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# The Effect of Interest Rate Options Hedging on Term-Structure Dynamics

- Beginning in the 1990s, the short-term dynamics of the yield curve changed in ways that appear to be related to the growth of the interest rate options market.

- When interest rates change, options dealers buy or sell securities to adjust the hedging positions that they have taken to offset their options exposures. Since the early 1990s, these trades have been, in aggregate, large enough to affect market liquidity.

- The net result of this trading activity can be to push interest rates further in the direction they were moving. Such "feedback" effects can alter the shape of the yield curve, especially when changes in interest rates are large.

- For this reason, analysts should use caution in interpreting short-run movements in the yield curve as signals of future economic developments.

Research has shown that the yield curve is a reasonably good predictor of economic activity, in part because it seems to reflect expectations of future economic fundamentals such as growth and inflation.[1] Accordingly, movements in the curve in response to economic and financial shocks are typically watched closely by market participants and policymakers. However, several recent episodes of market illiquidity, most notably the crisis in the fall of 1998, have shown that disruptions to liquidity can affect the short-term dynamics of interest rates and the shape of the curve independently of fundamentals.

In this article, we study the influence of market liquidity and dynamic trading strategies on the short-run dynamics of the yield curve. Specifically, we focus on the recent behavior of intermediate-maturity interest rates for evidence of market liquidity effects arising from the hedging of interest rate options. We base our approach in large part on the hypothesis that the hedging transactions of interest rate options dealers generate systematic trading flows in the underlying fixed-income markets following a shock to interest rates.

In the interest rate options market, dealers are net writers of options, and they manage or hedge their options exposures by taking offsetting positions in fixed-income instruments such as U.S. Treasury securities and Eurodollar futures contracts. As interest rates change, the dealers must buy or sell fixed-income securities to adjust these hedge positions. Consequently, in the aggregate, these hedging transactions can potentially affect the market prices of the hedging instruments themselves, thus leading to further changes in interest rates. Although the size

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of these hedging transactions in Treasuries and Eurodollar futures is usually relatively small, the transactions' systematic relationship to changes in interest rates suggests that they may still produce small but observable feedback patterns in the short-run movements of interest rates.

Our study reveals that the short-run dynamics in the intermediate maturities of the yield curve changed around 1990, with the appearance of positive feedback in weekly interest rate changes. For example, we find that after 1990, if the yield curve "bows" up at the five-year maturity, five-year interest rates are likely to rise further in subsequent weeks. The observed positive feedback is consistent with the effects of options dealers' hedging activity, and, notably, it is found only in the 1990s, after the market for interest rate options grew to a significant size. We also provide evidence indicating that the speed at which feedback effects move through the yield curve has increased in recent years. Not surprisingly, the market liquidity/positive-feedback effects are concentrated in the weeks following the largest changes in interest rates, but they are virtually nonexistent during periods of small changes in interest rates.

Our results also suggest that very short-run movements in the yield curve should be interpreted with caution, because such movements may reflect liquidity effects as well as changes in economic fundamentals. Moreover, we find that liquidity effects are likely to be larger when interest rate changes are large and thus when policymakers and market participants are most interested in monitoring yield curve movements closely. Reassuringly, the liquidity effects uncovered in this article are not long-lasting, suggesting that interpretation of yield curve movements over longer periods of time should not be affected by our findings.

Our analysis begins with a discussion of the role of liquidity risk and positive feedback in the short-run behavior of asset prices. We then consider how the hedging of interest rate options could produce liquidity effects in the medium-term segment of the yield curve, where market survey data suggest that dynamic hedging of options could have the largest impact on transaction flows and thus on market liquidity. Next, we test for evidence of liquidity effects at a weekly frequency in both the Treasury and swap (Eurodollar) yield curves. We conclude by considering our study's implications for risk management and policy.

## Liquidity Risk and Positive Feedback

Market liquidity risk is the price risk associated with executing large transactions or executing transactions quickly. The risk is manifested in a sharp movement of prices against a trader when making a large purchase or sale of a security, or, in an extreme case, when a trader is unable to execute a large trade at a reasonable price.[2] Thus, market liquidity refers to the degree to which transaction flows affect asset prices in a market separately from any change in the economic fundamentals that determine asset values.

The potential for liquidity risk to affect asset prices in ways that are distinct from the role of fundamental economic and financial variables is receiving more attention from economists and policymakers. In a recent paper, Longstaff (2001) describes how liquidity-constrained traders will make investment decisions that lead to illiquidity discounts in asset prices. The increasing role of tradable securities in the intermediation of risk and the allocation of capital is also drawing more attention to the determinants and dynamics of market liquidity. Two recent Bank for International Settlements reports (1999a, b) address the importance of market liquidity in the conduct of monetary policy and highlight the role of market liquidity in the financial market disruptions in the fall of 1998.

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Related literature examines the potential for positive-feedback trading to lead to sharp changes in or overshooting of asset prices. Positive-feedback traders who buy when prices rise and sell when prices fall have the potential to drive prices further in the same direction as the initial shock. Such trades occur in the presence of stop-loss risk management strategies, in the hedging of options, and as part of momentum trading strategies. Papers by Grossman (1988), DeLong et al. (1990), and Gennotte and Leland (1990) after the stock market crash of 1987 describe how positive feedback in asset prices can emerge and be self-sustaining, despite the presence of rational traders who might otherwise link market prices to the fundamentals. Although these papers have demonstrated how positive feedback can occur, until now little systematic evidence has been found.[3]

The growth of the interest rate options market to significant size by the early 1990s provides us with a naturally occurring experiment to test for liquidity effects in the yield curve. The over-the-counter interest rate options market grew from \$10 billion of outstanding contracts in 1986 to \$561 billion in 1990 and to \$3,704 billion in 1995.[4] If the options market affects market liquidity in the underlying fixed-income markets, then we may be able to find differences in the behavior of interest rates before and after 1990.

As interest rate options became widely available, they allowed market participants who had passively borne interest rate volatility risk to trade and transfer this risk to someone else. In practice, the "someone else" has proved to be the trading units of large financial intermediaries that acquire exposure to interest rate volatility by selling interest rate options to their customers and are more likely to hedge and manage volatility risk than to bear it passively. The nonlinear nature of an options exposure requires that its hedge position be adjusted as interest rates change. Thus, options dealers are exposed to market liquidity risk when executing the trades required by the these hedge adjustments. Furthermore, because dealers are generally net writers of options, they will execute similar hedge-related trades when rates change. The systematic relationship of such trades to interest rate changes presents us with an opportunity to look for signs of their impact on market liquidity in the short-run behavior of interest rates. In our analysis, we look for changes in the dynamics of interest rates around 1990 that are consistent with the predicted market impact of the dealers' options hedging in the aggregate. (More detailed information on interest rate options and how they are hedged can be found in the box.)

## Interest Rate Options and Their Hedging

**Over-the-Counter Interest Rate Options**

Most over-the-counter interest rate options are caps and floors on the level of interest rates; the remainder are swaptions, which are options on swaps contracts. In International Swaps and Derivatives Association market surveys from the mid-1990s, caps and floors amounted to more than 80 percent of outstanding contracts, while swaptions accounted for the rest.

Caps and floors are options on future short-term interest rates, usually six-month Eurodollar rates. In an *interest rate cap (floor)* contract, the buyer receives the difference between the market interest rate and a strike rate specified in the options contract when the market rate is above (below) the strike rate, and nothing otherwise. Most cap and floor contracts are written for several years, and thus they can be thought of as a string of call (put) options on future values of Eurodollar rates over the contract period. A dealer's portfolio of caps (floors) can therefore be thought of as a book of call (put) options on all six-month *forward interest rates* along the entire yield curve (out to the maturity of the longest maturity contract, which can be as long as ten years).

A *forward interest rate* is the interest rate for a future time period as implied by the current shape of the yield curve. For example, the three-to-five-year forward rate is the two-year interest rate for the period between three to five years in the future. In particular, it is the rate agreed to today for a two-year loan commencing three years in the future. In the case of the forward Treasury rate, it can be calculated directly from the current three- and five-year Treasury rates.

In practice, dealers do not manage their options books by directly hedging every single six-month forward interest rate exposure along the entire yield curve out to ten years. For maturities beyond two or three years, they hedge longer sections of the yield curve in blocks broken at those points where the markets in the underlying securities are most liquid. For example, an options dealer might hedge all of the six-month forward rates of between three and five years in terms of a single exposure to the three-to-five-year forward interest rate. Similarly, all six-month forward rates of between five and ten years would be hedged in terms of a single exposure to the five-to-ten-year forward interest rate.

