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# Fiscal Policy in a Monetary Union: Gains from Changing Institutions

**Susana Salvado**

Av. da República nº. 79 1050 – 243 Lisboa
Telf: (351) 217998150
Fax: (351) 217998154
**Web Site:** www.gee.min-economia.pt

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# Fiscal Policy in a Monetary Union: Gains from Changing Institutions[1]

**Susana Salvado**
FEUNL, GEE

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## Abstract

In a Monetary Union where individual monetary instruments are lost, fiscal policy becomes more important as a national policy. The question addressed in this article is whether fiscal policy should be decided at the country level or by a central decision maker, being in any case the fiscal instruments specific to each country. To answer this question, the focus is on the quantitative effect, since there are costs of implementing a supranational decision maker. While discussing the methodologies used in literature, we hereby propose a different one for quantifying gains from cooperation. We conclude that gains from fiscal coordination are significative, but gains that result from policy changes as a reaction to shocks are, by nature, very small. We also show that, symmetric shocks lead to coordination gains of the same magnitude than asymmetric ones.

[1] E-mail address: ssalvado@fe.unl.pt. The present work is part of my PhD research. I would like to thank Prof. Isabel Horta Correia for her comments and suggestions.

# Index

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Susana Salvado* †
FEUNL, GEE

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**Abstract**

In a Monetary Union where individual monetary instruments are lost, fiscal policy becomes more important as a national policy. The question addressed in this article is whether fiscal policy should be decided at the country level or by a central decision maker, being in any case the fiscal instruments specific to each country. To answer this question, the focus is on the quantitative effect, since there are costs of implementing a supranational decision maker. While discussing the methodologies used in literature, we hereby propose a different one for quantifying gains from cooperation. We conclude that gains from fiscal coordination are significative, but gains that result from policy changes as a reaction to shocks are, by nature, very small. We also show that, symmetric shocks lead to coordination gains of the same magnitude than asymmetric ones.

## 1 Introduction

When considering a Monetary Union, where monetary instruments are no longer an option at the country level, the importance of fiscal policy surfaces. An important issue to address is whether or not it is beneficial to coordinate such policies. Considering that the implementation of a supranational institution for fiscal policy implies significant costs, it is paramount to measure quantitatively those coordination gains. Moreover, most of the literature focuses on the loss of the stabilization role of national policies with the introduction of a monetary union and, therefore, the main issue becomes measuring welfare gains derived from fiscal policy coordination, which takes on the role of specific stabilization intruments.

This paper quantifies the increase in welfare within a Monetary Union following the introduction of a jointly decided fiscal policy. Going beyond the construction of a detailed model which could be a faithful replica of the past behavior of reality, on one hand it intends to build up a methodological approach. On the other hand, by understanding the main channels in a quite simple environment, those quantitative results can then be extended to any other environment. We decompose the fiscal coordination gain into two fundamental effects: the deterministic effect and the stochastic effect. The deterministic gain measures the increase in welfare deriving from a change of the steady-state. We call stochastic gain the one that corresponds to the increase in welfare from a different reaction to shocks. This last component is related with the one computed in the literature. We find that, by nature, this stochastic effect is always very small, by a measure developed in the article.

*E-mail address: ssalvado@fe.unl.pt

†The present work is part of my PhD research. I would like to thank Prof. Isabel Horta Correia for her comments and suggestions.

The model used is standard in the literature, except on the monetary transmission. We consider that each country produces a composite good that is traded among countries, that firms produce these goods using labor (that is immobile across countries) and we consider that linear technology is subject to shocks. Additionally, each government's consumption is limited to nationally produced goods and is financed by a distortionary tax on labor income. Households have a cash-in-advance constraint on the purchase of both goods. As regards debt, we consider that private debt is state-contingent and non-traded internationally and public debt is noncontingent and can be traded across countries.

This model inherits the four characteristics that Obstfeld and Rogoff (2002), Corsetti and Pesenti (2004) and Canzoneri and Diba (2005) describe as the characteristics that make a model tractable. That is, a log utility on consumption, constant expenditure shares on components of the composite consumption good, a cash in advance constraint (or a log specification of the utility of money in their cases) and a balanced current account[1]. Notice however, that not being crucial to impose in the methodology considered in this article, we use them for simplicity reasons.

We assume a monetary union comprised of two countries, where fiscal policy is implemented at a domestic level. As regards monetary policy, given that our objective is to quantify the effect of the fiscal policy, and in order to simplify the strategic interactions, we consider that the common monetary policy is implemented independently from individual fiscal policies. By fiscal policy we mean that domestic governments levy tax labor income for the purpose of financing public consumption which, in turn, will provide domestic economic agents with added utility. Notice that in this model, a direct relation between tax proceeds and government expenditures can be can established. Therefore, considering tax rates or public expenditure as the fiscal instrument is irrelevant for the analysis.

To implement fiscal policy in an independent manner, one country's government does not take into account the effect of its actions on the other country's government policy. Hence, our main objective is to compare the welfare level from this strategic behavior with the scenario where fiscal policy is implemented in a coordinated manner, *i.e.* where fiscal policy is decided by a supranational authority albeit implemented with individual national instruments. The difference between coordinated and non-coordinated fiscal policy derives from the fact that both countries have a strategic behavior in changing terms of trade when fiscal policy is implemented at a central level. Coordination gains exist, because in a coordination scenario each country has an incentive to reduce the level of labor effort and can do so by increasing the tax rate on labor, which is the same as increasing government expenditures. When one country increases its tax rate, the terms of trade increase and the partner country suffers a decline on the relative price of the good it produces, creating an incentive to increase the tax rate. When every country behaves this way, there is no impact in the terms of trade and given higher taxes it generates inefficient results.

As regards coordination gains, the literature of the 80's concerned deterministic static models and gains were driven by spillovers that occurred every single period. However, with the development of dynamic stochastic models, authors began to also study cyclical effects. As such, coordination gains were measured as a consequence of countries' reaction to shocks. The first studies concerned monetary

[1] That derives from the fact that we consider that each country holds an initial zero amount of debt.

policy, being fiscal policy coordination a secondary issue. They approached issues like flexible exchange rates versus fixed exchange rates, or the extent to which a coordinated monetary policy could react to individual country shocks.

In the beginning of the year 2000 and mainly due to the formation of the European Monetary Union, the literature started to focus on the role of fiscal policy. The main question was how, in the context of a Monetary Union, fiscal policy should react to shocks. Some studies focused on optimal fiscal policy. Others, proposed fiscal rules that could replicate some target and could be easily implemented at the country level. Coordination of fiscal policy is also an issue present in this branch of the literature. However, a question arrises which is not usually debated: how much are the gains from fiscal policy coordination? Moreover, which measure should be used to answer this question?

As it is going to be explained in the next section, in the literature we can find different methodologies which, if applied to the same framework, would deliver different results. In this paper we clarify those differences and suggest a more general methodology to measure gains from fiscal cooperation. This methodology can be used in the standard models of the literature and is not based in any kind of *a priori* approximation, delivering a more accurate result of the quantification of the coordination gain.

This paper is organized as follows. After debating methodology issues, we describe the model which is used to test such methodology. For simplicity, we first apply it to a model with flexible prices and then to one where firms set their prices one period in advance. We also consider that the only source of disturbance comes from shocks on technology. Afterwards, we compute welfare differences from a equilibrium where fiscal policy is decided by a common authority (the cooperative equilibrium) to a equilibrium where fiscal instruments are chosen at the country level (the Nash equilibrium). The welfare difference between these two equilibria is the gain from fiscal cooperation. Additionally, we decompose this gain in two effects: the deterministic effect and the one that occurs due to shocks.

We show that, out of the first best, deterministic fiscal coordination gains are always significative, representing an increase of more than 16% on average utility. Notice that, in line with the conclusions of Chari and Kehoe (1990), this result represents a lower boundary of the gains' magnitude as it is derived with only two economies. They show that when the number of economies increase and taxes are distorting, the wedge between the cooperative and the Nash equilibrium diverges, increasing the gains from cooperation.

On the other hand, we show that stochastic gains are, on average, small, representing less than 2% of the total gains. Despite the small magnitude of the stochastic gains, we also show that symmetric technology shocks lead to coordination gains with the same magnitude of asymmetric ones with similar mean. As the mean of the distribution of shocks is unaltered and we are considering that prices are set one period in advance, the reaction of one given country to a shock is independent from the other country's shock. Ex-ante, countries do not have the information of the shock that is going to happen, therefore tax rates in every country depend just on the shock of that country.

Hence, we can conclude that fiscal policy coordination should be considered in order to achieve a higher long-term welfare level, but it is not appealing as a short-run device for reacting to shocks.

Notice that this conclusion corroborates the earlier idea of Lucas (1988), where he claims that, for plausible risk aversion levels, stabilization gains are always small. Therefore, in our example, given the risk aversion levels considered, stabilization gains would not be very significative even if the possibility of accounting for a total stabilization of the effects of the shock existed. As considered by Lucas, the use of stabilization policies to reduce aggregate risks is not sufficient to eliminate all the risk that occurs at the individual level and those are not taken into account in the aggregated model used in this article.

## 2 Methodology

When comparing and evaluating the different methodologies used throughout the literature it is common to begin by a benchmark case where model specificities lead, not only to the elimination of gains from coordination, but also to a first best equilibria. This is important since this is an issue that is applied across methodologies.

As explained in Salvado (2009), to eliminate gains from cooperation that are derived by strategic interactions among countries and to achieve efficient results, the usual procedure is to neutralize a given distortion with another one. In the literature, where the source of the gain is in general generated by the existence of monopolistic competition, a subsidy on production is imposed to eliminate that distortion. However, as proposed in Salvado (2009), this non-cooperative-gains solution is just achieved in special conditions, that does not depend on the imposition of the labor subsidy *per se.* As such, it is proved that in the case of endogenous government expenditures, only when imposing consumption taxes and an unitary elasticity of substitution among goods, it is obtained a situation of inexistence of cooperative gains. Moreover, when lump-sum transfers are considered (which is the case throughout monetary policy literature), might coordination achieve efficiency.

However, when studying fiscal policy, it is worth analyzing these issues in more detail. Generally, in the literature it is assumed that government expenditures are endogenous. Moreover, the use of the labor subsidy to cancel monopolistic distortions is not sufficient to achieve a situation without coordination gains since the subsidy used in the cooperative case will be different from the one used in the Nash case which, *per se,* will be a source of distortion that is going to generate a coordination gain.

Additionally, in the majority of the models, the equality between the Nash, the cooperative and the First Best is appealing in order to consider the flexible price case as a benchmark. As known, the deterministic steady-state of a model with sticky pricing is equivalent to the deterministic steady-state of a model with flexible prices. Therefore, in a model where the cooperative equilibrium, the Nash equilibrium and the First Best coincide in the steady-state, the flexible price case can be a good benchmark.

Linked to coordination gains, the expression "stabilization gains" is recurrent in this kind of literature and has often different meanings from one paper to another, raising two issues. First, it is important to clarify the exact scope of the expression and how it relates to the "coordination gains" expression. Second, it is also important to understand how such gains are measured, that is, what is the underlying methodology.

To answer these questions, we decompose the expected value of the Union's utility into two parts: the deterministic effect and a function of the distribution of shocks ($f(\Omega)$).

$$E\left(U^U\right) = U^U_{steady\ state} + f(\Omega) \tag{1}$$

where $U^U_{steady\,state}$ represents the deterministic effect, that is, the Union's utility that is generated in the steady-state of the model, and $f(\Omega)$, the stochastic effect, that represents the utility that is generated due to a distribution of shocks. Notice that, the stochastic effect accounts to the direct effect of shocks and to an indirect effect, that occurs due to the impact of the volatility of shocks in the mean of the utility.

This decomposition allow us to easily compare and understand the different methodologies present in the literature. Notice that this decomposition is independent from coordination issues, since it is just a decomposition of the expected value of a variable that, in this case, is the utility of the Union, but could be the utility of a given country. However, applied to the difference between utility from coordination and non-coordination, gives us a measure of gains from coordination.

Another advantage of our method is that can be used with models that include distortionary taxation. Notice that, in the literature, distortionary taxation is not used because it implies an additional level of complexity. However, to our method the use of distortionary taxation is not a limitation. Moreover, as we explained previously, we consider two distinct situations: one, where countries in the Union choose their fiscal policy on their own (our Nash equilibrium) and, an other, where the fiscal policy is decided by an authority at the union level (our cooperative equilibrium). Then, welfare gains from moving from the Nash equilibrium to the cooperative equilibrium, are computed in the two cases.

First, we consider the deterministic steady-states of the Nash and the cooperative equilibrium and we compute the welfare gain that occurs in the steady-state. We name it the deterministic gain.

Second, we compute the gain that occurs considering a stochastic economy. Again, we compute the optimal welfare in the cooperative equilibrium and in the Nash equilibrium. From the difference of the two we obtain the total coordination gain.

