CentER

Discussion Paper

No. 2010–97

“THE VORACITY EFFECT” AND CLIMATE CHANGE: THE IMPACT OF CLEAN TECHNOLOGIES

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September 2010

ISSN 0924-7815

TILBURG UNIVERSITY

'The voracity effect' and climate change: the impact of clean technologies*

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Abstract

In the absence of a successful international cooperative agreement over the control of emissions there is a growing interest in the role that clean technologies may play to alleviate the climate change problem. Within a non-cooperative transboundary pollution game, we investigate, analytically and within a numerical example based on empirical evidence, the impact of the adoption of a cleaner technology (i.e., a decrease in the emission to output ratio). We show that countries may respond by increasing their emissions resulting in an increase in the stock of pollution that may be detrimental to welfare. This possibility is shown to arise for a significant and empirically relevant range of parameters. It is when the damage and/or the initial stock of pollution are relatively large and when the natural rate of decay of pollution is relatively small that the perverse effect of clean technologies is strongest. Cooperation over the control of emissions is necessary to ensure that the development of cleaner technologies does not exacerbate the free riding behavior that is at the origin of the climate change problem.

JEL classifications: Q20, Q54, Q55, Q58, C73.
Keywords: transboundary pollution, renewable resource, climate change, clean technologies, differential games

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[*] An earlier version of the paper was circulated under the title: "On Cleaner Technologies in a Transboundary Pollution Game".

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1 Introduction

We investigate whether the development and/or transfer of clean technologies can alleviate the consequences of failing to reach a global international agreement over greenhouse gas (GHG) emissions[1].

Recently governments, international organizations and academics have turned their attention towards the creation and sharing of clean technologies as a possible alternative means of alleviating climate change. In the United States (US), this has taken the form of new legislation. The "Investments for Manufacturing Progress and Clean Technology (IMPACT) Act of 2009," has been introduced to facilitate the development of domestic clean energy manufacturing and production[2]. International organizations, such as the United Nations (UN), are also actively encouraging countries to fund the development of clean technologies. In 2009, the UN Environmental Program urged countries to allocate one third of the $2.5 trillion planned stimulus package (spent by the developed world to boost the economy under the financial crisis) for investing on 'greening' the world economy. The G8 summit held in July 2009 included a commitment by the members to double public investment in the research and development of climate-friendly technologies by 2015.

A second related consensus among policy makers is the need to facilitate technology transfer from developed to developing countries. This transfer of technologies has emerged as a promising solution to deal with the asymmetric abilities of different countries to undertake these costs. On September 26, 2008, leading industrialized nations (including the United States, Britain and Japan) pledged more than US$6.1 billion to the Climate Investment Funds, a pair of international investment instruments designed

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[1] Large polluters, such as the US, remained outside the Kyoto Protocol. Others that ratified the Kyoto Protocol seem unable or unwilling to reach the targets they committed to. At the G8 summit held in July 2009, for example, there was widespread disappointment at the failure of countries to agree upon how they intended to achieve the emission targets (The Economist, July 10, 2009). More recent disappointment followed at the UN Climate Conference (COP15) held in Copenhagen in December 2009 which failed to set targets for emissions or to provide a mandate for a legally binding treaty (The Economist, December 19, 2009).

[2] The IMPACT Act will set up a two-year, $30 billion manufacturing revolving loan fund for small- and medium-sized manufacturers to expand production of clean energy products. It was integrated into the Waxman-Markey Act (also known as the American Clean Energy and Security Act) passed by the US House of Representatives in June 2009.

by the World Bank to provide interim, scaled-up funding to help developing countries in their efforts to mitigate increases in greenhouse gas emissions and adapt to climate change (World Bank Press Release No:2009/092/SDN). In addition, developed countries have promised to contribute funds to assist technology transfer to developing countries[3]. Developing countries such as India and China have demanded technology transfers (as is evident from the negotiations held at Copenhagen), and international agreements such as the Asia-Pacific Partnership on Clean Development and Climate (2006) have been signed. Partner countries of the Asia-Pacific Partnership on Clean Development and Climate (Australia, Canada, India, Japan, China, South Korea, and the United States) agreed to cooperate on development and transfer of technology which enables reduction of Green House Gas (GHG) emissions.[4] At the UN Climate Change Conference in Copenhagen in December 2009, it was agreed that $30 billion should be provided in the short run for funding projects in developing countries and a long term system should be set up whereby $100 billion is provided per annum from 2020 onwards (The Economist, 19 December 2009).

There is also increasing support in the academic literature for the view that innovative technology will play a central role to resolve the climate change predicament. Barrett (2009), argues that to stablize carbon concentration at levels that are compatible with a long-run goal of an increase of the earth's temperature by 2°C with respect to the pre-industrial era will require a 'technological revolution'. Galiana and Green (2009) similarly predict that reducing carbon emissions will require an energy-technology revolution and a global technology race[5].

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[3] For example, a bill was introduced in the US to set up the International Clean Technology Deployment Fund which would aid developing countries by promoting international deployment of US clean energy technology (US Fed News, 16 July 2008). The US president launched the Major Economies Forum (MEF) in March 2009, to initiate a dialogue among developed and emerging economies to combat climate change and promote clean energy (for further details, see the White House Fact Sheet on Clean Energy Technology Announcements (14 December 2009)).

[4] For further details, refer to http://www.asiapacificpartnership.org/english/default.aspx

[5] Barrett (2006) argues that even treaties on the development of breakthrough technologies will typically share the same fate as treaties on emissions control since, unless technological breakthroughs exhibit increasing returns to scale, these treaties will fail because of the incentive of countries to free ride. However Hoel and Zeeuw (2009) show that this pessimistic outcome can be overturned if one takes into account that the adoption costs of a breakthough technology vary with the level of R&D. They show that a large coalition can be both stable and result in a significant welfare improvement.

We investigate, analytically and through a numerical example using empirical evidence, the impact of adopting cleaner technologies within a framework that considers transboundary pollution emissions and where pollution emissions accumulate into a stock and therefore have lasting repercussions on the environment, two essential features of the GHG emissions' problem. Consider a world made of n countries or regions, we determine the non-cooperative emissions policies of each region and determine the impact of having all countries simultaneously adopt a cleaner technology (captured by a decrease in their emission to output ratio). Although we cover in detail the case of identical countries, our analysis allows the discussion of the case of asymmetric regions that differ with respect to their emissions per output ratio and where a 'clean' technology is being transferred to the regions that are using a 'dirty' technology. The case of asymmetric regions can be seen as a stylized model of the transfer of technology from developed countries to developing countries and where the technology transfer results in a decrease in the emissions to output ratio in the receiving country.

