What are the effects of monetary policy on output? Results from an agnostic identification procedure.\*

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# PRELIMINARY COMMENTS WELCOME

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#### Abstract

This paper proposes to estimate the effects of monetary policy shocks by a new "agnostic" method, imposing sign restrictions on the impulse responses of prices, nonborrowed reserves and the federal funds rate in response to a monetary policy shock. No restrictions are imposed on the response of real GDP to answer the key question in the title. We find that "contractionary" monetary policy shocks have an ambiguous effect on real GDP. Otherwise, the results found in the empirical VAR literature so far are largely confirmed. The results could be paraphrased as a new Keynesian-new classical synthesis: even though the general price level is sticky for a period of about a year, money may well be close to neutral.

We provide a counterfactual analysis of the early 80's, setting the monetary policy shocks to zero after December 1979, and recalculating the data. We found that the differences between observed real GDP and counterfactually calculated real GDP was not very large. Thus, the label "Volcker-recession" for the two recessions in the early 80's appears to be misplaced.

# 1 Introduction

What are the effects of monetary policy on output? This key question has been the focus of a substantial body of the literature. And the answer seems easy. The "Volcker recessions" at the beginning of the 80's have shown just how deep a recession a sudden tightening of monetary policy can produce. Alternatively, look at figure 1, which juxtaposes movements in the Federal Funds Rate from 1965 to 1996 with growth rates in real GDP, flipped upside-down for easier comparison. In particular for the first half of that sample, it is striking, how rises in the Federal Funds Rate are followed by falls in output (visible as *rises* in the dotted line, due to the upside-down flipping). This issue is closed.

Or is it? Eyeball econometrics such as figure 1 or case studies like the Volcker recessions can be deceptive: many things are going on simultaneously in the economy, and one may want to be careful to consider just a single cause-and-effect story. If the answer really is so obvious, it should emerge equally clearly from an analysis of multiple time series, which allows for additional channels of interaction and other explanations, at least in principle. Thus, many researchers have followed the lead of Sims (1972,1980,1986) and proceeded to analyze the key question in the title with the aid of vector autoregressions. Rapid progress has been made in the last ten years. Moving beyond a bivariate money-output structure, the federal funds rate in particular was recognized to be useful for gauging the stance of monetary policy, see Bernanke and Blinder (1992). The 'price puzzle', raised by Sims (1992), and other anomalies led to further refinements in the identification schemes and the inclusions of variables such as nonborrowed reserves, total reserves as well as a commodity price index in these VAR studies, see e.g. Eichenbaum (1992), Strongin (1992), Christiano and Eichenbaum (1992a,b), Leeper and Gordon (1992), Gordon and Leeper (1994), Christiano, Eichenbaum and Evans (1994, 1996a). Recently, Bernanke and Mihov (1996a,b) have reconciled a number of these approaches in a unifying framework, and Leeper, Sims and Zha (1996) have summarized the current state of the literature, while adding new directions on their own. Additional excellent surveys are in Canova (1995) and Christiano, Eichenbaum and Evans (1997).

There seems to be a growing agreement, that this literature has reached a healthy state: many "puzzles" have been resolved, and there are now a variety of identification schemes to choose from which make the impulse responses to a monetary policy shock look "reasonable". Based on these results, researchers have begun to develop theoretical models which replicate the findings of the VAR literature, aiming at a unified theory of both, see e.g. Christiano, Eichenbaum and Evans (1996b,1996c,1997) or Leeper and Sims (1994). In fact, Christiano, Eichenbaum and Evans (1996c) state directly, that "plausible models of the monetary transmission mechanism should be consistent with at least the following facts about the effects of a contractionary monetary policy shock: (i) Aggregate price level initially responds very little, (ii) Aggregate output falls, (iii) Interest rates initially rise, (iv) real wages decline by a modest amount, (v) profits fall." The conclusions of these authors, that these postulates are indeed "facts" stems from their analysis of the time series evidence, following and extending Christiano, Eichenbaum and Evans (1996a). In summary, this literature indeed reinforces the view emerging from the first simple look at the data as in figure 1.

We shall not dispute that the progress made hitherto has been great and that these conclusions are comforting. But perhaps a note of caution is appropriate nonetheless. There seems to be a degree of circularity in the way the literature currently draws its conclusions. A grossly oversimplified and satirized version of the style of argumentation runs as follows. According to many undergraduate textbooks in macroeconomics, a monetary contraction should raise the federal funds rate, lower prices and reduce real output. If a particular identification scheme does not accomplish this, then the observed responses are dubbed a "puzzle", and the search continues. If an identification scheme succeeds in matching undergraduate textbook intuition, it is called a success<sup>1</sup>. An argument is then made, that the identification scheme looks reasonable for some other reason by pointing, say, to the operating procedures of the Federal Reserve System, or by pointing to favourable comparisons to other modes of analysis such as Romer and Romer (1989,1990) or Boschen and Mills (1991). Finally, the estimated impulse responses, which happen to accord with undergraduate textbook intuition, are reported as "facts" about the monetary transmission mechanism, which a good theory ought to capture, see the quote of Christiano, Eichenbaum and Evans (1996c) above. There is a danger here that we just get out what we have stuck in, albeit a bit more polished and with numbers attached: namely the undergraduate textbook intuition. Gali (1992) therefore directly asks, whether the "IS-LM model fit[s] the postwar U.S. data" rather than indirectly presuming that this is the only model worth fitting<sup>2</sup>. There could be scope for asking whether monetary neutrality fits the postwar U.S. data as well: so far, the literature has not done that<sup>3</sup>.

Leeper, Sims and Zha (1996) defend this somewhat circular reasoning by arguing for the reasonableness of impulse responses as an "informal" identification criterion: "We use it informally, in that we focus attention on results that do not produce implausible impulse responses. Our criterion for plausibility is loose. We do not expect to see strongly positive responses from prices, output, or monetary aggregates to monetary contraction, nor strongly negative responses from interest rates. ... Our procedure differs from the standard practice of empirical researchers in economics only in being less apologetic. Economists adjust their models until they both fit the data and give 'reasonable' results. There is nothing unscientific or dishonest about this."

This paper proposes to push this idea all the way, and to identify the effects of monetary policy shocks via an "agnostic" method solely by directly imposing sign restrictions on the impulse responses of prices, nonborrowed reserves and the federal

<sup>&</sup>lt;sup>1</sup>What is described here is a specification search, see also Learner (1978)

<sup>&</sup>lt;sup>2</sup>See also the work by Gerlach and Smets, 1995.

<sup>&</sup>lt;sup>3</sup>It should be noted, however, that most authors do find, that monetary policy shocks explain only a modest fraction of the output fluctuations. Many estimates are in the ballpark of 20 percent or less, see e.g. Cochrane(1994), Kim (1996) and Leeper, Sims and Zha (1996).

funds rate in response to a monetary policy shock. Importantly, no restrictions are imposed on the response of real GDP. Thus, the central question in the title is left agnostically open: we will let the data decide. More specifically, we will assume that a "contractionary" monetary policy shock leads to no increase in prices, no increase in nonborrowed reserves, and no decrease in the federal funds rate for a certain period like six months, say, following a shock. While theories with different implications can fairly easily be constructed, these assumptions may enjoy broad support and in any case, are usually tacitly assumed in most of the VAR literature: our aim is to bring them directly out into the open and to make them subject to debate. By making them the sole tool for identifying monetary policy shocks, we avoid the often fragile assumptions about the timing of contemporaneous shocks, which permeates much of the identification discussions in the literature. The goal is to help break the circular reasoning in the literature pointed out above, and to deepen our understanding of the monetary transmission process as a result.

This will not be a free lunch, nor should one expect it to be. When imposing the sign restrictions, one needs to take a stand on for how long these restrictions ought to hold after a shock. Furthermore, one needs to take a stand on whether a strong response in the opposite direction is more desirable than a weak one. We will try out a variety of choices and look at the answers. We view these choices as proceeding along economically meaningful criteria and thus as an advantage over the usual "timing-of-shocks" discussions.

We find the following.

