Coordination and Stabilization Gains of Fiscal Policy in a Monetary Union

Susana Salvado

Coordination and Stabilization Gains of Fiscal Policy in a Monetary Union[1]

Susana Salvado
FEUNL, GEE

Classificação JEL codes: E61, E62, F41, F42

Keywords: Coordination, Fiscal Policy, Gains, Nash.

Abstract

The issue of fiscal coordination in a Monetary Union is recurrent as monetary policy can no longer be used as a national stabilization policy instrument. We measure the increase in welfare due to the coordination of fiscal policies in the typical Neo-Keynesian environment, where monetary policy would have significantive and persistent real effects. We propose a decomposition of coordination gains into a deterministic and a stochastic parcel. We show that the deterministic fiscal coordination gain is high but that the stochastic gain, often called stabilization gain, is very small generating, for our calibration, an increase of 0.0161 percentage points, measured in consumption equivalents.

Index

Coordination and Stabilization Gains of Fiscal Policy in a Monetary Union

Susana Salvado[*] [†]

FEUNL, GEE

March 30, 2009

Abstract

The issue of fiscal coordination in a Monetary Union is recurrent as monetary policy can no longer be used as a national stabilization policy instrument. We measure the increase in welfare due to the coordination of fiscal policies in the typical Neo-Keynesian environment, where monetary policy would have significantive and persistent real effects. We propose a decomposition of coordination gains into a deterministic and a stochastic parcel. We show that the deterministic fiscal coordination gain is high but that the stochastic gain, often called stabilization gain, is very small generating, for our calibration, an increase of 0.0161 percentage points, measured in consumption equivalents.

1 Introduction

In the Neo-Keynesian literature, the main objective of policy is the stabilization of economies that are subject to shocks. Moreover, gains that occur when two or more countries coordinate their policies are called stabilization gains. The motivation of this paper came from the idea that when countries join a monetary union they loose monetary policy as a stabilization policy instrument. Hence, it is important to substitute it by a fiscal instrument. To avoid spillovers that independent choices of policy could be undertaken, there are some who advise for a central decision on state contingent fiscal policies, namely when countries are subject to asymmetric shocks.

In this paper we aim not only to clarify the conceptual distinction between stabilization and coordination gains, but mainly to assess quantitatively the importance both concepts.

We extend the method developed in Salvado (2009). However, differently from that study, we use an environment more directly comparable with most of the Neo-Keynesian literature, where stabilization policy with monetary instruments have the highest possibilities. That is, we introduce nominal rigidities that have persistence effects due to pricing technologies that impose the setting of prices for more than one period.

The increase in welfare within a Monetary Union is measured, following the change from an independent fiscal policy in every country, to the decision of fiscal policy by a common policy maker, that is, the coordinated fiscal policy. We divide this increase in welfare into a deterministic effect, that accounts for the elimination of country strategic interactions that occur in the steady-state, and

---

a stochastic effect that measures the gain when policies take into account the shocks that occur in the economies. This last effect depends on the direct effect of the shock, that is usually related to stabilization, and also to the elimination of the strategic interaction among countries that is derived through the volatility of the shock. We show that deterministic gains represent an increase of around 17% in consumption equivalents[1] and that the stochastic component is very small, representing an increase of around 0.0161 p.p., in consumption equivalents.

We consider a model composed of two identical countries, where each country produces a tradeable composite good using labor (that is immobileacross countries) with a linear technology, that is subject to shocks. Prices are set *à la Calvo*. Households consume every good and are subject to a cash-in-advance constraint on the purchase of both goods. As such, we use a monetary model where money is used for unit of account and transaction, but we consider that monetary policy is decided at the union level such that the monetary distortion is minimized. Government consumption, in every country, is limited to national produced goods, and is financed by a distortionary tax on labor income. As said before these two countries belong to a monetary union, where fiscal policy is initially implemented at a domestic level.

With the purpose of simplification of the interaction analyzed in this model we consider that the monetary policy is implemented independently from fiscal policy[2]. Regarding fiscal policy, we first consider the case where each country's government does not take into account the effects of its actions on the other country's government policy (that represents the Nash equilibrium) and compare it with the case where fiscal policy is implemented in a coordinated manner (that represents the cooperative equilibrium). However, notice that in the coordinated case, we consider that fiscal policy is decided by a supranational authority that could implement different policies for different countries. Differently from monetary policy that is coordinated and harmonized in the monetary union, fiscal policy makers in both institutions are free to discriminate across countries.

Moreover, we describe the source of cooperative gains, that is, the welfare difference between the coordinated and the non-coordinated situations, from the fact that both countries have an incentive to deviate from the coordinated solution. If a country increases its tax rate it can increase its terms of trade in order to get a better trade gain. This will reduce labor effort. When every country follows the same strategy, terms of trade will not change and tax distortions increase leading to a lower welfare in the Nash equilibrium. As such, the main objective is to measure the amount of this loss in stochastic economies with nominal rigidities.

To compute coordination gains[3] we proceed as follows. For the measurement of the deterministic component, as nominal rigidities are not active, the economy works like the flexible price one. Since with flexible prices the model has a closed form solution, we can precisely measure the coordination gain, by computing the welfare differences from the Nash and the Cooperative equilibrium. Next, we compute the gain derived from the stochastic component when shocks are added to the economy, since the optimal solution will be solved numerically in deviations from the deterministic steady state.

---

[1]In comparison to the Nash case.

[2]We suppose that $\pi = 1$ is the target.

[3]We explain in detail this methodology in Salvado (2009).

Notice that, when imposing a Calvo price setting[4] the procedure to compute the coordination gain is not straightforward. We would like to highlight that we do not use any approximation prior to the optimization procedure, that is, prior to the conditions that define the choice of policies. As such, after defining the set of equations that characterize the equilibrium, we use this set of equations, mostly non-linear ones, as restrictions to our cooperative and Nash problems. It is only after deriving first order conditions of these problems, that we solve them by linear approximation. We consider a first order approximation of the variables around the corresponding optimal steady-state[5].

Our work is in line with the Ramsey literature, where the optimal fiscal policy is the one that results from a benevolent planner[6] that chose among all feasible equilibrium set of allocations.

In this literature, all the dynamics in the Ramsey equilibrium are solved by a first or $n-$order Taylor approximation towards the Ramsey steady-states. However, as argued in Schmitt-Grohé and Uribe (2005), "the exact solution is not significantly different from the one based on a first-order approximation". Hence, we consider a first-order approximation in our model. As these authors show, using the Taylor expansion, in the case of a linear approximation

$$\left(E\left(U^U \right) \simeq E\left(\overline{U^U} + \alpha \left(U^U - \overline{U^U} \right) \right) \right)$$

the unconditional mean of Union's utility $\left(\overline{U^U} \right)$ is equivalent to its deterministic steady-state, since α does not depend on the distribution of the shock. Therefore, in the presence of a shock, the certainty equivalent occurs and shock volatility does not contaminate average utility. When the approximation is done to the second degree we no longer have certainty equivalent and the unconditional mean $\left(\overline{U^U} \right)$ differs from the deterministic steady-state, not because α and β depend on shocks, but because the mean is going to be affected by shocks.

$$E\left(U^U \right) \simeq E\left(\overline{U^U} + \alpha \left(U^U - \overline{U^U} \right) + \beta \left(U^U - \overline{U^U} \right)^2 \right)$$

Parallel to these articles, there is the literature that uses the Linear Quadratic Method as an approximation technique. This method was used by Benigno and Benigno (2006) to measure gains from monetary cooperation, by Ferrero (2007) to measure the gain of pursuing debt stabilization and by Gali and Monacelli (2007) to measure fiscal stabilization gains in a Monetary Union. As explained in Salvado (2009) the problem of this method is that it measures stabilization as the quadratic difference towards a target that does not necessarily reflects the optimal level that can be pursued.