**Dynamic Hedging of Options**

Generally speaking, an option can be hedged by taking an offsetting position in the underlying asset, and the required size of this position varies with the price of the underlying asset. This variability of the hedge position results from the varying sensitivity of the option's value to the price of the underlying asset as its price changes. When the underlying asset price rises by a certain amount, a call option's value will increase by a smaller amount because of the possibility that the price of the underlying asset could still reverse direction by the time the contract matures, and even fall below the strike price, rendering the option worthless. As the underlying asset's price rises further, however, this prospect of a worthless outcome becomes less likely, and the option's value becomes more responsive to changes in the underlying asset's price.

This change in the price sensitivity of the option affects the size of the position in the underlying asset required to hedge the option. To compensate for the increase in the option's price sensitivity as the underlying asset price rises, the hedge position in the underlying asset must be made larger as well. Conversely, as the asset price falls, the hedge position should shrink. This adjustment of the hedge position's size as the underlying asset price changes is called *dynamic hedging*. Such adjustments involve buying the underlying security after its price has gone up and selling it after the price has fallen. This pattern of buying and selling introduces the potential for positive feedback in asset prices, as the transactions could introduce further upward (downward) pressure on prices after an initial upward (downward) shock to asset prices. See Hull (1993) for additional information on the pricing and hedging of interest rate options.

*Example of a Bond Market Hedge of an Interest Rate Cap*

The hedge of an interest rate cap involves a combination of long and short positions in fixed-income securities. A long position is a bond purchased with borrowed funds, while a short position is established by borrowing the bond and then selling it. The long position is closed out, or extinguished, by selling the bond and returning the borrowed funds, and the short position is closed out

## Implications of the Dynamic Hedging of Interest Rate Options

The 1995 central bank survey of the over-the-counter derivatives markets (Bank for International Settlements 1996)—the first detailed look at the structure of the markets—found that dealers had sold 50 percent more U.S. dollar interest rate options to customers than they had purchased. More recent data confirm that this asymmetry has persisted over time. This imbalance between end-user supply and demand is unique in the over-the-counter derivatives markets.[5] Generally speaking, options dealers do not leave themselves exposed to the interest rate risk in their net options positions; instead, they hedge this exposure by taking offsetting positions in other fixed-income securities. Indeed, the ability of dealers to trade in a broad range of fixed-income markets probably allows them to execute hedging transactions faster and at a lower cost than other market participants, making them more willing than others to absorb the market's net demand for interest rate options. Nevertheless, the dealers' need to adjust hedge positions as interest rates

### Interest Rate Options and Their Hedging (Continued)

by buying the bond and returning it to the bond's lender. The long position gains value when the bond price rises, while the short position gains value when the bond price falls because the bond can be repurchased at a price lower than its initial sale price.

An interest rate cap exposure to a forward interest rate can be hedged with a combination of bonds whose maturities straddle the maturity of the forward rate. The hedge consists of a long position in a bond whose maturity equals the beginning date of the forward rate and a short position in a bond whose maturity equals the ending date of the forward rate.

In the case of a Treasury market hedge of an interest rate cap on the three-to-five-year forward rate, the hedge consists of a long position in a three-year Treasury note and a short position in a five-year note. This position in the two notes is exposed only to forward rates of between three and five years because the long and short note positions extinguish exposures to interest rates of up to three years' maturity (see the table). In particular, the long position offsets the exposure of the short position to any interest rate that affects both notes—forward rates of three years' maturity or less. Meanwhile, the hedge position is exposed to longer maturity interest rates, because although the three-year note is not exposed to forward rates beyond three years, the five-year note does have exposure to longer term rates (up to its five-year maturity). Thus, the net hedge position is a short position in the five-year note with exposure only to forward rates in years four and five.

What happens, then, when interest rates change? An increase in interest rates causes the value of the caps to rise, increasing the dealer's exposure. (The dealer has sold caps.) At the same time, an increase in interest rates causes the prices of notes to fall. Thus, because the net hedge position is a short position in the five-year note (see the table), a fall in the value of the note due to an increase in rates will result in a gain in value of the short hedge position.[a] The overall effect is a rise in the value of the hedge position that offsets the increase in the dealer's interest rate cap exposure.

Following the increase in interest rates, dealers will readjust their hedge positions. As noted earlier, at the higher interest rate levels, the interest rate cap becomes more sensitive to further rate changes than the initial hedge position does. Thus, to maintain an appropriate hedge, dealers will increase the size of their positions in the three- and five-year notes. It is this dynamic hedging behavior that can potentially affect the prices of fixed-income securities.

#### Net Exposure of Hedge Position to Interest Rates
Change in Present Value Due to an Increase in Interest Rates

| Time (years) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Forward rates | $F_{0,1}$ | $F_{1,2}$ | $F_{2,3}$ | $F_{3,4}$ | $F_{4,5}$ |
| Impact of higher forward rates on short position in the five-year note | (+) | (+) | (+) | (+) | (+) |
| Impact of higher forward rates on long position in the three-year note | (-) | (-) | (-) | | |
| Net exposure to interest rates | | | | (+) | (+) |

Notes: A rise in interest rates will cause the present value of both the three- and five-year notes to fall. This fall in value, however, leads to a gain in the short position's value because the note can be repurchased and returned to the security lender at a price lower than its initial sale price. The gains and losses from exposure to forward rates of up to three years' maturity cancel each other, leaving only the short exposure in the five-year note to forward rates in years four and five.

[a]A short position is established by borrowing a security and selling it; the position is closed out by buying back the security at the prevailing market price and returning it to the lender. When the price of the security falls, the short position gains value because the bond can be purchased at a price lower than its original sale price.

change means that their exposure to interest rate volatility risk is converted to an exposure to market liquidity risk when executing the trades required by their hedge adjustments.

For U.S. dollar interest rate options, the most liquid instruments for hedging are Eurodollar futures contracts and U.S. Treasury securities and futures (see box). Previous estimates of the total volume of interest rate options dealers' hedging activity in the markets for these securities suggest that the hedging is likely to have the largest impact on market liquidity in the medium-term segment of the yield curve (Kambhu 1998). Although hedging activity is largest at the shortest maturities, the volume of hedging *relative to turnover volume* in Eurodollar futures is largest at the intermediate-maturity segment of the yield curve (Table 1). If dealers hedge with Eurodollar futures contracts, a 25-basis-point rise in forward rates all along the yield curve would lead to hedging transactions amounting to twice the average daily turnover volume of futures contracts at maturities of between three and five years. The same 25-basis-point rise in forward rates could generate hedging transactions of about five times the daily turnover volume at maturities of five years and beyond.[6]

The potentially large hedging impact at three-to-ten-year maturities in the Eurodollar futures market suggests that dealers will also be hedging in other liquid markets, most likely in the U.S. Treasury market. If options dealers were hedging in these markets, a 25-basis-point increase in forward rates would cause a hedge adjustment amounting to 7 percent of the daily turnover in the Treasury futures and cash markets of five- and ten-year-maturity securities (Kambhu 1998). A 75-basis-point increase in rates would cause hedge adjustments amounting to 21 percent of Treasury futures and cash turnover. Although these are not extraordinarily large shares of the Treasury market, they may be large enough to produce observable patterns in the behavior of the intermediate-maturity segment of the Treasury yield curve.[7]

Table 1

Estimated Volume of Hedging Activity Relative to Daily Turnover in the Eurodollar Futures Market

| | Volume as a Percentage of Turnover | |
|---|---|---|
| Maturity | For a 25-Basis-Point Rate Change | For a 75-Basis-Point Rate Change |
| Zero to one year | 5 | 24 |
| One to three years | 29 | 93 |
| Three to five years | 201 | 591 |
| Five to ten years | 513 | 1510 |

Source: Kambhu (1998).

At shorter maturities (less than three years), dealers' hedging of interest rate options is less likely to affect market liquidity. The shorter maturity fixed-income markets have substantially larger turnover volume and greater liquidity, and thus dealers' hedging activity can be easily accommodated. Therefore, we would not expect to find feedback effects at the short end of the yield curve.

## Implications for Intermediate-Maturity Interest Rates

Our review of hedging activity thus far suggests that we look for market liquidity effects in the three-to-ten-year segment of the term structure. In particular, we will look at how the five-year spot rate reacts to past changes in the three-to-five-year forward rate and the five-to-ten-year forward rate. These maturities were chosen because data on three-, five-, and ten-year Treasury rates are available for a relatively long time period and because the liquidity of these securities makes them attractive instruments for options hedging. We focus on the five-year spot rate and forward rates on either side of five years because the hedging of exposures to either of these forward rates will require trading in a five-year security (see box).

We formulate a testable hypothesis using the hedging of interest rate caps, because caps form the bulk of the over-the-counter interest rate options market. As described in the box, the hedging of a three-to-five-year forward interest rate cap involves taking a short position in the five-year note (as well as a long position in the three-year note). This hedge position is adjusted dynamically as interest rates change because the option's value increases at an escalating rate as forward rates rise. For example, a rise in the forward rate will require a larger short position in the five-year note. If many dealers attempt to sell short the five-year note at the same time, their actions could exert downward pressure on the price of the five-year Treasury note, translating into an increase in its yield.