- 1. "Contractionary" monetary policy shocks have an ambiguous effect on real GDP. With 2/3 probability, a typical shock will move real GDP by up to  $\pm 0.2$  percent<sup>4</sup>, consistent with the conventional view, but also consistent with e.g. monetary neutrality. Monetary policy shocks account for probably less than twentyfive percent of the k-step ahead prediction error variance of real output, and may easily account for less than three percent.
- 2. The GDP price deflator falls only slowly following a contractionary monetary policy shock, possibly indicating price stickiness. The commodity price index falls more quickly.
- 3. We also find, that monetary policy shocks account for only a small fraction of the forecast error variance in the federal funds rate, except at horizons shorter than half a year, as well as for prices.

While these observations confirm some of the results found in the empirical VAR literature so far, there are also some potentially important differences in particular with respect to our key question: "contractionary" monetary policy shocks do not necessarily seem to have contractionary effects on real GDP. The results could be paraphrased as a new Keynesian-new classical synthesis: even though the general

<sup>&</sup>lt;sup>4</sup>The usual label "contractionary" may thus be misleading, if output is moved *up*.

price level is sticky for a period of about a year, money may well be close to neutral. Our conclusion from these results: one should feel less comfortable with the conventional view and the current consensus of the VAR literature than has been the case so far.

We provide a counterfactual analysis of the early 80's, setting the monetary policy shocks to zero after December 1979, and recalculating the data. We found that the differences between observed real GDP and counterfactually calculated real GDP was not very large. Thus, the label "Volcker-recession" for the two recessions in the early 80's appears to be misplaced.

The new method introduced here is somewhat related to work by Lippi and Reichlin (1994a,b), Blanchard and Quah (1989), Faust (1999) or Dwyer (1997): these authors also impose restrictions on the impulse responses to particular shocks. However, here, we are just interested in the response to a monetary policy shock in a system with a number of variables. Also, we do not impose a particular shape of the impulse response as in Lippi and Reichlin (1994a) or Dwyer (1997). Instead, we are content with restrictions on the sign, making for substantial differences between their approach and ours. The approach here is also in the spirit of Bernanke and Mihov (1996a,b) in that we do not aim at a complete decomposition of the one-step ahead prediction error into all its components due to underlying structural shocks, but rather concentrate on identifying only one such shock, namely the shock to monetary policy. However, we achieve this solely by restricting the sign of the impulse responses directly.

Section 2 introduces the method with most of the technicalities postponed to the appendix A. Section 3 shows some results, based on the data set provided by Bernanke and Mihov (1996a,b). Section 4 provides a counterfactual analysis of the "Volcker recessions". Finally, section 5 concludes.

# 2 The Method

There is not much disagreement about how to estimate<sup>5</sup> VARs. A VAR is given by<sup>6</sup>

$$Y_t = B_{(1)}Y_{t-1} + B_{(2)}Y_{t-2} + \ldots + B_{(l)}Y_{t-l} + u_t, t = 1, \ldots, T$$
(1)

where  $Y_t$  is a  $m \times 1$  vector of data at date  $t = 1 - l, \ldots, T, B_{(i)}$  are coefficient matrices of size  $m \times m$  and  $u_t$  is the one-step ahead prediction error with variance-covariance matrix  $\Sigma$ .

<sup>&</sup>lt;sup>5</sup>There is some, but it is of lesser importance. Here, we have used a Bayesian procedure, in line with, say, Sims and Zha (1995,96) as well as practially the entire literature which relies on RATS to produce error bands for impulse responses, see Doan (1992), Example 10.1. For further details relevant to this paper, see appendix B.

 $<sup>^{6}</sup>$ We have excluded a constant as well as a trend term here as is common in large parts of the literature. They are easily included, see the appendix in Uhlig (1994) for details.

The disagreement starts when discussing, how to decompose the prediction error  $u_t$  into economically meaningful or "fundamental" innovations. This is necessary because one is typically interested in examining the impulse responses to such fundamental innovations, given the estimated VAR. In particular, much of the literature is interested in examining the impulse responses to a monetary policy innovation.

Suppose that there are a total of m fundamental innovations, which are mutually independent and normalized to be of variance 1: they can therefore be written as a vector v of size  $m \times 1$  with  $E[vv'] = I_m$ . What is needed is to find a matrix A such that  $u_t = Av_t$ . The j-th column of A (or its negative) then represents the immediate impact on all variables of the j-th fundamental innovation, one standard error in size. The only restriction on A thus far emerges from the covariance structure:

$$\Sigma = E[u_t u'_t] = AE[v_t v'_t]A' = AA'$$
<sup>(2)</sup>

Simple accounting shows that there are m(m-1)/2 "degrees of freedom" in specifying A, and hence, further restrictions are needed to achieve identification. Usually, these restrictions come from one of three procedures: from choosing A to be a Cholesky-factor of  $\Sigma$  and implying a recursive ordering of the variables as in Sims (1986), from some "structural" relationships between the fundamental innovations  $v_{t,i}$ ,  $i = 1, \ldots, m$  and the one-step ahead prediction errors  $u_{t,i}$ ,  $i = 1, \ldots, m$  as in Bernanke (1986), or Sims (1986), or from separating transitory from permanent components as in Blanchard and Quah (1989).

Here, we propose to proceed differently. We start from the observation, that much of the literature has taken the trouble of identifying all m fundamental innovations, although this is usually more than one is interested in. In particular for our purposes here, we are solely interested in the response to a monetary policy shock: we have no desire to also decompose the "other m - 1" components. Bernanke and Mihov (1996a,b) recognize this, and use a block-recursive ordering, to concentrate the identification exercise on only a limited set of variables which interact with the policy shock.

We propose to go all the way by only concentrating on finding the innovation corresponding to the monetary policy shock. This amounts to finding a single column  $a \in \mathbf{R}^m$  of the matrix A in equation (2), which satisfies some restrictions one might want to impose for identifying monetary policy shocks. It is useful to state a formal definition:

**Definition 1** The vector  $a \in \mathbf{R}^m$  is called an **impulse vector**, iff there is some matrix A, so that  $AA' = \Sigma$  and so that a is a column of A.

Proposition 1 in appendix A shows, that any impulse vector a can be characterized as follows. Let  $x_i, i = 1, ..., m$  be the eigenvectors of  $\Sigma$ , normalized to form an orthonormal basis of  $\mathbf{R}^m$ . Let  $\lambda_i, i = 1, ..., m$  be the corresponding eigenvalues. Then, a is an impulse vector if and only if there are coefficients  $\alpha_i, i = 1, ..., m$  with  $\sum_{i=1}^{m} \alpha_i^2 = 1$ , so that

$$a = \sum_{i=1}^{m} \left( \alpha_i \sqrt{\lambda_i} \right) x_i \tag{3}$$

Given an impulse vector a, it is easy to calculate the appropriate impulse response, see appendix B for details. Further, find a vector  $\tilde{b} \neq 0$  with

$$(\Sigma - aa')\tilde{b} = 0$$

normalized so that b'a = 1. Then, the real number

$$v_t^{(a)} = b' u_t \tag{4}$$

is the scale of the shock at date t in the direction of the impulse vector a, and  $v_t^{(a)}a$  is the part of  $u_t$  which is attributable to that impulse vector. With these tools, one can perform variance decompositions or counterfactual experiments.

To identify the impulse vector corresponding to monetary policy shocks, we impose, that a contractionary policy shock does not lead to an increase in prices or in nonborrowed reserves and does not lead to a decrease in the federal funds rate. These assumptions seem to be the least controversial implications of a contractionary monetary policy shock. Furthermore and crucially, these seem to be *distinguishing* characteristics of monetary policy shocks: it is hard to think of other shocks prominently proposed in the literature, which have the same implications.

The work lies in making these requirements *precise*. We shall pursue two related, but different approaches.