Gali and Monacelli (2007) has a very similar environment to the one used in this article and is therefore our benchmark. In their model the Monetary Union is composed by a continuum of countries and they consider lump-sum taxes and a Calvo price setting. They compare two types of policies: the optimal cooperative policy and the non-coordinated policy. They conclude that without fiscal coordination there is no stabilization of the output and fiscal gap at the aggregate level. As the fiscal gap is not stabilized, the aggregate price level has inefficient fluctuations which makes the Central Bank to inefficiently trade-off between output gap and inflation volatility.

---

[4]Or any other characteristic that generates a dynamic environment.

[5]To compute the policy functions, impulse responses and other moments we use the package of Christiano (1998).

[6]In this case, in the coordinated equilibrium the sum of the utilities of both countries is not the same of the representative agent, since he does not exist in general.

Another point worth mentioning is that these authors pay little attention to quantifying the gain derived from achieving coordination, that is, the loss reduction from moving from a non-coordinated policy to the optimal cooperative policy. These authors consider it to be "quantitatively small", but give no idea of its quantification. Moreover, considering that coordination gains are the difference from the stabilization gains that occur under coordination and non-coordination, it would be interesting to analyze the origin of its magnitude.

Ferrero (2007) does not account for the gain of a coordinated fiscal policy, he compares different fiscal policies always in a decentralized framework. That is, in his fiscal policy stabilization gain, he is concerned with the gain derived from using fiscal policy in order to stabilize a economy when it is hit by an asymmetric shock, relative to the situation of only using fiscal policy to obtain a balanced budget.

This paper is organized as follows. First we present the economy considering that firms set prices *à la Calvo*. Then, we compute the conditions that define the best policy when fiscal policy is chosen by a common authority and compare it to the conditions that characterize the best policy when fiscal instruments are chosen at the country level. The welfare difference of these two equilibria is the gain from fiscal cooperation. Additionally, computing the gain that would occur in the steady-state, allow us to decompose our gain in two effects: the deterministic effect and the effect that occurs due to shocks. We consider various scenarios for the occurrence of the shock and finally we measure and decompose the welfare gain from fiscal coordination.

2   The Economy

The world has two countries, Home (H) and Foreign (F) that have both linear technologies. Markets have a monopolistic competition structure. Each country is populated by a continuum of equal consumers with size one and identical preferences. In each period t the economy experiences one of finitely many events s_t. The initial realization s_0 is given. The set of all possible events in period t is denoted by S_t, the history of these events up to and including period t, which we call state at t, $(s_0, s_1, ..., s_t)$, is denoted by s^t, and the set of all possible states in period t is denoted by S^t. For the specific case analyzed, the distribution of these events is identical for every country. Each country sets their own fiscal policy, government expenditures and the tax rate paths, and the monetary policy is set by a common Central Bank.

2.1   The Households

Each country is inhabited by a representative household that maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\gamma) \ln C_t + \gamma \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right\} \tag{1}$$

where C_t, G_t, N_t, denote respectively private consumption, public consumption and labor. We assume the same preferences for the foreign household country, over C_t^*, G_t^*, N_t^*.

In its turn C_t is a composite consumption index defined by[7]:

$$C_t = 2C_{Ht}^{\frac{1}{2}}C_{Ft}^{\frac{1}{2}} \tag{2}$$

where C_{Ht} is an index of the home country's consumption of domestic goods given by the following CES function:

$$C_{Ht} = \left[\int_0^1 C_{Ht}(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \tag{3}$$

and C_{Ft} is an index of the home country's consumption of foreign goods:

$$C_{Ft} = \left[\int_0^1 C_{Ft}(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{4}$$

Note that the elasticity of substitution between home and foreign goods is equal to one. However, the elasticity of substitution between different varieties of a given good is given by $\theta > 1$. We assume that this parameter is the same for the two countries[8].

2.1.1 Demand Functions and Price Indexes

The optimal allocation of a given expenditure on each good produced in each country yields the following demand functions:

$$C_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} C_{Ht} \tag{5}$$

$$C_{Ft}(j) = \left(\frac{P_{Ft}^*(j)}{P_{Ft}^*}\right)^{-\theta} C_{Ft} \tag{6}$$

$$C_{Ht}^*(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} C_{Ht}^* \tag{7}$$

$$C_{Ft}^*(j) = \left(\frac{P_{Ft}^*(j)}{P_{Ft}^*}\right)^{-\theta} C_{Ft}^* \tag{8}$$

Where the price index of good H in country H is given by $P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ and the price index of good F in country F is given by $P_{Ft}^* = \left[\int_0^1 P_{Ft}^*(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$. Since this two goods are tradeables, in the Monetary Union we have that $P_H = P_H^*$ and $P_F = P_F^*$.

We can write the expenditure in each composite good purchased by the household of country H as[9],

$$\int_0^1 P_{Ht}(i)C_{Ht}(i)di = P_{Ht}C_{Ht}$$

$$\int_0^1 P_{Ft}(j)C_{Ft}(j)dj = P_{Ft}C_{Ft}$$

---

[7]For the foreign country this consumption index is given by $C_t^* = 2C_{Ht}^{*\frac{1}{2}}C_{Ft}^{*\frac{1}{2}}$.

[8]For the foreign country, C_{Ht}^* is an index of the foreign country consumption of H goods $\left(C_H^* = \left[\int_0^1 C_H^*(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}\right)$ and C_{Ft}^* is an index of the foreign country consumption of F goods $\left(C_F^* = \left[\int_0^1 C_F^*(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}\right)$.

[9]Equivalent expressions are obtained for the households of country F.

Additionally, from the optimal allocation of goods in each country and from the domestic price indexes $\left(P_t = P_t^* = P_{Ht}^{\frac{1}{2}} P_{Ft}^{\frac{1}{2}} \right)$, we obtain the following demand functions:

$$C_{Ht} = \frac{1}{2} \left(\frac{P_{Ft}}{P_{Ht}} \right)^{\frac{1}{2}} C_t \tag{9}$$

$$C_{Ft} = \frac{1}{2} \left(\frac{P_{Ht}}{P_{Ft}} \right)^{\frac{1}{2}} C_t \tag{10}$$

$$C_{Ht}^* = \frac{1}{2} \left(\frac{P_{Ft}}{P_{Ht}} \right)^{\frac{1}{2}} C_t^* \tag{11}$$

$$C_{Ft}^* = \frac{1}{2} \left(\frac{P_{Ht}}{P_{Ft}} \right)^{\frac{1}{2}} C_t^* \tag{12}$$

where $\frac{P_{Ft}}{P_{Ht}}$ $\left(\frac{P_{Ht}}{P_{Ft}} \right)$ represent the terms of trade in F (H).

