sum_{j=0}^{k-1} G^j_s + \sum_{j=k+1}^{m} t^j_s\Big)]; \tag{3.11}$$

$$t^k_{\hat{s}} = \max_{s \in S}\Big(\sum_{j=0}^{k-1} G^j_s + \sum_{j=k+1}^{m} t^j_s\Big) - \Big(\sum_{j=0}^{k-1} G^j_{\hat{s}} + \sum_{j=k+1}^{m} t^j_{\hat{s}}\Big) \tag{3.12}$$

where $t^j, j = k+1, \ldots m$ is any equilibrium transfer for the previous principals.

Note that for each principal the transfer is uniquely determined for two actions: the efficient action and the action where the maximum in (3.11) is achieved. The game has many equilibria. This is clear from the explicit solution for the transfers, given in the proposition (3.5). Now we characterize them. We denote:

$$T^k \equiv \sum_{j=k}^{m} t^j \text{ for every } k; \; T \equiv \sum_{j=1}^{m} t^j; \tag{3.13}$$

and for convenience we agree that

$$T^{m+1} = 0. \tag{3.14}$$

Then from (3.12):

$$t^k_{\hat{s}} = \max_{s}\Big(\sum_{j=0}^{k-1} G^j_s - \sum_{j=0}^{k-1} G^j_{\hat{s}} + T^{k+1}_s - T^{k+1}_{\hat{s}}\Big). \tag{3.15}$$

Now we denote:

$$\Delta^k_s \equiv \sum_{j=0}^{k} G^j_s - \sum_{j=0}^{k} G^j_{\hat{s}}; \tag{3.16}$$

and:

$$\Delta^k \equiv \max_s \Delta_s^k = (\max_{s \neq \hat{s}} \Delta_s^k)^+ = \max_{s \neq \hat{s}} (\Delta_s^k)^+ \geq 0. \qquad (3.17)$$

where $X^+ \equiv \max(X, 0)$. Then:

Lemma 3.6 For every $k = 1, \dots, m$,

$$T_{\hat{s}}^k = \max_{j=k-1,\dots,m-1} (\max_{s \neq \hat{s}} (\Delta_s^j + T_s^{j+2})^+). \qquad (3.18)$$

Proof. For every k,

$$\begin{aligned} T_{\hat{s}}^k &= T_{\hat{s}}^{k+1} + t_{\hat{s}}^k \\ &= T_{\hat{s}}^{k+1} + \max_s (\Delta_s^{k-1} + T_s^{k+1} - T_{\hat{s}}^{k+1}) \\ &= \max_s (\Delta_s^{k-1} + T_s^{k+1}) \\ &= \max(\max_{s \neq \hat{s}} (\Delta_s^{k-1} + T_s^{k+1}), T_{\hat{s}}^{k+1}) \\ &= \max(\max_{s \neq \hat{s}} (\Delta_s^{k-1} + T_s^{k+1})^+, T_{\hat{s}}^{k+1}) \qquad (3.19) \end{aligned}$$

(where the first equality is the definition of T^k, the second follows from (3.15) above and the definition of Δ_s^{k-1}, the third and fourth are clear, the fifth follows from the fact that $T_{\hat{s}}^{k+1} \geq 0$). This defines a relation between $T_{\hat{s}}^k$ and $T_{\hat{s}}^{k+1}$. Note that from (3.10)

$$T_{\hat{s}}^m = (\max_{s \neq \hat{s}} (\Delta_s^{m-1})^+)$$

directly from (3.19) and (3.14). Now an easy induction argument shows that (3.18) holds. Assume (3.18) for k. Then

$$\begin{aligned} T_{\hat{s}}^{k-1} &= \max(\max_{s \neq \hat{s}} (\Delta_s^{k-2} + T_s^k)^+, T_{\hat{s}}^k) \\ &= \max(\max_{s \neq \hat{s}} (\Delta_s^{k-2} + T_s^k)^+, \max_{j=k-1,\dots,m-1} (\max_{s \neq \hat{s}} (\Delta_s^j + T_s^{j+2})^+)) \\ &= \max(\max_{j=k-2,\dots,m-1} (\max_{s \neq \hat{s}} (\Delta_s^j + T_s^{j+2})^+)) \\ &= \max_{j=k-2,\dots,m-1} (\max_{s \neq \hat{s}} (\Delta_s^j + T_s^{j+2})^+), \qquad (3.20) \end{aligned}$$

(the first equality is (3.19) above, the second follows from the induction hypothesis, and the third and fourth are clear) which proves our claim. ■

4 Thrifty equilibria

From (3.9) and (3.9) we see that the outcome where

$$t_s^k = 0, \text{ for all } k \in M, \text{ and for all } s \neq \hat{s} \qquad (4.21)$$

is an equilibrium outcome. Take now any subgame $\Gamma(t^{k+1};\ldots;t^m)$. The equilibrium outcome action $\eta(t^{k+1};\ldots;t^m)$ in that subgame is determined as in proposition (3.1). There are many transfers that implement in equilibrium that action: but there is always a vector of transfers which is zero on all actions, except the action $\eta(t^{k+1};\ldots;t^m)$. If we make this choice of strategy on any subgame, we have a complete description of an equilibrium, which we call the thrifty.