- The pure-sign-restriction approach. Choose a horizon  $K \ge 0$ . Take a joint draw from both the posterior for the VAR parameters as well as a uniform distribution over the (m-1)-dimensional sphere  $(\alpha_1, \ldots, \alpha_{m-1}), \sum \alpha_i^2 = 1$ . Construct the impulse vector a, see equation (3) and calculate the impulse responses  $r_{k,j}$  at horizon  $k = 0, \ldots, K$  for the variables j, representing the GDP deflator, the commodity price index, nonborrowed reserves and the federal funds rate. If all these impulse responses satisfy the sign restrictions, keep the draw. Otherwise discard it. Repeat sufficiently often. Calculate statistics, based on the draws kept.
- The penalty function approach. Choose a horizon  $K \ge 0$ . Define the penalty function

$$f(x) = \begin{cases} x & \text{if } x \le 0\\ 100 * x & \text{if } x \ge 0 \end{cases},$$

which penalizes positive responses in linear proportion and rewards negative responses in linear proportion, albeit at a slope 100 times smaller than the slope for penalties on the positive side<sup>7</sup>. For the true VAR coefficients, let

<sup>&</sup>lt;sup>7</sup>Other choices for the penalty function are easy to conceive of. Analyzing the sensitivity to this choice is desirable and the topic of ongoing research, but is focussing on the econometric methodology of the method, and not the subject of this paper.

 $r_{k,j}, k = 0, \ldots, K$  be the impulse response of variable j and  $\sigma_j$  be the standard deviation of the first difference of the series for variable j. Let  $\iota_j = -1$ , if j is the index of the Federal Funds Rate in the data vector, and else, let  $\iota_j = 1$ . Define the monetary policy impulse vector as that impulse vector a, which minimizes the total penalty  $\Psi(a)$  for prices, nonborrowed reserves and (after flipping signs) the federal funds rate at horizons  $k = 0, \ldots, K$ ,

$$\Psi(a) = \sum_{\substack{i \in \\ j \in \\ minimized \\ mini$$

The rescaling by  $\sigma_j$  is necessary to make the deviations across different impulse responses comparable to each other. Note that the sign of the penalty direction is flipped for the Federal Funds Rate. Since the true VAR is not known, find the monetary policy impulse vector for each draw from the posterior. This requires numerical minimization, see appendix B for details. Keep all draws and accordingly calculated monetary policy impulse vectors, and calculate statistics based on these.

The pure-sign-restriction approach is "cleaner" in that it literally only imposes the sign restrictions: it does not require a somewhat arbitrary additional penalty function. As a drawback, the pure-sign-restriction is, in effect, *simultaneously* an estimation of the reduced-form VAR alongside the impulse vector: VAR parameter draws, which do not permit any impulse vector to satisfy the imposed sign restrictions, are discarded as "impossible". By contrast, the penalty-function approach leaves the reduced-form VAR untouched. Furthermore, it additionally prefers a large response away from the sign restriction to a small one, and it will typically identify a unique monetary policy impulse vector for any given VAR, rather than a range or none as in the pure-sign-restriction approach.

# 3 Results

In this section, we present some results, using our method. We have employed the data set used in Bernanke and Mihov (1996a,b), which contains the GDP, the GDP deflator, a commodity price index, total reserves, nonborrowed reserves and the federal funds rate for the U.S. at monthly frequencies from January 1965 to December 1996. To obtain monthly observations for all these series, some interpolation was required, see Bernanke and Mihov (1996a) and in particular their NBER 1995 working paper version for details. We have fitted a VAR with 12 lags in levels of the logs of the series except for using the federal funds rate directly. We did not include a constant or a

time trend. We used samples from the posterior to draw inferences, see appendix B for details.

For comparison, figure 2 shows results obtained from a "traditional" Cholesky decomposition of  $\Sigma$ , i.e. imposing lower triangularity on A. The Cholesky decomposition is popular in the literature because it is easy to compute. This method requires a choice regarding the ordering of the variables as well as the choice of the variable, whose innovations are to be interpreted as monetary policy shocks. Here, we identify the monetary policy shock with the innovations in the Federal Funds Rate ordered last. Put differently, with this identification, monetary policy shocks are assumed have an instantaneous effect only on the Federal Funds Rate (and to be the only shock to satisfy this).

Figure 2 shows impulse responses for a horizon of up to five years after the shock. The top rows contain the results for real GDP and total reserves, the middle row contains the results for the GDP price deflator and for nonborrowed reserves and the bottom row contains the results for the commodity price index and the federal funds rate. Here as well as in all other plots, we show the median as well as the 16% and the 84% quantiles for the sample of impulse responses: if the distribution was normal, these quantiles would correspond to a one standard deviation band<sup>8</sup>. The results are fairly "reasonable" in that they confirm conventional undergraduate textbook intuition. The "reasonableness" of figure 2 is not an accident: trying out all the possibilities of ordering variables and identifying a monetary policy shock, it should not be too surprising, that one can find one, which confirms the conventional wisdom. Without a good defense of the timing implicit in the particular Cholesky decomposition employed, the results of figure 2 may thus be tainted by the specification search described in the introduction.

One can also see the "price puzzle" pointed out by Sims (1992): the GDP deflator moves slightly above zero first before declining below zero after a monetary policy shock (see also the remarks in appendix B). The two "agnostic" identification approaches to be employed next will avoid the price puzzle by construction.

## 3.1 Results for the pure-sign-restriction approach.

Our benchmark result are contained in figure 3, showing the impulse responses from a pure-sign-restriction approach with K = 5. I.e., the responses of the GDP price deflator, the commodity price index and nonborrowed reserves have been restricted not to be positive and the federal funds rate not to be negative for the six months  $k, k = 0, \ldots, 5$  following the shock. The results can be described as follows:

 $<sup>^{8}</sup>$ A number of authors prefer two standard deviation bands, which would correspond to the 2.3% and the 97.7% quantiles. But given that we want to report the same statistics in all the figures and given that we based inference in the pure-sign-restriction approach on only one hundred draws for computational reasons, we felt that we could not report these quantiles precise enough. Furthermore, one standard deviation bands are popular in this literature as well.

- 1. With a 2/3 probability, the impulse response for real GDP is within a  $\pm 0.2$  percent confidence band around zero.
- 2. The GDP price deflator reacts very sluggishly, with prices dropping by about 0.2 percent within a year, and dropping by 0.5 percent within five years. The price puzzle is avoided by construction.
- 3. The commodity price index reacts swiftly, reaching a plateau of a 1.5 percent drop after about one year.
- 4. The Federal Funds Rate reacts large and positively immediately, typically rising by 30 basis points, then reversing course within a year, ultimately dropping by 10 basis points.
- 5. Nonborrowed reserves and total reserves both drop initially, with nonborrowed reserves dropping by more (around 0.5 percent) than total reserves (around 0.2 percent). After one to two years, a reversal sets in with reserves eventually expanding by around 0.4 percent.

The initial 6-months response for most of these variables look rather conventional except for real GDP. Indeed, one may conclude from this figure that the reaction of real GDP can as easily be positive as negative following a "contractionary" shock. While this is consistent with the textbook view of declining output after a monetary policy shock, the data does not seem to urge this view upon us. The answer to the opening question is: the effects of monetary policy shocks on real output are ambiguous. A one-standard deviation monetary policy shock may leave output unchanged or may drive output up or down by up to 0.2 percent in most cases, thus possibly triggering fairly sizeable movements of unknown sign.

The further course of all the responses looks perhaps less conventional, although not hard to explain. Here are some suggestions. Commodity prices react more quickly than the GDP deflator, since commodities are traded on markets with very flexible prices. As for reserves and interest rates, note that these impulse responses contain the endogenous reaction of monetary policy to its own shocks. The Federal Funds Rate reverses course and turns negative for perhaps one of the following two reason. First, this may reflect that monetary policy shocks really arise as errors of assessment of the economic situation by the Federal Reserve Bank. The Fed may typically try to keep the steering wheel steady: should they accidentally make an error and "shock" the economy, they will try to reverse course soon afterwards. Second, this may reflect a reversal from a liquidity effect to a Fisherian effect: with inflation declining, a decline in the *nominal* rate may nonetheless indicate a rise in the *real* rate. Looking at the responses of reserves, we favor the first view. Obviously, other reasonable interpretations can be found.

This identification of the monetary policy shock seems appealingly clean to us as it only makes use of a priori appealing and consensual views about the effects of monetary policy shocks on prices, reserves and interest rates. There is one degree of choice here, though: the horizon K for the sign restrictions. How precise does this horizon need to be specified, i.e. how sensitive are the results to changes in K? The answer is provided in figure 4, showing the impulse response functions for real GDP, when imposing a variety of choices for K. The left column shows the results for a 3-months (K=2) and a 6-months (K=5) horizon, while the right column shows the results for a 12-months (K=11) and a 24-months (K=23) horizon. The variation is remarkably moderate: essentially, all of these figures show again an error band of  $\pm 0.2$  percent around zero. As one moves from shorter to longer horizons K, that band seems to move up somewhat, however. A short-lived liquidity effect is better for the conventional view.