We consider the timing as in Lucas (1982). At the beginning of period t, households in country H, hold nominal wealth $\mathcal{W}_t$. In the asset markets they have asses to nominal balances, M_t, noncontingent debt issued by the two countries $B_{Ht} + B_{Ft}$ and private state-contingent debt $E_t \{Q_{t,t+1} B_{t+1}\}$ that cannot be traded among countries. The price of this last asset is $Q_{t,t+1}$, that represents the price at date t when the state of the world is s_t, of a bond paying one unit of currency at date $t+1$ if the state of the world is s_{t+1}. Thus,

$$M_t + B_{Ht} + B_{Ft} + E_t \{Q_{t,t+1} B_{t+1}\} \leq \mathcal{W}_t \tag{13}$$

Afterwards, good markets open and they buy consumption goods, restricted to the following cash-in-advance constraint:

$$P_{Ht} C_{Ht} + P_{Ft} C_{Ft} \leq M_t$$

Finally, at the end of period t, they receive labor income net of taxes, $(1 - \tau_t) W_t N_t$, seigniorage revenues from the central monetary authority, Z_t, profits from the monopolistic firms ($\int_0^1 \Gamma_t(i) \, di$) and all asset returns. Therefore, wealth in the beginning of next period is:

$$\mathcal{W}_{t+1} = M_t + (B_{Ht} + B_{Ft}) R_t + B_{t+1} + (1 - \tau_t) W_t N_t + Z_t + \int_0^1 \Gamma_t(i) \, di - P_{Ht} C_{Ht} - P_{Ft} C_{Ft}$$

Therefore, in country H households choose $\{C_{Ht}, C_{Ft}, N_t, M_t, B_{Ht}, B_{Ft}, B_{t+1}\}_{t=0}^{\infty}$ in order to maximize its utility $E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1-\gamma}{2} \ln C_{Ht} + \frac{1-\gamma}{2} \ln C_{Ft} + \gamma \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right\}$, subject to the following budget constraint,

$$P_{Ht} C_{Ht} + P_{Ft} C_{Ft} + M_{t+1} + B_{Ht+1} + B_{Ft+1} + E_{t+1} \{Q_{t+1,t+2} B_{t+2}\} \leq$$

$$\leq M_t + (B_{Ht} + B_{Ft}) R_t + B_{t+1} + (1 - \tau_t) W_t N_t + Z_t + \int_0^1 \Gamma_t(i) \, di \tag{14}$$

and to the cash-in-advance condition:

$$P_{Ht} C_{Ht} + P_{Ft} C_{Ft} \leq M_t$$

The households in country F have an identical problem.

From the first order conditions of the households problem we obtain the Euler equation (equation (15)), the intertemporal condition of the state contingent debt (equation (16)) and two intratemporal conditions (equations (17) and (18)).

$$\frac{1}{P_{Ht}C_{Ht}} = \beta E_t \frac{R_t}{P_{Ht+1}C_{Ht+1}} \tag{15}$$

$$Q_{t,t+1} = \beta \frac{P_{Ht}C_{Ht}}{P_{Ht+1}C_{Ht+1}} \tag{16}$$

$$N_t^\varphi C_{Ht} = \frac{1-\gamma}{2} \frac{1-\tau_t}{R_t} \frac{W_t}{P_{Ht}} \tag{17}$$

$$\frac{C_{Ht}}{C_{Ft}} = \frac{P_{Ft}}{P_{Ht}} \tag{18}$$

From (15) and (16) and taking expectations we can get the usual non-arbitarge condition $R_t = \frac{1}{E_t Q_{t,t+1}}$.

2.2 National Fiscal Authorities

The fiscal authority in the home country taxes labor income at the rate τ. The budget constraint of the home fiscal authority[10] is:

$$\tau_t W_t N_t + B_{Ht} = P_{Ht}G_t + B_{Ht-1}R_{t-1} \tag{19}$$

where G_t represent public consumption of the domestically produced good. For any given level of G_t the government optimizes the expenditures across national goods, yielding the following government demand function [11]:

$$G_t(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} G_t \tag{20}$$

The foreign fiscal authority has a similar budget and the same fiscal instruments.

2.3 Central Monetary Authority

Each period, the central monetary authority sets the interest rate, R_t, issues money, M_t^U and allocates the seigniorage revenue Z_t, Z_t^* to the two countries.

2.4 The Firms

In country H, each firm has the following production function:

$$Y_t(i) = A_t N_t(i) \tag{21}$$

where $Y_t(i)$ is the production of good i that can be used for private consumption in the home and in the foreign country (C_{Ht}, C_{Ht}^*) and for public consumption in the home country (G_t). A_t is a random

---

[10] For the foreign country, the budget constraint of the fiscal authority is given by
$\tau_t^* W_t^* N_t^* + B_{Ft} + B_{Ft}^* = P_{Ft}G_t^* + (B_{Ft-1} + B_{Ft-1}^*) R_{t-1}$

[11] For the foreign country the demand function is $G_t^*(j) = \left(\frac{P_{Ft}(j)}{P_{Ft}}\right)^{-\theta} G_t^*$.

variable that represents aggregate technology in country H. Country F has an analogous production function $(Y_t^*(j) = A_t^* N_t^*(j))$, where $Y_t^*(j)$ can be used for private consumption in the home and in the foreign country (C_{Ft}, C_{Ft}^*) and for public consumption in the foreign country (G_t^*).

From the clearing of the market of good i in country H and equations (5), (7) and (20), we can write that:

$$
\begin{aligned}
Y_t(i) &= C_{Ht}(i) + C_{Ht}^*(i) + G_t(i) \Leftrightarrow \\
Y_t(i) &= \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} [C_{Ht} + C_{Ht}^* + G_t]
\end{aligned}
$$

Summing for all goods i we obtain the following aggregation:

$$
Y_t = C_{Ht} + C_{Ht}^* + G_t \tag{22}
$$

where $Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$ is aggregate output index for country H.

Doing the same for good j, we obtain:

$$
Y_t^* = C_{Ft} + C_{Ft}^* + G_t^* \tag{23}
$$

We consider that each firm is a monopolistic producer of one of the differentiated goods produced in each country. The price-setting is à la Calvo (1983) and the fraction of firms that set prices optimally in a given period is given by $1 - \mu$. Hence, when a firm has the opportunity to set a new price in period t, it maximizing the expected discount value of its profits. For the rest of the firms, we consider that they just adjust the pre set prices according to the steady-state inflation rate[12] π_H, π_F for firms in the home country and for firms in the foreign country, respectively. Hence,

$$
\begin{aligned}
P_{Ht}(m) &= \pi_H P_{Ht-1}(m), \text{ for firms in country } H \\
P_{Ft}(n) &= \pi_F P_{Ft-1}(n), \text{ for firms in country } F
\end{aligned}
$$

The problem of the firms (in country H) that get the chance to optimize their price $(\widetilde{P}_{Ht})$, can be described as:

$$
\max_{\widetilde{P}_{Ht}} \quad E_t \sum_{k=0}^{\infty} (\beta\mu)^k v_{t+k} \left[\widetilde{P}_{Ht} Y_{t+k}(m) - \frac{W_{t+k}}{A_{t+k}} Y_{t+k}(m)\right]
$$

subject to

$$
Y_{t+k}(m) = \left(\frac{P_{Ht+k}}{P_{Ht}}\right)^{\theta} Y_{t+k}
$$

Therefore, the optimal choice of $\widetilde{P}_{Ht}$ is given by:

$$
\frac{\widetilde{P}_{Ht}}{P_{Ht}} = \frac{E_t \sum_{k=0}^{\infty} (\beta\mu)^k \frac{1-\gamma}{2} \frac{Y_{t+k}}{C_{Ht+k}} X_{t,k}^{-\theta} \frac{\theta}{\theta-1} \frac{W_{t+k}}{P_{Ht+k}} \frac{1}{A_{t+k}}}{E_t \sum_{k=0}^{\infty} (\beta\mu)^k \frac{1-\gamma}{2} \frac{Y_{t+k}}{C_{Ht+k}} X_{t,k}^{1-\theta}}
$$

where $X_{t,k} = \frac{X_{t+1,k-1}}{\pi_{Ht+1}}$, or:

$$
X_{t,k} = \begin{cases} \frac{1}{\pi_{Ht+1}\pi_{Ht+2}\ldots\pi_{Ht+k}}, & k \geq 1 \\ 1, & k = 0 \end{cases}
$$

---

[12]Definitions of inflation rates are given by $\pi_{Ht} = P_{Ht}/P_{Ht-1}$ and $\pi_{Ft} = P_{Ft}/P_{Ft-1}$.