As we do not have a closed solution for our model, the description of the Nash problem is not of direct formulation, specially given the particulars of the models used. Gali and Monacelli (2005), Forlati (2007), among others, consider that countries integrated in a monetary union are too small to influence the terms of trade. Therefore, in the description of the Nash problem, they only consider the restrictions on equilibrium regarding each country and the conditions for the market clearing of goods. Conversely, in the present paper, we consider that the union is composed by two countries with the same weight and thus they have the ability to change the terms of trade with their actions, generating strategic interactions which, otherwise, would not take place. This way, in the description of the Nash equilibrium, we consider that both countries are restricted by all the conditions that describe the equilibrium. Therefore, the key is to consider as restrictions to both problems (Nash and Cooperative) all the conditions that define the equilibrium.

Finally, the stochastic gain is computed by eliminating the deterministic gain from the total gain. It represents the gain that is derived by a distribution of exogenous shocks that move around the optimal steady-states. It is worth mentioning that, as our deterministic gain is computed by differences in the two optimal steady-states (the coordinated and the Nash one), our stochastic gain measures the true gain that derives from a situation of uncertainty. Therefore we can write that,

$$E\left(U^{U^{Coop}} - U^{U^{Nash}}\right) = \underbrace{E\left(U^{U^{Coop}}_{steady\,state} - U^{U^{Nash}}_{steady\,state}\right)}_{deterministic\ effect} + \underbrace{E\left(U^{U^{Coop}}_{shock} - U^{U^{Nash}}_{shock}\right)}_{pure\ stochastic\ effect} \quad (2)$$

The advantage of this decomposition comes from the fact that our deterministic effect is measured taking into consideration the variations in welfare derived from differences that emerge from the steady-state of the two optimal policies (cooperative and Nash). Hence, our deterministic effect does not take into consideration changes in the steady-state that would occur via changes in the volatility of Union's utility. Therefore, it captures the true effect that follows only from the strategic interactions of countries without any type of distortions that would occur in a stochastic environment. Moreover, this effect of the volatility in the mean is considered in the stochastic gain.

The objective of this section is to compare our methodology to the different ones used in the literature. Therefore, and for the purpose of explanation, we divide the literature into two major groups: one that is based in models that allow for a closed form solution and other that is based on numerical approximations.

Within the closed-solution models, there is a sub-group[2] that assumes that shocks have a lognormal distribution. In conjunction with some restrictions on functional forms, this hypothesis generates endogenous variables which are lognormally distributed. We call this methodology the "Lognormal Distribution Method". In this literature, in addition to having a closed form solution, that only depends on the exogenous variables and policy instruments, they allow for a particular case of the decomposition made in condition 2. The stabilization effect is in this case equivalent to the stochastic effect. Moreover, with this kind of models, volatility directly influences the expected value of a variable, changing its expected value. Hence,

$$E\left(U^{U^{Coop}} - U^{U^{Nash}}\right) = \underbrace{\left(\overline{U^{U}_{Coop}} - \overline{U^{U}_{Nash}}\right)}_{mean\ effect} + \underbrace{\frac{\sigma^2_{U^{U^{Coop}}} - \sigma^2_{U^{U^{Nash}}}}{2}}_{stochastic\ effect} \quad (3)$$

being $\overline{U^U}$ the mean and $\sigma^2$ the variance of the distribution.

The key fact is that it leads to lognormal distributed endogenous variables, which implies a very strong restriction in the structure of the models.

In this group of the literature, to our knowledge, it is rare to introduce fiscal policy, with the analysis being limited to monetary policy.

Obstfeld and Rogoff (2002) create the grounds for some of the more recent work. In an environment with money in the utility function[3], where asset markets are incomplete - as well as efficient risk

[2] In other sub-group, we find the work of Corsetti and Pesenti (2001), where they use the mentioned four characteristics that allow for a closed form solution of the model, in addition to lump-sum taxes. They analyze monetary and fiscal transmission mechanisms and conclude that the welfare effects of an expansionary policy are driven from monopolistic distortions in production supply and in a country incentive to influence terms of trade. However, they do not quantify these effects.

[3] However, they eliminate the monetary cost making the parameter associated to money in the utility function tend to zero ($\chi \rightarrow 0$).

sharing in consumption of tradeable goods, wages set one period in advance, flexible prices, and where monetary policy is defined by a simple rule for each country[4], - two types of gains are analyzed: stabilization and cooperation. The stabilization gain corresponds to "the gain from monetary policies that target flexible-wage equilibrium compared with policies that hold money supply constant". The cooperation gain corresponds to "the gain from moving from an independent monetary policy to a cooperative monetary policy". They show that the optimal cooperative policy is the one that replicates the flexible-wage allocation that, by its turn, is constrained Pareto efficient ex-ante. Moreover, they separate the effects between the various risk-aversion levels. They conclude that stabilization gains decrease with the risk-aversion level, given that for higher levels of $\rho$[5], a lower adjustment in wages is necessary, and coordination gains are much smaller[6]. Notwithstanding the above, when the utility function is logarithmic in consumption ($\rho = 1$), they conclude that stabilization gains represent 1.01% of output, and no coordination gains exist. In other words, the Nash equilibrium is the same as the cooperative equilibrium which, in turn, is not a first best. Notice that this result occurs because, as they are analyzing monetary interactions without costs of holding money, they do not have the distortionary effects of taxes and they do not allow for country strategic interactions via monopolistic distortions.

More recently, Evers (2007) using the same environment as Obstfeld and Rogoff but introducing a cash in advance constraint, considers that monetary policy can be implemented with two instruments which are demonstrated to be independent: by setting the nominal interest rate and by establishing rules for the money supply. By studying of the monetary policy interactions, this author concludes that gains driven by the money supply rule create second-order effects, that is, in the variance, and that coordination gains originated in the interest-rate side (derived from incentives to manipulate terms of trade) are in the unconditional mean. That is, money supply affects the terms of trade by changing the nominal exchange rate ex-post and it is used to decrease macro variability. Nominal interest rate affects the terms of trade by changing expected inflation ex-ante, which leads to an inefficient inflation tax that reduces the average welfare. Hence, he concludes that, in a situation of low trade and when the elasticity of intertemporal substitution is 1/2, coordination gains represent 0.217% of consumption equivalents, decomposed in 0.155% from the mean effect and 0.062% from the stochastic effect. Notice that Evers, by giving a dual objective to monetary policy, has two instruments that can be used in two distinct policies. Therefore, monetary policy can account for both the variability of prices and inflation.

Canzoneri and Diba (2005) use a similar setup of Obstfeld and Rogoff (2002). As they impose a log utility of consumption, a log specification of the utility of money, constant expenditure shares and a balanced current account, the model does not generate any terms-of-trade externalities. Hence, the flexible price solution does not account for any monetary coordination gains. However, as they relax the hypothesis that productivity shocks are perfectly correlated, the Nash equilibrium no longer achieves the flexible price solution, which generates coordination gains. Therefore, they conclude that

[4] In this rule the monetary instrument is money supply, where the stochastic component can react to world shocks (symmetric) and to asymmetric shocks.

[5] Being $1/\rho$ the elasticity of intertemporal substitution.

[6] Notice that these results are consistent to the expected because monetary policy has a small impact on real activity.

depending on the size and the correlation of shocks, coordination gains can be large.

Devereux and Engel (2003), in an economy where prices are set one period in advance, compare the welfare of fixed and floating exchange-rate regimes depending on whether prices are set in the currency of producers or consumers. They conclude that in the case of the producer currency pricing, if exchange rates are flexible (which is the same as a Nash equilibrium in monetary policy), a lower consumption variance is generated. However, interest-rate volatility produces a lower (average) level of consumption. In this case, using their terminology, monetary policy will generate stabilization losses (in the variance) and coordination gains (in the mean).

In the second group of the literature we find standard Neo-Keynesian models which cannot be solved analytically. Therefore, authors use different numerical methods to solve those models. As they are mainly interested in measuring the reaction to shocks, all variables are treated in deviations from the steady-state. Therefore, it is clear that their aim is to study policies that react to shocks. When they analyze coordination gains they call it stabilization gains, which can generate some confusion between the two definitions. As deterministic coordination is rarely considered, they measure the gain that occur due to stabilization.

Notice that we would have a direct relation between this "stabilization gain" and the "stochastic effect" if economies have sufficient instruments so as to cancel the changes in the mean driven from volatility changes, which is not a regular procedure. Additionally, in Salvado (2009) we identify specific cases where the coordination gain driven from the steady-state does not exist, that is, when the steady-state of the Nash equilibrium is the same as steady-state of the cooperative equilibrium. However, regarding distortionary taxation, it is proved that those cases never reach efficiency.

Within the set of papers that are based in numerical approximations, we find a branch in the literature that uses the Linear Quadratic (LQ) Method. Here models are reduced to a small number of equations in log-deviations (first-order) and welfare is transformed in a loss function of second order, following the Woodford method, 2003. They then use the second-order approximation of the model equations to replace the linear terms in the loss function with quadratic terms.

$$\begin{aligned} f(x;y) - f(\overline{x};\overline{y}) &\simeq \alpha_1 (x-\overline{x}) + \alpha_2 (y-\overline{y}) + \beta_1 (x-\overline{x})^2 + \beta_2 (y-\overline{y})^2 \Rightarrow \\ &\Rightarrow Loss = \underbrace{\gamma_1 (x-\overline{x})^2 + \gamma_2 (y-\overline{y})^2}_{stabilization} \end{aligned}$$

Subsequently, they minimize the quadratic loss function subject to linear constraints, implied by the first order approximation to the model. This leads to a linear quadratic policy problem with an analytical solution that is the linear approximation to the optimal policy function.

It is also worth mentioning that in the majority of this literature, the comparison between coordination and non-coordination is not analyzed, and the quantification of coordination gains is therefore rare.

The original idea of Rotemberg and Woodford (1997) was that, with flexible prices, if monopolistic distortions are compensated with any kind of production subsidy in the steady-state, the cooperative equilibrium replicates the first best and coincides with the Nash equilibrium. Hence, flexible prices are a good benchmark and are used as target of approximation. Notice that, any way, the stabilization gains measured differ from ours, and correspond instead to what we call pure stochastic gains.

In the second generation of this literature, equality of the three equilibria[7] are not imposed and some articles consider non-distortionary taxation, allowing for second-best equilibria. However, they continue to use the flexible price case as benchmark, not accounting for two problems. The first concerns the fact that some papers set off the monopolistic distortions with subsidies in the steady-state, but these subsidies are different in the cooperative and in the Nash case. The second problem derives from the fact that flexible prices can be a source of volatility. Actually, they can account for more volatility than a sticky-price model. Therefore, when those models measure stability gains in comparison with the flexible price case, they do not disaggregate the effects of the volatility in the mean from the stabilization effect.

To be more precise, in this literature (Benigno and Woodford (2005), Gali and Monacelli (2007), Forlati (2007) among others) gains are transformed in losses from a target level and can be described as follows:

$$Loss = \alpha \left(\widehat{y}_t - \widetilde{y}_t\right)^2 + \beta \pi_t^2 + \gamma \left(\widehat{p}_t - \widetilde{p}_t\right)^2 + t.i.p.$$

where $\widehat{x}_t - \widetilde{x}_t$ represents the deviation of each variable from its target level, $y$ is output, $p$ represent terms-of-trade, $\pi$ the inflation rate and $t.i.p$ are terms independent of policy. Usually, target inflation rate is zero.

Hence, when measuring stability gains as the gain that would occur if the economy could reach the flexible price case, they do not evaluate the true stabilization gain. However, the problem arrives because they can overestimate or underestimate the true stabilization gain depending on the target and the type of the model they are considering. Actually, if we consider the stabilization gain to be the gain that would emerge from a realization of a shock coming from a certain distribution, this methodology is not able to replicate the true stochastic gain.

The work of Benigno and Woodford (2005) is related to those aspects. In fact, in their work they show how a LQ problem of a model with distortionary taxation and monopolistic distortions can be derived in order to take into account the effects of stabilization, characterized by the variances of endogenous variables. They consider the conditions under which price stability is optimal and they conclude that the introduction of tax distortions (production tax) does not create an extra channel, it only affects the weights on these objectives and the proper definition of the output target.

Gali and Monacelli (2007) present a model where the Monetary Union is made up of a continuum of countries and lump-sum taxes exist. In their framework, labor subsidies are introduced so as to cancel out monopolistic distortions in the steady-state, with the purpose of recreating an efficient flexible-price equilibrium[8]. They use the "Linear Quadratic Method", where they compare two types of policies: the optimal cooperative policy and the non-coordinated policy. In the optimal cooperative policy, inflation is stabilized at the union level (with monetary policy) and fiscal policy is only used for stabilization of asymmetric shocks. Again, the idea is that fiscal policy decisions remain at the

[7] Cooperative, Nash and First Best.

[8] With these characteristics the model could be solved with the "Lognormal Distribution Method", since it allows for a closed form solution.

national level but countries act simultaneously posing no inflationary pressure on the union. In the non-coordinated policy, they conclude that joint actions lead to a suboptimal outcome forcing the Central Bank to choose between inflation and output gap stabilization at the union level. In their work, they are interested in analyzing the stabilization of the main aggregates towards the efficient allocation (that is characterized by the flexible price case) in these two types of policies.