Proposition 4.1 For any $k = 1;\ldots;m$ the transfers at the thrifty equilibrium are such that:

$$T^k_{\hat{s}} = \max_{j=k-1;\ldots;m-1} \Delta^j; \tag{4.22}$$

and

$$t^k_{\hat{s}} = (\Delta^k - \max_{j=k;\ldots;m} (\Delta^j))^+ \tag{4.23}$$

Proof. The first equality (4.22) follows from setting $T^{j+2}_s = 0$ in (3.18) for $s \neq \hat{s}$, which gives

$$T^k_{\hat{s}} = \max_{j=k-1;\ldots;m-1} (\max_{s\neq\hat{s}} (\Delta^j_s)^+):$$

To prove (4.23),

$$\begin{aligned}
t^k_{\hat{s}} &= \max_s (\Delta^{k-1}_s + T^{k+1}_s - T^{k+1}_{\hat{s}}) \\
&= \max(0; \max_{s\neq\hat{s}} (\Delta^{k-1}_s - T^{k+1}_{\hat{s}})) \\
&= (\max_{s\neq\hat{s}} (\Delta^{k-1}_s - T^{k+1}_{\hat{s}}))^+ \\
&= (\max_{s\neq\hat{s}} (\Delta^{k-1}_s) - T^{k+1}_{\hat{s}})^+ \\
&= (\max_{s\neq\hat{s}} (\Delta^{k-1}_s)^+ - T^{k+1}_{\hat{s}})^+ \\
&= (\max_s \Delta^{k-1}_s - T^{k+1}_{\hat{s}})^+ \\
&= (\Delta^{k-1} - T^{k+1}_{\hat{s}})^+ \\
&= (\Delta^{k-1} - \max_{j=k;\ldots;m} (\Delta^j))^+
\end{aligned} \tag{4.24}$$

where the first equality follows from (3.15) and the definition of Δ^{k-1}_s, the second is clear, the third is notational, the fourth is clear, the fifth follows by considering the different cases, the sixth is (3.17), the seventh is notational, the last follows from (4.22).

In the thrifty equilibrium, principals only make the transfers that are strictly necessary. So the aggregate transfers are minimal: the following proposition follows directly from the lemma (3.6):

Proposition 4.2 Let $(t^j)_{j=0,\ldots,m}$ be the vector of transfers of any SPE, and $\hat{t}^j$, for $j = 1,\ldots,m$ the corresponding vector at the thrifty equilibrium. Then for any k:

$$\sum_{j=k} t_s^j \geq \sum_{j=k} \hat{t}_s^j \text{ for any } s.$$

Consider the transfer $\hat{t}_s^j$ made at the thrifty equilibrium by the principal j for the efficient action s. It might be tempting to think that for all other equilibria the transfer made at equilibrium by that principal is less that $\hat{t}_s^j$. This is not true.

The reason for this is the following. Take a principal k who has an action giving a gross payoff to him higher than the efficient action. Suppose that some other principal n, moving earlier than him, makes a positive transfer on that action. This transfer improves the bargaining position of k: he might his favorite action relying on the transfer of n, so get t. In the resulting equilibrium, his transfers may be smaller than in the thrifty equilibrium.

This is shown precisely in the next example. Consider the game:

	s_1	s_2	s_3
G_s^0	0	0	5
G_s^1	8	10	0
G_s^2	0	10	0
G_s^3	10	10	0

(4.25)

In the thrifty equilibrium, principals 3 and 2 make zero transfers, and 1 makes a transfer $(0;5;0)$. But there is another equilibrium, where the principal 3, the first to move, makes a transfer $(10;0;0)$. The money offered on the first action improves the position of the principal 1, who may get a payoff of 8 from the first action, unless principal 2 counters the offer of 3 on the first action with a transfer of at least 8 on the third. Now principal 2 has only to offer the amount 2 needed to match the 10 in the first action, and that is the amount he pays at this equilibrium, where $t^2 = (0;8;0)$ and $t^3 = (0;2;0)$.

Simultaneous and sequential common agency

In the next proposition we provide a comparison of the totals transfers in the two games. Since both games have many equilibria, we focus on the truthful equilibrium for the simultaneous game (which is also the coalition proof equilibrium) and the thrifty equilibrium in the sequential game.

Proposition 4.3 The sum of transfers in the truthful equilibrium of the simultaneous common agency is larger than the total transfers in the thrifty equilibrium of the sequential game, for any order of move of the principals.