Additionally, the aggregate price can be written as:

$$P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \Leftrightarrow$$

$$P_{Ht} = \left[(1-\mu) \widetilde{P}_{Ht}^{1-\theta} + \mu P_{Ht-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

$$\widetilde{p}_{Ht} = \left[\frac{1}{1-\mu} - \frac{\mu}{1-\mu} \pi_{Ht}^{\theta-1} \right]^{\frac{1}{1-\theta}}, \quad with \quad \widetilde{p}_{Ht} = \frac{\widetilde{P}_{Ht}}{P_{Ht}} \tag{24}$$

In the special case of flexible prices, the price chosen by every firm is identical and a constant markup over marginal costs:

$$P_{Ht} = P_{Ht}(i) = \frac{\theta}{\theta-1} \frac{W_t}{A_t} \tag{25}$$

2.5   Market Clearing

For each state of the world, s_t and date t, we have the following market clearing conditions:

- Goods market:

$$Y_t(i) = C_{Ht}(i) + C_{Ht}^*(i) + G_t(i) \tag{26}$$

$$Y_t^*(j) = C_{Ft}(j) + C_{Ft}^*(j) + G_t^*(j) \tag{27}$$

- Labor market:

$$\int_0^1 N_t(i) \, di = N_t \tag{28}$$

$$\int_0^1 N_t^*(j) \, dj = N_t^* \tag{29}$$

- Money market:

$$M_t^U \le P_{Ht}(C_{Ht} + C_{Ht}^*) + P_{Ft}(C_{Ft} + C_{Ft}^*) \tag{30}$$

- State contingent nominal bonds market:

$$\int_0^1 B_t(i) \, di = 0; \quad \int_0^1 B_t^*(j) \, dj = 0 \tag{31}$$

- Non state contingent bonds market:

$$\int_0^1 (B_{Ht}(i) + B_{Ft}(i)) \, di + \int_0^1 B_{Ft}^*(j) \, dj = 0 \tag{32}$$

3   The equilibrium

The assumption taken in this paper, that the two countries are identical in structure and distribution of shocks, imply a symmetric equilibrium and an identical expected welfare whatever is the equilibrium with symmetric fiscal policy instruments. In addition, since we take as given a common monetary policy and identical distribution of seigniorage, the influences that a choice of one policy in one country can have on the equilibrium of the other country, are only determined through terms of

trade and nominal prices, given the nominal rigidity. Since we consider that the nominal interest rate is defined ex-ante and therefore constant, this implies that the channel of external assets is reduced in its importance. This reasoning leads us to simplify the analysis imposing no change of assets either across countries or between government and households. Therefore we can write the equilibrium of the trade balance in every date and state as an additional restriction on the equilibrium. It can be written as,

$$P_{Ht}C_{Ht}^* = P_{Ft}C_{Ft} \tag{33}$$

The equilibrium with sticky prices can be defined as the sequence of allocations and prices $\left\{C_{Ht}, C_{Ft}, C_{Ht}^*, C_{Ft}^*, N_t, N_t^*, Y_t, Y_t^*, G_t, G_t^*, \frac{P_{Ft}}{P_{Ht}}, \frac{W_t}{P_{Ht}}, \frac{W_t^*}{P_{Ft}}, K_t, F_t, K_t^*, F_t^*, \pi_{Ht}, \pi_{Ft}\right\}_{t=0}^{\infty}$ and policies $\left\{R_t, M_t^U, Z_t, Z_t^*, \tau_t, \tau_t^*\right\}_{t=0}^{\infty}$ that satisfy the following conditions:

$$Y_t = C_{Ht} + C_{Ht}^* + G_t \tag{34}$$

$$Y_t^* = C_{Ft} + C_{Ft}^* + G_t^* \tag{35}$$

$$G_t = \tau_t \frac{W_t}{P_{Ht}} N_t \tag{36}$$

$$G_t^* = \tau_t^* \frac{W_t^*}{P_{Ft}} N_t^* \tag{37}$$

$$Y_t = A_t N_t \tag{38}$$

$$Y_t^* = A_t^* N_t^* \tag{39}$$

$$\frac{P_{Ft}}{P_{Ht}} = \frac{C_{Ht}}{C_{Ft}} \tag{40}$$

$$\frac{P_{Ft}}{P_{Ht}} = \frac{C_{Ht}^*}{C_{Ft}^*} \tag{41}$$

$$E_t \frac{C_{Ht} R_t}{\pi_{Ht+1} C_{Ht+1}} = \frac{1}{\beta} \tag{42}$$

$$E_t \frac{C_{Ft}^* R_t}{\pi_{Ft+1} C_{Ft+1}^*} = \frac{1}{\beta} \tag{43}$$

$$\frac{1-\gamma}{2} \frac{1}{C_{Ht}} = \frac{N_t^{\varphi}}{1-\tau_t} \frac{R_t}{\frac{W_t}{P_{Ht}}} \tag{44}$$

$$\frac{1-\gamma}{2} \frac{1}{C_{Ft}^*} = \frac{N_t^{*\varphi}}{1-\tau_t^*} \frac{R_t}{\frac{W_t^*}{P_{Ft}}} \tag{45}$$

$$K_t = \frac{1-\gamma}{2} \frac{Y_t}{C_{Ht}} \frac{\theta}{\theta-1} \frac{W_t}{P_{Ht}} \frac{1}{A_t} + \beta\mu E_t \left(\frac{1}{\pi_{Ht+1}}\right)^{-\theta} K_{t+1} \tag{46}$$

$$K_t^* = \frac{1-\gamma}{2} \frac{Y_t^*}{C_{Ft}^*} \frac{\theta}{\theta-1} \frac{W_t^*}{P_{Ft}} \frac{1}{A_t^*} + \beta\mu E_t \left(\frac{1}{\pi_{Ft+1}}\right)^{-\theta} K_{t+1}^* \tag{47}$$

$$F_t = \frac{1-\gamma}{2} \frac{Y_t}{C_{Ht}} + \beta\mu E_t \left(\frac{1}{\pi_{Ht+1}}\right)^{1-\theta} F_{t+1} \tag{48}$$

$$F_t^* = \frac{1-\gamma}{2} \frac{Y_t^*}{C_{Ft}^*} + \beta\mu E_t \left(\frac{1}{\pi_{Ft+1}}\right)^{1-\theta} F_{t+1}^* \tag{49}$$

$$\left(\frac{K_t}{F_t}\right)^{1-\theta} = \frac{1}{1-\mu} - \frac{\mu}{1-\mu} \pi_{Ht}^{\theta-1} \tag{50}$$

$$\left(\frac{K_t^*}{F_t^*}\right)^{1-\theta} = \frac{1}{1-\mu} - \frac{\mu}{1-\mu} \pi_{Ft}^{\theta-1} \tag{51}$$

$$\frac{P_{Ft}}{P_{Ht}} = \frac{C^*_{Ht}}{C_{Ft}} \tag{52}$$

Notice that we can substitute $\frac{P_{Ft}}{P_{Ht}}$ of the trade balance (equation (52)) into equations (40) and (41) in order to eliminate this variable from the system. We, then obtain the following two equations:

$$C_{Ht} = C^*_{Ht} \tag{53}$$

$$C_{Ft} = C^*_{Ft} \tag{54}$$

The above two equations represent risk sharing in aggregate consumption. Due to the structure of the economy, imposing balanced trade in each moment in time, is equivalent to imposing consumption levels of each good equal across countries.