For an interest rate cap on the five-to-ten-year forward rate, the hedge involves a long position in the five-year note (and a short position in the ten-year note) that must also be adjusted dynamically. Consequently, a rise in the five-to-ten-year forward rate will require a larger long position in the five-year note, which in turn will lead to additional purchases of the five-year note. A large quantity of such purchases could place upward pressure on five-year note prices and thus exert downward pressure on five-year rates. Because of sizable transaction costs, we assume that such hedge adjustments are not instantaneous, but instead occur over a number of days. Below, we look for such effects at a weekly frequency.

The combination of these hedging transactions gives us the following testable hypothesis: a rise in the three-to-five-year forward rate will lead to a future rise in the five-year spot rate, and a rise in the five-to-ten-year forward rate will be followed by a fall in the five-year spot rate. We test for the presence of this relationship using the following equation:

$$(1)\quad \Delta r5 = c + \alpha Z(-1) + \beta_1 \Delta F_{3,5}(-1,-t) + \beta_2 \Delta F_{5,10}(-1,-t) + \beta_3 \Delta r5(-1,-t) + \varepsilon ,$$

where

$r5$ is the five-year interest rate;

$F_{3,5}$ is the three-to-five-year forward rate;

$F_{5,10}$ is the five-to-ten-year forward rate;

$\Delta r5 = r5 - r5(-1)$ is the one-week change in the five-year interest rate;

$\Delta F_{g,h}(-1,-t) = F_{g,h}(-1) - F_{g,h}(-t)$ is the $t$-1-week change in the forward rate, lagged one week, where the subscripts denote the interval of the forward rate;

$\Delta r5(-1,-t) = r5(-1) - r5(-t)$ is the $t$-1-week change in the five-year spot rate, lagged one week; and

$Z(-1) = r5(-1) - c - a\, F_{3,5}(-1) - bF_{5,10}(-1)$ is an error-correction term from the cointegration relationship in the levels of the five-year and forward rates, lagged one week.[8]

If hedging by options dealers has an impact on the five-year interest rate, as hypothesized above, the coefficients on the changes in forward rates will have the signs $\beta_1 > 0$ and $\beta_2 < 0$.[9]

## Were the 1990s Different from Earlier Periods?

If the hedging of interest rate options affects intermediate-maturity interest rates, then the behavior of these rates in the 1990s should be different from their behavior in earlier decades. To look for this change, we begin with an analysis of Treasury interest rates. We then examine the relationship between spot rates and lagged forward rates during the 1990s using forward Eurodollar rates from the interest rate swap yield curve.

Despite the fact that the Eurodollar rate is the benchmark rate in the caps market and dealers may first look to the Eurodollar futures market to hedge their options exposures, two factors drive us to use Treasury rates initially. First, a sufficiently long sample period for the years before the 1990s is available only with Treasury interest rates, and second, arbitrage opportunities between Eurodollar rates and Treasury rates make the two interest rates very highly correlated. For the analysis using Treasury rates, we end the sample period at 1999. We end it there because new issuances of three-year Treasury notes were discontinued in 1998, and because the relationship between long-term Treasury rates and other market rates appears to have changed in response to the Treasury buy-back program, which began in early 2000. (Our analysis using swaps rates, however, employs a data set that includes 2000.)

### Results Using Forward Rates from the Treasury Curve

To determine whether a change in the dynamics of interest rates occurred around 1990, we estimate regressions of equation 1 in sets of seven-year sample periods, rolling forward in one-year increments from 1965 to 1999. The data consist of weekly interest rates, with observations on the Wednesday of each week.[10] Each regression estimates the relationship between changes in the spot five-year interest rate and past changes in the three-to-five-year forward rate and the five-to-ten-year forward rate. We use two versions of these regressions, one with two-week changes in forward rates and the other with five-week changes in forward rates. (Additional details on data and estimations can be found in Appendix A.)

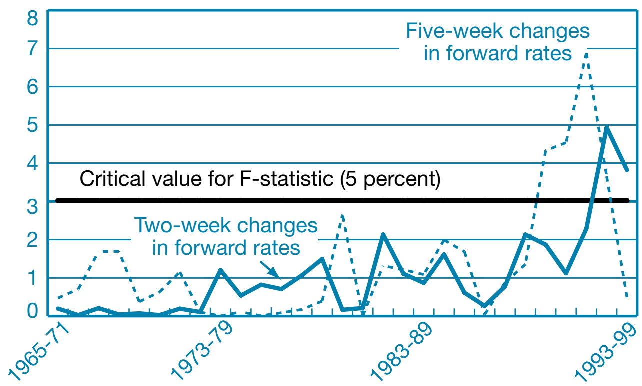
*If the hedging of interest rate options affects intermediate-maturity interest rates, then the behavior of these rates in the 1990s should be different from their behavior in earlier decades.*

The estimation results are summarized in Charts 1 and 2. Chart 1 depicts the statistical significance of the relationship between the spot and lagged forward rates, while Chart 2 depicts the direction and size of the relationship. The charts show a distinct difference in the relationship before and after 1990.

Chart 1 measures the strength of the statistical relationship between the spot and lagged forward rates over the 1965-99 period. It plots two lines: one for the effects of two-week changes in forward rates and the other for the effects of five-week changes in forward rates. The first point on each line is from the first rolling regression (1965-71) and the last point is for the final regression (1993-99). The test statistic shown is an F-statistic for the joint distribution of the coefficients for the two forward rates, $\beta_1$ and $\beta_2$. The F-statistic is nonstandard and is above the critical value only if both of the forward rates are statistically significant predictors of the change in the five-year rate. (For more on the test, see Appendix A.)

Chart 1

F-Statistics for the Joint Distribution of the Coefficients $\beta_1$ and $\beta_2$ from 1965 to 1999




Notes: The dependent variable is the Treasury rate; the explanatory variables are forward rates from the Treasury curve. F-statistics are from twenty-nine rolling regressions, each with seven-year sample periods. The F-statistic is for the test $H0$: $\beta_1 = 0$ or $\beta_2 = 0$; $H1$: $\beta_1 \neq 0$ and $\beta_2 \neq 0$.

Chart 1 shows that a statistically significant relationship between the spot rates and both forward rates appears only during the 1990s, and that the equations with five-week changes in forward rates are significant in the early 1990s while those with two-week changes in forward rates are significant in the latter part of the 1990s. These results suggest that changes in forward rates affect subsequent spot rates only during the 1990s, and that the impact occurs faster in the latter part of the decade. Later, we explore further the speed and duration of the forward rates' influence on the spot rate.

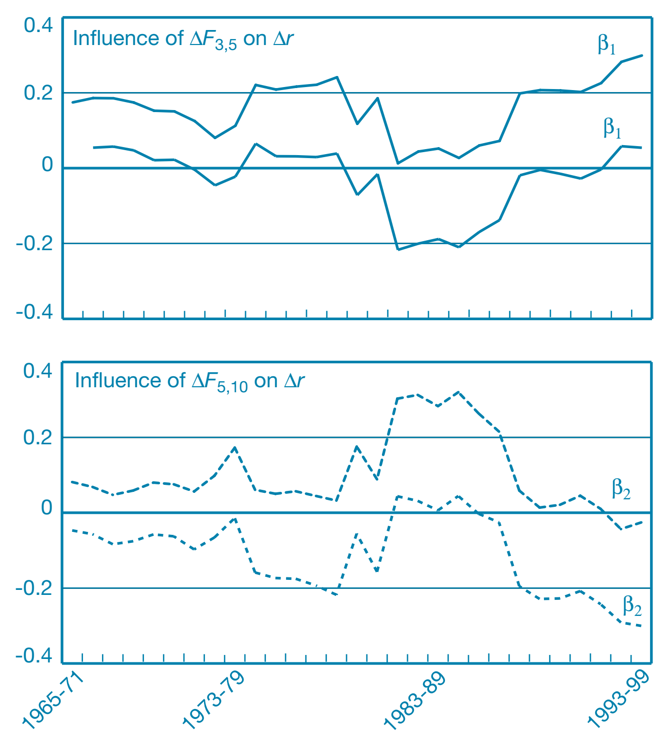
While Chart 1 summarizes only the statistical significance of the relationship between the spot and forward rates, Chart 2 shows the size and direction of the relationship. That chart provides confidence bands for the coefficients in the regressions with the two-week change in forward rates. Both coefficients are significant simultaneously and have signs consistent with hedging-related liquidity effects only after 1992 (a positive coefficient for $\Delta F_{3,5}$ and a negative coefficient for $\Delta F_{5,10}$).[11] For the periods before 1992, the lagged three-to-five-year forward rate occasionally has a significant positive effect on the five-year spot rate, but the change in the five-to-ten-year forward rate is rarely significant, and then only with a positive coefficient rather than the postulated negative coefficient.[12] Regressions using five-week changes in forward rates (not shown) produced similar results, with the notable exception being that both forward rates were significant only during the early 1990s. (The statistics from a set of representative regressions are provided in Appendix B.)

