The Value of Interest Rate Smoothing: How the Private Sector Helps the Federal Reserve

By Jeffery D. Amato and Thomas Laubach

Most central banks conduct monetary policy by setting targets for overnight interest rates. During the 1990s, central banks have tended to move these interest rates in small steps without reversing direction quickly, a practice called interest rate smoothing. For example, the majority of Federal Reserve policy moves in the last decade and a half have come in a sequence of 25-basis-point moves, in striking contrast to the early 1980s, when short-term interest rates fluctuated widely. In light of this historical contrast, it is natural to ask whether interest rate smoothing is a beneficial way to conduct monetary policy.

This article argues that interest rate smoothing is beneficial because the private sector is forward-looking. The private sector bases its decisions on expectations of the future. Thus, a monetary policy move today will be more effective if it is expected to persist over time. By smoothing interest rates, the size of changes in interest rates required to reduce fluctuations in the economy can be smaller than would otherwise be necessary.

The first section of this article describes interest rate smoothing. The second section presents evidence that the Federal Reserve has smoothed interest rates in the past and reviews a traditional argument that may explain this apparent behavior. The third section offers an alternative explanation for interest rate smoothing—based on the forward-looking behavior of the private sector—and provides evidence on the benefits of smoothing.

I. WHAT IS INTEREST RATE SMOOTHING?

Central banks can smooth interest rates at various frequencies. For example, three frequencies at which the Federal Reserve arguably has smoothed interest rates are seasonal, event, and day to day. Seasonal smoothing means that the central bank eliminates all calendar patterns in interest rates. Event smoothing means that, when a crisis occurs that puts sudden upward pressure on interest rates, the central bank provides liquidity to the market to avoid large interest rate

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changes. Day-to-day smoothing means that the average level of the interest rate over the span of a few days is close to the target level desired by the central bank.[1] Economists have provided evidence that the Federal Reserve has engaged in each of these three types of smoothing.[2]

The focus of this article is a fourth type of smoothing – the smoothing of changes in the central bank's *target* for the short-term interest rate. Smoothing of this kind means that decisions about the target explicitly depend on recent past decisions about the target; that is, target changes are purposely damped. For example, in recent years, the Federal Reserve has typically considered changes in its target for the federal funds rate at regular meetings of the Federal Open Market Committee (FOMC), which occur roughly every six weeks. But it actually changes the target relatively infrequently. From 1994 to 1998, for example, the FOMC changed the target at 12 of 40 meetings. In addition, though, the FOMC has occasionally changed its target for the federal funds rate *between* regular meetings. Whether target changes occur at or between regular meetings of the FOMC, the changes tend to be damped. The FOMC's intentional smoothing of its federal funds rate *target* over a sequence of target changes, as opposed to damping changes in the federal funds rate *between* target changes, is the focus of this article.

There is evidence that the Federal Reserve has engaged in this type of smoothing. To approximate the interval at which the Federal Reserve has made target decisions, this article presents empirical evidence based on U. S. data at monthly and quarterly frequencies. For example, Chart 1 plots monthly values of the federal funds rate from January 1965 to December 1997.[3] The chart indicates that some periods are characterized by a smooth federal funds rate path – for example, the period in the third panel (January 1987 to December 1997). On the other hand, the federal funds rate is less smooth from October 1979 to October 1982.

II. INTEREST RATE SMOOTHING: EVIDENCE AND A TRADITIONAL EXPLANATION

This section provides evidence that the Federal Reserve has pursued a policy of smoothing and offers a traditional explanation for this behavior. Evidence on smoothing takes the form of a federal funds rate that changes slowly over time due largely to Federal Reserve policy. Traditionally, economists have explained this inertia in the federal funds rate using theories that do not rely on forward-looking behavior of private agents. One of these explanations is that central banks respond cautiously to an uncertain policy environment.