Another point worth mentioning is that these authors pay little attention to quantifying the gain derived from achieving coordination, that is, the loss reduction from moving from a non-coordinated policy to the optimal cooperative policy. These authors conclude that gains are "quantitatively small", but give no comment on this quantification. Moreover, considering that coordination gains are the difference from the stabilization gains that occur under coordination and non-coordination, it would be relevant to analyze the origin of its magnitude.

Another question that arises is that these differences are only in deviations from the steady-state, not accounting for deterministic differences. Even cancelling out monopolistic distortions with a constant labor subsidy is not sufficient to obtain a comparable situation between the Nash and the cooperative steady-states. They rationalize it because it allows to simultaneously offset the market power and the terms of trade externality. However, we notice that this subsidy differs in the cooperative and in the Nash situations, and it is, *per se*, an instrument that is not taken into consideration when computing the total of the cooperative gains, since terms of trade are a source of spillovers that influence gains from stabilization.

In a similar setup, Forlati (2007) compares the case of coordination, where a common authority chooses timeless optimal monetary and fiscal policies, with the case of non-coordination, where fiscal authorities are not coordinated neither among each other, nor with the Central Bank. Although not computing the gain from moving from one situation to the other, she makes a careful explanation on how differences in the intertemporal elasticities of private and public consumption may generate differences in the reaction of the aggregates following a technology or a markup shock.

However, her analysis is very similar to Gali and Monacelli in what entails the method (LQ) and the particulars of the model used. Therefore, when computing the coordinated equilibrium, she considers that the target for stabilization is the first best because she imposes a subsidy on production to eliminate the monopoly distortions[9]. In the non-coordination situation, the target for the fiscal authorities is the flexible price allocation under floating exchange rates. Therefore, if she were to compute the stabilization gain, it would be the gain obtained by the approximation of each allocation to their respective target in both situations (coordination and non-coordination), and the coordination gain would be the gain from moving from a situation of non-coordination to a situation of coordination. As regards the stabilization procedure, she shows that the inefficient steady-state distribution of resources across private and public consumption generates inefficient variations in output. That is, as fiscal policymakers can affect the terms of trade in the steady-state, this leads to distortions when reacting to shocks.

Gali and Monacelli (2007) and Forlati (2007) claim that one important extension would be to

[9] However, as proved in Salvado (2009), the introduction of a labor subsidy is not sufficient to achieve the First Best.

include distortionary taxation and study the reaction to shocks, which give us another reason to introduce labor income taxes in our model.

Ferrero (2007) also uses this LQ approach but does not consider a coordinated fiscal policy. In fact, he considers that there is a common monetary policy, but fiscal policy is uncoordinated. This author does not eliminate markup distortions from his model, and considers only a distortionary tax on production. The objective of this work is to compute the welfare costs of policies that aim to achieve strict price stability and are constrained by a balanced budget requirement. The conclusions that he takes from the model are the same as the above two authors: in the optimal equilibrium, monetary policy is used to achieve price stability through a flexible inflation targeting rule and fiscal policy is used to stabilize asymmetric shocks through changes in government debt. However, as he introduces distortionary taxation, he can go a step further in obtaining an extra channel of effects. He shows that optimal fiscal policy must take into account the intratemporal response to inflation among countries as well as the intertemporal smoothing of tax distortions via changes in terms of trade.

From a numerical point of view, Ferrero measures the welfare gain of pursuing debt stabilization. He shows that the relative gain[10] of obtaining monetary stabilization is about 1.17% and the relative gain[11] of pursuing fiscal stabilization is 42.4% which, in consumption equivalents[12], represents 7.58% and 4.42%, respectively. That is, monetary policy has a small impact on the real side of the economy, but fiscal policy, by moving from balanced budget rules to optimal flexible debt targeting, improves welfare. Nonetheless, notice that these stabilization gains are not in line with the aim of the present paper.

Benigno and Benigno (2006), compute the gains from monetary cooperation. They do not introduce fiscal policy in this work, and just consider the existence of a tax on sales that offset the monopolistic distortion in the steady state. They show that, in general, there are gains from monetary cooperation because of the existence of the externality on terms of trade.

Beetsma and Jensen (2005), follow a setup identical to Gali and Monacelli. However, the method for analyzing the stabilization gains is somewhat different. These authors consider that the Union may solve the following two problems: i) a full optimization problem where the Union simultaneously chooses the monetary policy (through interest rate deviations) and the fiscal policy of each country (through government spending deviations from a flexible-prices scenario); ii) a problem where the fiscal policy is passive and the Union merely chooses the monetary policy. Fiscal stabilization gains derive from the difference in welfare between the two scenarios. One of the main conclusions of these authors is that, because fluctuations depend only on the differences in shocks between the two countries, welfare differences are proportional to the relative shock variance. In their benchmark parametrized case, fiscal stabilization gains represent 0.42% in consumption equivalents. They also study the effect in changing some of the parameters and conclude that stabilization gains varies as a result of relative price changes. Moreover, relative prices increase with the reduction of labor-supply elasticity because the fluctuations in production effort associated with relative price movements are more costly. They

[10] The gain is relative to the case of pursuing a strict price stability rule.
[11] The gain is relative to the case of pursuing a strict price stability rule.
[12] Of the nominal private consumption expenditure per capita in 2005, in the United States.

also increase with i) the elasticity of government-spending, since it allows for more active relative fiscal policy responses; ii) with a reduction in the consumption share, as it reduces the share of output that is sensitive to relative price movements; and, finally, iii) with the increase in price rigidity, because it increases the role of stabilization policies.

Summarizing the above, the purpose of the present paper is to allow for the measurement of total cooperation gains (both deterministic gains and stochastic) in a manner equivalent to the first group of authors above mentioned, but with the ability to use more complex models similar to those used by the second group of authors. Opposed to the first group of authors, the methodology used, as described below, allows for a simpler computation of coordination gains as well as for a clearer separation of such gains from stabilization gains.

# 3 The Economy

The world has two identical countries, Home ($H$) and Foreign ($F$) that have the same technology and market structure. Each country is populated by a continuum of equal consumers with size one. Consumption is composed by two traded goods, one produced domestically and the other produced abroad. The production of each good is linear and only depends on labor, which is immobile across countries. Technology is a random variable which can take different realizations in each country. Fiscal policy is characterized by a tax rate on labor and by government expenditures that are welfare improving. Monetary policy is set by a common Central Bank.

## 3.1 The Households

Each country is inhabited by a representative household with identical preferences that maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, G_t, N_t\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\gamma) \ln C_t + \gamma \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right\} \tag{4}$$

where $C_t$, $N_t$ denote private consumption and labor and $G_t$ public consumption. We assume that in the case of the foreign country, these variables are denoted by a star (for example $C_t^*$ is private consumption in country $F$).

We use this type of utility function[13] because it is the one found in the majority of the literature (Gali and Monacelli (2006), Beetsma and Jensen (2005), among others) allowing for a direct comparison of results. Additionally, as we will explain later, this utility function simplifies our understanding in the differences of the spillovers from the flexible price case and the sticky price case. In the flexible price case, technology shocks do not affect directly labor. Therefore, differences in welfare are just driven from differences in private and public consumption. As the later two variables appear in logarithms in the utility function, the effect of the technology shocks in the cooperative gain can be linearly decomposed. However, when prices are sticky, labor will change with technology and hence cooperative gains are no longer linear in technology.

[13] Particularly, with an additive structure in the logarithm of $C$ and $G$ and also depending on hours.

$C_t$ is a composite consumption index defined by[14]:

$$C_t = 2C_{Ht}^{\frac{1}{2}} C_{Ft}^{\frac{1}{2}} \tag{5}$$

where $C_{Ht}$ is an index of the home country's consumption of domestic goods given by the following CES function:

$$C_{Ht} = \left[\int_0^1 C_{Ht}(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \tag{6}$$

and $C_{Ft}$ is an index of the home country's consumption of foreign goods:

$$C_{Ft} = \left[\int_0^1 C_{Ft}(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{7}$$

The elasticity of substitution between different varieties of a given good is given by $\theta > 1$. We assume that this parameter is the same for the two aggregate goods[15].

As we choose to study identical countries and aggregate consumption is a Cobb-Douglas function it significantly simplifies the exercise.

### 3.1.1 Demand Functions and Price Indexes

The optimal allocation of a given expenditure on each good produced in each country yields the following demand functions:

$$C_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} C_{Ht} \tag{8}$$

$$C_{Ft}(j) = \left(\frac{P_{Ft}^*(j)}{P_{Ft}^*}\right)^{-\theta} C_{Ft} \tag{9}$$

$$C_{Ht}^*(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} C_{Ht}^* \tag{10}$$

$$C_{Ft}^*(j) = \left(\frac{P_{Ft}^*(j)}{P_{Ft}^*}\right)^{-\theta} C_{Ft}^* \tag{11}$$

The price index of good $H$ in country $H$ is given by $P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ and the price index of good $F$ in country $F$ is given by $P_{Ft}^* = \left[\int_0^1 P_{Ft}^*(i)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$. Since this two economies belong to a Monetary Union and all goods are tradeable, we have that $P_{Ht} = P_{Ht}^*$ and $P_{Ft} = P_{Ft}^*$.

Additionally, from the optimal allocation of goods in each country and from the domestic price

[14] For the foreign country this consumption index is given by $C_t^* = 2C_{Ht}^{*\frac{1}{2}} C_{Ft}^{*\frac{1}{2}}$.

[15] For the foreign country, $C_{Ht}^*$ is an index of the foreign country consumption of $H$ goods $\left(C_H^* = \left[\int_0^1 C_H^*(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}\right)$ and $C_{Ft}^*$ is an index of the foreign country consumption of $F$ goods $\left(C_F^* = \left[\int_0^1 C_F^*(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}\right)$.

indexes $\left(P_t = P_t^* = P_{Ht}^{\frac{1}{2}} P_{Ft}^{\frac{1}{2}}\right)$, we can write demand functions in the following way:

$$
\begin{aligned}
C_{Ht} &= \frac{1}{2}\left(\frac{P_{Ft}}{P_{Ht}}\right)^{\frac{1}{2}} C_t & (12)\\
C_{Ft} &= \frac{1}{2}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{\frac{1}{2}} C_t & (13)\\
C_{Ht}^* &= \frac{1}{2}\left(\frac{P_{Ft}}{P_{Ht}}\right)^{\frac{1}{2}} C_t^* & (14)\\
C_{Ft}^* &= \frac{1}{2}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{\frac{1}{2}} C_t^* & (15)
\end{aligned}
$$

where $\frac{P_{Ft}}{P_{Ht}}$ represent the terms of trade between country $F$ and country $H$.

### 3.1.2 Households Problem

We consider the timing as in Lucas (1982). At the beginning of period $t$, households in country $H$ hold nominal wealth $\mathcal{W}_t$. In the asset markets, households can trade nominal balances, $M_t$, noncontingent debt issued by the two countries $B_{Ht} + B_{Ft}$ and private state-contingent debt $E_t\left\{Q_{t,t+1}B_{t+1}\right\}$. The price of this last asset is $Q_{t,t+1}$, that represents the price at date $t$ when the state of the world is $s_t$, of a bond paying one unit of currency at date $t+1$ if the state of the world is $s_{t+1}$. Thus,

$$M_t + B_{Ht} + B_{Ft} + E_t\left\{Q_{t,t+1}B_{t+1}\right\} \leq \mathcal{W}_t$$

Afterwards, goods markets open and households buy consumption goods, restricted to the following cash-in-advance constraint:

$$P_t C_t \leq M_t$$

Finally, at the end of period $t$, they receive labor income net of taxes, $(1-\tau_t) W_t N_t$, seigniorage revenues from the central monetary authority, $Z_t$, profits from the monopolistic firms $\int_0^1 \Gamma_t(i)\, di$ and all asset returns. Therefore, wealth in the beginning of next period is:

$$\mathcal{W}_{t+1} = M_t + (B_{Ht} + B_{Ft}) R_t + B_{t+1} + (1-\tau_t) W_t N_t + Z_t + \int_0^1 \Gamma_t(i)\, di - P_t C_t$$

where $R_t$ is the return of non-contingent assets.

Therefore, in country $H$ households choose $\left\{C_t, N_t, M_{t+1}, B_{Ht+1}, B_{Ft+1}, B_{t+2}\right\}_{t=0}^{\infty}$ in order to maximize its utility (given by equation (4)), subject to the following budget constraint,

$$
\begin{aligned}
& P_t C_t + M_{t+1} + B_{Ht+1} + B_{Ft+1} + E_{t+1}\left\{Q_{t+1,t+2}B_{t+2}\right\} \leq \\
& \leq M_t + (B_{Ht} + B_{Ft}) R_t + B_{t+1} + (1-\tau_t) W_t N_t + Z_t + \int_0^1 \Gamma_t(i)\, di \qquad (16)
\end{aligned}
$$

and to the cash-in-advance condition:

$$P_t C_t \leq M_t$$

The problem of the households in country $F$ is identical.