Although the lagged forward rates exert a statistically significant effect on the spot rate, they explain only a small part of the variation in the spot rate. This is evident from the small $R^2$ of the regression equations in Appendix B. The low explanatory power of our regressions is reflected in the fact that our regression results cannot be used to create profitable trading strategies in the presence of transaction costs. Our regression results suggest that the five-year Treasury yield can be predicted using information on past changes in forward rates. Thus, one might assume that there are unexploited profitable trading opportunities in the five-year-note market. In light of the depth and liquidity of the U.S. Treasury market, however, the existence of such unexploited profit opportunities would seem unlikely. Indeed, trading strategies based on our regression results were not consistently profitable.[13] Other implications of this low explanatory power are examined later.

Chart 2

Confidence Bands for the Coefficients $\beta_1$ and $\beta_2$ from 1965 to 1999

For Two-Week Changes in Forward Rates




Notes: The dependent variable is the Treasury rate; the explanatory variables are forward rates from the Treasury curve. Depicted are 95 percent confidence bands (one-tailed) of the coefficients from twenty-nine rolling regressions, each with seven-year sample periods.

To explore further whether these results are related to market liquidity effects, we estimated regressions similar to those above using shorter maturity interest rates. We reasoned that the much larger turnover volume and greater liquidity of the shorter maturity fixed-income markets should easily accommodate options hedging activity with little effect on shorter maturity interest rates. Indeed, regressions of changes in the one-year spot rate on lagged changes in the six-month-to-one-year forward rate and the one-to-two-year forward rate produced no evidence of positive feedback.[14]

## Results with Forward Rates from the Interest Rate Swap Yield Curve

Because the predominant benchmark rate in the caps market is the Eurodollar rate, we would expect positive-feedback effects, if any, to appear first in the Eurodollar market. Thus, we repeat the analysis above using forward rates derived from the interest rate swap yield curve. The swaps data consist of one-week changes in rates from Wednesday to Wednesday, where the rates are the fixed rate in fixed-for-floating Eurodollar interest rate swaps. Because reliable data on rates of long-dated swaps are available only from the late 1980s, our sample begins in 1989. As before, our analysis consists of a series of regressions, each with a seven-year sample period rolling forward in one-year increments. (More information on our data and sources can be found in Appendix A.)

The results found using forward Eurodollar rates are the same as those found using the Treasury forward interest rates. The confidence bands for the estimated coefficients $\beta_1$ and $\beta_2$ for two-week changes in forward rates are presented in Chart 3. The top panel shows the effect on five-year Treasury rates and the bottom panel shows the effect on five-year swap rates, where the explanatory variables in both cases are forward Eurodollar rates from the swap curve. As before, the three-to-five-year forward rate has a positive coefficient and the five-to-ten-year forward rate has a negative coefficient. The two-week changes in forward rates are always statistically significant in the latter part of the decade. Similar results are obtained with the five-week changes in forward rates (not shown), except that statistically significant coefficients are found only in the earlier part of the decade.

Like the influence of Treasury forward rates, the influence of forward Eurodollar rates on the spot rate is also consistent with the predicted effect of the dynamic hedging of options.

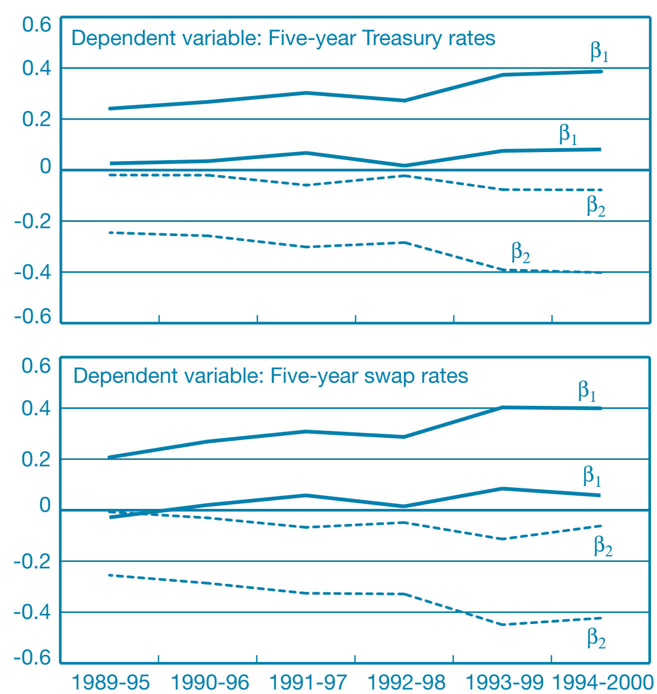
The regression results in Appendix B provide additional details on our finding that changes in forward rates affect spot rates faster toward the end of the decade. Moreover, the impact also appears to have remained at least as strong. In fact, in all the representative regressions in Tables B1 and B2 of Appendix B, the estimated coefficients for the two-week changes in forward rates in the second half of the decade are consistently larger than the coefficients for the five-week changes in forward rates in the first half of the decade.

One interpretation of the faster impact of changes in the forward rates toward the end of the 1990s is that options dealers adjusted their hedge positions faster at the end of the decade than they did at the beginning. This change in behavior could be due either to lower transaction costs or a recognition that delayed hedge adjustments, requiring larger transactions, can be costly and difficult to execute, because they strain market liquidity. Although the classic options pricing models assume continuous rebalancing of hedge positions, in practice

Chart 3

Confidence Bands for the Coefficients $\beta_1$ and $\beta_2$ for Forward Rates from the Interest Rate Swap Curve from 1989 to 2000

For Two-Week Changes in Forward Rates




Notes: Depicted are 95 percent confidence bands (one-tailed) of the coefficients from six rolling regressions, each with seven-year sample periods. $\beta_1$ is the influence of $\Delta F_{3,5}$ on $\Delta r$; $\beta_2$ is the influence of $\Delta F_{5,10}$ on $\Delta r$.

options dealers face a trade-off in the timing of their hedge adjustments. Faster or more frequent hedge adjustments produce hedge positions that more effectively match an option's exposure to price risk, but they do so at the cost of higher cumulative transaction costs over the life of the option (see, for example, Toft [1996]). Thus, a change in the trade-off from lower transaction costs would lead to faster or more frequent hedge adjustments.

## Only the Big Changes Matter

Facing transaction costs, dealers might choose to adjust their hedge positions only for changes in forward rates above a certain threshold. If this asymmetry in behavior is present, then regressions with subsamples of small and large changes in forward rates should produce different results. The samples for these regressions are constructed by partitioning the data into subsamples of roughly equal size, with smaller changes in forward rates in one subsample and larger changes in the other.[15] The results of these regressions confirm the presence of the asymmetric behavior (Table 2). The feedback effects from forward rates to spot rates appear to be present only in large changes in forward rates. Further evidence that feedback effects are stronger when rate changes are large is presented in Table B3, where the effect is estimated for periods of large sustained changes in the five-year rate.

## How Long Does It Last?

If our empirical results are due to hedging-related liquidity effects, we would expect them to be relatively short-lived. To examine the duration of the influence of forward rates on the spot rate, we estimate a series of regressions using changes in forward rates ranging from one to thirteen weeks. These regressions estimate the effect of changes in forward rates of up to thirteen weeks on the one-week change in the spot Treasury rate. (Appendix A provides further details of the estimation.) The results are summarized in Chart 4, which shows the strength of the statistical significance of the relationship between the spot and forward rates, and Chart 5, which depicts the direction and size of the relationship.

Chart 4 presents F-statistics measuring the statistical significance of the changes in forward rates for changes ranging from one to thirteen weeks. Results are shown for regressions using both the five-year Treasury rate and the five-year swaps rate as the dependent variable. In both regressions, the explanatory variables are forward rates from the swap curve. The regressions are estimated separately for the periods 1990 to 1995 (top panel)

Table 2

### Regressions with Large and Small Changes in Forward Rates

| | Large Changes $\lvert \Delta F_{3,5} \rvert \geq 26\ bp$ or $\lvert \Delta F_{5,10} \rvert \geq 26\ bp$ | Small Changes $\lvert \Delta F_{3,5} \rvert < 26\ bp$ and $\lvert \Delta F_{5,10} \rvert < 26\ bp$ |
|---|---|---|
| $Z$(-1) | 0.012 | 0.003 |
| | (P=0.102) | (P=0.607) |
| $\Delta F_{3,5}$ (-1,-6) | 0.157* | -0.013 |
| | (P=0.003) | (P=0.873) |
| $\Delta F_{5,10}$ (-1,-6) | -0.120* | -0.110 |
| | (P=0.035) | (P=0.184) |
| $R^2$ | 0.046 | 0.011 |
| Adjusted $R^2$ | 0.036 | -0.001 |
| Number of observations | 276 | 298 |

Source: Authors' calculations.