The adoption of a cleaner technology reduces the marginal cost of production (measured in terms of pollution damages) thereby giving an incentive to each country to increase its production. We show that the increase in emissions associated with the increase in production can outweigh the positive environmental impact of adopting a 'cleaner' technology. The benefit of the extra consumption from the adoption of the 'clean' technology can be outweighed by the loss in welfare due to the increase in pollution. The positive shock of implementing a cleaner technology results in a more 'aggressive' and 'selfish' behavior of countries that exacerbates the efficiency loss due to the presence of the pollution externality. The qualitative result and intuition extends to the case of the transfer of a 'clean' technology which can result in an increase in the pollution emissions of the receiver country and the level of the stock of pollution. Thus, a technology transfer can result in an increase of environmental damages and a decrease of the donor countries as well as the receivers' welfare.

Our findings can be related to those of Long and Sorger (2006) that builds on Tornell and Lane (1999) to consider the impact of the cost of appropriation on growth in an economy with weak or absent property rights. Rival groups can accumulate a private asset and a common property asset. Tornell and Lane (1999) show that an increase in the rate of return of the common asset can exacerbate the rent seeking behavior of the competing groups, and may end up reducing the rate of growth of the economy. They coined the term 'voracity effect' to describe the increase in rent seeking behavior. Ploeg (2010) considers the case where the common resource is exhaustible and investigates the impact of the absence of property rights and competing rival groups on the Hartwick (1977) rule for reinvesting natural resource rents. Long and Sorger (2006) introduce a private appropriation cost, e.g. cost of money laundering or lobbying, into Tornell and Lane (1999)'s model and generalize the utility function to allow agents to derive utility from wealth as well as from consumption[6]. They show that an increase in the appropriation cost reduces the growth rate of the public capital stock and thus obtain "the striking result that high costs of money laundering are detrimental to economic growth". In our model, it is the emissions per output ratio (which can be interpreted as the cost of production in terms of emissions) that can be negatively related to the stock of pollution and countries' welfare.

We use the seminal transboundary pollution game model in Dockner and Long (1993) and Ploeg and Zeeuw (1992). In contrast with Ploeg and Zeeuw (1992) and Jorgensen and Zaccour (2001), we have taken the ratio of emissions to output as exogenously given. This captures situations where a cleaner technology is readily available in the more advanced country. Our analysis thus captures the impact of a transfer of technology only. Ploeg and Zeeuw (1992) (section 8) and Jorgensen and Zaccour (2001) consider the case where the ratio of emissions to output is endogenous and is a decreasing function of the level of the stock of clean technology. While Ploeg and Zeeuw (1992) assume that the stock of clean technology is public knowledge, Jorgensen and Zaccour (2001) consider the case where the stock of clean technology, also referred to as the stock of abatement capital, is country specific. Each country can invest in the abatement capital in addition to its control of emissions[7]. We have opted to consider exogenously given levels of ratios of emissions to output to focus on the cases where it is a transfer of a technology that is readily available. The fact that a transfer of technology may have

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[6] This feature of Long and Sorger (2006) 's model make it closer to our model where countries instantaneous objectives depend on the flow of production as well as the stock of pollution.

[7] Ploeg and de Zeeuw (1992) compare the outcome under international policy coordination and the open loop equilibrium when there is no coordination. They show that the level of production and the stock of clean technology are both higher under the non-cooperative equilibrium.

Jorgensen and Zaccour (2001) consider an asymmetric game where there exist two regions facing a pure downstream problem. They design a transfer scheme that induces the cooperative levels of abatement and satisfies overall individual rationality for both regions.

counterintuitive effects is even more striking in this simple case where the technology is readily available and free. Our conclusions definitely suggest that incentives to invest in abatement technologies need to be reevaluated in the face of the possibility of sharing the new technology with other countries.

The main policy recommendation that can be taken from this analysis is that developing cleaner technologies and sharing available clean technologies, cannot be a substitute for the difficult task of agreeing on emission restraints and finding commitment devices that ensure that agreements, such as the Kyoto Protocol and the post-Kyoto Protocol, are enforced. Facilitating the transfer of available clean technologies need to be accompanied with enforceable agreements to limit pollution emissions. A more rigorous pricing of carbon will not only give the proper incentives to initiate R&D race and the technology 'revolution' necessary to control green house gas emissions, as argued, for instance, in Barrett (2009) and Galiana and Green (2009), but it is also necessary to prevent the implementation of the innovations from exacerbating the climate change problem.

Section 2 presents the model. Section 3 defines the Markov perfect equilibrium of the model that we use. We study analytically the impact of the adoption of a cleaner technology in section 4 and offer a numerical analysis based on empirical evidence of the model parameters in section 5. Section 6 contains a discussion of the impact of a transfer of clean technologies between asymmetric regions and section 7 offers concluding remarks.

2 The Model

Consider n countries indexed by $i = 1, .., n$. Each country produces a single consumption good, ϕ_i. Production generates pollution emissions.

Let ε_i denote country i's emissions of pollution. We have:

$$\varepsilon_i = \theta_i \phi_i \tag{1}$$

where θ_i is an exogenous parameter that represents country i's ratio of emissions to output[8]. The implementation of a cleaner technology in country i is represented by a fall in θ_i.

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[8] For $n = 2$ and $\theta_1 = \theta_2 = 1$, our model is equivalent to Dockner and Long (1993).

Emissions of pollution accumulate into a stock, $P(t)$, according to the following transition equation:

$$\dot{P}(t) = \Sigma_{i=1}^{n} \varepsilon_i(t) - kP(t) \tag{2}$$

with

$$P(0) = P_0 \tag{3}$$

where $k > 0$ represents the rate at which the stock of pollution decays naturally.

For notational convenience, the time argument, t, is generally omitted throughout the paper although it is understood that all variables may be time dependent.

The instantaneous net benefits of country $i = 1, .., n$ are given by

$$b_i(\phi_i, P) = U_i(\phi_i) - D_i(P) \tag{4}$$

with

$$U_i(\phi_i) = A\phi_i - \frac{B}{2}\phi_i^2, \quad A > 0$$

and

$$D_i(P) = \frac{s}{2}P^2, \quad s > 0. \tag{5}$$

The objective of country i's government is to choose a production strategy, $Q_i(t)$ (or equivalently a pollution control strategy), that maximizes the discounted stream of net benefits from consumption:

$$\max_{Q_i} \int_0^\infty e^{-rt} b_i(\phi_i(t), P(t))\, dt \tag{6}$$

subject to the accumulation equation (2) and the initial condition (3). The discount rate, r, is assumed to be constant and identical for all countries. We define below a subgame perfect Nash equilibrium of this n-player differential game.

3 The Markov perfect equilibrium

Countries use Markovian strategies: $\phi_i(.) = Q_i(P, .)$ with $i = 1, .., n$. The n-tuple $(Q_1^*, .., Q_n^*)$ is a Markov Perfect Nash equilibrium, MPNE, if for each $i \in \{1, .., n\}$, $\{\phi_i(t)\} = \{Q_i^*(P(t), t)\}$ is an optimal control path of the problem (6) given that $\phi_j(.) = Q_j^*(P, .)$ for $j \in \{1, .., n\}, j \neq i$.