From the first order conditions of the households problem we obtain the Euler equation (17), the pricing condition of the state contingent debt (18) and the intratemporal condition (19).

$$\frac{1}{P_t C_t} = \beta E_t \frac{R_t}{P_{t+1} C_{t+1}},\ t \geq 0 \tag{17}$$

$$Q_{t,t+1} = \beta \frac{P_t C_t}{P_{t+1} C_{t+1}},\ t \geq 0 \tag{18}$$

$$N_t^{\varphi} = (1-\gamma) \frac{1-\tau_t}{R_t C_t} \frac{W_t}{P_t},\ t \geq 0 \tag{19}$$

From equations (17) and (18) and taking expectations we can observe as usually that $R_t = \frac{1}{E_t Q_{t,t+1}}$.

Since we consider a monetary environment, the cash-in-advance constraint imposes a monetary cost on consumption, as it is shown in the presence of $R_t$ in equation (19). As such, this condition is different from the usual one found in cashless economies models.

## 3.2 The Firms

In country $H$, each firm has the following production function:

$$Y_t(i) = A_t N_t(i) \tag{20}$$

where $Y_t(i)$ is the production of good $i$ that can be used for private consumption in the home and in the foreign country ($C_{Ht}$, $C_{Ht}^*$) and for public consumption in the home country ($G_t$). $A_t$ is an aggregate technology shock. Country $F$ has an analogous production function ($Y_t^*(j) = A_t^* N_t^*(j)$), where $Y_t^*(j)$ can be used for private consumption in the home and in the foreign country ($C_{Ft}$, $C_{Ft}^*$) and for public consumption in the foreign country ($G_t^*$).

### 3.2.1 Price setting

Firms are assumed to set prices one period in advance, that is, whatever the realization of the productivity, prices remain unchanged for one period.

In country $H$ they choose, at $t-1$, the price $P_{Ht}(i)$ in order to maximize the expected value of their profits[16] and taking into account the production function and the demand functions already derived.

$$\max_{P_{Ht}(i)} \quad E_{t-1}\left[Q_{t-1,t} Q_{t,t+1} \left(P_{Ht}(i) Y_t(i) - W_t N_t(i)\right)\right] \Leftrightarrow$$

$$\max_{P_{Ht}(i)} \quad E_{t-1}\left[\frac{1}{R_t} \frac{1}{C_{t+1} P_{t+1}} \left(P_{Ht}(i) Y_t(i) - W_t N_t(i)\right)\right]$$

As we consider that all firms are equal, they all set the same price:

$$P_{Ht} = P_{Ht}(i) = \frac{\theta}{\theta - 1} E_{t-1}\left[\eta_t \frac{W_t}{A_t}\right] \tag{21}$$

where

$$\eta_t = \frac{\frac{1}{R_t} \frac{Y_t}{C_{t+1} P_{t+1}}}{E_{t-1}\left[\frac{1}{R_t} \frac{Y_t}{C_{t+1} P_{t+1}}\right]} \tag{22}$$

[16] The price of profits in period $t$ is characterized by $Q_{t-1,t} Q_{t,t+1}$ which is the same as $Q_{t-1,t+1}$. That is, profits are received in the end of the period $t$ (represented by $t-1$) and are used for consumption one period later ($t+1$).

In the special case of flexible prices, the price chosen contemporaneously by firms is a constant markup over marginal costs:

$$P_{Ht} = P_{Ht}(i) = \frac{\theta}{\theta - 1}\frac{W_t}{A_t} \tag{23}$$

In country $F$, firms face an identical problem.

### 3.3 National Fiscal Authorities

The fiscal authority in the home country taxes labor income at the rate $\tau$. The period by period budget constraint of the home fiscal authority is:

$$\tau_t W_t N_t + B_{H_{t+1}} = P_{Ht} G_t + B_{Ht} R_t \tag{24}$$

where $G_t$ represent public consumption.

For the foreign country, the budget constraint of the fiscal authority is given by

$$\tau_t^* W_t^* N_t^* + B_{Ft} + B_{Ft}^* = P_{Ft} G_t^* + \left(B_{Ft-1} + B_{Ft-1}^*\right) R_{t-1}$$

Notice that for the representation of all the noncontingent debt in both economies we only need three variables: $B_{Ht}$, $B_{Ft}$ and $B_{Ft}^*$[17].

For any given level of $G_t$, the government of country $H$ optimizes the expenditures across national goods, yielding the following government demand function [18]:

$$G_t(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} G_t \tag{25}$$

Later we will suppose that the government will have a balanced budget in each period, therefore infinite accumulation of debt can be ruled out.

### 3.4 Central Monetary Authority

Each period, the central monetary authority sets an interest rate $R_t$, issues money $\left(M_t^U\right)$ and allocates the seigniorage revenue $\left(Z_t^U\right)$ to the two countries.

In the aggregate, the union is subject to the following cash-in-advance constraint:

$$\frac{M_t^U}{P_t} \leq C_t + C_t^* \tag{26}$$

### 3.5 Market Clearing

For each state of the world and date $t$, we have the following market clearing conditions:

- Goods market:

$$Y_t(i) = C_{Ht}(i) + C_{Ht}^*(i) + G_t(i) \tag{27}$$

$$Y_t^*(j) = C_{Ft}(j) + C_{Ft}^*(j) + G_t^*(j) \tag{28}$$

[17]The inclusion of the variable $B_{Ht}^*$ is irrelevant because we can trace all the debt flows between the two countries from the other three variables.

[18]For the foreign country the demand function is $G_t^*(j) = \left(\frac{P_{Ft}^*(j)}{P_{Ft}^*}\right)^{-\theta} G_t^*$.

- Labor market:

$$\int_0^1 N_t(i)\,di = N_t \quad (29)$$
$$\int_0^1 N_t^*(j)\,dj = N_t^* \quad (30)$$

- Money market:

$$Z_t^U = M_{t+1}^U - M_t^U$$

- Risk free nominal bonds market:

$$\int_0^1 (B_{Ht}(i) + B_{Ft}(i))\,di + \int_0^1 B_{Ft}^*(j)\,dj = 0 \quad (31)$$

- State contingent nominal bonds market:

$$\int_0^1 B_t(i)\,di = 0; \quad \int_0^1 B_t^*(j)\,dj = 0 \quad (32)$$

# 4 Aggregate Demand

From the clearing of the market of good $i$ in country $H$ and equations (8), (10), (12), (14) and (25) we obtain:

$$\begin{aligned} Y_t(i) &= C_{Ht}(i) + C_{Ht}^*(i) + G_t(i) \Leftrightarrow \\ Y_t(i) &= \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta}\left[\frac{1}{2}\left(\frac{P_{Ft}}{P_{Ht}}\right)^{\frac{1}{2}}(C_t + C_t^*) + G_t\right] \end{aligned}$$

Summing for all goods $i$ we obtain the following aggregation across goods $i$:

$$Y_t = \frac{1}{2}\left(\frac{P_{Ft}}{P_{Ht}}\right)^{\frac{1}{2}}(C_t + C_t^*) + G_t \quad (33)$$

where $Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}$ is the aggregate output index for country $H$.

Doing the same for good $j$, we obtain:

$$Y_t^* = \frac{1}{2}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{\frac{1}{2}}(C_t + C_t^*) + G_t^* \quad (34)$$

# 5 Flexible price equilibrium

From the market clearing conditions (equations (27) and (28)), the two government budget constraints and the demand functions (equations (12) to (15)) we obtain the following two equations:

$$\left(1 - \frac{\theta-1}{\theta}\tau_t\right)A_t N_t = \frac{1}{2}\left(\frac{P_{Ft}}{P_{Ht}}\right)^{\frac{1}{2}}(C_t + C_t^*) \quad (35)$$
$$\left(1 - \frac{\theta-1}{\theta}\tau_t^*\right)A_t^* N_t^* = \frac{1}{2}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{\frac{1}{2}}(C_t + C_t^*) \quad (36)$$

Using the above expressions we obtain the price ratio as a function of technology, labor and tax rates in both economies.

$$\frac{P_{Ft}}{P_{Ht}} = \frac{A_t}{A_t^*}\frac{N_t}{N_t^*}\frac{\theta-(\theta-1)\tau_t}{\theta-(\theta-1)\tau_t^*} \tag{37}$$

In this model, we have incomplete markets because assets that are traded among countries ($B_H$, $B_F$ and $B_F^*$) are not state contingent. However, a strategy to solve our model is to consider no initial debt.

**Lemma 1** *If state contingent nominal bonds could be traded across countries, considering that both countries have in moment $t=0$ zero debt, the model generates consumption risk sharing.*

***Proof.*** *From equation (18) and the foreign counterpart, we can write that*

$$\begin{aligned} Q_{0,t} &= \beta^t \frac{P_0 C_0}{P_t C_t} \\ Q_{0,t}^* &= \beta^t \frac{P_0 C_0^*}{P_t C_t^*} \end{aligned}$$

*therefore if state contingent nominal bonds could be traded across countries $\left(Q_{0,t}=Q_{0,t}^*\right)$, we obtain that,*

$$C_t = \frac{C_0}{C_0^*} C_t^* \tag{38}$$

*Using both actualized households budget constraint combined with firms's profit, considering that in moment zero, each country hold zero contingent debt, we obtain that*

$$\begin{aligned} \sum_{t=0}^{\infty} C_t &= \sum_{t=0}^{\infty} \frac{1}{\theta}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{\frac{1}{2}} A_t N_t \left[\theta-(\theta-1)\tau_t\right] + B_{H0} + B_{F0} \\ \sum_{t=0}^{\infty} C_t^* &= \sum_{t=0}^{\infty} \frac{1}{\theta}\left(\frac{P_{Ft}}{P_{Ht}}\right)^{\frac{1}{2}} A_t^* N_t^* \left[\theta-(\theta-1)\tau_t^*\right] + B_{F0}^* \end{aligned}$$

*Applying equation (37) and considering that in moment zero each country hold zero debt we obtain that,*

$$\begin{aligned} \sum_{t=0}^{\infty} C_t - B_{H0} - B_{F0} &= \sum_{t=0}^{\infty} C_t^* - B_{F0}^* \Leftrightarrow \\ \sum_{t=0}^{\infty} C_t &= \sum_{t=0}^{\infty} C_t^* \end{aligned}$$

*Applying equation (38), $C_0 = C_0^*$. Hence*

$$C_t = C_t^*$$

■

**Proposition 2** *For the defined structure, considering identical countries is equivalent to consider that the current account is always balanced.*

***Proof.*** *From Lemma (1) we have consumption risk sharing and from demand equations there is balanced trade in each period,*

$$\begin{aligned} C_t^* &= C_t \Leftrightarrow \\ P_{Ht}\frac{1}{2}\left(\frac{P_{Ft}}{P_{Ht}}\right)^{\frac{1}{2}} C_t^* &= P_{Ft}\frac{1}{2}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{\frac{1}{2}} C_t \Leftrightarrow \\ P_{Ht} C_{Ht}^* &= P_{Ft} C_{Ft} \end{aligned}$$

*Hence, in each period debt is not traded. Therefore, the non-existence of contingent debt is irrelevant for the equilibrium.* ■

Using expressions (35) and (36) and knowing that $C_t = C_t^*$ we obtain the price ratio and the aggregate consumption as a function of technology, labor and tax rates in both economies.

$$C_t = \frac{1}{\theta}\left[A_t N_t A_t^* N_t^* \left(\theta - (\theta - 1)\tau_t\right)\left(\theta - (\theta - 1)\tau_t^*\right)\right]^{\frac{1}{2}} \tag{39}$$

Therefore, differences in the terms of trade are due to differences in technology, labor and the labor tax chosen by each country of the union.

Combining the labor supply condition and the foreign counterpart with equations (37) and (39), after some algebra, we obtain an expression for labor as a function of the home tax rate, that does not depend on technology shocks. This implies that, changes in labor are just due to changes in the tax rates. This occurs because we consider a separable (log) utility function.

$$N_t = \left[\frac{(1-\gamma)(\theta-1)(1-\tau_t)}{\theta - (\theta-1)\tau_t}\right]^{\frac{1}{\varphi+1}} \tag{40}$$

And the same for labor in country F:

$$N_t^* = \left[\frac{(1-\gamma)(\theta-1)(1-\tau_t^*)}{\theta - (\theta-1)\tau_t^*}\right]^{\frac{1}{\varphi+1}} \tag{41}$$

Then, the consumption allocations are given by:

$$C_t = \frac{\left[(1-\gamma)(\theta-1)\right]^{\frac{1}{2(\varphi+1)}}}{\theta}\left(A_t A_t^*\right)\left[(1-\tau_t)(1-\tau_t^*)\right]^{\frac{1}{2(\varphi+1)}}\left[\left(\theta-(\theta-1)\tau_t\right)\left(\theta-(\theta-1)\tau_t^*\right)\right]^{\frac{\varphi}{2(\varphi+1)}} \tag{42}$$

Notice that, $C_t = C_t^*$.