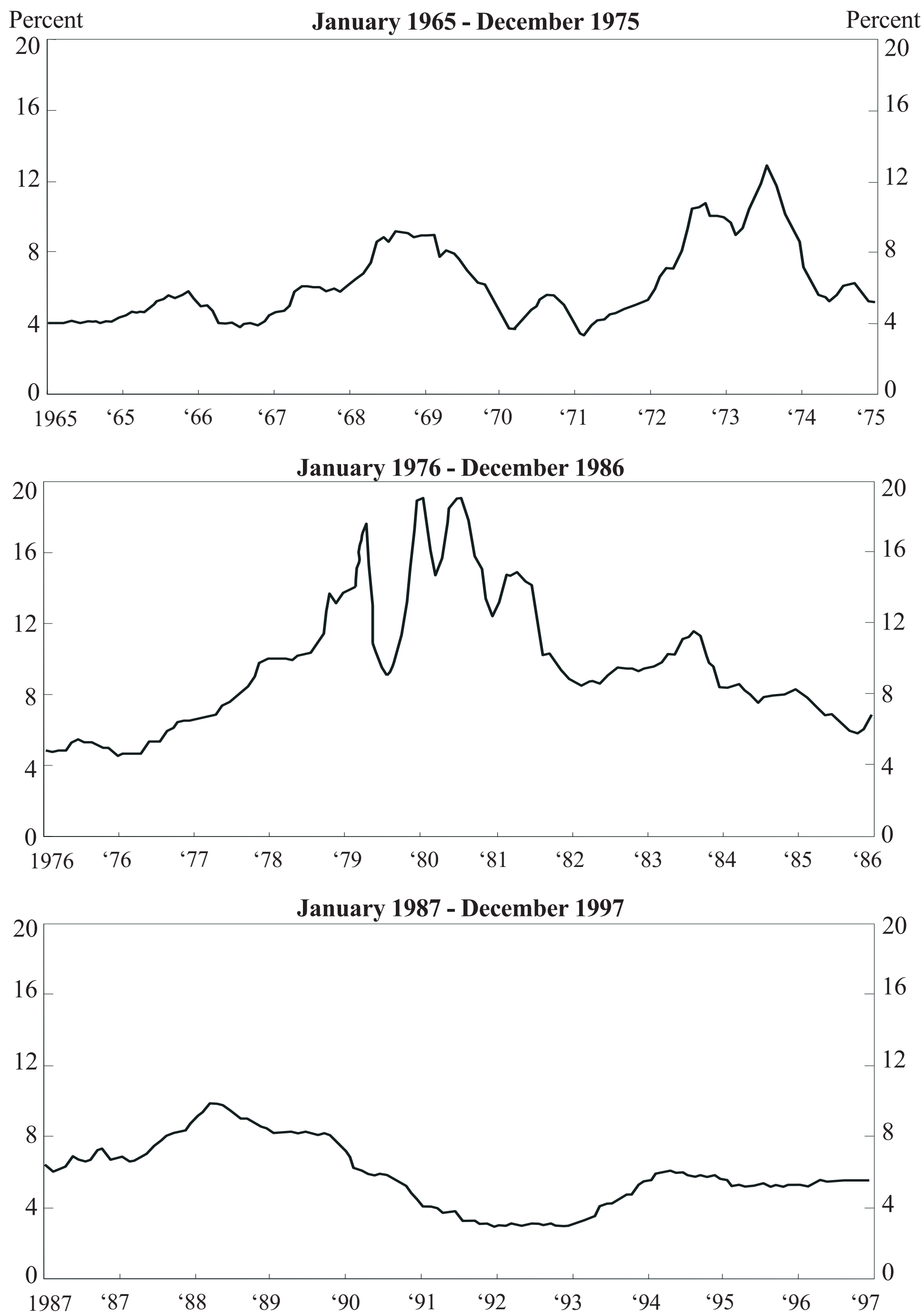
Evidence of interest rate smoothing

Evidence of smoothing is based on two observations: First, the federal funds rate is highly correlated over time. Second, an empirical characterization of recent Federal Reserve behavior suggests much of the observed smoothness in the funds rate is due to the Federal Reserve deliberately damping fluctuations in the federal funds rate target.

Correlations of the federal funds rate. As suggested in Chart 1, the federal funds rate appears to have moved smoothly over most of the last three decades, especially since the mid-1980s. Several measures show how smoothly these movements have been over this period. One measure is the simple correlation between federal funds rates observed at different points in time. For instance, if values of the federal funds rate one quarter apart tend to be similar, then the estimated correlation of federal funds rates one quarter apart should be close to 1. A correlation close to 1 would be consistent with interest rate smoothing.

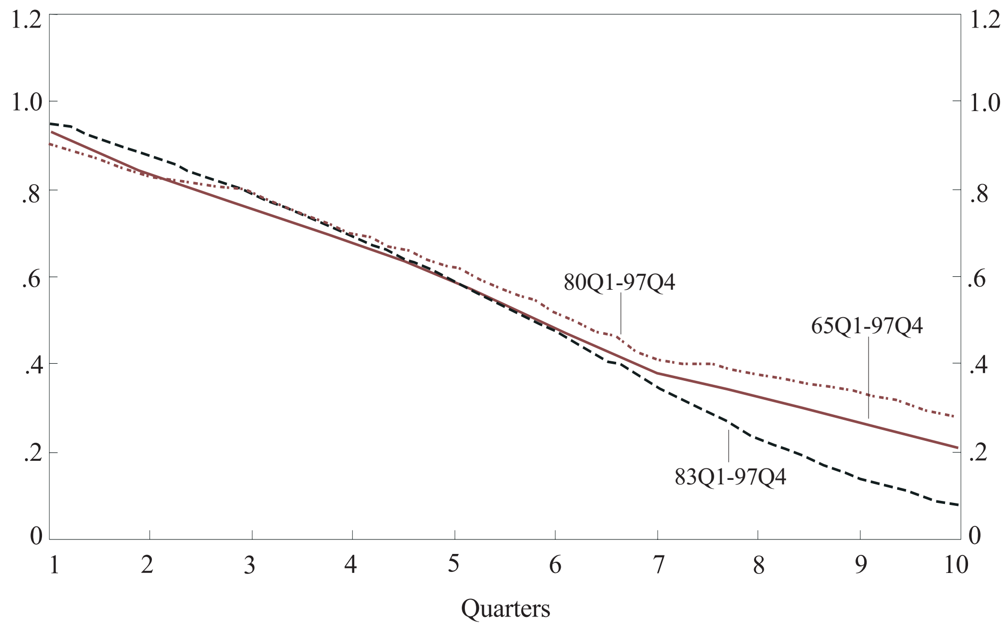
Chart 2 shows estimates of the correlations between federal funds rates at various quarters apart. The lines in the chart reflect estimates cal-

Chart 1
THE FEDERAL FUNDS RATE



Source: Board of Governors of the Federal Reserve System.

Chart 2
CORRELATIONS OF FEDERAL FUNDS RATES



Note: Data are correlations in the federal funds rate 1 to 10 quarters apart.

culated using data over different sample periods. The shortest sample (1983:Q1 to 1997:Q4) corresponds to the period immediately after the Federal Reserve's switch from targeting nonborrowed reserves to targeting the federal funds rate. The second sample (1980:Q1 to 1997:Q4) covers most of Paul Volcker's tenure as Chairman of the Federal Reserve Board and the first ten years of Chairman Greenspan's tenure, a period over which the Federal Reserve pursued a disinflationary monetary policy. The largest sample (1965:Q1 to 1997:Q4) spans the period in which the federal funds market has been fully operational and liquid.

Regardless of the sample period used, values of the federal funds rate as far as six quarters apart have a high positive correlation (that is, above 0.5). In addition, the gradual decline of the correlations toward 0 implies that the path of the interest rate exhibits substantial persistence.[4]

Estimates of high positive correlations alone, however, do not prove that the Federal Reserve has intrinsically smoothed interest rates. Central bank policy can move slowly, or *inertially*, without any explicit desire to smooth interest rates. Many factors influence how the central bank's target is determined. If any of these factors changes gradually over time, then that persistence may also cause interest rates to change gradually over time. For example, if the Federal Reserve reacts vigorously to inflationary impulses, but these impulses tend to be persistent, the federal funds rate would exhibit substantial persistence. It is therefore necessary to determine whether the smooth path of interest rates is due to explicit smoothing of policy or to slowly moving factors that influence policy. To separate an explicit smoothing motive from other factors that may have determined the federal funds rate, it is necessary to estimate the effect of past interest rates on current interest rates after accounting for other factors that influence current interest rates.