In the following section, we analyze the case where countries are identical, that is $\theta_1 = .. = \theta_n = \theta$. In this case, such a game admits a unique linear equilibrium and a continuum of equilibria with non-linear strategies (Dockner and Long (1993)). The linear equilibrium is globally defined and, therefore, qualifies as a Markov perfect equilibrium. The non-linear equilibria are typically locally defined, i.e. over a subset of the state space. We focus in this analysis on the linear strategies equilibrium. Since our contribution is to highlight an a priori unexpected outcome from the adoption of a "cleaner" technology, we wish to make sure that our result is not driven by the fact that countries are using highly "sophisticated" strategies.

Proposition 1: *The vector* $(Q, .., Q)$

$$Q_i^*(P) = Q(P) \equiv \frac{1}{B}(A - \beta\theta - \alpha\theta P), \;\; i = 1, .., n \tag{7}$$

constitutes a Markov perfect linear equilibrium and discounted net welfare is given by

$$W_i(P) = -\frac{1}{2}\alpha P^2 - \beta P - \mu, \;\; i = 1, 2 \tag{8}$$

where

$$\alpha = \frac{\sqrt{B\left(B(2k+r)^2 + (2n-1)\,4s\theta^2\right)} - (2k+r)\,B}{2(2n-1)\,\theta^2}$$

$$\beta = \frac{An\alpha\theta}{B(k+r) + (2n-1)\,\alpha\theta^2}$$

$$\mu = -\frac{(A-\beta\theta)(A-(2n-1)\,\beta\theta)}{2Br}$$

The steady state level of pollution

$$P_{SS}(\theta) = \frac{n\theta(A-\theta\beta)}{Bk + n\alpha\theta^2} > 0 \tag{9}$$

is globally asymptotically stable.

Proof: We use the undetermined coefficient technique (see Dockner et al (2000) Chapter 4) to derive the linear Markov perfect equilibrium. The details are omitted. (See Proposition 1 of Dockner and Long (1993) for the case where $\theta = 1$). ■

We note that $Q_i > 0$ iff $P < \bar{P}(\theta) \equiv \frac{1}{\theta\alpha}(A - \theta\beta)$. It is straightforward to show that $\bar{P}(\theta) > P_{SS}(\theta)$ for all $\theta \geq 0$.

4 Adoption of a cleaner technology

We consider the case where $\theta_1 = .. = \theta_n = \theta$. The implementation of a cleaner technology is captured by a decrease in the emissions to output ratio, θ, and affects all countries. Throughout this section, without loss of generality, we normalize B to 1.

It will be useful to rewrite[9] the equilibrium production strategy as

$$Q\left(P\right)=\frac{1}{2n-1}\left(\left(n-1+2n\frac{k+r}{\Omega+r}\right)A-\frac{1}{\theta}\frac{\Omega-2k-r}{2}P\right)\tag{10}$$

where

$$\Omega\equiv\sqrt{\left(2k+r\right)^{2}+\left(2n-1\right)4s\theta^{2}}.$$

The impact of a cleaner technology on equilibrium steady state pollution stock and equilibrium emissions turns out to be ambiguous. More precisely:

Proposition 2: *For any* $\theta>0$ *there exists* $\bar{s}>0$ *such that for all* $s>\bar{s}$ *we have*

$$\frac{\partial P_{SS}}{\partial\theta}<0$$

a decrease in the emissions to output ratio results in a larger stock of pollution at the steady state.

Proof: We now evaluate $\frac{\partial P_{SS}}{\partial\theta}$. We show in the appendix that

$$\lim_{\Omega\to\infty}\left(\left(\frac{n\Omega-nr-k}{2n}\right)^{2}\frac{\partial P_{SS}}{\partial\theta}\right)=-\frac{1}{2}A\frac{k\left(2n^{2}+n-1\right)+\left(3n-1\right)nr}{n}<0\tag{11}$$

and therefore

$$\lim_{s\to\infty}\frac{\partial P_{SS}}{\partial\theta}<0\blacksquare$$

Let $E\left(P\right)\equiv\theta Q\left(P\right)$, i.e. $E\left(P\right)$ denotes the emissions that are associated with the equilibrium production strategy $Q\left(P\right)$.

Proposition 3: *There exists* $\tilde{P}$ *such that*

$$E_{\theta}\left(P\right)\leq(>)0\text{ for all }P\geq(<)\tilde{P}$$

Moreover $\tilde{P}<\bar{P}$ *and* $\tilde{P}>0$.

Proof: See appendix.

The adoption of a cleaner technology results in a decrease of emissions in the short-run only when the stock of pollution is below a certain level $\tilde{P}$. The results of Propositions 2 and 3 are illustrated in Figure 1 for a discrete change of θ from θ' to $\theta''<\theta'$: there exists $\tilde{P}_{\theta'}$ such that for $P>\tilde{P}_{\theta'}$, the adoption of a clean technology results in a higher level of emissions in the short-run. When the damage caused by the stock of pollution is large enough, a cleaner technology results in an increase of emissions in the short-run as well as at the steady state (when $P>\tilde{P}_{\theta'}$).

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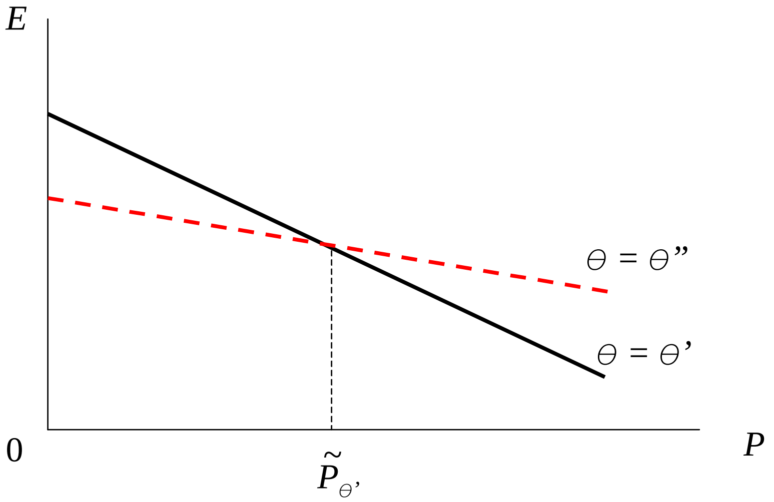
[9] For details see the appendix.

Figure 1: Emissions as a function of *P* as Θ changes from Θ' to Θ" < Θ'



Following the adoption of a cleaner technology each country increases its production. The resulting increase in emissions outweighs the positive shock of a decrease in the emissions to output ratio and can ultimately increase the stock of pollution. Proposition 2 also establishes that the adoption of a cleaner technology results in an increase of the long-run (the steady state) level of emissions.