## **3.2** Results from the penalty-function approach.

Figures 5 and 6 provide the same results, now using the penalty-function approach rather than the pure-sign-restriction approach. First, compare the results for the 6-months horizon, K = 5, when using the penalty function approach in figure 5 to those of the pure-sign-restriction approach in figure 3. The results look qualitatively largely the same. The magnitudes are slightly larger, and the confidence bands somewhat sharper, in particular immediately after the shock, compared to the pure-sign restriction approach. The greatest difference obtains for the impulse response for real GDP, i.e. for our central question. Here, one can perhaps see some evidence for the conventional view: real output, after a short initial and puzzling increase by around 0.05 percent, then declines by a tenth of a percent or more within a year, on average gradually recovering after that.

The differences between these two approaches in figures 3 and 5 are easy to explain. While the pure-sign-restriction approach is very agnostic about the size of the impulse response away from the sign restriction, a larger responses is "rewarded" by the penalty-function approach at least as long as this does not generate sign-violations elsewhere. Instead of a range of impulse vectors consistent with the sign restriction, the penalty function approach seeks a unique monetary policy impulse vector by searching e.g. for a large initial reaction of the Federal Funds Rate. Indeed, this reaction is now fairly sharply estimated to be about 30 basis points, quickly rising by another 10 basis points. One obtains similarly sharp error bands elsewhere. The monetary policy impulse vector uniquely identified by the penalty function is an element in the set of the vectors admitted by the pure-sign-restriction approach, given a draw for the VAR coefficients, provided that set is not empty. One would therefore expect the range of impulse responses of the penalty function approach to be contained in the range of impulse responses of the pure-sign-restriction approach. Indeed, this seems to be the case: the 64% range for the real GDP response, for example, never seems to venture outside the  $\pm 0.2$  percent error band around zero calculated for the pure-sign-restriction approach.

One can thus either view the results in figure 5 as a *sharpening* of the results in

figure 3, due to additionally desirable properties imposed on the restricted impulse responses, or as a *distortion* of the results in figure 3 due to additional ad-hoc restrictions. Since the aim is to impose the sign restrictions and nothing else, we find the pure-sign-restriction approach to be more appealing. The results of the second approach are nonetheless informative in that they show the additional mileage obtained from additional, potentially desirable restrictions, opening the door to more detailed investigations.

The results of the penalty function approach are also more sensitive to the choice of the restriction horizon K, as a comparison of figure 6 with figure 4 shows. This is not surprising: as the restriction horizon increases, it is increasingly harder to keep the sign restrictions satisfied. With increasing k, the search for the monetary policy impulse vector increasingly seeks to avoid penalties for sign violations rather than rewards for movements in the opposite direction. The 64% percent confidence band for the penalty-function approach keeps staying in the  $\pm 0.2$  percent band, and again moves up into positive ranges with larger K: in fact, the results for a 24-month horizon (K=23) practically *rules out* the conventional view. This reinforces the conclusion drawn in the previous subsection, that the conventional view requires a short-lived liquidity effect or, alternatively, monetary policy shocks as policy errors which are quickly reversed.

## 3.3 How much variation do monetary policy shocks explain?

Having identified the monetary policy shock, it is then interesting to find out, how much of the variation these shocks explain. What fraction of the unexpected k-step ahead variance in, say, real GDP, prices and interest rates, are accounted for by monetary policy shocks? These questions are answered by figure 7 for the benchmark experiment, i.e. using a pure sign restriction approach with a 6-months restriction (K=5). Results for the other experiments look fairly similar. The figure shows the fraction of the k-step ahead variance in the six variables explained by monetary policy shocks, with the variables ordered as in figure 3.

According to the median estimates, shown as the middle lines in this figure, monetary policy shocks account for 10 percent of the variations in real GDP at all horizons, for up to 30 percent of the long-horizon variations in prices and 25 percent of the variation in interest rates at the short horizon, falling off after that. Explaining just three or so percent of the real GDP variations at any horizon is within the 64% confidence band: it thus seems fairly likely, that monetary policy has very little effect on real GDP. This may either be due to monetary policy shocks having little real effect, or due to a Federal Reserve Bank keeping a steady hand on the wheel, as argued by Woodford (1994) or Bernanke (1996).

Among the six series, the largest fraction at the long end is explained for prices, reemphasizing Friedmans dictum, that inflation is always and everywhere a monetary phenomenon. For interest rates, the largest fraction of variation explained by monetary policy is at the short horizon, providing further support to the view, that monetary policy shocks are accidental errors by the Federal Reserve Bank, which are quickly reversed. The remaining variations in prices and interest rates may still be due to monetary policy, but then it needs to be due to the *endogenous* part of monetary policy: by systematically responding to shocks elsewhere, monetary policy may end up being responsible for 100% of the movements in prices. Only 30% percent of these movements can directly be ascribed to shocks generated by monetary policy itself. These results are rather similar to the results found in the empirical VAR literature so far, see the surveys cited in the introduction.

## 3.4 A summary.

The results could be paraphrased as a new Keynesian-new classical synthesis: even though the general price level is sticky for a period of about a year (see also Blinder, 1994), monetary policy has ambiguous real effects. These observations largely confirm the results found in the empirical VAR literature so far, see in particular Leeper, Sims and Zha (1996), except for the ambiguity regarding the effect on output: this exception is, of course, a rather important difference. We also agree with these authors that it makes sense that variation in monetary policy accounts only for a small fraction of the variation in any of these variables: good monetary policy should be predictable policy, and from that perspective, monetary policy in the US during this time span has been successful indeed.

## 4 The Volcker recessions, which weren't.

So, what about the Volcker recessions at the beginning of 1980? In August 1979, Paul Volcker became chairman of the Federal Reserve: in order to quelch inflation, he let interest rates rise to unprecedented levels, plunging the economy into two deep recessions as a result. At least, this is the usual story, see e.g. Krugman (1990). Shouldn't this case study be viewed as sufficient evidence that monetary policy surprises can have dramatic real effects?

With our results, one can analyze this issue by providing a counterfactual analysis. To do so, we set all the monetary policy shocks to zero, after Volcker became chairmen in August 1979, i.e. we simulated the VAR, using  $u_t - v_t^{(a)}a$  rather than  $u_t$  as date-t one-step ahead prediction error, where  $v_t^{(a)}$  is the scale of the monetary policy shock at date t, see equation (4). In light of the well-known Lucas Critique (1976), one obviously needs to interpret the results with some caution. Leeper, Sims and Zha (1996) provide a clear discussion of these issues for the analysis of VAR's.

The results can be seen in figures 8: we have concentrated our focus on real GDP. The top panel in figure 8 has been based on the pure-sign-restriction approach, whereas the penalty function approach has been employed for the middle panel: for both, a 6-months restriction (K = 5) was used. Both figures do not provide support for the standard story. With the counterfactual experiment, the recessions are some-

what diminuished, but they certainly do not go away. For comparison, the bottom panel of figure 8 provides results, when using the "traditional" Cholesky decomposition with the Federal Funds Rate ordered last: even with this identification, the first recession cannot be attributed to monetary policy shocks, although perhaps the second recession might be.

In sum, something else but monetary policy shocks must be the explanation for the recession of 1980 and probably also the recession of 1982: the Volcker recessions weren't due to Paul Volcker. This answer may appear somewhat unsatisfactory: it provides a negative answer, not an explanation. This is comparable to a murder trial, in which the prime suspect is pronounced "not guilty" without having somebody else provide a confession. Rounding up the other usual suspects is something which we have explicitly *not* focussed on in this exercise. There are a number of other candidates, though. For example, it is remarkably easy to explain the movements in real GDP in that time period with Hansen's (1985) simple benchmark real business cycle model, using observed labor productivities as exogenous input for a model simulation. It is admittedly hard to also explain the large interest rate movements this way. Some have proposed oil shocks as a leading explanation. Finally, it may be that the recessions were due to monetary policy, but caused by the endogenous feedback part of predictably changing interest rates in responses to shocks originating elsewhere. While blaming monetary policy, an explanation along such lines would nonetheless have to be quite different from the standard story. Whatever the truth: we believe that the analysis here supports a "not guilty" verdict for monetary policy shocks as a cause for the 1980 recession, in contrast to the conventional consensus view.