Notes: The sample period is 1990-2000. The dependent variable is the Treasury rate; the explanatory variables are forward rates from the swap curve. An asterisk indicates that the coefficient is significant at the 5 percent level and with the predicted sign. The term $Z$ in each regression is estimated using the sample in Table B2: 1989-2000. The estimated cointegrating equation is $Z = r5 - 0.032 - 7.698\ F_{3,5} + 7.007\ F_{5,10}$.

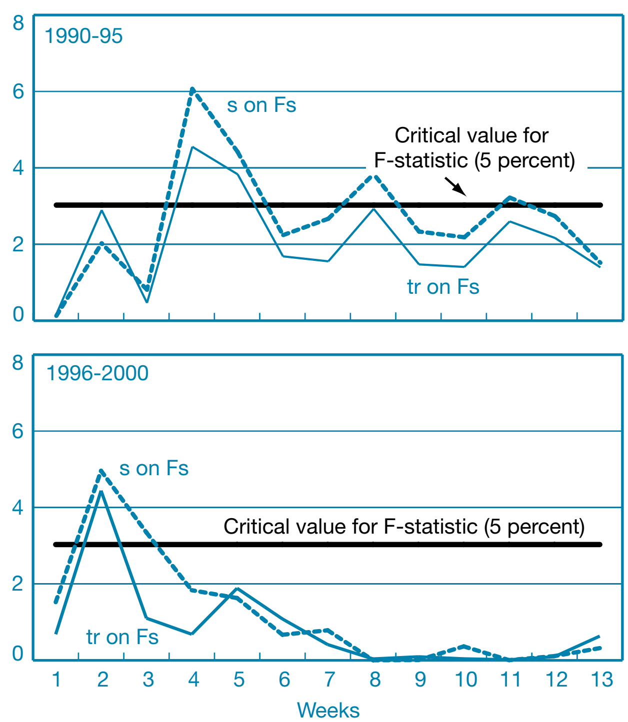
and 1996 to 2000 (bottom panel) to allow for differences between the beginning and end of the 1990s.[16] The results suggest that the statistical significance of the influence of forward rates on the spot five-year rate weakens after six weeks. In addition, a comparison of the two panels further supports our finding that the influence of lagged forward rates on the spot rate occurs more quickly in the second half of the decade; moreover, the effect appears to dissipate faster.

Chart 5 gives the confidence bands for the coefficients $\beta_1$ and $\beta_2$ for the regressions with spot Treasury rates as the dependent variable for the 1996-2000 period. The confidence bands are estimates of the size and direction of the influence of lagged changes in forward rates on the spot five-year rate. The chart shows that a two-week change in forward rates influences the Treasury spot rate in the predicted direction (a positive effect for $F_{3,5}$ and a negative effect for $F_{5,10}$). In the estimates using the full sample (top panel), the effect has weakened enough to no longer be statistically significant by about three weeks, and by nine weeks, the effect has disappeared. In a sample restricted to large rate changes (bottom panel), the feedback effects are larger and longer lasting, remaining statistically significant for as long as five to six weeks. As in the full sample, the effects disappear by about nine weeks.

The absence of a long-lasting influence of the forward rates on the five-year spot rate in these results suggests that we are observing market liquidity effects. In particular, the results are consistent with prices returning to prevailing levels after the market absorbs dealers' hedging transactions.

Chart 4

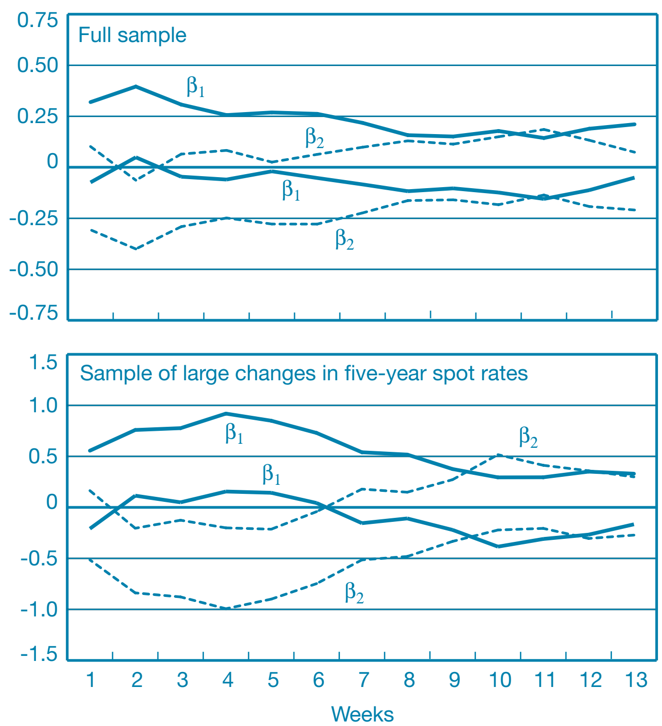
**F-Statistics for the Joint Distribution of the Coefficients $\beta_1$ and $\beta_2$ for Changes in Forward Rates from One to Thirteen Weeks**




Notes: The explanatory variables are forward rates from the swap curve: s on Fs is the regression of the spot swaps rate on forward rates from the swap curve; tr on Fs is the regression of the spot Treasury rate on forward rates from the swap curve. F-statistics are from thirteen regressions, each with a change in forward rates over the interval indicated on the horizontal axis—from one to thirteen weeks. The F-statistic is for the test $H0$: $\beta_1 = 0$ or $\beta_2 = 0$ ; $H1$: $\beta_1 \neq 0$ and $\beta_2 \neq 0$.

Chart 5

**Confidence Bands for the Coefficients $\beta_1$ and $\beta_2$ for Changes in Forward Rates from One to Thirteen Weeks**




Notes: The sample period is 1996-2000. $\beta_1$ is the influence of $\Delta F_{3,5}$ on $\Delta r$; $\beta_2$ is the influence of $\Delta F_{5,10}$ on $\Delta r$. The dependent variable is the spot Treasury rate; the explanatory variables are forward rates from the swap curve. Depicted are 95 percent confidence bands (one-tailed) from thirteen regressions, each with a change in forward rates over the interval indicated on the horizontal axis—from one to thirteen weeks. In the bottom panel, the regression sample is restricted to those periods in which the subsequent two-month change in the five-year Treasury rate lies within the largest 25 percent of rate changes.

## Do Liquidity Effects Spill Over from the Swap Curve to the Treasury Curve?

Having analyzed the length of the relationship between lagged forward rates and the spot interest rate, we can explore the direction of the influence on the relationship between the Treasury and swap yield curves. In the regressions above, we obtain similar statistical results regardless of whether we regress five-year Treasury rates on forward Treasury rates or forward swaps rates. Similarly, forward swaps rates predict similar liquidity effects on the five-year Treasury rate and the five-year swaps rate. However, if we use forward Treasury rates to predict swaps rates, the statistical results are not as strong. Forward Treasury rates influence the five-year swaps rate only in the early part of the 1990s. In the latter half of the decade, forward Treasury rates are not statistically significant predictors of the five-year swaps rate. Thus, in the latter part of the decade, it appears that the direction of "causality" for positive-feedback effects is from the swaps market to the Treasury market.

One interpretation of this result is that the growing liquidity of the swaps market and the wider use of more refined pricing models have shifted the focus of hedging decisions toward the swap curve. At the beginning of the 1990s, more dealers may have been using earlier generations of pricing and hedging models that did not differentiate strongly between swaps rates and Treasury rates at the longer maturities. Toward the end of the decade, however, more refined pricing models based on the swap yield curve became more widely used, causing lagged Treasury forward rates to lose their explanatory power.[17]

## Implications for Risk Management and Policy

At first glance, the feedback effects highlighted above appear to be second-order. The small explanatory power of the regressions indicates that the postulated market liquidity effect of the hedging of interest rate options explains only a small portion of typical fluctuations in the yield curve. In addition, the magnitude of the feedback effect is normally not large. For example, during 1996-2000, only 20 to 25 percent of the change in lagged forward rates was transmitted to the five-year spot rate (Appendix B, Tables B1 and B2). During this period, the average weekly change (absolute value) in the five-year rate was about 9.5 basis points, only 2 basis points of which could be attributed to the combined changes in the two forward rates.

If our interpretation is correct, the small impact on the yield curve is reassuring, as it implies that options dealers' liquidity risk arising from their need to adjust hedge positions should be manageable under normal circumstances. Nevertheless, the market disruptions in the fall of 1998 were a reminder that normally manageable liquidity risk can turn large in ways that surprise even experienced market participants.[18] Many institutions that relied on market liquidity to execute dynamic risk management strategies found themselves exposed to far higher risks than they had anticipated. This heightened sense of risk, in turn, caused many participants to withdraw from markets, further impairing liquidity. Although the dynamic hedging strategies examined in this article were not particularly stressed during the fall of 1998 (because the benchmark interest rate environment remained comparatively benign), the suddenness of the 1998 liquidity crisis is a warning about any market's vulnerability to dynamic risk management strategies.