The welfare implications of adopting a cleaner technology is thus not straightforward since the increase of production is associated with an increase of pollution. We show that implementing a cleaner technology may end up reducing social welfare, (8), in each country: $W_i(P)$ may be an increasing function of θ. From the optimality condition of a best response of a single player we have from the Hamilton Jacobi Bellman equation associated to a player's problem

$$rW(P) = U(Q) - D(P) + W'(P)(n\theta Q - kP)$$

The impact of a change in θ is thus

$$rW_\theta(P) = (U'(Q) + n\theta W')Q_\theta + W'_\theta(n\theta Q - kP) + nQW'$$

From the first order conditions of the single player's problem we have

$$U'(Q) + \theta W' = 0$$

and thus

$$rW_\theta(P) = (n-1)\,\theta W' Q_\theta + W'_\theta(n\theta Q - kP) + nQW'$$

We evaluate W_θ at $P = P_{SS}(\theta)$

$$r\left.W_\theta\right|_{P=P_{SS}(\theta)} = (n-1)\,\theta W' Q_\theta + nQW'$$

$$r\left.W_\theta\right|_{P=P_{SS}(\theta)} = (n-1)\,(\theta Q_\theta + Q)\,W' + QW'$$

$$r\left.W_\theta\right|_{P=P_{SS}(\theta)} = (n-1)\,E_\theta W' + QW'$$

When production remains unchanged, for an infinitesimal decrease in θ there is a decrease of emissions by Q which results in an increase of welfare since $W' < 0$. Thus the second term of the right hand side of $r\left.W_\theta\right|_{P=P_{SS}(\theta)}$ is positive and reflects the positive impact of a decrease in θ due the reduction of the country's own emissions if production is left unchanged. The first term of the right hand side reflects the impact on a country of the reaction of the other $n-1$ countries to the change in θ. If the decrease in θ results in a decrease of emissions then the first term is negative and the impact of a clean technology on welfare is unambiguously positive. However if $E_\theta < 0$ then the sign of W_θ is indeterminate. We show below that W_θ may well be positive. The expression of W_θ is too cumbersome to allow a determination of the sign of W_θ for all parameter values. In this section we show analytically, in a limit case (i.e. when the damage from pollution is large enough), that W_θ is positive: a decrease of the emissions per output ratio reduces welfare. In the next section, we investigate the sign of W_θ numerically using plausible values of the parameters and show that there exist a range of realistic values of the parameters under which W_θ is positive.

For an analytical analysis of the sign of $\left.W_\theta\right|_{P=P_{SS}(\theta)}$ rewrite

$$r\left.W_\theta\right|_{P=P_{SS}(\theta)} = (n-1)\,\theta W' Q_\theta + nQW'$$

as

$$r\left.W_\theta\right|_{P=P_{SS}(\theta)} = \left(\frac{\theta Q_\theta}{Q} + \frac{n}{n-1}\right)(n-1)\,QW'$$

Recall that the equilibrium production strategy is given by

$$Q = \frac{1}{2n-1}\left(\left(n-1+2n\frac{k+r}{\Omega+r}\right)A - \frac{1}{\theta}\frac{\Omega-2k-r}{2}P\right) \tag{12}$$

Taking the derivative of Q wrt θ gives

$$Q_\theta = -2A\frac{n}{(2n-1)}\frac{k+r}{(\Omega+r)^2}\Omega_\theta + \frac{1}{\theta^2}\frac{\Omega-2k-r}{2(2n-1)}P - \frac{1}{\theta}\frac{\Omega_\theta}{2(2n-1)}P \tag{13}$$

Evaluating $\frac{Q_\theta}{Q}$ at the steady state where $Q = \frac{kP_{SS}}{n\theta}$ gives

$$\frac{Q_\theta}{Q} = -2A\frac{n}{(2n-1)}\frac{k+r}{(\Omega+r)^2}\Omega_\theta\frac{n\theta}{kP_{SS}} + \frac{1}{\theta}\frac{n}{2(2n-1)k}(\Omega-2k-r-\theta\Omega_\theta) \tag{14}$$

which after using the facts that

$$\theta\Omega_\theta = \frac{\Omega^2-(2k+r)^2}{\Omega} = \Omega - \frac{(2k+r)^2}{\Omega}$$

that

$$\lim_{s\to\infty}\Omega = \infty$$

and that[10]

$$\lim_{s\to\infty}\Omega P_{SS} = 2\theta(n-1)A \tag{15}$$

gives after substitution and simplification

$$\lim_{s\to\infty}\left(\frac{\theta Q_\theta}{Q} + \frac{n}{n-1}\right) = -\frac{nr}{2k}\frac{3n-1}{(n-1)(2n-1)} < 0$$

Main Proposition: *For any $n > 1$, there exists $\bar{s} > 0$ such that $W_\theta|_{P=P_{SS}(\theta)} > 0$ for all $s > \bar{s}$.*

The positive shock of a cleaner technology results in a more "aggressive" or "voracious" behavior of countries that exacerbates the efficiency loss due to the presence of the pollution externality. The intuition behind this result is similar to the one behind the 'voracity effect' in Tornell and Lane (1999) and Long and Sorger (2006), obtained in the context of growth under weak or absent property rights. The main feature of the equilibrium that drives this 'voracity' effect is the fact that the non-cooperative emissions strategies are downward sloping functions of the stock of pollution[11]. Unlike static

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[10] See the appendix for details.

[11] This feature is related to the notion of intertemporal strategic substitutability at the steady state of the MPNE, used in Jun and Vives (2004) which provides a taxonomy for possible strategic interactions in continuous-time dynamic duopoly models. Unlike the duopoly games covered in Jun and Vives (2004), in our model there is one state variable. Intertemporal strategic substitutability at the steady state of the MPNE, corresponds to a situation where an increase in the state variable of one firm decreases the action of its rival.

games or dynamic games where countries would choose emissions paths, when Markovian strategies are considered to construct a Nash equilibrium, a country can still influence its rival's action path even though it is taking its rival's strategy as given. When the rival's emission strategy is a downward sloping function of the stock of pollution, the action of increasing one's emissions bears an additional benefit: increase in one's emissions would result ceteris paribus in a larger level of the stock of pollution which would in turn induce one's rival to reduce her emissions. This possibility to influence rival's emissions' path results in an overall more polluted world than would prevail if each country takes the rival's actions as given. The response of each country to a positive shock such as a reduction of the emissions to output ratio can be to expand its output. In this 'aggressive' setup, the extent of the increase in output is such that the increase in pollution that follows and the damage it creates outweigh the benefits from the additional consumption.