# 5 Conclusions

This paper proposed to estimate the effects of monetary policy by a new "agnostic" method, imposing sign restrictions on the impulse responses of prices, nonborrowed reserves and the federal funds rate in response to a monetary policy shock. No restrictions are imposed on the response of real GDP. It turned out that

- 1. "Contractionary" monetary policy shocks have an ambiguous effect on real GDP, moving it up or down by up to  $\pm 0.2$  percent with a probability of 2/3. Monetary policy shocks accounts for probably less than twentyfive percent of the k-step ahead prediction error variance of real output, and may easily account for less than three percent.
- 2. The GDP price deflator falls only slowly following a contractionary monetary policy shock, possibly indicating price stickiness. The commodity price index falls more quickly.
- 3. Monetary policy shocks account for only a small fraction of the forecast error variance in the federal funds rate, except at horizons shorter than half a year.

They account for about one third of the variation in prices at longer horizons.

While these observations confirm some of the results found in the empirical VAR literature so far, there are also some potentially important differences in particular with respect to our key question. "Contractionary" monetary policy shocks do not necessarily seem to have contractionary effects on real GDP. The results could be paraphrased as a new Keynesian-new classical synthesis: even though the general price level is sticky for a period of about a year, money may well be close to neutral. One should therefore feel less comfortable with the conventional view and the current consensus of the VAR literature than has been the case so far.

We provided a counterfactual analysis of the early 80's, setting the monetary policy shocks to zero after December 1979, and recalculating the data. We found that the differences between observed real GDP and counterfactually calculated real GDP was not very large. Thus, the label "Volcker-recession" for the two recessions in the early 80's appears to be misplaced.

# Appendix

# A Characterizing Impulse Vectors

Let u be the one-step ahead prediction error in a VAR of n variables and let v be the vector of fundamental innovations, related to u via some matrix A,

$$u = Av$$

Let  $\Sigma$  be the variance-covariance matrix of u, assumed to be nonsingular, while the identity matrix is assumed to be the variance-covariance matrix of v. If  $v = e_1$ , i.e. the vector with zeros everywhere except for its first entry, equal to unity, then  $u = Ae_1$  equals  $a_1$ , the first column of A. Hence, the j-th column of A describes the j-th "impulse vector", i.e. the representation of an innovation in the j-th structural variable as a one-step ahead prediction error. Put differently, the j-th column of A describes the immediate impact on all variables of an innovation in the j-th structural variable. Our aim is to characterize all possible impulse vectors. One can do so, using the observation that any two decompositions  $\Sigma = AA'$  and  $\Sigma = \tilde{A}\tilde{A}'$  have to satisfy that

$$\tilde{A} = AQ$$

for some orthogonal matrix Q, i.e. QQ' = I, see also Faust (1999) and Uhlig (1999). We find the following proposition useful, which we shall prove directly. We follow the general convention that all vectors are to be interpreted as columns.

**Proposition 1** Let  $\Sigma$  be a positive definite matrix. Let  $x_i, i = 1, ..., m$  be the eigenvectors of  $\Sigma$ , normalized to form an orthonormal basis of  $\mathbf{R}^m$ . Let  $\lambda_i, i = 1, ..., m$  be the corresponding eigenvalues. Let  $a \in \mathbf{R}^m$  be a vector. Then, the following three statements are equivalent:

1. There are coefficients  $\alpha_i, i = 1, ..., m$  with  $\sum_{i=1}^m \alpha_i^2 = 1$ , so that

$$a = \sum_{i=1}^{m} \left( \alpha_i \sqrt{\lambda_i} \right) x_i$$

- 2.  $\tilde{\Sigma} = \Sigma aa'$  is positive semidefinite and singular.
- 3. The vector a is an impulse vector, i.e., there is some matrix A, so that  $AA' = \Sigma$ and so that a is a column of A.
- 4. Let  $\tilde{A}\tilde{A}' = \Sigma$  for some matrix  $\tilde{A} = [\tilde{a}_1, \ldots, \tilde{a}_m]$ . Then there are coefficients  $\tilde{\alpha}_i, i = 1, \ldots, m$  with  $\sum_{i=1}^m \tilde{\alpha}_i^2 = 1$ , so that

$$a = \sum_{i=1}^{m} \tilde{\alpha}_i \tilde{a}_i$$

Note that there are m-1 "degrees of freedom" in picking an impulse vector, and that impulse vectors cannot be arbitrarily long: the Cauchy-Schwarz inequality implies that

$$\mid a \mid \mid \leq \sqrt{\sum_{i=1}^{m} \mid \lambda_i \mid \mid \mid x_i \mid \mid^2,}$$

for example.

**Proof:** We first show the equivalence of the first two statements. Note, that one can write  $\Sigma$  as

$$\Sigma = \sum_{i=1}^{m} \lambda_i x_i x_i'$$

Let v be any vector. Find coefficients  $\gamma_i$ , so that

$$v = \sum_{i=1}^{m} \frac{\gamma_i}{\sqrt{\lambda_i}} x_i$$

Note that

$$v'\tilde{\Sigma}v = v'\Sigma v - (v'a)^2$$
$$= \sum_{i=1}^m \gamma_i^2 - \left(\sum_{i=1}^m \gamma_i \alpha_i\right)^2$$

Thus,  $\tilde{\Sigma}$  is positive semidefinite if and only if

$$\sum_{i=1}^{m} \gamma_i^2 \ge \left(\sum_{i=1}^{m} \gamma_i \alpha_i\right)^2$$

for all  $(\gamma_i)_{i=1}^m$  in  $\mathbf{R}^m$  with equality for at least one such  $(\gamma_i)_{i=1}^m$  different from the zero vector. It follows from the Cauchy-Schwarz inequality, that this is equivalent  $\sum_{i=1}^m \alpha_i^2 = 1$ . To see this, note that the Cauchy-Schwarz inequality generally implies

$$\left(\sum_{i=1}^{m} \gamma_i^2\right) \left(\sum_{i=1}^{m} \alpha_i^2\right) \ge \left(\sum_{i=1}^{m} \gamma_i \alpha_i\right)^2$$

with equality for  $\gamma_i \equiv \alpha_i$ . Hence, our first two statements are equivalent.

We show now that the third statement implies the second statement. To that end, write  $A = [a_1 \dots a_m]$  in form of its columns, and note that

$$\Sigma = AA' = \sum_{i=1}^{m} a_i a'_i$$

Assume w.l.o.g., that a is the first column,  $a = a_1$ . Then,  $\tilde{\Sigma} = \sum_{i=2}^{m} a_i a'_i$ , which is positive semidefinite and singular, since each of the matrices  $a_i a'_i$  are of rank 1.

Next, we show that the second statement implies the third. Find the nonzero eigenvalues  $\tilde{\lambda}_i, i = 2, \ldots, m$  and its corresponding eigenvectors  $\tilde{x}_i, i = 2, \ldots, m$  for the positive semidefinite matrix  $\tilde{\Sigma} = \Sigma - aa'$ , noting that  $\tilde{\Sigma}$  must be of rank m - 1, since  $\Sigma$  is of rank m. Let

$$A = \begin{bmatrix} a, & \sqrt{\tilde{\lambda}_2} \tilde{x}_2, & \sqrt{\tilde{\lambda}_3}, \tilde{x}_3 & \dots & \sqrt{\tilde{\lambda}_m} \tilde{x}_m \end{bmatrix}$$

A simple calculation shows that indeed  $\Sigma = AA'$ .

To see that the third statement implies the last, note that  $A = \tilde{A}Q$  for some matrix Q with QQ' = I, as remarked before this proposition. The coefficients  $\alpha$  can now be found in the first column of Q. Conversely, given any such vector  $\alpha$  of unit length, complement it to an orthogonal basis to form the matrix Q. Then, let  $A = \tilde{A}Q$ .