Estimates of smoothing in a reaction function. If a central bank acts systematically, it may be possible to capture central bank behavior in an equation that relates the variable that the central bank controls (such as the federal funds rate) to the goal variables about which central banks ultimately care (inflation and output). Economists call this equation a reaction function.[5] Estimates of such an equation are presented below.

Many factors influence monetary policy. Among the most important are inflation and output. Accordingly, the estimated reaction functions determine settings for the current federal funds rate (r_t) as a function of current inflation (π_t), output minus its long-run trend, or *detrended output* (z_t), and the past value of the federal funds rate (r_{t-1}):

$$r_t = c + a\pi_t + b z_t + d r_{t-1} + e_t, \qquad (1)$$

where c is a constant and e_t is a residual term.

The systematic component of policy is captured by the inflation, detrended output, and lagged interest rate terms on the right-hand side of the equation. The coefficients a and b measure the Federal Reserve's response to an increase in inflation and detrended output, respectively. The estimated values of a and b are expected to be positive. For example, an increase in inflation should lead the Federal Reserve to increase its federal funds rate target. Smoothing is represented by the lagged interest rate term alone. High values of the coefficient d imply a high degree of smoothing; a value of d equal to 0 means that policy does not involve smoothing at all.[6]

The residual accounts for variables not explicitly included in the reaction function that may occasionally influence Federal Reserve decisions. The residual itself, however, is not expected to exhibit a systematic pattern.

Table 1 presents estimates of d for alternative measures of inflation and detrended output, and for four sample periods. (More detailed estimation results, with discussion, are in Appendix A.) As mentioned above, the degree of persistence in inflation and detrended output is potentially important in explaining the persistence in the federal funds rate. Two measures of each of these variables can help assess the robustness of estimates of equation (1). The two inflation series are the percentage change in the consumer price index less its food and energy components (commonly referred to as core CPI) and the percentage change in the implicit price deflator for nonfarm business output. The two output series are deviations from trend in real gross domestic product (GDP) and nonfarm business output. A linear deterministic trend is used as a proxy for the long-run trend for each measure of output.[7]

The table presents estimates of the smoothing coefficient for the three sample periods described earlier in the discussion of Chart 1. It is also helpful to consider a fourth, shorter sample (1988:Q1 to 1997:Q4), which covers the first ten years of Chairman Greenspan's tenure. All of the entries in Table 1 present a clear and uniform message: the coefficient on the lagged federal funds rate is large (close to 1) and significantly different than 0. This result holds, regardless of how inflation and output are measured or which sample period is used.[8] These estimates suggest that the Federal Reserve purposely smoothed interest rates in the past.[9]

A traditional explanation of smoothing behavior

The high correlations of the federal funds rate over time provide convincing evidence that the

Table 1

ESTIMATES OF THE SMOOTHING COEFFICIENT IN THE REACTION FUNCTION

$$r_t = c + a\pi_t + b\,z_t + d\,r_{t-1} + e_t,$$

Specification			
Inflation measure	Output measure	Sample	d
Core CPI	GDP	65Q1-97Q4	.838
		80Q1-97Q4	.778
		83Q1-97Q4	.916
		88Q1-97Q4	.812
Nonfarm business output deflator	Nonfarm business output	65Q1-97Q4	.871
		80Q1-97Q4	.803
		83Q1-97Q4	.911
		88Q1-97Q4	.807

federal funds rate evolved smoothly over most of the past three decades. The large coefficients on the lagged interest rate term in the estimated reaction functions suggest that the Federal Reserve has intentionally followed a policy of interest rate smoothing. Still, these estimates do not shed light on *why* the Federal Reserve has smoothed interest rates.

Economists have provided various explanations why central banks smooth interest rates. One explanation is that central banks are simply being cautious because they have limited knowledge about the economy. Brainard first argued that, in the face of uncertainty about how the economy works, it may be best for policymakers to make a more muted response to new observations of data than if they knew the economy's true structure.[10] By moving cautiously, policymakers can avoid generating larger fluctuations in economic outcomes.

Building on Brainard's earlier contribution, Sack argued that, if a central bank believes the structure of the economy is constantly changing, then the best policy strategy is to smooth interest rate changes. Policymakers can observe (albeit, imperfectly) the effect of recent policy actions on the economy. They have more information about the effect of recent levels of the short-term interest rate than about levels that are much different. The most assured response is to minimize changes – that is, to smooth interest rates. This argument implies that the coefficient on the lagged federal funds rate in equation (1) should be large and positive, similar to the estimates in Table 1.

II. SMOOTHING UNDER FORWARD-LOOKING BEHAVIOR

As discussed in the previous section, a traditional explanation of inertia in short-term interest rates is based on uncertainty about the economy. More recently, economists have offered an alternative explanation based on the interaction of a systematic monetary policy with a forward-looking private sector. This section shows how interest rate smoothing increases the potency of monetary policy and provides evidence on how smoothing reduces macroeconomic volatility.

Why smoothing increases policy potency

Under interest rate smoothing, small interest rate changes have a relatively big effect on economic activity. This is true because a given interest rate change has stronger economic effects the longer it is expected to persist. Furthermore, systematic monetary policy that involves interest rate smoothing leads the public to expect that a given change in short-term interest rates will be long lasting.[11]

The role of forward-looking behavior. Interest rate smoothing is attractive to policymakers because the private sector makes decisions based in part on its outlook for the future. Forecasts of the future economy and, in particular, of future monetary policy, affect the decisions of consumers, investors, workers, and firms.