Proof. Leave for the moment the order of the principals arbitrary. We denote by $\hat{t}$ the transfers in the thrifty equilibrium of the sequential game and by $\bar{t}$ the transfers in the truthful equilibrium in the simultaneous game. The action profile of the agents is the same, and is denoted by $\hat{s}$. The total transfers in the thrifty equilibrium are:

$$T^1_{\hat{s}} = \sum_{j=1}^{m} \hat{t}^j_{\hat{s}} = \max_{k=0,\ldots,m-1} \Delta^k; \tag{4.26}$$

where we recall that

$$\Delta^k \equiv \max_s \sum_{j=0}^{k} G^j_s - \sum_{j=0}^{k} G^j_{\hat{s}}. \tag{4.27}$$

Our claim is that

$$\sum_{j=1}^{m} t^{1j}_{\hat{s}} \geq T^1_{\hat{s}} \text{ for any order of move of the principals.} \tag{4.28}$$

In view of (4.26) and (4.27) proving (4.28) is equivalent to proving that $\sum_{j=1}^{m} t^{1j}_{\hat{s}} \geq \max_s \sum_{j=0}^{k} G^j_s - \sum_{j=0}^{k} G^j_{\hat{s}}$ for any k and for any order of move of the principals. If we use the notation $\sum_{j \in J} G^j + G^0 \equiv G^J$ (note that the sum includes the preferences of the agent) this is equivalent to:

$$\sum_{j=1}^{m} t^{1j}_{\hat{s}} \geq \max_s G^J_s - G^J_{\hat{s}} \text{ for any } J \subseteq M. \tag{4.29}$$

But now recall that transfers are non negative, and that in a truthful equilibrium:

$$\sum_{j \notin J} t^{1j}_{\hat{s}} \geq \max_s G^J_s - G^J_{\hat{s}} \text{ for any } J \subseteq M. \tag{4.30}$$

The inequality 4.30 is proved in Bernheim and Whinston [1, p. 28] as follows. Take any $s(J) \in \operatorname{argmax}_{s \in S} G^J_s$, and $J^c \equiv M \setminus J$; then:

$$\begin{aligned}
\sum_{j \in M} \hat{t}^j_{\hat{s}} &= \sum_{j \in J} \hat{t}^j_{\hat{s}} + \sum_{j \notin J} \hat{t}^j_{\hat{s}} \\
&\geq \sum_{j \in J} \hat{t}^j_{s(J)} + \sum_{j \notin J} \hat{t}^j_{s(J)} + G^0_{s(J)} - G^0_{\hat{s}} \\
&\geq \sum_{j \in J} \hat{t}^j_{s(J)} + G^0_{s(J)} - G^0_{\hat{s}} \\
&\geq G^J_{s(J)} - G^J_{\hat{s}} + \sum_{j \in J} \hat{t}^j_{\hat{s}}
\end{aligned} \tag{4.31}$$

where the first equality is obvious, the first inequality because agents optimize, the second one is obvious, the third one follows because the equilibrium is truthful. Cancelling terms gives (4.30), and our claim. ∎

5 Truthful Equilibria

The definition of truthful equilibrium used in simultaneous common agency has a natural extension to the sequential case:

Definition 5.1 A truthful equilibrium $(\bar t, \bar s)$ of the sequential game is a subgame-perfect equilibrium in which, for all $k \in M$, the transfer matrix $\bar t$ satisfies

$$\bar t_s^k = \max(0, G_s^k - G_{\bar s}^k + \bar t_{\bar s}^k) \tag{5.32}$$

for any principal k and any action s.

Proposition 5.2 A sequential common agency game has a truthful equilibrium.

Proof. Let $\bar s$ be an efficient action and let the transfer matrix $\bar t$ be defined recursively as follows: For $k = 1, \ldots, m$,

$$\bar t_{\bar s}^k = \max_{s \in S}\Big(\sum_{j=0}^{k-1} G_s^j + \sum_{j=k+1}^{m} \bar t_s^j\Big) - \Big(\sum_{j=0}^{k-1} G_{\bar s}^j + \sum_{j=k+1}^{m} \bar t_{\bar s}^j\Big)$$

and, for $s \neq \bar s$,

$$\bar t_s^k = \max(0, G_s^k - G_{\bar s}^k + \bar t_{\bar s}^k).$$

We want to show that $\bar t$ satisfies the conditions of Proposition 3.5. Suppose that $\bar t$ satisfies 3.11 for principals $k+1$ to m. By definition, $t_{\bar s}^k$ satisfies 3.11. To tackle the case $s \neq \bar s$, notice that, by Proposition 3.1,

$$G_s^k - G_{\bar s}^k \quad \Big(\sum_{j=0}^{k-1} G_{\bar s}^j + \sum_{j=k+1}^{m} \bar t_{\bar s}^j\Big) - \Big(\sum_{j=0}^{k-1} G_s^j + \sum_{j=k+1}^{m} \bar t_s^j\Big).$$