Remark: The Main Proposition's content mirrors the comparative static result in oligopoly theory that an increase in firms' costs may end-up increasing firms' profits (see Seade (1983) and Dixit (1986)). In our framework countries' instantaneous payoffs do not depend on each other's flows of emissions directly; they are interrelated through the damage from the stock of pollution, a stock to which they all contribute. A decrease of the emissions per output ratio is analogous to a decrease in the damage from the production of a unit of output. However in our context, the dynamic dimension, an essential feature of a climate change model, brings an additional level of interaction between players, compared to a static or repeated game, that contributes to our result. Indeed, if one considers the simple case of two countries and where the damage arises from the flow of the sum of pollution (i.e., if the cost of pollution were $\frac{1}{2}s\left(E_1+E_2\right)^2$ instead of $\frac{1}{2}sP^2$), it can easily be shown that a decrease of the emissions to output ratio is always welfare improving, for any arbitrarily large value of the damage parameter s, in sharp contrast with the Main Proposition. One can possibly retrieve the 'voracity' effect, present in the MPNE, in a static framework using a conjectural variations approach. Dockner (1992) considered a dynamic oligopoly in the presence of adjustment costs and has shown that any steady state subgame-perfect equilibrium of the dynamic game can be viewed as a conjectural variations equilibrium of a corresponding static game[12]. However,

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[12] Dockner (1992) shows that, in the case of a differential game with linear demand and quadratic costs, that the dynamic conjectures consistent with closed-loop steady state equilibria are negative, constant and symmetric.

the analysis of the full fledged differential game allows to, first capture the intertemporal nature of the pollution game under consideration, and second take into account the transition dynamics when determining the impact of a decrease in the emissions per output ratio.

5 Numerical example

We investigate the sign of W_θ numerically, using 'plausible' values of the parameters based on empirical evidence. We also present the effect of non-marginal changes in the emissions per output parameter on equilibrium welfare.

We would like to emphasize the absence of consensus in the literature about precise values of the parameters of the model. This is partly due to the large uncertainty surrounding the economic repercussions of climate change. After a brief description of the ranges within which each parameter may fall, we start by presenting the impact of a change in the emissions per output ratio in a benchmark case and then conduct sensitivity analysis with respect to parameter values.

The value of the discount rate is the subject of important debates: The Stern Review uses 1.4%, Nordhaus uses 3 to 4%, others view discounting as unethical and that the rate of discount should be nil (Heal (2009)). Most Integrated Assessment Models (IAMs) (for example, the DICE model (Nordhaus (1994)), the RICE model (Nordhaus and Yang (1996)), the ENTICE model (Popp (2003))) could have up to 20 regions but usually consider between 8 to 15 regions. Following Nordhaus (1994), Hoel and Karp (2001) among others, we use the natural rate of decay $k = 0.005$.

The damage parameter is derived from estimates of the damage caused by a doubling of the stock of GHG. Let x denote the percentage of world GDP lost due to a change in temperature if the stock of pollution doubles. The value of x is undoubtedly the subject of heated debates on the political and academic arena and is crucial to define the extent and the pace at which climate change related policies need to be implemented. In a recent study Tol (2009) conducts the difficult task of aggregating the results of fourteen studies on climate change's economic repercussions and gives a relationship between the increase of temperature and the damage using different scenarios. The upper bound of the 95 percent confidence interval of x is approximately 10% under the assumption that temperature would rise by 2.5°C and 12.5% if the temperature increases by 3°C.[13] Based on experts opinions reported in Nordhaus (1994), Karp and Zhang (2010) use 21% as the maximum value for x. As Tol (2009) points out, most of the studies do not give any estimation of x for changes in temperature that exceed 3°C, do not look at a time horizon beyond 2100 and their estimates typically ignore important non-market impacts such as extreme climate scenarios, biodiversity loss or political violence due to the increasing scarcity of resources induced by climate change. Taking into account market and non-market impacts, Heal (2009) estimates that the cost could be 10% of world income. Taking into account the risk of catastrophe, the Stern Review estimates the 95th percentile to be 35.2% loss in global per-capita GDP by 2200. Thus, although for example the Stern Review uses 5% as an estimate of x, it considers it as a conservative estimate. We will use 2.5% in the benchmark case, and conduct a sensitivity analysis with respect to x, using $x = 5\%$ and $x = 10\%$.

We start by describing the benchmark case with $n = 10$, $x = 0.025$, $k = 0.005$ and $r = 0.025$. We define the function

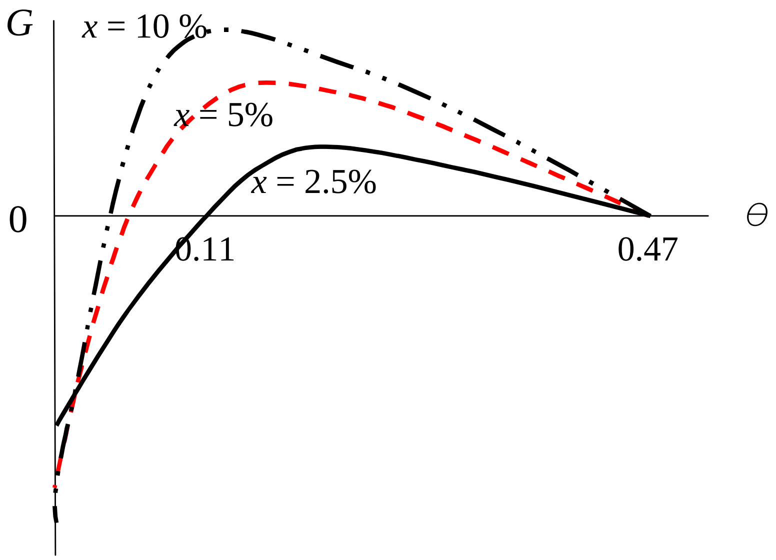
$$G(P, \theta, \theta_0) = \frac{W(P)|_{\theta} - W(P)|_{\theta=\theta_0}}{W(P)|_{\theta=\theta_0}}$$

which represents the relative change in welfare as θ changes from θ_0 to θ and the stock of pollution is P. We plot in Figure 2, $G(P_{SS}(\theta_0), \theta, \theta_0)$ where θ_0 is set to 0.47 kg of CO2/\$ of GDP (World GDP was estimated at \$ 61.1 trillion and emissions of CO2 at 28.5 billions of metric tons) and set $B = \theta_0^2$ so that when $\theta_0 = 1$ we retrieve the same specification of the linear quadratic models of transboundary pollution where instantaneous utility U is expressed in terms of emissions (e.g., Dockner and Long (1993), Ploeg and Zeeuw (1992), List and Mason (2001), Hoel and Karp (2001)). We can observe that a decrease of the emissions per output from θ_0 can result in a loss in welfare: the welfare loss from the increase in pollution emissions outweighs the welfare gains from an increase in consumption. Note that in Figure 2, $W(P)|_{\theta=\theta_0} < 0$ and therefore when $G(P, \theta, \theta_0) > 0$ we have $W(P)|_{\theta} - W(P)|_{\theta=\theta_0} < 0$.

---

[13] In Nordhaus (1994) the 95 percent confidence interval of x is $(-30.0, 0)$ under the assumption that temperature rises by 3°C.