The conclusion that liquidity risk is manageable under normal circumstances thus leaves room for questions about more extreme circumstances. In our empirical results, the feedback effects are stronger when interest rate changes are large (Table 2 and Chart 5). As a result, estimates based on the full sample may understate potential distortions to the yield curve during periods of large changes in interest rates. Indeed, during periods of large sustained changes in rates that continue for several weeks, the change in forward rates accounts for a relatively large proportion of the change in the five-year spot rate. For instance, if we examine periods during 1996-2000, when the five-year rate changed by more than 68 basis points over a two-month period (the largest 10 percent of such changes), more than 70 percent of the change in lagged forward rates was transmitted to the five-year spot rate (Table B3). Furthermore, changes in lagged forwards account for nearly half of the variation in the five-year spot rate during such periods. Finally, the feedback effect is also more persistent during periods of large rate changes, as we see in Chart 5.

*The times when market participants and policymakers are most interested in extracting from the yield curve a signal about economic fundamentals are precisely the times when changes in the curve may be distorted by liquidity effects.*

Our finding of larger feedback effects during episodes of large changes in interest rates suggests that dealers' hedging demands might run up against more severe liquidity constraints if the volatility of rates were to rise sharply. Overall, the interest rate environment of the 1990s was relatively benign by historical standards. In a more volatile environment—such as the one experienced in the late 1970s and early 1980s—dynamic hedging might introduce more disruptive positive-feedback effects if reduced market liquidity and dealers' hedging demands interacted to amplify market shocks.[19]

Furthermore, the potential for positive-feedback effects has implications for how short-run yield curve movements are interpreted by market participants and policymakers. In recent years, these movements have been followed closely for several reasons. For example, the yield curve has been shown by studies to be a relatively good predictor of economic activity, in part because it appears to reflect expectations of future economic fundamentals (see, for example, Estrella and Mishkin [1998], Estrella and Hardouvelis [1991], and Stock and Watson [1989]). In addition, the curve reflects one component of the monetary policy transmission mechanism, from short-term to long-term interest rates.

If yield curve movements over short time periods are influenced by liquidity effects as well as by expectations of economic fundamentals and policy, these movements may have to be interpreted more carefully. Yield curve changes tend to be monitored most closely when large economic and financial shocks occur or when significant policy changes are made. However, our empirical results suggest that liquidity effects in the yield curve are largest when shocks to interest rates are large. Thus, the times when market participants and policymakers are most interested in extracting from the yield curve a signal about economic fundamentals are precisely the times when changes in the curve may be distorted by liquidity effects.

## Conclusion

In this article, we have examined the influence of market liquidity and dynamic trading strategies on the short-run behavior of the yield curve. Motivating our analysis was the hypothesis that dynamic hedging by sellers of interest rate options could generate transaction flows that affect market liquidity and thus produce systematic patterns in interest rate movements. The growth of the over-the-counter interest rate options market to significant size in the late 1980s allowed us to identify potential changes in interest rate dynamics that followed the development of this market. Indeed, we found a distinct difference in the dynamics of the term structure before and after 1990 that is consistent with the predicted impact of dynamic hedging of interest rate options.

Previous research on the structure of the interest rate options market has found that the largest impact of dealers' dynamic hedging on trading volume in the underlying fixed-income markets likely occurs along the intermediate-maturity section of the yield curve. For this segment of the curve, the hedging of options exposures to the three-to-five-year forward rate and the five-to-ten-year forward rate could have an effect on the five-year spot interest rate. Beginning around 1990, we find that the five-year spot rate does tend to behave as predicted following changes in the forward rates. In contrast, the relationship between the spot rate and the forward rates does not appear in the data before the 1990s.

We interpret the observed behavior of five-year interest rates as the product of short-term liquidity effects. This conclusion is based on several findings. First, the predicted relationship between forward rates and spot rates does not persist beyond a few weeks, nor can it be profitably exploited in a systematic way. Both results suggest that short-term liquidity forces rather than economic fundamentals are likely to be driving the results. In addition, and in contrast to the behavior of medium-maturity rates, shorter maturity interest rates show no evidence of such feedback effects. The ample liquidity of the markets for short-term interest rate products, where market turnover is large relative to hedging demands, makes them an unlikely site for any evidence of positive-feedback effects. Finally, forward rates predict spot rates in the medium-term segment of the yield curve only in the weeks when rate changes are relatively large. This finding is also consistent with liquidity effects, since large interest rate changes cause large adjustments to options hedges, which in turn induce trading flows that will be large relative to normal market turnover.

Although we find evidence of market liquidity effects consistent with dynamic hedging at the medium-term segment of the yield curve, the relationship accounts for only a small part of the variation in rates. The relatively small impact on the yield curve suggests that the U.S. dollar fixed-income markets are liquid and deep enough to absorb dealers' hedging transactions under normal market conditions, and that the liquidity risk arising from their need to adjust hedge positions dynamically should be manageable. However, during periods when interest rates are changing rapidly or periods of market stress when interest rate volatility jumps, liquidity effects could be significantly larger. It is exactly during such times that short-term yield curve movements may be most affected by hedge-related trading and may move in ways that are unrelated to economic fundamentals.

# Appendix A: Data and Estimation Details

## Data

Our analysis uses weekly changes in interest rates; we do not use daily data because transaction costs can make it uneconomical to adjust options hedges completely on a daily basis. Our choice of interest rate data is based on two criteria. First, we wish to use market rates that reflect as closely as possible the actual transaction prices at which options dealers are trading. Second, to evaluate whether interest rate dynamics have changed over time, we choose data that had a relatively long history.

For Treasury securities, these two criteria lead us to use constant-maturity Treasury rates. These rates are nearly identical to on-the-run Treasury yields (with an adjustment to maintain a fixed maturity), and they are available on a daily basis going back more than thirty years. As a check, we also performed our analysis using on-the-run interest rate data from dealer quotes reported by Bloomberg. The results were the same as those using the constant-maturity data. We elected not to use estimated zero-coupon yields, such as those in McCulloch and Kwon (1993), because such yields are based on imputed prices, not transaction prices, and because they are calculated from less liquid, off-the-run Treasury securities, which are unlikely hedging vehicles for options dealers.

For forward Eurodollar rates, we use forward rates derived from the interest rate swap yield curve. Swaps rates are the fixed rate in fixed-for-floating interest rate swaps, where the floating rate is indexed to a short-term Eurodollar interest rate (often a three-month rate). This index and the wide use of the swaps market for trading and hedging make the swap yield curve a reasonable source for forward Eurodollar rates. The rates are Reuters quotes, obtained from DRI-WEFA.

To check for day-of-week and overlapping-day effects, we also estimated equation 1 using Tuesday to (prior) Wednesday changes in rates. This alternative model specification had no effect on our results.

## Estimation

Consistent with previous research, we find that the *levels* of interest rates along the yield curve are cointegrated. In other words, interest rates are generally nonstationary integrated time series, but there exists a linear combination of these rates that is stationary. As noted by Engle and Granger (1987), time-series regressions involving relationships between the changes in cointegrated variables should include a lagged cointegration term in order to control for correlation between the contemporaneous levels of the regression variables that would otherwise interfere with consistent estimation of the equation coefficients. Using standard regression techniques, we estimated a cointegrating relationship between the five-year Treasury rate and the three-to-five-year and five-to-ten-year forward rates. This relationship produced the error-correction term $Z$ in equation 1. The cointegrating relationship was estimated with a constant term, but without a time trend, while controlling for thirteen lags of changes in rates.

In equation 1, the lagged changes in the five-year spot rate were never statistically significant in any estimation of the equation. This result suggests that the lagged changes in the five-year rate affect changes in the current five-year rate only through the lagged changes in the forward rates. (By definition, changes in the three-to-five-year and the five-to-ten-year forward rates are affected by changes in the five-year spot rate in the same observation period.) Consequently, all of our charts and tables report regression results without the lagged change in the five-year rate.

## Rolling Regressions

The first set of estimates, in Charts 1 and 2, consists of twenty-nine regressions, each over seven-year sample periods rolling forward in one-year increments from 1965 to 1999. We use two versions of equation 1, corresponding to two- and five-week changes in forward rates:

$$\text{(A1)} \quad \Delta r5 = c + \alpha Z(-1) + \beta_1 \Delta F_{3,5}(-1,-3) + \beta_2 \Delta F_{5,10}(-1,-3) + \varepsilon \text{ and}$$

$$\text{(A2)} \quad \Delta r5 = c + \alpha Z(-1) + \beta_1 \Delta F_{3,5}(-1,-6) + \beta_2 \Delta F_{5,10}(-1,-6) + \varepsilon .$$

The estimation is least squares and the residuals terms are well behaved. The estimation was performed both with and without Newey-West heteroskedasticity and autocorrelation (HAC) consistent covariance estimates, with similar results produced in both cases. Charts 1-3 show the results without the Newey-West covariance estimates. The error-correction term $Z$ in each regression is estimated using a sample that begins and ends three years before and after the regression sample (except at the end points of the full sample).