Then, by substituting $\bar t_{\bar s}^k$,

$$G_s^k - G_{\bar s}^k - \bar t_s^k \quad \max_{s \in S}\Big(\sum_{j=0}^{k-1} G_s^j + \sum_{j=k+1}^{m} t_s^j\Big) - \Big(\sum_{j=0}^{k-1} G_s^j + \sum_{j=k+1}^{m} t_s^j\Big).$$

Therefore, $\bar t_s^k$ satisfies 3.11 for principal k. Then, by recursion, $\bar t$ is a subgame-perfect equilibrium of the sequential common agency game. ∎

What is the connection between the truthful equilibrium of the sequential game as defined here and the truthful equilibrium of the simultaneous game as defined by Bernheim and Whiston? Let M be a set of principals and let P be a particular ordering of this set of principals. Let $\Gamma(M, S, G, P)$ be the sequential common agency game defined by M, S, G, and P. Let $\tilde\Gamma(M, S, G)$ be the simultaneous game defined by M, S, and G. We say that $\tilde\Gamma(M, S, G)$ is the simultaneous game corresponding to $\Gamma(M, S, G, P)$.

Clearly, the set of efficient actions is the same in a sequential game and in its corresponding simultaneous game. From Theorem 3.4 of this paper we know that any subgame perfect equilibrium, and therefore any truthful equilibrium, of the sequential game selects an efficient action. The same is true for the simultaneous common agency game (Theorem 2 of Bernheim and Whinston). Then, all truthful equilibria both in the simultaneous game and in the sequential game (for any order P) support actions in $\hat{S}$. If $\hat{S}$ has one element —which is true generically— all truthful equilibria, whether in the sequential game or in the simultaneous game, support the same action.

It remains to understand the relation between equilibrium transfers in the two contexts.

Simultaneous common agency

We recall that in the simultaneous game a pair $(\bar{t}, \hat{s})$ is an equilibrium if and only if the following conditions are satisfied:

i. for every $j = 1, \ldots, m$ and for every $s \in S$

$$G^j_{\hat{s}} + \sum_{i \neq j} \bar{t}^i_{\hat{s}} \geq G^j_s + \sum_{i \neq j} \bar{t}^i_s; \tag{5.33}$$

ii. for every $j = 1, \ldots, m$,

$$\bar{t}^j_{\hat{s}} = \max_{s \in S}(G^0_s + \sum_{i \neq j} \bar{t}^i_s) - (G^0_{\hat{s}} + \sum_{i \neq j} \bar{t}^i_{\hat{s}}) \tag{5.34}$$

and for every $s \in S$

$$\bar{t}^j_s \leq \max_{s' \in S}(G^0_{s'} + \sum_{i \neq j} \bar{t}^i_{s'}) - (G^0_s + \sum_{i \neq j} \bar{t}^i_s) \tag{5.35}$$

iii. for every $s \in S$,

$$G^0_{\hat{s}} + \sum_{i=1}^{m} \bar{t}^i_{\hat{s}} \geq G^0_s + \sum_{i=1}^{m} \bar{t}^i_s \tag{5.36}$$

The conditions are intuitively clear: the condition (5.33) determines the equilibrium action as the action giving the highest net payoff to the principal j; (5.34) and (5.35) determine the minimum cost for that principal to implement the action $\hat{s}$; finally (5.36) insures that the agent chooses the action $\hat{s}$.

The condition (5.34) implies $\sum_{i=1}^{m} \bar{t}^i_{\hat{s}} + G^0_{\hat{s}} = \max_{s \in S}(G^0_s + \sum_{i \neq j} \bar{t}^i_s)$ and therefore for every j there is an $s(j) \neq \hat{s}$ such that $\bar{t}^j_{s(j)} = 0$, and $s(j)$ gives maximum transfer to the agent:

$$\sum_{i=1}^{m} \bar{t}^i_{\hat{s}} + G^0_{\hat{s}} = G^0_{s(j)} + \sum_{i \neq j} \bar{t}^i_{s(j)} \text{ for every } j.$$

In addition, $(\bar{t}, \hat{s})$ is a truthful equilibrium if and only if $\hat{s} \in \hat{S}$, the condition (5.32) is satisfied and

$$G^0_{\hat{s}} + \sum_{j=1}^{m} \bar{t}^j_{\hat{s}} = \max_{s \in S}\left(G^0_s + \sum_{j \neq k} \bar{t}^j_s\right) \tag{5.37}$$

for $k = 1, \ldots, m$ (See Dixit, Grossman, and Helpman [5, Proposition 3]).