Figure 2: G at $P = P_{SS}(\Theta_0)$ as a function of Θ

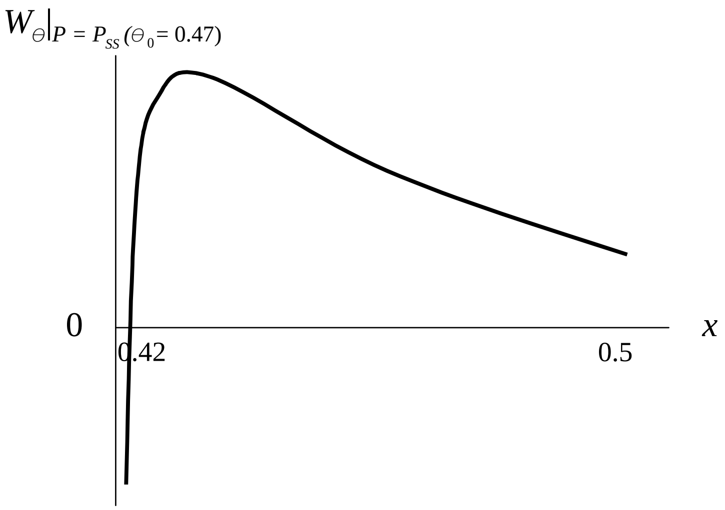


The benchmark case represents a relatively optimistic scenario in terms of the damage of pollution. The sensitivity analysis that follows demonstrates that the consideration of less optimistic parameter values strengthens the 'voracity' effect that follows a reduction in the emissions per output ratio.

The emissions per output ratio has to decrease below $\tilde{\theta}_0 = 0.11$ (i.e., a decrease of 76.28%) for the decrease to be welfare enhancing. The threshold $\tilde{\theta}_0$ falls to 0.0699 (i.e., a decrease of 85.1%) when we use $x = 5\%$ and to 0.0455 (i.e., a decrease of 90.3%) when $x = 10\%$.

We plot $W_\theta|_{P=P_{SS}(\theta_0=0.47)}$ as a function of x.

Figure 3: $W_{\theta}|_{P = P_{SS}(\theta_0 = 0.47)}$ as a function of x



For the benchmark case, for all $x > 0.42\%$ we have $W_\theta|_{P=P_{SS}(\theta_0=0.47)} < 0$. A marginal decrease in emissions per out ratio reduces welfare. The relationship of $W_\theta|_{P=P_{SS}(\theta_0=0.47)}$ with respect to x (which is a proxy for s) mirrors the result obtained analytically for the behavior of $W_\theta|_{P=P_{SS}(\theta)}$ in the limit case where $s \to \infty$. The larger the damage parameter the more likely a decrease of the emissions per output ratio will be welfare reducing.

Figure 4 gives that the graph of $W_\theta|_{P=Z*P_{SS}(\theta_0=0.47)}$ is a strictly increasing function of Z, where Z is parameter that sets the initial level of the stock pollution relative to the steady state stock pollution.

Figure 4: $W_{\Theta}|_{P\,=\,Z\,*\,P_{SS}(\Theta_0=\,0.47)}$ as a function of Z

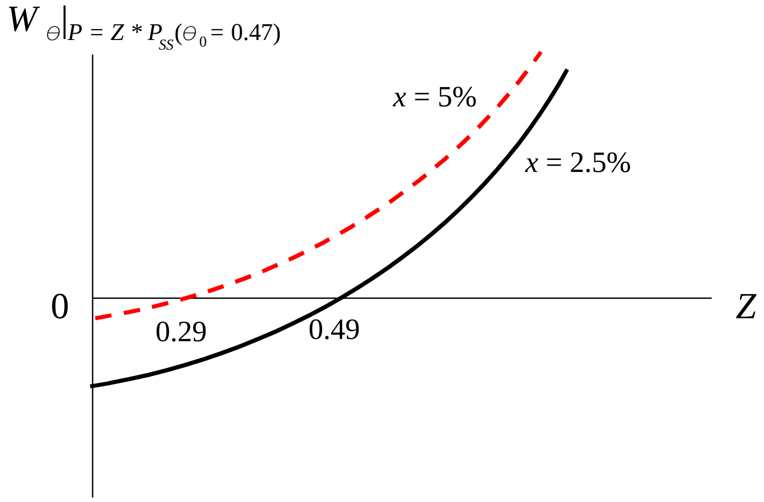


Figure 4 shows that $W_\theta|_{P=Z*P_{SS}(\theta_0=0.47)}$ is positive for $Z > \tilde{Z} = 0.49$. The larger the stock of pollution at which we introduce a cleaner technology the more likely this will result in a welfare loss. The value of $\tilde{Z}$ decreases to 0.38 when $x = 5\%$ and to 0.29 when $x = 10\%$.

Similarly, one can shown that $W_\theta|_{P=P_{SS}(\theta_0=0.47)}$ is a strictly decreasing function of k and is positive for $k < \tilde{k} = 0.014$. The smaller the rate of decay the more likely the implementation of a clean technology can reduce all players welfare. The threshold $\tilde{k}$ increases to 0.019 when $x = 5\%$ and to 0.027 when $x = 10\%$.

These results represent rather pessimistic conclusions about the ability of technology to alleviate the tragedy of the commons, since it is when the damage is important and/or the stock of pollution is large enough, and nature is least able to absorb pollution, that a decrease of the emissions per output ratio is mitigated by the increase in pollution emissions of each player to the point where welfare diminishes.

Moreover, it can be shown that $W_\theta|_{P=P_{SS}(\theta_0=0.47)}$ is a strictly decreasing function of r and is positive for $r < 0.252$. It is when players are the most patient that a clean technology can reduce all players welfare. This is a rather surprising result which may appear to conflict with the intuition gained from the folk theorem. From the folk theorem, in a repeated game, the larger the discount rate the less possible it is to sustain cooperation. Here players are not using trigger strategies to sustain cooperation, only the non-cooperative scenario is analyzed. Moreover the impact of r is on the change on welfare due to a change in θ and not welfare itself.

This numerical example has demonstrated that the 'voracity' effect is not a mere theoretical possibility. It is shown to be strong for a significant and empirically relevant range of parameters. It is when the damage is relatively large and/or the initial stock of pollution are relatively large and when the natural rate of decay of pollution is relatively 'small', i.e. precisely the situations where the tragedy of the commons is at its worse, that the 'voracity' effect prevails.

6 Transfer of clean technologies

Consider now the case of an asymmetric pollution game where countries differ with respect to their emissions per output ratios: we no longer assume that $\theta_i = \theta_j$ for $i, j = 1, .., n$. The analysis of the previous sections can be reproduced. However, since the intuition of these results obtained in the sections above carry over to this case, we refrain from doing so and just give the description of the results.