This finishes the proof.  $\bullet$ 

Given an impulse vector a, one would like to calculate the part of the one-step ahead prediction error  $u_t$  which is attributable to shocks proportional to that vector. If the entire matrix A was available and a was the, say, first column, one would simply calculate  $v_t = A^{-1}u_t$  and use  $v_{t,1}$  as the scale of the shock attributable to a. Motivated by this reasoning, define:

**Definition 2** Given an impulse vector a and a one-step ahead prediction error  $u \in \mathbf{R}^m$ ,  $v^{(a)} \in \mathbf{R}$  is called the scale of a shock attributable to a, if there exists a matrix A with  $A'A = \Sigma$ , of which a is the j-th column for some j, so that  $v^{(a)} = (A^{-1}u)_j$ .

It turns out that this ties down the scale uniquely, provided  $\Sigma$  is not singular.

**Proposition 2** Given an impulse vector a and a one-step ahead prediction error u, the scale of the shock  $v^{(a)}$  attributable to a is unique and can be calculated as follows. Let  $b \in \mathbf{R}^m$  solve the two equations

$$0 = (\Sigma - aa') b$$
  

$$1 = b'a$$

The solution b exists and is unique. Then,

$$v^{(a)} = b'u$$

**Proof:** Suppose, A was available and assume w.l.o.g., that a is its first column. Thus, A can be partitioned as  $A = [a|A_2]$ . Likewise, partition  $B = A^{-1}$  into

$$B = \left[ \begin{array}{c} b' \\ B_2 \end{array} \right]$$

as well as  $v = A^{-1}u$  into

$$v = \left[ \begin{array}{c} v^{(a)} \\ V_2 \end{array} \right]$$

Clearly,  $v^{(a)} = b'u$ : thus, the task is to characterize b. Note first that

$$\Sigma = AA' = aa' + A_2A_2' \tag{5}$$

Next, note that

$$I_m = BA = \begin{bmatrix} b'a & b'A_2 \\ B_2a & B_2A_2 \end{bmatrix}$$

Hence, b'a = 1 and  $b'A_2 = 0$ . The latter equality implies together with equation (5)

$$0 = b'A_2A'_2b = b'(\Sigma - aa')b$$

Since  $\Sigma - aa'$  is symmetric, this is equivalent to  $(\Sigma - aa')b = 0$ . Note, that there is unique one-dimensional subspace of vectors b satisfying  $(\Sigma - aa')b = 0$ , since  $\Sigma$ is assumed to be regular. Also, because  $\Sigma$  is regular,  $a'b \neq 0$  for any  $b \neq 0$  which satisfies this equation. Thus, there is a unique b, which also satisfies b'a = 1.

With  $v^{(a)}$  it is now furthermore clear, that the part of u which is attributable to the shock proportional to the impulse vector a is given by  $v^{(a)}a$ .

# **B** Estimation and Inference

For convenience, we collect here the main tools for estimation and inference. We use a Bayesian approach since it is computationally simple and since it allows for a conceptually clean way of drawing error bands for statistics of interest such as impulse responses, see Sims and Zha (1995) for a clear discussion on this point. Note that draws from the posterior are "candidate truths". Thus, if e.g. the true impulse response for prices should not violate the imposed sign restriction, then this should also literally be true for any draw from the posterior. Thus e.g. the "price puzzle" in figure 2 is a violation by "candidate truths", and worrisome. With a classical approach, by contrast, considerations of significance would enter: a violation may be considered admissable if insignificant, requiring further judgement. Put differently, a Bayesian approach is more convenient and cleaner to justify. The reader who rather wishes to pursue a classical approach and inference regarding impulse response functions in vector autoregressions is referred to the work by Mittnik and Zadrozny (1993) as well as Kilian (1996a,b,c).

Using monthly data, we fixed the number of lags at l = 12 as in Bernanke and Mihov (1996a, 1996b). Stack the system (1) as

$$\mathbf{Y} = \mathbf{X}\mathbf{B} + \mathbf{u} \tag{6}$$

where  $X_t = [Y'_{t-1}, Y'_{t-2}, \dots, Y'_{t-l}]'$ ,  $\mathbf{Y} = [Y_1, \dots, Y_T]'$ ,  $\mathbf{X} = [X_1, \dots, X_T]'$ ,  $\mathbf{u} = [u_1, \dots, u_T]'$ and  $\mathbf{B} = [B_{(1)}, \dots, B_{(l)}]'$ . To compute the impulse response to an impulse vector a, let  $\mathbf{a} = [a', 0_{1,m(l-1)}]'$  as well as

$$\Gamma = \begin{bmatrix} \mathbf{B} \\ I_{m(l-1)} & \mathbf{0}_{m(l-1),m} \end{bmatrix}$$

and compute  $r_{k,j} = (\Gamma^k \mathbf{a})_j, k = 0, 1, 2, \ldots$  to get the response of variable j at horizon k. The variance of the k-step ahead forecast error due to an impulse vector a is obtained by simply squaring its impulse responses. Summing again over all  $a_j$ , where  $a_j$  is the j-th column of some matrix A with  $AA' = \Sigma$  delivers the total variance of the k-step ahead forecast error.

We assume that the  $u_t$ 's are independent and normally distributed. The MLE for  $(\mathbf{B}, \Sigma)$  is given by

$$\hat{B} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}, \ \hat{\Sigma} = \frac{1}{T}(\mathbf{Y} - \mathbf{X}\hat{B})'(\mathbf{Y} - \mathbf{X}\hat{B})$$
(7)

Our prior and posterior for  $(\mathbf{B}, \Sigma)$  belongs to the Normal-Wishart family, whose properties are further discussed in Uhlig (1994). A proper Normal-Wishart distribution is parameterized by a "mean coefficient" matrix  $\bar{B}$  of size  $k \times m$ , a positive definite "mean covariance" matrix S of size  $m \times m$  as well as a positive definite matrix N of size  $l \times l$  and a "degrees of freedom" real number  $\nu \geq 0$  to describe the uncertainty about  $(\mathbf{B}, \Sigma)$  around  $(\bar{B}, S)$ . The Normal-Wishart distribution specifies, that  $\Sigma^{-1}$  follows a Wishart distribution<sup>9</sup>  $\mathcal{W}_m(S^{-1}/\nu, \nu)$  with  $E[\Sigma^{-1}] = S^{-1}$ , and that, conditionally on  $\Sigma$ , the coefficient matrix in its columnwise vectorized form, vec $(\mathbf{B})$ follows a Normal distribution  $\mathcal{N}(\text{vec}(\bar{B}), \Sigma \otimes N^{-1})$ .

Proposition 1 on p. 670 in Uhlig (1994) states, that if the prior is described by  $\bar{B}_0$ ,  $N_0$ ,  $S_0$  and  $\nu_0$ , then the posterior is desribed by  $\bar{B}_T$ ,  $N_T$ ,  $S_T$  and  $\nu_T$ , where

$$\nu_{T} = T + \nu_{0}$$

$$N_{T} = N_{0} + \mathbf{X}'\mathbf{X}$$

$$\bar{B}_{T} = N_{T}^{-1}(N_{0}\bar{B}_{0} + \mathbf{X}'\mathbf{X}\hat{B})$$

$$S_{T} = \frac{\nu_{0}}{\nu_{T}}S_{0} + \frac{T}{\nu_{T}}\hat{\Sigma} + \frac{1}{\nu_{T}}(\hat{B} - \bar{B}_{0})'N_{0}N_{T}^{-1}\mathbf{X}'\mathbf{X}(\hat{B} - \bar{B}_{0})$$

We use a "weak" prior, and use  $N_0 = 0$ ,  $\nu_0 = 0$ ,  $S_0$  and  $\bar{B}_0$  arbitrary. Then,  $\bar{B}_T = \hat{B}$ ,  $S_T = \hat{\Sigma}$ ,  $\nu_T = T$ ,  $N_T = \mathbf{X}'\mathbf{X}$ , which is also the form of the posterior used in the RATS manual for drawing error bands, see example 10.1 in Doan (1992).