Comparison of the equilibria

Proposition 5.3 If $(\hat{t}, \hat{s})$ is a truthful equilibrium of a sequential game, then $(\hat{t}, \hat{s})$ is a truthful equilibrium of the corresponding simultaneous game.

Proof. Let $(\hat{t}, \hat{s})$ be a truthful equilibrium of the sequential game. Then, $\hat{s} \in \hat{S}$ and, by Proposition 3.1, for $k = 1, \ldots, m$,

$$\hat{t}^k_{\hat{s}} = \max_{s \in S} \left(\sum_{j=0}^{k} G^j_s + \sum_{j=k+1}^{m} \hat{t}^j_s \right) - \left(\sum_{j=0}^{k} G^j_{\hat{s}} + \sum_{j=k+1}^{m} \hat{t}^j_{\hat{s}} \right).$$

Truthfulness implies that, for any $k \in M$ and any $s \in S$, $G^k_s - G^k_{\hat{s}} \geq \hat{t}^k_s - \hat{t}^k_{\hat{s}}$. Hence,

$$\hat{t}^k_{\hat{s}} \geq \max_{s \in S} \left(G^0_s + \sum_{j \neq k} \hat{t}^j_s \right) - \left(G^0_{\hat{s}} + \sum_{j \neq k} \hat{t}^j_{\hat{s}} \right). \tag{5.38}$$

However, if 5.38 held as a strict inequality, then

$$\max_{s \in S} \left(G^0_s + \sum_{j=1}^{m} \hat{t}^j_s \right) > G^0_{\hat{s}} + \sum_{j=1}^{m} \hat{t}^j_{\hat{s}},$$

in which case the agent would not choose $\hat{s}$. Thus, 5.38 must hold as an equality, which implies 5.37. ∎

A truthful equilibrium of a sequential game is always a truthful equilibrium of the corresponding simultaneous game. One may wonder whether a truthful equilibrium in the simultaneous game is always a truthful equilibrium of a sequential game, at least for some ordering P. This is not true. In some cases, there is a truthful equilibrium of $\tilde{\Gamma}(M, S, G)$, given which there exists no ordering P such that it is also a truthful equilibrium (or just a subgame-perfect equilibrium) of $\Gamma(M, S, G, P)$. Consider for instance a game with three actions and in which the agent cares only about transfers:

	s_1	s_2	s_3
G^1_s	3	0	0
G^2_s	1	0	2
G^3_s	1	2	0

This game, if played simultaneously, has a continuum of truthful equilibria such that $\hat{s} = s_1$ and

	s_1	s_2	s_3
$\hat{t}^1_s$	x	0	0
$\hat{t}^2_s$	$1 - x$	0	$2 - x$
$\hat{t}^3_s$	$1 - x$	$2 - x$	0

where $x \in [0, 1]$. On the contrary, in the sequential game, given an ordering of principals, there exists a unique truthful equilibrium. If 1 is the last to play, then the unique truthful equilibrium is

	s_1	s_2	s_3
$\hat{t}^1_s$	1	0	0
$\hat{t}^2_s$	0	0	1
$\hat{t}^3_s$	0	1	0

For all other orderings,

	s_1	s_2	s_3
$\hat{t}^1_s$	0	0	0
$\hat{t}^2_s$	1	0	2
$\hat{t}^3_s$	1	2	0

Therefore, of the continuum of truthful equilibria in the simultaneous game, only the two extremes ($x = 0$ and $x = 1$) are also truthful equilibria of a sequential game. For $0 < x < 1$, the truthful equilibrium of the simultaneous game is not an equilibrium of any sequential game.[1] Notice that the total transfer for the efficient action s_1 varies from a truthful equilibrium to the other.

However, there exists a very special case in which there is a one-to-one correspondence between equilibria of the simultaneous game and equilibria of the sequential game:[2]

Proposition 5.4 *Suppose that there are only two principals and that the agent is indifferent among actions. Then, for any P, $(\hat{t}; \hat{s})$ is a truthful equilibrium of a sequential game if and only if it is a truthful equilibrium of the corresponding simultaneous game.*

Proof. It is easy to verify that, both in the simultaneous and in the sequential game for any P, an equilibrium $(\hat{s}; \hat{t})$ is truthful if and only if

$$\hat{s} \in \operatorname{argmax}_{s \in S}(G^1_s + G^2_s)$$
$$\hat{t}^1_s = \max(0; G^1_s - G^1_{\hat{s}} + (\max_{s' \in S} G^2_{s'}) - G^2_{\hat{s}}) \text{ for all } s \in S$$
$$\hat{t}^2_s = \max(0; G^2_s - G^2_{\hat{s}} + (\max_{s' \in S} G^1_{s'}) - G^1_{\hat{s}}) \text{ for all } s \in S.$$

■

[1] The example above involves some symmetry between Principal 2 and Principal 3. This structure has been chosen to simplify the analysis. However, the underlying result is robust. A more generic example is:

	s_1	s_2	s_3
G^1_s	4	0	0
G^2_s	2	0	5
G^3_s	3	4	0

In this example as well the simultaneous game has a continuum of truthful equilibria while the sequential game has a unique equilibrium for each ordering.