4 Strategy to solve the cooperative and non-cooperative equilibria

Most studies in the Neo-Keynesian literature which cannot be solved in closed form (see Salvado 2009) approximate the set of equilibria by loglinearizing and then, depending on the institutions, compute the optimal equilibrium. Namely, the Linear Quadratic approach in models with distortionary taxation can be described with the following procedure as in Beningo and Woodford (2005): first is computed a second-order approximation to the model structural equations. Then, those approximated equations are used to solve for the expected discounted value of output as a quadratic function. This solution can then be used to substitute for the terms proportional to expected discounted output in the quadratic approximation to expected utility. In this way, it is obtained an approximation to expected utility which is purely quadratic, that can be seen as a loss function. This loss function can then be evaluated to second order using an approximate solution for the endogenous variables of the model that is accurate only to first order. One is then able to compute a linear approximation to optimal policy using a simple linear-quadratic methodology.

Here instead we obtain an optimum choice of policies in every institution prior to performing any type of approximation[13]. Only afterwards, when we derive all the equations that define the cooperative equilibrium and the Nash equilibrium, do we perform a linear approximation around the corresponding steady-state, to be able to get a numerical solution. Notice that we use a linear approximation since we rely on the proof of Schmitt-Grohé and Uribe (2005), where they show that the exact solution of a model is not significantly different from the one based on first-order approximations.

We proceed to the approximation around the steady-state of the non-stochastic cooperative (and Nash[14]) equilibrium of the set of equations that define the optimum. That is, we take the first order Taylor expansion of all the equations that define the cooperative (and Nash) equilibrium. We consider that each variable $\widehat{x}_t \equiv X_t - X$, being X the steady-state of a given variable[15].

Given the values for the monetary policy and the choice in this problem of fiscal policies, the state is defined in this problem by $s_t = \{A_t, A^*_t\}$.

---

[13]In both problems we consider a timeless perspective as defined in Woodford (2003).

[14]We would like to notice that this procedure is done to the cooperative equilibrium and to the Nash equilibrium in separate.

[15]We use the numerical method of Christiano (1998) to solve the linear system.

We assume that shocks evolve according to

$$\widehat{s}_t = P\widehat{s}_{t-1} + \varepsilon_t$$

where $\widehat{s}_t = \begin{bmatrix} \widehat{a}_t \\ \widehat{a}_t^* \end{bmatrix}$, $P = \begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix}$ with $\rho < 1$ and ε_t is uncorrelated over time and over shocks. For the numerical simulations we consider that the serial correlation of the technology shock is $\rho = 0.9$, that the standard deviation of both shocks is 0.05 and that the shocks between countries are uncorrelated.

Therefore, the solution to this problem can be described by

$$\widehat{z}_t = A\widehat{z}_{t-1} + B\widehat{s}_t$$

where z_t is the vector of all variables in deviations from the steady-state.

We develop the exercise for identical countries: identical technology, identical preferences, identical initial conditions and identical distributions of state variables.

In all the numeric simulations we use a parametrization common to the literature. As such, we consider a labor supply elasticity of 2 ($\varphi = 0.5$), a markup over marginal costs of 1.2 ($\theta = 6$) and that the probability of a firm optimizing prices is 25% ($\mu = 0.75$). Additionally, we consider $\gamma = 0.25$, which is coincident to the average ratio of government consumption over GDP in the major developed economies. Finally we parametrize $\beta = 0.96$, and $A = A^* = 10$.

We consider that monetary policy is chosen independently of the way fiscal authorities choose tax rates. That is, monetary policy is exogenous and given by $R_t = R_{steady\,state}$.

4.1 The cooperative equilibrium

The policy objective of the central policy maker is the choice of both fiscal instruments (τ_t, τ_t^*) that maximize the expected sum of life time utilities of both countries at time 0. Hence, the common fiscal policy maker chooses
$\left\{ C_{Ht}, C_{Ft}, C_{Ht}^*, C_{Ft}^*, N_t, N_t^*, Y_t, Y_t^*, G_t, G_t^*, \frac{W_t}{P_{Ht}}, \frac{W_t^*}{P_{Ft}}, K_t, F_t, K_t^*, F_t^*, \pi_{Ht}, \pi_{Ft}, \tau_t, \tau_t^* \right\}_{t=0}^{\infty}$ that maximizes

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \left[\frac{1-\gamma}{2}\ln C_{Ht} + \frac{1-\gamma}{2}\ln C_{Ft} + \gamma\ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \\ + \left[\frac{1-\gamma}{2}\ln C_{Ht}^* + \frac{1-\gamma}{2}\ln C_{Ft}^* + \gamma\ln G_t^* - \frac{N_t^{*1+\varphi}}{1+\varphi} \right] \end{array} \right\}$$

s.t.

Equations (34) to (39); (42) to (51); (53) and (54)

The optimal steady-state[16] allocations are:

$$C_H^C = 2.6231; \quad C_H^{*C} = 2.6231$$

$$C_F^C = 2.6231; \quad C_F^{*C} = 2.6231$$

$$G^C = 1.5792; \quad G^{*C} = 1.5792$$

$$N^C = 0.68253; \quad N^{*C} = 0.68253$$

$$\tau^C = 0.27765; \quad \tau^{*C} = 0.27765$$

$$\pi_H^C = 1; \quad \pi_F^C = 1; \quad \frac{P_F^C}{P_H^C} = 1$$

$$U^C = 0.9814; \quad U^{*C} = 0.9814$$

$$U^{U,C} = 0.9814$$

Given the hypothesis of symmetry that we impose, the optimal steady state allocations are identical in every country. This assumption and the symmetry of both aggregate goods, C_H and C_F, (C_H^* and C_F^*) in the definition of aggregate consumption[17], leads to a relative price in the steady-state equal to one.

4.2 The Nash equilibrium

The Nash equilibrium can be described by taking simultaneously the system of equations that define the problem of the fiscal policymaker in country H, and the equations that define the problem of the fiscal policymaker in country F, as those include clearing conditions. A natural way to represent the Nash equilibrium of each country was to include as restrictions to the maximization problem, all the equations that define the competitive equilibrium. However, to simplify the computations we first analyze which restrictions are necessary to include in the problem of each country.

Proposition 1 *Considering that the competitive equilibrium of a generic economy can be summarized as the following system of 3 equations, where (X, P, τ) represents allocations, prices and policies,*

$$F_1(X, P, \tau) = 0$$

$$F_2(X^*, P, \tau^*) = 0$$

$$F_3(P, \tau, \tau^*) = 0$$

The Nash equilibrium can be written as usual as (problem A):

$$\max_{\tau} \quad U(X(\tau, \tau^*)), \quad \text{for country } H$$

$$\max_{\tau^*} \quad U^*(X^*(\tau, \tau^*)), \quad \text{for country } F$$

or as, (problem B):

$$\max_{\substack{\tau, X, P \\ s.t.}} \quad U(X)$$

$$F_1(X, P, \tau) = 0$$

$$F_3(P, \tau, \tau^*) = 0$$

---

[16] That coincides with the steady-state of the cooperative equilibrium with flexible prices.