No attempt has been made to impose more specific prior knowledge such as the "no change forecast" of the Minnesota prior, see Doan, Litterman and Sims (1984),

<sup>&</sup>lt;sup>9</sup>To draw from this distribution, use e.g.  $\Sigma = (R * R')^{-1}$ , where R is a  $m \times \nu$  matrix with each column an independent draw from a Normal distribution  $\mathcal{N}(0, S^{-1}/\nu)$  with mean zero and variance-covariance matrix  $S^{-1}$ .

special treatments of roots near unity, see the discussion in Sims and Uhlig (1991) as well as Uhlig (1994), or to impose the more sophisticated priors of Leeper, Sims and Zha (1996) or Sims and Zha (1996). Also, we have not experimented with regimeswitching as in Bernanke and Mihov (1996a,b) or with stochastic volatility as in Uhlig (1997).

## B.1 The pure-sign-restriction approach

To draw inferences from the posterior for the pure-sign-restriction approach, we take  $n_1$  draws from the VAR posterior and, for each of these draws  $n_2$  draws  $\alpha$  from the *m*-dimensional unit sphere. From  $\Sigma$  and  $\alpha$ , we construct the impulse vector, using the characterization (3) or, alternatively, the forth characterization in proposition 1 with  $\tilde{A}$  the Cholesky decomposition. For each draw, we calculate the impulse responses, and check, whether the sign restrictions are satisfied. If they are, we keep the draw. If not, we proceed to the next. Finally, error bands etc. are calculated, using all the draws which have been kept. For the calculations, we have chosen  $n_1 = n_2$  and high enough, so that a couple of hundred joint draws satisfied the sign restriction. E.g. for the sixth months restriction (K = 5), we used  $n_1 = n_2 = 200$ .

## **B.2** The penalty-function approach

To draw inference from the posterior for the penalty function approach, we take n draws from it, employing a Monte-Carlo method: because optimizing over the shape of the impulse responses is time-consuming, we usually took n = 100. For each of these draws, we calculate the impulse responses and the variance decomposition and collect them. Thus, after 100 draws, we have 100 draws for each point on an impulse response function we may wish to estimate: it is now easy to calculate their median and their 68% error band.

To do the numerical minimization of the criterion function  $\Psi$  for each draw from the posterior, we needed to parameterize the space of vectors  $(\alpha_j)_{j=1}^6$  of unit length: we found the parameterization

$$\alpha = \begin{bmatrix} \cos(\gamma_1)\cos(\gamma_2)\cos(\gamma_3)\\ \cos(\gamma_1)\cos(\gamma_2)\sin(\gamma_3)\\ \cos(\gamma_1)\sin(\gamma_2)\\ \sin(\gamma_1)\cos(\gamma_4)\cos(\gamma_5)\\ \sin(\gamma_1)\cos(\gamma_4)\sin(\gamma_5)\\ \sin(\gamma_1)\sin(\gamma_4) \end{bmatrix}, (\gamma_j)_{j=1}^5 \in \mathbf{R}^5$$

particularly convenient. We have coded all our routines in MATLAB, and used its general purpose minimizer fmins.m to perform the minimization task numerically. It turned out that fmins sometimes stopped the search before converging to the optimal solution: we thus performed fmins.m three times in a row, starting it first at a randomly selected  $(\gamma_j)_{j=1}^5 \in \mathbf{R}^5$  and then starting it successively at the previously

found optimum. Now, the minimization seemed to miss the minimum in safely less than 5 percent of all cases. To achieve near-certain convergence, we did this procedure twice, starting it from two different initial random vectors  $(\gamma_j)_{j=1}^5 \in \mathbf{R}^5$ , and selecting the best of the two minimas found. That way, the chance of missing the optimum was safely below 0.3 percent. To calculate this for a single draw from the posterior took around four minutes on a Pentium-based machine. We used a sample of 100 draws from the posterior for inference.

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# Figures

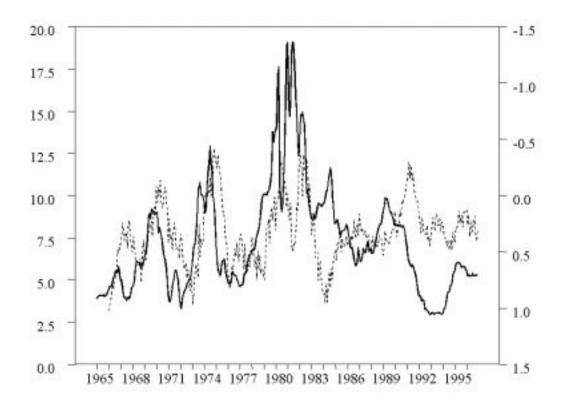


Figure 1: This figure contrasts movments in the Federal Funds Rate, shown as a thick, solid line with the scale on the left, with real GDP growth rates, shown as a thinner, dotted line with the scale on the right. To aid the visual comparison, the real GDP growth rates have been put "upside down", i.e., peaks in the figure are actually particularly low values for the growth rate. "Eyeball econometrics" suggests a strong cause-and-effect from Federal Funds Rate movements to real GDP: whenever interest rates rise, growth rates fall (i.e. the dotted line rises) shortly afterwards. This is particularly visible for 1968 through 1983. It seems easy to conclude from this picture, that the question about the effects of monetary policy on output is answered clearly: contractionary monetary policy leads to contractions in real GDP.

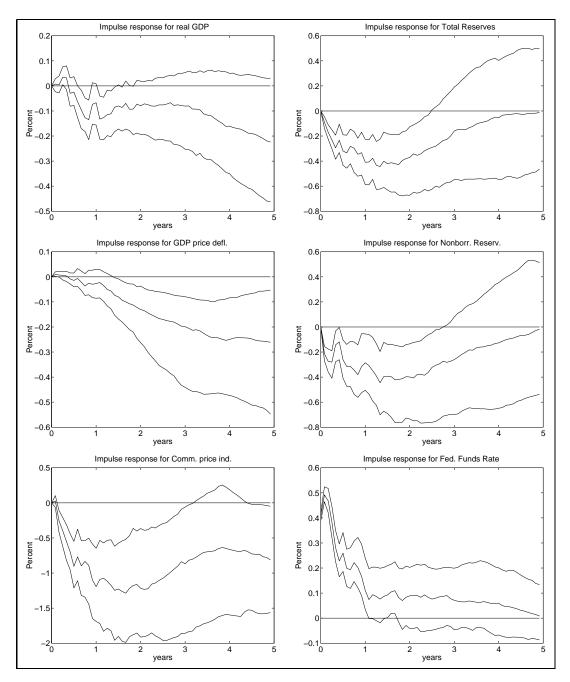


Figure 2: Impulse responses to a contractionary monetary policy shock one standard deviation in size, identified as the innovation in the Federal Funds Rate ordered last in a Cholesky decomposition. This "conventional" identification exercise is provided for comparison. The three lines are the 16% quantile, the median and the 16% quantile of the posterior distribution. The first column shows the responses of real GDP, the GDP deflator and the commodity price index. The second column shows the responses of total reserves, nonborrowed reserves, and the Federal Funds Rate. This identification mostly generates "reasonable" results, but also the price puzzle: the GDP deflator rises first before falling.

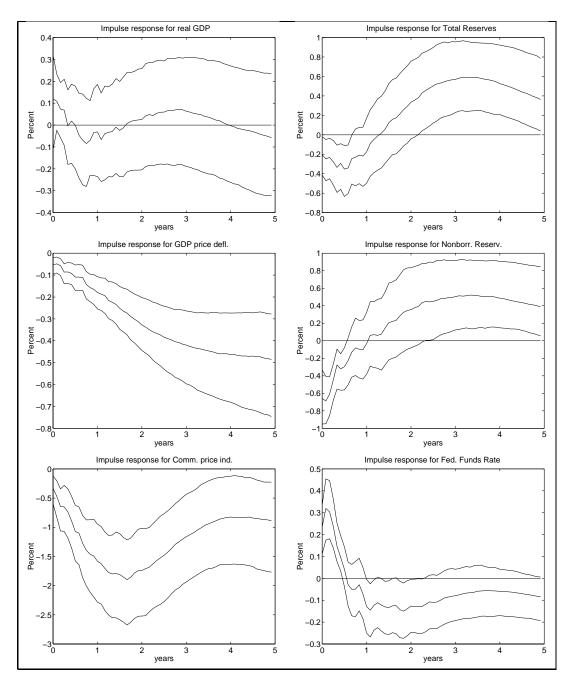


Figure 3: Impulse responses to a contractionary monetary policy shock one standard deviation in size, using the pure-sign-restriction approach with K = 5. I.e., the responses of the GDP price deflator, the commodity price index and nonborrowed reserves have been restricted not to be positive and the federal funds rate not to be negative for months k, k = 0, ..., 5 after the shock. The impulse response for real GDP is a  $\pm 0.2$  percent confidence band around zero: while consistent with the textbook view of declining output after a monetary policy shock, it is also consistent with e.g. monetary neutrality.