Finally, government expenditures can be written as:

$$G_t = \frac{\theta-1}{\theta}\tau_t A_t N_t \tag{43}$$

$$G_t = \frac{\theta-1}{\theta}\tau_t A_t \left[\frac{(1-\gamma)(\theta-1)(1-\tau_t)}{\theta-(\theta-1)\tau_t}\right]^{\frac{1}{\varphi+1}} \tag{44}$$

$$G_t^* = \frac{\theta-1}{\theta}\tau_t^* A_t^* \left[\frac{(1-\gamma)(\theta-1)(1-\tau_t^*)}{\theta-(\theta-1)\tau_t^*}\right]^{\frac{1}{\varphi+1}} \tag{45}$$

For a given value of $M_t^U$ and from the equilibrium allocations $C_t$ and $C_t^*$, we can derive the price level from the cash-in-advance condition $\left(\frac{M_t^U}{P_t} = C_t + C_t^*\right)$.

As fiscal policy is based on taxes on labor income, differences in production due to asymmetric technology shocks are totally absorbed by government consumption, which is not traded among countries. Additionally, as we do not allow for government debt, this means that all the variability of the shock is going to be absorbed in that period and hence, real government consumption will not be smoothed over time. As the value of government expenditures is going to be smoothed that justifies our imposition of zero government debt. That is, relative prices allow for the smoothing of private consumption, labor and the value of government expenditures.

## 5.1 Monetary authority decision

The central monetary authority is going to choose monetary policy independent of the way fiscal authorities choose tax rates. Therefore, we consider that the interest rate is exogenous for both fiscal problems. Hence, our measure of cooperative gains is transversal to the strategic interaction among the monetary and fiscal authorities. Since for most of the exercise we are interested in the optimum, we consider for this model an equilibrium interest rate constant and equal to zero (in our case $R_t = 1$).

## 5.2 Fiscal policy

**Lemma 3** *When consumption and government expenditures are additive in logarithms in the utility function, the optimal cooperative equilibrium in the flexible price economy is characterized by policy instruments* $(\tau_t, \tau_t^*)$ *that are independent of the technology shocks.*

***Proof.*** *As seen in equations (40) to (45) the equilibrium allocations are multiplicative in* $A_t$ *and* $A_t^*$*. Hence, each country's utility function can be written as the sum of two main parts. The first one depends on both tax rates and the second one only depends on the technology shocks and parameters. Therefore, the utility of country* $H$[19] *can be decomposed as,*

$$U_t = \mathcal{U}_1(\tau_t, \tau_t^*) + \mathcal{U}_2(A_t, A_t^*)$$

*where,*

$$\begin{aligned}
\mathcal{U}_1(\tau_t, \tau_t^*) &= \frac{1}{2(\varphi+1)}\ln(1-\tau_t) + \frac{1-\gamma}{2(\varphi+1)}\ln(1-\tau_t^*) + \gamma\ln(\tau_t) \\
&+\frac{(1-\gamma)\varphi - 2\gamma(\varphi+1)}{2(\varphi+1)}\ln(\theta-(\theta-1)\tau_t) + \\
&+\frac{(1-\gamma)\varphi}{2(\varphi+1)}\ln(\theta-(\theta-1)\tau_t^*) + \\
&-\frac{(1-\gamma)(\theta-1)}{\varphi+1}\frac{1-\tau_t}{\theta-(\theta-1)\tau_t} \\
\mathcal{U}_2(A_t, A_t^*) &= \ln A_t + (1-\gamma)\ln A_t^* + \Phi \\
\Phi &= \frac{1+\gamma}{2(\varphi+1)}\left[\ln(1-\gamma) + \ln(\theta-1)\right] + \\
&+\gamma\ln(\theta-1) - \ln(\theta)
\end{aligned}$$

*Hence, the Union's Utility can be written as*

$$U_t^U = \left[\mathcal{U}_1(\tau_t, \tau_t^*) + \mathcal{U}_1^*(\tau_t, \tau_t^*)\right] + \left[\mathcal{U}_2(A_t, A_t^*) + \mathcal{U}_2^*(A_t, A_t^*)\right]$$

*Therefore, in both problems (Nash and Cooperative) maximizing the utility in order to the tax rates is equivalent to maximizing the first component of the utility. Hence optimal tax rates* $(\tau_t, \tau_t^*)$ *do not depend on the technology shocks.* ■

### 5.2.1 The cooperative equilibrium

Having determined the equilibrium of the economy that depends on $(\tau_t, \tau_t^*)$, we compute here the fiscal cooperative equilibrium which corresponds to the equilibrium when the fiscal policy is chosen op-

[19] The decomposition of country's $F$ utility is similar: $U_t^* = \mathcal{U}_1^*(\tau_t, \tau_t^*) + \mathcal{U}_2^*(A_t, A_t^*)$.

timally by a central authority. The objective is to compare this equilibrium with the Nash equilibrium and measure the welfare gain of moving from the decentralized to the coordinated fiscal policy.

When national fiscal policy is coordinated among the two countries, it is as if there was a central authority maximizing the sum of the equally weighted agents' utility in order to choose the two tax rates $(\tau_t, \tau_t^*)$.

$$\begin{aligned} &\max_{\tau_t, \tau_t^*} \quad E\left[U_t^U\right] = E\left[U\left(C_t, N_t, G_t\right) + U^*\left(C_t^*, N_t^*, G_t^*\right)\right] \\ &s.t. \quad eq \quad (40) \quad to \quad (45) \end{aligned}$$

**Proposition 4** *With flexible prices, optimal fiscal policy instruments are constant across states. Therefore, utility deviations from the steady-state equilibrium can be written as,*

$$\widehat{U_t^U} \equiv U^{U^C}\left(\tau_t^C, \tau_t^{*C}, A_t, A_t^*\right) - U^{U^C}\left(\tau^C, \tau^{*C}, A, A^*\right) = \widehat{\mathcal{U}_2^C}\left(A_t, A_t^*, A, A^*\right) \tag{46}$$

***Proof.*** *After some simplifications to the first order conditions of the cooperative problem, the cooperative equilibrium is determined by the tax rate* $\tau_t^C = \tau_t^{*C} = \tau^C$*, constant across dates and states, that solves the following equation:*

$$\frac{\gamma}{\tau^C} - \frac{1}{\varphi+1}\frac{1}{1-\tau^C} + \frac{\gamma-(1-\gamma)\varphi}{\varphi+1}\frac{(\theta-1)}{\theta-(\theta-1)\tau^C} + \frac{1-\gamma}{\varphi+1}\frac{(\theta-1)}{\left[\theta-(\theta-1)\tau^C\right]^2} = 0 \tag{47}$$

*Notice that this Lemma occurs because this problem aggregates two important features: a separable utility function (logarithmic) and an unitary elasticity of substitution between home and foreign goods.* ■

Notice that, as the utility function is logarithmic in consumption and government expenditures, and as labor allocations does not depend on shocks, the optimal cooperative tax rate is constant.

### 5.2.2 The Nash Equilibrium

The fiscal authority of country $H$ is going to choose the tax rate $\tau$ that maximizes the utility of its country households, assuming that its decision will not affect fiscal decisions of the other countries. Therefore, it maximizes

$$\begin{aligned} &\max_{\tau_t | \tau_t^*} \quad E\left[U\left(C_t, N_t, G_t\right)\right] \\ &s.t. \quad eq \quad (40), \quad (42) \quad and \quad (44) \end{aligned}$$

The fiscal authority of the other country faces an identical problem.

$$\begin{aligned} &\max_{\tau_t^* | \tau_t} \quad E\left[U^*\left(C_t, N_t^*, G_t^*\right)\right] \\ &s.t. \quad eq \quad (41), \quad (42) \quad and \quad (45) \end{aligned}$$

**Proposition 5** *With flexible prices, the fiscal policy instruments in the Nash equilibrium are constant across states. Therefore, utility deviations from the steady-state Nash equilibrium can be written as,*

$$\widehat{U_t^U}^N \equiv U^{U^N}\left(\tau_t^N, \tau_t^{*N}, A_t, A_t^*\right) - U^{U^N}\left(\tau^N, \tau^{*N}, A, A^*\right) = \widehat{\mathcal{U}_2^N}\left(A_t, A_t^*, A, A^*\right) \tag{48}$$

***Proof.*** *Applying the same procedure as before, the Nash equilibrium is the tax rate* $\tau_t^N = \tau_t^{*N} = \tau^N$ *that solves the following equation:*

$$\frac{\gamma}{\tau^N} - \frac{1}{2(\varphi+1)}\frac{1}{1-\tau^N} + \frac{2\gamma-(1-\gamma)\varphi}{2(\varphi+1)}\frac{(\theta-1)}{\theta-(\theta-1)\tau^N} + \frac{1-\gamma}{\varphi+1}\frac{(\theta-1)}{[\theta-(\theta-1)\tau^N]^2} = 0$$

*Notice again that the value of the optimal tax rate is constant over the time and states.* ■

The same reasoning as in the cooperative equilibrium applies here. That is, as the utility function is logarithmic in consumption and government expenditures, and as labor allocations does not depend on shocks, the optimal Nash tax rates are independent of shocks.

### 5.2.3 Comparing the Cooperative with the Nash

**Proposition 6** *With flexible prices, stabilization gain is zero.*

***Proof.*** *From equations (46) and (48) we can write the stabilization gain as a function that does not depend on policy instruments.*

$$Stabilization\ Gain \equiv \widehat{U_t^U}^C - \widehat{U_t^U}^N = \widehat{\mathcal{U}_2^C}(A_t, A_t^*, A, A^*) - \widehat{\mathcal{U}_2^N}(A_t, A_t^*, A, A^*) = 0$$

■

For the functional forms used and with flexible prices, optimal tax rates are not state contingent, which generates inexistence of fiscal coordination gains in a stochastic environment. As such, fiscal coordination gains are identical to the deterministic case. As, in the steady-state those tax rates are different in the cooperative and the Nash equilibrium, there exist a deterministic gain. Moreover, as we saw differences among the two equilibria derive from differences in optimal labor, private and government consumption.

To quantify the gains we use standard parameter values considered in the literature. That is, we consider a labor supply elasticity of 2, which implies that $\varphi = \frac{1}{2}$ and a markup over marginal costs of 1.2 which implies that $\theta = 6$. Additionally, we consider $\gamma = 0.25$, which is coincident with the average ratio of government expenditures over GDP in major developed economies.

Observing figure 1, it is evident that, for identical shocks in both countries, the difference between the cooperative and the Nash equilibrium is constant for each level of the technology shock. As this difference is the same for each $A$, we call it deterministic gain.

Notice that obtaining efficient results is not an objective of the present work, since the introduction of just distortionary taxes without any kind of other compensation achieves a trivial inefficient result. Hence, we are not interested in explaining the difference between the cooperative equilibrium and the first-best that is present in figure 1.

### 5.2.4 Results

Table 1 represents the tax rates and the cooperative gains for a given symmetric ($A = A^* = e$) shock. Deterministic gains of moving from a competitive tax system into a cooperative tax system represent an increase of 16.8% in total Union's utility and 17.42% in consumption equivalents.

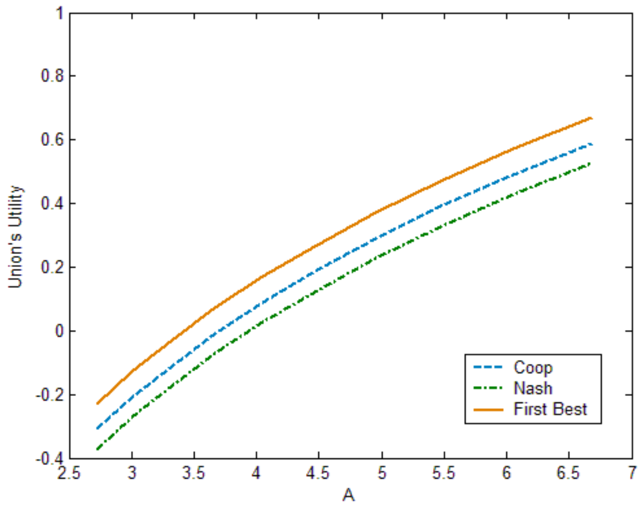


Figure 1: Union's utility in the Cooperative, Nash and First Best equilibria for different levels of the technology shock ($A = A^*$).

| | Cooperative | Nash | Utility Gain | Consumption Equivalents |
|---|---|---|---|---|
| | $\tau^C = \tau^{*C}$ | $\tau^N = \tau^{*N}$ | $\frac{U^{U^C} - U^{U^N}}{U^{U^N}}$ | % |
| $A = A^* = e$ | 27.87% | 46.53% | 16.8% | 17.42% |

Table 1: Cooperative and Nash results under flexible prices

These gains occur because when fiscal policy is chosen independently, each country has an incentive to increase its tax rate in order to reduce the labor effort. As such, we observe that in the Nash equilibrium the tax rate is higher than in the cooperative equilibrium. As a result, in the Nash case private consumption is smaller and public consumption is higher. Even obtaining equality in the terms of trade (because the two countries have the same characteristics), there is an higher level of prices in both countries[20].

We have also performed this analysis with different parameter values. In figure 2, we observe that deterministic gains increase with the elasticity of labor supply. That is, when agents become more elastic in their decisions between labor and leisure, the gain is going to increase. In other words, as this elasticity increases, the amount of labor increases which, in turn, will affect relative prices. Thus, from a coordination point of view, this change in relative prices leads to an higher incentive in changing terms of trade between the two countries resulting in an overall increase in the Union's utility gain when compared with the Nash equilibrium.