[2] The truthful equilibrium of Proposition 5.4 is generically unique.

6 Re¯nements

We have seen that sequential common agency has in general multiple subgame perfect equilibria. All of them select an e±cient action but the equilibrium transfer vector varies from one equilibrium to the other. One may wonder whether some equilibria are more plausible than others. This section considers two equilibrium re¯nements: trembling-hand perfection and coalition-proofness. In the simultaneous game, Bernheim and Whinston show that perfection does not help while coalition-proofness selects the truthful equilibrium. As we will see, also in the sequential game perfection has little bite. On the other hand, coalition-proofness signi¯cantly restricts the set of equilibria. An equilibrium is coalition-proof only if it is payo®-equivalent to the thrifty equilibrium. Thus, the connection between truthfulness and coalition-proofness that Bernheim and Whinston found for the simultaneous game does not carry over to the sequential game.

6.1 Perfect Equilibria

Some of the equilibria, including the truthful ones, may seem fragile. Consider for instance the common agency game with payo®s as in (1.2). One of the equilibria has the transfers equal to

$$t^1 = (6; 0; 1; 2); \; t^1 = (0; 0; 5; 0):$$

This equilibrium may seem not very robust. The principal who moves ¯rst is o®ering a large transfer on the ¯rst action, counting on the fact that in equilibrium he will not pay it. But he will not pay because the second principal will o®er exactly 5 on the third action, and the agent will choose the third action, even if he is indi®erent between the ¯rst and the third action. Any small mistake of any of the two players that come after him will cost him dearly.

We may formalize this idea with the perfectness criterion. Suppose that with some positive probability, possibly very small, the principal 2 may make a mistake. This mistake may take di®erent forms. To ¯x ideas, assume that when he does a mistake the principal fails to show up, and makes a zero transfer on all the actions. Suppose that also the agent may make a mistake, also with a small probability. Again this mistake may take di®erent forms: suppose the agent chooses randomly, with uniform probability, one of the actions.

If we look for the equilibria in the perturbed game where the second principal and the agent may make mistakes, and then let the probability of the mistakes go to zero we ¯nd only one equilibrium: the thrifty equilibrium, with equilibrium transfers $t^1 = (0; 0; 1; 0)$ and $t^2 = (0; 0; 0; 0)$. The reason is clear: the redundant transfers on the ¯rst and fourth action by the ¯rst principal are cost-less in the game where nobody makes mistakes, but they become costly in the perturbed game.

This result depends in a critical way on the speci¯cation we have chosen for the mistakes. If we change the speci¯cation, the equilibrium that is selected will also change. In general, the issue of the form of the mistakes becomes critical in games, like the common agency games, with infinite action space. A detailed discussion of this issue is in the Simon and Stinchcombe paper ([14]; see also [13]). Since there seem to be no natural way to restrict the type of mistake, we do not introduce any restriction.

To make this discussion formal, we introduce a definition of perfect equilibria for this game, which is the standard definition of perfect equilibrium for an extensive form game (see for instance van Damme, [15], chapter 6.4, page 111). We only have to adapt it to the our situation, in which the principals have a strategy space which has infinite cardinality.

Definition 6.1 A perturbation is

i. a vector $(\epsilon^0, \epsilon^1, \ldots, \epsilon^m)$ of real numbers in $(0, 1)$

ii. for every k and for every vector of transfers $(t^{k+1}, \ldots, t^m)$, a probability distribution $\psi^k(t^{k+1}, \ldots, t^m)$ that gives positive probability to every open subset of the strategy space of the principal k, or, when $k = 0$, of the agent.

The interpretation of a perturbation is the following. Every player makes a mistake with probability ϵ^k, and when he does he then chooses a strategy according to the probability distribution ψ^k.

The perturbed game is completely described by the perturbation as follows:

Definition 6.2 For the sequential common agency game defined in section 2, a perturbed game assigns to every strategy $(\sigma \equiv (\sigma_0, \sigma_1, \ldots, \sigma_m)$ the strategy profile

$$\begin{aligned} \Phi(\sigma, \epsilon, \psi) &= \sigma^k \text{ with probability } 1 - \epsilon^k \\ &= \psi^k \text{ with probability } \epsilon^k \end{aligned} \tag{6.39}$$

The payoff in the perturbed game for the strategy profile σ is the payoff in the original game to the perturbed strategy profile $\Phi(\sigma, \epsilon, \psi)$.