The regressions with forward rates from the swap curve (Charts 3-5 and Table 2) have similar structures. In these regressions, the error-correction term is estimated using the full sample of swaps data (1989-2000).

## F-Statistics

The test for statistical significance, the results of which appear in Chart 1, is a nonstandard F-test of the joint distribution of the coefficients $\beta_1$ and $\beta_2$,

$$H0\colon \beta_1 = 0 \text{ or } \beta_2 = 0$$

$$H1\colon \beta_1 \neq 0 \text{ and } \beta_2 \neq 0 .$$

In geometric terms, the test asks whether the ninety-fifth percentile confidence ellipse of the estimated coefficients (centered on the estimated values of $\beta_1$ and $\beta_2$) intersects either of the axes $\beta_1 = 0$ or $\beta_2 = 0$. If it does not, both estimated coefficients are nonzero at a 95 percent confidence level (5 percent critical value).[20]

To perform the test, rather than finding all values of $\beta_1$ and $\beta_2$ on the confidence ellipse (that is, all combinations of $\beta_1$ and $\beta_2$ for the 5 percent critical value of the F-statistic) and seeing whether these are in the interior of the parameter space, we construct F-statistics for the estimated coefficients along the axes for $\beta_1$ and $\beta_2$ and ask whether the F-statistics exceed the 5 percent critical value. If they do, the ninety-fifth percentile confidence ellipse must be in the interior of the parameter space. Specifically, we calculate F-statistics for the joint distribution of the estimated coefficients $\beta_1$ and $\beta_2$ along the axis $\beta_1 = 0$ and the axis $\beta_2 = 0$, with the distribution centered on the estimated values of $\beta_1$ and $\beta_2$. The F-statistic chosen for the test is the smallest of these F-statistics, and the null hypothesis is rejected when this minimum value exceeds the critical value. This F-statistic is presented in Chart 1.

## The Duration of the Relationship between the Spot Rate and Lagged Forward Rates

To determine the duration of the influence of the forward rates on the spot rate, we estimate the equation

$$\text{(A3)} \qquad \Delta r5 = c + \alpha Z(-1) + \beta_1 \Delta F_{3,5}(-1, -t) + \beta_2 \Delta F_{5,10}(-1, -t) + \varepsilon$$

for $t = 2, 3, \ldots, 14$, giving us a set of thirteen regressions of the effect of changes in forward rates ranging from a one-week change to a thirteen-week change. As in the earlier case, $\Delta r5$ is the one-week change in the spot Treasury rate and $Z(.)$ is the error-correction term from the cointegration relationship in the level of rates, and is estimated as described above. The test for statistical significance of the relationship between the spot and forward rates uses the same test procedure described earlier. The F-statistics for the joint distribution of $\beta_1$ and $\beta_2$ are shown in Chart 4; confidence bands for the $\beta_1$ and $\beta_2$ estimates are presented in Chart 5. The test statistics in these charts were computed with Newey-West HAC consistent covariances because of heteroskedastic residuals in the shorter sample periods, particularly 1996-2000.

# Appendix B: Representative Regression Results

## Results Using Treasury Interest Rates

The statistics for a set of regressions illustrating the differences between the early and later parts of the 1990s are presented in Table B1. These regressions are performed for two subperiods: the six-year period from 1990 to 1995 and the four-year period from 1996 to 1999. Two regressions are estimated in each subperiod, one with two-week changes in forward rates (equation A1) and the other with five-week changes in forward rates (equation A2).

The statistics in Table B1 show that for the early 1990s, only the five-week changes in forward rates (equation A2) are statistically significant with the anticipated signs (a positive sign for changes in the three-to-five-year forward rate and a negative sign for changes in the five-to-ten-year forward rate). For the late 1990s, only the two-week changes (equation A1) are statistically significant with the anticipated signs. These findings, plus the absence of significant results for the other regressions, are consistent with Charts 1 and 2: lagged changes in forward rates affect five-year yields more quickly in the later part of the 1990s than in the earlier years of the decade. In the earlier period, only the regression with the five-week change in forward rates has significant coefficients, while in the later period, only the regression with the two-week change in forward rates has significant coefficients.

## Results Using Forward Rates from the Swap Curve

Illustrative regression statistics from equations using swaps interest rates are shown in Table B2. The explanatory variables in the regressions are forward rates derived from the swap yield curve, while the dependent variable is the spot Treasury rate in

Table B1

Regression of the Spot Treasury Rate on Forward Treasury Rates

| | 1990-95 | | 1996-99 | |
|---|---|---|---|---|
| | Equation A1: Two-Week Change in Forward Rates | Equation A2: Five-Week Change in Forward Rates | Equation A1: Two-Week Change in Forward Rates | Equation A2: Five-Week Change in Forward Rates |
| $Z(-1)$ | 0.009 | 0.017 | 0.019 | 0.023 |
| | (P=0.270) | (P=0.020) | (P=0.032) | (P=0.006) |
| $\Delta F_{3,5}$ (-1,-3) | 0.081 | | 0.197* | |
| | (P=0.269) | | (P=0.005) | |
| $\Delta F_{5,10}$ (-1,-3) | -0.080 | | -0.195* | |
| | (P=0.247) | | (P=0.028) | |
| $\Delta F_{3,5}$ (-1,-6) | | 0.160* | | 0.079 |
| | | (P=0.001) | | (P=0.148) |
| $\Delta F_{5,10}$ (-1,-6) | | -0.123* | | -0.043 |
| | | (P=0.016) | | (P=0.523) |
| $R^2$ | 0.006 | 0.034 | 0.032 | 0.027 |
| Adjusted $R^2$ | -0.003 | 0.025 | 0.017 | 0.022 |

Source: Authors' calculations.

Notes: An asterisk indicates that the coefficient of the change in forward rates is significant at the 5 percent level (two-tailed) and with the predicted sign. Equations A1 and A2 for 1990-95 are estimated using least squares and Newey-West HAC consistent covariance, while an EGARCH(2,2) correction was used in the estimation of equations A1 and A2 for 1996-99. The term $Z$ in each regression is estimated using the sample in Table B2: 1989-2000. The estimated cointegrating equation is $Z = r5 - 0.006 - 6.566\, F_{3,5} + 5.57\, F_{5,10}$.

Table B2

## Regressions Using Forward Eurodollar Rates

| | 1990-95 | | 1996-2000 | |
|---|---|---|---|---|
| | Equation A1: Two-Week Change in Forward Rates | Equation A2: Five-Week Change in Forward Rates | Equation A1: Two-Week Change in Forward Rates | Equation A2: Five-Week Change in Forward Rates |
| **Panel A: Regression of Spot Treasury Rate on Forward Eurodollar Rates** | | | | |
| $Z(-1)$ | 0.002 | 0.006 | 0.033 | 0.032 |
| | (P=0.645) | (P=0.269) | (P=0.004) | (P=0.011) |
| $\Delta F_{3,5}$ (-1,-3) | 0.143 | | 0.222* | |
| | (P=0.070) | | (P=0.036) | |
| $\Delta F_{5,10}$ (-1,-3) | -0.130 | | -0.232* | |
| | (P=0.090) | | (P=0.024) | |
| $\Delta F_{3,5}$ (-1,-6) | | 0.141* | | 0.124 |
| | | (P=0.007) | | (P=0.157) |
| $\Delta F_{5,10}$ (-1,-6) | | -0.104* | | -0.127 |
| | | (P=0.051) | | (P=0.171) |
| $R^2$ | 0.012 | 0.029 | 0.041 | 0.033 |
| Adjusted $R^2$ | 0.003 | 0.019 | 0.029 | 0.022 |
| **Panel B: Regression of Spot Swaps Rate on Forward Eurodollar Rates** | | | | |
| $Z(-1)$ | 0.014 | 0.020 | 0.079 | 0.076 |
| | (P=0.260) | (P=0.089) | (P=0.015) | (P=0.032) |
| $\Delta F_{3,5}$ (-1,-3) | 0.126 | | 0.223* | |
| | (P=0.156) | | (P=0.027) | |
| $\Delta F_{5,10}$ (-1,-3) | -0.141 | | -0.240* | |
| | (P=0.116) | | (P=0.024) | |
| $\Delta F_{3,5}$ (-1,-6) | | 0.136* | | 0.119 |
| | | (P=0.010) | | (P=0.186) |
| $\Delta F_{5,10}$ (-1,-6) | | -0.117* | | -0.122 |
| | | (P=0.036) | | (P=0.203) |
| $R^2$ | 0.011 | 0.021 | 0.031 | 0.023 |
| Adjusted $R^2$ | 0.002 | 0.012 | 0.020 | 0.011 |

Source: Authors' calculations.