Consider, for example, a firm that is planning an investment project. The firm's reaction to an increase in short-term interest rates will depend on how long it expects the interest rate increase to persist. If it believes the interest rate increase will be short-lived, the firm's response will be muted, whether the investment project is financed by borrowing short term and rolling over the debt periodically or by borrowing long term. If it borrows short term, the firm would expect to be able to roll over the debt at a lower rate in the near future. If it borrows long term, it would likely face largely unchanged long-term interest rates because, with efficient markets, a temporary increase in short-term rates would have little effect on long-term rates. Either way, the firm's financing costs would largely be unaffected by the increase in short-term rates, as would its decision to undertake the investment project.

In contrast, if the firm believes the increase in short-term interest rates will persist over time, it might scale back or cancel the investment project. Again, this response could occur whether the firm was planning to finance the investment project with short-term or long-term debt. With short-term financing, the firm would expect to continue paying, for some time into the future, the higher interest rate as it rolled over its debt. With long-term financing, the firm would likely have to pay a higher long-term interest rate. According to the expectations theory of the yield curve, an increase in short-term rates that is expected to persist will have a bigger impact on long-term interest rates than an increase that is not expected to persist. Thus, under either short-term or long-term financing, the firm will likely respond more aggressively to a increase in short-term rates that is likely to persist.

The need for systematic monetary policy. For a change in short-term interest rates to have relatively large effects, economic agents must expect it to persist. A monetary policy involving interest rate smoothing will generate expectations of persistence in rates only if the smoothing is predictable. A systematic monetary policy helps ensure this predictability.

The economic benefits of smoothing are most apparent when the central bank acts systematically by committing itself to a *rule*. A rule describes how a central bank determines a short-term interest rate as a function of goal variables, such as current and past values of inflation and output, as well as past values of the short-term interest rate itself. Thus, a rule is a transparent way of conducting systematic monetary policy. Apart from helping agents form better forecasts of future policy, rules help ensure that central banks remain committed to achieving their long-run goals.[12]

This article assumes that central banks are able to commit themselves to following a rule when setting interest rates. Assuming that central banks can commit themselves to rules makes it easier to distinguish the benefits of smoothing from a policy that does not involve smoothing. With a well-established rule, the private sector will know whether or not the central bank is smoothing interest rates.

The benefits of smoothing

With a systematic monetary policy, interest rate smoothing reduces macroeconomic volatility with only small interest rate changes because even small interest rate changes can have relatively large effects on inflation and output. This result can be seen with the aid of a small and highly stylized economic model.[13]

The goals of policy. Until recently, much of the analysis of monetary policy focused on how the central bank can achieve a low average level of inflation in the long run. The argument in this article, as put forth strongly by Woodford (1999), is that *stabilization* gains can be made by conducting monetary policy in a systematic way that involves interest rate smoothing. Therefore, this article assumes that the central bank, in addition to pursuing a goal of a low long-run average of inflation, aims at minimizing fluctuations in inflation around this average. Furthermore, it is assumed that monetary policymakers aim at minimizing fluctuations in output around a maximum sustainable, or efficient, level. The difference between this efficient level of output and the actual level is called the output gap.

Because nominal interest rates cannot fall below zero, a policy aimed at a low long-run average of inflation cannot involve a highly variable interest rate. In the analysis that follows, therefore, the goal of low interest rate variability is added to the goals of stabilizing inflation and output. It is important to distinguish between a policy of low interest rate variability and one involving interest rate smoothing. A policy may involve very small, but short-lived, interest rate changes—that is, interest rate variability is low, but interest rates are not smoothed. Alternatively, a policy may involve large and long-lived interest rate fluctuations—that is it involves smoothing, but the resulting interest rate is very variable. Adding the goal of low interest rate variability rules out policies involving large interest rate fluctuations, because such policies would require a high average inflation rate, which is undesirable, or they would require the nominal interest rate to fall below zero, which is impossible.

In practice, these assumed goals of monetary policy are consistent with the objectives of most central banks. For example, the Federal Reserve Act states that the Federal Reserve shall "promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates." The latter two objectives are closely related. Nominal interest rates are equal to the sum of real interest rates and inflation premia, so that low average inflation rates will lead to moderate nominal interest rates.

Evidence from simulations. The effects of smoothing can be seen using an economic model, which is explained in detail in Appendix B. The model embodies three relationships that explain the behavior of inflation, output, and short-term interest rates over time. The first relationship, based on firms' pricing decisions, states that when high demand pushes output above its efficient level, or when expectations of future inflation rise, inflation today increases. The second relationship states that firms' and households' demand for goods today depends negatively on their expectations of the long-term real interest rate (which is the average of expected future short-term rates less expected inflation). The third relationship is an interest rate rule of the kind discussed earlier, explaining how the central bank adjusts the short-term interest rate in response to inflation and output.

Chart 3 shows how smoothing influences the behavior over time of inflation, the output gap, and the short-term interest rate in response to a positive demand shock that gradually dies out over several quarters.[14] The solid lines in the chart show how the economy responds under interest rate smoothing. The top panel shows that the interest rate response is initially small (relative to the size of the demand shock), but drawn out. The interest rate slowly returns to its