For simplicity consider the case of two groups of countries: $\theta_i = \theta_l$ with $i = 1, ..n_C$ and $\theta_i = \theta_h > \theta_l$ with $i = n_C + 1, .., n$. Countries are identical in all respects except for the emissions per output ratios. Clearly if $\frac{n_C}{n}$ is small enough then, any transfer of clean technologies from the group of clean countries to the group of dirty countries, captured by a decrease in θ_h can lead to an increase of the dirty countries' emissions and a smaller welfare worldwide.

This possibility can also be shown to arise by considering the limit case where: $\theta_l = 0$. In that case the group of clean countries cannot condition their action on the stock of pollution even though they are impacted by it. Each clean country chooses to produce at a rate A. The objective of each of a 'dirty' country's government is to choose a production strategy, $Q_i(t)$ (or equivalently a pollution control strategy), that maximizes the discounted stream of net benefits from consumption subject to the accumulation equation

$$\dot{P}(t) = \Sigma_{i=n_C+1}^{n} \varepsilon_i(t) - kP(t) \tag{16}$$

and the initial condition (3).

Clearly if technology were fully transferable we would have a decrease of the dirty countries' emissions per output ratio from θ_h to 0 and therefore a transfer of technology results in a decrease of emissions and an increase in all countries' welfare. However, technologies are typically only partially transferable and the case $\theta_l = 0$ is considered here only for an illustration.

Even though this is an asymmetric differential game, it is still analytically tractable and one can follow identical steps used in the sections above to show that a decrease in θ_h may result in an increase in emissions of pollution, therefore reducing the clean countries' welfare. Moreover, if the increase in emissions is large enough, this may result in all countries' welfare diminishing following a 'partial' transfer of clean technologies to the dirty countries.

7 Concluding Remarks

Given the unsuccessful attempts of multilateral efforts to control emissions and slow down the human contribution to climate change, the development and use of cleaner technologies is often invoked as the way out of the 'brink'. This paper shows that the failure of coordination over emissions may prevent the international community from ripping any benefit from the creation and adoption of a cleaner technology and may even result in exacerbating the tragedy of the commons.

The decrease of the emissions per output ratio has two components, the direct effect which is a decrease of emissions if the quantity produced by each player remains unchanged and the indirect effect since quantity produced changes and so do the emissions. Emissions may increase following the adoption of a cleaner technology, and the resulting increase in pollution damages can be substantial enough to annihilate the positive impact of the direct effect on welfare. We have shown that this may arise for a wide range of 'realistic' values of the parameters of the model. Moreover, the possibility that emissions per output ratio and world emissions can evolve in opposite directions is supported by recent anecdotal evidence. While the world's emissions per output ratio decreased from 0.54 (kilograms of CO2 per 1$ of GDP (PPP)) in 1990, to 0.50 in 2000 and 0.47 in 2007, world's emissions of CO2 increased from 21,899 millions of metric tons in 1990 to 24,043 in 2000 and 29,595 in 2007 (see The Millennium Development Goals Report 2010 (United Nations)).

Our results extend to the case of an asymmetric pollution game where countries differ with respect to their emissions per output ratios: a transfer of clean technologies from 'clean' countries to 'dirty' countries may result in a loss of welfare for the clean countries and even a loss of welfare for all countries.

This brings into question the effectiveness of the World Bank's Clean Technology Fund to which developed countries have pledged over 6 billion dollars and proposed legislation such as the IMPACT Act 2009 in the US and other policy measures that are currently being pursued by countries to develop and spread clean technologies. Our analysis shows that it may only be possible to reap the benefits arising from such measures if the implementation of clean technologies is accompanied with enforceable agreements to limit pollution emissions.

The results of this paper should not be interpreted as supporting the use of dirtier of technologies. The main policy recommendation is that the efforts of discovering and using clean technologies should not be viewed as a substitute for the need to succeed in a multilateral coordination of emissions. Similarly, the potential negative effects on welfare of transfers of clean technologies should be interpreted as a recommendation to accompany technology transfers with agreements over limitations on emissions of the receiving country.

The effort of creating and transferring clean technologies and the effort of coordinating the control over emissions should be pursued jointly. Intuition would suggest that the potential negative impact of clean technologies would not take place if the adoption of a clean technology were accompanied with a well designed limit over emissions. Although this is intuitive, this idea deserves to be carefully studied as the impact of quotas in dynamic games are far from trivial (see, e.g., Dockner and Haug (1990 and 1991)).

Barrett (2009) and Galiana and Green (2009), among others, argue that a substantial and comprehensive change in technology is required to stabilize atmospheric concentrations and that market incentives are insufficient to induce the necessary technological change. Barrett (2009) concludes that 'international cooperation is needed to set a carbon penalty, to increase R&D spending,...'. Our analysis shows that the international cooperation over a carbon penalty or emissions control is not only needed as an incentive to induce R&D and innovation, it is necessary to ensure that the development of cleaner technologies does not exacerbate the free riding behavior that is at the origin of the climate change problem. We have considered the impact of an exogenous change in the emissions per output ratio. The results of this paper suggest that the analysis of a model that embeds this framework and where investment in R&D to reduce emissions per output is taken into account, can be a promising line of future research.

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Appendix

Proof of Proposition 3

Proposition 3: *There exists* $\tilde{P}$ *such that*

$$E_\theta\left(P\right) \leq (>)0 \text{ for all } P \geq (<)\tilde{P}$$

Moreover $\tilde{P} < \bar{P}$ *and* $\tilde{P} > 0$.

Proof:

$$E_\theta\left(P\right) = \frac{1}{2n-1}\left(n-1+2n\frac{k+r}{\Omega+r}-2n\frac{k+r}{\left(\Omega+r\right)^2}\Omega_\theta\theta\right)A-\frac{1}{2n-1}\frac{\Omega_\theta}{2}P$$

Using the fact that

$$\theta\Omega_\theta = \frac{\Omega^2-\left(2k+r\right)^2}{\Omega} = \Omega-\frac{\left(2k+r\right)^2}{\Omega} > 0$$

gives

$$E_\theta\left(P\right) = E_\theta\left(0\right)-\frac{1}{2n-1}\frac{1}{2\theta}\left(\Omega-\frac{\left(2k+r\right)^2}{\Omega}\right)P$$

with

$$E_\theta\left(0\right) = \frac{1}{2n-1}\left(n-1+2n\frac{k+r}{\Omega+r}-2n\frac{k+r}{\left(\Omega+r\right)^2}\left(\Omega-\frac{\left(2k+r\right)^2}{\Omega}\right)\right)A.$$

Thus

$$E_\theta\left(P\right) < 0$$

iff

$$\frac{2\left(2n-1\right)\theta E_\theta\left(0\right)}{\left(\Omega-\frac{\left(2k+r\right)^2}{\Omega}\right)} = \tilde{P} < P.$$

After simplification of $E_\theta\left(0\right)$ we have

$$E_\theta\left(0\right) = \frac{1}{2n-1}\left(n-1+2nr\frac{\left(k+r\right)}{\left(\Omega+r\right)^2}\left(1+\frac{\left(2k+r\right)^2}{\Omega}\right)\right)A > 0 \tag{17}$$

thus showing that $\tilde{P} > 0$.