[17] This is equivalent with identical preferences to, no home bias.

$$\max_{\substack{\tau^*, X^*, P \\ s.t.}} \quad U^*(X^*)$$

$$F_2(X^*, P, \tau^*) = 0$$

$$F_3(P, \tau, \tau^*) = 0$$

Proof. In appendix. ∎

Applying the structure of problem B, the fiscal policymaker of country H is going to choose $\left\{ C_{Ht}, C_{Ft}, C_{Ht}^*, N_t, Y_t, G_t, \frac{W_t}{P_{Ht}}, K_t, F_t, \pi_{Ht}, \tau_t \right\}_{t=0}^{\infty}$ that maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1-\gamma}{2} \ln C_{Ht} + \frac{1-\gamma}{2} \ln C_{Ft} + \gamma \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right\}$$

s.t.

Equations (34), (36), (38), (42), (44), (46), (48), (50), (53) and (54)

At the same time the fiscal policymaker of country F is going to choose $\left\{ C_{Ft}, C_{Ht}^*, C_{Ft}^*, N_t^*, Y_t^*, G_t^*, \frac{W_t^*}{P_{Ft}^*}, K_t^*, F_t^*, \pi_{Ft}, \tau_t^* \right\}_{t=0}^{\infty}$ that maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1-\gamma}{2} \ln C_{Ht}^* + \frac{1-\gamma}{2} \ln C_{Ft}^* + \gamma \ln G_t^* - \frac{N_t^{*1+\varphi}}{1+\varphi} \right\}$$

s.t.

Equations (35), (37), (39), (43), (45), (47), (49), (51), (53) and (54)

The main steady-state[18] allocations are:

$$C_H^N = 2.0019; \quad C_H^{*N} = 2.0019$$

$$C_F^N = 2.0019; \quad C_F^{*N} = 2.0019$$

$$G^N = 2.5037; \quad G^{*N} = 2.5037$$

$$N^N = 0.65075; \quad N^{*N} = 0.65075$$

$$\tau^N = 0.4617; \quad \tau^{*N} = 0.4617$$

$$\pi_H^N = 1; \quad \pi_F^N = 1; \quad \frac{P_F^N}{P_H^N} = 1$$

$$U^N = 0.9199; \quad U^{*N} = 0.9199$$

$$U^{U,N} = 0.9199$$

As seen before, and by the same reasoning described in the optimum cooperative equilibrium, we observe that allocations are identical in both countries and relative price equals one. However when we compare the steady state equilibrium in the cooperative case and in the Nash, we can observe that they are quite different. In the Cooperative equilibrium, countries have an incentive to reduce the level of labor effort, increasing the tax rate on labor, which is the same as increasing government expenditures. This interaction is inefficient, creating lower consumption levels in the steady-state of the Nash equilibrium. Moreover, the reduction of labor effort and the increase in government expenditures is not sufficient to compensate the reduction in consumption levels, generating a lower level of utility as compared to the cooperative equilibrium. This utility difference represents the deterministic gain.

---

[18] That coincides with the steady-state of the Nash equilibrium with flexible prices.

4.3 Understanding the gain

4.3.1 Impulse response functions

To understand the response of optimal allocations to shocks we develop some exercises: first we consider a common positive technology shock for both countries (represented in figures 1 and 2) and then we consider a positive technology shock in country H (represented in figures 3 and 4). The blue lines describe the Cooperative equilibrium and the green dotted lines the Nash equilibrium.

For the case of a common technology shock, we observe an increase in both tax rates in the moment of the shock, followed by a reduction to a level below the steady-state. Finally, both tax rates converge to their initial level. This path in common to the Cooperative and the Nash equilibrium. However, the cooperative equilibrium tax rate is characterized by an higher initial increase, compared to the Nash case. This occurs because in the Nash equilibrium each country does not take into account the other country fiscal policy, which imply a smaller movement of the tax rate in the Nash case. By nature, government consumption mimics the path of the tax rates.

Notice that, as shocks are identical in both countries and we are considering equality among countries, the effects on relative prices are symmetric, which imply constancy of terms of trade.

As regards labor, it slightly decreases in reaction to the shock because of the initial positive income effect, then it increases above the steady state and it converges again to the initial level. We also observe an increase in the consumption of both goods by both countries. Finally, the Union's Utility is characterized by an initial increase of less than 1% and my a smooth decrease towards the steady-state level. However, the difference between the Nash and the cooperative equilibrium path of this variable is not very large, which give us an hint for the small magnitude of the stochastic gain.

In figures 3 and 4 we plot the impulse response function for a positive shock in the home country (H). Again, labor slightly decreases in the more productive country (H), then it increases above the steady state and it converges again to the initial level. We also observe an increase in the consumption of good H (the good produced in the country that was subject to the shock). However, due to the type of utility function used, consumption of good F does not change. Since we are considering a logarithmic utility function in both types of consumption, the marginal utility of consuming a good is independent of the other. Hence, the income effect does not propagate from one consumption good to the other. However, the positive effect of the shock is passed to the other country (in this case F) through the consumption of good H, which implies a slightly increase in relative prices in the moment of the shock.

As regards fiscal policy, we observe that the optimal strategy for the common fiscal policymaker is to increase labor taxes in the country that occurred the shock and to keep constant taxes in the other country in order to obtain the same pattern of government expenditures. In the end, utility in both countries temporarily increase.

Notice that although with differences in magnitude, the reactions to a shock in the Nash case are similar to the coordinated case. This is so because with this setup the only variable that changes in country F is consumption. As labor remains constant, country F does not have any incentive in changing taxes, which implies that strategic interactions among individual fiscal policymakers are
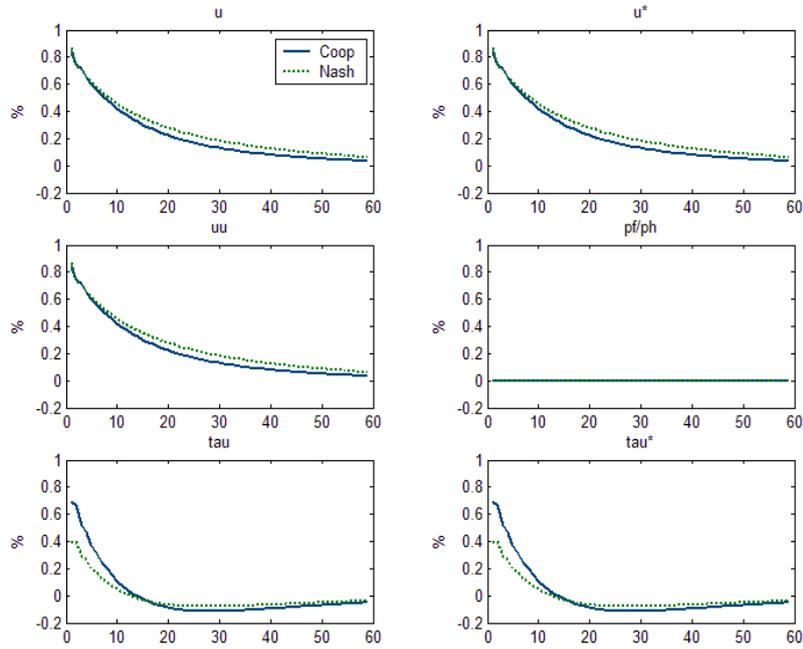
Figure 1: Cooperative and Nash equilibrium impulse response functions of a simultaneous positive technology shock in both countries.
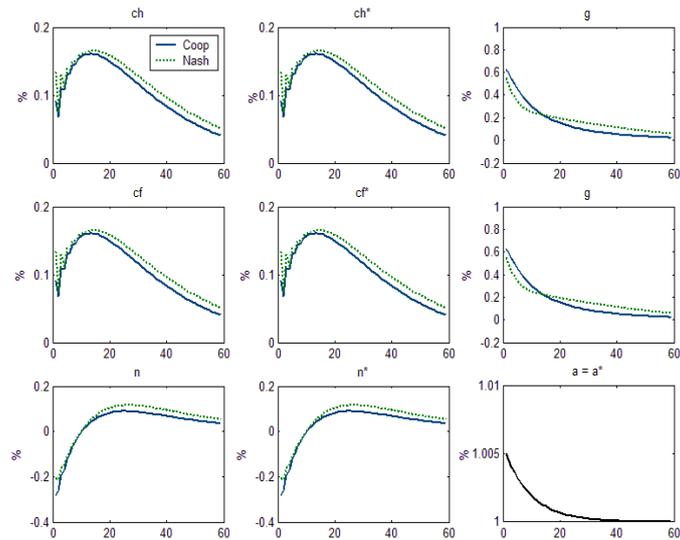