Chart 3
RESPONSES OF THE ECONOMY TO A DEMAND SHOCK

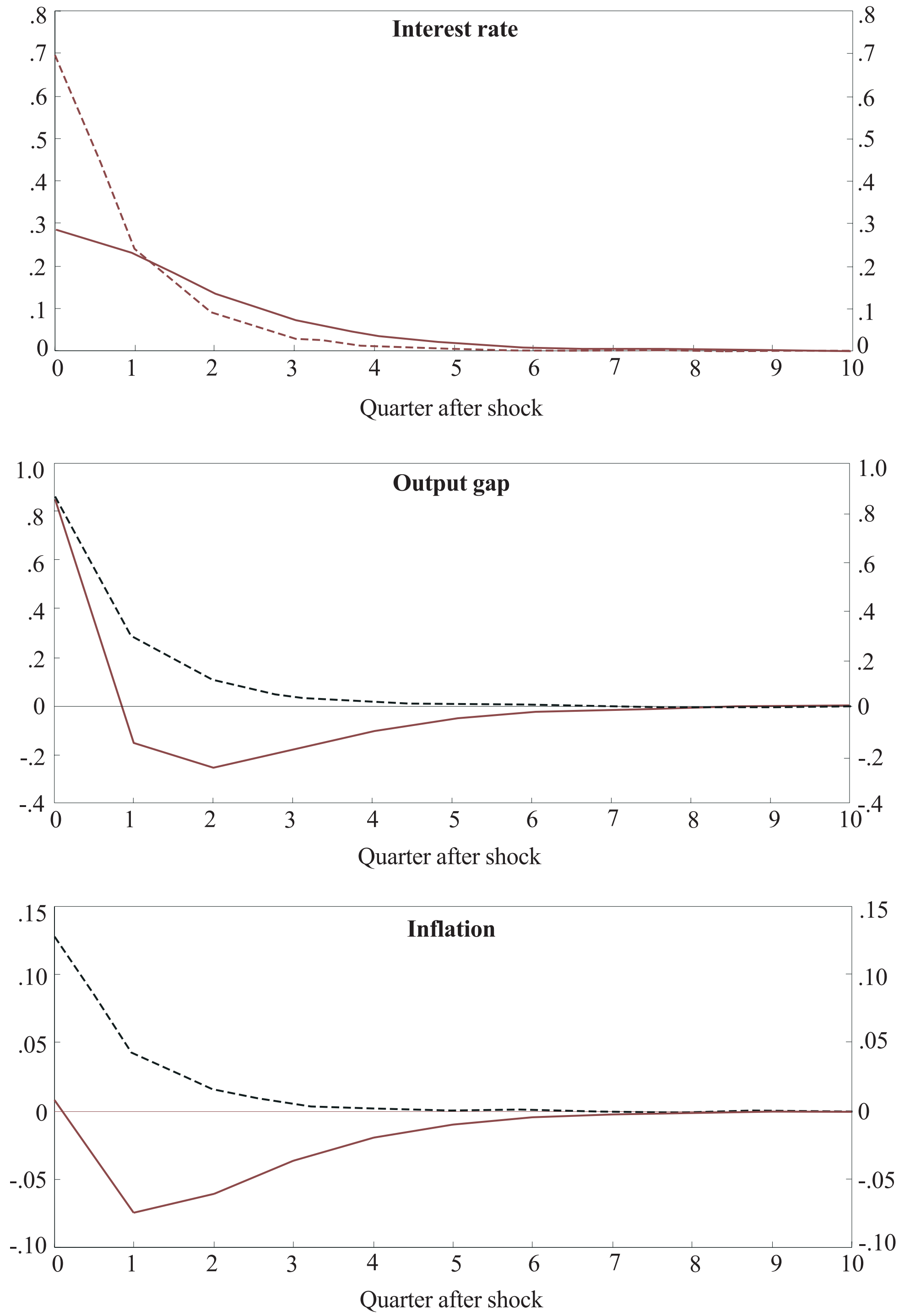
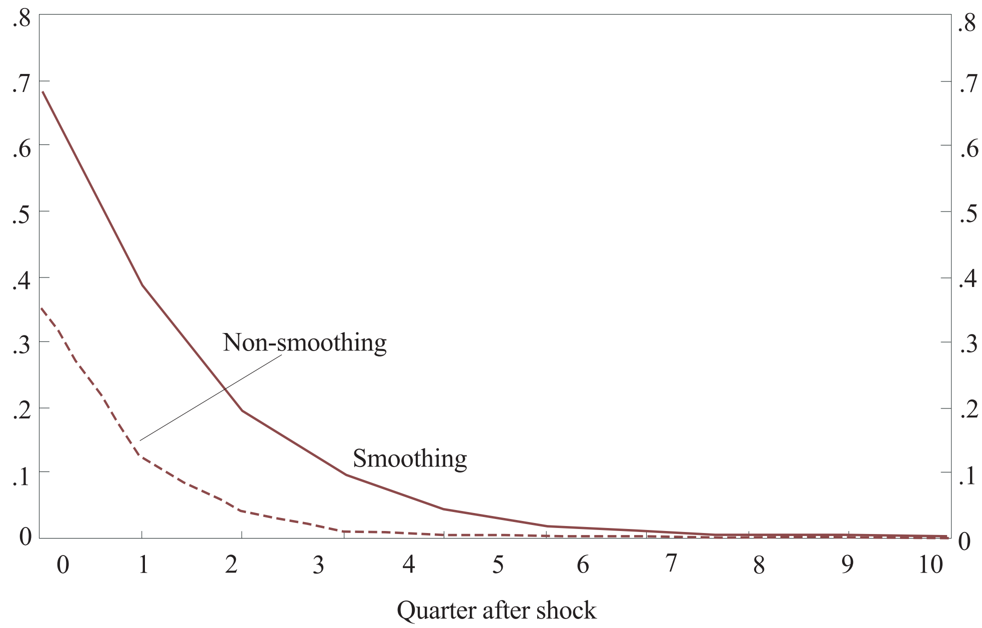


Chart 4
IMPLIED CORRELATIONS OF FEDERAL FUNDS RATES



Note: Data are correlations in the federal funds rate 1 to 10 quarters apart.

long-run level after about seven quarters. As shown in the second panel, the smaller response of policy allows the output gap to increase in the quarter of the shock. In subsequent quarters, a persistently tight policy – through both contemporaneous and expected effects – forces the output gap to turn negative and adjust slowly back to its long-run level.

The bottom panel shows that, in the quarter of the shock, the inflationary impact of a positive output gap is offset by private sector expectations of negative output gaps in the future. In subsequent quarters, inflation falls and then gradually adjusts back to its long-run level. Overall, the departures of each of the variables from their long-run levels are small, but persistent. The result is that business cycles and inflationary episodes are longer under a policy of smoothing, but more importantly, less severe.

In contrast, consider a policy that does not involve smoothing. Any inertia in short-term interest rates is entirely attributable to the persistence of shocks that hit the economy. The dashed lines in Chart 3 show the behavior of the economy under a policy without smoothing given the same demand shock as described above. With no smoothing at work, the interest rate follows the evolution of the shock, as do the output gap and inflation since there is no smoothing at work (Appendix B). Relative to economic performance under a policy of smoothing, shocks have a greater impact on inflation and a similar impact on the output gap.