This decomposition in the ϵ and ψ components is of course without loss of generality, and is made simply for convenience. Then we say:

Definition 6.3 An equilibrium of the sequential common agency problem is perfect if it is the limit point of a sequence of equilibria in the sequence of games perturbed by some sequence $\{(\epsilon^k_n, \psi^k)_{k=0,\ldots,m}, n = 1, 2, \ldots\}$, where $\lim_{n \to \infty} \epsilon^k_n = 0$ for every k.

One can then prove that a subgame perfect equilibrium is also perfect. The proof is omitted for simplicity, since it is not directly related to the main point of this paper. The intuition is clear: the definition of perfect equilibrium in 6.3 leaves open a wide choice of perturbations. For any subgame perfect equilibrium it is possible to find the appropriate perturbation that makes the equilibrium choice of the principals approximately the best response in the perturbed game.

Proposition 6.4 *All the subgame perfect equilibria are perfect.*

The proposition (6.4) depends critically on the fact that the probability distributions ϕ^k may depend on the transfers of the previous players. If additional restrictions are imposed on the possible mistakes of the players, then the set of prefect equilibria is a proper subset of the subgame perfect equilibria.

A special case is, of course, the thrifty equilibrium. This equilibrium is perfect however, even if the probabilities ϕ^k do not depend on the transfers. In fact, it is enough that the probability that an agent makes a mistake is higher than the probability that a principal does, and that his mistakes are uniform.

Proposition 6.5 *The thrifty equilibrium is perfect.*

Proof. It suffices to show that for some vector of probabilities $\phi^k; k = 0, \ldots, m$ and a sequence of mistakes $\{\epsilon^k_n\}, k = 0, 1, \ldots, m; n = 1, 2, \ldots$, the equilibrium of the game converges to the thifty equilibrium. Take any fixed vector $\phi^k; k = 1, \ldots, m$, let ϕ_0 be the uniform distribution on actions. Then let the sequence of mistakes ϵ^k_n satisfy:

$$\lim_{n \to \infty} \frac{\epsilon^k_n}{\epsilon^0_n} = 0. \qquad (6.40)$$

It is now easy to see that for this sequence of mistakes the sequence of equilibria converges to the thrifty equilibrium. From the condition (6.40) it is clear that the principles only take into account, in the limit, the mistake of the agent. But the ϕ_0 is independent of the transfers. So any transfer with positive components for actions different from the efficient only adds to the expected cost, without altering the choice of the agent. So principals choose transfers which are zero for actions different from the efficient. ■

6.2 Coalition-Proof Equilibria

The refinement suggested in [1] for common agency problem is the concept of Coalition-Proof. Since the game we consider is sequential, we need to introduce and discuss the extension of this concept to games in extensive form. An extension is presented in [3], and the corresponding equilibrium concept is called Perfectly Coalition-Proof Nash equilibrium.

The concept of Perfectly Coalition-Proof Nash equilibrium requires that there is no other self-enforcing strategy that Pareto dominates it. It is important to note that the concept of Pareto dominance used in the definition in [3], page 10, is strict: each player in the deviating coalition must be strictly better off.

Now consider our game. In any subgame $\Gamma(t^{k+1}, \ldots, t^m)$, once the principal k has determined the action $\eta(t^{k+1}, \ldots, t^m)$ he has no choice in the transfers that he is going to pay. In particular the payoff of the principal who moves first is the same in all the equilibria. Now every coalition will have a principal who moves first, and the payoff for this principal is fixed. Hence, if we adopt the definition of Perfectly

Coalition-Proof Nash Equilibrium given in [3], page 10, then all equilibria in the sequential common agency problem are perfectly coalition-proof.

This de¯nition ignores the fact that each principal has a direct in°uence on the transfer paid by those who move after him, and he can always choose this to be a minimum amount. It seems natural to adopt a slightly weaker de¯nition, in our context, of perfect coalition-proof, which replaces the strict Pareto dominance with the weaker requirement that each player is at least as well o®, and at least one strictly better o®. Therefore, we adopt in the following the natural modi¯cation of the concept of Perfectly Coalition-Proof when the Pareto dominance criterion is the weak criterion, and we call this concept Strongly Perfect Coalition-Proof. With this more restrictive concept, the only equilibria are the thrifty, at least as payo®s are concerned. Formally, we say as usual that two equilibria are payo® equivalent if the payo® at equilibrium for each player is the same. Then:

Proposition 6.6 A SPE is strongly perfect coalition proof only if it is payo® equivalent to the thrifty equilibrium.