Figure 2: Cooperative and Nash equilibrium impulse response functions of a simultaneous positive technology shock in both countries (continuation).
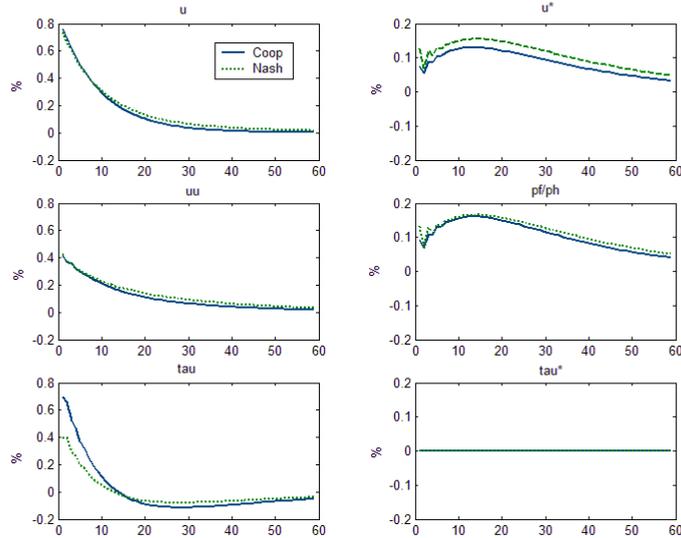
Figure 3: Cooperative and Nash equilibrium impulse response functions of a positive technology shock in country H.

	Total Gain (%)	Deterministic		Stochastic	
		Gain (%)	%	Gain (p.p.)	%
Consumption Equivalents	**17.17**%	17.16%	99.9%	0.0161	0.1%
Utility Gain	**6.70**%	6.69%	99.8%	0.0137	0.2%

Table 1: Decomposition of the cooperative gains

inexistent.

Therefore, we conclude that as the shape of the impulse response functions are very similar in the Nash and in the cooperative case, fiscal policy does not appear crucial as a stabilization device.

4.3.2 Simulations

We follow the method used in Salvado (2009) which consists of splitting the cooperative gain into two main components: the deterministic gain and the stochastic gain. The stochastic gain is computed as the mean of the gains that occur within 5.000 simulations[19] of the approximate two linearized models (the cooperative and the Nash) with the stochastic process for the state variables, defined in section 4.1.

We present the results in table 1. We observe that the deterministic gain accounts for more than 99.9% of total gains and represents an increase of 17.16% in consumption equivalents. However, the stochastic component is small, representing an increase of 0.0161 p.p. in consumption equivalents, that only represent 0.1% of total gains.

Therefore we see that, as shown in Salvado (2009) for a static model, the dynamics will not reverse the results. The gain from using coordination of policies as a reaction to shocks is small. With this

---

[19]The results do not change significantly if we increase the number of simulations.
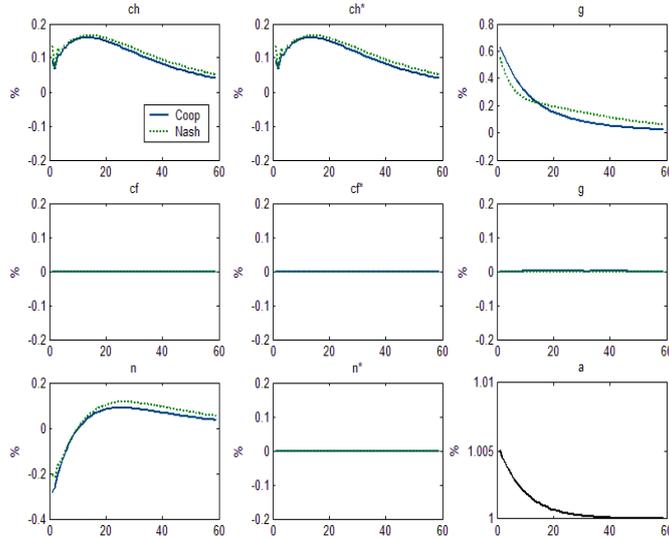
Figure 4: Cooperative and Nash equilibrium impulse response functions of a positive technology shock in country H (continuation).

example done here we can see how small this gain is.[20]

5 Conclusions

In this paper we compute the fiscal coordination gain that would emerge in a Monetary Union. Considering a Calvo price setting, and separating stabilization from the deterministic component, we show that, the deterministic fiscal coordination gains are always significative, whereas stabilization gains are, on average, almost zero, representing 0.1% of total coordination gains, measured in consumption equivalents. With this measurement we are able to quantify the gain that would occur in a Monetary Union if fiscal policy would be coordinated over the cycle. The cost of constructing a common fiscal authority and the incentives needed to impose harmonized policies instead of the coordinated solution would for sure be higher. Therefore we conclude that, as is well known, changing volatility even when in comes from models with nominal rigidities has a small effect on welfare. This result should then be robust to more complex environments. However, at the same time, as we conclude that deterministic gains are high, fiscal policy coordination on the choice and on the average level of fiscal instruments should be part of a common economic policy in a monetary union.

References

[1] Beetsma, R. and H. Jensen, 2005. "Monetary and Fiscal Policy Interactions in a Micro-Founded Model of a Monetary Union", Journal of International Economics 67 (2): 320-352.

---

[20]We have also performed some sensitive analysis for different risk aversion values as regards private consumption, however, the stochastic gain remained very small.

[2] Benigno, G. and P. Benigno, 2000. "Monetary Policy Rules and the Exchange Rate", Berkeley; New Jersey: Princeton University.

[3] Benigno, G. and P. Benigno, 2003. "Price Stability in Open Economies", Review of Economic Studies 70 (4), 743-765.

[4] Benigno, G. and P. Benigno, 2006. "Designing Targeting Rules for International Monetary Policy Cooperation", Journal of Monetary Economics 53, 473-506.

[5] Benigno, P. and M. Woodford, 2005. "Inflation Stabilization and Welfare: The Case of a Distorted Steady State", Journal of the European Economic Association No. 6 (3), 1185-1236.

[6] Benigno, P. 2002. "A Simple Approach to International Monetary Policy Coordination", Journal of International Economics 57, 177-196.

[7] Betts, C. and M. Devereux, 2000. "International Monetary Policy Coordination and Competitive Depreciation", Journal of Money, Credit and Banking 32, 4: 722-745.

[8] Calvo, Guillermo, 1983. "Staggered Prices in a Utility-Maximizing Framework", Journal of Monetary Economics, 12(3): 383-98.

[9] Christiano, L., 1998. "Solving Dynamic Equilibrium Models by a Method of Undetermined Coefficients", NBER working paper No. 225.

[10] Cole, H. and M. Obstfeld, 1991. "Commodity trade and international risk sharing: How much financial markets matter?", Journal of Monetary Economics 28: 3-24.