Table 2

ECONOMIC PERFORMANCE WITH AND WITHOUT SMOOTHING

	Variances of:			Central bank objective function
	Inflation	Output gap	Interest rate	
Smoothing	.135	10.606	1.935	1.085
No smoothing	.214	9.911	6.745	2.251

The behavior of the interest rate in the two cases can be compared more directly. Chart 4 shows the correlation between the interest rate in one quarter with the interest rates in various other quarters. The chart shows that smoothing induces much more persistence in interest rates, as represented by uniformly higher correlations up to seven quarters apart.[15]

Under the assumption that the goals of monetary policy are to stabilize inflation around some low long-run average and output around its efficient level, the simulations illustrate the benefits of smoothing. In terms of Chart 3, policy's goal is to minimize the square of the distance of the paths of output and inflation from the zero line.[16] Under smoothing, each of the variables departs from its long-run value longer than without smoothing, but the overall magnitude of the squared discrepancies is smaller, implying a smaller variance. Thus, there is a small tradeoff between the persistence and severity of fluctuations in the economy depending on whether interest rates are smoothed. Nevertheless, interest rate smoothing will lead to lower overall volatility in the economy.

Building on this intuition, the benefits of smoothing are captured more precisely by two sets of measures of economic performance (Table 2). The first set consists of the variances of inflation, the output gap, and the interest rate implied by the model under monetary policies with and without smoothing.[17] These variances incorporate the effects over time of the demand shock shown in Chart 3 as well as the variance of the demand shock itself. The second set consists of the value of the central bank's objective function, again under policies with and without smoothing. Consistent with the goals of monetary policy discussed earlier, this objective function is a combination of the variances of the variables, so policies that produce smaller values of the function are preferable (Appendix B).

As can be seen in the table, inflation and the interest rate are much less volatile with smoothing than without it, while the output gap is only slightly more volatile. Under smoothing, inflation is 37 percent less volatile and the interest rate is 71 percent less volatile, while the output gap is only 7 percent more volatile.

A policy that involves smoothing is better than one without it because the reductions in the variances of inflation and the interest rate more than make up for the increase in the variance of the output gap. The value of the central bank objective function is 52 percent smaller under smoothing. All of these results are qualitatively similar to those from a slightly richer model that has stronger empirical support when applied to U.S. data (Rotemberg and Woodford). Thus, these results suggest that interest rate smoothing can be beneficial.

III. SUMMARY

This article presents evidence that Federal Reserve policy over the past three decades has involved systematic interest rate smoothing. A traditional explanation for this behavior is that the Federal Reserve reacts cautiously in the presence of uncertainty about the structure or the state of the economy.

The argument for interest rate smoothing presented in this article is instead based on the forward-looking nature of the private sector. With smoothing, a policy move today has a bigger impact because consumers and firms expect it will last well into the future, and these expectations affect their current behavior. Simulation results from a small, stylized model suggest that interest rate smoothing reduces macroeconomic fluctuations and is therefore beneficial.

APPENDIX A

ESTIMATION OF THE REACTION FUNCTION

This appendix provides more details on the estimation of the reaction function, equation (1) in the text. For convenience, this equation is reproduced here:

$$r_t = c + a\,\pi_t + b\,z_t + d\,r_{t-1} + e_t, \quad (1)$$

Table A1 presents estimation results in two panels. The panels are distinguished by the method used for estimating the long-run trend of output. In the top panel, the long-run trend is estimated as a deterministic linear trend; in the bottom panel, the long-run trend is estimated using the Hodrick-Prescott filter. As in the text, each panel presents results for two measures of inflation and output, and for four different sample periods.

Two sets of ordinary least squares estimates of equation (1) are presented. For each sample period, the first line shows estimates of *a*, *b*, and *d*; the second line shows estimates of *a* and *b* when *d* is restricted to be 0. The last two columns of the panels are summary statistics: Theil's adjusted coefficient of determination ($\bar{R}^2$) and the Box-Ljung Q-statistic ($Q(8)$), which tests for serial correlation in the residual, e_t.

The estimates of *d* in the top panel repeat the information in Table 1 in the text. As can be seen in the bottom panel, the estimates of *d* are insensitive to the method used for constructing the long-run trend of output. This result reinforces the conclusion that the Federal Reserve smoothed interest rates in the past.

There are three striking differences between the estimated equations when *d* is estimated or is set equal to 0. One difference is that the estimate of the coefficient on inflation, *a*, increases substantially—in magnitude and statistical significance—when *d* is restricted to be 0. The highly significant estimates of *d*, however, suggest there is not a multicollinearity problem. The second difference is the sizes of the $\bar{R}^2$ statistics. Including the lagged interest rate substantially improves the fit of the equation. The third difference is the sizes of the $Q(8)$ statistics. The large values of the statistics across the cases when *d* is set equal to 0 suggest that neither inflation nor detrended output individually contributes much to the persistence of the federal funds rate. The large discrepancies between the cases when *d* is estimated and restricted to be 0 point to the difficulty in identifying whether inertia in the federal funds rate is due to smoothing or serially correlated errors. However, even when the lagged interest rate is included, there still appears to be serial correlation in the error term.

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APPENDIX A - continued

Table A1

ESTIMATES OF THE REACTION FUNCTION

A. Linearly detrended output

Specification: Inflation measure	Specification: Output measure	Sample	a	b	d	$\bar{R}^2$	Q(8)
Core CPI	GDP	65Q1-97Q4	.120	.158**	.838**	.899	25.370**
			.788**	.101	---	.499	410.881**
		80Q1-97Q4	.289**	.073	.778**	.914	16.273*
			1.111**	-.088	---	.639	129.299**
		83Q1-97Q4	.086	.046	.916**	.935	22.442**
			1.273**	.155	---	.430	124.561**
		88Q1-97Q4	.132	.235**	.812**	.958	26.187**
			1.119**	.783**	---	.777	32.577**
Nonfarm business output deflator	Nonfarm business output	65Q1-97Q4	.036	.163**	.871**	.898	25.276**
			.644**	.209	---	.415	445.804**
		80Q1-97Q4	.255*	.104**	.803**	.907	17.853*
			1.258**	.012	---	.662	124.654**
		83Q1-97Q4	.092	.059	.911**	.938	20.346**
			.896**	.240	---	.328	140.622**
		88Q1-97Q4	.1579*	.176**	.807**	.964	24.173**
			.853**	.531**	---	.673	36.983**