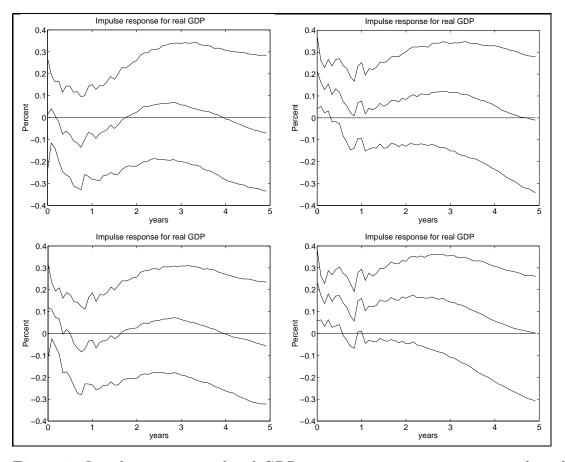


Figure 4: Impulse responses of real GDP to a contractionary monetary policy shock one standard deviation in size, using the pure-sign-restriction approach. For the left column, K = 2 and K = 5 were used, whereas K = 11 and K = 23 have been used in the right column. Essentially, all of these figures show again an error band of  $\pm 0.2$ percent around zero. As one moves from shorter to longer horizons K, that band seems to move up. Overall, the evidence in favor of the conventional view of a fall in output after a "contractionary" monetary policy shock seems to weak at best.

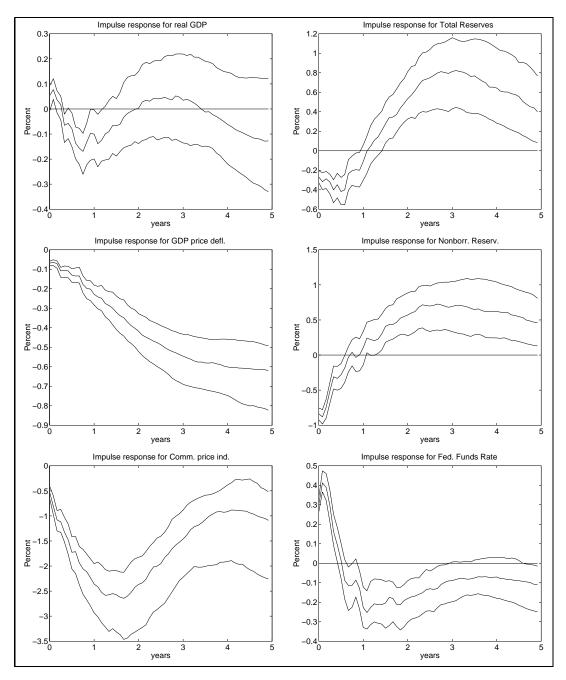


Figure 5: Impulse responses to a contractionary monetary policy shock one standard deviation in size, using the penalty-function approach with K = 5. I.e., the responses of the GDP price deflator, the commodity price index, nonborrowed reserves and the negative of the Federal Funds Rate have been penalized for positive values and slightly rewarded for negative values in the months k, k = 0, ..., 5 following the shock: the shock was identified by minimizing total penalties. The error bands are now much sharper than in figure 3. While the real GDP response is still within the  $\pm 0.2$  error band around zero estimated before, there now seems to be a piece between one and 12 month, showing a conventional response.

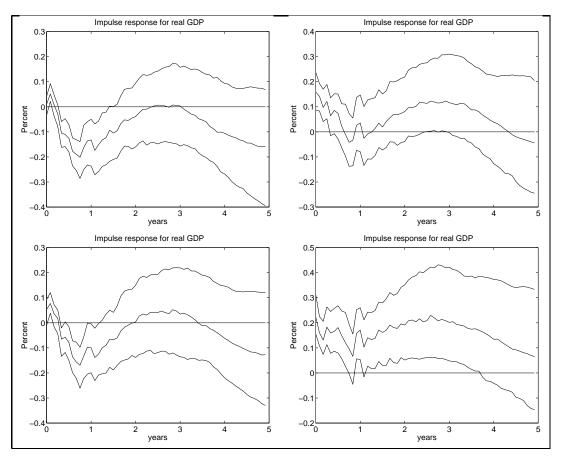


Figure 6: Impulse responses of real GDP to a contractionary monetary policy shock one standard deviation in size, using the penalty-function approach, imposing sign restriction for the months k = 0, ..., K after the shock. For the left column, K = 2and K = 5 were used, whereas K = 11 and K = 23 have been used in the right column. Compared to figure 4, the results are now sharper, but also more sensitive to variations in K. Only low values of K are fairly consistent with the conventional view of a decline in real GDP following a "contractionary" monetary policy shock.

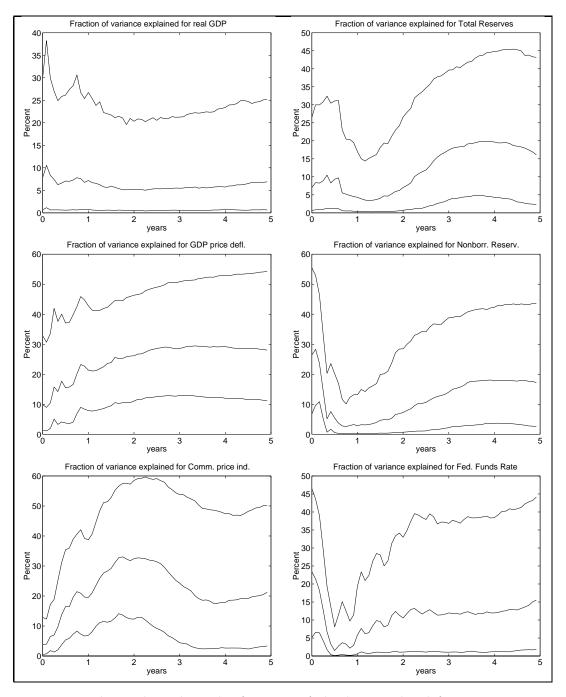


Figure 7: These plots show the fraction of the k-step ahead forecast error variance explained by a monetary shock, using a pure-sign restriction approach with K = 5, as in figure 3. The three lines are the 16% quantile, the median and the 16% quantile of the posterior distribution. According to the median estimates, monetary policy shocks account for 10 percent of the variations in real GDP at all horizons, for up to 30 percent of the long-horizon variations in prices and for 25 percent of the variation in interest rates at the short horizon, falling off after that.

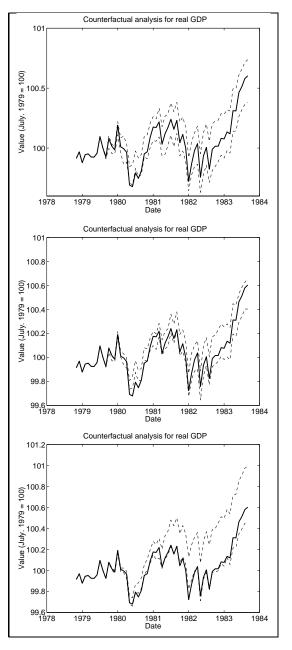


Figure 8: These plots compare the actual data for real GDP to a "counterfactual" experiment, in which it is assumed that there hadn't been any monetary policy shocks after Volcker became the chairman of the Federal Reserve: all monetary policy shocks after August 1979 have been set to zero. For the top figure, the pure-sign-restriction approach has been used, whereas the penalty-function approach was taken for the middle figure, both with K = 5. The bottom figure has been calculated from a Cholesky decomposition with monetary policy shocks identified as innovations in the Federal Funds Rate ordered last. Note that certainly the 1980 recession does not disappear in any of these three plots.