[11] Corsetti, G. and P. Pesenti, 2001. "Welfare and Macroeconomic Interdependence", Quarterly Journal of Economics 116, 2: 421-445.

[12] Corsetti, G. and P. Pesenti, 2004. "International Dimensions of Optimal Monetary Policy", NBER working paper number 8230.

[13] Ferrero, A., 2007. "Fiscal and Monetary Rules for a Currency Union", FED of New York working paper.

[14] Gali, J. and T. Monacelli, 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", Review of Economic Studies 72: 707-734.

[15] Gali, J. and T. Monacelli, 2006. "Optimal Monetary and Fiscal Policy in a Currency Union", mimeo.

[16] Jensen, H., 1996. "The Advantage of International Fiscal Cooperation under Alternative Monetary Regimes", European Journal of Political Economy 12: 485-504.

[17] Kollmann, R., 2004. "Welfare Effects of a Monetary Union: The Role of Trade Openness", Journal of the European Economic Association 2: 289-301.

[18] Lambertini, L., 2004. "Fiscal Cooperation in a Monetary Union", Boston College, mimeo.

[19] Lambertini, L., 2006. "Fiscal Rules in a Monetary Union", Boston College, mimeo.

[20] Liu, Z. and E. Pappa, 2005. "Gains from International Monetary Policy Coordination: Does it pay to be different", ECB working paper 514.

[21] Lombardo, G. and A. Sutherland, 2004. "Monetary and Fiscal Policy Interactions in Open Economies", Journal of Macroeconomics 26: 319-347.

[22] Lucas, Robert E., Jr., 1982. "Interest Rates and the Currency Prices in a Two-Country World", Journal of Monetary Economics, 10: 335-359.

[23] Obstfeld, M. and K. Rogoff, 2002. "Global Implications of Self-Oriented National Monetary Rules", Quarterly Journal of Economics 117: 503-535.

[24] Salvado, S., 2009. "Fiscal Policy in a Monetary Union: Gains from Changing Institutions".

[25] Schmitt-Grohé, S. and M. Uribe, 2005. "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model", NBER working paper 11417.

[26] Sutherland, A., 2002. "International Monetary Policy Coordination and Financial Market Integration", Federal Reserve Bank discussion paper 751.

[27] Tchakarov, I., 2004. "The Gains from International Monetary Cooperation Revisited", International Monetary Fund, working paper 04/01.

[28] Woodford, M., 2003. "Interest and Prices: Foundations of a Theory of Monetary Policy", Princeton University Press.

A   Appendix

A.1   The Nash equilibrium

A.1.1   Selection of the subset of restrictions

First let us consider for simplicity that we can describe the indirect utility function of a country as depending only of a variable that *per se* depends on both countries policy instruments.

The problem of country H can be reduced to:

$$\max_{\tau} \quad U\left(X\left(\tau, \tau^*\right)\right)$$

The first order condition of this problem is:

$$\frac{\partial U}{\partial X}\frac{\partial X}{\partial \tau} = 0 \tag{55}$$

The problem of country F can be reduced to:

$$\max_{\tau^*} \quad U^*\left(X^*\left(\tau, \tau^*\right)\right)$$

The first order condition of this problem is:

$$\frac{\partial U^*}{\partial X^*}\frac{\partial X^*}{\partial \tau^*} = 0 \tag{56}$$

At the same time, we can describe the Nash problem of the country H as maximizing the utility of country H subject to all the equilibrium conditions that define the competitive equilibrium of the model considered. For simplicity we reduced it to three equations. Hence, we can write it as:

$$\max_{\tau, X, X^*, P} \quad U(X)$$

s.t.

$$F_1(X, P, \tau) = 0$$
$$F_2(X^*, P, \tau^*) = 0$$
$$F_3(P, \tau, \tau^*) = 0$$

That is:

$$\mathcal{L} = U(X) + \lambda_1 F_1(X, P, \tau) + \lambda_2 F_2(X^*, P, \tau^*) + \lambda_3 F_3(P, \tau, \tau^*)$$

The optimum of this problem can be defined as:

$$\frac{\partial \mathcal{L}}{\partial \tau} = 0 \Leftrightarrow \lambda_1 \frac{\partial F_1}{\partial \tau} + \lambda_3 \frac{\partial F_3}{\partial \tau} = 0 \tag{57}$$

$$\frac{\partial \mathcal{L}}{\partial N} = 0 \Leftrightarrow \frac{\partial U}{\partial X} + \lambda_1 \frac{\partial F_1}{\partial X} = 0 \tag{58}$$

$$\frac{\partial \mathcal{L}}{\partial N^*} = 0 \Leftrightarrow \lambda_2 \frac{\partial F_2}{\partial X^*} = 0 \tag{59}$$

$$\frac{\partial \mathcal{L}}{\partial P} = 0 \Leftrightarrow \lambda_1 \frac{\partial F_1}{\partial P} + \lambda_2 \frac{\partial F_2}{\partial P} + \lambda_3 \frac{\partial F_3}{\partial P} = 0 \tag{60}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = 0 \Leftrightarrow F_1(X, P, \tau) = 0 \tag{61}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = 0 \Leftrightarrow F_2(X^*, P, \tau^*) = 0 \tag{62}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = 0 \Leftrightarrow F_3(P, \tau, \tau^*) = 0 \tag{63}$$

From equation 59 we obtain that $\lambda_2 = 0$ because $\frac{\partial F^*}{\partial X^*} \neq 0$. From equation 57 we obtain that $\lambda_1 = -\frac{\frac{\partial U}{\partial X}}{\frac{\partial F_1}{\partial X}} = -\frac{\partial U}{\partial X}\frac{\partial X}{\partial F_1}$, then from 58

$$\lambda_1 \frac{\partial F_1}{\partial \tau} + \lambda_3 \frac{\partial F_3}{\partial \tau} = 0 \Leftrightarrow$$

$$-\frac{\partial U}{\partial X}\frac{\partial X}{\partial F_1}\frac{\partial F_1}{\partial \tau} + \lambda_3 \frac{\partial F_3}{\partial \tau} = 0 \Leftrightarrow$$

$$\lambda_3 = \frac{\partial U}{\partial X}\frac{\partial X}{\partial F_1}\frac{\partial F_1}{\partial \tau}\frac{\partial \tau}{\partial F_3}$$

Finally, from 60 we obtain that,

$$\lambda_1 \frac{\partial F_1}{\partial P} + \lambda_2 \frac{\partial F_2}{\partial P} + \lambda_3 \frac{\partial F_3}{\partial P} = 0 \Leftrightarrow$$

$$-\frac{\partial U}{\partial X}\frac{\partial X}{\partial F_1}\frac{\partial F_1}{\partial P} + \frac{\partial U}{\partial X}\frac{\partial X}{\partial F_1}\frac{\partial F_1}{\partial \tau}\frac{\partial \tau}{\partial F_3}\frac{\partial F_3}{\partial P} = 0 \Leftrightarrow$$

$$-\frac{\partial U}{\partial X}\frac{\partial X}{\partial F_1}\frac{\partial F_1}{\partial P} + \frac{\partial U}{\partial X}\frac{\partial X}{\partial F_1}\frac{\partial F_1}{\partial P}\frac{\partial \tau}{\partial F_3}\frac{\partial F_3}{\partial \tau} = 0 \Leftrightarrow$$

$$0 = 0$$

Therefore we can conclude that the restriction $F_2\left(X^*, P, \tau^*\right) = 0$ is not active in this problem and hence we do not need to include it to obtain the same result.