B. H-P detrended output

Specification: Inflation measure	Specification: Output measure	Sample	a	b	d	$\bar{R}^2$	Q(8)
Core CPI	GDP	65Q1-97Q4	.142*	.290**	.850**	.909	25.400**
			.809**	.164	---	.501	419.631**
		80Q1-97Q4	.259*	.196*	.787**	.917	16.777*
			1.114**	-.095	---	.637	128.482**
		83Q1-97Q4	.112	.242**	.878**	.944	18.869**
			1.199**	.748*	---	.523	85.261**
		88Q1-97Q4	-.015	.274**	.913**	.952	28.315**
			1.035**	1.024**	---	.589	47.689**
Nonfarm business output deflator	Nonfarm business output	65Q1-97Q4	.075	.260**	.894**	.906	26.836**
			.721**	.202	---	.397	454.094**
		80Q1-97Q4	.208*	.226**	.821**	.912	19.500**
			1.258**	.006	---	.662	124.625**
		83Q1-97Q4	.083	.220**	.885**	.947	16.656*
			.838**	.670*	---	.396	121.313**
		88Q1-97Q4	.073	.263**	.873**	.958	28.997**
			.799**	.759**	---	.522	46.461**

** Significant at 1 percent level.
* Significant at 5 percent level.

APPENDIX B

DESCRIPTION OF THE MODEL USED IN THE SIMULATIONS

This appendix describes the model used to perform the simulations in the text. It first lays out formally the objectives the central bank seeks to achieve. It then provides the equations of a model that captures forward-looking behavior of the private sector. The appendix ends with a discussion of how optimal policy involves interest rate smoothing.

The central bank's objective function

As stated in the text, this article assumes that central banks seek to minimize the variance of inflation around a target of 0 and the variance of the output gap (x_t). These goals of monetary policy are captured by the following objective function:

$$\mathrm{var}(\pi_t) + \lambda_x \ \mathrm{var}(x_t) + \bar{\pi}^2, \quad \text{(B1)}$$

where $x_t \equiv y_t - y_t^n$, y_t is output, y_t^n is the efficient level of output, $\bar{\pi}$ is the long-run average of inflation, var(·) is the unconditional variance of a variable, and λ reflects the weight the central bank puts on stabilizing fluctuations in the output gap relative to stabilizing fluctuations in inflation. Inclusion of $\bar{\pi}^2$ in (B1) reflects the central bank's goal to keep inflation low over the long run in addition to minimizing fluctuations in inflation.

The policy strategy that minimizes the variances of inflation and the output gap may call for the central bank to induce sharp swings in the short-term interest rate. However, monetary policy is constrained by the fact that nominal interest rates have a lower bound of zero. One way to reduce the chance of hitting this bound is to generate a high average rate of inflation. Since the nominal interest rate is the sum of the real interest rate and inflation, high inflation rates on average will result in nominal interest rates that are typically far away from zero. Of course, this sort of policy runs counter to the objective of achieving a low rate of inflation in the long run.

An alternative way to express the objective function (B1) that also explicitly recognizes the zero lower bound for nominal interest rates is to replace $\bar{\pi}^2$ with the variance of the short-term interest rate:

$$\mathrm{var}(\pi_t) + \lambda_x \ \mathrm{var}(x_t) + \lambda_r \ \mathrm{var}(r_t), \quad \text{(B2)}$$

where λ is the weight the central bank puts on minimizing the variance of the short-term interest rate relative to the variances of inflation and the output gap. The parameter λ_r should be chosen sufficiently large to minimize the probability that the optimal policy calls for the interest rate to hit its zero lower bound.

It is important to note that adding the term $\lambda_r \ \mathrm{var}(r_t)$ to the objective function does not automatically imply that the optimal policy entails interest rate smoothing. This term means that extreme values of the interest rate are undesirable. However, it does not rule out a policy that completely disregards past values of the short-term interest rate.

The values of λ_x and λ_r used in the simulations in section three are shown in Table

continued . . .

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B1. These values are taken from Woodford (1999).[18]

The model of the economy

The type of economy that underlies the analysis in the text can be formalized as a dynamic stochastic general equilibrium model with monopolistic competition. The economy consists of consumers who have preferences over a continuum of goods and supply labor in a competitive factor market. The goods market is populated with a continuum of monopolistically competitive firms, and prices are sticky. The equations of the model can be derived as the result of optimizing behavior by the private sector (Woodford 1996).

The first equation is a "forward-looking" IS equation. Output depends on next period's expected output, the *ex ante* real interest rate, and a demand shock (g_t):[19]

$$y_t = E_t(y_{t+1}) - \sigma^{-1}[r_t - E_t(\pi_{t+1})] + g_t, \quad \text{(B3)}$$

where $E_t(\cdot)$ is the conditional expectation operator, based on information available at time *t*. The parameter σ^{-1} is the intertemporal elasticity of substitution.

The second equation is a "New Keynesian" Phillips curve (aggregate supply equation). Inflation in one period is determined by the amount of inflation expected next period and the output gap:

$$\pi_t = \beta \ E_t(\pi_{t+1}) + \kappa \ x_t, \quad \text{(B4)}$$

where the parameter β is the private sector's subjective discount factor and the parameter κ is a decreasing function of the average length of time that firms keep prices fixed and an increasing function of the sensitivity of marginal cost to changes in the output gap.