Proof. Take the SPE, and denote by σ^{*j*} its strategies, by t^{*j} its transfers, by T^{*k} its aggregate transfers. Also denote by t^j and T^k the corresponding quantities in the thrifty equilibrium. Since the two equilibria are not payo® equivalent,

$$T_{\bar{s}}^{*} > T_{\bar{s}}. \tag{6.41}$$

(It has to be the case that $T_{\bar{s}}^{*k} \geq T_{\bar{s}}^{k}$, by the proposition (4.2); and the equality holds for every k, then the transfers of each principal have to be the same in the two equilibria.) Now take the principal with the largest index k_0 such that the following two conditions are satis¯ed:

i. for some $s_0 \neq \bar{s}$, $t_{s_0}^{*k_0} > 0$;

ii. in the equilibrium where the transfer $t_{s_0}^{*k_0} > 0$ is replaced by $t_{s_0}^{**k_0} = 0$ the total transfer of the principals is strictly less than $T_{\bar{s}}^{*}$.

Recall that setting $t_{s_0}^{k_0} = 0$ and leaving all the other transfers equal to t^{*j} gives a set of transfers of a subgame-perfect equilibrium, by the (3.5). Also a principal with this property must exist, because of (6.41). Since the total transfers are smaller in this new equilibrium, there is also a principal k_1 who is paying an amount $t_{\bar{s}}^{**k_1}$ strictly smaller than $t_{\bar{s}}^{*k_1}$. Now take a coalition of the two principals k_0 and k_1, adopting the strategies which are equal to σ^{*}, except at the equilibrium path, where they set $t_{s_0}^{**k_0}$ and $t_{\bar{s}}^{*k_1}$ respectively. For ¯xed strategies $\sigma^{*j}; j \neq k_0; k_1$, this is a self-enforcing strategy pro¯le for these two principals, hence σ^{*} is not strongly perfect coalition proof. ∎

The thrifty equilibrium may fail to be strongly coalition proof. In the example (4.25), the strategy pro¯le where the principal 3 makes a positive o®er on the ¯rst action is self enforcing, and strictly improves the payo® of the principal 1. In fact this example and the previous result (6.6) show that a strongly perfect coalition proof equilibrium may fail to exist. We report this observation formally as:

Proposition 6.7 A strongly perfect coalition proof equilibrium may not exist.

Proof. Consider again the game described in the example (4.25). We claim it does not have strongly perfect coalition proof equilibria. If such an equilibrium exists, it must be payo® equivalent to the thrifty equilibrium by the proposition (6.6). But such an equilibrium must have:

i. $t_2^3 = t_2^2 = 0$, since these are the transfers on the equilibrium action of the thrifty equilibrium;

ii. $t_3^3 = t_3^2 = t_3^1 = 0$, because if any of these tranfers is positive the amount on the second action should be larger than 5, violating the payo® equivalence condition;

iii. $t_1^3 + t_1^2 + t_1^1$ 5, by the same argument as before.

But now the same argument that shows that the thrifty equilibrium is not strongly perfect coalition proof shows that any such equilibrium is not. Hence no equilibrium with this property can exists. ∎

7 Conclusions

Common agency is a way of modeling lobbying. Previous work has assumed that lobbies make their o®ers simultaneously. However, we have argued that some existing political arrangements involve lobbies acting in a public and sequential manner. It is thus important to study sequential common agency.

Our two main results relate to e±ciency and total amount of trasfers. The ¯rst result is that in a sequential game all subgame-perfect equilibria select an e±cient action. In a simultaneous game all coalition-proof equilibria select an e±cient action. As subgame-perfection is generally viewed as a much weaker requirement than coalition-proofness, our result strengthens the conclusion that in a common agency game one can expect an e±cient outcome to arise. As Besley and Coate [4, page 32-40] point out, if in a simultaneous common agency game lobbies do not play truthful, there can be welfare losses. However, this paper has shown that such nontruthful equilibria do not carry over to the sequential case.

The second set of results relates to the transfer vector. While the equilibrium outcome is generically unique, there exists a continuum of equilibrium transfers. Within this continuum we single out the truthful equilibrium and the thrifty equilibrium. The truthful equilibrium corresponds to one of the truthful equilibria of the simultaneous game. In the sequantial game, the truthful equilibrium is not coalition-proof. Only the thrifty equilibrium can be coalition-proof. This result contrasts with Bernheim and Whinston's result that in a simultaneous game only a truthful equilibrium can be coalition-proof. Also, we show that the agent is worse o® in the thrifty equilibrium of the sequential game than in the truthful equilibrium of the simulatneous game.

Our results on equilibrium transfers have direct relevance for lobbying. The politician is better o® if o®ers are made simultaneously. If he can choose, he will therefore favor an institutional setting in which lobbies' contribution schedules are unobservable (of course, he will have to ¯nd a way to solve the enforcement problem implied by secrecy). Instead lobbies are in general better o® in a sequential model. Thus, if the institutional choice is left to lobbies, they will select a setting in which o®ers are public and sequential.

Common agency assumes that there are no transaction costs. Instead there may be a waste associated with campaign contributions (due, for instance, to distortions created by campaign ¯nance regulations). If this waste is increasing with the amount of transfers, then clearly a sequential institutional setting is preferred to a simultaneous setting.

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