Notes: An asterisk indicates that the coefficient of the change in forward rates is significant at the 5 percent level (two-tailed) and with the predicted sign. Equations are estimated using least squares with Newey-West HAC consistent covariance. The cointegration term $Z$ in each regression is estimated using the full sample of swaps data: 1989-2000. The estimated cointegrating equations are $Z = r5 - 0.032 - 7.698\ F_{3,5} + 7.007\ F_{5,10}$ for panel A, and $Z = r5 - 0.019 - 4.575\ F_{3,5} + 3.764\ F_{5,10}$ for panel B.

panel A and the spot swaps rate in panel B. The results in these panels are similar to those arrived at using forward rates from the Treasury curve. As noted, changes in forward rates affect five-year yields more quickly toward the end of the 1990s.

## Results for Periods of Large Changes in the Five-Year Treasury Rate

The regressions in Table B3 estimate the relationship between the spot five-year rate and lagged forward rates in periods leading up to episodes of large changes in the five-year rate. The regression sample is restricted to periods in which the subsequent two-month change in the five-year rate lies within the largest 10 percent of rate changes. In these regressions, the changes in forward rates have a substantially stronger influence on the spot five-year rate than they do in the full-sample regressions in Tables B1 and B2. For instance, during 1996-2000, the predicted one-week change in the spot five-year rate is more than 70 percent of the two-week change in forward rates—compared with only 20 percent in the full sample. In addition, during this period, the $R^2$ of the regression indicates that almost half of the variability in the spot five-year rate can be attributed to the change in forward rates.

Table B3

Regressions for Periods of Large Changes in the Five-Year Treasury Rate

| | 1990-2000 | 1996-2000 |
|---|---|---|
| $Z$(-1) | 0.062 | 0.110 |
| | (P=0.001) | (P=0.003) |
| $\Delta F_{3,5}$ (-1,-3) | | 0.720 |
| | | (P=0.057) |
| $\Delta F_{5,10}$ (-1,-3) | | -0.956 |
| | | (P=0.008) |
| $\Delta F_{3,5}$ (-1,-6) | 0.522 | |
| | (P=0.000) | |
| $\Delta F_{5,10}$ (-1,-6) | -0.443 | |
| | (P=0.002) | |
| $R^2$ | 0.26 | 0.41 |
| Adjusted $R^2$ | 0.22 | 0.32 |
| Average value of dependent variable (basis points) | | |
| Predicted | 8 | 7 |
| Actual | 13 | 10 |
| Number of observations | 57 | 25 |

Source: Authors' calculations.

Notes: The sample is the largest 10 percent of two-month changes in the Treasury rate in the indicated period. In the regression equation $\Delta r5 = c + \alpha Z(-1) + \beta_1 \Delta F_{3,5}(-1,-t) + \beta_2 \Delta F_{5,10}(-1,-t) + \varepsilon$, all terms are as defined in equation 1. The regression sample is defined by $|r5(8) - r5(-1)| \geq 77$ basis points for 1990-2000 and $|r5(8) - r5(-1)| \geq 68$ basis points for 1996-2000, where $r5(8) - r5(-1)$ is the leading two-month change in the spot five-year Treasury rate. All equations are estimated using least squares and Newey-West HAC consistent covariance. The explanatory variables are forward rates derived from the swap yield curve. The term $Z$ in each regression is estimated using the sample in Table B2: 1989-2000. The estimated cointegrating equation is $Z = r5 - 0.032 - 7.698\, F_{3,5} + 7.007\, F_{5,10}$.

# Endnotes

1. The yield curve depicts interest rates of different maturities at a point in time. For more on the yield curve's role as a predictor of economic activity, see, for example, Estrella and Mishkin (1998), Estrella and Hardouvelis (1991), and Stock and Watson (1989).

2. In the presence of liquidity risk, traders have a choice when making a large trade: they can either accept a disadvantageous price in return for immediate trade execution, or they can spread the trade over a series of smaller transactions and sacrifice immediacy of execution while bearing exposure to price risk until all transactions are completed.

3. An exception is Kodres (1994).

4. Figures are from International Swaps and Derivatives Association survey data.

5. See Kambhu (1998).

6. A 25-basis-point change in forward rates is slightly less than the largest daily change and represents approximately the ninetieth percentile of one-week changes in forward rates during the 1990s.

7. Other dynamic trading activity also has the potential to affect market liquidity in the fixed-income markets. For example, the hedging of mortgage-backed securities involves the dynamic adjustment of hedge positions as mortgage prepayment rates change in response to interest rate changes. Adjustments of mortgage-backed-securities hedges may have influenced the shape of the yield curve following the Federal Reserve's interest rate hikes in 1994 (Fernald, Keane, and Mosser 1994).

8. Equation 1 can be interpreted as an error-correction model for the five-year spot rate as a function of the lagged forward rates and the lagged five-year rate. Because the forward rates are functions of the three-, five-, and ten-year spot rates, the equation could be rewritten as an error-correction model of the five-year rate as a function of lagged three-, five-, and ten-year rates. To focus on the possible feedback effects of options trading, we chose to write the model in terms of forward rates. The coefficients $a$, $b$, and $c$ in the cointegration term are estimated separately. See Appendix A for more details.

9. Our regression is different from that usually used for testing the expectations hypothesis of the term structure of interest rates. In the usual expectations-hypothesis framework, one-week changes in the five-year yield would be written as a function of the lagged slope of the term structure (from one week to five years) plus an (unforecastable) error term representing the one-week excess return on the five-year note (see Campbell [1995]). The lagged changes in rates should not be significant. Although our regressions are not directly comparable to the traditional expectations-hypothesis framework, we find that lagged changes in forward rates do affect changes in the five-year rate during the 1990s. Moreover, if we augment our regression by adding the lagged spread between the five-year rate and a one-week (LIBOR) rate, we still find that lagged forward rates predict the weekly change in the five-year yield during the 1990s.

10. The data are constant-maturity Treasury (CMT) yields. Forward rates are derived from the CMT rates. See Appendix A for details.

11. Although the two forward rates are correlated, their collinearity does not seem to account for the relationship depicted in these charts. The correlation between the forward rates in the same sample periods as the regressions in Charts 1 and 2 bears no relationship to the results in those charts. The periods during the 1980s in which the degree of correlation was the same as in the 1990s did not have the same regression results as in the latter period.

12. Indeed, during the 1980s, there is evidence of mild negative feedback (a negative coefficient for $\Delta F_{3,5}$ and a positive coefficient for $\Delta F_{5,10}$) from the forward rates to the five-year spot rate, although it is not statistically significant.

13. The trades consisted of either a long or short position in the five-year Treasury note, depending on the signal from the forward rates, and an offsetting duration-matched position in the two- and ten-year Treasury securities. This "butterfly" trade created a position with exposure to the five-year spot rate but without exposure to changes in the level of the yield curve (see Garbade [1996, Chapter 14] for more on butterfly trades and the yield curve). Profits were calculated net of the transaction cost of 0.75 basis point per trade. Several strategies were tested, each of which required a different level of the signal from the forward rates before a trade was undertaken. Of those tested, the best performing strategy was profitable only at the very end of the 1990s.

14. The data used were Treasury bill rates for the six-month and one-year rates and constant-maturity rates for the two-year Treasury note. We found some evidence in the late 1980s that lagged forwards had predictive power for the one-year-note rate, but the relationship proved to be the opposite of what would be expected from the liquidity effects of hedging activity.

15. The partition was achieved by increasing a cutoff value for the change in both forward rates in 1-basis-point increments until the subsample of changes in forward rates that exceeds the cutoff value was smaller than the other subsample.

16. Although the regression samples for the early and later parts of the 1990s are split at the middle of the decade (1995 and 1996), similar results are still found with other partitions. In other words, our results are robust with respect to how the decade is split into early and later periods.

17. At the very end of the decade, reductions in the supply of Treasury securities along with the Treasury buy-back program probably exacerbated this trend.

18. See Bank for International Settlements (1999b) and Fleming (2000) for measures of the degree of illiquidity in what were normally liquid markets.

19. This possibility, however, does not imply that restrictions on dynamic hedging or option-like products are warranted. Indeed, restrictions would be undesirable for two reasons. First, dynamic hedging is not disruptive under normal market conditions. Second, restrictions on financial products whose risks are managed dynamically would limit the use of financial innovations that provide benefits to a wide range of economic agents, from residential mortgage borrowers to institutional investors. A more appropriate policy and risk management response would be prudent risk-based capital levels and robust liquidity management.

20. See Dhrymes (1978, pp. 80-3) for further discussion.

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