Equation (B3) can be rewritten in terms of the output gap:

$$x_t = E_t(x_{t+1}) - \sigma^{-1}[r_t - r_t^n - E_t(\pi_{t+1})], \quad \text{(B5)}$$

where $r_t^n \equiv \sigma \ \{g_t + [E_t(y^n{}_{t+1}) - y^n{}_t]\}$ is the natural rate of interest (Woodford 1999). Since the central bank seeks to stabilize fluctuations in x_t, r_t^n is the relevant combination of the exogenous shocks to which the central bank wishes to react. The natural rate of interest is modeled as an AR(1):

$$r_t^n = \rho \ r_{t-1}^n + \varepsilon_t, \quad \text{(B6)}$$

where $|\rho| < 1$ and $\varepsilon_t \sim iid(0, \text{var}(\varepsilon))$. For values of $\rho \neq 0$, the variables of the model exhibit persistence even in the absence of interest rate smoothing. The values of the parameters used in the simulations are shown in Table B1, which are also taken from Woodford (1999).

Solving for the optimal policy

The optimal path of interest rates can be derived as the solution to the social planner's problem. Minimizing a Lagrangian formed by the central bank's objective function and the model's equations produces a set of first-order conditions that characterize the optimal path of the model's three endogenous variables and the two Lagrange multipliers. These first order conditions involve both current and lagged Lagrange multipliers, and therefore imply that the optimal policy involves smoothing. For details on the

APPENDIX B - continued

Table B1

PARAMETER VALUES USED IN THE SIMULATIONS

β	.990
σ	.157
κ	.024
ρ	.350
var(ε)	12.145
λ_x	.047
λ_r	.233

solution to this problem, see Woodford 1999.

This article takes a slightly different approach in comparing optimal policies with and without smoothing. The approach here assumes that the central bank determines policy according to an interest rate rule. Rules with smoothing include a lagged interest rate term, whereas rules without smoothing do not. Attention is restricted to simple rules that include only current values of inflation and the output gap, and in the case of smoothing, the interest rate lagged one period. The optimal values for the parameters in the rules are obtained by minimizing the objective function (B2) subject to the model's equations (B4) - (B6) and the policy rule.

ENDNOTES

[1] Day to day smoothing is a natural consequence of conducting policy using interest rates; otherwise it would be meaningless to say a central bank sets a target for the interest rate.

[2] For example, economists have shown that one of the consequences of early Federal Reserve policy was the elimination of a seasonal cycle in market interest rates due to the harvest (Miron). Economists have also shown that, for over much of the last three decades, the Federal Reserve has pegged the federal funds rate for short intervals at a time (Bernanke and Mihov).

[3] The monthly data are averages of daily values of the effective federal funds rate, expressed as annualized percentages.

[4] An important limitation of this method to measure "smoothness" is suggested by the fact that, according to Chart 2, the correlation of federal funds rates more than four quarters apart are higher over the sample 1980 to 1997 than over the sample excluding the early 1980s. Visual inspection of Chart 1 suggests exactly the opposite. The measure of smoothing presented next avoids this problem.

[5] Reaction functions are closely related to policy rules. Rules are usually interpreted as normative prescriptions to the central bank, whereas the reaction functions considered here are understood as empirical descriptions of past monetary policy.

[6] As mentioned above, even when d is 0, the federal funds rate may be persistent if either inflation or the output gap evolves slowly over time. Inflation and the output gap are driven to some extent by shocks that hit the economy. If these shocks are persistent, then inflation and the output gap will show persistence, which in turn induces persistence in the federal funds rate. The next section presents a model in which shocks gradually die out. Inflation and output exhibit persistence that is directly attributable to the shocks, and therefore, by responding to these variables, so does policy.

[7] The inflation series are constructed as differences in log-levels, expressed as annualized percentages. The detrended output series are expressed as quarterly percentage deviations. The federal funds rate is the quarterly average of daily values, expressed as annualized percentages.

Measuring inflation using the overall CPI or the GDP deflator does not alter the results.

8 All of the estimates are statistically significantly greater than 0.5, at a five percent level of significance.

9 A different explanation of the evidence presented in Table 1 is that large and positive estimates can be misleading if equation (1) omits some important variables that influence policy. Central banks may place no weight on past interest rates when setting a target for the current interest rate, but the *estimate* of the coefficient on the lagged interest rate term in the reaction function may still be large if some omitted variables are serially correlated, that is, persistent. Interpreting the large estimates of d as evidence of smoothing is therefore based on the assumption that no persistent variables that affect Federal Reserve behavior systematically are omitted from equation (1).

10 Along similar lines, since the initial releases of new observations on many variables are often subjected to many revisions, it may be advisable to move cautiously until better quality data are available.

11 This basic idea of why smoothing is beneficial dates back to Goodfriend. It has been revived recently by Woodford (1999).

12 John Taylor of Stanford University offered an example of a rule that has received much attention of economists recently. In Taylor's rule, the federal funds rate is set equal to a weighted sum of four components: the long-run real interest rate, inflation over the past year, the deviation of inflation from a long-run target, and detrended output. As Taylor acknowledges, in reality rules are too restrictive to be the sole determinate of interest rates. Monetary policy cannot be put on autopilot because the world is a more complicated place than can be captured in a small model of the economy. But even if a central bank cannot strictly adopt a rule, a rule can serve as a benchmark to guide policymaking in the relevant direction.

13 See Appendix B for an example of a model in which it can be proved that the best policy involves smoothing.

14 The shock is normalized to have an initial impact equal to a 1-percent perturbation in the natural rate of interest, so that the initial size of the demand shock is σ^{-1} (Appendix B).

15 Comparison of Charts 2 and 4 might suggest that the federal funds rate has been much smoother than would be optimal according to the model. This comparison is misleading, because the historical shocks to which monetary policy responded may have been much more persistent than the shock series used in the model

16 Eventually, each variable moves towards its long-run value of 0. The inflation target is assumed to be 0 and both output and the efficient level of output revert to their common long-run trend (Appendix B). For convenience, the interest rate is plotted as a deviation from its long-run value. This long-run value is partly determined by the target around which the central bank minimizes interest rate fluctuations, that is r^*. (See equation B2 in Appendix B.)

17 Table 2 reports the unconditional variances of the variables implied by the model economy laid out in Appendix B combined with, respectively, policies with and without smoothing.

18 See Woodford (1999) for a discussion of these choices.

19 This description abstracts from government purchases and net exports. Changes in each of these variables are interpreted as demand shocks.

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