We now compare $\tilde{P}$ to $\bar{P}$. We have

$$\bar{P} = \frac{2\theta\left(n-1+2n\frac{k+r}{\Omega+r}\right)A}{\Omega-2k-r}$$

and thus

$$\frac{\tilde{P}}{\bar{P}} = \frac{2\left(2n-1\right)\theta E_{\theta}\left(0\right)}{\left(\Omega - \frac{(2k+r)^2}{\Omega}\right)} \frac{\Omega - 2k - r}{2\theta\left(n - 1 + 2n\frac{k+r}{\Omega+r}\right)A}$$

or

$$\frac{\tilde{P}}{\bar{P}} = \frac{\Omega}{\left(\Omega + 2k + r\right)} \frac{\left(2n-1\right)E_{\theta}\left(0\right)}{\left(n - 1 + 2n\frac{k+r}{\Omega+r}\right)A}.$$

After simplification and substitution of $E_{\theta}\left(0\right)$ we have

$$\frac{\left(2n-1\right)E_{\theta}\left(0\right)}{\left(n - 1 + 2n\frac{k+r}{\Omega+r}\right)A} = \frac{n - 1 + 2n\frac{k+r}{\Omega+r} - 2n\frac{k+r}{(\Omega+r)^2}\left(\Omega - \frac{(2k+r)^2}{\Omega}\right)}{n - 1 + 2n\frac{k+r}{\Omega+r}}$$

or

$$\frac{\left(2n-1\right)E_{\theta}\left(0\right)}{\left(n - 1 + 2n\frac{k+r}{\Omega+r}\right)A} = 1 - \frac{2n\frac{k+r}{(\Omega+r)^2}\left(\Omega - \frac{(2k+r)^2}{\Omega}\right)}{n - 1 + 2n\frac{k+r}{\Omega+r}} < 1$$

implying

$$\frac{\tilde{P}}{\bar{P}} < 1 \blacksquare$$

Derivation of (10)

The equilibrium production strategy is

$$Q = A - \frac{An\alpha\theta^2}{k + r + (2n-1)\,\alpha\theta^2} - \frac{\alpha\theta^2}{\theta} P \tag{18}$$

where

$$\theta^2\alpha = \frac{-2k - r + \sqrt{(2k+r)^2 + (2n-1)\,4s\theta^2}}{2\,(2n-1)}$$

Let

$$\Omega = \sqrt{(2k+r)^2 + (2n-1)\,4s\theta^2}$$

so

$$\alpha\theta^2 = \frac{\Omega - 2k - r}{2\,(2n-1)}.$$

Substitute into the equilibrium production strategy

$$Q = A - \frac{An\frac{\Omega-2k-r}{2(2n-1)}}{k + r + (2n-1)\,\frac{\Omega-2k-r}{2(2n-1)}} - \frac{\frac{\Omega-2k-r}{2(2n-1)}}{\theta} P \tag{19}$$

$$Q = A - \frac{An\frac{\Omega-2k-r}{2(2n-1)}}{k + r + \frac{\Omega-2k-r}{2}} - \frac{1}{\theta}\frac{\Omega - 2k - r}{2\,(2n-1)} P \tag{20}$$

$$Q = A - \frac{An\frac{\Omega-2k-r}{2(2n-1)}}{\frac{\Omega+r}{2}} - \frac{1}{\theta}\frac{\Omega - 2k - r}{2\,(2n-1)} P \tag{21}$$

which after simplification gives (10).

Derivation of (11)

The steady state is determined as the solution to

$$A\frac{n-1}{2n-1} + 2A\frac{n}{(2n-1)}\frac{k+r}{\Omega+r} = \left(\frac{k}{n\theta} + \frac{1}{\theta}\frac{\Omega - 2k - r}{2\,(2n-1)}\right) P \tag{22}$$

which after simplification yields

$$A\left(n - 1 + 2n\frac{k+r}{\Omega+r}\right)\theta = \left(\frac{n\Omega - nr - k}{2n}\right) P \tag{23}$$

Taking the derivative with respect to θ and multiplying each side by $\left(\frac{n\Omega-nr-k}{2n}\right)$ and using (23) gives

$$
\begin{aligned}
& A\left(n-1+2n\frac{k+r}{\Omega+r}\right)\left(\frac{n\Omega-nr-k}{2n}\right)+A\left(-2n\frac{k+r}{(\Omega+r)^2}\right)\left(\frac{n\Omega-nr-k}{2n}\right)\theta\Omega_\theta \\
= & \frac{1}{2}A\left(n-1+2n\frac{k+r}{\Omega+r}\right)\theta\Omega_\theta+\left(\frac{n\Omega-nr-k}{2n}\right)^2 P_\theta \qquad (24)
\end{aligned}
$$

Using the facts that

$$
\theta\Omega_\theta=\frac{\Omega^2-(2k+r)^2}{\Omega}=\Omega-\frac{(2k+r)^2}{\Omega}
$$

gives

$$
\begin{aligned}
& \left(\frac{n\Omega-nr-k}{2n}\right)^2 P_\theta \\
= & A\left(n-1+2n\frac{k+r}{\Omega+r}\right)\left(\frac{n\Omega-nr-k}{2n}\right)+ \\
& A\left(-2n\frac{k+r}{(\Omega+r)^2}\frac{n\Omega-nr-k}{2n}-\frac{1}{2}\left(n-1+2n\frac{k+r}{\Omega+r}\right)\right)\frac{\Omega^2-(2k+r)^2}{\Omega}
\end{aligned}
$$

The RHS can be written as a fraction of two polynomials of degree 3 in Ω. We can therefore determine

$$
\lim_{\Omega\to\infty}\left(\left(\frac{n\Omega-nr-k}{2n}\right)^2 P_\theta\right)
$$

as the fraction of the two monomials of degree 3 in the numerator and the denominator respectively, which gives (11).

Derivation of (15)

The steady state is given by

$$
A\left(n-1+2n\frac{k+r}{\Omega+r}\right)\theta=\left(\frac{n\Omega-nr-k}{2n}\right)P \qquad (25)
$$

which can be rewritten as

$$
A\left(n-1+2n\frac{k+r}{\Omega+r}\right)\theta=\left(\frac{1}{2}-\frac{nr+k}{2n}\right)\Omega P \qquad (26)
$$

or

$$\Omega P = \frac{A\left(n-1+2n\frac{k+r}{\Omega+r}\right)\theta}{\left(\frac{1}{2}-\frac{nr+k}{2n}\right)} \tag{27}$$

therefore

$$\lim_{\Omega\to\infty}\Omega P = 2A\left(n-1\right)\theta.$$

Since

$$\lim_{s\to\infty}\Omega = \infty$$

we have

$$\lim_{s\to\infty}\Omega P = 2A\left(n-1\right)\theta.$$