In figure 3 we observe that coordination gains are not monotonic with the private consumption share $(1-\gamma)$ represented in the utility function. We verify that, for the low-end range, that is, for very high shares of private consumption, an increase in gamma will lead to an increase in the deterministic gain. This is so because, as we increase gamma, we increase the importance of public expenditure in the utility, thereby increasing the distortion between the cooperative and the Nash equilibrium. However, this link is reversed at $\gamma = 0.3$, the point in which public expenditure represents 27.48% of total consumption. Therefore, once private consumption becomes less then 72.52%, deterministic

[20] See scenario 6 in figure 7, in appendix A.1.

coordination gains have a decreasing connection with $\gamma$, given that the positive effect of the distortion is more than off-set by the negative effect of the increase in the Government's relative weight.

We also present the behavior of deterministic cooperative gains for different markup-over-marginal-cost levels. In figure 4 we observe one of the most discussed ideas in literature: as the market distortion reduces, *i.e.*, as the markup converges towards 1, coordination gains become smaller. It should be noted that, even in this case, we continue to have substantial coordination gains. This is so because, as previously discussed, this model incorporates distortionary taxes. On the other hand, we may also observe that for plausible markup-over-marginal-cost values, increasing deterministic cooperation, gains occur. This happens because, the higher the markup, the greater the distortion on relative prices, which generates a higher deterministic gain. Therefore, we can conclude that deterministic gains are very sensitive to changes in markup values, when compared to changes in the other parameters.

We also perform some sensitive analysis for different risk aversion values. To do so, we apply the above methodology to the following utility function:

$$U = (1-\gamma)\frac{C_t^{1-\rho}}{1-\rho} + \gamma \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi}$$

Figure 5 plots the deterministic gains in terms of the Union's utility for different risk aversion levels. We observe that, as risk aversion increases, gains decrease, implying that the difference between the cooperative solution and the Nash solution fades away. Therefore, coordination does not act as risk compensation.

# 6 Prices set in advance

The model used above is very clear in delivering results since it has a closed form solution. However, it lacks one of the characteristics common in the literature: the presence of nominal rigidities. As such, in this section, we consider that firms set prices on period in advance, introducing the rigidity that was absent in our framework.

## 6.1 Solving the competitive equilibrium

Combining the labor supply (equation (19)) with the labor demand obtained when prices are set one period in advance (equation (21)), after some algebra we obtain the following price setting equation for firms in country $H$[21]:

$$E_{t-1}\left[\left(\frac{P_{Ht}}{P_{Ft}}\right)^{\frac{1}{2}}\frac{A_t N_t}{C_t}\right] = \frac{\theta}{(\theta-1)(1-\gamma)} E_{t-1}\left[\frac{N_t^{\varphi+1}}{1-\tau_t}\right] \tag{49}$$

Firms in country $F$ face a similar equation:

$$E_{t-1}\left[\left(\frac{P_{Ft}}{P_{Ht}}\right)^{\frac{1}{2}}\frac{A_t^* N_t^*}{C_t^*}\right] = \frac{\theta}{(\theta-1)(1-\gamma)} E_{t-1}\left[\frac{N_t^{*\varphi+1}}{1-\tau_t^*}\right] \tag{50}$$

[21] We still consider that $R_t = 1$.

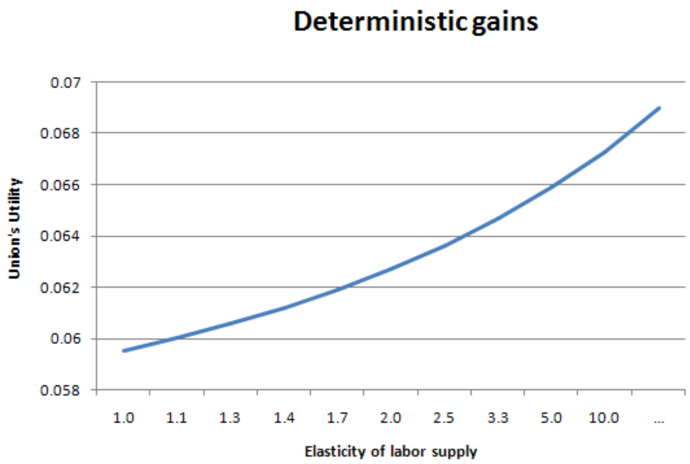


Figure 2: Deterministic gain for different values of the labor supply elasticity.

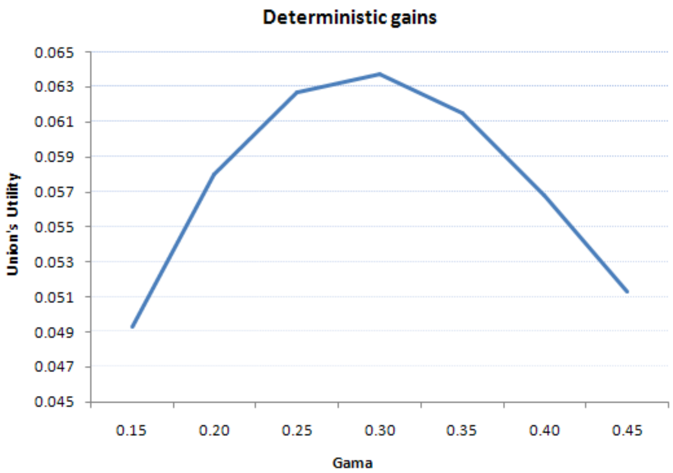


Figure 3: Deterministic gain for different consumption shares.

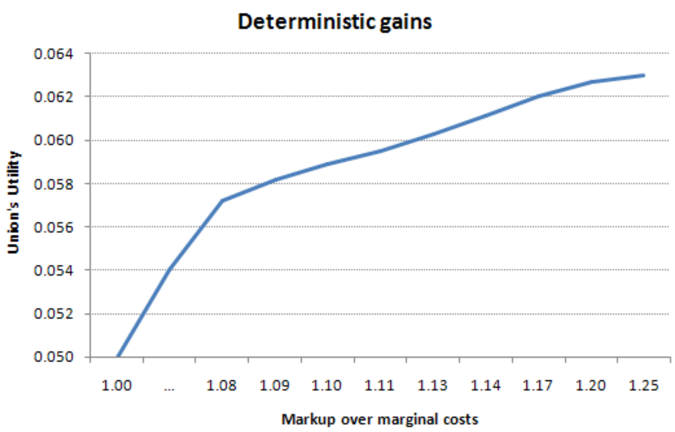


Figure 4: Deterministic gain for different markup levels.

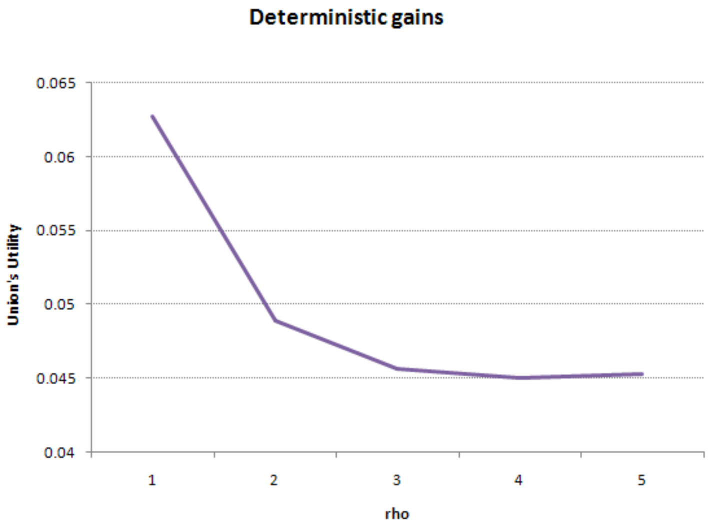


Figure 5: Deterministic gain for different levels of risk aversion.

From the aggregate demand equations and the government budget constraints, we obtain the following two aggregate conditions, where $s$ represent the state:

$$A_s N_s = \frac{1}{2}\left(\frac{P_F}{P_H}\right)^{\frac{1}{2}}\left[C_s\left(1+\frac{2}{1-\gamma}\frac{\tau_s}{1-\tau_s}N_s^{\varphi+1}\right)+C_s^*\right], \quad \forall s \tag{51}$$

$$A_s^* N_s^* = \frac{1}{2}\left(\frac{P_H}{P_F}\right)^{\frac{1}{2}}\left[C_s+C_s^*\left(1+\frac{2}{1-\gamma}\frac{\tau_s^*}{1-\tau_s^*}N_s^{*\varphi+1}\right)\right], \quad \forall s \tag{52}$$

And finally, we use Lemma 1 to consider consumption risk sharing and we assume that money supply is constant in each state of the nature.

$$C_s = C_s^*, \quad \forall s \tag{53}$$

$$\frac{M^U}{P_H^{\frac{1}{2}}P_F^{\frac{1}{2}}} = C_s + C_s^*, \quad \forall s \tag{54}$$

Hence, the equilibrium will be composed by equations (49), (50), and equations (51), (52), (53) and (54) (one for each state).

Notice that the symmetric situation that we derived in the flexible price case does not apply here. Now, $N_s$ will be different from $N_s^*$, and as a result, in the sticky price equilibrium we will have an extra channel of effects. For example, in the presence of a negative shock, individuals will work harder and the impact of the shock will be amplified through the distortionary effects of the tax.

## 6.2 The cooperative equilibrium

The central authority is going to maximize the sum of the equally weighted agents' utility in order to choose the two tax rates in each state $(\tau_s, \tau_s^*)$. As equilibrium allocations derive from the solution of the system of equations (49) to (54), we must include them in this problem, and this way the central fiscal authority is going to choose the two tax rates and $(P_H, P_F, C_s, C_s^*, N_s, N_s^*)_{\forall s}$.

After deriving the cooperative equilibrium problem it is worth arranging the utility function in a simpler way.

$$\begin{aligned} E(U^U) &= \sum_s \pi_s \left\{ \begin{array}{c} \left[(1-\gamma)\ln C_s + \gamma \ln G_s - \frac{N_s^{1+\varphi}}{1+\varphi}\right] \\ +\left[(1-\gamma)\ln C_s^* + \gamma \ln G_s^* - \frac{N_s^{*1+\varphi}}{1+\varphi}\right] \end{array} \right\} = \\ &= \sum_s \pi_s \left\{ \begin{array}{c} \left[ \begin{array}{c} \ln C_s + \gamma \ln \tau_s - \gamma \ln(1-\tau_s) + \frac{\gamma}{2}\ln P_F - \frac{\gamma}{2}\ln P_H \\ +\gamma(\varphi+1)\ln N_s - \frac{N_s^{1+\varphi}}{1+\varphi} - \gamma \ln(1-\gamma) \end{array} \right] \\ + \left[ \begin{array}{c} \ln C_s^* + \gamma \ln \tau_s^* - \gamma \ln(1-\tau_s^*) - \frac{\gamma}{2}\ln P_F + \frac{\gamma}{2}\ln P_H \\ +\gamma(\varphi+1)\ln N_s^* - \frac{N_s^{*1+\varphi}}{1+\varphi} - \gamma \ln(1-\gamma) \end{array} \right] \end{array} \right\} \end{aligned}$$

Therefore, the cooperative problem can be described as,

$$\max_{\{\tau_s,\tau_s^*,P_H,P_F,C_s,C_s^*,N_s,N_s^*\}_{\forall s}} \quad E(U^U) = \sum_s \pi_s \{U(C_s,N_s,\tau_s,P_H,P_F) + U^*(C_s^*,N_s^*,\tau_s^*,P_H,P_F)\}$$
$$s.t. \quad eq \quad 49 \quad to \quad 54$$

The solution to this problem is quite different from the solution in the flexible price case. When prices were flexible we saw that, in equilibrium, labor did not change with technology because the substitution and the income effects where completely offset due to the functional forms chosen in the exercise. However, when prices are set one period in advance, even with the particulars of those functional forms, we obtain a response of labor to technology. For example, in the presence of a positive shock in both countries, we observe that labor is reduced. Now, ex-post real wage is higher from its ex-ante value[22]. Households should work harder to obtain the most of the shock. Additionally, the common fiscal policymaker increases taxes to increase government consumption. Private consumption increases but is smoothed across states.

### 6.3 The Nash Equilibrium

In the Nash equilibrium, the fiscal authority of each country is going to choose its policy instrument in order to maximize the expected value of utility, independent from the choice of the other country. However, as in this case we cannot write the equilibrium allocations explicitly, we include all the equations that define the equilibrium as restrictions to each country's problem.

Given the above, the fiscal authority of country $H$, $\max_\tau \quad E[U(C(\tau,\tau^*), N(\tau,\tau^*), G(\tau,\tau^*))]$ is equivalent to:

$$\max_{\{\tau_s,P_H,P_F,C_s,C_s^*,N_s,N_s^*|\tau_s^*\}_{\forall s}} \quad E(U) = \sum_s \pi_s \{U(C_s,N_s,\tau_s,P_H,P_F)\}$$
$$s.t. \quad eq \quad 49 \quad to \quad 54$$

And the same for the fiscal authority of country $F$ :

$$\max_{\{\tau_s^*,P_H,P_F,C_s,C_s^*,N_s,N_s^*|\tau_s\}_{\forall s}} \quad E(U^*) = \sum_s \pi_s \{U^*(C_s^*,N_s^*,\tau_s^*,P_H,P_F)\}$$
$$s.t. \quad eq \quad 49 \quad to \quad 54$$

[22] It is also higher than the wage rate in the flexible price case.

| State | Probability | Shock in $H$ | Shock in $F$ |
|---|---|---|---|
| $s=1$ | $\pi_1$ | $A_1$ | $A_1^*$ |
| $s=2$ | $\pi_2$ | $A_2$ | $A_2^*$ |
| $s=3$ | $\pi_3$ | $A_3$ | $A_3^*$ |

Table 2: Distribution of shocks

The solution to this Nash problem will be given by combining all first order conditions of the above defined two maximization problems. Notice that the subset of the first order conditions of country $H$'s problem represents its reaction function, and the same is true for country $F$.

The Nash equilibrium has the characteristics of the cooperative equilibrium mainly in what concerns the reaction of labor to shocks and the smoothing of consumption across states. However, compared to the cooperative equilibrium, there is an extra effect that derives from the strategic interaction of both fiscal policymakers. The rationale is the following: as each policymaker does not consider the effects of changing their own tax rate in the other country's policy, in the margin he increases its tax rate in order to reduce labor (that creates disutility) and increase government consumption. This increase in the tax rate creates an upward pressure of terms of trade in each country. However, in the Nash equilibrium, considering cases with symmetric shocks, terms-of-trade do not change and, in the end, the result is welfare-reducing for both countries.

### 6.4 Comparing the Cooperative with the Nash solution

Using the same parametrization as in the flexible price case and considering several distributions of shocks, we can compute the cooperative gains when prices are set one period in advance.

For simplicity, we consider that technology shocks follow a generic discrete distribution with 3 states of nature, described in table 2.

As described in the introduction, the gain in the Union's utility can be decomposed in two main components: the deterministic gain and a pure stochastic gain, the later being equivalent to the cooperative gain in deviations from the steady-state.

$$\text{Pure Stochastic Gain} = \text{Total Gain - Deterministic Gain}$$

where

$$\begin{aligned} \text{Deterministic Gain} &= U^{U^C}_{steady\ state} - U^{U^N}_{steady\ state} \\ \text{Total Gain} &= E\left(U^{U^C}\right) - E\left(U^{U^N}\right) \end{aligned}$$

In table 3 we report the results for some scenarios, considering that $\pi_1 = 0.75$ and $\pi_2 = \pi_3 = 0.125$. We use the replication of the deterministic case, as a benchmark. We observe that it captures the same equilibrium results of the flexible price case. Moreover, we observe that stochastic gains represent between 1.44% and 0.2% of total gains, which in consumption equivalents, represent an increase between 0.03 and 0.19 percentage points, in relation to the deterministic gain. Additionally, we observe that stochastic gains are identical in scenario 1 (symmetric shocks) and 2 (asymmetric shocks), with the following particulars:

| | State | Cooperative | | Nash | | Deterministic gain | |
|---|---|---|---|---|---|---|---|
| | | $\tau$ | $\tau^*$ | $\tau$ | $\tau^*$ | Utility Gain (%.) | Consumption Equiv. (%.) |
| $A_1 = A_1^* = e$<br>$A_2 = A_2^* = e$<br>$A_3 = A_3^* = e$ | $s_1$<br>$s_2$<br>$s_3$ | 27.87%<br>27.87%<br>27.87% | 27.87%<br>27.87%<br>27.87% | 46.53%<br>46.53%<br>46.53% | 46.53%<br>46.53%<br>46.53% | 16.8% | 17.42% |
| | State | Cooperative | | Nash | | Stochastic gain | |
| | | $\tau$ | $\tau^*$ | $\tau$ | $\tau^*$ | In % of Total Gains | Consumption Equiv. (p.p.) |
| Scenario 1<br>$A_1 = A_1^* = e$<br>$A_2 = A_2^* = e^{0.5}$<br>$A_3 = A_3^* = e^2$ | $s_1$<br>$s_2$<br>$s_3$ | 29.05%<br>10.77%<br>75.60% | 29.05%<br>10.77%<br>75.60% | 47.83%<br>23.76%<br>83.86% | 47.83%<br>23.76%<br>83.86% | 1.22% | 0.1908 |
| Scenario 2<br>$A_1 = A_1^* = e$<br>$A_2 = A_3^* = e^{0.5}$<br>$A_3 = A_2^* = e^2$ | $s_1$<br>$s_2$<br>$s_3$ | 29.05%<br>10.77%<br>75.60% | 29.05%<br>75.60%<br>10.77% | 47.83%<br>23.76%<br>83.86% | 47.83%<br>83.86%<br>23.76% | 1.22% | 0.1908 |
| Scenario 3<br>$A_1 = A_1^* = e$<br>$A_2 = A_3 = e^{0.5}$<br>$A_2^* = A_3^* = e^2$ | $s_1$<br>$s_2$<br>$s_3$ | 35.89%<br>14.63%<br>14.63% | 21.44%<br>69.44%<br>69.44% | 54.51%<br>29.87%<br>29.87% | 39.14%<br>79.96%<br>79.96% | 0.82% | 0.1294 |
| Scenario 4<br>$A_1 = A_1^* = e$<br>$A_2 = A_2^* = e^2$<br>$A_3 = A_3^* = e^2$ | $s_1$<br>$s_2$<br>$s_3$ | 21.44%<br>69.44%<br>69.44% | 21.44%<br>69.44%<br>69.44% | 39.14%<br>79.96%<br>79.96% | 39.14%<br>79.96%<br>79.96% | 1.44% | 0.2276 |
| Scenario 5<br>$A_1 = A_1^* = e$<br>$A_2 = A_2^* = e^{0.5}$<br>$A_3 = A_3^* = e^{0.5}$ | $s_1$<br>$s_2$<br>$s_3$ | 35.89%<br>14.63%<br>14.63% | 35.89%<br>14.63%<br>14.63% | 54.51%<br>29.87%<br>29.87% | 54.51%<br>29.87%<br>29.87% | 0.2% | 0.0312 |

Table 3: Deterministic and Stochastic Cooperative Gains

- both economies suffer a negative shock in one state ($s = 2$) and a positive shock in another ($s = 3$), and
- country H suffers a negative shock and country F suffers a positive shock in state 2 and vice-versa in state 3.

This occurs because the mean of the distribution of shocks is unaffected and as we are considering that prices are set one period in advance, the reaction of one given country to a shock is independent from the other country's shock. Ex-ante, countries do not have the information of the shock that is going to happen, and since these shocks are *iid*, prices are the same for the same shock. Therefore, policy affects terms of trade just by the mean of marginal costs and not by strategic redistribution.

**Proposition 7** *Symmetric technology shocks lead to coordination gains with the same magnitude of asymmetric ones with similar mean.*

In figure 6 we disaggregate total cooperative gains into deterministic and stochastic components for each state of the world in the case of a symmetric and a asymmetric shock. In the first panel we report the results for the symmetric shock named scenario 1 with the baseline parametrization in table 3. That is, it depicts the gains that would have happened in a given state. Those gains are very different from what we would obtain had we considered the repetition of each state independently. We observe that, in each state of the world, the stochastic component is not so small as it appears on average. For example, when $s = 2$, the absolute value of the stochastic component accounts for $-149\%$ of the deterministic component and when $s = 3$ this percentage goes up to 111%. It is also important to notice that negative stochastic gains derive from both economies reaction to a bad state, which in this scenario is represented by state $s = 2$.

In the case of an asymmetric shock, represented by scenario 2, in the second panel of figure 6 we observe that in both states 2 and 3, stochastic gains are small, but negative. However, when both countries face an asymmetric shock with the same expected value, the average result of the stochastic component is positive and equal to the case of an equivalent[23] symmetric shock.

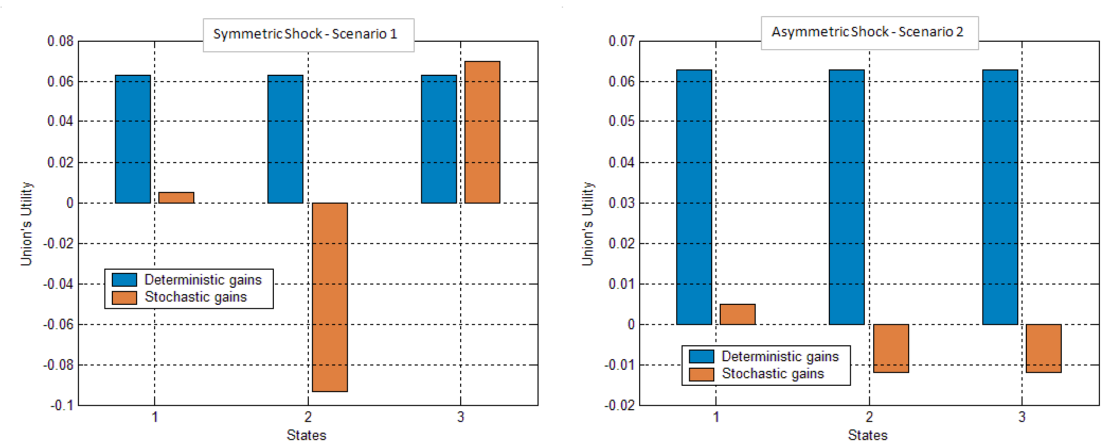


Figure 6: Deterministic and stochastic gains of cooperation in case of a symmetric (scenario 1) and asymmetric shock (scenario 2).

On the other hand, when we look to scenario 3 (in table 3), where one country suffers a negative shock in both states and the other suffers a positive shock, we verify that the stochastic gain, albeit being smaller (0.82%) still exists. Hence, even changing the mean of the distribution of shocks in favor of coordination, we still obtain a stochastic gain.

# 7 Conclusions

In a Monetary Union where monetary stabilization instruments are lost at the country level, the question is how fiscal instruments should be used as means to stabilize shocks that have asymmetric transmissions throughout the Union.

In this paper we develop a simple methodology to measure gains from fiscal cooperation and to separate these gains in two parts: the deterministic and the stochastic gain. We show that deterministic fiscal coordination gains are positive and significative and stochastic coordination gains are very small namely when compared to deterministic ones. Moreover, the dimension of the stochastic gain does

[23]Equivalent meaning that both shocks have the same expected value.

not change significantly if shocks are symmetric or asymmetric. As shown by Lucas (1988), normal risk aversion levels generate small stabilization gains. Hence, we conclude that stabilization fiscal policies should be conducted from a decentralized institutional environment, although the existence of a supranational institution can be sustained in order to coordinate the deterministic component of those policies.

In the next paper we continue this line of investigation studying the robustness of these findings to models where we introduce a Calvo price setting and therefore measure stabilization gains in an environment where shocks and policies have persistent effects.

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# A Appendix

## A.1 Allocation values

| | Cooperative | | | | | | Nash | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | s1 | | s2 | | s3 | | s1 | | s2 | | s3 | |
| | H | F | H | F | H | F | H | F | H | F | H | F |
| **Baseline** | *exp(1)* | *exp(1)* | *exp(1)* | *exp(1)* | *exp(1)* | *exp(1)* | *exp(1)* | *exp(1)* | *exp(1)* | *exp(1)* | *exp(1)* | *exp(1)* |
| **Consumption** | 1.4634 | 1.4634 | 1.4634 | 1.4634 | 1.4634 | 1.4634 | 1.1115 | 1.1115 | 1.1115 | 1.1115 | 1.1115 | 1.1115 |
| **Labor** | 0.7012 | 0.7012 | 0.7012 | 0.7012 | 0.7012 | 0.7012 | 0.6679 | 0.6679 | 0.6679 | 0.6679 | 0.6679 | 0.6679 |
| **Gov. Exp** | 0.4427 | 0.4427 | 0.4427 | 0.4427 | 0.4427 | 0.4427 | 0.704 | 0.704 | 0.704 | 0.704 | 0.704 | 0.704 |
| **Ph or Pf** | 0.3417 | 0.3417 | 0.3417 | 0.3417 | 0.3417 | 0.3417 | 0.4498 | 0.4498 | 0.4498 | 0.4498 | 0.4498 | 0.4498 |
| **Pf/ph** | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | |
| **Real wage** | 2.2652 | 2.2652 | 2.2652 | 2.2652 | 2.2652 | 2.2652 | 2.2651 | 2.2651 | 2.2651 | 2.2651 | 2.2651 | 2.2651 |
| **Tau** | 0.2787 | 0.2787 | 0.2787 | 0.2787 | 0.2787 | 0.2787 | 0.4653 | 0.4653 | 0.4653 | 0.4653 | 0.4653 | 0.4653 |
| **Utility** | | | -0.3096 | | | | | | -0.3723 | | | |
| **Stochastic gain (%)** | | | | | | 0 | | | | | | |
| **Scenario 1** | *exp(1)* | *exp(1)* | *exp(0.5)* | *exp(0.5)* | *exp(2)* | *exp(2)* | *exp(1)* | *exp(1)* | *exp(0.5)* | *exp(0.5)* | *exp(2)* | *exp(2)* |
| **Consumption** | 1.4214 | 1.4214 | 1.4214 | 1.4214 | 1.4214 | 1.4214 | 1.0761 | 1.0761 | 1.0761 | 1.0761 | 1.0761 | 1.0761 |
| **Labor** | 0.6846 | 0.6846 | 1.001 | 1.001 | 0.3745 | 0.3745 | 0.6486 | 0.6486 | 0.8744 | 0.8744 | 0.4303 | 0.4303 |
| **Gov. Exp** | 0.4396 | 0.4396 | 0.2291 | 0.2291 | 1.3456 | 1.3456 | 0.6871 | 0.6871 | 0.3656 | 0.3656 | 2.1037 | 2.1037 |
| **Ph or Pf** | 0.3518 | 0.3518 | 0.3518 | 0.3518 | 0.3518 | 0.3518 | 0.4647 | 0.4647 | 0.4647 | 0.4647 | 0.4647 | 0.4647 |
| **Pf/ph** | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | |
| **Real wage** | 2.2101 | 2.2101 | 2.1250 | 2.1250 | 4.7533 | 4.7533 | 2.2149 | 2.2149 | 1.7598 | 1.7598 | 5.8314 | 5.8314 |
| **Tau** | 0.2905 | 0.2905 | 0.1077 | 0.1077 | 0.756 | 0.756 | 0.4783 | 0.4783 | 0.2376 | 0.2376 | 0.8386 | 0.8386 |
| **Utility** | | | -0.3129 | | | | | | -0.3764 | | | |
| **Stochastic gain (%)** | | | | | | 1.22% | | | | | | |
| **Scenario 2** | *exp(1)* | *exp(1)* | *exp(0.5)* | *exp(2)* | *exp(2)* | *exp(0.5)* | *exp(1)* | *exp(1)* | *exp(0.5)* | *exp(2)* | *exp(2)* | *exp(0.5)* |
| **Consumption** | 1.4214 | 1.4214 | 1.4214 | 1.4214 | 1.4214 | 1.4214 | 1.0761 | 1.0761 | 1.0761 | 1.0761 | 1.0761 | 1.0761 |
| **Labor** | 0.6846 | 0.6846 | 1.0011 | 0.3745 | 0.3745 | 1.0011 | 0.6486 | 0.6486 | 0.8744 | 0.4303 | 0.4303 | 0.8744 |
| **Gov. Exp** | 0.4396 | 0.4396 | 0.2291 | 1.3456 | 1.3456 | 0.2291 | 0.6871 | 0.6871 | 0.3656 | 2.1037 | 2.1037 | 0.3656 |
| **Ph or Pf** | 0.3518 | 0.3518 | 0.3518 | 0.3518 | 0.3518 | 0.3518 | 0.4647 | 0.4647 | 0.4647 | 0.4647 | 0.4647 | 0.4647 |
| **Pf/ph** | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | |
| **Real wage** | 2.2101 | 2.2101 | 2.1251 | 4.7533 | 4.7533 | 2.1251 | 2.2149 | 2.2149 | 1.7598 | 5.8314 | 5.8314 | 1.7598 |
| **Tau** | 0.2905 | 0.2905 | 0.1077 | 0.756 | 0.756 | 0.1077 | 0.4783 | 0.4783 | 0.2376 | 0.8386 | 0.8386 | 0.2376 |
| **Utility** | | | -0.3129 | | | | | | -0.3764 | | | |
| **Stochastic gain (%)** | | | | | | 1.22% | | | | | | |
| **Scenario 3** | *exp(1)* | *exp(1)* | *exp(0.5)* | *exp(2)* | *exp(0.5)* | *exp(2)* | *exp(1)* | *exp(1)* | *exp(0.5)* | *exp(2)* | *exp(0.5)* | *exp(2)* |
| **Consumption** | 1.4474 | 1.4474 | 1.4474 | 1.4474 | 1.4474 | 1.4474 | 1.0969 | 1.0969 | 1.0969 | 1.0969 | 1.0969 | 1.0969 |
| **Labor** | 0.6253 | 0.7746 | 0.8999 | 0.4098 | 0.8999 | 0.4098 | 0.612 | 0.7112 | 0.8072 | 0.4534 | 0.8072 | 0.4534 |
| **Gov. Exp** | 0.4582 | 0.4184 | 0.2421 | 1.3412 | 0.2421 | 1.3412 | 0.7212 | 0.6564 | 0.3882 | 2.0737 | 0.3882 | 2.0737 |
| **Ph or Pf** | 0.4027 | 0.2964 | 0.4027 | 0.2964 | 0.4027 | 0.2964 | 0.5304 | 0.3917 | 0.5304 | 0.3917 | 0.5304 | 0.3917 |
| **Pf/ph** | 0.7360 | | 0.7360 | | 0.7360 | | 0.7385 | | 0.7385 | | 0.7385 | |
| **Real wage** | 1.7520 | 1.5913 | 1.5784 | 2.9755 | 1.5784 | 2.9755 | 1.8574 | 1.4966 | 1.3837 | 3.6291 | 1.3837 | 3.6291 |
| **Tau** | 0.3589 | 0.2144 | 0.1463 | 0.6944 | 0.1463 | 0.6944 | 0.5451 | 0.3914 | 0.2987 | 0.7996 | 0.2987 | 0.7996 |
| **Utility** | | | -0.2989 | | | | | | -0.363 | | | |
| **Stochastic gain (%)** | | | | | | 0.82% | | | | | | |
| **Scenario 4** | *exp(1)* | *exp(1)* | *exp(2)* | *exp(2)* | *exp(2)* | *exp(2)* | *exp(1)* | *exp(1)* | *exp(2)* | *exp(2)* | *exp(2)* | *exp(2)* |
| **Consumption** | 1.6872 | 1.6872 | 1.6872 | 1.6872 | 1.6872 | 1.6872 | 1.2765 | 1.2765 | 1.2765 | 1.2765 | 1.2765 | 1.2765 |
| **Labor** | 0.7746 | 0.7746 | 0.4098 | 0.4098 | 0.4098 | 0.4098 | 0.7112 | 0.7112 | 0.4534 | 0.4534 | 0.4534 | 0.4534 |
| **Gov. Exp** | 0.4184 | 0.4184 | 1.3412 | 1.3412 | 1.3412 | 1.3412 | 0.6564 | 0.6564 | 2.0737 | 2.0737 | 2.0737 | 2.0737 |
| **Ph or Pf** | 0.2964 | 0.2964 | 0.2964 | 0.2964 | 0.2964 | 0.2964 | 0.3917 | 0.3917 | 0.3917 | 0.3917 | 0.3917 | 0.3917 |
| **Pf/ph** | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | |
| **Real wage** | 2.5202 | 2.5202 | 4.7124 | 4.7124 | 4.7124 | 4.7124 | 2.3584 | 2.3584 | 5.7188 | 5.7188 | 5.7188 | 5.7188 |
| **Tau** | 0.2144 | 0.2144 | 0.6944 | 0.6944 | 0.6944 | 0.6944 | 0.3914 | 0.3914 | 0.7996 | 0.7996 | 0.7996 | 0.7996 |
| **Utility** | | | -0.1373 | | | | | | -0.201 | | | |
| **Stochastic gain (%)** | | | | | | 1.44% | | | | | | |
| **Scenario 5** | *exp(1)* | *exp(1)* | *exp(0.5)* | *exp(0.5)* | *exp(0.5)* | *exp(0.5)* | *exp(1)* | *exp(1)* | *exp(0.5)* | *exp(0.5)* | *exp(0.5)* | *exp(0.5)* |
| **Consumption** | 1.2416 | 1.2416 | 1.2416 | 1.2416 | 1.2416 | 1.2416 | 0.9426 | 0.9426 | 0.9426 | 0.9426 | 0.9426 | 0.9426 |
| **Labor** | 0.6253 | 0.6253 | 0.8999 | 0.8999 | 0.8999 | 0.8999 | 0.612 | 0.612 | 0.8072 | 0.8072 | 0.8072 | 0.8072 |
| **Gov. Exp** | 0.4582 | 0.4582 | 0.2421 | 0.2421 | 0.2421 | 0.2421 | 0.7211 | 0.7211 | 0.3882 | 0.3882 | 0.3882 | 0.3882 |
| **Ph or Pf** | 0.4027 | 0.4027 | 0.4027 | 0.4027 | 0.4027 | 0.4027 | 0.5304 | 0.5304 | 0.5304 | 0.5304 | 0.5304 | 0.5304 |
| **Pf/ph** | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | |
| **Real wage** | 0.8223 | 0.8223 | 0.7408 | 0.7408 | 0.7408 | 0.7408 | 1.1466 | 1.1466 | 0.8540 | 0.8540 | 0.8540 | 0.8540 |
| **Tau** | 0.3589 | 0.3589 | 0.1463 | 0.1463 | 0.1463 | 0.1463 | 0.5452 | 0.5452 | 0.2987 | 0.2987 | 0.2987 | 0.2987 |
| **Utility** | | | -0.4622 | | | | | | -0.525 | | | |
| **Stochastic gain (%)** | | | | | | 0.20% | | | | | | |
| **Scenario 6** | *exp(1)* | *exp(2)* | *exp(1)* | *exp(2)* | *exp(1)* | *exp(2)* | *exp(1)* | *exp(2)* | *exp(1)* | *exp(2)* | *exp(1)* | *exp(2)* |
| **Consumption** | 2.4127 | 2.4127 | 2.4127 | 2.4127 | 2.4127 | 2.4127 | 1.8326 | 1.8326 | 1.8326 | 1.8326 | 1.8326 | 1.8326 |
| **Labor** | 0.7012 | 0.7012 | 0.7012 | 0.7012 | 0.7012 | 0.7012 | 0.6679 | 0.6679 | 0.6679 | 0.6679 | 0.6679 | 0.6679 |
| **Gov. Exp** | 0.4427 | 1.2035 | 0.4427 | 1.2035 | 0.4427 | 1.2035 | 0.704 | 1.9136 | 0.704 | 1.9136 | 0.704 | 1.9136 |
| **Ph or Pf** | 0.3417 | 0.1257 | 0.3417 | 0.1257 | 0.3417 | 0.1257 | 0.4498 | 0.1655 | 0.4498 | 0.1655 | 0.4498 | 0.1655 |
| **Pf/ph** | 0.3679 | | 0.3679 | | 0.3679 | | 0.3679 | | 0.3679 | | 0.3679 | |
| **Real wage** | 1.3738 | 1.3738 | 1.3738 | 1.3738 | 1.3738 | 1.3738 | 1.3741 | 1.3741 | 1.3741 | 1.3741 | 1.3741 | 1.3741 |
| **Tau** | 0.2787 | 0.2787 | 0.2787 | 0.2787 | 0.2787 | 0.2787 | 0.4653 | 0.4653 | 0.4653 | 0.4653 | 0.4653 | 0.4653 |
| **Utility** | | | 0.1904 | | | | | | 0.1277 | | | |
| **Stochastic gain (%)** | | | | | | 0 | | | | | | |
| **Scenario 7** | *exp(1)^2* | *exp(1)^2* | *exp(1)^2* | *exp(1)^2* | *exp(1)^2* | *exp(1)^2* | *exp(1)^2* | *exp(1)^2* | *exp(1)^2* | *exp(1)^2* | *exp(1)^2* | *exp(1)^2* |
| **Consumption** | 3.9779 | 3.9779 | 3.9779 | 3.9779 | 3.9779 | 3.9779 | 3.0215 | 3.0215 | 3.0215 | 3.0215 | 3.0215 | 3.0215 |
| **Labor** | 0.7012 | 0.7012 | 0.7012 | 0.7012 | 0.7012 | 0.7012 | 0.6679 | 0.6679 | 0.6679 | 0.6679 | 0.6679 | 0.6679 |
| **Gov. Exp** | 1.2035 | 1.2035 | 1.2035 | 1.2035 | 1.2035 | 1.2035 | 1.9136 | 1.9136 | 1.9136 | 1.9136 | 1.9136 | 1.9136 |
| **Ph or Pf** | 0.1257 | 0.1257 | 0.1257 | 0.1257 | 0.1257 | 0.1257 | 0.1655 | 0.1655 | 0.1655 | 0.1655 | 0.1655 | 0.1655 |
| **Pf/ph** | 1.0000 | | 1.0000 | | 1.0000 | | 1.0000 | | 1.0000 | | 1.0000 | |
| **Real wage** | 6.1574 | 6.1574 | 6.1574 | 6.1574 | 6.1574 | 6.1574 | 6.1575 | 6.1575 | 6.1575 | 6.1575 | 6.1575 | 6.1575 |
| **Tau** | 0.2787 | 0.2787 | 0.2787 | 0.2787 | 0.2787 | 0.2787 | 0.4653 | 0.4653 | 0.4653 | 0.4653 | 0.4653 | 0.4653 |
| **Utility** | | | 0.6904 | | | | | | 0.6277 | | | |
| **Stochastic gain (%)** | | | | | | 0 | | | | | | |

Figure 7: Allocation values for different scenarios.